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Moving fractional branes with background fields: Interaction and tachyon condensation

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We calculate the bosonic boundary state corresponding to a moving fractional Dp-brane in a partially orbifoldized spacetime $\mathbb{R}^{1,d-5} \times \mathbb{C}^2/\mathbb{Z}_2$ in the presence of the Kalb-Ramond field, the $U(1)$ gauge potential and the tachyon field. Using this boundary state, we obtain the interaction amplitude of two parallel moving Dp-branes with the above background fields. Various properties of the interaction will be investigated. In addition, we study effects of the tachyon condensation on a moving fractional Dp-brane with the above background fields through the boundary state formalism.

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I. INTRODUCTION

Boundary states, which first appeared in the literature in Refs. [1,2], have a central role in string theory and D-branes. They have been used to study D-brane properties and their interactions [3,4]. Precisely, the interaction between two D-branes can be described in two different ways: the open and closed string channels. In the open string channel, the interaction amplitude is given by the one-loop diagram of the open string, stretched between two D-branes, [5–7]; hence, it is a quantum process. In the closed string channel, one can describe the interaction between the branes via the tree-level exchange of a closed string that is emitted from the first brane then propagates toward the second one and is absorbed there [8–11]; thus, it is a classical process. In this approach each brane couples to all closed string states via the boundary state corresponding to the brane. This is because the boundary state encodes all properties of the D-branes. However, these two approaches of interaction between the branes are equivalent, and this equivalence is called the open/closed string duality [12].

On the other hand, the D-branes with nonzero background internal fields have shown several interesting properties [13–19]. Therefore, the boundary state formalism for various setups of D-branes in the presence of background fields such as $B_{\mu\nu}$, the $U(1)$ gauge field and tachyon field in the compact spacetime have been investigated. However, among the various setups with two D-branes, the systems with fractional branes have some interesting behaviors [20–25]. For example, in [25] the gauge/gravity correspondence is derived from the open/closed string duality for a system of fractional branes.

Another important issue concerning the D-branes is the stability of them. The stability (instability) of D-branes can be investigated via the open string tachyon condensation [26,27]. This condensation usually leads to the instability and collapse of the D-branes. That is, an unstable D-brane decays into a lower-dimensional unstable D-brane as an intermediate state and, finally, to the closed string vacuum. These concepts have been studied by various methods [28–31]. Since the boundary state completely comprises all properties of the brane, it can be used to investigate the time evolution of the brane through the tachyon condensation process [32–35].

In this paper we use the boundary state method to obtain the interaction amplitude between two parallel moving fractional Dp-branes in a factorizable spacetime with the orbifold structure $\mathbb{R}^{1,d-5} \times \mathbb{C}^2/\mathbb{Z}_2$. We shall consider the Kalb-Ramond field $B_{\mu\nu}$, the $U(1)$ gauge potential and the tachyon field on the world volumes of the branes. In addition, the branes are moving along a common axis which is perpendicular to both of them. Thus, in this setup the generality of the system has been exerted, which drastically affects the interaction of the branes. We shall also study long-time behavior of the interaction amplitude. Besides, we shall investigate effects of tachyon condensation on the stability of a moving fractional D-branes. We shall observe that condensation of the tachyon drastically reduces the dimensions of such branes.

The paper is organized as follows. In Sec. II, we compute the boundary state associated with a moving fractional Dp-brane with various background fields. In Sec. III, we find the interaction amplitude of two parallel such branes, and its behavior for large distances of the branes. In Sec. IV, we examine a moving fractional Dp-brane with various fields under the experience of the tachyon condensation. Section V is devoted to the conclusions.

II. THE BOUNDARY STATE OF D*p*-BRANE

Consider a fractional Dp-brane which lives in the d-dimensional spacetime, including the orbifold $\mathbb{C}^2/\mathbb{Z}_2$, where the $\mathbb{Z}_2$ group acts on the coordinates

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$\{x^a | a = d-4, d-3, d-2, d-1\}$. This orbifold is noncompact, so its fixed points are located at $x^a = 0$. The Dp-brane is stuck at these fixed points.

