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Phases of $SU(3)$ gauge theories with fundamental quarks via Dirac spectral density

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We propose that, in $SU(3)$ gauge theories with fundamental quarks, confinement can be inferred from spectral density of the Dirac operator. This stems from the proposition that its possible behaviors are exhausted by three distinct types (Fig. 1). The monotonic cases are standard and entail confinement with valence chiral symmetry breaking (A) or the lack of both (C,C'). The bimodal (*anomalous*) option (B) was frequently regarded as an artifact (lattice or other) in previous studies, but we show for the first time that it persists in the continuum limit, and conclude that it informs of a nonconfining phase with broken valence chiral symmetry. This generalization rests on the following. (α) We show that bimodality in $\mathrm{N}_f = 0$ theory past deconfinement temperature T_c is stable with respect to removal of both infrared and ultraviolet cutoffs, indicating that anomalous phase is not an artifact. (β) We demonstrate that transition to bimodality in $\mathrm{N}_f = 0$ is simultaneous with the loss of confinement: anomalous phase occurs for $T_c < T < T_{ch}$, where T_{ch} is the valence chiral restoration temperature. (γ) Evidence is presented for thermal anomalous phase in $\mathrm{N}_f = 2 + 1$ QCD at *physical* quark masses, whose onset too coincides with the conventional "crossover T_c." We conclude that the anomalous regime $T_c < T < T_{ch}$ is very likely a feature of nature's strong interactions. (δ) Our past studies of zero-temperature $\mathrm{N}_f = 12$ theories revealed that bimodality also arises via purely light-quark effects. As a result, we expect to encounter the anomalous phase on generic paths to valence chiral restoration. We predict its existence also for N_f massless flavors ($T = 0$) in the range $\mathrm{N}_f^c < \mathrm{N}_f < \mathrm{N}_f^{ch}$, where N_f^c could be quite low. Conventional arguments would associate N_f^{ch} with the onset of conformal window.

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I. INTRODUCTION

$SU(3)$ gauge theories with quarks in fundamental representation are important in elementary particle physics. Indeed, the strong dynamics of "real-world" quarks and gluons is believed to be described within this context, and the near-conformal infrared behavior in theories with many massless flavors (just below conformal window) may be relevant for theories of technicolor type. In addition, the systems within the conformal window are attractive test beds of conformal dynamics in four-dimensional quantum field theory.

Since the quarks of nature are not massless, it is desirable to consider all theories of the above type, i.e. including any number of arbitrarily massive quarks. Denoting such theory space at zero temperature as $\mathcal{T}_0$ and at any temperature as $\mathcal{T} \supset \mathcal{T}_0$, this wide landscape involves various kinds of dynamics, expected to be well distinguished by corresponding vacuum properties.[1] In that regard, confinement and spontaneous chiral symmetry breaking (SChSB) dominate the thinking about strongly interacting dynamics. Nevertheless, classifying elements of $\mathcal{T}$ in these terms involves both conceptual and practical issues.

One problem is that SChSB is well defined only when at least a pair of quarks is massless. However, the vacuum of any theory in $\mathcal{T}$ can be probed for its ability to support long-range order via *valence* Goldstone pions (see e.g. [1,2]). Formally, a pair of valence quarks and a pair of action-compensating pseudofermions is added to the system [3]. In the massless valence limit, flavored chiral rotations of valence fields $\eta, \bar{\eta}$ are elevated to symmetries, which the vacuum either respects or not. The latter possibility entails valence spontaneous chiral symmetry breaking (vSChSB), indicated by nonzero value of $\langle \bar{\eta}\eta \rangle$. SChSB and vSChSB are identical notions whenever SChSB is meaningful, but vSChSB provides for a symmetry-based distinction among vacua over the whole set $\mathcal{T}$.

The situation is different in the case of confinement, whose meaning is quite intuitive throughout $\mathcal{T}$, but a satisfactory consensus on its precise definition in such a general context (even within $\mathcal{T}_0$) is lacking. Moreover, existing definitions are technically difficult to verify in a given theory. Convenient symmetry-based interpretation is only available when quarks do not affect vacuum correlations: in pure glue ($\mathrm{N}_f = 0$) theories at finite temperature

[1]Note that, in what follows, we will not differentiate between "vacuum" (zero-temperature concept) and "equilibrium state" (finite-temperature concept). Path integral description of the state (via typical configurations) makes little difference between the two at the technical level.

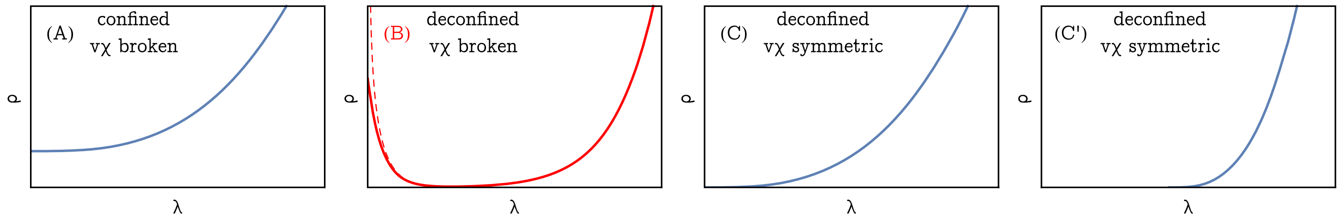

FIG. 1 (color online). Confinement/vSChSB structure of $\mathcal{T}$ via infrared behavior of Dirac spectral density.

