On the description of nonunitary neutrino mixing

F. J. Escrihuela,[1,*] D. V. Forero,[2,†] O. G. Miranda,[3,‡] M. Tórtola,[1,§] and J. W. F. Valle[1,‖]

[1]*AHEP Group, Institut de Física Corpuscular—C.S.I.C./Universitat de València,*
Parc Cientific de Paterna, C/Catedratico José Beltrán, 2 E-46980 Paterna, València, Spain
[2]*Center for Neutrino Physics, Virginia Tech, Blacksburg, Virginia 24061, USA*
[3]*Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN*
Apdo. Postal 14-740 07000 Mexico, Distrito Federal, Mexico
(Received 11 May 2015; published 17 September 2015)

Neutrino oscillations are well established and the relevant parameters determined with good precision, except for the *CP* phase, in terms of a unitary lepton mixing matrix. Seesaw extensions of the Standard Model predict unitarity deviations due to the admixture of heavy isosinglet neutrinos. We provide a complete description of the unitarity and universality deviations in the light-neutrino sector. Neutrino oscillation experiments involving electron or muon neutrinos and antineutrinos are fully described in terms of just three new real parameters and a new *CP* phase, in addition to the ones describing oscillations with unitary mixing. Using this formalism we describe the implications of nonunitarity for neutrino oscillations and summarize the model-independent constraints on heavy-neutrino couplings that arise from current experiments.

I. INTRODUCTION

Neutrino masses, without which current neutrino oscillation data cannot be understood [1], are here to stay [2]. It has been long noted that small neutrino masses can arise from an effective lepton number violation dimension-five operator $\mathcal{O}_5 \propto LL\Phi\Phi$, which may arise from unknown physics beyond that of the $\mathrm{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y}$ model. Here L denotes one of the three lepton doublets and Φ is the Standard Model scalar doublet [3]. After electroweak symmetry breaking takes place through the nonzero vacuum expectation value (vev) $\langle\Phi\rangle$ such operator leads to Majorana neutrino masses. In contrast to the charged fermion masses, which arise directly from the coupling of the scalar Higgs, neutrino masses appear in second order in $\langle\Phi\rangle$ and imply lepton number violation by two units ($\Delta L = 2$) at some large scale. This fact accounts for the smallness of neutrino masses relative to those of the Standard Model charged fermions. This is all we can say from first principles about the operator $\mathcal{O}_5$ in Fig. 1. In general we have no clue on the *mechanism* giving rise to $\mathcal{O}_5$, nor its associated mass *scale*, nor the possible details of its *flavor structure*.

One may assume that $\mathcal{O}_5$ is induced at the tree level by the exchange of heavy "messenger" particles, whose mass lies at a scale associated to the violation of the global lepton number symmetry by new physics, beyond that of the $\mathrm{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y}$ model,

$$m_\nu = \lambda_0 \frac{\langle\Phi\rangle^2}{M_X},$$

where λ_0 is some unknown dimensionless constant. For example gravity, which in a sense "belongs" to the SM, could induce the dimension-five lepton number violation operator $\mathcal{O}_5$ [4,5]. In such a minimalistic scenario [6] the large scale M_X in the denominator is the Planck scale and hence the neutrino mass that results is too small to account for current neutrino oscillation data. Hence we need genuine "new physics" in order to generate neutrino masses this way.

Neutral heavy leptons (NHLs) arise naturally in several extensions of the Standard Model. Their possible role as messengers of neutrino mass generation constitutes one of their strongest motivations and a key ingredient of the type-I seesaw mechanism [7–11] in any of its variants. If realized at the Fermi scale [12–20], it is likely that the
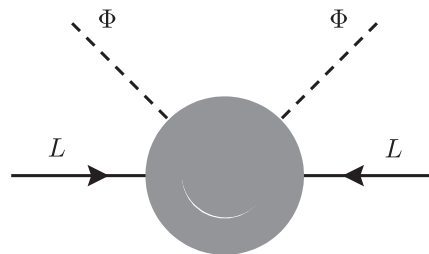
*[*]franesfe@alumni.uv.es
[†]dvanegas@vt.edu
[‡]omr@fis.cinvestav.mx
[§]mariam@ific.uv.es
[‖]valle@ific.uv.es

FIG. 1.   Dimension-five operator responsible for neutrino mass.

"seesaw messengers" responsible for inducing neutrino masses would lead to a variety of phenomenological implications. These depend on the assumed gauge structure. Here for definiteness and simplicity, we take the minimal $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ structure which is well tested experimentally. In this case one can have, for example,

(1) Light isosinglet leptons are usually called "sterile." If they lie in the eV range they could help accommodate current neutrino oscillation anomalies [21,22] by taking part in the oscillations. Sterile neutrinos at or above the keV range might show as distortions in weak decay spectra [23] and be relevant for cosmology [24].

(2) Heavy isosinglet leptons below the Z mass could have been seen at LEP I [25–27]. Likewise, TeV NHLs might be seen in the current LHC experiment, though in the latter case rates are not expected to be large in the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ theory.

(3) Whenever NHLs are too heavy to be emitted in weak decay processes, the corresponding decay rates would decrease, leading to universality violation [28].

(4) The admixture of NHLs in the charged current weak interaction would affect neutrino oscillations, since they would not take part in oscillations. These would be effectively described by a nonunitary mixing matrix [29].

(5) Majorana-type NHLs would modify rates for lepton number violation processes such as neutrinoless double beta ($0\nu\beta\beta$) decays through long-range contributions (mass mechanism), as well as induce short-range contributions [30–32].

(6) NHLs would induce charged lepton flavor violation processes [29,33]. However the corresponding restrictions depend on very model-dependent rates.

In what follows we consider the generic structure of the lepton mixing matrix relaxing the unitarity approximation.[1] We show that their most general form is factorizable, so that current experiments involving only electron and muon neutrinos or antineutrinos can be effectively described in terms of just three new real parameters and one new *CP* violation phase. We illustrate how these parameters affect oscillations and discuss the main restrictions on such a generalized mixing structure that follow from universality tests. For logical completeness we also present a brief compilation of various model-independent constraints on NHL mixing parameters within the same parametrization, including those that follow from the possibility of direct NHL production at high energy accelerator experiments.

II. THE FORMALISM

Isosinglet neutral heavy leptons couple in the weak charged current through mixing with the standard isodoublet neutrinos. The most general structure of this mixing matrix has been given in the symmetric parametrization in Ref. [8]. Here we consider an equivalent presentation of the lepton mixing matrix which manifestly factorizes the parameters associated to the heavy leptons from those describing oscillations of the light neutrinos within the unitarity approximation. Here we present its main features; details are given in the Appendix.[2]

For the case of three light neutrinos and $n-3$ neutral heavy leptons, one can break up the matrix $U^{n \times n}$ describing the diagonalization of the neutral mass matrix as [35]

$$U^{n \times n} = \begin{pmatrix} N & S \\ V & T \end{pmatrix}, \qquad (1)$$

where N is a 3×3 matrix in the light-neutrino sector, while S describes the coupling parameters of the extra isosinglet states, expected to be heavy (for a perturbative expansion for $U^{n \times n}$ see [9]). As shown in the Appendix, the matrix N can be expressed most conveniently[3] as

$$N = N^{NP}U = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U, \qquad (2)$$

where U is the usual unitary form of the 3×3 leptonic mixing matrix probed in neutrino oscillation studie[4] corrected by the left triangle prefactor matrix, N^{NP}, characterizing unitarity violation.

Note that Eq. (2) provides a most convenient, general and complete description of the propagation of solar, atmospheric and terrestrial neutrinos from reactors, radioactive sources and accelerators beams, relaxing the unitarity approximation. Due to the zeros in the first two rows of the prefactor matrix in Eq. (2) it is clear that there are only four extra parameters beyond those characterizing unitary mixing: the two real parameters α_{11} and α_{22} plus the complex parameter α_{21} which contains a single *CP* phase.

---

[1]In Secs. II–VI we mainly consider isosinglet neutrinos above 100 GeV or so, which are hence too heavy to take part in oscillations or low energy weak decay processes.

[2]We consider stable neutrinos; neutrino decays were discussed, for instance, in Ref. [34].

[3]There are other forms for the light-neutrino mixing matrix, where the prefactor off-diagonal zeros are located at different entries. However Eq. (2) is the most convenient to describe current neutrino experiments.

[4]As discussed in Ref. [36], this may, for example, be parametrized in the original symmetric way or equivalently as prescribed by the Particle Data Group.

Indeed the existence [37] and possible effects [38] of extra CP phases associated to the admixture of NHLs in the charged leptonic weak interaction have already been noted in an earlier paper [8]. The new point here is that, despite the proliferation of phase parameters, only one combination enters the "relevant" neutrino oscillation experiments. This holds irrespective of the number of extra heavy isosinglet neutrino states present. Other studies, such as [39–41], appear as particular cases with a fixed number of extra heavy isosinglet neutrino states, any of which can be expressed in terms of the same set of parameters α_{ij}. Similarly, the matrix U may be expressed in different ways, such as in particle data group form or in our fully symmetric description, which is particularly useful for

phenomenological analyses. The diagonal elements, α_{ii}, are real and expressed in a simple way as

$$
\begin{aligned}
\alpha_{11} &= c_{1n}c_{1n-1}c_{1n-2}\ldots c_{14}, \\
\alpha_{22} &= c_{2n}c_{2n-1}c_{2n-2}\ldots c_{24}, \\
\alpha_{33} &= c_{3n}c_{3n-1}c_{3n-2}\ldots c_{34},
\end{aligned}
\tag{3}
$$

in terms of the cosines of the mixing parameters [8], $c_{ij} = \cos\theta_{ij}$.