We start with the following sigma-model action for the closed string,

$$S = -\frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma(\sqrt{-h}h^{ab}G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + \epsilon^{ab}B_{\mu\nu}\partial_a X^\mu\partial_b X^\nu) + \frac{1}{2\pi\alpha'}\int_{\partial\Sigma} d\sigma\left(A_\alpha \partial_\sigma X^\alpha + \frac{i}{2}U_{\alpha\beta}X^\alpha X^\beta\right), \tag{2.1}$$

where the set $\{x^\alpha | \alpha = 0, 1, \ldots, p\}$ represents the brane directions, Σ indicates the world sheet of the closed string, and $\partial\Sigma$ is the boundary of it. The metrics of the world sheet and the d-dimensional spacetime are h_{ab} and $G_{\mu\nu}$, respectively. For simplifying the equations we select the Kalb-Ramond field $B_{\mu\nu}$ to be constant and $G_{\mu\nu} = \eta_{\mu\nu} = \mathrm{diag}(-1, 1, \ldots, 1)$. The tachyon profile is chosen as $T(X) = \frac{i}{4\pi\alpha'}U_{\alpha\beta}X^\alpha X^\beta$ with constant symmetric matrix $U_{\mu\nu}$. We chose the tachyon field only in the world volume of the Dp-brane. For the $U(1)$ gauge potential A_α, which lives on the world volume of the brane, we consider the gauge $A_\alpha = -\frac{1}{2}F_{\alpha\beta}X^\beta$ where the field strength is constant. Note that the gauge and tachyon fields are in the open string spectrum; hence, their open string state counterparts adhere to the brane.

The vanishing variation of this action defines the following boundary state equations for the closed string,

$$(\partial_\tau X^\alpha + \mathcal{F}^\alpha{}_\beta \partial_\sigma X^\beta - iU^\alpha{}_\beta X^\beta)_{\tau=0}|B_x\rangle = 0,$$
$$(X^I - y^I)_{\tau=0}|B_x\rangle = 0, \tag{2.2}$$

where the coordinates $\{x^I | I = p+1, \ldots, d-1\}$ refer to the directions perpendicular to the brane world volume and the parameters $\{y^I\}$ specify the location of the brane. For more simplification, we assumed that the mixed elements $B^\alpha{}_I$ are zero. The total field strength possesses the definition $\mathcal{F}_{\alpha\beta} = F_{\alpha\beta} - B_{\alpha\beta}$.

Note that because the brane is stuck at the orbifold fixed points, the presence of the orbifold directions puts some prominent constraints on its dimension and motion. In the d-dimensional spacetime, the brane can possess the maximum dimension d-5. Besides, along the orbifoldized directions, it cannot move. Therefore, for adding a velocity to the brane along the perpendicular directions $\{x^i | i = p+1, \ldots, d-5\}$, we apply a boost on the Eqs. (2.2),

$$[\gamma\partial_\tau(X^0 - v^i X^i) + \mathcal{F}^0{}_{\bar\alpha}\partial_\sigma X^{\bar\alpha} - iU^0{}_0\gamma(X^0 - v^i X^i) - iU^0{}_{\bar\alpha}X^{\bar\alpha}]_{\tau=0}|B_x\rangle = 0,$$
$$[\partial_\tau X^{\bar\alpha} + \gamma\mathcal{F}^{\bar\alpha}{}_0\partial_\sigma(X^0 - v^i X^i) + \mathcal{F}^{\bar\alpha}{}_{\bar\beta}\partial_\sigma X^{\bar\beta} - iU^{\bar\alpha}{}_0\gamma(X^0 - v^i X^i) - iU^{\bar\alpha}{}_{\bar\beta}X^{\bar\beta}]_{\tau=0}|B_x\rangle = 0,$$
$$[\gamma(X^i - v^i X^0) - y^i]_{\tau=0}|B_x\rangle = 0,$$
$$[X^a - y^a]_{\tau=0}|B_x\rangle = 0, \tag{2.3}$$

where $\gamma = 1/\sqrt{1 - v^i v^i}$, the set $\{x^{\bar\alpha}\}$ shows the directions of the brane, and the set $\{x^i\}$ indicates the directions perpendicular to its world volume except the orbifoldized directions. Since the branes are stuck at the orbifold fixed points, we have $y^a = 0$.