[4,5]. In this restricted case, the breakdown of Z_3 symmetry, signaled by nonzero Polyakov line $\langle L\rangle \neq 0$ or its magnitude $\langle |L|\rangle > 0$, distinguishes confined and deconfined dynamics.

Despite issues with its formal definition over $\mathcal{T}$, confinement can have simple and unambiguous dynamical signatures. Indeed, since the phenomenon shapes dynamics in a crucial manner, it is reasonable to expect that even simple vacuum observables are qualitatively affected. While details of this influence depend on the unknown specifics of the confinement mechanism, the logic here is that of reverse engineering: if simple signatures can be empirically developed, consistently with existing results of lattice simulations and with the theoretically clean $N_f = 0$ case, then a valuable insight into the mechanism is obtained. In addition, practical benefits for investigating the phase structure of $\mathcal{T}$ are likely to be acquired.

Following such rationale, we propose that the vSChSB/confinement structure of $\mathcal{T}$ may be inferred from low-energy behavior of Dirac spectral density $\rho(\lambda)$ in these theories. Gauge invariant $\rho(\lambda)$ is thus viewed as a vacuum object already known to encode vSChSB via its "infinitely infrared" behavior [$\rho(\lambda \to 0)$ in infinite volume] and Banks-Casher equivalence [6]. The suggestion is that properties of $\rho(\lambda)$ in the *finite* infrared regime reflect vSChSB/confinement combination as shown in Fig. 1: case (A) corresponds to vSChSB with confinement, (B) to vSChSB without confinement, and (C,C') to valence chiral symmetry and no confinement. Note that option (A) represents generic behavior observed in low-temperature real-world QCD simulations, and (C,C') are standard possibilities for the chirally symmetric vacuum.[2] We refer to the atypical case (B) as *anomalous*. There are a few explanatory comments to make.

(i) Dependencies of Fig. 1 refer to limits approached in asymptotically large volumes. Nonmonotonicities arising e.g. in small volumes at fixed topological charge are not invoked.

(ii) While the Banks-Casher relation is a kinematical constraint, the above connection between vacuum properties and $\rho(\lambda)$ is dynamical, arising due to the nature of strong force.

(iii) Note that confinement without vSChSB is not among the possibilities in $\mathcal{T}$, aligning with conclusions of Ref. [7].

(iv) We implicitly assume a fully chiral lattice definition of valence quark dynamics via the overlap Dirac operator [8]. This makes vSChSB and the proposed classification of phases sharply defined at the regularized level. Note that the nature of phase identification is such that it is immaterial whether bare or renormalized [9] spectral density is used. It is equally immaterial that $\rho(\lambda \to 0)$ (valence chiral condensate) may diverge even at the regularized level in some theories [10]: this just means that the system is necessarily in phase (B).

(v) Bimodality in $\rho(\lambda)$ was first observed on $N_f = 0$ backgrounds [11] above deconfinement temperature T_c. However, its cutoff dependencies have not been systematically studied amidst suspicions (see e.g. [12]) that it may be a lattice artifact. Consequently, substantiating the existence of anomalous phase in $\mathcal{T}$ is central to our discussion.

(vi) Recent observation of anomalous $\rho(\lambda)$ in zero-temperature dynamics with many light flavors [2,13] provided an important impetus for current generalization: since both thermal and light-quark effects induce anomalous phase, it is reasonable to expect its occurrence along generic paths in $\mathcal{T}$ leading to valence chiral symmetry restoration.

(vii) In the current context, the only role of light valence quarks is to probe the vacuum: while not influencing it in any way, their infrared dynamics is sensitive to its long-range properties. Contrasted with type (A), the anomalous density (B) then signifies a different mechanism for Goldstone-like correlations, relevant when confinement is absent.

In what follows, we first discuss new results from lattice QCD needed for this proposal, i.e. points (α)–(γ) outlined in the Abstract. We then proceed to generalize (Fig. 6), taking into account point (δ), and suggest the existence of anomalous phase preceding the conformal window. Selected aspects of our results, conclusions and their consequences, are elaborated upon in the last section.

II. REALITY OF THE ANOMALOUS PHASE

The basic requirement for viability of the proposed general scenario is establishing the existence of deconfined

[2]Case (C') involving strict "gap" in density is very difficult to infer from any practical simulation and may or may not exist in $\mathcal{T}$. However, the distinction between (C) and (C') is immaterial for our purposes.