Now the off-diagonal terms α_{21} and α_{32} are expressed as a sum of $n-3$ terms

$$
\begin{aligned}
\alpha_{21} &= c_{2n}c_{2n-1}\ldots c_{25}\eta_{24}\bar{\eta}_{14} + c_{2n}\ldots c_{26}\eta_{25}\bar{\eta}_{15}c_{14} + \cdots + \eta_{2n}\bar{\eta}_{1n}c_{1n-1}c_{1n-2}\ldots c_{14}, \\
\alpha_{32} &= c_{3n}c_{3n-1}\ldots c_{35}\eta_{34}\bar{\eta}_{24} + c_{3n}\ldots c_{36}\eta_{35}\bar{\eta}_{25}c_{24} + \cdots + \eta_{3n}\bar{\eta}_{2n}c_{2n-1}c_{2n-2}\ldots c_{24},
\end{aligned}
\tag{4}
$$

where $\eta_{ij} = e^{-i\phi_{ij}}\sin\theta_{ij}$ and its conjugate $\bar{\eta}_{ij} = -e^{i\phi_{ij}}\sin\theta_{ij}$ contain all of the CP violating phases. Finally, by neglecting quartic terms in $\sin\theta_{ij}$, with $j = 4, 5, \ldots$ one finds a similar expression for α_{31},

$$
\begin{aligned}
\alpha_{31} &= c_{3n}c_{3n-1}\ldots c_{35}\eta_{34}c_{24}\bar{\eta}_{14} + c_{3n}\ldots c_{36}\eta_{35}c_{25}\bar{\eta}_{15}c_{14} + \cdots \\
&\quad + \eta_{3n}c_{2n}\bar{\eta}_{1n}c_{1n-1}c_{1n-2}\ldots c_{14}.
\end{aligned}
\tag{5}
$$

In summary, by choosing a convenient ordering for the products of the complex rotation matrices ω_{ij} (see the Appendix), one obtains a parametrization that separates all the information relative to the additional leptons in a simple and compact form, containing three zeros. We will now concentrate on this specific parametrization.

III. NONUNITARY NEUTRINO MIXING MATRIX

Given the above considerations and the chiral nature of the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ model, we notice that the couplings of the n neutrino states in the charged current weak interaction can be described by a rectangular matrix [8]

$$
K = (N \quad S),
\tag{6}
$$

with N a 3×3 matrix described by Eq. (2) and S a $3 \times (n-3)$ matrix. This can be parametrized in the symmetric form or as prescribed in the Particle Data Group. The relative pros and cons of the two presentations are considered in Ref. [36].

The presence of extra heavy fermions that mix with the active light neutrinos would imply the effective nonunitarity of the 3×3 light-neutrino mixing matrix, hence modifying several SM observables. For example, note that the unitarity condition will take the form

$$
KK^{\dagger} = NN^{\dagger} + SS^{\dagger} = I,
\tag{7}
$$

with

$$
NN^{\dagger} = \begin{pmatrix}
\alpha_{11}^2 & \alpha_{11}\alpha_{21}^* & \alpha_{11}\alpha_{31}^* \\
\alpha_{11}\alpha_{21} & \alpha_{22}^2 + |\alpha_{21}|^2 & \alpha_{22}\alpha_{32}^* + \alpha_{21}\alpha_{31}^* \\
\alpha_{11}\alpha_{31} & \alpha_{22}\alpha_{32} + \alpha_{31}\alpha_{21}^* & \alpha_{33}^2 + |\alpha_{31}|^2 + |\alpha_{32}|^2
\end{pmatrix}.
\tag{8}
$$

We will show that, with the parametrization discussed here, one can, at least in principle, introduce all of the information of the extra $n-3$ states into the α_{ij} parameters in a simple compact form. The method is completely general and includes all the relevant CP phases. In what follows we will consider different direct or indirect tests of the existence of the extra heavy fermions, expressing the relevant observables in terms of these parameters, in order to derive the relevant constraints.

IV. UNIVERSALITY CONSTRAINTS

First one notes that if, as generally expected due to their gauge singlet nature, the heavy leptons cannot be kinematically emitted in various weak processes such as muon or beta decays, these decays will be characterized by different effective Fermi constants, hence breaking universality. One can now apply the above formalism in order to describe the various weak processes and to derive the corresponding experimental sensitivities. We first discuss the universality constraint, already reported in the literature [28,42–49], in order to cast it within the above formalism. Comparing muon and beta decays one finds

$$G_\mu = G_F \sqrt{(NN^\dagger)_{11}(NN^\dagger)_{22}} = G_F \sqrt{\alpha_{11}^2(\alpha_{22}^2 + |\alpha_{21}|^2)}, \quad (9)$$

and

$$G_\beta = G_F \sqrt{(NN^\dagger)_{11}} = G_F \sqrt{\alpha_{11}^2}. \quad (10)$$

Therefore, all the observables related to the Fermi constant will be affected by this change, for instance, the quark Cabibbo-Kobayashi-Maskawa (CKM) matrix elements [42]. In particular, the CKM matrix elements V_{ud} and V_{us} are proportional to G_μ. These matrix elements are measured in β-decay, K_{e3} decay, and hyperon decays. The effect on G_μ, therefore, modifies V_{ui} and the unitarity constraint for the first row of the CKM is now expressed as [42,43]

$$\sum_{i=1}^{3} |V_{ui}|^2 = \left(\frac{G_\beta}{G_\mu}\right)^2 = \left(\frac{G_F\sqrt{(NN^\dagger)_{11}}}{G_F\sqrt{(NN^\dagger)_{11}(NN^\dagger)_{22}}}\right)^2$$
$$= \frac{1}{(NN^\dagger)_{22}}, \quad (11)$$

where Eq. (9) has been used in the last equality. Following the previous equation one gets [50]

$$\sum_{i=1}^{3} |V_{ui}|^2 = \frac{1}{\alpha_{22}^2 + |\alpha_{21}|^2} = 0.9999 \pm 0.0006, \quad (12)$$

and, therefore, $1 - (NN^\dagger)_{22} = (SS^\dagger)_{22} = 1 - \alpha_{22}^2 - |\alpha_{21}|^2 < 0.0005$ at 1σ.

There are other universality tests that give constraints on these α parameters. For example, universality implies that the couplings of the leptons to the gauge bosons are flavor independent, a feature that emerges in the Standard Model without heavy leptons. In the presence of heavy isosinglets, these couplings will be flavor dependent; the ratios of these couplings can be extracted from weak decays and they are expressed as [42]

$$\left(\frac{g_a}{g_\mu}\right)^2 = \frac{(NN^\dagger)_{aa}}{(NN^\dagger)_{22}} \quad a = 1, 3. \quad (13)$$

For $a = 1$, this ratio can be constrained by comparing the experimental measurement and the theoretical prediction of the pion decay branching ratio [45]:

$$R_\pi = \frac{\Gamma(\pi^+ \to e^+ \nu)}{\Gamma(\pi^+ \to \mu^+ \nu)}. \quad (14)$$

One obtains [45,51]

$$r_\pi = \frac{R_\pi}{R_\pi^{SM}} = \frac{(NN^\dagger)_{11}}{(NN^\dagger)_{22}} = \frac{\alpha_{11}^2}{\alpha_{22}^2 + |\alpha_{21}|^2}$$
$$= \frac{(1.230 \pm 0.004) \times 10^{-4}}{(1.2354 \pm 0.0002) \times 10^{-4}} = 0.9956 \pm 0.0040 \quad (15)$$

which implies $1 - \alpha_{11}^2 < 0.0084$ at 1σ for the least conservative case of $\alpha_{22}^2 + |\alpha_{21}|^2 = 1$. This procedure was adopted in Ref. [47]. However, in general, $[(NN^\dagger)_{22}] \neq 1$, and it can be estimated using the unitarity constraints on the CKM matrix discussed above. Combining both constraints [from Eqs. (12) and (15)] we obtain the results shown in Fig. 2, restricting the parameter combinations shown in the plot. These translate in the constraints

$$1 - \alpha_{11}^2 < 0.0130,$$
$$1 - \alpha_{22}^2 - |\alpha_{21}|^2 < 0.0012, \quad (16)$$

at 90% C.L. for 2 d.o.f. One can make use of a third observable in order to have constraints for every
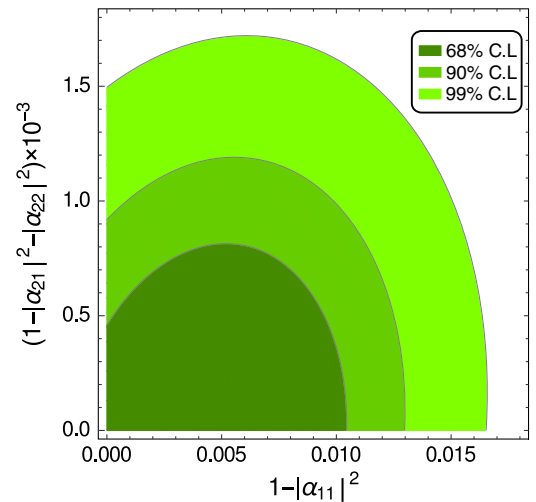

FIG. 2 (color online). Constraints on the deviations from unitarity.

independent parameter. This will be discussed in the next section.

For the sake of completeness we now show the constraints coming from the $\mu - \tau$ universality which, using Eq. (13), give the bound:

$$\frac{(NN^{\dagger})_{33}}{(NN^{\dagger})_{22}} = 0.9850 \pm 0.0057. \qquad (17)$$

This implies $1 - (NN^{\dagger})_{33} = (SS^{\dagger})_{33} < 0.0207$ at 1σ for the least conservative case of $(SS^{\dagger})_{22} = 0$. The experimental value was taken from Ref. [52]. We now turn to neutrino oscillations.

V. NONUNITARITY EFFECT ON NEUTRINO OSCILLATIONS

In this section we focus on neutrino oscillation experiments. First we obtain general expressions for neutrino survival and conversion probabilities in this parametrization and confront them with the existing experimental data. The general expressions will be relatively simple, especially if we neglect cubic products of α_{21}, $\sin\theta_{13}$, and $\sin(\frac{\Delta m_{21}^2}{4E})$, which is a reasonable approximation for many applications. The results of this approach for the three probabilities discussed in this section are shown in Eqs. (21), (27) and (33).

