# Studies of two-solar-mass hybrid stars within the framework of Dyson-Schwinger equations

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In this paper we introduce an equation of state (EOS) of quark matter within the framework of Dyson-Schwinger equations to study the structure of compact stars. The smooth crossover from hadronic matter to quark matter in the hybrid star is studied. We compare different strategies to obtain crossover EOSs and find a new way to construct two-solar-mass hybrid stars with even a relatively soft quark EOS, while earlier works showed that the quark EOS should be stiff enough to support a massive hybrid star.

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## I. INTRODUCTION

The equation of state (EOS) of quark matter is crucial for the study of quark stars and compact stars. Once the EOS is obtained, the radius and gravitational mass of a compact star with a given central mass density can be calculated by solving the Tolman-Oppenheimer-Volkoff (TOV) equations. By changing the central density, we can draw the mass-radius relationship curve that is compared with the astronomical observations. Thus, the astronomical observations can be used to constrain the parameter sets of the EOSs and even rule out some EOSs. However, on the one hand, the perturbative QCD method fails in the low-energy region, and it is very difficult to obtain a reliable EOS from the first principles of QCD. Some approximate methods that incorporate basic features of QCD are adopted to calculate the EOS of quark matter, such as the MIT bag model and the Nambu-Jona-Lasinio (NJL) model. But all these models have their own weaknesses. The MIT bag model violates chiral symmetry even in the limit of a massless quark. Thus, some authors have developed advanced bag models such as the so-called chiral MIT bag model [1]. This model restores the chiral symmetry by introducing the coupling of a pion and a quark. The NJL model assumes that the interaction between quarks is point-like, so this model is not renormalizable, and it cannot incorporate quark confinement [2]. On the other hand, the observational maximum mass of compact stars is increasing. For example, the PSR $J0348 + 0432$ was reported to have a mass of $2.01 \pm 0.04$ solar masses in 2013 [3]. Only very stiff EOSs can support such a large maximum mass. Nonetheless, the EOS is often softened when hyperon mixing, deconfined quark matter, or a kaon condensate is taken into consideration in the core of a compact star. Thus, Ozel even argued that condensates and unconfined quarks may not exist in the centers of neutron stars [4].

To solve these problems, people tried to find stiffer EOSs that can support massive quark stars and neutron stars. For instance, in a pure hadronic framework, the introduction of a "universal three-body force" acting on all the baryons or an effective Lagrangian with quartic terms in the meson fields can make hyperon-mixed neutron stars possible [5,6]. In the MIT bag model, some authors also introduced a density-dependent bag parameter to get stiff EOSs [7]. Besides, the smooth crossover from hadronic matter to quark matter has been recently used to construct hybrid stars with high mass. Certainly, the order of the hadron-quark phase transition at zero temperature is still an open problem and the first-order phase transition (Maxwell or Gibbs construction) has been widely adopted in some recent works [8,9], and the existence of the so-called "mass twins" in the mass-radius relationship for compact stars has been presented as potential evidence of a first-order phase transition [10]. However, we know that the radius of a compact star is very difficult to determine exactly, so it is not easy to find "mass twins," and there are also some arguments that the "mass twins" also exist in the smooth phase transition case [11]. Besides, lattice QCD (LQCD) studies show that the transition line for low net baryon is a crossover [12–15], but the results for low temperature and large baryon density are still model dependent. Many people suppose the existence of a critical end point (CEP), but others do not. Actually, on the theoretical side there is still an ambiguity, not only for the location of a CEP

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but also for the existence of a CEP [16]. For example the authors of Ref. [17] argued that if the vector interaction is strong enough, the transition is a crossover in the whole phase diagram. Furthermore, treating the point-like hadron as an independent degree of freedom in the Gibbs condition is not fully justified in the transition region because all hadrons are extended objects composed of quarks and gluons. The study of smooth phase transitions in hybrid stars is thus also necessary. The picture of a gradual onset of quark degrees of freedom in dense matter associated with the percolation of finite-size hadrons has been discussed in some seminal works such as Refs. [18,19]. Recently, the smooth crossover from hadronic matter to quark matter was used to construct massive hybrid stars [20–22]. In these works, based on the percolation picture, the authors utilized different interpolation functions to do some phenomenological studies on hybrid stars compatible with two solar mass. Obviously, the strategies of interpolation are different, but in each paper the authors concentrated on a certain interpolation strategy.