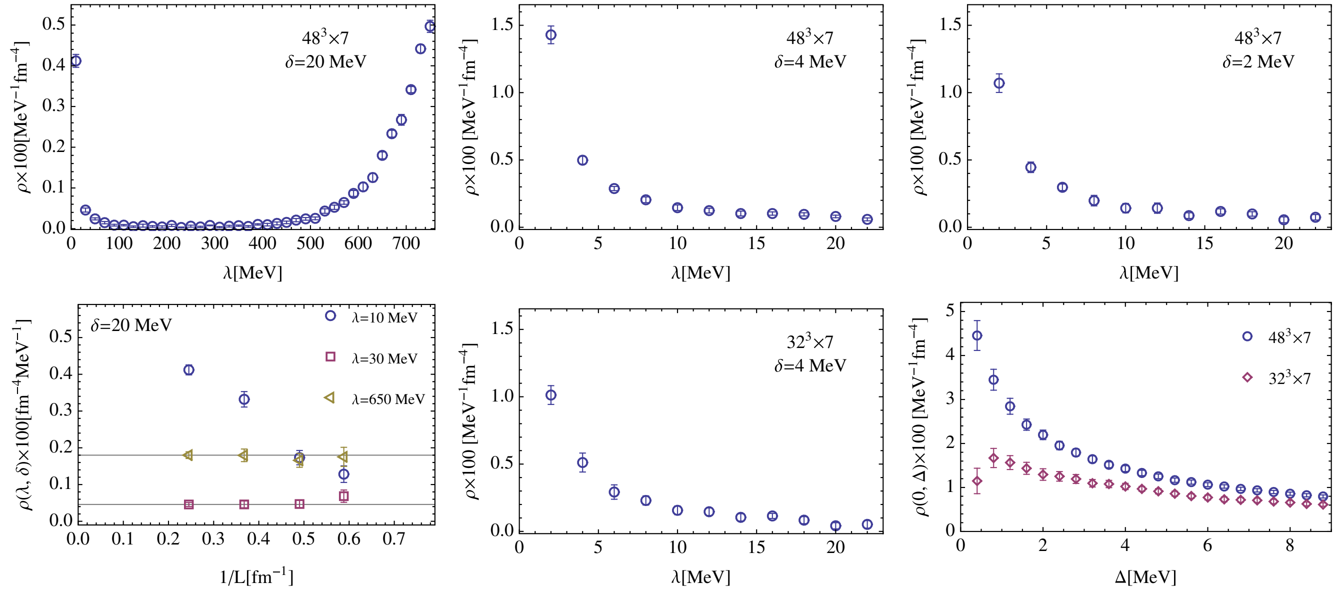

FIG. 2 (color online). Anomalous phase in $N_f = 0$ theory at $T = 1.12T_c$ and lattice spacing $a = 0.085$ fm.

dynamics with vSChSB and anomalous spectral density anywhere in $\mathcal{T}$. Thermal $N_f = 0$ theory is ideal for this purpose since all three elements involved are well defined and lattice accessible. Working with Wilson's lattice theory at $T/T_c = 1.12$ (fixed by $r_0 T_c$ [14]), where we observed bimodality in spectral density previously [1,2], our aim is to perform crucial stability tests with respect to infrared and ultraviolet cutoffs. Our preliminary work in this direction was reported in Ref. [15]. The results below refer to Z_3-broken vacuum with "real Polyakov line" [16], which gets selected when deforming the theory away from $N_f = 0$.

The overlap Dirac operator ($\rho = 26/19$) constructed from its Wilson counterpart ($r = 1$) is used to define valence quark dynamics throughout this paper. Spectral density $\rho(\lambda)$ is the right derivative of cumulative function $\sigma(\lambda) \equiv \langle \sum_{0<\lambda_i<\lambda} 1 \rangle / V$, where V is the 4-volume and λ_i (real numbers) have magnitudes of Dirac eigenvalues and signs of their imaginary parts. Indicated exclusion of exact zeromodes from counting is harmless since their effect vanishes in the infinite-volume limit. Given the finite statistics of any simulation, coarse-graining of $\rho(\lambda)$ is unavoidable. In the absence of suspicion for singularity, we use the symmetric definition away from the origin, namely

$$\rho(\lambda, \delta) \equiv \frac{\sigma(\lambda + \delta/2) - \sigma(\lambda - \delta/2)}{\delta} \quad \lambda \geq \delta/2. \tag{1}$$

When estimating the infinite-volume value of $\rho(\lambda \to 0)$ from finite systems, it is convenient to work with the "right derivative" form, i.e. $\rho(\lambda = 0, \Delta) \equiv \sigma(\Delta)/\Delta$. Valence chiral condensate with overlap is identical to $\pi \lim_{\lambda \to 0} \lim_{V \to \infty} \rho(\lambda, V)$, as is formally in the continuum.