For the case of the muon neutrino conversion probability into electron neutrinos we have

$$P_{\mu e} = \sum_{i,j}^{3} N_{\mu i}^{*} N_{ei} N_{\mu j} N_{ej}^{*} - 4\sum_{j>i}^{3} \text{Re}[N_{\mu j}^{*} N_{ej} N_{\mu i} N_{ei}^{*}] \sin^{2}\left(\frac{\Delta m_{ji}^2 L}{4E}\right) + 2\sum_{j>i}^{3} \text{Im}[N_{\mu j}^{*} N_{ej} N_{\mu i} N_{ei}^{*}] \sin\left(\frac{\Delta m_{ji}^2 L}{2E}\right). \qquad (18)$$

And now, instead of the usual unitarity condition for the 3×3 case, we must use the condition given in Eqs. (7) and (8), arriving at the expression

$$P_{\mu e} = \alpha_{11}^2 |\alpha_{21}|^2 - 4\sum_{j>i}^{3} \text{Re}[N_{\mu j}^{*} N_{ej} N_{\mu i} N_{ei}^{*}] \sin^{2}\left(\frac{\Delta m_{ji}^2 L}{4E}\right) + 2\sum_{j>i}^{3} \text{Im}[N_{\mu j}^{*} N_{ej} N_{\mu i} N_{ei}^{*}] \sin\left(\frac{\Delta m_{ji}^2 L}{2E}\right). \qquad (19)$$

Using Eq. (2) one can substitute the values of $N_{\alpha i}$ in terms of $U_{\alpha i}$ and α_{ij} to obtain

$$P_{\mu e} = \alpha_{11}^2 |\alpha_{21}|^2 \left(1 - 4\sum_{j>i}^{3} |U_{ej}|^2 |U_{ei}|^2 \sin^{2}\left(\frac{\Delta m_{ji}^2 L}{4E}\right)\right) - (\alpha_{11}\alpha_{22})^2 4\sum_{j>i}^{3} \text{Re}[U_{\mu j}^{*} U_{ej} U_{\mu i} U_{ei}^{*}] \sin^{2}\left(\frac{\Delta m_{ji}^2 L}{4E}\right)$$

$$+ (\alpha_{11}\alpha_{22})^2 2\sum_{j>i}^{3} \text{Im}[U_{\mu j}^{*} U_{ej} U_{\mu i} U_{ei}^{*}] \sin\left(\frac{\Delta m_{ji}^2 L}{2E}\right)$$

$$- 4\alpha_{11}^2 \alpha_{22} \sum_{j>i}^{3} \text{Re}[\alpha_{21}|U_{ei}|^2 U_{\mu j}^{*} U_{ej} + \alpha_{21}^{*}|U_{ej}|^2 U_{\mu i} U_{ei}^{*}] \sin^{2}\left(\frac{\Delta m_{ji}^2 L}{4E}\right)$$

$$+ 2\alpha_{11}^2 \alpha_{22} \sum_{j>i}^{3} \text{Im}[\alpha_{21}|U_{ei}|^2 U_{\mu j}^{*} U_{ej} + \alpha_{21}^{*}|U_{ej}|^2 U_{\mu i} U_{ei}^{*}] \sin\left(\frac{\Delta m_{ji}^2 L}{2E}\right). \qquad (20)$$

Substituting the terms $U_{\alpha i}$ in our parametrization, and neglecting cubic products of α_{21}, $\sin\theta_{13}$, and Δm_{21}^2, one obtains

$$P_{\mu e} = (\alpha_{11}\alpha_{22})^2 P_{\mu e}^{3\times 3} + \alpha_{11}^2 \alpha_{22} |\alpha_{21}| P_{\mu e}^{I} + \alpha_{11}^2 |\alpha_{21}|^2, \qquad (21)$$

where we have denoted the standard three-neutrino conversion probability $P_{\mu e}^{3\times 3}$ as [2,53,54]

$$P_{\mu e}^{3\times 3} = 4\left[\cos^2\theta_{12}\cos^2\theta_{23}\sin^2\theta_{12}\sin^2\left(\frac{\Delta m_{21}^2 L}{4E_\nu}\right) + \cos^2\theta_{13}\sin^2\theta_{13}\sin^2\theta_{23}\sin^2\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right)\right]$$

$$+ \sin(2\theta_{12})\sin\theta_{13}\sin(2\theta_{23})\sin\left(\frac{\Delta m_{21}^2 L}{2E_\nu}\right)\sin\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right)\cos\left(\frac{\Delta m_{31}^2 L}{4E_\nu} - I_{123}\right), \qquad (22)$$

while $P_{\mu e}^{I}$ refers to a term that depends on the 3×3 mixing angles, plus an extra CP phase:

$$P_{\mu e}^I = -2 \left[\sin(2\theta_{13}) \sin\theta_{23} \sin\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right) \sin\left(\frac{\Delta m_{31}^2 L}{4E_\nu} + I_{NP} - I_{123}\right) \right]$$

$$- \cos\theta_{13} \cos\theta_{23} \sin(2\theta_{12}) \sin\left(\frac{\Delta m_{21}^2 L}{2E_\nu}\right) \sin(I_{NP}), \tag{23}$$

with $\;I_{123} = -\delta_{CP} = \phi_{12} - \phi_{13} + \phi_{23}\;$ and $\;I_{NP} = \phi_{12} - \mathrm{Arg}(\alpha_{21})$.

Notice that the conversion probability depends on just two phases, the standard one, $I_{123} = -\delta$ and another phase describing the new physics, I_{NP}. This new phase contains the information of the imaginary part of α_{21}, that is, the overall effect of all the additional phases associated with the heavy states. Notice that, besides the standard CP term in Eq. (23), two new CP phase-dependent terms appear; the first involves the difference between standard and non-standard phase, $I_{123} - I_{NP}$, while the second one depends only on I_{NP}. One sees in Eq. (23) that the first term is proportional to $\sin\theta_{13}$, while the second one depends on the solar mass difference Δm_{21}^2 and, therefore, both terms should be small. In order to illustrate their impact upon current neutrino data analysis, we show in Fig. 3 how this new phase parameter influences the conversion probability. In this figure we compare the standard three neutrino probability (with a "best-fit" phase $\delta = -I_{123} = 3\pi/2$), with the case of an additional neutral heavy lepton with overall contribution given by $\alpha_{11} = 1$, $\alpha_{22} = 0.997$, $|\alpha_{21}| = 0.078$, and for the particular new physics phase parameter of $\pi/2$ or $3\pi/2$ (left panel) or $0, \pi$ (right panel). One sees that the effect of the additional phase in future oscillation appearance experiments could be sizable and, depending on the specific value of this new phase, the survival probability could either increase or decrease.

For the sake of completeness, we also give the expression for the survival probability $P_{\mu\mu}$:

$$P_{\mu\mu} = \sum_i^3 |N_{\mu i}|^2 |N_{\mu i}|^2 + \sum_{j>i}^3 2|N_{\mu j}|^2 |N_{\mu i}|^2 \cos\left(\frac{\Delta m_{ji}^2}{2E} L\right), \tag{24}$$

$$P_{\mu\mu} = (|\alpha_{21}|^2 + \alpha_{22}^2)^2 - 4\sum_{j>i}^3 |N_{\mu j}|^2 |N_{\mu i}|^2 \sin^2\left(\frac{\Delta m_{ji}^2}{4E} L\right), \tag{25}$$

$$P_{\mu\mu} = (|\alpha_{21}|^2 + \alpha_{22}^2)^2 - 4\sum_{j>i}^3 |\alpha_{21} U_{ej}$$
$$+ \alpha_{22} U_{\mu j}|^2 |\alpha_{21} U_{ei} + \alpha_{22} U_{\mu i}|^2 \sin^2\left(\frac{\Delta m_{ji}^2}{4E} L\right), \tag{26}$$

so that, neglecting cubic products of α_{21}, $\sin\theta_{13}$, and Δm_{21}^2, we will obtain

$$P_{\mu\mu} = \alpha_{22}^4 P_{\mu\mu}^{3\times3} + \alpha_{22}^3 |\alpha_{21}| P_{\mu\mu}^{I_1} + 2|\alpha_{21}|^2 \alpha_{22}^2 P_{\mu\mu}^{I_2} \tag{27}$$

with $P_{\mu\mu}^{3\times3}$, the standard oscillation formula, given by
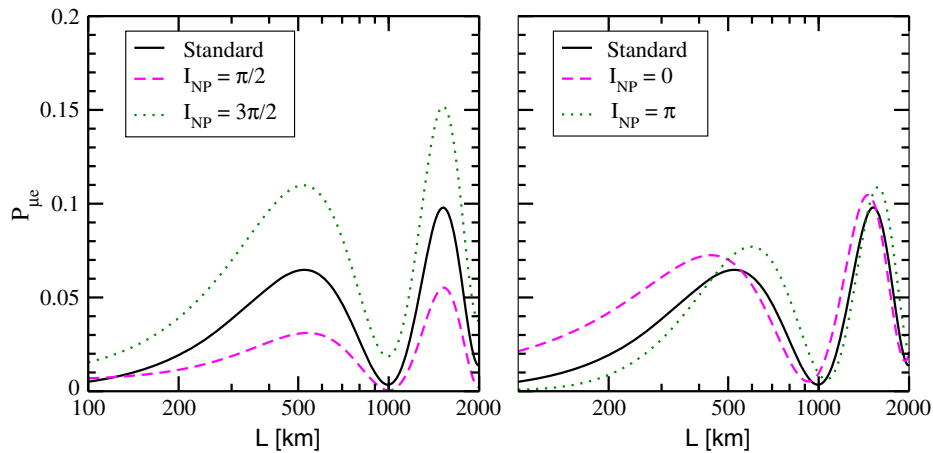
FIG. 3 (color online). Conversion probability for a fixed neutrino energy $E_\nu = 1$ GeV. The solid (black) curve shows the standard conversion probability, with $\delta = -I_{123} = 3\pi/2$. The nonunitary case is illustrated for $\alpha_{11} = 1$, $\alpha_{22} = 0.997$, and $|\alpha_{21}| = 0.078$. In the left panel, two values for the new CP phase parameter I_{NP} are considered: $\pi/2$ (dashed/magenta line) and $3\pi/2$ (dotted/green line), while in the right panel we take $I_{NP} = 0$ (dashed/magenta line) and π (dotted/green line).

$$P_{\mu\mu}^{3\times3} \approx 1 - 4[\cos^2\theta_{23}\sin^2\theta_{23} - \cos(2\theta_{23})\sin^2\theta_{23}\sin^2\theta_{13}]\sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)$$

$$+ 2[\cos^2\theta_{12}\cos^2\theta_{23}\sin^2\theta_{23} - \cos(I_{123})\cos\theta_{23}\sin(2\theta_{12})\sin^3\theta_{23}\sin\theta_{13}]\sin\left(\frac{\Delta m_{31}^2 L}{2E}\right)\sin\left(\frac{\Delta m_{21}^2 L}{2E}\right)$$

$$- 4\left[\cos^2\theta_{12}\cos^2\theta_{23}\sin^2\theta_{23}\cos\left(\frac{\Delta m_{31}^2 L}{2E}\right) + \cos^2\theta_{12}\cos^4\theta_{23}\sin^2\theta_{12}\right]\sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right), \tag{28}$$

while the extra terms in the oscillation probability are given by

$$P_{\mu\mu}^{I_1} \approx -8[\sin\theta_{13}\sin\theta_{23}\cos(2\theta_{23})\cos(I_{123} - I_{NP})]\sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)$$

$$+ 2[\cos\theta_{23}\sin(2\theta_{12})\sin^2\theta_{23}\cos(I_{NP})]\sin\left(\frac{\Delta m_{31}^2 L}{2E}\right)\sin\left(\frac{\Delta m_{21}^2 L}{2E}\right), \tag{29}$$

$$P_{\mu\mu}^{I_2} \approx 1 - 2\sin^2\theta_{23}\sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right). \tag{30}$$

As for the conversion probability, $P(\nu_\mu \to \nu_e)$, we also compute the muon neutrino survival probability and show its behavior in Fig. 4. As one can see, this disappearance channel is also sensitive to the new CP phase. The computations were performed for the same parameter values used in the previous figure, that is, $\alpha_{11} = 1$, $\alpha_{22} = 0.997$, $|\alpha_{21}| = 0.078$, and an overall phase of either $\pi/2$ or $3\pi/2$ as well as 0 or π. The Standard Model phase was fixed to be $\delta = -I_{123} = 3\pi/2$.