The mode expansion of the closed string coordinates along the nonorbifold directions x^α and x^i has the feature

$$X^\lambda(\sigma,\tau) = x^\lambda + 2\alpha' p^\lambda \tau + \frac{i}{2}\sqrt{2\alpha'}\sum_{m\neq 0}\frac{1}{m}(\alpha^\lambda_m e^{-2im(\tau-\sigma)} + \tilde\alpha^\lambda_m e^{-2im(\tau+\sigma)}), \qquad \lambda \in \{\alpha, i\}, \tag{2.4}$$

and for the orbifold directions takes the form

$$X^a(\sigma,\tau) = \frac{i}{2}\sqrt{2\alpha'}\sum_{r\in\mathbb{Z}+\frac{1}{2}}\frac{1}{r}(\alpha^a_r e^{-2ir(\tau-\sigma)} + \tilde\alpha^a_r e^{-2ir(\tau+\sigma)}). \tag{2.5}$$

Now, for simplification, we suppose $U_{0\alpha} = U_{\alpha 0} = 0$. Using the above mode expansions, the boundary state equations (2.3) can be written in terms of the string oscillators and zero modes,

$$[\gamma(\alpha^0_m - v^i\alpha^i_m) - \mathcal{F}^0{}_{\bar\alpha}\alpha^{\bar\alpha}_m + \gamma(\tilde\alpha^0_{-m} - v^i\tilde\alpha^i_{-m}) + \mathcal{F}^0{}_{\bar\alpha}\tilde\alpha^{\bar\alpha}_{-m}]|B_{\mathrm{osc}}\rangle = 0,$$
$$\left[\alpha^{\bar\alpha}_m - \gamma\mathcal{F}^{\bar\alpha}{}_0(\alpha^0_m - v^i\alpha^i_m) - \mathcal{F}^{\bar\alpha}{}_{\bar\beta}\alpha^{\bar\beta}_m + \frac{1}{2m}U^{\bar\alpha}{}_{\bar\beta}\alpha^{\bar\beta}_m + \tilde\alpha^{\bar\alpha}_{-m} + \gamma\mathcal{F}^{\bar\alpha}{}_0(\tilde\alpha^0_{-m} - v^i\tilde\alpha^i_{-m}) + \mathcal{F}^{\bar\alpha}{}_{\bar\beta}\tilde\alpha^{\bar\beta}_{-m} - \frac{1}{2m}U^{\bar\alpha}{}_{\bar\beta}\tilde\alpha^{\bar\beta}_{-m}\right]|B_{\mathrm{osc}}\rangle = 0,$$
$$[\alpha^i_m - v^i\alpha^0_m - \tilde\alpha^i_{-m} + v^i\tilde\alpha^0_{-m})]|B_{\mathrm{osc}}\rangle = 0,$$
$$(\alpha^a_r - \tilde\alpha^a_{-r})|B_{\mathrm{osc}}\rangle = 0, \tag{2.6}$$

$$(\hat{p}^0 - v^i \hat{p}^i)|B\rangle^{(0)} = 0,$$
$$[2\alpha' \hat{p}^{\bar{\alpha}} - iU^{\bar{\alpha}}{}_{\bar{\beta}} \hat{x}^{\bar{\beta}}]|B\rangle^{(0)} = 0,$$
$$(\hat{p}^i - v^i \hat{p}^0)|B\rangle^{(0)} = 0,$$
$$[\gamma(\hat{x}^i - v^i \hat{x}^0) - y^i]|B\rangle^{(0)} = 0. \tag{2.7}$$

Note that we decomposed the boundary state as $|B_x\rangle = |B_{\rm osc}\rangle \otimes |B\rangle^{(0)}$. Since the closed string is emitted (absorbed) at the brane position $x^a = 0$, the zero-mode equations do not have any contribution from X^a's. The second equation of (2.7), in terms of the eigenvalues, implies the relation

$$p^{\bar{\alpha}} = \frac{i}{2\alpha'} U^{\bar{\alpha}}{}_{\bar{\beta}} x^{\bar{\beta}}. \tag{2.8}$$

Thus, in the brane volume the momentum of the emitted (absorbed) closed string depends on its center of mass position. Thus, we deduce that the tachyon field inspires a peculiar potential on the closed string.

Using the coherent state method, the oscillating part of the boundary state possesses the solution

$$|B_{\rm osc}\rangle = \prod_{n=1}^{\infty} [\det M_{(n)}]^{-1} \exp\left[-\sum_{m=1}^{\infty}\left(\frac{1}{m}\alpha^{\lambda}_{-m} S_{(m)\lambda\lambda'} \tilde{\alpha}^{\lambda'}_{-m}\right)\right] \times \exp\left[-\sum_{r=1/2}^{\infty}\left(\frac{1}{r}\alpha^{a}_{-r}\tilde{\alpha}^{a}_{-r}\right)\right]|0\rangle_{\alpha}|0\rangle_{\tilde{\alpha}}, \tag{2.9}$$

where the infinite product comes from path integral and can be learned by the Refs. [36,37]. Note that $\lambda, \lambda' \in \{\alpha, i\}$. The matrix $S_{(m)}$ is defined as $S_{(m)} = M^{-1}_{(m)} N_{(m)}$ with

$$M^0_{(m)\lambda} = \gamma(\delta^0_\lambda - v^i \delta^i_\lambda) - \mathcal{F}^0{}_{\bar{\alpha}} \delta^{\bar{\alpha}}_\lambda,$$
$$M^{\bar{\alpha}}_{(m)\lambda} = \delta^{\bar{\alpha}}_\lambda - \gamma \mathcal{F}^{\bar{\alpha}}{}_0 (\delta^0_\lambda - v^i \delta^i_\lambda) - \left(\mathcal{F}^{\bar{\alpha}}{}_{\bar{\beta}} - \frac{1}{2m} U^{\bar{\alpha}}{}_{\bar{\beta}}\right)\delta^{\bar{\beta}}_\lambda,$$
$$M^i_{(m)\lambda} = \delta^i_\lambda - v^i \delta^0_\lambda.$$
$$N^0_{(m)\lambda} = \gamma(\delta^0_\lambda - v^i \delta^i_\lambda) + \mathcal{F}^0{}_{\bar{\alpha}} \delta^{\bar{\alpha}}_\lambda,$$
$$N^{\bar{\alpha}}_{(m)\lambda} = \delta^{\bar{\alpha}}_\lambda + \gamma \mathcal{F}^{\bar{\alpha}}{}_0 (\delta^0_\lambda - v^i \delta^i_\lambda) + \left(\mathcal{F}^{\bar{\alpha}}{}_{\bar{\beta}} - \frac{1}{2m} U^{\bar{\alpha}}{}_{\bar{\beta}}\right)\delta^{\bar{\beta}}_\lambda,$$
$$N^i_{(m)\lambda} = -\delta^i_\lambda + v^i \delta^0_\lambda. \tag{2.10}$$

Equation (2.9) elaborates that a boundary state describes creation of all closed string states from vacuum, or equivalently it represents a source for closed strings, emitted by the D-brane.

In fact, the coherent state method on the boundary state (2.9) imposes the constraint $S_{(m)} S^T_{(-m)} = \mathbf{1}$, which introduces some relations among the parameters $\{v^i, U_{\bar{\alpha}\bar{\beta}}, \mathcal{F}_{\alpha\beta}\}$; hence, it reduces the number of independent parameters.