Focusing first on the infrared, we simulated $N^3 \times 7$ systems at $N = 20, 24, 32, 48$. To set $T = 1.12T_c$ requires lattice spacing $a = 0.085$ fm ($\beta = 6.054$, $r_0 = 0.5$ fm), giving linear size $L = 4.1$ fm to the largest system. Low-lying overlap eigensystems were computed and bimodal $\rho(\lambda)$, such as in the top left plot of Fig. 2, was found at all volumes. The spectrum exhibits excellent volume scaling, exemplified in the bottom left plot,[3] except for the most infrared point, which follows a growing trend ensuring the anomalous shape (B) in the infinite-volume limit. The coarse graining $\delta = 20$ MeV is sufficiently fine except for the most infrared bin (0,20) MeV. Here $\rho(\lambda)$ changes rapidly, as the closeup with $\delta = 4$ MeV (top middle) shows. Further lowering of δ (top right) only affects the most infrared point, and we have thus arrived at the resolution revealing the shape of the anomalous peak. Importantly, this shape is also stable under the change of infrared cutoff: comparing $N = 32$ to $N = 48$ (lower middle vs top middle), only accumulation in the lowest bin changes, growing again. We conclude that, in the infinite volume, $\rho(\lambda)$ has a positive (possibly infinite) local maximum at $\lambda = 0$, following behavior (B), with anomalous peak of *finite width* (few MeV).

While anomalous behavior (B) alone implies it, we address valence chiral symmetry breaking explicitly (lower right plot of Fig. 2) via Δ-dependence of $\rho(\lambda = 0, \Delta)$. Note that, in any finite volume, $\lim_{\Delta \to 0} \rho(0, \Delta, V) = 0$, and the associated downturn, shown for $N = 32$, will occur at sufficiently small Δ. However, as expected in broken theory, the break is pushed toward zero with increasing

[3]The horizontal lines guiding the eye are fits to a constant from the three largest systems.

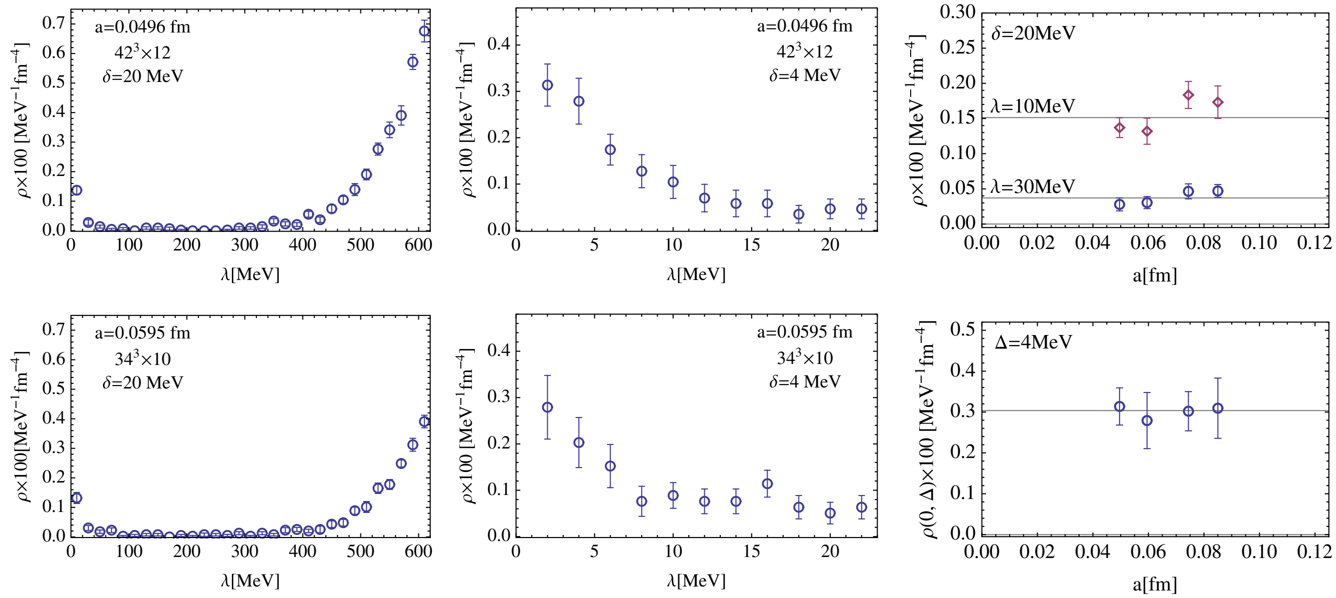

FIG. 3 (color online). Ultraviolet cutoff and anomalous behavior: $\mathrm{N}_f = 0$ theory at $T = 1.12T_c$, $L = 2.0$ fm.

volume. Moreover, the estimates at any fixed Δ are growing, making vSChSB all but certain. At $T = 1.12T_c$ the system is deconfined, and thus all elements of the proposed connection are in place for this case.

Turning to ultraviolet cutoff, we fixed the physical volume of $N = 24$ theory ($L = 2.04$ fm), and drove the system toward continuum limit, while keeping $T/T_c = 1.12$. This resulted in sequence $N \times N_t = 24\times 7, 28\times 8, 34\times 10, 42\times 12$ at $a = 0.0850, 0.0744, 0.0595, 0.0496$ fm respectively. The choice of moderate physical volume was chiefly motivated by computational considerations. While data presented in Fig. 2 indicate volume effects in the infrared, the qualitative behavior is still very clearly anomalous at this volume, and the setup allows us to explore the system rather deep into the continuum limit which is essential.