We now turn our attention to oscillations of electron neutrinos or antineutrinos relevant, say, for the description of solar neutrino experiments, as well as terrestrial experiments using reactors or radioactive sources. The electron (anti)neutrino survival probability (in vacuum) is given by the following expression:

$$P_{ee} = \sum_i^3 |N_{ei}|^2|N_{ei}|^2 + \sum_{j>i}^3 2|N_{ej}|^2|N_{ei}|^2 \cos\left(\frac{\Delta m_{ji}^2}{2E}L\right), \tag{31}$$

and, using Eq. (2), it is easy to see that $N_{ei} = \alpha_{11}U_{ei}$ which leads to the expression

$$P_{ee} = \alpha_{11}^4\left[\sum_i^3 |U_{ei}|^2|U_{ei}|^2 + \sum_{j>i}^3 2|U_{ej}|^2|U_{ei}|^2 \cos\left(\frac{\Delta m_{ji}^2}{2E}L\right)\right]. \tag{32}$$

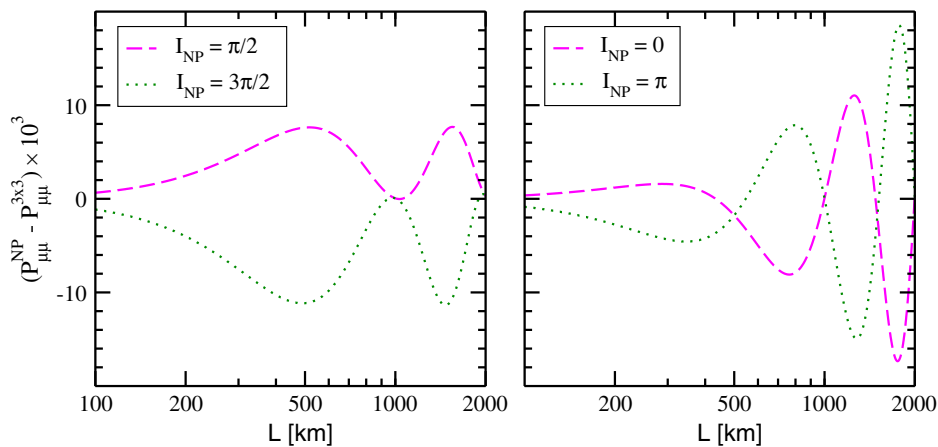This transforms, in a straightforward way, to the equation

FIG. 4 (color online).   Correction to the standard muon neutrino survival probability for different values of the new CP phase parameter I_{NP}, with the remaining parameters fixed as in Fig. 3.

$$P_{ee} = \alpha_{11}^4[1 - \cos^4\theta_{13}\sin^2(2\theta_{12})\sin^2(\Delta_{12})$$
$$- \sin^2(2\theta_{13})\sin^2(\Delta_{13})], \qquad (33)$$

with $\Delta_{ij} = \frac{\Delta m_{ij}^2}{4E}L$. Notice that in this case, the effect of a neutral heavy lepton will be an overall factor that accounts for the violation of unitarity: α_{11}^4, which is unlikely to produce visible effects in oscillations of, say, reactor neutrinos, given the strong universality restrictions derived in Fig. 2.

For completeness we mention that, should the extra neutrino states be light enough to take part in oscillations, they could potentially play a role [55,56] in the anomalies reported by the MiniBooNE Collaboration [22] or the reactor neutrino experiments [57]. We will not consider this possibility here.

VI. NONSTANDARD INTERACTIONS

From the previous formulas for the oscillation probabilities one sees that, even at zero distance, the survival and conversion probabilities differ from one and zero, respectively. This is a well-known behavior and it is a consequence of the effective nonunitarity of the 3×3 leptonic mixing matrix [58]. We can express these probabilities, for the zero distance case, as

$$P_{ee} = [\alpha_{11}]^4 = [(NN^\dagger)_{11}]^2 = [1 - (SS^\dagger)_{11}]^2,$$
$$P_{\mu\mu} = [|\alpha_{21}|^2 + \alpha_{22}^2]^2 = [(NN^\dagger)_{22}]^2 = [1 - (SS^\dagger)_{22}]^2,$$
$$P_{\mu e} = \alpha_{11}^2|\alpha_{21}|^2 = [(NN^\dagger)_{21}]^2 = [(SS^\dagger)_{21}]^2. \qquad (34)$$

In order to make a quick estimate of the constraints on the new parameters, we write these expressions in a different way, in order to compare them with the corresponding expressions for a light sterile neutrino in the limit of $\Delta m_{ij}^2 L/(4E) \gg 1$ ($\langle\sin^2(\Delta m_{ij}^2 l/(4E))\rangle = 1/2$). The result for our case can be expressed in an analogous way as in the case of extra light neutrinos [59]:

$$P_{ee} = 1 - \frac{1}{2}[\sin^2(2\theta_{ee})]_{\text{eff}},$$
$$P_{\mu\mu} = 1 - \frac{1}{2}[\sin^2(2\theta_{\mu\mu})]_{\text{eff}},$$
$$P_{\mu e} = \frac{1}{2}[\sin^2(2\theta_{\mu e})]_{\text{eff}}, \qquad (35)$$

with

$$[\sin^2(2\theta_{ee})]_{\text{eff}} = 2(1 - \alpha_{11}^4),$$
$$[\sin^2(2\theta_{\mu\mu})]_{\text{eff}} = 2[1 - (|\alpha_{21}|^2 + \alpha_{22}^2)^2],$$
$$[\sin^2(2\theta_{\mu e})]_{\text{eff}} = 2\alpha_{11}^2|\alpha_{21}|^2. \qquad (36)$$

We can compare these expressions with the current constraints on light sterile neutrinos in order to get the following 3σ limits [56]:

$$[\sin^2(2\theta_{ee})]_{\text{eff}} \leq 0.2,$$
$$[\sin^2(2\theta_{\mu\mu})]_{\text{eff}} \leq 0.06,$$
$$[\sin^2(2\theta_{\mu e})]_{\text{eff}} \leq 1 \times 10^{-3}. \qquad (37)$$

Apart from the MiniBooNE anomaly [22], there are strong constraints on the existence of a fourth neutrino from ν_μ to ν_e oscillation experiments. The stronger bound comes from the NOMAD experiment [60]. Translated into the parametrization under discussion, this constraint takes the form

$$\alpha_{11}^2|\alpha_{21}|^2 \leq 0.007 \quad (90\% \text{ C.L.}). \qquad (38)$$

If we combine this limit with those coming from universality at Eq. (16), the following 90% C.L. bounds are obtained:

$$\alpha_{11}^2 \geq 0.987, \quad \alpha_{22}^2 \geq 0.9918, \quad |\alpha_{21}|^2 \leq 0.0071. \qquad (39)$$

VII. COMPILING CURRENT NHL CONSTRAINTS

Nonstandard features such as unitarity violation in neutrino mixing could signal new physics responsible for neutrino mass. For example, they could shed light upon the properties of neutral heavy leptons such as right-handed neutrinos, which are the messengers of neutrino mass generation postulated in seesaw schemes. In many such schemes the smallness of neutrino masses severely restricts the magnitudes of the expected NHL signatures. However these limitations can be circumvented within a broad class of low-scale seesaw realizations [12–20]. For this reason in this section we will present a compilation of model-independent NHL limits, which do not require them to play the role of neutrino mass messenger in any particular seesaw scheme. Results of this section are not original, but they are included for logical completeness.

Isosinglet neutrinos have been searched for in a variety of experiments. For example, if they are very light they may be emitted in weak decays of pions and kaons. Heavier ones, but lighter than the Z boson, would have been copiously produced in the first phase of the LEP experiment should the coupling be appreciable [25,26]. Searches have been negative, including those performed at the higher, second phase energies [27].

A summary of constraints for the direct production of neutral heavy leptons is shown in Figs. 5–7. In most cases, experiments have looked for a resonance in a given energy window, for a given mixing of the additional state, described in this case by the submatrix S of Eq. (1). Although the constraints for the mixing in these cases are stronger, in most of the cases they rely upon extra assumptions on how the heavy neutrino should decay.
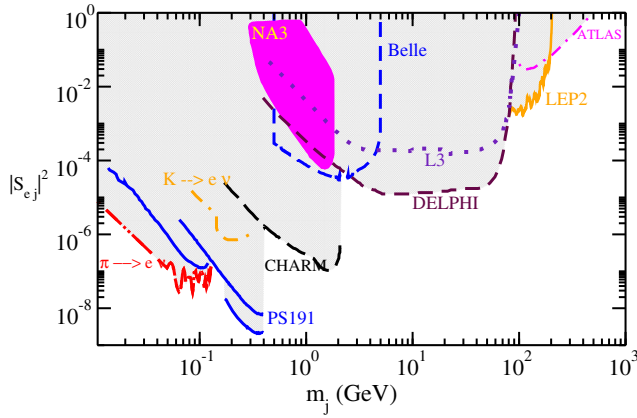
FIG. 5 (color online). Bounds on the component of a heavy isosinglet lepton of mass m_j in the electron neutrino.

FIG. 7 (color online). Bounds on the component of a heavy isosinglet lepton of mass m_j in the tau neutrino.