The aim of this paper is to find a proper EOS of quark matter to calculate the structure of compact stars and discuss the possibility of a two-solar-mass hybrid star based on the smooth phase transition between hadronic matter and quark matter. So, we perform a model study on the hybrid star from the point of view of the smooth crossover. For the quark phase, we adopt the EOS with strangeness based on the Dyson-Swinger equations (DSEs), because the DSEs can simultaneously address both confinement and dynamical chiral symmetry breaking [23,24], and it has been applied successfully to hadron physics [25–30]. We discuss different interpolation strategies to construct hybrid EOSs and compare their influence on the final results, that is to say, the maximum mass of the hybrid stars. We find a new way to construct an EOS that is sufficiently stiff to include stable hybrid stars compatible with two solar masses and our conclusion is an excellent complement to the conclusion in Ref. [20].

This paper is organized as follows. In Sec. II, we introduce the EOS of quark matter. In Sec. III, we use several methods to try to construct hybrid EOSs from the point of view of the smooth crossover phase transition. A new way to construct massive hybrid stars even with a soft EOS of quark matter is proposed. Finally, a brief discussion and conclusion is presented in Sec. V.

## II. THE EOS OF QUARK MATTER AND THE DSEs

In this section, we briefly introduce our EOS of quark matter based on the DSE approach. According to Ref. [31], we start from the zero-temperature and zero-quark-chemical-potential version of the quark propagator DSE which reads

$$S(p)^{-1} = Z_2(i\not{p} + Z_m m) + g^2 Z_{1F} \int_q \frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma^a_\nu(p,q) D_{\mu\nu}(p-q), \quad (1)$$

where $S(p)$ is the dressed quark propagator, $Z_2$ is the field-strength renormalization constant, $Z_m$ is the mass renormalization constant with current quark mass $m$, $g$ is the coupling constant, $Z_{1F}$ is the quark-gluon-vertex renormalization constant, $\int_q = \int \frac{d^4q}{(2\pi)^4}$ is a symbol that represents a Poincaré-invariant regularization of the four-dimensional Euclidean integral, $\lambda^a$ are the Gell-Mann matrices, $\Gamma^a_\nu(p,q)$ is the quark-gluon vertex, and $D_{\mu\nu}(p-q)$ is the dressed gluon propagator. The general form of the quark propagator at zero temperature and zero chemical potential reads

$$S(p)^{-1} = i\not{p}A(p^2) + B(p^2), \quad (2)$$

where $A(p^2)$ and $B(p^2)$ are scalar functions of $p^2$. The renormalization condition is

$$A(\zeta^2) = 1, \quad (3)$$

$$B(\zeta^2) = m \quad (4)$$

at sufficiently large space-like $\zeta^2$ [32,33]. We choose the renormalization point to be $\zeta = 19$ GeV, as used in Refs. [32,33]. Another popular choice is $\xi = 2$ GeV which has been used in recent studies (see, e.g., Ref. [34]). Actually, DSEs make up a renormalizable theory, so the choices of $\zeta$ do not affect the physical results. Equation (1) is the exact result from the first principles of QCD, but we cannot solve it directly unless concrete truncations are performed. Rainbow truncation is used in this work, which means a bare vertex is adopted,

$$\Gamma^a_\nu(p,q) = \frac{\lambda^a}{2}\gamma_\nu, \quad (5)$$

and the Qin-Chang gluon propagator model [35] is specified by a choice of the effective interaction in Landau gauge. Finally, one can obtain the following equations for the two dressing functions $A(p^2)$ and $B(p^2)$:

$$A(p^2) = Z_2 + \frac{4}{3p^2}\int_q \frac{\mathcal{G}(k^2)}{k^2}\frac{A(q^2)}{q^2A^2(q^2)+B^2(q^2)} \times \left(p\cdot q + 2\frac{(k\cdot p)(k\cdot q)}{k^2}\right), \quad (6)$$

$$B(p^2) = Z_4 m + 4\int_q \frac{\mathcal{G}(k^2)}{k^2}\frac{B(q^2)}{q^2A^2(q^2)+B^2(q^2)}. \quad (7)$$

The coupled Eqs. (6) and (7) can then be solved by direct iteration. The DSE solution for the dressed quark propagator can be well fitted with three pairs of complex-conjugate poles with the representation [36]

$$S(p) = \sum_{n=1}^{3}\left(\frac{z_n}{i\not{p}+m_n}+\frac{z_n^*}{i\not{p}+m_n^*}\right). \tag{8}$$

We fit $u$, $d$, and $s$ quarks, where the requirement that the quark propagator in the ultraviolet region should tend to the free-quark propagator is employed:

$$\sum_{k=1}^{3}(z_k+z_k^*) = 1. \tag{9}$$

In this paper, we choose a new parameter set that not only fits the propagator well but also leads to a stiff EOS. The corresponding parameters for $u$ and $d$ quarks are

$$\begin{aligned} m_1 &= 285+121i\ \text{MeV}, & z_1 &= 0.308+0.618i,\\ m_2 &= -1120+59i\ \text{MeV}, & z_2 &= 0.111-0.193i,\\ m_3 &= 1214+429i\ \text{MeV}, & z_3 &= 0.081, \end{aligned} \tag{10}$$

and for the $s$ quark,

$$\begin{aligned} m_1 &= 509+236i\ \text{MeV}, & z_1 &= 0.327+0.449i,\\ m_2 &= -1166+616i\ \text{MeV}, & z_2 &= 0.101+0.00178i,\\ m_3 &= 1572+660i\ \text{MeV}, & z_3 &= 0.072+0.017i. \end{aligned} \tag{11}$$

If we compare these parameters with those in Ref. [31], we will find that the smaller real parts of $m_1$ and $z_1$ make the EOS stiffer. Then, the propagator at zero temperature and zero chemical potential can be generalized to nonzero temperature and chemical potential by the following replacement [37]:

$$p^4 \to \tilde{\omega}_n = (2n+1)\pi \mathrm{T} + \mathrm{i}\mu, \tag{12}$$

which is widely used within the rainbow truncation of DSEs in thermal and dense QCD, and $\tilde{\omega}_n$ are the Matsubara frequencies. Hence, the quark propagator at nonzero $T$ and $\mu$ is

$$S(\vec{p},\tilde{\omega}_n) = \sum_{k=1}^{3}\left(\frac{z_k}{i\not{p}+\mathrm{i}\gamma_4\tilde{\omega}_n+m_k}+\frac{z_k^*}{i\not{p}+\mathrm{i}\gamma_4\tilde{\omega}_n+m_k^*}\right). \tag{13}$$

Then, the well-known formula for the quark number density is [37,38]

$$\rho(\mu,T) = -N_cN_fT\sum_{n=-\infty}^{+\infty}\int\frac{d^3\vec{p}}{(2\pi)^3}\mathrm{tr}[S(\vec{p},\tilde{\omega}_n)\gamma_4]. \tag{14}$$

After a series of calculations and taking the limit $T\to 0$, the final result for the quark number density at zero temperature can be obtained:

$$\rho(\mu,T=0) = \frac{N_cN_f}{3\pi^2}\sum_{k=1}^{3}(z_k+z_k^*)\theta(\mu-\mu_k^0) \times\left(\mu^2-\frac{d_k^2}{4\mu^2}-c_k\right)^{\frac{3}{2}}, \tag{15}$$

where $\mu_k^0 = |\mathrm{Re}(m_k)|$ and $c_k, d_k$ are defined by $m_k^2 = c_k + d_k i$. The quark number density dependence on $\mu$ at $T=0$ for the $s$ quark is displayed in Fig. 1, and from this figure we can see that the quark number density of the $s$ quark becomes nonzero at $\mu_c = 520$ MeV, which is smaller than in Ref. [31] because of our new parameter set. So, this EOS is more appropriate for quark matter with strangeness. Finally, we take the chemical equilibrium and electric charge neutrality conditions into consideration to constrain different quark chemical potentials. The conditions read

$$\mu_d = \mu_u + \mu_e, \tag{16}$$

$$\mu_s = \mu_u + \mu_e, \tag{17}$$

$$\frac{2}{3}\rho_u - \frac{1}{3}\rho_d - \frac{1}{3}\rho_s - \rho_e = 0. \tag{18}$$

Then there is only one independent chemical potential due to these constraints. We choose $\mu_u$ in this work. For a definite quark chemical potential, the EOS of QCD at $T=0$ reads [38,39]

$$P(\mu) = P(\mu=0) + \int_0^{\mu}\rho(\mu')d\mu', \tag{19}$$

and the relation between the energy density and the pressure of the corresponding system is [40,41]

$$\epsilon = -P + \sum_i \mu_i\rho_i, \tag{20}$$

where $\mu_i$ and $\rho_i$ represent the chemical potential and the particle number density for each component in the system.

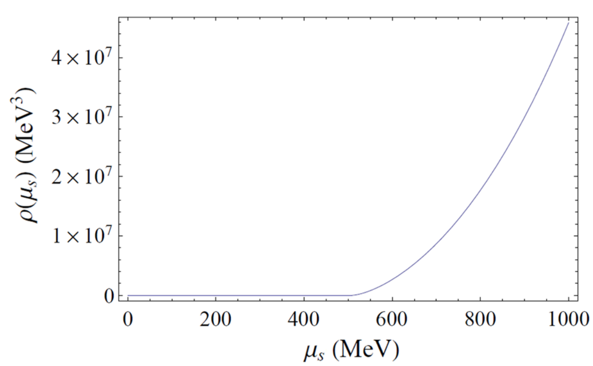


FIG. 1 (color online). The quark number density of the $s$ quark.

## III. THE SMOOTH CROSSOVER AND THE STRUCTURE OF HYBRID STARS

As usual, one can obtain the structure of a bare quark star by directly integrating the TOV equations with the equation of state:

$$\frac{dP(r)}{dr} = -\frac{G(\varepsilon + P)(M + 4\pi r^3 P)}{r(r - 2GM)},$$
$$\frac{dM(r)}{dr} = 4\pi r^2 \varepsilon. \tag{21}$$

However, in Eq. (19) we can see that $P(\mu = 0)$ is only a $\mu$-independent constant. It is the pressure of the vacuum. Although a method to calculate $P(\mu = 0)$ self-consistently is given in Ref. [37], there is actually no model-independent way to reliably calculate $P(\mu = 0)$ from the first principles of QCD. In this paper, analogous to the MIT bag model, we reconsider the effect of this term and assume that there is negative pressure at zero chemical potential in the vacuum which manifests the confinement of QCD. Namely, we identify $P(\mu = 0)$ with $-B$, where $B$ is the vacuum bag constant. In this paper we take it as a phenomenological parameter only. Our EOS for nuclear matter is the APR EOS with the $A18 + \delta v + UIX^*$ interaction from Ref. [42]. This EOS is based on the Argonne v18 two-body potential and the Urbana IX three-body interaction, which includes charge neutrality and beta equilibrium. The $\delta v$ indicates the inclusion of relativistic corrections. For simplicity, the APR model includes only nucleonic degrees of freedom, and does not take hyperons into account, whose interactions with nucleons and among themselves are not well determined. Then, we choose $B = (110\ \mathrm{MeV})^4$ to ensure that the energy density of the quark matter should always be higher than that of hadronic matter in the low-density region. But with this value of $B$, the maximum mass of a pure quark star is just about 1.65 solar masses. If we choose a smaller $B$, the EOS will become stiffer and the maximum mass will become larger, but such a small $B$ is unreasonable. Thus, we simply try to find a way to construct massive hybrid stars from the point of view of a smooth crossover phase transition with a soft EOS for quark matter. In order to facilitate analysis and compare with the first-order phase transition, the pressure as a function of the baryon chemical potential of both the quark matter and the hadronic matter is shown in Fig. 2. From Fig. 2 we can see that the intersection in the $P - \mu_B$ plane is just the first-order transition point. However, the corresponding baryon number density at this point is over 5 times that of normal nuclear matter ($\rho_0 = 0.17\ \mathrm{fm}^{-3}$). The hadronic EOS is usually not reliable at such a high density. Besides, the system must be strongly interacting in the transition region, so that it can be described by neither an extrapolation of the hadronic EOS from the low-density side nor an extrapolation of the quark EOS from the high-density side [43]. We should point that this is not a special result in our hadronic and quark EOS, because experiments show that even in a relatively large region of chemical potential the quark-gluon plasma is still strongly interacting, so the pressure of the quark EOS tends slowly to the free-quark gas in the $P - \mu_B$ plane. This means that the pressure increases slowly in the $P - \mu_B$ plane, and the intersection between quark matter and hadronic matter usually occurs at high chemical potential and high number density if we choose a modern model to calculate the EOS for quark matter rather than the MIT bag or a perturbative model. Now, we start to construct an EOS for hybrid stars from the point of view of the smooth crossover. From Refs. [20–22] we can see that the strategy is not unique, but it should meet the requirements of physics. As in the model of Ref. [20]—the so-called "three window model"—we construct the hybrid EOS from the $P - \rho_B$ plane. The pressure as a function of the baryon number density is shown in the Fig. 3. The shaded region is the possible crossover region that is between $2\rho_0$ and $4\rho_0$. We adopt the interpolation function from Ref. [20] to make a smooth connection between the two EOSs in the $P - \rho_B$ plane:

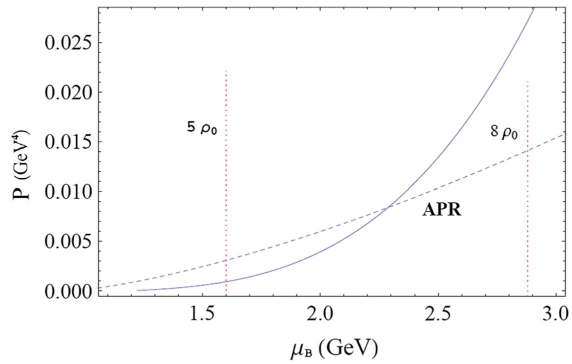


FIG. 2 (color online). Pressure as a function of baryon chemical potential. The dashed line is the EOS of hadronic matter, and the solid line represents the EOS of quark matter. The vertical dashed lines show the corresponding baryon number density of hadronic matter. The baryon number density is scaled by the baryon number density of normal nuclear matter $\rho_0 = 0.17\ \mathrm{fm}^{-3}$.

$$P = P_H \times f_- + P_Q \times f_+, \tag{22}$$

$$f_\pm = \frac{1}{2}\left(1 \pm \tanh\left(\frac{\rho - \bar{\rho}}{\Gamma}\right)\right), \tag{23}$$

where $P_H$ and $P_Q$ are the pressures in the hadronic matter and the quark matter, respectively. The window $\bar{\rho} - \Gamma \lesssim \rho \lesssim \bar{\rho} + \Gamma$ characterizes the crossover region in which both hadrons and quarks are strongly interacting, so that neither a pure hadronic EOS nor a pure quark EOS is reliable. From the thermodynamical relation $P = \rho^2 \partial(\epsilon/\rho)/\partial\rho$, we obtain

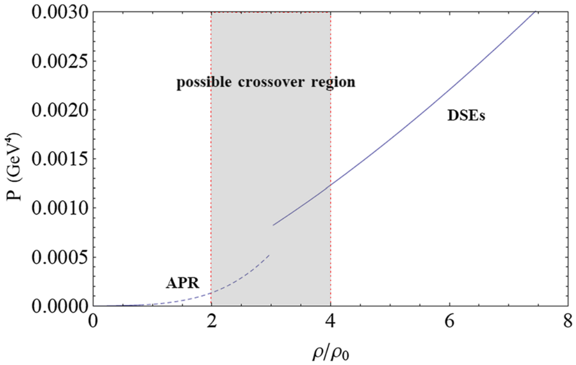


FIG. 3 (color online). The pressure as a function of the baryon number density. The shaded region is a possible crossover region, the solid line is the EOS of quark matter, and the dashed line is the APR EOS. The $x$ axis is the baryon number density scaled by the nuclear number density and the $y$ axis is the pressure.