The bimodal $\rho(\lambda)$ was indeed found at all cutoffs, with global view for the system closest to the continuum displayed in Fig. 3 (top left). The associated closeup with proper resolution is also shown (top middle). Note that, contrary to changing the volume, varying ultraviolet cutoff is not expected to affect very infrared scales significantly, unless a qualitative change of dynamics (phase transition) occurs, separating lattice and continuum-like behaviors. It is decisively the former scenario that is observed, and illustrated by comparisons to the situation at smaller cutoff (lower left and middle plots). The scaling of first and second data point at $\delta = 20$ MeV (top right) indicates that the bimodal shape of $\rho(\lambda)$ will be preserved in the continuum limit at this resolution. Moreover, the accumulation of very near zeromodes, i.e. $\rho(0, \Delta)$ with resolution well within the natural width of the anomalous peak ($\Delta = 4$ MeV), is stable with cutoff (lower right plot), offering no hint of a qualitative change.

Given the demonstrated dramatic volume effects leading to anomalous phase (B) at physically relevant cutoff, and the stability of bimodality at fixed volume under ultraviolet cutoff, we conclude that *continuum* $\mathrm{N}_f = 0$ theory at $T/T_c = 1.12$ is in the anomalous phase.

III. CONFINEMENT AND ANOMALOUS SPECTRAL DENSITY

An important aspect of the proposed general scheme is that the transition from confined to deconfined theory in $\mathcal{T}$ *coincides* with the transition from regular type (A) of $\rho(\lambda)$ to anomalous type (B). To show this for the $\mathrm{N}_f = 0$ thermal transition, we selected a system just below T_c. Replacing $N_t = 7$ of our volume study with $N_t = 8$ (keeping $\beta = 6.054$, $a = 0.085$ fm) corresponds to temperature $T = 0.98T_c$. This nominal assignment uses universal continuum value $r_0 T_c = 0.7498(50)$ of Ref. [14], and T/T_c at finite cutoff is expected to shift slightly upwards. We thus checked the behavior of Polyakov loop in large volumes up to $N = 48$, and confirmed that the system at hand is indeed confined and at the very edge of the transition.

Figure 4 (left) shows spectral density for theory so tuned, on the $20^3 \times 8$ lattice. Type (A) behavior is found, as predicted, with flat dependence at low energies indicating a system on the verge of the transition. To compare this to the situation at temperature just above deconfinement point, we increased the coupling slightly ($\beta = 6.0783$, $a = 0.0817$ fm) to set $T = 1.02T_c$. Spectral density for such a theory on the $36^3 \times 8$ lattice (middle) shows a dramatic change to sharp bimodal behavior: there is little doubt that the Z_3 deconfinement transition coincides with the transition to anomalous phase. Note that we also show

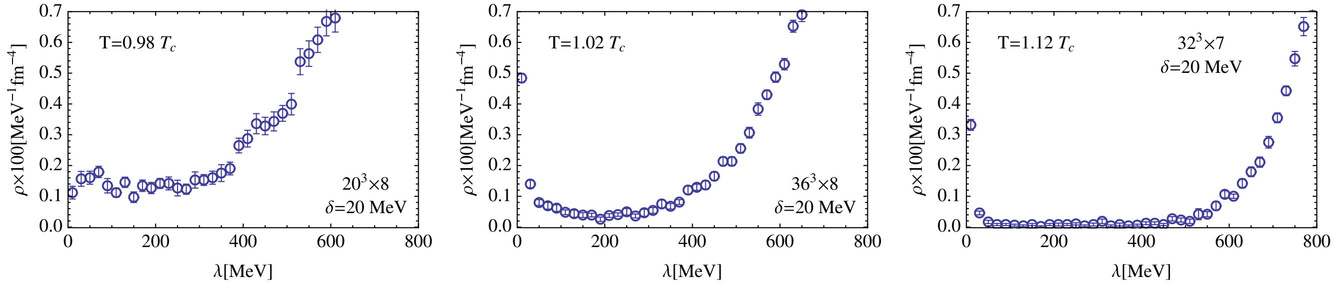

FIG. 4 (color online). Thermal transitions to deconfined and anomalous phase coincide in $N_f = 0$ theory.