In particular, in Fig. 5, we summarize the constraints on $|S_{ej}|^2$ for a mass range from 10^{-2} to 10^2 GeV coming from the experiments TRIUMF [61,62] (denoted as $\pi \to e\nu$ and $K \to e\nu$ in the plot), PS191 [63], NA3 [64], CHARM [65], Belle [66], the LEP experiments DELPHI [27], L3 [67], LEP2 [68], and the recent LHC results from ATLAS [69,70]. Future experimental proposals, such as DUNE [71] and ILC, expect to improve these constraints [72].

In Fig. 6 we show the corresponding constraints for the case of the mixing of a neutral heavy lepton with a muon neutrino. In this case we show the experimental results coming again from PS191, NA3, and Belle, from the LEP experiments L3, DELPHI, and from the LHC experiment ATLAS; we also show the bounds coming from KEK [73,74] (denoted as $K \to \mu\nu$ in the plot), CHARM II [75], FMMF [76], BEBC [77], NuTeV [78], E949 [79], and from the LHC experiments CMS [80] and LHCb [81]. Finally, for the less studied case of the mixing of a neutral heavy lepton with a tau
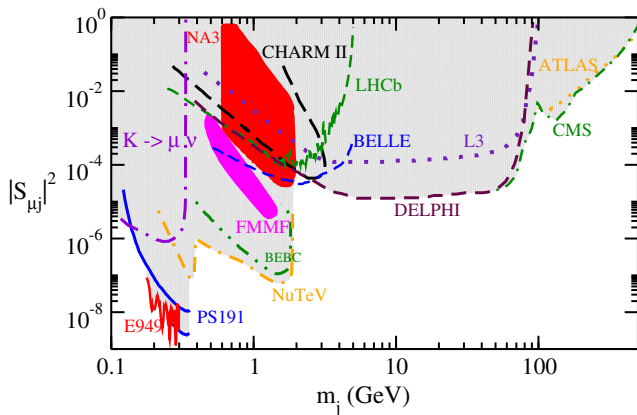
neutrino, the known constraints, coming from NOMAD [82], CHARM [83], and DELPHI [27], are shown in Fig. 7.

Heavier neutrinos in the TeV range, natural in the context of low-scale seesaw, can also be searched for at the LHC. However, within the standard $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ model such heavy, mainly isosinglet, neutrinos would be produced only through small mixing effects. Indeed, it can be seen from Figs. 5, 6 and 7 that restrictions are rather weak. In contrast, this limitation can be avoided in extended electroweak models. In such case a production portal involving extra kinematically accessible gauge bosons, such as those associated with left-right symmetric models, can give rise to signatures at high energies, such as processes with lepton flavor violation [84,85].

A. Neutrinoless double beta decay

If neutrinos have Majorana nature, as expected on theoretical grounds, neutrinoless double beta decay is expected to occur at some level [30]. We start our discussion by remembering the definition of the effective Majorana neutrino mass [86],

$$\langle m \rangle = \left| \sum_j (U_{ej}^{n \times n})^2 m_j \right|, \qquad (40)$$

where the sum runs only for the light neutrinos coupling to the electron and the W-boson.

From Eq. (2) one sees that, in the presence of the heavy neutrinos, the three light SM neutrino charged current couplings will be modified to $U_{ei}^{n \times n} = \alpha_{11} U_{ei}$, with i = 1, 2, 3, and their contribution to neutrinoless double beta decay will change correspondingly.

Moreover, the heavy states will induce also a short-range or contact contribution to neutrinoless double beta decay involving the exchange of the heavy Majorana neutrinos. Since these are $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

FIG. 6 (color online). Bounds on the component of a heavy isosinglet lepton of mass m_j in the muon neutrino.

singlets they couple only through the mixing coefficients S_{ej}. The general form of the amplitude is proportional to

$$\mathcal{A} \propto \frac{m_j}{q^2 - m_j^2}, \qquad (41)$$

where q is the virtual neutrino momentum transfer. Clearly there are two main regimes for this amplitude; for $q^2 \gg m_j^2$, we have

$$\mathcal{A}_{\text{light}} \propto m_j, \qquad (42)$$

while for $q^2 \ll m_j^2$

$$\mathcal{A}_{\text{heavy}} \propto \frac{1}{m_j}. \qquad (43)$$

This behavior can be seen in the corresponding estimated sensitivity curve shown in Fig. 8. This line is obtained for ^{76}Ge assuming a single massive isosinglet neutrino [87]. The change in slope takes place for masses close to the typical nuclear momentum, around 100–200 MeV. Both light and heavy contributions must be folded in with the appropriate nuclear matrix elements [88] whose uncertainties are still large. As a result it is not possible to probe the indirect NHL effect upon the light-neutrino contribution to the effective mass in Eq. (40) which amounts to a multiplicative factor α_{11}^2 in the amplitude, a difference well below current sensitivities. Notice that, in contrast to bounds discussed in Figs. 5, 6 and 7 the restriction from the neutrinoless double beta decay in Fig. 8 holds only if neutrinos have Majorana nature.

B. Charged lepton flavor violation

Virtual exchange of NHLs would also induce charged lepton flavor violation processes both at low
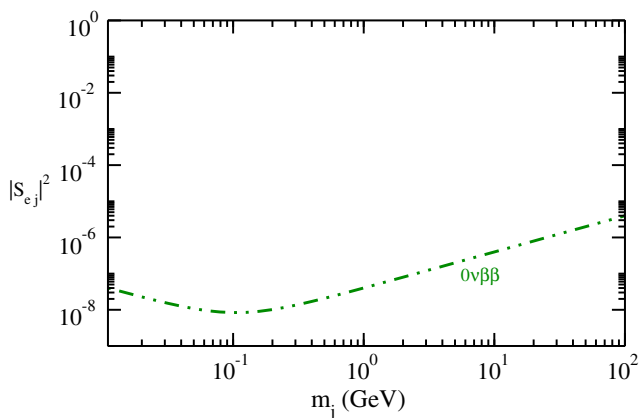

FIG. 8 (color online).   Sensitivity of neutrinoless double beta decay to isosinglet mass m_j in the electron neutrino.

energies [29] and at the high energies provided by accelerator experiments [33]. However rates would depend on additional flavor parameters and upon details on the seesaw mechanism providing masses to neutrinos. The possibility of probing it at hadronic colliders such as the LHC may be realistic in low-scale seesaw models with additional TeV-scale gauge bosons beyond those of the SM gauge structure and with lighter NHLs [84,85,89,90]. However we do not consider this possibility any further here because the corresponding rates depend on very model-dependent assumptions.

VIII. SUMMARY

Simplest seesaw extensions of the Standard Model predict unitarity deviations in the leptonic mixing matrix describing the charged current leptonic weak interaction. This is due to the admixture of heavy isosinglet neutrinos, such as "right-handed neutrinos," which are the "messengers" whose exchange generates small neutrino masses. Low-scale realizations of such schemes suggest that such NHLs may be light enough as to be accessible at high energy colliders such as the LHC or, indirectly, induce sizable unitarity deviations in the "effective" lepton mixing matrix. In this paper we used the general symmetric parametrization of lepton mixing of Ref. [8] in order to derive a simple description of unitarity deviations in the light-neutrino sector. Most experiments employ neutrinos or anti-neutrinos of the first two generations. Their description becomes especially simple in our method, Eq. (2), as it involves only a subset of parameters consisting of three real effective parameters plus a single *CP* phase. We have illustrated the impact of nonunitary lepton mixing on weak decay processes as well as neutrino oscillations. For logical completeness we have also recompiled the current model-independent constraints on heavy-neutrino coupling parameters arising from various experiments in this notation. In short, our method will be useful in a joint description of NHL searches as well as upcoming precision neutrino oscillation studies, and will hopefully contribute to shedding light on the possible seesaw origin of neutrino mass.

APPENDIX: NEUTRINO MIXING AND HEAVY ISOSINGLETS

As already explained, heavy gauge singlet neutrinos arise naturally in several extensions of the Standard Model. The general form of the mixing matrix describing their charged current weak interaction has been given in [8]. Here we will further develop the formalism so as to describe not only the couplings of the additional heavy neutrinos but also their effects in the light-neutrino sector in a convenient but complete way, with no assumptions about CP conservation. Using Okubo's notation [91], we can construct the rotation matrix $U^{n \times n}$ as

$$U^{n \times n} = \omega_{n-1n}\omega_{n-2n}...\omega_{1n}\omega_{n-2n-1}\omega_{n-3n-1}...\omega_{1n-1}$$
$$...\omega_{23}\omega_{13}\omega_{12}, \qquad (A1)$$

where each $\omega_{ij}(i < j)$ stands for the usual complex rotation matrix in the ij plane [36]:

$$\omega_{13} = \begin{pmatrix} c_{13} & 0 & e^{-i\phi_{13}}s_{13} \\ 0 & 1 & 0 \\ -e^{i\phi_{13}}s_{13} & 0 & c_{13} \end{pmatrix}, \qquad (A2)$$

with $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$. This matrix can be expressed in general as

$$(\omega_{ij})_{\alpha\beta} = \delta_{\alpha\beta}\sqrt{1 - \delta_{\alpha i}\delta_{\beta j}s_{ij}^2 - \delta_{\alpha j}\delta_{\beta i}s_{ij}^2} + \eta_{ij}\delta_{\alpha i}\delta_{\beta j}$$
$$+ \bar{\eta}_{ij}\delta_{\alpha j}\delta_{\beta i}, \qquad (A3)$$

where $i < j$ and $s_{ij}^2 = \sin^2\theta_{ij}, \eta_{ij} = e^{-i\phi_{ij}}\sin\theta_{ij}$ and $\bar{\eta}_{ij} = -e^{i\phi_{ij}}\sin\theta_{ij}$, generalizing the matrix in Eq. (A2) as

$$\omega_{ij} = \begin{pmatrix} 1 & 0 & & \cdots & 0 & \cdots & & & 0 \\ 0 & 1 & & & & & & & \vdots \\ \vdots & & c_{ij} & \cdots & 0 & \cdots & \eta_{ij} & & \\ & & \vdots & \ddots & & & \vdots & & \\ & & 0 & & 1 & & 0 & & \\ & & \vdots & & & \ddots & \vdots & & \\ & & \bar{\eta}_{ij} & \cdots & 0 & \cdots & c_{ij} & & \vdots \\ \vdots & & & & & & & 1 & 0 \\ 0 & & & \cdots & 0 & \cdots & & 0 & 1 \end{pmatrix}. \qquad (A4)$$