The zero-mode part of the boundary state, i.e., the solution of Eqs. (2.7), is given by

$$|B\rangle^{(0)} = \frac{T_p}{2\sqrt{\det(U/4\pi\alpha')}} \times \int_{-\infty}^{\infty} \prod_{\lambda} dp^{\lambda} \exp[-\alpha'(U^{-1})_{\bar{\alpha}\bar{\beta}} p^{\bar{\alpha}} p^{\bar{\beta}}] \times \prod_i \delta\left(\hat{x}^i - v^i \hat{x}^0 - \frac{1}{\gamma} y^i\right) \prod_i |p^i\rangle \prod_{\alpha} |p^{\alpha}\rangle. \tag{2.11}$$

The total boundary state associated with the Dp-brane is exhibited by the following direct product,

$$|B\rangle = |B_{\rm osc}\rangle \otimes |B\rangle^{(0)} \otimes |B_{\rm gh}\rangle,$$

where $|B_{\rm gh}\rangle$ is the boundary state of the anticommuting ghosts,

$$|B_{\rm gh}\rangle = \exp\left[\sum_{m=1}^{\infty}(c_{-m}\tilde{b}_{-m} - b_{-m}\tilde{c}_{-m})\right] \times \frac{c_0 + \tilde{c}_0}{2} |q = 1\rangle |\tilde{q} = 1\rangle. \tag{2.12}$$

Since the ghost fields do not interact with the matter part, their contribution to the boundary state is not affected by the orbifold projection, the brane velocity and the background fields.

III. INTERACTION OF THE Dp-BRANES

In this section we calculate the interaction amplitude between two parallel-moving fractional Dp-branes through the closed string exchange. For this, we compute the overlap of the two boundary states via the closed string propagator, i.e., $\mathcal{A} = \langle B_1 | D | B_2 \rangle$, where $|B_1\rangle$ and $|B_2\rangle$ are the total boundary states corresponding to the branes, and D is the closed string propagator which is accurately defined by

$$D = 2\alpha' \int_0^{\infty} dt e^{-tH_{\rm closed}}.$$

The closed string Hamiltonian is the sum of the Hamiltonians of the matter part and ghost part. For the matter part, there is

$$H_{\mathrm{matter}} = \alpha' p^{\lambda} p_{\lambda} + 2\left(\sum_{n=1}^{\infty}(\alpha^{\lambda}_{-n}\alpha_{n\lambda} + \tilde{\alpha}^{\lambda}_{-n}\tilde{\alpha}_{n\lambda}) + \sum_{r=1/2}^{\infty}(\alpha^{a}_{-r}\alpha_{ra} + \tilde{\alpha}^{a}_{-r}\tilde{\alpha}_{ra})\right) - \frac{d-4}{6}. \qquad (3.1)$$

The difference of the constant term with the conventional case is a consequence of the orbifold projection on the four directions.

For simplicity we suppose that the branes are moving along the same alignment with the velocities v_1^i and v_2^i. The result of the calculations reveals the following elegant interaction amplitude,

$$\mathcal{A} = \frac{T_p^2 \alpha' V_{\bar{\alpha}}}{2(2\pi)^{d-p-5}} \frac{\prod_{n=1}^{\infty}[\det(M_{(n)1}M_{(n)2})]^{-1}}{\sqrt{\det(U_1/4\pi\alpha')\det(U_2/4\pi\alpha')}} \int_0^{\infty} dt \Bigg[(\det \mathbf{A})^{-1/2} e^{\frac{d-4}{6}t} \times \left(\sqrt{\frac{\pi}{\alpha' t}}\right)^{d-p-5} \exp\left(-\frac{1}{4\alpha' t}\sum_i \left(\frac{y_1^i}{\gamma_1} - \frac{y_2^i}{\gamma_2}\right)^2\right)\Bigg] \times \prod_{n=1}^{\infty}(\det[1 - S_{(n)1}S^T_{(n)2}e^{-4nt}]^{-1}(1-e^{-4nt})^{-2}), \qquad (3.2)$$

where $V_{\bar{\alpha}}$ is the common volume of the branes, and

$$\mathbf{A}_{\bar{\alpha}\bar{\beta}} = 2\alpha' t \delta_{\bar{\alpha}\bar{\beta}} - 2\alpha'[(U_1^{-1})_{\bar{\alpha}\bar{\beta}} - (U_2^{-1})_{\bar{\alpha}\bar{\beta}}]. \qquad (3.3)$$

In the second line the exponential term indicates a damping factor concerning the distance of the branes. In the last line, the determinant part, accompanied by the factor $\prod_{n=1}^{\infty}(1-e^{-4nt})^{-4}$, is the contribution of the oscillators while the advent of $\prod_{n=1}^{\infty}(1-e^{-4nt})^{2}$ is due to the conformal ghosts. The overall factor behind the integral, which depends on the parameters of the system, clarifies a portion of the interaction strength.

A. Interaction of the distant branes

In any interaction theory, the behavior of the interaction amplitude, after a long enough time, gives a trusty long-range forces of the theory. On the other hand, for the distant branes, the massless closed string states make a considerable contribution to the interaction, while the contributions of all massive states, except the tachyon state, are damped.

The orbifold projection specifies some new effects on the large-distance amplitude. This interaction is constructed via the limit $t \to \infty$ of the oscillating part of the general amplitude (3.2). Therefore, the contribution of the graviton, Kalb-Ramond, dilaton and tachyon states on the interaction in the 26-dimensional spacetime is determined by

$$\mathcal{A}_0 = \frac{T_p^2 \alpha' V_{\bar{\alpha}}}{2(2\pi)^{d-p-5}} \frac{\prod_{n=1}^{\infty}[\det(M_{(n)1}M_{(n)2})]^{-1}}{\sqrt{\det(U_1/4\pi\alpha')\det(U_2/4\pi\alpha')}} \times \int^{\infty} dt \left(\sqrt{\frac{\pi}{\alpha' t}}\right)^{d-p-5} \exp\left(-\frac{1}{4\alpha' t}\sum_i\left(\frac{y_1^i}{\gamma_1} - \frac{y_2^i}{\gamma_2}\right)^2\right) \times (\det \mathbf{A})^{-1/2}(e^{11t/3} + [2 + \mathrm{Tr}(S_1 S_2^T)]e^{-t/3}). \qquad (3.4)$$

We applied the limit only on the third line of Eq. (3.2). This is due to the fact that the other factors do not originate from the exchange of the massless and tachyon states. For example, the exponential factor is related to the position of the branes. Appearance of the divergent part is a subsequent of the exchange of the closed string tachyon, due to its negative mass squared. At the limit $t \to \infty$ the second factor in the last line vanishes. This demonstrates that the massless states, i.e., the gravitation, dilaton and Kalb-Ramond, notably do not possess any contribution in the long-distance interaction. In other words, orbifold projection quenches the long-range force. More precisely, this projection manipulated the zero-point energy of the Hamiltonian; hence, this unconventional result was created. Note that the massless states, similar to the massive ones, for usual distances of the branes contribute to the interaction.

One can see that in the noncritical dimension $d = 28$ the quenching factor $e^{-t/3}$ will be removed, and the long-range force is restored. In this case the modified zero-point energy of the Hamiltonian is exactly balanced by two extra dimensions.

IV. INSTABILITY OF A Dp-BRANE UNDER THE TACHYON CONDENSATION

One of the main important aspects of studying the D-branes is determining their stability or instability, which drastically leads to finding the time evolution of them. Generally, adding the tachyonic mode of the open string spectrum to a single D-brane or to a system of D-branes usually makes them unstable. This phenomenon is known as tachyon condensation [26,27]. During this process the dimension of the brane is consecutively reduced, and at the end we receive only closed strings. In this section we examine the behavior of our Dp-brane under the experience of the condensation of the tachyon. Our aim is to see the effects of the fractionality, transverse motion and background fields on the stability of the brane.

Tachyon condensation occurs when some of the elements of the tachyon matrix become infinity. We exhibit the condensation via the limit $U_{pp} \to \infty$. To obtain evolution of the Dp-brane, we apply this limit on the corresponding boundary state. At first we observe that since there is no tachyon matrix element in the orbifold part of the boundary state, the condensation of the tachyon has no effect on this part. This elaborates that fractionality of the brane on its instability is inactive.