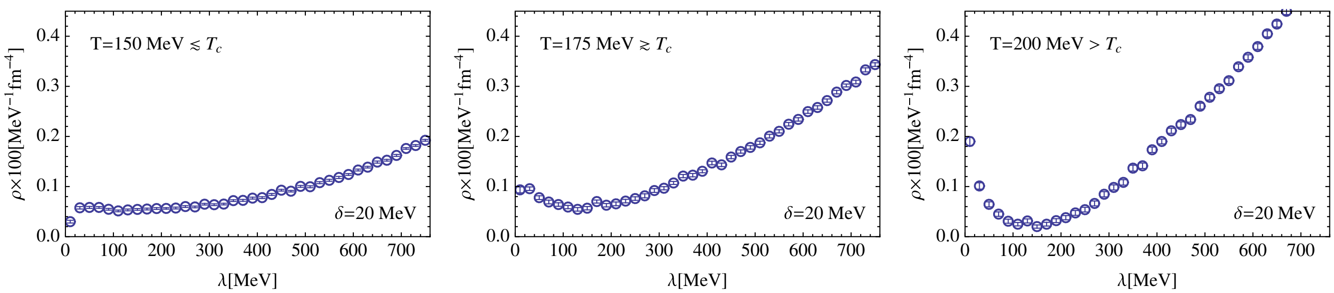

FIG. 5 (color online). Anomalous phase across the thermal crossover in $N_f = 2+1$ QCD at the physical point.

the result for system at $T = 1.12T_c$ ($32^3 \times 7, \beta = 6.054$) in a volume comparable to the $T = 1.02T_c$ case, exemplifying the situation well inside the anomalous region (right).

IV. THERMAL ANOMALOUS PHASE OF "REAL WORLD" QCD

The above evidence of anomalous phase in $N_f = 0$ QCD, and its association with deconfinement, gives an essential impetus for its existence in other corners of $\mathcal{T}$. It is of particular interest to clarify whether anomalous dynamics is part of physical reality for nature's quarks and gluons. We address this via $N_f = 2+1$ QCD at *physical* quark masses, i.e. light mass of $(m_u + m_d)/2$ and heavy of m_s, since it is well established that this theory provides for very precise representation of relevant strongly interacting physics. Among other things, calculations in this context led to the conclusion that thermal transition of strong interactions is a crossover [17].

To probe the crossover region, we utilize $N_t = 8$ lattice ensembles of the Wuppertal-Budapest group (see [18] and references therein), used in precise determination of transition temperatures. On a technical side, the simulated theory involves Symanzik-improved (tree level) gauge action and stout-improved staggered fermions. The physical line of constant physics was defined via fixing f_K/m_π, f_K/m_K and both scaling violations and taste-splitting staggered fermion effects were shown to be small at gauge couplings involved. Due to its crossover nature, transition temperature is not a unique concept and depends both on the observable used and the defining condition chosen. For reference, the temperatures associated with inflection points of (light) scalar density ("condensate") and Polyakov line were reported as $T_c \approx 155$ and 170 MeV respectively. Given that, we selected ensembles at temperatures $T = 150, 175$ and 200 MeV, to be close to both ends of the crossover, and to examine the possible presence of anomalous phase past the transition region.

The resulting overlap spectra are shown in Fig. 5. On the lower edge of the crossover (left plot), spectral density is extremely flat in the infrared, but follows the standard (A) behavior. On the other hand, just past the Polyakov line crossover temperature (middle plot), the anomalous behavior clearly sets in. There is thus little doubt that the appearance of anomalous phase closely follows the conventional measures for expected transition to deconfinement, i.e. conventional "T_c." At $T = 200$ MeV, well above the established transition region, the anomalous phase fully develops, featuring prominent anomalous peak and the large degree of depletion at intermediate scales.

Given the extensive checks performed on the ensembles used, both with respect to continuum limit and sufficiency of volumes[4] (see [18] and references therein), we conclude that the anomalous phase $T_c < T < T_{ch}$, i.e. deconfined phase with broken valence chiral symmetry, exists in high-temperature dynamics of strong interactions, and follows the general scheme proposed here.

[4]The smallest linear size involved here is $L = 3.9$ fm at $T = 200$ MeV.

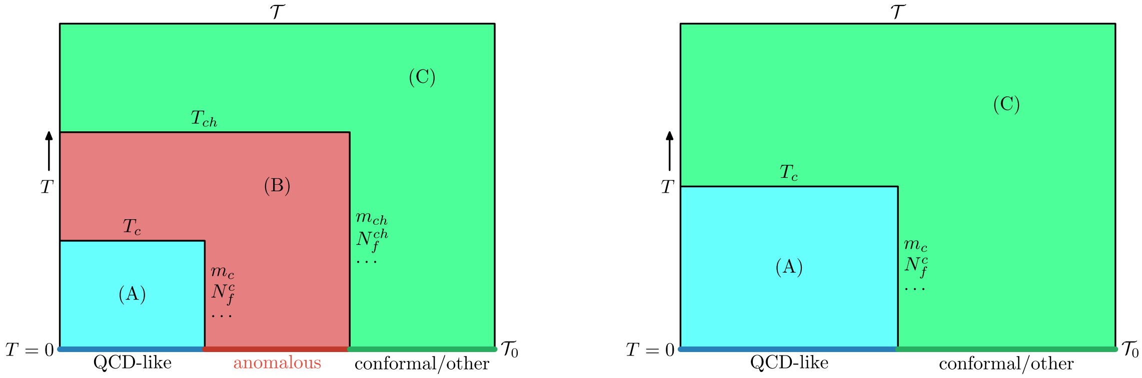

FIG. 6 (color online). The structure of set $\mathcal{T}$ proposed here (left) and the conventional one (right).