In general, one can decompose Eq. (A1) in the following way:

$$U^{n \times n} = U^{n-N}U^N, \qquad (A5)$$

with

$$U^N = \omega_{N-1N}\omega_{N-2N}...\omega_{1N}, \qquad (A6)$$

$$U^{n-N} = \omega_{n-1n}\omega_{n-2n}...\omega_{1n}\omega_{n-1n-1}\omega_{n-2n-1}...\omega_{1N+1}, \qquad (A7)$$

so that the matrix decomposition will be given by

$$U^{n-N}U^N = \begin{pmatrix} \alpha_{11} & 0 & \cdots & 0 & \vdots & & \\ \alpha_{21} & \alpha_{22} & \ddots & \vdots & \vdots & & \\ \vdots & & \ddots & 0 & \vdots & & S \\ \alpha_{N1} & \cdots & & \alpha_{NN} & \vdots & & \\ \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots \\ & & & & \vdots & & \\ & & V' & & \vdots & & T \\ & & & & \vdots & & \end{pmatrix} \begin{pmatrix} U_{11}^N & U_{12}^N & \cdots & U_{1N}^N & \vdots & \\ U_{21}^N & U_{22}^N & & & \vdots & \vdots \\ \vdots & & \ddots & & \vdots & 0 \\ U_{N1}^N & \cdots & & U_{NN}^N & \vdots & \\ \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots \\ & & & & \vdots & \\ & & 0 & & \vdots & I \\ & & & & \vdots & \end{pmatrix}, \qquad (A8)$$

which turns out to be very convenient. The 3×3 neutrino mixing matrix, $U^{3 \times 3}$, determined in oscillation experiments could be unitary, or it could be just a nonunitary submatrix of the larger mixing matrix $U^{n \times n}$ described in Eq. (A1).

Therefore, when dealing with more than three neutrinos, we can write $U^{n \times n}$ as the product of two matrices:

$$U^{n \times n} = U^{\text{NP}}U^{\text{SM}}, \qquad (A9)$$

where NP means new physics and SM stands for the Standard Model matrix,

$$U^{\mathrm{NP}} = \omega_{n-1n}\omega_{n-2n}\ldots\omega_{3n}\omega_{2n}\omega_{1n}\omega_{n-2n-1}$$

$$\ldots\omega_{3n-1}\omega_{2n-1}\omega_{1n-1}\ldots\omega_{34}\omega_{24}\omega_{14}, \qquad (A10)$$

$$U^{\mathrm{SM}} = \omega_{23}\omega_{13}\omega_{12}. \qquad (A11)$$

The complete $n \times n$ matrix, $U^{n\times n}$, may be written as [35]

$$U^{n\times n} = \begin{pmatrix} N & S \\ V & T \end{pmatrix}, \qquad (A12)$$

where N is the 3×3 matrix with the standard neutrino terms. From Eq. (A9) one sees that N can always be parametrized as

$$N = N^{\mathrm{NP}}U^{3\times3} = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U^{3\times3}, \qquad (A13)$$

where the zero triangle submatrix characterizes this decomposition. It is useful to see how the components α_{ij} of this matrix can be found. First notice that $\omega_{ij}\omega_{kl}$ commutes when $i \neq k, l$ and $j \neq k, l$; therefore, Eq. (A10) can be rewritten as

$$U^{\mathrm{NP}} = \omega_{n-1n}\omega_{n-2n}\ldots\omega_{4n}\omega_{n-2n-1}\ldots\omega_{4n-1}\ldots\omega_{45}\omega_{3n}\omega_{2n}\omega_{1n}$$

$$\times \omega_{3n-1}\omega_{2n-1}\omega_{1n-1}\ldots\omega_{34}\omega_{24}\omega_{14}. \qquad (A14)$$

Clearly, the first line of this equation has no influence in the submatrices N and S. On the other hand, the second line of the above equation is a set of products of the form $\omega_{3j}\omega_{2j}\omega_{1j}$, each of them having the form

$$\alpha^j = \omega_{3j}\omega_{2j}\omega_{1j} = \begin{pmatrix} c_{1j} & 0 & 0 & \vdots & 0 & \eta_{1j} & 0 \\ \eta_{2j}\bar\eta_{1j} & c_{2j} & 0 & \vdots & 0 & \eta_{2j}c_{1j} & 0 \\ \eta_{3j}c_{2j}\bar\eta_{1j} & \eta_{3j}\bar\eta_{2j} & c_{3j} & \vdots & 0 & \eta_{3j}c_{2j}c_{1j} & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \vdots & I & 0 & 0 \\ c_{3j}c_{2j}\bar\eta_{1j} & c_{3j}\bar\eta_{2j} & \bar\eta_{3j} & \vdots & 0 & c_{3j}c_{2j}c_{1j} & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & I \end{pmatrix} = \begin{pmatrix} \alpha^j_{11} & 0 & 0 & \vdots & 0 & \alpha^j_{1j} & 0 \\ \alpha^j_{21} & \alpha^j_{22} & 0 & \vdots & 0 & \alpha^j_{2j} & 0 \\ \alpha^j_{31} & \alpha_{32} & \alpha^j_{33} & \vdots & 0 & \alpha_{3j} & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \vdots & I & 0 & 0 \\ \alpha^j_{j1} & \alpha^j_{2j} & \alpha^j_{j3} & \vdots & 0 & \alpha^j_{jj} & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & I \end{pmatrix}.$$

$$(A15)$$

We can see that the expression for N^{NP} depends only on products of the type $\alpha^n\alpha^{n-1}\cdots\alpha^5\alpha^4$. After performing the multiplication one notes that the diagonal entries of the matrix N^{NP} are in general given by

$$\alpha_{11} = \alpha^n_{11}\alpha^{n-1}_{11}\alpha^{n-2}_{11}\ldots\alpha^4_{11} = c_{1n}c_{1n-1}c_{1n-2}\ldots c_{14},$$

$$\alpha_{22} = \alpha^n_{22}\alpha^{n-1}_{22}\alpha^{n-2}_{22}\ldots\alpha^4_{22} = c_{2n}c_{2n-1}c_{2n-2}\ldots c_{24},$$

$$\alpha_{33} = \alpha^n_{33}\alpha^{n-1}_{33}\alpha^{n-2}_{33}\ldots\alpha^4_{33} = c_{3n}c_{3n-1}c_{3n-2}\ldots c_{34},$$

while the off-diagonal entries α_{ij} are given as

$$\alpha_{21} = \alpha^n_{21}\alpha^{n-1}_{11}\ldots\alpha^4_{11} + \alpha^n_{22}\alpha^{n-1}_{21}\ldots\alpha^4_{11} + \cdots + \alpha^n_{22}\alpha^{n-1}_{22}\alpha^{n-2}_{22}\ldots\alpha^4_{21},$$

$$\alpha_{32} = \alpha^n_{32}\alpha^{n-1}_{22}\ldots\alpha^4_{22} + \alpha^n_{33}\alpha^{n-1}_{32}\ldots\alpha^4_{22} + \cdots + \alpha^n_{33}\alpha^{n-1}_{33}\alpha^{n-2}_{33}\ldots\alpha^4_{32},$$

$$\alpha_{31} = \alpha^n_{31}\alpha^{n-1}_{11}\ldots\alpha^4_{11} + \alpha^n_{33}\alpha^{n-1}_{31}\ldots\alpha^4_{11} + \cdots + \alpha^n_{33}\alpha^{n-1}_{33}\alpha^{n-2}_{33}\ldots\alpha^4_{31}$$

$$+ \alpha^n_{32}(\alpha^{n-1}_{21}\alpha^{n-2}_{11}\ldots\alpha^4_{11} + \alpha^{n-1}_{22}\alpha^{n-2}_{21}\ldots\alpha^4_{11} + \cdots + \alpha^{n-1}_{22}\alpha^{n-2}_{22}\ldots\alpha^4_{21})$$

$$+ \alpha^n_{33}\alpha^{n-1}_{32}(\alpha^{n-2}_{21}\alpha^{n-3}_{11}\ldots\alpha^4_{11} + \cdots + \alpha^{n-2}_{22}\alpha^{n-3}_{22}\ldots\alpha^4_{21}) + \cdots$$

$$+ \alpha^n_{33}\alpha^{n-1}_{33}\alpha^{n-2}_{32}(\alpha^{n-3}_{21}\alpha^{n-4}_{11}\ldots\alpha^4_{11} + \cdots + \alpha^{n-3}_{22}\alpha^{n-4}_{22}\ldots\alpha^4_{21}) + \cdots$$

$$+ \alpha^n_{33}\alpha^{n-1}_{33}\alpha^{n-2}_{33}\ldots\alpha^5_{32}\alpha^4_{21}, \qquad (A16)$$

or, more explicitly,

$$\alpha_{21} = c_{2n}c_{2n-1}\ldots c_{25}\eta_{24}\bar{\eta}_{14} + c_{2n}\ldots c_{26}\eta_{25}\bar{\eta}_{15}c_{14} + \cdots + \eta_{2n}\bar{\eta}_{1n}c_{1n-1}c_{1n-2}\ldots c_{14},$$

$$\alpha_{32} = c_{3n}c_{3n-1}\ldots c_{35}\eta_{34}\bar{\eta}_{24} + c_{3n}\ldots c_{36}\eta_{35}\bar{\eta}_{25}c_{24} + \cdots + \eta_{3n}\bar{\eta}_{2n}c_{2n-1}c_{2n-2}\ldots c_{24},$$

$$\alpha_{31} = c_{3n}c_{3n-1}\ldots c_{35}\eta_{34}c_{24}\bar{\eta}_{14} + c_{3n}\ldots c_{36}\eta_{35}c_{25}\bar{\eta}_{15}c_{14} + \cdots + \eta_{3n}c_{2n}\bar{\eta}_{1n}c_{1n-1}c_{1n-2}\ldots c_{14}$$

$$+ c_{3n}c_{3n-1}\ldots c_{35}\eta_{35}\bar{\eta}_{25}\eta_{24}\bar{\eta}_{14} + c_{3n}\ldots c_{36}\eta_{36}\bar{\eta}_{26}c_{25}\eta_{24}\bar{\eta}_{14}$$

$$+ \ldots + \eta_{3n}\bar{\eta}_{2n}\eta_{2n-1}\bar{\eta}_{1n-1}c_{1n-2}\ldots c_{14}. \tag{A17}$$

With these formulas, and the known expression for $U^{3\times3}$, we already have the explicit description of Eq. (A13) for any number of extra neutrino states. Before concluding this appendix, we would like to remark that the position of the three off-diagonal zeros in N^{NP} was chosen to conveniently make the matrix lower triangular. This simplifies the form of the nonunitary lepton mixing matrix describing most situations of phenomenological interest, involving solar, atmospheric, reactor and accelerator neutrinos. By choosing alternative factor orderings, one can have different parametrizations, with the zeros located at different off-diagonal entries.

1. Application to $3 + 1$ seesaw scheme

We will conclude this appendix by showing the expressions for α_{ij} in the case of one and three additional neutrinos. For the case of just one additional neutrino, the mixing matrix is given by

$$U^{4\times4} = \begin{pmatrix} N_{3\times3} & S_{3\times1} \\ T_{1\times3} & V_{1\times1} \end{pmatrix}. \tag{A18}$$

The corresponding expressions for the parameters α_{ij} will be given by

$$\alpha_{11} = c_{14},$$

$$\alpha_{22} = c_{24},$$

$$\alpha_{33} = c_{34},$$

$$\alpha_{21} = \eta_{24}\bar{\eta}_{14},$$

$$\alpha_{32} = \eta_{34}\bar{\eta}_{24},$$

$$\alpha_{31} = \eta_{34}c_{24}\bar{\eta}_{14}. \tag{A19}$$

2. Application to $3 + 3$ seesaw scheme

In this case the full mixing matrix will have the following structure:

$$U^{6\times6} = \begin{pmatrix} N_{3\times3} & S_{3\times3} \\ T_{3\times3} & V_{3\times3} \end{pmatrix}, \tag{A20}$$

with the α parameters given by

$$\alpha_{11} = c_{16}c_{15}c_{14},$$

$$\alpha_{22} = c_{26}c_{25}c_{24},$$

$$\alpha_{33} = c_{36}c_{35}c_{34},$$

$$\alpha_{21} = \eta_{26}\bar{\eta}_{16}c_{15}c_{14} + c_{26}\eta_{25}\bar{\eta}_{15}c_{14} + c_{26}c_{25}\eta_{24}\bar{\eta}_{14},$$

$$\alpha_{32} = c_{36}c_{35}\eta_{34}\bar{\eta}_{24} + c_{36}c_{35}\bar{\eta}_{25}c_{24} + \eta_{36}\bar{\eta}_{26}c_{25}c_{24},$$

$$\alpha_{31} = c_{36}c_{35}c_{34}\eta_{34}c_{24}\bar{\eta}_{14} + c_{36}\eta_{35}c_{24}\bar{\eta}_{15}c_{14} + \eta_{36}c_{26}\bar{\eta}_{16}c_{15}c_{14}$$

$$+ c_{36}\eta_{35}\bar{\eta}_{25}\eta_{24}\bar{\eta}_{14} + \eta_{36}\bar{\eta}_{26}c_{25}\eta_{24}\bar{\eta}_{14} + \eta_{36}\bar{\eta}_{26}\eta_{25}\bar{\eta}_{15}c_{14}. \tag{A21}$$

[1] D. V. Forero, M. Tortola, and J. W. F. Valle, Global status of neutrino oscillation parameters after Neutrino-2012, Phys. Rev. D **86**, 073012 (2012).

[2] H. Nunokawa, S. J. Parke, and J. W. F. Valle, *CP* violation and neutrino oscillations, Prog. Part. Nucl. Phys. **60**, 338 (2008).

[3] S. Weinberg, Varieties of baryon, and lepton nonconservation, Phys. Rev. D **22**, 1694 (1980).

[4] S. B. Giddings and A. Strominger, Loss of incoherence and determination of coupling constants in quantum gravity, Nucl. Phys. **B307**, 854 (1988).

[5] T. Banks and N. Seiberg, Symmetries and strings in field theory and gravity, Phys. Rev. D **83**, 084019 (2011).

[6] A. de Gouvea and J. W. F. Valle, Minimalistic neutrino mass model, Phys. Lett. B **501**, 115 (2001).

[7] J. W. F. Valle and J. C. Romao, *Neutrinos in High Energy and Astroparticle Physics*, 1st ed. (Wiley-VCH, Berlin, 2015).

[8] J. Schechter and J. W. F. Valle, Neutrino masses in SU(2) × U(1) theories, Phys. Rev. D **22**, 2227 (1980).

[9] J. Schechter and J. W. F. Valle, Neutrino decay and spontaneous violation of lepton number, Phys. Rev. D **25**, 774 (1982).

[10] M. Gell-Mann, P. Ramond, and R. Slansky, Complex spinors and unified theories, Conf. Proc. C **790927**, 315 (1979).

[11] R. N. Mohapatra and G. Senjanovic, Neutrino masses and mixings in gauge models with spontaneous parity violation, Phys. Rev. D **23**, 165 (1981).

[12] R. N. Mohapatra and J. W. F. Valle, Neutrino mass and baryon number nonconservation in superstring models, Phys. Rev. D **34**, 1642 (1986).

[13] M. C. Gonzalez-Garcia and J. W. F. Valle, Fast decaying neutrinos and observable flavor violation in a new class of Majoron models, Phys. Lett. B **216**, 360 (1989).

[14] E. K. Akhmedov, M. Lindner, E. Schnapka, and J. W. F. Valle, Left-right symmetry breaking in NJL approach, Phys. Lett. B **368**, 270 (1996).

[15] E. K. Akhmedov, M. Lindner, E. Schnapka, and J. W. F. Valle, Dynamical left-right symmetry breaking, Phys. Rev. D **53**, 2752 (1996).

[16] M. Malinsky, J. C. Romao, and J. W. F. Valle, Novel Super-symmetric SO(10) Seesaw Mechanism, Phys. Rev. Lett. **95**, 161801 (2005).

[17] S. M. Boucenna, S. Morisi, and J. W. F. Valle, The low-scale approach to neutrino masses, Adv. High Energy Phys. **2014**, 831598 (2014).

[18] P. S. B. Dev and R. N. Mohapatra, TeV scale inverse seesaw in SO(10) and leptonic non-unitarity effects, Phys. Rev. D **81**, 013001 (2010).

[19] C. H. Lee, P. S. Bhupal Dev, and R. N. Mohapatra, Natural TeV-scale left-right seesaw mechanism for neutrinos and experimental tests, Phys. Rev. D **88**, 093010 (2013).

[20] M. Drewes and B. Garbrecht, Experimental and cosmological constraints on heavy neutrinos, arXiv: 1502.00477.

[21] A. Aguilar *et al.* (LSND Collaboration), Evidence for neutrino oscillations from the observation of anti-neutrino (electron) appearance in a anti-neutrino(muon) beam, Phys. Rev. D **64**, 112007 (2001).

[22] A. A. Aguilar-Arevalo *et al.* (MiniBooNE Collaboration), Improved Search for $\bar{\nu}_\mu \to \bar{\nu}_e$ Oscillations in the Mini-BooNE Experiment, Phys. Rev. Lett. **110**, 161801 (2013).

[23] R. E. Shrock, General theory of weak leptonic and semi-leptonic decays. 1. Leptonic pseudoscalar meson decays, with associated tests for, and bounds on, neutrino masses and lepton mixing, Phys. Rev. D **24**, 1232 (1981).

[24] T. Asaka, S. Blanchet, and M. Shaposhnikov, The nuMSM, dark matter and neutrino masses, Phys. Lett. B **631**, 151 (2005).

[25] M. Dittmar, A. Santamaria, M. C. Gonzalez-Garcia, and J. W. F. Valle, Production mechanisms and signatures of isosinglet neutral heavy leptons in Z^0 decays, Nucl. Phys. **B332**, 1 (1990).

[26] M. Z. Akrawy *et al.* (OPAL Collaboration), Limits on neutral heavy lepton production from Z^0 decay, Phys. Lett. B **247**, 448 (1990).

[27] P. Abreu *et al.* (DELPHI Collaboration), Search for neutral heavy leptons produced in Z decays, Z. Phys. C **74**, 57 (1997); **75**, 580(E) (1997).

[28] M. Gronau, C. N. Leung, and J. L. Rosner, Extending limits on neutral heavy leptons, Phys. Rev. D **29**, 2539 (1984).

[29] D. V. Forero, S. Morisi, M. Tortola, and J. W. F. Valle, Lepton flavor violation and non-unitary lepton mixing in low-scale type-I seesaw, J. High Energy Phys. 09 (2011) 142.

[30] J. Schechter and J. W. F. Valle, Neutrinoless double beta decay in SU(2) × U(1) theories, Phys. Rev. D **25**, 2951 (1982).

[31] M. Hirsch, Phenomenology of neutrinoless double beta decay, Nucl. Phys. B, Proc. Suppl. **221**, 119 (2011).

[32] W. Rodejohann, Neutrino-less double beta decay, and particle physics, Int. J. Mod. Phys. E **20**, 1833 (2011).

[33] J. Bernabeu, A. Santamaria, J. Vidal, A. Mendez, and J. W. F. Valle, Lepton flavor nonconservation at high-energies in a superstring inspired Standard Model, Phys. Lett. B **187**, 303 (1987).

[34] J. M. Berryman, A. de Gouvêa Hernández, and R. L. N. Oliviera, Non-unitary neutrino propagation from neutrino decay, Phys. Lett. B **742**, 74 (2015).

[35] H. Hettmansperger, M. Lindner, and W. Rodejohann, Phenomenological consequences of sub-leading terms in see-saw formulas, J. High Energy Phys. 04 (2011) 123.

[36] W. Rodejohann and J. W. F. Valle, Symmetrical parametrizations of the lepton mixing matrix, Phys. Rev. D **84**, 073011 (2011).

[37] G. C. Branco, M. N. Rebelo, and J. W. F. Valle, Leptonic *CP* violation with massless neutrinos, Phys. Lett. B **225**, 385 (1989).

[38] N. Rius and J. W. F. Valle, Leptonic *CP* violating asymmetries in Z^0 decays, Phys. Lett. B **246**, 249 (1990).

[39] S. Antusch, C. Biggio, E. Fernandez-Martinez, M. B. Gavela, and J. Lopez-Pavon, Unitarity of the leptonic mixing matrix, J. High Energy Phys. 10 (2006) 084.

[40] Z. z. Xing, Correlation between the charged current interactions of light, and heavy Majorana neutrinos, Phys. Lett. B **660**, 515 (2008).