$$\epsilon(\rho) = \epsilon_H(\rho)f_-(\rho) + \epsilon_Q(\rho)f_+(\rho) + \Delta\epsilon, \qquad (24)$$

$$\Delta\epsilon = \rho \int_{\bar{\rho}}^{\rho} (\epsilon_H(\rho') - \epsilon_Q(\rho')) \frac{g(\rho')}{\rho'} d\rho', \qquad (25)$$

with $g(\rho) = \frac{2}{\Gamma}(e^X + e^{-X})^{-2}$ and $X = \frac{\rho - \bar{\rho}}{\Gamma}$. Here $\epsilon_H(\epsilon_Q)$ is the energy density obtained from the hadronic EOS and quark EOS. $\Delta\epsilon$ is an extra term which guarantees thermodynamic consistency. Correspondingly, if we start from the energy density and deduce the pressure (this is called $\epsilon$ interpolation in Ref. [20]), there will be a $\Delta P$ term in the expression of the pressure. But we find that the $\Delta P$ term often leads to a fluctuation in the hybrid EOS and makes the sound velocity ($v_s = \sqrt{\frac{dP}{d\epsilon}}$) become larger than that of light. This is obviously unreasonable, so we do not try this. The final result for this hybrid EOS is shown in Fig. 4, compared with the quark EOS and hadronic EOS. From Fig. 4 and the relation between the energy density and pressure, we can evaluate the stiffness of the EOS easily. The APR EOS has a well-known stiff tail in the large-density region. So, in Fig. 4 we can see that when the pressure and energy density are large, the slope of the APR EOS becomes much lower than that of the DSE EOS, while in the low-density region the quark EOS is often stiffer than the hadronic EOS. The hybrid EOS based on a first-order phase transition is equal to the hadronic EOS in the low-density region, and in the large-density region it is equal to the quark EOS. So, the hybrid EOS based on a first-order phase transition is often softer than both the hadronic EOS and the quark EOS. However, if we construct the hybrid EOS from a smooth crossover, the EOS will be influenced by the quark EOS in the low-density region and by the hadronic EOS in the large-density region. So, the result will be different. Unfortunately, our hybrid EOS is still very soft if we utilize this interpolation strategy, because our quark EOS is soft. This is in accordance with the conclusion in Ref. [20] that the maximum mass of hybrid stars can exceed two solar masses only if the quark matter has a stiff equation of state and the crossover takes place at around 3 times the normal nuclear matter density. However, we find another way to construct two-solar-mass hybrid stars with a soft quark EOS. We apply the same interpolation function to the $P - \mu_B$ plane and then calculate the energy density from thermodynamical relations. The pressure, energy density, and EOS are shown in Figs. 5, 6, and 7. The chemical potential in the crossover region is between 1.4 and 3.4 GeV because the first-order phase transition point is around 2.4 GeV. In order to quantify the stiffness of the EOSs, the sound velocities are shown in Figs. 8 and 9. A comparison can be found in Ref. [20]. The EOS obtained by the interpolation from the $P - \rho_B$ plane is soft, while the EOS obtained by the interpolation from the $P - \mu_B$ plane is much stiffer. From Fig. 8 we can see that the sound velocity is always smaller than 0.4 times of the velocity of light, while the sound velocity in Fig. 9 is much larger.