V. GENERIC ANOMALOUS PHASE, N_f^c AND N_f^{ch}

We have presented evidence that anomalous phase with claimed properties *exists* in $\mathcal{T}$, as well as evidence that thermal effects lead to bimodal $\rho(\lambda)$ rather generically. Regarding the latter, apart from results presented here, observation of bimodal behavior in $SU(N)$ theories was reported under varied but always thermal circumstances, e.g. in Refs. [1,2,10–12,19–21]. An important additional ingredient is provided by the recent finding that the presence of light dynamical quarks, without any thermal agitation, also leads to anomalous phases at a sufficiently large number of flavors [2,13]. Thermal and light-quark effects thus appear to be analogous in this regard. The relevance of this stems from the fact that thermal effects (increasing T), light-quark effects (decreasing masses, increasing N_f), and their combinations, are the only available freedoms in $\mathcal{T}$ to facilitate the transition from broken valence chiral symmetry to its restoration. This leads us to propose that anomalous phases commonly occur on paths to chirally symmetric vacua. Our evidence on simultaneity of transitions to deconfinement and anomalous $\rho(\lambda)$ (in known cases) completes the rational basis for the proposed picture.

Note that for $\mathrm{N}_f = 12$ theories, the above scenario was seen [2,13] to imply the existence of a mass m_c below which the anomalous phase appears. The phase may either extend down to $m = 0$ or to nonzero $m_{ch} < m_c$, namely the possible point of valence chiral restoration. Here we wish to point out another special case of the above general argument, concerning the "path" in $\mathcal{T}$ parametrized by the number of massless quark species N_f at $T = 0$. Indeed, it is widely believed that the $\mathrm{N}_f = 2$ case represents a confining, chirally broken theory with type (A) spectral density. At the same time, since the work of Banks and Zaks [22], it is expected that there is a critical number of flavors $\mathrm{N}_f^{cr} < 16.5$, such that theories in the window $\mathrm{N}_f^{cr} < \mathrm{N}_f < 16.5$ are both asymptotically free, and controlled at low energy by a conformal infrared fixed point.[5] Theories in the conformal window cannot generate low-energy scales (at least not below the scale of conformality), and the standard scenario is that N_f^{cr} marks the common transition to deconfined, chirally symmetric phase with $\rho(\lambda)$ of type (C).

However, based on the proposed picture, we predict that when increasing the number of flavors from $\mathrm{N}_f = 2$, quark-gluon dynamics will first lose confinement (at N_f^c), and only then chiral symmetry breaking (at N_f^{ch}), generating the anomalous phase for

$$2 < \mathrm{N}_f^c < \mathrm{N}_f < \mathrm{N}_f^{ch} = \mathrm{N}_f^{cr}. \tag{2}$$

The last equality holds if the current view associating the onset of conformal window with chiral symmetry restoration is valid. It is important in this regard that $\mathrm{N}_f = 12$ may be in the anomalous phase [2,13]. Indeed, the setup with yet fewer flavors is even more conducive to the possibility of anomalous vSChSB surviving masslessness (SChSB), since less of a condensate-destructive light-quark effect is generated. N_f^c could thus be quite low.

Schematic structure of the set $\mathcal{T}$, we are proposing, is shown in Fig. 6 (left). At $T = 0$ (set $\mathcal{T}_0$) there are theories with all three types of $\rho(\lambda)$ and associated vacuum properties. We refer to type (A) as QCD-like for this purpose. Type (C) is expected to contain theories from conformal window and e.g. theories without asymptotic freedom. Heating up a QCD-like system generically involves both deconfinement (T_c) and separately valence chiral restoration (T_{ch}), while anomalous type (B) theory only undergoes the latter, and type (C) neither. Moving within $\mathcal{T}$ at fixed temperature involves analogous transitions. Note that other kinds of changes, unrelated to vSChSB and confinement, may occur within the three regions. Also, we do not imply that phases (A) and (C) are strictly disconnected, with the

[5]Note that we consider N_f to be an integer, while boundary values such as N_f^{cr}, N_f^c and N_f^{ch} half integers.

anomalous phase occurring along every path between them: the intention is to convey a typical situation.

VI. IMPORTANT POINTS

In this paper, we have proposed a specific association between infrared behavior of Dirac spectral density, and confinement/valence chiral symmetry breaking structure of $SU(3)$ gauge theories with fundamental quarks (Fig. 1). There are a few points we wish to discuss and highlight in this regard.

(I) A significant conceptual step implied by our analysis and represented by Fig. 6, is that valence chiral symmetry breaking without confinement is commonplace for $SU(3)$ gauge theories with fundamental quarks: the schematic view on the left, with many new transitions, is proposed to be a good representation of the situation, as opposed to the standard view on the right. The *existence* of anomalous phase in $\mathcal{T}$, shown here, is pivotal for this transformation.