[41] Z. z. Xing, A full parametrization of the 6 × 6 flavor mixing matrix in the presence of three light or heavy sterile neutrinos, Phys. Rev. D **85**, 013008 (2012).

[42] E. Nardi, E. Roulet, and D. Tommasini, Limits on neutrino mixing with new heavy particles, Phys. Lett. B 327, 319 (1994).

[43] P. Langacker and D. London, Mixing between ordinary and exotic fermions, Phys. Rev. D 38, 886 (1988).

[44] M. C. Gonzalez-Garcia, A. Santamaria, and J. W. F. Valle, Isosinglet neutral heavy lepton production in Z decays and neutrino mass, Nucl. Phys. B342, 108 (1990).

[45] A. Abada, D. Das, A. M. Teixeira, A. Vicente, and C. Weiland, Tree-level lepton universality violation in the presence of sterile neutrinos: Impact for R_K and R_π, J. High Energy Phys. 02 (2013) 048.

[46] A. Abada, A. M. Teixeira, A. Vicente, and C. Weiland, Sterile neutrinos in leptonic and semileptonic decays, J. High Energy Phys. 02 (2014) 091.

[47] A. Atre, T. Han, S. Pascoli, and B. Zhang, The search for heavy Majorana neutrinos, J. High Energy Phys. 05 (2009) 030.

[48] A. Abada, V. De Romeri, and A. M. Teixeira, Effect of sterile states on lepton magnetic moments and neutrinoless double beta decay, J. High Energy Phys. 09 (2014) 074.

[49] S. Antusch and O. Fischer, Non-unitarity of the leptonic mixing matrix: Present bounds and future sensitivities, J. High Energy Phys. 10 (2014) 94.

[50] K. A. Olive et al. (Particle Data Group Collaboration), Review of particle physics, Chin. Phys. C 38, 090001 (2014).

[51] G. Czapek, A. Federspiel, A. Fluckiger, D. Frei, B. Hahn, C. Hug, E. Hugentobler, W. Krebs et al., Branching Ratio for the Rare Pion Decay into Positron and Neutrino, Phys. Rev. Lett. 70, 17 (1993).

[52] B. Aubert et al. (BABAR Collaboration), Measurements of Charged Current Lepton Universality and $|V_{us}|$ using Tau Lepton Decays to $e^-\bar\nu_e\nu_\tau$, $\mu^-\bar\nu_\mu\nu_\tau$, $\pi^-\nu_\tau$, and $K-\nu_\tau$, Phys. Rev. Lett. 105, 051602 (2010).

[53] M. Freund, Analytic approximations for three neutrino oscillation parameters, and probabilities in matter, Phys. Rev. D 64, 053003 (2001).

[54] E. K. Akhmedov, R. Johansson, M. Lindner, T. Ohlsson, and T. Schwetz, Series expansions for three flavor neutrino oscillation probabilities in matter, J. High Energy Phys. 04 (2004) 078.

[55] C. Giunti, M. Laveder, Y. F. Li, Q. Y. Liu, and H. W. Long, Update of short-baseline electron neutrino and antineutrino disappearance, Phys. Rev. D 86, 113014 (2012).

[56] C. Giunti, M. Laveder, Y. F. Li, and H. W. Long, Pragmatic view of short-baseline neutrino oscillations, Phys. Rev. D 88, 073008 (2013).

[57] G. Mention, M. Fechner, T. Lasserre, T. A. Mueller, D. Lhuillier, M. Cribier, and A. Letourneau, The reactor antineutrino anomaly, Phys. Rev. D 83, 073006 (2011).

[58] J. W. F. Valle, Resonant oscillations of massless neutrinos in matter, Phys. Lett. B 199, 432 (1987).

[59] T. K. Kuo and J. T. Pantaleone, Neutrino oscillations in matter, Rev. Mod. Phys. 61, 937 (1989).

[60] P. Astier et al. (NOMAD Collaboration), Search for $\nu_\mu \to \nu_e$ oscillations in the NOMAD experiment, Phys. Lett. B 570, 19 (2003).

[61] D. I. Britton, S. Ahmad, D. A. Bryman, R. A. Burnbam, E. T. H. Clifford, P. Kitching, Y. Kuno, J. A. Macdonald et al., Measurement of the $\pi^+ \to e^+\nu$ Neutrino Branching Ratio, Phys. Rev. Lett. 68, 3000 (1992).

[62] D. I. Britton, S. Ahmad, D. A. Bryman, R. A. Burnham, E. T. H. Clifford, P. Kitching, Y. Kuno, J. A. Macdonald et al., Improved search for massive neutrinos in $\pi^+ \to e^+\nu$ neutrino decay, Phys. Rev. D 46, R885 (1992).

[63] G. Bernardi, G. Carugno, J. Chauveau, F. Dicarlo, M. Dris, J. Dumarchez, M. Ferro-Luzzi, J.-M. Levy et al., Further limits on heavy neutrino couplings, Phys. Lett. B 203, 332 (1988).

[64] J. Badier et al. (NA3 Collaboration), Mass and lifetime limits on new longlived particles in 300 GeV/c π^- interactions, Z. Phys. C 31, 21 (1986).

[65] J. Dorenbosch et al. (CHARM Collaboration), A search for decays of heavy neutrinos in the mass range 0.5–2.8 GeV, Phys. Lett. 166B, 473 (1986).

[66] D. Liventsev et al. (Belle Collaboration), Search for heavy neutrinos at Belle, Phys. Rev. D 87, 071102 (2013).

[67] O. Adriani et al. (L3 Collaboration), Search for isosinglet neutral heavy leptons in Z^0 decays, Phys. Lett. B 295, 371 (1992).

[68] P. Achard et al. (L3 Collaboration), Search for heavy isosinglet neutrino in e^+e^- annihilation at LEP, Phys. Lett. B 517, 67 (2001).

[69] ATLAS Collaboration, Search for Majorana neutrino production in pp collisions at $\sqrt{s} = 7$ TeV in dimuon final states with the ATLAS detector, Report Nos. ATLAS-CONF-2012-139, ATLAS-COM-CONF-2012-165.

[70] J. Klinger, Search for heavy Majorana neutrinos in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector, Ph.D. thesis, University of Manchester, 2014.

[71] C. Adams et al. (LBNE Collaboration), The long-baseline neutrino experiment: Exploring fundamental symmetries of the Universe, arXiv:1307.7335.

[72] F. F. Deppisch, P. S. B. Dev, and A. Pilaftsis, Neutrinos and collider physics, New J. Phys. 17, 075019 (2015).

[73] R. S. Hayano, T. Taniguchi, T. Yamanaka, T. Tanimori, R. Enomoto, A. Ishibashi, T. Ishikawa, S. Sato et al., Heavy Neutrino Search Using $K_{\pi 2}$ Decay, Phys. Rev. Lett. 49, 1305 (1982).

[74] A. Kusenko, S. Pascoli, and D. Semikoz, New bounds on MeV sterile neutrinos based on the accelerator and Super-Kamiokande results, J. High Energy Phys. 11 (2005) 028.

[75] P. Vilain et al. (CHARM II Collaboration), Search for heavy isosinglet neutrinos, Phys. Lett. B 343, 453 (1995); 351, 387 (1995).

[76] E. Gallas et al. (FMMF Collaboration), Search for neutral weakly interacting massive particles in the Fermilab Tevatron wide band neutrino beam, Phys. Rev. D 52, 6 (1995).

[77] A. M. Cooper-Sarkar et al. (WA66 Collaboration), Search for heavy neutrino decays in the BEBC beam dump experiment, Phys. Lett. 160B, 207 (1985).

[78] A. Vaitaitis et al. (NuTeV and E815 Collaborations), Search for Neutral Heavy Leptons in a High-Energy Neutrino Beam, Phys. Rev. Lett. 83, 4943 (1999).

[79] A. V. Artamonov *et al.* (E949 Collaboration), Search for heavy neutrinos in $K^+ \to \mu^+ \nu_H$ decays, Phys. Rev. D **91**, 052001 (2015); **91**, 059903(E) (2015).

[80] V. Khachatryan *et al.* (CMS Collaboration), Search for heavy Majorana neutrinos in $\mu^\pm \mu^\pm +$ jets events in proton-proton collisions at $\sqrt{s} = 8$ TeV, Phys. Lett. B **748**, 144 (2015).

[81] R. Aaij *et al.* (LHCb Collaboration), Search for Majorana Neutrinos in $B^- \to \pi^+ \mu^- \mu^-$ Decays, Phys. Rev. Lett. **112**, 131802 (2014).

[82] P. Astier *et al.* (NOMAD Collaboration), Search for heavy neutrinos mixing with tau neutrinos, Phys. Lett. B **506**, 27 (2001).

[83] J. Orloff, A. N. Rozanov, and C. Santoni, Limits on the mixing of tau neutrino to heavy neutrinos, Phys. Lett. B **550**, 8 (2002).

[84] J. A. Aguilar-Saavedra, F. Deppisch, O. Kittel, and J. W. F. Valle, Flavour in heavy neutrino searches at the LHC, Phys. Rev. D **85**, 091301 (2012).

[85] S. P. Das, F. F. Deppisch, O. Kittel, and J. W. F. Valle, Heavy neutrinos and lepton flavour violation in left-right symmetric models at the LHC, Phys. Rev. D **86**, 055006 (2012).

[86] F. F. Deppisch, M. Hirsch, and H. Pas, Neutrinoless double beta decay and physics beyond the Standard Model, J. Phys. G **39**, 124007 (2012).

[87] M. Mitra, G. Senjanovic, and F. Vissani, Neutrinoless double beta decay and heavy sterile neutrinos, Nucl. Phys. **B856**, 26 (2012).

[88] F. Simkovic, J. Vergados, and A. Faessler, Few active mechanisms of the neutrinoless double beta-decay and effective mass of Majorana neutrinos, Phys. Rev. D **82**, 113015 (2010).

[89] F. F. Deppisch, N. Desai, and J. W. F. Valle, Is charged lepton flavor violation a high energy phenomenon?, Phys. Rev. D **89**, 051302 (2014).

[90] C. Y. Chen, P. S. B. Dev, and R. N. Mohapatra, Probing heavy-light neutrino mixing in left-right seesaw models at the LHC, Phys. Rev. D **88**, 033014 (2013).

[91] S. Okubo, Note on unitary symmetry in strong interaction. II Excited states of baryons, Prog. Theor. Phys. **28**, 24 (1962).