In the $P - \mu_B$ plane, we can clearly find the difference between the phase transition of a smooth crossover and the Maxwell construction. Although the pressure tends to that of quark matter in the high-chemical-potential region and tends to that of hadronic matter in the low-chemical-potential

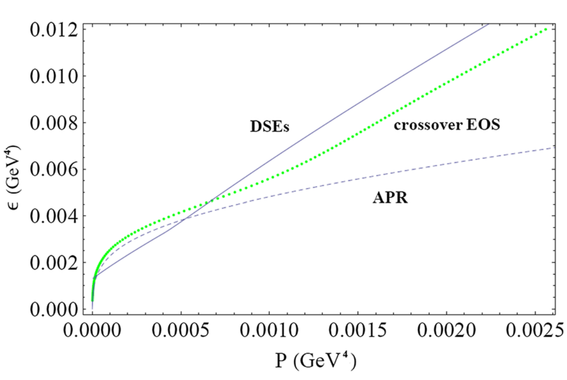


FIG. 4 (color online). The interpolation EOS (green dotted line) compared with the quark EOS (solid line) and hadronic EOS (dashed line).

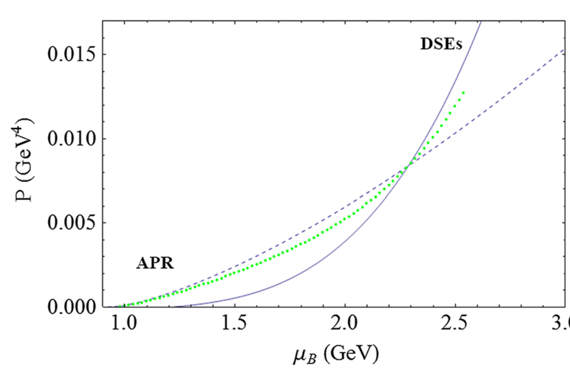


FIG. 5 (color online). The pressure as a function of the baryon chemical potential. The green dotted line shows the pressure of the interpolation EOS, the solid line is the pressure of the quark matter, and the pressure of the hadronic matter is the dashed line.

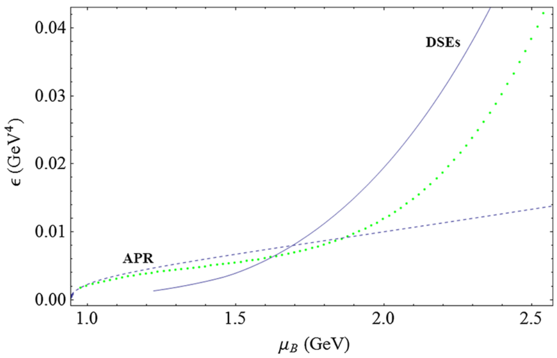


FIG. 6 (color online). The energy density as a function of the baryon chemical potential. The green dotted line shows the energy density of the interpolation EOS, the solid line is the energy density of the quark matter, and the energy density of the hadronic matter is the dashed line.

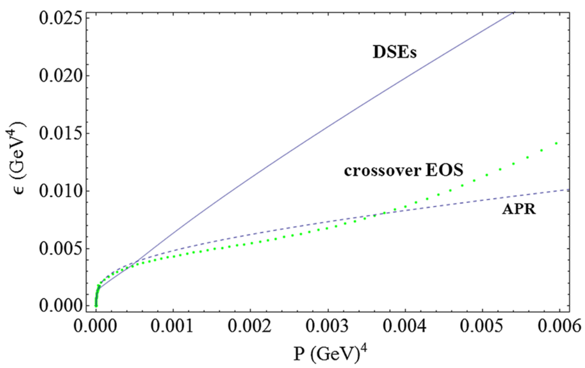


FIG. 7 (color online). The interpolation EOS (green dotted line) compared with the quark EOS (solid line) and hadronic EOS (dashed line).

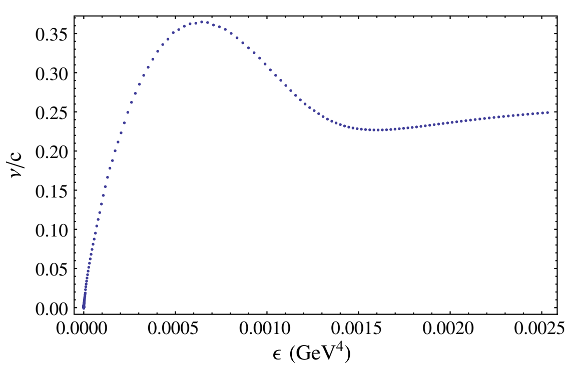


FIG. 8 (color online). The sound velocity of the hybrid EOS obtained by the interpolation from the $P-\rho_B$ plane.