(II) The proposed association of Dirac spectral density with vacuum properties turns it into a valuable tool for investigating the phase structure of $\mathcal{T}$. Indeed, recall that standard methods, such as those used in thermal lattice QCD, are "transitional" in nature, requiring the study of *changes* in the theory space. On the other hand, utilizing $\rho(\lambda)$ is as convenient as order parameter at hand: it provides a definite answer for any standalone theory. A single-number indicator of anomalous phase ("order parameter") can be defined as follows. Let λ_{an} be the largest scale such that $\rho(\lambda, V \to \infty)$ strictly decreases on $[0, \lambda_{an})$. Then

$$\Omega_{an} \equiv \sigma(\lambda_{an}) > 0 \tag{3}$$

i.e. a nonzero volume density of anomalous modes, identifies the theory in the anomalous phase.[6] This definition makes Ω_{an} analogous in nature to Ω, the volume density of chirally polarized modes, which was proposed to be a valid order parameter of vSChSB [1,2].

(III) The importance of concluded anomalous (deconfined with broken valence chiral symmetry) phase in physics of quarks and gluons is hard to overstate. Indeed, the temperature regime involved is relevant for strongly interacting dynamics associated with the creation of the plasmalike state currently studied at RHIC and LHC. While the bimodal aspect of $\rho(\lambda)$ did not remain completely unnoticed with regard to quark-gluon plasma phenomenology [23], the reality of the anomalous phase, and the associated details, provide a strong rationale for detailed exploration of compatible models. We will focus on such issues in a separate study.

The emerging evidence for strong interactions generating two "high-temperature" phases, rather than one, complicates the associated physics in an intriguing way. This complexity can be seen in chiral polarization properties of Dirac modes. Indeed, our results imply that, past the thermal crossover, quark dynamics generates rich multilayered chiral structure in Dirac spectrum, shown in Fig. 17 of Ref. [2] (left plot).

(IV) If the proposed anomalous phase separating QCD-like behavior and infrared conformality materializes, it is expected to be instrumental for understanding of massless quark dynamics, and of transition to conformality in particular. It may also complicate some options being considered for realization of the walking technicolor scenario [24,25]. While it is exactly the near-conformal theories that are of interest for that purpose, one should emphasize that dynamics in the anomalous phase is still strongly coupled, and that deconfinement does not imply a complete absence of hadronic bound states. The theories from "anomalous window" could thus still be relevant in the context of walking technicolor.

[6] Note that λ_{an} itself is such an indicator.

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Note added.—A few days before the first version of this manuscript was released, the work relevant for point (γ) of the Abstract also appeared [26]. While the discussion of [26] is carried out in the context of the $U_A(1)$ restoration problem, their results in $N_f = 2 + 1$ theories with light mass not far from physical, are consistent with and corroborate our conclusion that thermal effects generically lead to the anomalous phase (as defined here).

[1] A. Alexandru and I. Horváth, Phys. Lett. B **722**, 160 (2013).
[2] A. Alexandru and I. Horváth, Nucl. Phys. **B891**, 1 (2015).
[3] A. Morel, J. Phys. II (France) **48**, 1111 (1987).
[4] A. M. Polyakov, Phys. Lett. B **72**, 477 (1978).
[5] B. Svetitsky and L. G. Yaffe, Nucl. Phys. **B210**, 423 (1982).
[6] T. Banks and A. Casher, Nucl. Phys. **B169**, 103 (1980).
[7] A. Casher, Phys. Lett. **83B**, 395 (1979).
[8] H. Neuberger, Phys. Lett. B **417**, 141 (1998); **427**, 353 (1998).
[9] L. Del Debbio, L. Giusti, M. Lüscher, R. Petronzio, and N. Tantalo, J. High Energy Phys. 02 (2006) 011.
[10] J. E. Kiskis and R. Narayanan, Phys. Rev. D **64**, 117502 (2001).
[11] R. G. Edwards, U. M. Heller, J. E. Kiskis, and R. Narayanan, Phys. Rev. D **61**, 074504 (2000).
[12] P. V. Buividovich, E. V. Luschevskaya, and M. I. Polikarpov, Phys. Rev. D **78**, 074505 (2008).
[13] A. Alexandru and I. Horváth, arXiv:1412.1777.
[14] S. Necco, Nucl. Phys. **B683**, 137 (2004).
[15] A. Alexandru and I. Horváth, *Proc. Sci.*, Lattice2014 (2014) 336 [arXiv:1409.7094].
[16] S. Chandrasekharan and N. Christ, Nucl. Phys. B, Proc. Suppl. **47**, 527 (1996).
[17] Y. Aoki, G. Endrődi, Z. Fodor, S. D. Katz, and K. K. Szabó, Nature (London) **443**, 675 (2006).
[18] S. Borsányi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, C. Ratti, and K. K. Szabó, J. High Energy Phys. 09 (2010) 073.
[19] G. Cossu, S. Aoki, H. Fukaya, S. Hashimoto, T. Kaneko, H. