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Hadron fragmentation inside jets in hadronic collisions

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We present an analytical next-to-leading-order QCD calculation of the partonic cross sections for the process $pp \to (\text{jet}h)X$, for which a specific hadron is observed inside a fully reconstructed jet. In order to obtain the analytical results, we assume the jet to be relatively narrow. We show that the results can be cast into a simple and systematic form based on suitable universal jet functions for the process. We confirm the validity of our calculation by comparing to previous results in the literature for which the next-to-leading-order cross section was treated entirely numerically by Monte Carlo integration techniques. We present phenomenological results for experiments at the LHC and at RHIC. These suggest that $pp \to (\text{jet}h)X$ should enable very sensitive probes of fragmentation functions, especially of the one for gluons.

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I. INTRODUCTION

Final states produced at high transverse momentum (p_T), such as jets, single hadrons, or prompt photons, have long been regarded as sensitive and well-understood probes of short-distance QCD phenomena. Recently, a new "hybrid" type of high-p_T jet/hadron observable has been proposed and explored theoretically [1–5]. It is defined by an identified specific hadron found inside a fully reconstructed jet, giving rise to a same-side hadron-jet momentum correlation. This correlation may for example be described in terms of the variable $z_h \equiv p_T/p_T^{\text{jet}}$, where p_T and p_T^{jet} are the transverse momenta of the hadron and the jet, respectively. The production of identified hadrons in jets was first considered for the case of e^+e^- annihilation [1–3] and subsequently also for pp scattering [4]. Experimental studies have been pioneered in $p\bar{p} \to (\text{jet}h)X$ at the Tevatron [6]. At the LHC, the ATLAS [7,8] and CMS [9] experiments have studied $pp \to (\text{jet}h)X$, and measurements are being carried out by ALICE [10]. Measurements of the cross section (and, perhaps, spin asymmetries) should also be possible at RHIC.

There are several reasons why it is interesting to study the production of hadrons inside jets. Perhaps most importantly, the observable provides an alternative window on fragmentation functions [4]. The latter, denoted here by $D_c^h(z,\mu)$, describe the formation of a hadron h from a parent parton $c = q, \bar{q}, g$. The variable z is the fraction of the parton's momentum transferred to the hadron, and μ denotes the factorization scale at which the fragmentation function is probed. Usually, fragmentation functions for a hadron h are determined from the processes $e^+e^- \to hX$ or $ep \to ehX$. The power of these processes lies in the fact that they essentially allow direct scans of the fragmentation functions as functions of z. The reason for this is that to leading order (LO) in QCD, it turns out that z is identical to a kinematic (scaling) variable of the process. For instance, in $e^+e^- \to hX$ one has $z = 2p_h \cdot q/q^2$ to LO, where p_h is the momentum of the observed hadron and q is the momentum of the virtual photon that is produced by the e^+e^- annihilation. NLO corrections dilute this direct "local" sensitivity only a little. A drawback of $e^+e^- \to hX$ or $ep \to ehX$ is on the other hand that the gluon fragmentation function can be probed only indirectly by evolution or higher-order corrections.

Being universal objects, the same fragmentation functions are also relevant for describing hadron production in pp scattering. So far, one has been using the process $pp \to hX$ as a further source of information on the $D_c^h(z,\mu)$ [11–13]. Although this process does probe gluon fragmentation, its sensitivity to fragmentation functions is much less clear-cut than in the case of $e^+e^- \to hX$ or $ep \to ehX$. This is because for the single-inclusive process $pp \to hX$ the fragmentation functions arise in a more complex convolution with the partonic hard-scattering functions, which involves an integration over a typically rather wide range of z already at LO. As a result, information on the $D_c^h(z,\mu)$ is smeared out and not readily available at a given fixed value of z.

The process $pp \to (\text{jet}h)X$ allows one to overcome this shortcoming. As it turns out, if one writes its cross section differential in the variable z_h introduced above, then to LO the hadron's fragmentation function is to be evaluated at $z = z_h$. This means that by selecting z_h one can "dial" the value at which the $D_c^h(z,\mu)$ are probed, similarly to what is available in $e^+e^- \to hX$ or $ep \to ehX$. Thanks to the fact that in pp scattering different weights are given to the various fragmentation functions than compared to $e^+e^- \to hX$ and $ep \to ehX$, it is clear that $pp \to (\text{jet}h)X$ has the potential to provide complementary new information on the $D_c^h(z,\mu)$, especially on gluon fragmentation. Data for $pp \to (\text{jet}h)X$ should thus become valuable input to global QCD analyses of fragmentation functions. At the very least, they should enable novel tests of the universality of fragmentation functions. We note that similar opportunities are expected to arise when the hadron is produced on the "away side" of the jet, that is, basically back to back with the jet [14], although the kinematics is somewhat more elaborate in this case.

The production of specific hadrons inside jets may also provide new insights into the structure of jets and the hadronization mechanism. Varying z_h and/or the hadron species, one can map out the abundances of specific hadrons in jets. Particle identification in jets becomes particularly interesting in a nuclear environment in AA scattering, where distributions of hadrons may shed further light on the phenomenon of "jet quenching." Knowledge of fragmentation functions in jet production and a good theoretical understanding of the process $pp \to (\text{jet}h)X$ are also crucial for studies of the Collins effect [15–17], an important probe of spin phenomena in hadronic scattering [18].

In the present paper, we perform a new next-to-leading-order (NLO) calculation of $pp \to (\text{jet}h)X$. In contrast to the previous calculation [4] which was entirely based on a numerical Monte Carlo integration approach, we will derive analytical results for the relevant partonic cross sections. Apart from providing independent NLO predictions in a numerically very efficient way, this offers several advantages. In the context of the analytical calculation, one can first of all explicitly check that the final-state collinear singularities have the structure required by the universality of fragmentation functions, meaning that the same fragmentation functions occur for $pp \to (\text{jet}h)X$ as for usual single-inclusive processes such as $pp \to hX$. We note that to our knowledge this has not yet been formally proven beyond NLO. Also, as we shall see, the NLO expressions show logarithmic enhancements at high z_h, which recur with increasing power at every order in perturbation theory, eventually requiring resummation to all orders. Having explicit analytical results is a prerequisite for such a resummation. In Ref. [3], considering the simpler case of e^+e^--annihilation, such resummation calculations for large z_h were presented.

Technically, we will derive our results by assuming the jet to be relatively narrow, an approximation known as the "narrow jet approximation" (NJA). This technique was used previously for NLO calculations of single-inclusive jet production in hadronic scattering, $pp \to \text{jet}X$ [19–23]. The main idea is to start from NLO "inclusive-parton" cross sections $d\hat{\sigma}^c_{ab}$ for the processes $ab \to cX$, which are relevant for the cross section for $pp \to hX$. They are *a priori* not suitable for computing a jet cross section, which is evident from the fact that the $d\hat{\sigma}^c_{ab}$ require collinear subtraction of final-state collinear singularities, whereas a jet cross section is infrared safe as far as the final state is concerned. Instead, it depends on the algorithm adopted to define the jet and thereby on a generic jet (size) parameter $\mathcal{R}$. As was shown in Refs. [19–23], at NLO one may nonetheless go rather straightforwardly from the single-inclusive parton cross sections to the jet ones, for any infrared-safe jet algorithm. The key is to properly account for the fact that at NLO two partons can fall into the same jet, so that the jet needs to be constructed from both. In fact, within the NJA, one can derive the translation between the $d\hat{\sigma}^c_{ab}$ and the partonic cross sections for jet production analytically. We note that the NJA formally corresponds to the limit $\mathcal{R} \to 0$, but it turns out to be accurate even at values $\mathcal{R} \sim 0.4 - 0.7$ relevant for experiment. In the NJA, the structure of the NLO jet cross section is of the form $\mathcal{A}\log(R) + \mathcal{B}$; corrections to this are of $\mathcal{O}(\mathcal{R}^2)$ and are neglected. In this paper, we apply the NJA to the case of $pp \to (\text{jet}h)X$, using it to derive the relevant NLO partonic cross sections. In the course of the explicit NLO calculation, we find that the partonic cross sections for $pp \to \text{jet}X$ and $pp \to (\text{jet}h)X$ may be very compactly formulated in terms of the single-inclusive parton ones $d\hat{\sigma}^c_{ab}$, convoluted with appropriate perturbative "jet functions." These functions are universal in the sense that they only depend on the type of the outgoing partons that fragment and/or produce the jet, but not on the underlying partonic hard-scattering function. On the basis of the jet functions, the NLO partonic cross sections for $pp \to \text{jet}X$ and $pp \to (\text{jet}h)X$ take a very simple and systematic form. In fact, it turns out that for $pp \to (\text{jet}h)X$ the jet functions have a "two-tier" form, with a first jet function describing the formation of the jet and a second one the fragmentation of a parton inside the jet. We note that the concept of jet functions for formulating jet cross sections is not new but was introduced in the context of soft-collinear effective theories (SCET) [1–3,24–26], although applications to $pp \to (\text{jet}h)X$ have to our knowledge not been given. Jet functions in a more general context of SCET or QCD resummation have been considered in Refs. [27] and [28], for example. We also note that in Ref. [14] the NLO corrections for the case of away-side jet-hadron correlations were presented in the context of a Monte Carlo integration code.

Our paper is organized as follows. In Sec. II we present our NLO calculation. In particular, Sec. II C contains our main new result, the formulation of $pp \to (\text{jet}h)X$ in terms of suitable jet functions. Section III presents phenomenological results for $pp \to (\text{jet}h)X$ for the LHC and RHIC. We finally conclude our work in Sec. IV. The Appendices collect some technical details of our calculations.