The limit $U_{pp} \to \infty$ implies that

$$\lim_{U_{pp}\to\infty}(U^{-1})_{p\bar{\alpha}} = \lim_{U_{pp}\to\infty}(U^{-1})_{\bar{\alpha}p} = 0. \qquad (4.1)$$

Therefore, the dimensional reduction on the exponential factor of Eq. (2.11) takes place; i.e., the matrix $(U^{-1})_{\alpha'\beta'}$ with $\alpha', \beta' \neq p$, which is $(p-1)\times(p-1)$, appears.

The prefactor of the total boundary state is

$$\frac{T_p}{2}\frac{\prod_{n=1}^{\infty}[\det M_{(n)}]^{-1}}{\sqrt{\det(U/4\pi\alpha')}}. \qquad (4.2)$$

Now we find the evolution of this factor after condensation of the tachyon. Thus, we have

$$\lim_{U_{pp}\to\infty}\det U_{p\times p} = U_{pp}\det\tilde{U}_{(p-1)\times(p-1)},$$

where the matrix $\tilde{U}$ is completely similar to U without the last row and the last column. In the same way, for the matrix $M_{(n)}$ we acquire

$$\lim_{U_{pp}\to\infty}\det(M_{(n)})_{(d-4)\times(d-4)} = \frac{1}{2n}U_{pp}\det(\tilde{M}_{(n)})_{(d-5)\times(d-5)}.$$

Again the matrix $\tilde{M}_{(n)}$ is completely similar to $M_{(n)}$ without the $(p+1)$th row and $(p+1)$th column. Adding all these together, we receive the following satisfactory limit for the prefactor (4.2):

$$\frac{T_p}{2}\lim_{U_{pp}\to\infty}\frac{\prod_{n=1}^{\infty}[\det M_{(n)}]^{-1}}{\sqrt{\det(U/4\pi\alpha')}} \to \frac{T_{p-1}}{2}\frac{\prod_{n=1}^{\infty}[\det\tilde{M}_{(n)}]^{-1}}{\sqrt{\det(\tilde{U}/4\pi\alpha')}}. \qquad (4.3)$$

Note that for accomplishing this limit we used the regulation formula $\prod_{n=1}^{\infty}(na) \to \sqrt{2\pi/a}$, and also we introduced the prominent relation between the tensions of a Dp-brane and a D$(p-1)$-brane, i.e., $T_{p-1} = 2\pi\sqrt{\alpha'}T_p$. The Eq. (4.3) clarifies that the total prefactor of the boundary state does not resist against the collapse of the brane.

Now we demonstrate that the matrix $S_{(n)\lambda\lambda'}$ also respects the dimensional reduction of the Dp-brane. To investigate this, for simplicity we suppose that the velocity has only one component along the x^{p+1} direction. In this case, after tachyon condensation all elements of the $(p+1)$th row and $(p+1)$th column of the matrix $S_{(n)\lambda\lambda'}$ vanish, except the element $S_{(n)pp}$ which tends to -1. However, because of the velocity and background fields, elements of the $(p+2)$th row and $(p+2)$th column remain nonzero. We deduce that this part of the boundary state also does not prevent elimination of the x^p direction of the Dp-brane.

For example, the matrix $S_{(n)}$ for a fractional D2-brane, parallel to the x^1x^2 plane with the velocity v along the x^3 direction, at the infrared fixed point $U_{22}\to\infty$ possesses the following feature,

$$\lim_{U_{22}\to\infty}S_{(n)} = \begin{pmatrix}(\Gamma_{(n)})_{4\times4} & 0\\ 0 & -\mathbf{1}_{(d-8)\times(d-8)}\end{pmatrix},$$

$$\Gamma_{(n)} = \begin{pmatrix}\Gamma_{(n)0}{}^{0} & \Gamma_{(n)1}{}^{0} & 0 & \Gamma_{(n)3}{}^{0}\\ \Gamma_{(n)0}{}^{1} & \Gamma_{(n)1}{}^{1} & 0 & \Gamma_{(n)3}{}^{1}\\ 0 & 0 & -1 & 0\\ \Gamma_{(n)0}{}^{3} & \Gamma_{(n)1}{}^{3} & 0 & \Gamma_{(n)3}{}^{3}\end{pmatrix}, \qquad (4.4)$$

where the matrix elements are given by

$$\Gamma_{(n)0}{}^{0} = \frac{\gamma^2(1+v^2)(1+\frac{1}{2n}U_{11}) + E_1^2}{1+\frac{1}{2n}U_{11} - E_1^2},$$

$$\Gamma_{(n)1}{}^{0} = -\frac{2\gamma E_1}{1+\frac{1}{2n}U_{11} - E_1^2},$$

$$\Gamma_{(n)0}{}^{1} = -\frac{2\gamma E_1}{1+\frac{1}{2n}U_{11} - E_1^2},$$

$$\Gamma_{(n)1}{}^{1} = \frac{1-\frac{1}{2n}U_{11} + E_1^2}{1+\frac{1}{2n}U_{11} - E_1^2},$$

$$\Gamma_{(n)3}{}^{0} = -\frac{2\gamma^2 v(1+\frac{1}{2n}U_{11})}{1+\frac{1}{2n}U_{11} - E_1^2},$$

$$\Gamma_{(n)3}{}^{1} = \frac{2\gamma v E_1}{1+\frac{1}{2n}U_{11} - E_1^2},$$

$$\Gamma_{(n)0}{}^{3} = \frac{\gamma^2 v[(1+\frac{1}{2n}U_{11}) + 2E_1^2]}{1+\frac{1}{2n}U_{11} - E_1^2},$$

$$\Gamma_{(n)1}{}^{3} = -\frac{2\gamma v E_1}{1+\frac{1}{2n}U_{11} - E_1^2},$$

$$\Gamma_{(n)3}{}^{3} = \frac{-\gamma^2(1+v^2)(1+\frac{1}{2n}U_{11}) + E_1^2}{1+\frac{1}{2n}U_{11} - E_1^2}.$$

The electric field component is defined by $E_1 = \mathcal{F}_{01}$. In the static case, i.e., $v = 0$, the matrix $\Gamma_{(n)}$ finds the conventional feature; that is, the elements of its last row and last column, except $\Gamma_{(n)33}$, vanish, and the element $\Gamma_{(n)33}$ tends to -1.

V. CONCLUSIONS

In this article we constructed the boundary state of a bosonic closed string, emitted (absorbed) by a moving fractional Dp-brane in the orbifoldized spacetime $\mathbb{R}^{1,d-5} \times \mathbb{C}^2/\mathbb{Z}_2$ in the presence of the Kalb-Ramond field, a $U(1)$ gauge potential and the open string tachyon field. The boundary state equations reveal that in the brane volume the tachyon field induces an exotic potential on the center-of-mass of the closed string.

The interaction amplitude of two parallel moving fractional branes with the same dimension, in the presence of various background fields, was acquired. The variety of the adjustable parameters, i.e., the background fields, velocities, the spacetime and branes dimensions, and the orbifoldized directions, elaborates a generalized amplitude and an adjustable strength for the branes' interactions.

For the large distances of the branes, the behavior of the interaction amplitude was studied. We observed that for the critical dimension $d = 26$, in the large times the contribution of the mediated massless states quickly vanishes. This is purely an effect of the orbifold projection. In the special noncritical dimension, i.e., $d = 28$, the contribution of the massless states reduces to the conventional case; i.e., in this dimension we receive a long-range force. In fact, for each number of the orbifoldized directions one can demonstrate that the damping of the long-range force is compensated by a specific dimension of the noncritical spacetime, while for the other dimensions the long-range force is removed. That is, for some dimensions it is drastically quenched, while for the other dimensions it is divergent.

At the end we specified the effects of the tachyon condensation phenomenon on a moving fractional Dp-brane with various background fields via its corresponding boundary state. We observed that the advent of the fractionality, transverse motion and background fields cannot protect the brane against collapse and dimensional reduction.

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