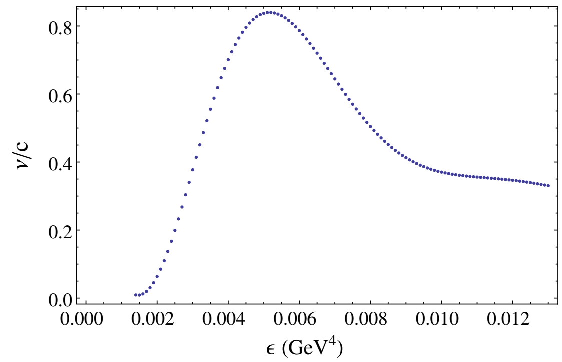


FIG. 9 (color online). The sound velocity of the hybrid EOS obtained by the interpolation from the $P-\mu_B$ plane.

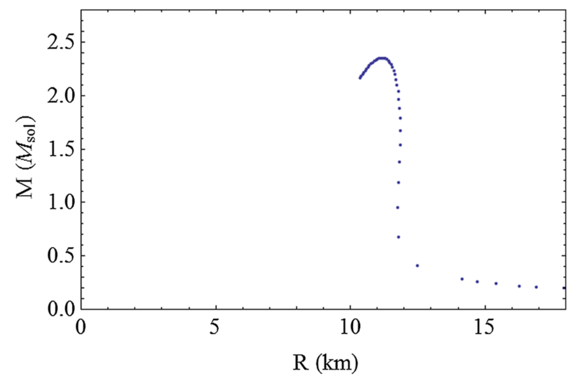


FIG. 10 (color online). The mass-radius relation of the hybrid stars based on the smooth crossover EOS. The maximum mass is over two solar masses.

region, it is different from both quark matter and hadronic matter in the crossover region. Finally, by integrating the TOV equations with the EOS, we get the mass-radius relation of the hybrid stars. This is shown in the Fig. 10. The maximum mass is 2.35 times the solar mass. There are also other interpolation functions to make the interpolation EOS. For example, a polynomial function was used in Ref. [22]. But we find that different interpolation functions cannot make an appreciable difference. If we adopt the polynomial function to construct the EOS, the difference of the maximum mass is less than 5 percent. This result is apprehensible because the interpolation functions should be smooth at the boundaries of the interpolating interval. Thus, the values of different interpolation functions in the crossover region do not differ widely.

## IV. DISCUSSION

In this paper we introduced our quark EOS with three flavors of quarks based on the framework of DSEs to calculate the structure of a hybrid star. For the hadronic phase we adopted the APR EOS and we calculated the mass-radius relationship of the hybrid stars from the point of view of the smooth crossover phase transition from hadronic matter to quark matter. The common belief is that the EOS for the quark phase should be very stiff in order to construct a two-solar-mass hybrid star. For example, in Ref. [20] the authors performed the interpolation in the $P-\rho$ plane and claimed that no matter what kind of hadronic EOS is adopted, the maximum mass of neutron stars can only exceed two solar masses if the crossover takes place at around 3 times the normal nuclear matter density and the quark matter is strongly interacting in the crossover region and has a stiff EOS. Nonetheless, we find that if we start from the $P-\mu$ plane, the interpolation function can generate a stiff EOS and finally construct a massive hybrid star compatible with two solar masses, even though our quark EOS is relatively soft.

For simplicity, the hyperons were not included because the interaction of hyperons with nucleons and among themselves are not well determined. The consideration of the hyperon will soften the EOS for hadronic matter, but in our result the maximum mass of the hybrid stars is larger than the maximum mass of both the pure quark stars and the pure hadronic stars. So, the extent of the influence of hyperons will be model dependent and needs further study.

## ACKNOWLEDGMENTS

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