II. ASSOCIATED JET-PLUS-HADRON PRODUCTION IN THE NJA

A. Single-inclusive hadron production in hadronic collisions

Our formalism is best developed by first considering the process $H_1H_2 \to hX$, where a hadron h is observed at large transverse momentum p_T, but no requirement of a reconstructed hadronic jet is made. This is of course a standard reaction, for which the NLO corrections have been known for a long time [29,30]. The factorized cross section at given hadron p_T and rapidity η reads

$$\frac{d\sigma^{H_1H_2\to hX}}{dp_T d\eta} = \frac{2p_T}{S}\sum_{abc}\int_{x_a^{\min}}^{1}\frac{dx_a}{x_a}f_a^{H_1}(x_a,\mu_F) \times \int_{x_b^{\min}}^{1}\frac{dx_b}{x_b}f_b^{H_2}(x_b,\mu_F) \times \int_{z_c^{\min}}^{1}\frac{dz_c}{z_c^2}\frac{d\hat{\sigma}_{ab}^{c}(\hat{s},\hat{p}_T,\hat{\eta},\mu_F,\mu_F',\mu_R)}{v dv dw} \times D_c^h(z_c,\mu_F'), \tag{1}$$

with the usual parton distribution functions f_a^H, the fragmentation functions D_c^h, and the hard-scattering cross sections $d\hat{\sigma}_{ab}^c$ for the partonic processes $ab\to cX'$, with X' denoting an unobserved partonic final state. Defining

$$V \equiv 1-\frac{p_T}{\sqrt{S}}e^{-\eta}, \qquad W\equiv \frac{p_T^2}{SV(1-V)}, \tag{2}$$

where $\sqrt{S}$ is the hadronic c.m. system energy, we have

$$x_a^{\min} = W, \quad x_b^{\min} = \frac{1-V}{1-VW/x_a}, \quad z_c^{\min} = \frac{1-V}{x_b}+\frac{VW}{x_a}. \tag{3}$$

The $d\hat{\sigma}_{ab}^c$ are functions of the partonic c.m. system energy $\hat{s}=x_a x_b S$, the partonic transverse momentum $\hat{p}_T = p_T/z_c$ and the partonic rapidity $\hat{\eta}=\eta-\frac{1}{2}\log(x_a/x_b)$. Since only $\hat{p}_T$ depends on z_c, the last integral in Eq. (1) takes the form of a convolution. The variables v and w in Eq. (1) are the partonic counterparts of V and W:

$$v\equiv 1-\frac{\hat{p}_T e^{-\hat{\eta}}}{\sqrt{\hat{s}}}, \qquad w\equiv\frac{\hat{p}_T^2}{\hat{s}v(1-v)}. \tag{4}$$

One customarily expresses $\hat{p}_T$ and $\hat{\eta}$ by v and w:

$$\hat{p}_T^2 = \hat{s}vw(1-v), \qquad \hat{\eta}=\frac{1}{2}\log\left(\frac{vw}{1-v}\right). \tag{5}$$

Finally, the various functions in Eq. (1) are tied together by their dependence on the initial- and final-state factorization scales, μ_F and $\mu_F{}'$, respectively, and the renormalization scale μ_R.

The partonic hard-scattering cross sections may be evaluated in QCD perturbation theory. We write the perturbative expansion to NLO as

$$\frac{d\hat{\sigma}_{ab}^c}{dvdw} = \frac{d\hat{\sigma}_{ab}^{c,(0)}}{dv}\delta(1-w) + \frac{\alpha_s(\mu_R)}{2\pi}\frac{d\hat{\sigma}_{ab}^{c,(1)}}{dvdw} + \mathcal{O}(\alpha_s^2(\mu_R)), \tag{6}$$

where we have used that $w=1$ for leading-order (LO) kinematics (since the unobserved partonic final state X' consists of a single parton), equivalent to $2\hat{p}_T\cosh(\hat{\eta})/\sqrt{\hat{s}}=1$. The NLO terms $d\hat{\sigma}_{ab}^{c,(1)}$ have been presented in Refs. [29,30].

B. Translation to single-inclusive jet cross section via jet functions

As shown in Refs. [19–22], one can transform the cross section for single-inclusive hadron production to a single-inclusive jet one. References [19–22] explicitly constructed this translation at NLO. We may write the jet cross section as

$$\frac{d\sigma^{H_1H_2\to \text{jet}X}}{dp_T^{\text{jet}}d\eta^{\text{jet}}} = \frac{2p_T^{\text{jet}}}{S}\sum_{ab}\int_{x_a^{\min}}^{1}\frac{dx_a}{x_a}f_a^{H_1}(x_a,\mu_F) \times \int_{x_b^{\min}}^{1}\frac{dx_b}{x_b}f_b^{H_2}(x_b,\mu_F) \times \frac{d\hat{\sigma}_{ab}^{\text{jet,algo}}(\hat{s},p_T^{\text{jet}},\hat{\eta},\mu_F,\mu_R,\mathcal{R})}{vdvdw}, \tag{7}$$

where p_T^{jet} and η^{jet} are the jet's transverse momentum and rapidity, and where $\mathcal{R}$ denotes a parameter specifying the jet algorithm. For the jet cross section we still have

$$x_a^{\min}=W, \qquad x_b^{\min}=\frac{1-V}{1-VW/x_a}, \tag{8}$$

as in Eq. (3), but with V and W now defined by

$$V\equiv 1-\frac{p_T^{\text{jet}}}{\sqrt{S}}e^{-\eta^{\text{jet}}}, \qquad W\equiv\frac{(p_T^{\text{jet}})^2}{SV(1-V)}. \tag{9}$$

Likewise, v and w are as in Eq. (4) but with $\hat{p}_T\to p_T^{\text{jet}}$. Furthermore, in analogy with the inclusive-hadron case, $\hat{\eta}=\eta^{\text{jet}}-\frac{1}{2}\log(x_a/x_b)$. We note that the partonic cross sections $d\hat{\sigma}_{ab}^{\text{jet,algo}}$ relevant for jet production depend on the algorithm used to define the jet. They do not carry any dependence on a final-state factorization scale.

In order to go from the inclusive-parton cross sections $d\hat{\sigma}_{ab}^c$ to the jet ones $d\hat{\sigma}_{ab}^{\text{jet}}$, the idea is to apply proper correction terms to the former. The $d\hat{\sigma}_{ab}^c$ have been integrated over the full phase space of all final-state partons other than c. Therefore, they contain contributions where a second parton in the final state is so close to parton c that the two should jointly form the jet for a given jet definition. One can correct for this by subtracting such contributions from $d\hat{\sigma}_{ab}^c$ and adding a piece where they actually do form the jet together. At NLO, where there can be three partons c, d, e in the final state, one has after suitable summation over all possible configurations

$$d\hat{\sigma}_{ab}^{\rm jet} = [d\hat{\sigma}_{ab}^{c} - d\hat{\sigma}_{ab}^{c(d)} - d\hat{\sigma}_{ab}^{c(e)}] + [d\hat{\sigma}_{ab}^{d} - d\hat{\sigma}_{ab}^{d(c)} - d\hat{\sigma}_{ab}^{d(e)}] + [d\hat{\sigma}_{ab}^{e} - d\hat{\sigma}_{ab}^{e(c)} - d\hat{\sigma}_{ab}^{e(d)}] + d\hat{\sigma}_{ab}^{cd} + d\hat{\sigma}_{ab}^{ce} + d\hat{\sigma}_{ab}^{de}. \tag{10}$$

Here $d\hat{\sigma}_{ab}^{j(k)}$ is the cross section where parton j produces the jet, but parton k is so close that it should be part of the jet, and $d\hat{\sigma}_{ab}^{jk}$ is the cross section when both partons j and k jointly form the jet. The decomposition (10) is completely general to NLO. It may be applied for any jet algorithm, as long as the algorithm is infrared safe. As mentioned before, a property of the $d\hat{\sigma}_{ab}^{\rm jet}$ is that all dependence on the final-state factorization scale μ_F', which was initially present in the $d\hat{\sigma}_{ab}^{j}$, must cancel. This cancellation comes about in Eq. (10) because the $d\hat{\sigma}_{ab}^{jk}$ possess final-state collinear singularities that require factorization. This introduces dependence on μ_F' in exactly the right way as to compensate the μ_F' dependence of the $d\hat{\sigma}_{ab}^{j}$.

In the NJA, the correction terms $d\hat{\sigma}_{ab}^{j(k)}$ and $d\hat{\sigma}_{ab}^{jk}$ may be computed analytically. At NLO, they both receive contributions from real-emission $2 \to 3$ diagrams only. For the NJA one assumes that the observed jet is rather collimated. This in essence allows one to treat the two outgoing partons j and k as collinear. The relevant calculations for the standard cone[1] and (anti-)k_t [32–34] algorithms were carried out in Refs. [21,22], while Ref. [23] addressed the case of the "J_{E_T}" algorithm proposed in Refs. [35,36]. We note that we always define the four-momentum of the jet as the sum of four-momenta of the partons that form the jet. This so-called "E recombination scheme" [37] is the most popular choice nowadays.

By close inspection of Eq. (10), we have found that in the NJA the jet cross section may be cast into a form that makes use of the *single-inclusive parton* production cross sections $d\hat{\sigma}_{ab}^{c}$:

$$\frac{d\sigma^{H_1H_2\to {\rm jet}X}}{dp_T^{\rm jet}d\eta^{\rm jet}} = \frac{2p_T^{\rm jet}}{S}\sum_{abc}\int_{x_a^{\rm min}}^{1}\frac{dx_a}{x_a}f_a^{H_1}(x_a,\mu_F) \times \int_{x_b^{\rm min}}^{1}\frac{dx_b}{x_b}f_b^{H_2}(x_b,\mu_F) \times \int_{z_c^{\rm min}}^{1}\frac{dz_c}{z_c^2}\frac{d\hat{\sigma}_{ab}^{c}(\hat{s},\hat{p}_T,\hat{\eta},\mu_F,\mu_F',\mu_R)}{v dv dw} \times \mathcal{J}_c\left(z_c,\frac{\mathcal{R}p_T^{\rm jet}}{\mu_F'},\mu_R\right), \tag{11}$$

with *inclusive jet functions* $\mathcal{J}_q$ and $\mathcal{J}_g$. We have $\hat{p}_T = p_T^{\rm jet}/z_c$, and $x_a^{\rm min}, x_b^{\rm min}, z_c^{\rm min}$ and v, w are now as in Eqs. (3) and (4), respectively. Equation (11) thus states that one can go directly from the cross section for single-hadron production to that for jet production by replacing the fragmentation functions D_c^h in Eq. (1) by the jet functions $\mathcal{J}_c$. The latter are such that any dependence on μ_F' disappears from the cross section. They depend on the jet algorithm and hence on a jet parameter $\mathcal{R}$. For the cone and (anti-)k_t algorithms $\mathcal{R}$ is just given by the usual jet size parameter R introduced for these algorithms, while for the jet algorithm of Refs. [35,36] we have $\mathcal{R} = 1/\sqrt{\beta z_c}$ with β the "maximization" parameter defined for this algorithm. In the NJA we generally assume $\mathcal{R} \ll 1$ and neglect $\mathcal{O}(\mathcal{R}^2)$ contributions. The jet functions then read explicitly

$$\begin{aligned}
\mathcal{J}_q\left(z,\lambda\equiv\frac{\mathcal{R}p_T^{\rm jet}}{\mu_F{}'},\mu_R\right) &= \delta(1-z) - \frac{\alpha_s(\mu_R)}{2\pi}\left[2C_F(1+z^2)\left(\frac{\log(1-z)}{1-z}\right)_+ + P_{qq}(z)\log(\lambda^2) + \delta(1-z)I_q^{\rm algo} + C_F(1-z)\right] \\
&\quad - \frac{\alpha_s(\mu_R)}{2\pi}[P_{gq}(z)\log(\lambda^2(1-z)^2) + C_F z], \\
\mathcal{J}_g\left(z,\lambda\equiv\frac{\mathcal{R}p_T^{\rm jet}}{\mu_F'},\mu_R\right) &= \delta(1-z) - \frac{\alpha_s(\mu_R)}{2\pi}\left[\frac{4C_A(1-z+z^2)^2}{z}\left(\frac{\log(1-z)}{1-z}\right)_+ + P_{gg}(z)\log(\lambda^2) + \delta(1-z)I_g^{\rm algo}\right] \\
&\quad - \frac{\alpha_s(\mu_R)}{2\pi}2n_f[P_{qg}(z)\log(\lambda^2(1-z)^2) + z(1-z)],
\end{aligned} \tag{12}$$

where $C_F = 4/3$, $C_A = 3$ and n_f is the number of active flavors, and where the LO splitting functions $P_{ij}(z)$ as well as the "plus" distribution are defined in Appendix A. The dependence on the jet algorithm is reflected in the terms $I_q^{\rm algo}$ and $I_g^{\rm algo}$, which are just numbers that we also collect in Appendix A.

[1]Here we have in mind primarily the "seedless infrared safe cone" (SISCone) algorithm introduced in Ref. [31] which represents the only cone-based jet definition known to be strictly infrared safe. However, for single-inclusive jet cross sections, the lack of infrared safety of other cone-type algorithms occurs first at next-to-next-to-leading order in perturbation theory and hence is not an issue here.

Equation (11) evidently exhibits a factorized structure in the final state for the jet cross section in the NJA. Its physical interpretation is essentially that the hard scattering produces a parton c that "fragments" into the observed jet via the jet function $\mathcal{J}_c$, the jet carrying the fraction z_c of the produced parton's momentum. At NLO, the factorization is in fact rather trivial. To get a clear sense of it, it is instructive to see how one recovers Eqs. (7) and (10) from Eq. (11). To this end, we combine Eqs. (6) and (12) and expand to first order in the strong coupling. The products of the $d\hat{\sigma}^c_{ab}$ with the LO $\delta(1-z)$ terms in $\mathcal{J}_c$ just reproduce the single-inclusive parton cross sections at $\hat{p}_T = p_T^{\rm jet}$, i.e. the terms $d\hat{\sigma}^c_{ab}, d\hat{\sigma}^d_{ab}, d\hat{\sigma}^e_{ab}$ in Eq. (10). The only other terms surviving in the expansion to $\mathcal{O}(\alpha_s)$ are the products of the LO terms $\delta(1-w)d\hat{\sigma}^{c,(0)}_{ab}/dv$ of Eq. (6) with the $\mathcal{O}(\alpha_s)$ terms in the jet functions. These precisely give the remaining contributions $d\hat{\sigma}^{cd}_{ab} - d\hat{\sigma}^{c(d)}_{ab} - d\hat{\sigma}^{d(c)}_{ab}$ (plus the other combinations) in Eq. (10). Because of the convolution in z_c in Eq. (11), the $\delta(1-w)$ function in the Born cross section actually fixes z_c to the value $z_c = 2p_T^{\rm jet}\cosh(\hat{\eta})/\sqrt{\hat{s}}$. Based on our NLO calculation, we evidently cannot prove the factorization shown in Eq. (11) to beyond this order. We note, however, that similar factorization formulas have been derived using SCET techniques [24–26], for the case of jet observables in e^+e^- annihilation. In particular, functions closely related to our inclusive jet functions $\mathcal{J}_{q,g}$ may be found in Ref. [25], where they are termed "unmeasured" quark (or gluon) jet functions. We shall return to comparisons with SCET results below.

C. Hadrons produced inside jets

We are now ready to tackle the case that we are really interested in, $H_1H_2 \to ({\rm jet}h)X$ where the hadron is observed inside a reconstructed jet and is part of the jet. Our strategy for performing an analytical NLO calculation will be to use the NJA and the same considerations as those that gave rise to Eq. (10). Subsequently, we will again phrase our results in a simple and rather general way in terms of suitable jet functions.

The cross section we are interested in is specified by the jet's transverse momentum $p_T^{\rm jet}$ and rapidity $\eta^{\rm jet}$, and by the variable

$$z_h \equiv \frac{p_T}{p_T^{\rm jet}}, \tag{13}$$

where as in Sec. II A p_T refers to the transverse momentum of the produced hadron. As we are working in the NJA, we consider collinear fragmentation of the hadron inside the jet. Thus, the observed hadron and the jet have the same rapidities, $\eta = \eta^{\rm jet}$, since differences in rapidity are $\mathcal{O}(\mathcal{R}^2)$ effects and hence suppressed in the NJA.

The factorized jet-plus-hadron cross section is written as

$$\begin{aligned}\frac{d\sigma^{H_1H_2\to({\rm jet}h)X}}{dp_T^{\rm jet}d\eta^{\rm jet}dz_h} = &\frac{2p_T^{\rm jet}}{S}\sum_{a,b,c}\int_{x_a^{\rm min}}^1\frac{dx_a}{x_a}f_a^{H_1}(x_a,\mu_F)\\ &\times\int_{x_b^{\rm min}}^1\frac{dx_b}{x_b}f_b^{H_2}(x_b,\mu_F)\\ &\times\int_{z_h}^1\frac{dz_p}{z_p}\frac{d\hat{\sigma}^{({\rm jet}\,c)}_{ab}(\hat{s},p_T^{\rm jet},\hat{\eta},\mu_F,\mu_F',\mu_R,\mathcal{R},z_p)}{vdvdwdz_p}\\ &\times D_c^h\left(\frac{z_h}{z_p},\mu_F'\right),\end{aligned} \tag{14}$$

where $x_a^{\rm min}$, $x_b^{\rm min}$, and $\hat{\eta} = \eta^{\rm jet} - \frac{1}{2}\log(x_a/x_b)$ are as for the single-inclusive jet cross section, and where z_p is the partonic analog of z_h. In other words, the $d\hat{\sigma}^{({\rm jet}\,c)}_{ab}$ are the partonic cross sections for producing a final-state jet (subject to a specified jet algorithm), inside of which there is a parton c with transverse momentum $p_T^c = z_p p_T^{\rm jet}$ that fragments into the observed hadron. The argument of the corresponding fragmentation functions is fixed by $p_T = zp_T^c$ and hence, using Eq. (13), is given by $z = z_h/z_p$. Thus the new partonic cross sections are in convolution with the fragmentation functions. Note that all other variables V, W and v, w have the same definitions as in the single-inclusive jet case; see Eq. (9).

At lowest order, there is only one parton forming the jet, and this parton also is the one that fragments into the observed hadron, implying $z_p = 1$. The partonic cross sections hence have the perturbative expansions

$$\begin{aligned}\frac{d\hat{\sigma}^{({\rm jet}\,c)}_{ab}}{dvdwdz_p} = &\frac{d\hat{\sigma}^{c,(0)}_{ab}}{dv}\delta(1-w)\delta(1-z_p) + \frac{\alpha_s(\mu_R)}{\pi}\frac{d\hat{\sigma}^{({\rm jet}\,c),(1)}_{ab}}{dvdwdz_p}\\ &+\mathcal{O}(\alpha_s^2(\mu_R)),\end{aligned} \tag{15}$$

with the same Born terms $d\hat{\sigma}^{c,(0)}_{ab}/dv$ as in Eq. (6).

In order to derive the NLO partonic cross sections $d\hat{\sigma}^{({\rm jet}\,c),(1)}_{ab}$, we revisit Eq. (10). Since we now "observe" a parton c in the final state (the fragmenting one), we must not sum over all possible final states, but rather consider only the contributions that contain parton c:

$$d\hat{\sigma}^c_{ab} - d\hat{\sigma}^{c(d)}_{ab} - d\hat{\sigma}^{c(e)}_{ab} + d\hat{\sigma}^{cd}_{ab} + d\hat{\sigma}^{ce}_{ab}. \tag{16}$$

However, for each term we now need to derive its proper dependence on z_p before combining all terms. For the terms $d\hat{\sigma}^c_{ab}$ and $d\hat{\sigma}^{c(d)}_{ab}$, $d\hat{\sigma}^{c(e)}_{ab}$ this is trivial since for all of these terms parton c alone produces the jet and also is the parton that fragments. As a result, all these terms simply acquire a factor $\delta(1-z_p)$. This becomes different for the pieces $d\hat{\sigma}^{cd}_{ab}, d\hat{\sigma}^{ce}_{ab}$. Following Refs. [21–23], in the NJA we may write the NLO contribution to any $d\hat{\sigma}^{cd}_{ab}$ as

$$\frac{d\hat{\sigma}_{ab}^{cd,(1)}}{dvdw} = \frac{\alpha_s}{\pi}\mathcal{N}_{ab\to K}(v,w,\varepsilon)\delta(1-w) \times \int_0^1 dz_p z_p^{-\varepsilon}(1-z_p)^{-\varepsilon}\tilde{P}_{cK}^{<}(z_p) \times \int_0^{m^2_{\mathrm{max,algo}}} \frac{dm^2_{\mathrm{jet}}}{m^2_{\mathrm{jet}}} m_{\mathrm{jet}}^{-2\varepsilon}, \quad (17)$$

where we have used dimensional regularization with $D = 4-2\varepsilon$ space-time dimensions. Equation (17) is derived from the fact that the leading contributions in the NJA come from a parton K splitting into partons c and d "almost" collinearly in the final state. We therefore have an underlying Born process $ab \to KX$ (with some unobserved recoil final state X), whose D-dimensional cross section is contained in the "normalization factor" $\mathcal{N}_{ab\to K}$, along with some trivial factors. The integrand then contains the D-dimensional LO splitting functions $\tilde{P}_{cK}^{<}(z)$, where the superscript "<" indicates that the splitting function is strictly at $z<1$, that is, without its $\delta(1-z)$ contribution that is present when $c=K$. The functions are defined in Eq. (A3) in Appendix A. The argument of the splitting function is the fraction of the intermediate particle's momentum (equal to the jet momentum) transferred in the splitting. In the NJA it therefore coincides with our partonic variable z_p. In the second integral in Eq. (17) m_{jet} is the invariant mass of the jet. The explicit factor m^2_{jet} in the denominator represents the propagator of the splitting parton K. The integral over the jet mass runs between zero and an upper limit $m_{\mathrm{max,algo}}$, which in the NJA is formally taken to be relatively small. As indicated, $m_{\mathrm{max,algo}}$ depends on the algorithm chosen to define the jet. We have [21–23]

$$m^2_{\mathrm{max,algo}} = \begin{cases} (p_T^{\mathrm{jet}}R)^2 \min\left(\frac{z_p}{1-z_p},\frac{1-z_p}{z_p}\right) & \text{cone algorithm,} \\ (p_T^{\mathrm{jet}}R)^2 z_p(1-z_p) & \text{(anti-)}k_t \text{ algorithm,} \\ \frac{(p_T^{\mathrm{jet}})^2}{\beta}\min(z_p,1-z_p) & J_{E_T} \text{ algorithm.} \end{cases} \quad (18)$$

To make the cross section $d\hat{\sigma}_{ab}^{cd,(1)}$ differential in z_p we now just need to drop the integration over z_p in Eq. (17). We next expand the resulting expression in ε. The m^2_{jet} integration produces a collinear singularity in $1/\varepsilon$. It also contributes a factor $(1-z_p)^{-\varepsilon}$ at large z_p which may be combined with the explicit factor $(1-z_p)^{-\varepsilon}$ in Eq. (17). In the presence of a diagonal splitting function in the integrand we hence arrive at a term $(1-z_p)^{-1-2\varepsilon}$, which may be expanded in ε to give a further pole in $1/\varepsilon$ and "plus" distributions in $1-z_p$. The double poles $1/\varepsilon^2$ arising in this way cancel against double poles in $d\hat{\sigma}_{ab}^{c(d)}, d\hat{\sigma}_{ab}^{c(e)}$. The remaining single poles are removed by collinear factorization into the fragmentation function for parton c. For nondiagonal splitting functions there are only single poles which are directly subtracted by factorization. We note that the original $d\hat{\sigma}_{ab}^{cd,(1)}$ is in fact needed both for the cross section with parton c fragmenting and also for the one where d fragments. This is reflected in the fact that the z_p integral in Eq. (17) runs from 0 to 1, while for $d\hat{\sigma}_{ab}^{cd,(1)}$ the limit $z_p \to 0$ is never reached as long as $z_h > 0$. For parton d fragmenting, however, we need to use $d\hat{\sigma}_{ab}^{dc,(1)}$ which differs from $d\hat{\sigma}_{ab}^{cd,(1)}$ only by a change of the splitting function. In the case of a quark splitting into a quark and a gluon, this change is from $\tilde{P}_{qq}^{<}(z)$ for an observed quark to $\tilde{P}_{gq}^{<}(z)$ for an observed gluon. Because of $\tilde{P}_{gq}^{<}(z) = \tilde{P}_{qq}^{<}(1-z)$ one precisely recovers the old expression for the inclusive-jet cross sections when all final states are summed over. Likewise, if a gluon splits into a $q\bar{q}$ or gg, the relevant splitting functions $\tilde{P}_{qg}^{<}(z)$, $\tilde{P}_{gg}^{<}(z)$ are by themselves symmetric under $z \leftrightarrow 1-z$.

From this discussion, and combined with Eqs. (15) and (16), we obtain to NLO in the NJA

$$\frac{d\hat{\sigma}_{ab}^{(\mathrm{jet}\,c)}}{dvdwdz_p} = \left[\frac{d\hat{\sigma}_{ab}^{c}}{dvdw} - \frac{d\hat{\sigma}_{ab}^{c(d)}}{dvdw} - \frac{d\hat{\sigma}_{ab}^{c(e)}}{dvdw}\right]\delta(1-z_p) + \frac{d\hat{\sigma}_{ab}^{cd}}{dvdwdz_p} + \frac{d\hat{\sigma}_{ab}^{ce}}{dvdwdz_p}. \quad (19)$$

Computing and inserting all ingredients of this expression, we find that the cross section may be cast into a form that again makes use of the *single-inclusive parton* production cross sections $d\hat{\sigma}_{ab}^{c}$, similar to the case of inclusive-jet production in Eq. (11):

$$\frac{d\sigma^{H_1H_2\to(\mathrm{jet}h)X}}{dp_T^{\mathrm{jet}}d\eta^{\mathrm{jet}}dz_h} = \frac{2p_T^{\mathrm{jet}}}{S}\sum_{a,b,c}\int_{x_a^{\min}}^1 \frac{dx_a}{x_a} f_a^{H_1}(x_a,\mu_F) \times \int_{x_b^{\min}}^1 \frac{dx_b}{x_b} f_b^{H_2}(x_b,\mu_F) \times \int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2}\frac{d\hat{\sigma}_{ab}^{c}(\hat{s},\hat{p}_T,\hat{\eta},\mu_F,\mu_F',\mu_R)}{vdvdw} \times \sum_{c'}\int_{z_h}^1 \frac{dz_p}{z_p}\mathcal{K}_{c\to c'}\left(z_c,z_p;\frac{\mathcal{R}p_T^{\mathrm{jet}}}{\mu_F'},\frac{\mathcal{R}p_T^{\mathrm{jet}}}{\mu_F''},\mu_R\right) \times D_{c'}^{h}\left(\frac{z_h}{z_p},\mu_F''\right), \quad (20)$$

where $x_a^{\min}$, $x_b^{\min}$ and $z_c^{\min}$ are as given in Eq. (3), with V and W defined in terms of jet transverse momentum and rapidity. Furthermore, as in Eq. (11) we have $\hat{p}_T = p_T^{\mathrm{jet}}/z_c$. The (jet-algorithm-dependent) functions $\mathcal{K}_{c\to c'}$ are new "semi-inclusive" jet functions that describe the production of a fragmenting parton c' inside a jet that results from a parton c produced in the hard scattering. For the "transition" $q \to q$ we find

$$\mathcal{K}_{q\to q}\left(z, z_p, \lambda = \frac{\mathcal{R} p_T^{\rm jet}}{\mu_F'}, \kappa = \frac{\mathcal{R} p_T^{\rm jet}}{\mu_F''}, \mu_R\right) = \delta(1-z)\delta(1-z_p) + \frac{\alpha_s(\mu_R)}{2\pi}\left[-\delta(1-z_p)\left\{2C_F(1+z^2)\left(\frac{\log(1-z)}{1-z}\right)_+ + P_{qq}(z)\log(\lambda^2) + C_F(1-z)\right\} + \delta(1-z)\left\{2C_F(1+z_p^2)\left(\frac{\log(1-z_p)}{1-z_p}\right)_+ + P_{qq}(z_p)\log(\kappa^2) + C_F(1-z_p) + \mathcal{I}_{qq}^{\rm algo}(z_p)\right\}\right], \tag{21}$$

where $\mathcal{I}_{qq}^{\rm algo}(z_p)$ is a function that depends on the jet algorithm. Since we will write our new jet functions in a more compact form below, we do not present the other functions $\mathcal{K}_{c\to c'}$ here but collect them in Appendix B, along with the $\mathcal{I}_{c'c}^{\rm algo}(z_p)$.

As indicated, $\mathcal{K}_{q\to q}$ carries dependence on two (final-state) factorization scales, μ_F' and μ_F''. The former is the same as we encountered in the case of single-inclusive jets in Eqs. (11) and (12). It was originally introduced in the collinear factorization for the single-inclusive parton cross sections, but now has to cancel exactly between the $d\hat{\sigma}_{ab}^c$ and the $\mathcal{K}_{c\to c'}$. As in the case of single-inclusive jets, the cancellation of dependence on μ_F' is just a result of the fact that we foremost define our observable by requiring a jet in the final state. In this sense, μ_F' is simply an artifact of the way we organize the calculation and is not actually present in the final answer. The scale μ_F'', on the other hand, arises because we now also require a hadron in the final state. Technically it arises when we subtract collinear singularities from the $d\hat{\sigma}_{ab}^{cd,(1)}$. The logarithms in μ_F'' are thus just the standard scale logarithms that compensate the evolution of the fragmentation functions at this order. We also note that there are two sum rules that connect the inclusive and the semi-inclusive jet functions [1,2]:

$$\int_0^1 dz_p z_p[\mathcal{K}_{q\to q}(z, z_p; \lambda, \kappa, \mu_R) + \mathcal{K}_{q\to g}(z, z_p; \lambda, \kappa, \mu_R)] = \mathcal{J}_q(z, \lambda, \mu_R),$$
$$\int_0^1 dz_p z_p[\mathcal{K}_{g\to g}(z, z_p; \lambda, \kappa, \mu_R) + \mathcal{K}_{g\to q}(z, z_p; \lambda, \kappa, \mu_R)] = \mathcal{J}_g(z, \lambda, \mu_R). \tag{22}$$

Both are fulfilled by our expressions. Furthermore, $\int_0^1 dz_p \mathcal{K}_{q\to q}(z, z_p; \lambda, \kappa, \mu_R)$ reproduces the quark splitting contributions to $\mathcal{J}_q$, i.e. the first two lines in Eq. (12).

We may actually go one step further and decompose the functions $\mathcal{K}_{c\to c'}$ into products of jet functions that separate the dependence on z and z_p. We define two sets of functions:

$$j_{q\to q}(z, \lambda, \mu_R) \equiv \delta(1-z) - \frac{\alpha_s(\mu_R)}{2\pi}\left[2C_F(1+z^2)\left(\frac{\log(1-z)}{1-z}\right)_+ + P_{qq}(z)\log(\lambda^2) + \delta(1-z) I_q^{\rm algo} + C_F(1-z)\right],$$
$$j_{q\to g}(z, \lambda, \mu_R) \equiv -\frac{\alpha_s(\mu_R)}{2\pi}[P_{gq}(z)\log(\lambda^2(1-z)^2) + C_F z],$$
$$j_{g\to g}(z, \lambda, \mu_R) \equiv \delta(1-z) - \frac{\alpha_s(\mu_R)}{2\pi}\left[\frac{4C_A(1-z+z^2)^2}{z}\left(\frac{\log(1-z)}{1-z}\right)_+ + P_{gg}(z)\log(\lambda^2) + \delta(1-z) I_g^{\rm algo}\right],$$
$$j_{g\to q}(z, \lambda, \mu_R) \equiv -\frac{\alpha_s(\mu_R)}{2\pi}[P_{qg}(z)\log(\lambda^2(1-z)^2) + z(1-z)], \tag{23}$$

(where as before $\lambda = \mathcal{R} p_T^{\rm jet}/\mu_F{}'$), and

$$\tilde{j}_{q\to q}(z_p,\kappa,\mu_R) \equiv \delta(1-z_p) + \frac{\alpha_s(\mu_R)}{2\pi}\left[2C_F(1+z_p^2)\left(\frac{\log(1-z_p)}{1-z_p}\right)_+ + P_{qq}(z_p)\log(\kappa^2) + C_F(1-z_p) + \mathcal{I}_{qq}^{\rm algo}(z_p) \right.$$
$$\left. + \delta(1-z_p) I_q^{\rm algo}\right],$$
$$\tilde{j}_{q\to g}(z_p,\kappa,\mu_R) \equiv \frac{\alpha_s(\mu_R)}{2\pi}[P_{gq}(z_p)\log(\kappa^2(1-z_p)^2) + C_F z_p + \mathcal{I}_{gq}^{\rm algo}(z_p)],$$
$$\tilde{j}_{g\to g}(z_p,\kappa,\mu_R) \equiv \delta(1-z_p) + \frac{\alpha_s(\mu_R)}{2\pi}\left[\frac{4C_A(1-z_p+z_p^2)^2}{z_p}\left(\frac{\log(1-z_p)}{1-z_p}\right)_+ + P_{gg}(z_p)\log(\kappa^2) + \mathcal{I}_{gg}^{\rm algo}(z_p) + \delta(1-z_p) I_g^{\rm algo}\right],$$
$$\tilde{j}_{g\to q}(z_p,\kappa,\mu_R) \equiv \frac{\alpha_s(\mu_R)}{2\pi}[P_{qg}(z_p)\log(\kappa^2(1-z_p)^2) + z_p(1-z_p) + \mathcal{I}_{qg}^{\rm algo}(z_p)], \tag{24}$$

where again $\kappa = \mathcal{R}p_T^{\rm jet}/\mu_F''$ and the $I_{q,g}^{\rm algo}$ are as given in Appendix A for the inclusive-jet case. To the order we are considering we then have

$$\mathcal{K}_{c\to c'}(z,z_p;\lambda,\kappa,\mu_R) = \sum_e j_{c\to e}(z,\lambda,\mu_R)\tilde{j}_{e\to c'}(z_p,\kappa,\mu_R), \tag{25}$$

and hence from (20)

$$\frac{d\sigma^{H_1H_2\to({\rm jet}h)X}}{dp_T^{\rm jet}d\eta^{\rm jet}dz_h} = \frac{2p_T^{\rm jet}}{S}\sum_{a,b,c}\int_{x_a^{\rm min}}^1 \frac{dx_a}{x_a} f_a^{H_1}(x_a,\mu_F)\int_{x_b^{\rm min}}^1 \frac{dx_b}{x_b} f_b^{H_2}(x_b,\mu_F)\int_{z_c^{\rm min}}^1 \frac{dz_c}{z_c^2}\frac{d\hat{\sigma}_{ab}^c(\hat{s},\hat{p}_T,\hat{\eta},\mu_F,\mu_F',\mu_R)}{vdvdw}$$
$$\times \sum_e j_{c\to e}\left(z_c,\frac{\mathcal{R}p_T^{\rm jet}}{\mu_F'},\mu_R\right)\sum_{c'}\int_{z_h}^1 \frac{dz_p}{z_p}\tilde{j}_{e\to c'}\left(z_p,\frac{\mathcal{R}p_T^{\rm jet}}{\mu_F''},\mu_R\right)D_{c'}^h\left(\frac{z_h}{z_p},\mu_F''\right). \tag{26}$$

In other words, in the NJA the production of a jet with an observed hadron factorizes into the production cross section for parton c, a jet function $j_{c\to e}$ describing the formation of a jet "consisting" of parton e which has taken the fraction z_c of the parent parton's momentum, another jet function $\tilde{j}_{e\to c'}$ describing a "partonic fragmentation" of parton e to parton c' inside the jet, and finally a regular fragmentation function $D_{c'}^h$. This picture is sketched in Fig. 1. It is interesting to see that the structure of the first part of Eq. (26) is very similar to that of the inclusive-jet cross section (11) when formulated in terms of the jet functions $\mathcal{J}_c$. In fact, if we drop the terms starting with $\sum_{c'}$ in (26) and perform the sum over parton-type e, we will exactly arrive at Eq. (11), since

$$j_{q\to q}(z,\lambda,\mu_R) + j_{q\to g}(z,\lambda,\mu_R) = \mathcal{J}_q(z,\lambda,\mu_R),$$
$$2n_f j_{g\to q}(z,\lambda,\mu_R) + j_{g\to g}(z,\lambda,\mu_R) = \mathcal{J}_g(z,\lambda,\mu_R). \tag{27}$$

The terms starting with $\sum_{c'}$ in Eq. (26) thus describe the production of an identified hadron in the jet.

We note that at the level of our NLO computation we cannot prove the factorization in Eq. (26) to all orders. In fact, at $\mathcal{O}(\alpha_s)$ we can move terms between $j_{c\to e}$ and $\tilde{j}_{e\to c'}$. On the other hand, it seems very natural that the jet functions that we encountered in the single-inclusive jet case should play a role also in this case in the "first step" of the formation of the final state described by the $j_{c\to e}$. Also, our jet functions $\tilde{j}_{e\to c'}$ are identical to the corresponding functions found in the SCET study [3] of hadrons in jets produced in e^+e^- collisions, except for end-point

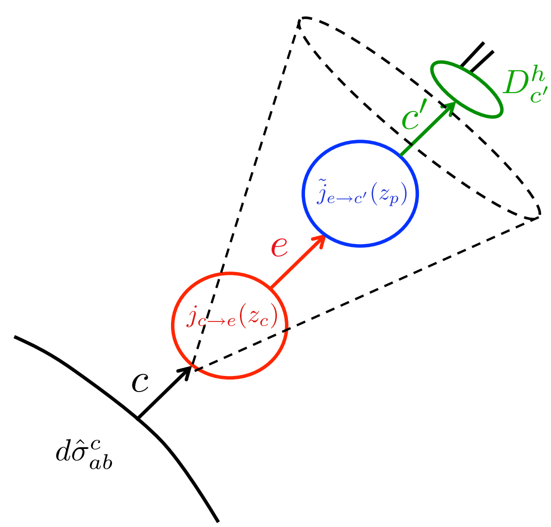

FIG. 1 (color online). Sketch of the production of an observed hadron inside a jet, described in terms of the jet functions $j_{c\to e}$ and $\tilde{j}_{e\to c'}$ (see text).

contributions $\propto \delta(1-z_p)$ that are necessarily different in the SCET formalism due to the presence of a soft function.

We finally note that the cross section (26) may also be expressed in terms of the hadron kinematics, using the relation

$$\frac{d\sigma^{H_1H_2\to(\text{jet}\,h)X}}{dp_T d\eta dz_h}(p_T,\eta,z_h) = \frac{1}{z_h}\frac{d\sigma^{H_1H_2\to(\text{jet}\,h)X}}{dp_T^{\text{jet}} d\eta^{\text{jet}} dz_h}\left(p_T^{\text{jet}} = \frac{p_T}{z_h},\eta,z_h\right). \tag{28}$$

III. PHENOMENOLOGICAL RESULTS

We now present some phenomenological results for associated jet-plus-hadron production. First, we compare our analytical calculation in the NJA with the one of Ref. [4], where the NLO cross section was obtained numerically by Monte Carlo integration techniques. As in that paper, we consider the case of charged hadrons produced in pp collisions at the LHC with center-of-mass energy $\sqrt{S}=8$ TeV. We define the jet by the anti-k_t algorithm with jet parameter $R=0.4$. The renormalization and initial-state factorization scales are set equal to the transverse momentum of the jet, $\mu_R=\mu_F=p_T^{\text{jet}}$, while the final-state factorization scale is chosen as $\mu_F''=Rp_T^{\text{jet}}$. The latter choice serves to sum logarithms of R to all orders [38,39], although this only becomes necessary for jet sizes much smaller than $R=0.4$. As in Ref. [4] we use the CTEQ6.6M parton distributions [40] and the "de Florian-Sassot-Stratmann" (DSS07) fragmentation functions of Ref. [11]. Our results refer to (summed) charged hadrons, i.e. $h\equiv h^+ + h^-$.

In Fig. 2 we show the ratio of the cross section in the NJA with that obtained numerically in Ref. [4]. The ratio is shown as a function of z_h, where the cross sections have been integrated over $|\eta|<1$ and $30\text{ GeV} < p_T < 200$ GeV in hadron rapidity and transverse momentum.

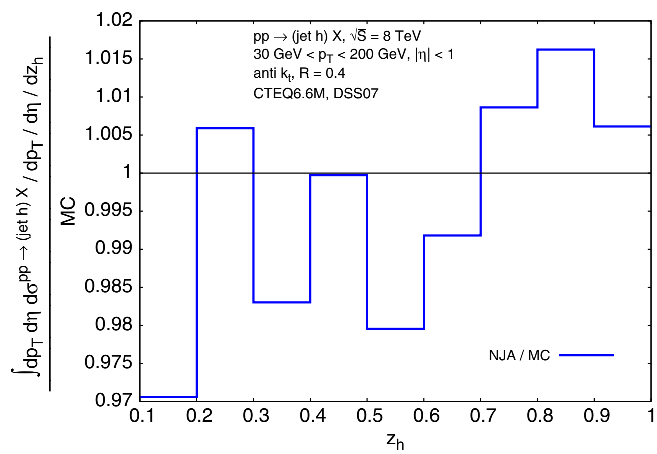

FIG. 2 (color online). Comparison of our results in the NJA to the ones of Ref. [4] for LHC kinematics.

As one can see, the agreement of the two NLO calculations is very good. The deviations are smaller than 3% everywhere, which demonstrates the good accuracy of the NJA. We note that in Ref. [4] a closely related variable Z_h is considered, which is defined as

$$Z_h \equiv \frac{\vec{p}_T\cdot\vec{p}_T^{\,\text{jet}}}{|\vec{p}_T^{\,\text{jet}}|^2}. \tag{29}$$

This definition differs from Eq. (13) only by $\mathcal{O}(\mathcal{R}^2)$ corrections, which are anyway neglected in the NJA. In the limit $z_h\to 1$, the two definitions become equivalent. This explains why the ratio in Fig. 2 is even closer to unity for larger values of z_h. The excellent accuracy of the NJA observed in the figure is consistent with similar comparisons for the case of single-inclusive jet production in the NJA [21,22].

Next, we show some results for the kinematics relevant for the ongoing studies at ALICE [10]. We consider pp collisions at $\sqrt{S}=7$ TeV and fragmentation into charged pions ($\pi\equiv\pi^+ + \pi^-$). For the rapidity interval we choose $|\eta|<0.5$ and we restrict the jet transverse momentum to $15\text{ GeV} < p_T^{\text{jet}} < 20$ GeV. As before, the jet is defined by the anti-k_t algorithm with $R=0.4$. We now use more modern sets for the parton distributions, CT10 [41], and fragmentation functions, DSS14 [12]. All scales are set equal to the transverse momentum of the jet, $\mu_R=\mu_F=\mu_F''=p_T^{\text{jet}}\equiv\mu$. Figure 3 shows the LO (dashed) and NLO (solid) cross sections for associated jet-plus-pion production differential in the transverse momentum of the pion. Note that the variable z_h is determined as p_T/p_T^{jet} and hence

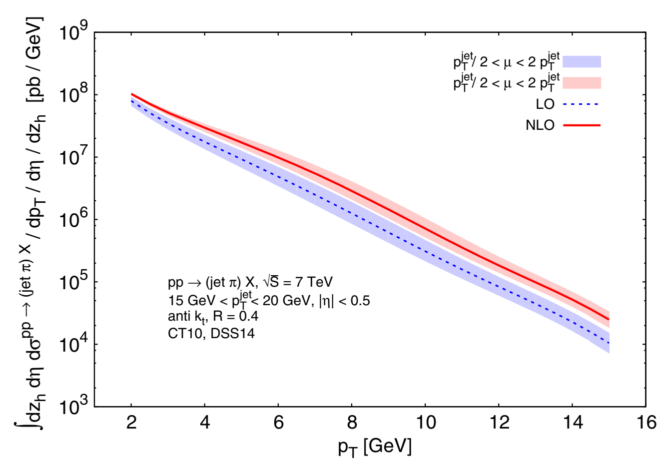

FIG. 3 (color online). LO (dashed) and NLO (solid) cross sections for $pp\to(\text{jet}\pi)X$ for ALICE conditions, as functions of pion p_T. The bands show the scale dependence of the cross section for variations of the scale between $p_T^{\text{jet}}/2$ (upper end of bands) and $2p_T^{\text{jet}}$ (lower end of bands). The factorization and renormalization scales have all been set equal and varied simultaneously.

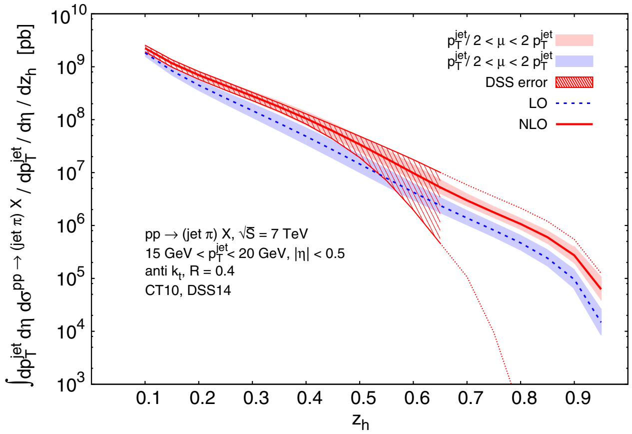

FIG. 4 (color online). Same as in Fig. 3, but as a function of z_h. As before the solid bands show the scale uncertainty. The hatched band displays the uncertainty of the cross section related to the fragmentation functions. This band is only reliable up to $z_h = 0.65$ and extrapolated beyond (see text).

is varied upon integration over $p_T^{\rm jet}$. The bands show the changes of the cross sections when the scales are varied in the range $p_T^{\rm jet}/2 < \mu < 2p_T^{\rm jet}$. As one can see, the scale dependence of the cross sections improves somewhat when going from LO to NLO, although not as much as one would have hoped. This feature was also observed for single-inclusive hadron production in hadronic scattering [30].

For the same kinematical setup we also show the cross section differential in z_h; see Fig. 4. A fixed value of z_h implies that the hadron's transverse momentum varies as we integrate over $p_T^{\rm jet}$. As discussed in the Introduction, this arguably is the most interesting distribution for $pp \to ({\rm jet}\pi)X$ since it allows direct scans of the fragmentation functions. Apart from the scale variation, we also show in the figure the uncertainty related to the fragmentation functions, which we compute using the Hessian error sets provided in the DSS14 set [12]. Note that the resulting uncertainty band is reliable only up to $z_h \approx 0.65$, beyond which there are presently hardly any hadron production data available for e^+e^- annihilation or ep scattering. We hence stop the main uncertainty band there and only sketch its possible extrapolation to higher z_h. It is clear from the figure that precise measurements of the cross section as a function of z_h have the potential to provide new information on fragmentation functions that is complementary to—and in some respects better than—that available from e^+e^- annihilation.

An interesting question is of course which of the fragmentation functions are primarily probed when the cross section for $pp \to ({\rm jet}\pi)X$ is studied as a function of z_h. Depending on kinematics, different initial states may dominate the contributions to the cross section, resulting also in different weights with which the fragmentation functions for the various parton species enter. Given how little information on gluon fragmentation is available from $e^+e^- \to hX$ and $ep \to ehX$, it is especially interesting to see how strongly the cross section for $pp \to ({\rm jet}\pi)X$ depends on D_g^h. It is known that for LHC energies, channels with gluonic initial states (especially gg) typically make important contributions to cross sections. In order to explore whether this allows probes of D_g^h at the LHC, we investigate in Fig. 5 the relative contributions of quark/antiquark (summed over all flavors) and gluon fragmentation to the cross section for $pp \to ({\rm jet}\pi)X$ at ALICE (as shown in the previous Fig. 4). We normalize the contributions to the full cross section, so that the quark and gluon contributions add up to unity. We use both the DSS07 and DSS14 sets. As one can see, for $z_h \lesssim 0.5$ the two sets give similar results and show that the cross section is strongly dominated by gluon fragmentation here. This is already interesting, since it implies that in this regime clean probes of D_g^h should be possible that should be much more sensitive than e^+e^- annihilation. Beyond $z_h = 0.5$, the two sets of fragmentation functions show very different behavior. For DSS07, gluon fragmentation continues to dominate all the way up to $z_h \sim 0.9$, whereas for DSS14 the quarks take over at $z_h \sim 0.7$. We stress again that the uncertainties of the fragmentation functions become very large at such values of z, as we saw in the previous figure, and are in fact hard to quantify reliably. It is evident that information from $pp \to ({\rm jet}\pi)X$ in this regime will be most valuable, regardless of whether quark or gluon fragmentation dominates. Detailed measurements for various bins in transverse momentum and rapidity will likely help in disentangling fragmenting quarks and gluons.

As mentioned in the Introduction, measurements of charged hadrons produced in jets are already available from ATLAS [7,8] and CMS [9]. ATLAS has published measurements at $\sqrt{S} = 7$ TeV [7] and presented preliminary data [8] also at $\sqrt{S} = 2.76$ TeV. The two analyses

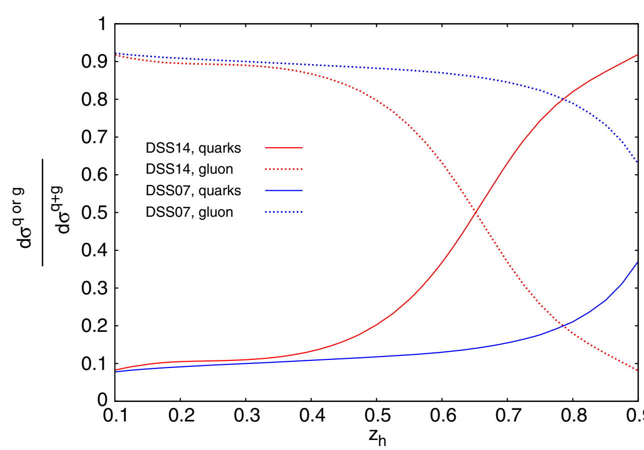

FIG. 5 (color online). Normalized quark (solid) and gluon (dashed) contributions to the cross section differential in z_h for the kinematic conditions chosen for Fig. 4. We show results for DSS07 [11] and DSS14 [12] fragmentation functions.

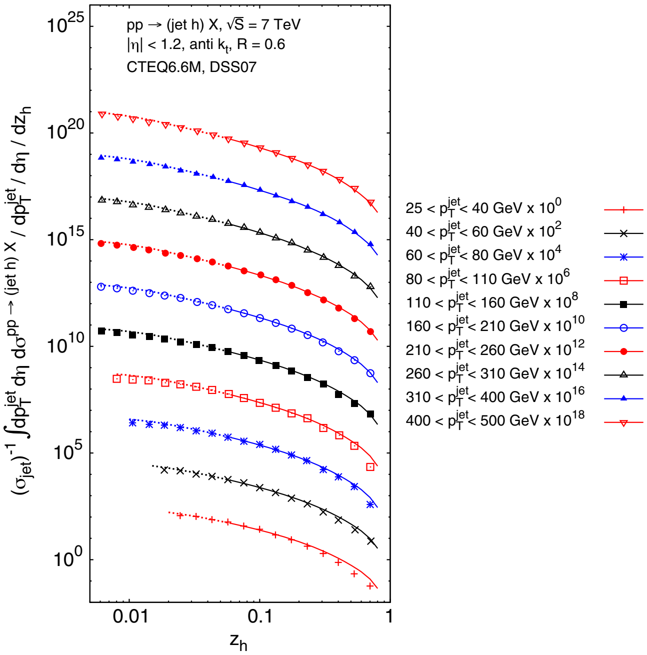

FIG. 6 (color online). NLO cross section for $pp \to (\mathrm{jet}h)X$ as a function of z_h at $\sqrt{S} = 7$ TeV, compared to the ATLAS data [7] for charged hadron production in the leading jet. The cross section is normalized to the total jet rate. In the region outside the validity of the DSS07 set the theory curves are extrapolated and plotted as dotted lines.

each use a slightly different definition of z_h which however both coincide with our z_h in the NJA limit. Figures 6 and 7 present comparisons of our NLO calculations to the ATLAS data for the two energies. We have now gone back to the DSS07 set, since unspecified charged-hadron fragmentation functions are not available in the more recent DSS14 set. As one can see, there is overall a very good agreement. Note that this agreement extends even down to values of $z_h < 0.05$, well outside the region of validity of the DSS sets. The figures clearly demonstrate the potential of the data to further pin down the charged-hadron fragmentation functions.

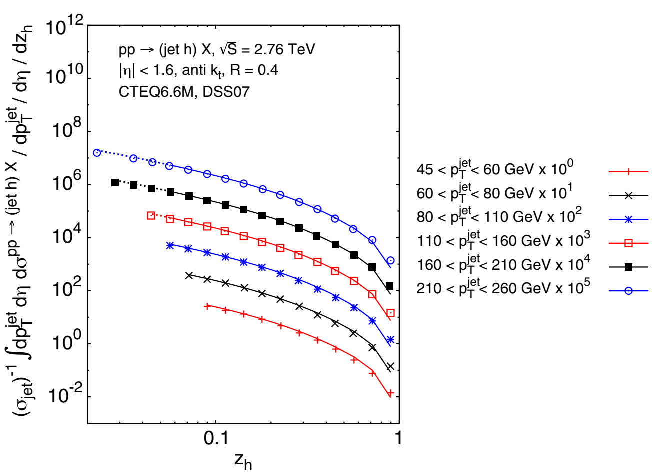

FIG. 7 (color online). Same as Fig. 6 but at $\sqrt{S} = 2.76$ TeV, compared to the preliminary ATLAS data [8].

The CMS analysis [9] started from a dijet sample and then studied charged-hadron production inside either the leading jet (which is required to have $p_T^{\mathrm{jet}} > 100$ GeV) or the subleading jet (with $p_T^{\mathrm{jet}} > 40$ GeV). As such, these conditions are different from the single-inclusive jet situation we consider in this paper, and strictly speaking we cannot compare to the CMS data. On the other hand, it turns out that the CMS data for hadron production in the leading and the subleading jet are in remarkable agreement for $z_h \gtrsim 0.05$, when one normalizes each of them individually to the corresponding total (leading or subleading) jet event rate. This finding clearly indicates that fragmentation inside jets is really independent of the underlying event topology and happens in the same way in any jet. Therefore, the overall reservation notwithstanding, we show in Fig. 8 the comparison of the normalized one-jet rate differential in $\log(1/z_h)$ to the CMS data for hadron production in the leading jet. We show the theoretical curve down to $z_h \sim 0.1$. As one can see, the agreement with the data is very good in this regime. We have found that quark and gluon fragmentation contribute roughly in equal parts to the cross section.

We finally note that measurements of $pp \to (\mathrm{jet}\pi)X$ should readily be feasible at RHIC, especially in the STAR experiment where both inclusive jet [42] and pion cross sections [43,44] have been measured. Figure 9 shows our NLO predictions as functions of z_h for pp collisions at $\sqrt{S} = 200$ GeV and $\sqrt{S} = 510$ GeV. For the former, we have integrated the jet transverse momentum over $5\ \mathrm{GeV} < p_T^{\mathrm{jet}} < 40$ GeV, while for $\sqrt{S} = 510$ GeV we have used $10\ \mathrm{GeV} < p_T^{\mathrm{jet}} < 80$ GeV. In both cases we integrate over $|\eta| < 1$. The jet is defined by the anti-k_t

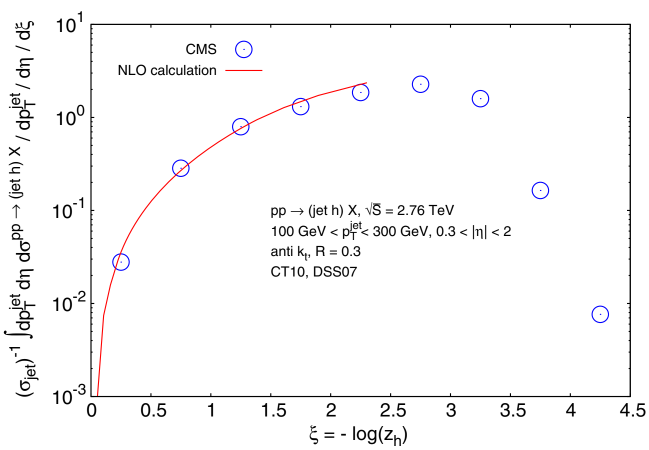

FIG. 8 (color online). NLO cross section for the $pp \to (\mathrm{jet}h)X$ differential in $\xi \equiv \log(1/z_h)$ at $\sqrt{S} = 2.76$ TeV, compared to the CMS data [9] for hadron production in the leading jet. The cross section is normalized to the total jet rate.

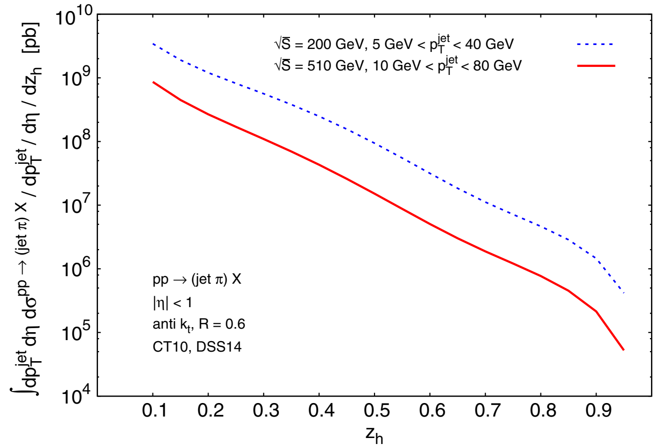

FIG. 9 (color online). NLO cross section for the $pp \to (\mathrm{jet}h)X$ differential in z_h, for STAR kinematics with $\sqrt{S} = 200$ GeV (dashed) and $\sqrt{S} = 510$ GeV (solid).

algorithm with $R = 0.6$. As before we use CT10 and DSS14 and set all scales equal to the jet transverse momentum.

IV. CONCLUSIONS AND OUTLOOK

We have considered the process $pp \to (\mathrm{jet}h)X$, for which a specific hadron is observed inside a fully reconstructed jet. Using the approximation of relatively narrow jets, we have performed an analytical next-to-leading-order calculation of the partonic cross sections for this process. We have found that the NLO partonic cross sections may be systematically formulated in terms of simple jet functions for the process. These functions are universal; that is, they only depend on the types of partons producing the jet and fragmenting into the observed hadron. We note that in the process of computing the jet functions we needed to perform subtractions of the final-state collinear singularities. These take the same form as the corresponding subtractions in single-inclusive hadron production (without a reconstructed jet). This demonstrates that the fragmentation functions are universal to NLO in the sense that the same functions appear in $pp \to (\mathrm{jet}h)X$ as in $pp \to hX$. Essentially, all effects of the fact that a jet is reconstructed along with the hadron factorize into a perturbatively computable factor, the jet function. The factorized structure in terms of jet functions we find at NLO suggests that this statement is true to all orders. Our finding is in line with the result of Ref. [3].

Our numerical results are in very good agreement with those obtained by Monte Carlo integration techniques in Ref. [4]. We have presented phenomenological results for the NLO cross section for the kinematics relevant for forthcoming measurements at ALICE and for previous ones by ATLAS and CMS. These results show that $pp \to (\mathrm{jet}h)X$ should enable very sensitive probes of fragmentation functions. In particular, the cross section differential in z_h probes the fragmentation functions almost "locally" at the momentum fraction z_h. The combination of fragmentation functions that is probed depends on the mix of initial-state parton distributions and hard-scattering functions that dominates. We find that, in contrast to the standard process $e^+e^- \to hX$ that is customarily used for extractions of fragmentation functions, the process $pp \to (\mathrm{jet}h)X$ should offer detailed insights into gluon fragmentation. Also, information at very large z_h might become accessible, although here it may become necessary to perform resummations of large logarithmic terms in the jet functions. We note that at high z_h typical particle multiplicities in the jet are very low, so that power corrections and nonperturbative phenomena will become important here as well. As has been discussed in Ref. [45], hadronization corrections to inclusive-jet production may exhibit a scaling with $1/R$, making them especially relevant in the case of rather narrow jets. Although these corrections are at the same time suppressed by an inverse power of transverse momentum, it will be an interesting and important task to investigate their structure in the case of the hadron-plus-jet observable where two separate transverse momenta are present.

There are various other possible extensions of our work that we hope to address in the future. As is well known, hadron production in jets has important applications in studies of spin phenomena in QCD in terms of the Collins effect [18], where the azimuthal distribution of a hadron around the jet axis is considered. Studies of the effect in pp scattering [15–17] will require a detailed theoretical understanding of the process, to which we hope we have contributed in this paper by computing the NLO corrections for the denominator of the spin asymmetry. We expect that our method based on jet functions is also applicable to the spin-dependent case. Finally, we mention that also photon fragmentation in jets could be interesting as a means to constrain the poorly known photon fragmentation functions (see Ref. [46] for related work on e^+e^- annihilation and ep scattering).

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APPENDIX A: DETAILS FOR JET FUNCTIONS IN THE SINGLE-INCLUSIVE CASE

In our results [Eq. (12)] for the single-inclusive jet functions we have the standard LO splitting functions

$$P_{qq}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right],$$
$$P_{gq}(z) = C_F\frac{1+(1-z)^2}{z},$$
$$P_{gg}(z) = 2C_A\frac{(1-z+z^2)^2}{z(1-z)_+} + \frac{\beta_0}{2}\delta(1-z),$$
$$P_{qg}(z) = \frac{1}{2}(z^2+(1-z)^2), \tag{A1}$$

with $\beta_0 = \frac{11}{3}C_A - \frac{2}{3}n_f$. The "plus" distribution is defined as usual by

$$\int_0^1 dz f(z)[g(z)]_+ \equiv \int_0^1 dz(f(z)-f(1))g(z). \tag{A2}$$

Dropping the δ-function contributions and ignoring the "plus" distributions, we obtain the splitting functions $P_{ij}^{<}(z)$ at $z<1$. For our calculations, we actually need these functions computed in dimensional regularization in $D = 4-2\varepsilon$ dimensions, where they are denoted as $\tilde{P}_{ij}^{<}(z)$. We have

$$\tilde{P}_{ij}^{<}(z) = P_{ij}^{<}(z) + \varepsilon P_{ij}^{(\varepsilon)}(z), \tag{A3}$$

with

$$P_{qq}^{(\varepsilon)}(z) = -C_F(1-z), \qquad P_{gq}^{(\varepsilon)}(z) = -C_F z,$$
$$P_{qg}^{(\varepsilon)}(z) = -z(1-z), \qquad P_{gg}^{(\varepsilon)}(z) = 0. \tag{A4}$$

We note in passing that the pieces in Eq. (12) that are independent of the jet algorithm may be constructed following a simple rule: each of the jet functions $\mathcal{J}_c$ contains the combination

$$-\frac{\alpha_s}{2\pi}\sum_i [P_{ij}^{<}(z)\log(\lambda^2(1-z)^2) - P_{ij}^{(\epsilon)}(z)], \tag{A5}$$

up to regularization by distributions at $z=1$.

The algorithm-dependent terms I_q^{algo} and I_g^{algo} in Eq. (12) may be determined from the calculations presented in Refs. [22,23]. For cone algorithms we have

$$I_q^{\mathrm{cone}} = C_F\left(-\frac{7}{2} + \frac{\pi^2}{3} - 3\log 2\right),$$
$$I_g^{\mathrm{cone}} = C_A\left(-\frac{137}{36} + \frac{\pi^2}{3} - \frac{11}{3}\log 2\right) + \frac{n_f}{2}\left(\frac{23}{18} + \frac{4}{3}\log 2\right), \tag{A6}$$

while for the (anti-)k_t algorithms

$$I_q^{k_t} = C_F\left(-\frac{13}{2} + \frac{2\pi^2}{3}\right),$$
$$I_g^{k_t} = C_A\left(-\frac{67}{9} + \frac{2\pi^2}{3}\right) + \frac{23}{18}n_f. \tag{A7}$$

Finally, for the "J_{E_T}" algorithm

$$I_q^{J_{E_T}} = C_F\left(-5 + \frac{\pi^2}{2} - \frac{3}{2}\log 2\right),$$
$$I_g^{J_{E_T}} = C_A\left(-\frac{45}{8} + \frac{\pi^2}{2} - \frac{11}{6}\log 2\right) + \frac{n_f}{2}\left(\frac{23}{12} + \frac{2}{3}\log 2\right). \tag{A8}$$

Interestingly, we find

$$I_j^{J_{E_T}} = \frac{1}{2}(I_j^{\mathrm{cone}} + I_j^{k_t}). \tag{A9}$$

APPENDIX B: JET FUNCTIONS FOR THE SEMI-INCLUSIVE CASE

In addition to $\mathcal{K}_{q\to q}^{\mathrm{algo}}$ in Eq. (21) we have

$$\mathcal{K}_{q\to g}^{\mathrm{algo}}(z,z_p,\lambda,\kappa,\mu_R) = \frac{\alpha_s(\mu_R)}{2\pi}[-\delta(1-z_p)\{P_{gq}(z)\log(\lambda^2(1-z)^2) + C_F z\} + \delta(1-z)\{P_{gq}(z_p)\log(\kappa^2(1-z_p)^2) + C_F z_p + \mathcal{I}_{gq}^{\mathrm{algo}}(z_p)\}], \tag{B1}$$

$$\mathcal{K}_{g\to g}^{\mathrm{algo}}(z,z_p,\lambda,\kappa,\mu_R) = \delta(1-z)\delta(1-z_p) + \frac{\alpha_s(\mu_R)}{2\pi}\left[-\delta(1-z_p)\left\{\frac{4C_A(1-z+z^2)^2}{z}\left(\frac{\log(1-z)}{1-z}\right)_+ + P_{gg}(z)\log(\lambda^2)\right\} + \delta(1-z)\left\{\frac{4C_A(1-z_p+z_p^2)^2}{z_p}\left(\frac{\log(1-z_p)}{1-z_p}\right)_+ + P_{gg}(z_p)\log(\kappa^2) + \mathcal{I}_{gg}^{\mathrm{algo}}(z_p)\right\}\right], \tag{B2}$$

$$\mathcal{K}^{\rm algo}_{g\to q}(z, z_p, \lambda, \kappa, \mu_R) = \frac{\alpha_s(\mu_R)}{2\pi}[-\delta(1-z_p)\{P_{qg}(z)\log(\lambda^2(1-z)^2) + z(1-z)\} + \delta(1-z)\{P_{qg}(z_p)\log(\kappa^2(1-z_p)^2) + z_p(1-z_p) + \mathcal{I}^{\rm algo}_{qg}(z_p)\}], \tag{B3}$$

where as before $\lambda = \mathcal{R}p_T^{\rm jet}/\mu_F'$, $\kappa = \mathcal{R}p_T^{\rm jet}/\mu_F''$, and where the algorithm-dependent terms are

$$\mathcal{I}^{\rm algo}_{c'c}(z) = \begin{cases} 2P_{c'c}(z)\log\left(\frac{z}{1-z}\right)\Theta(1/2-z) & \text{cone algorithm,} \\ 2P_{c'c}(z)\log z & \text{(anti-)}k_t \text{ algorithm,} \\ P_{c'c}(z)\left[\log(z) + \log\left(\frac{z}{1-z}\right)\Theta(1/2-z)\right] & J_{E_T} \text{ algorithm.} \end{cases} \tag{B4}$$

We note that a closely related result for the cone algorithm was obtained in Ref. [3]. Again, similar to Eq. (A9), we have

$$\mathcal{I}^{J_{E_T}}_{c'c}(z) = \frac{1}{2}(\mathcal{I}^{\rm cone}_{c'c} + \mathcal{I}^{k_t}_{c'c}). \tag{B5}$$

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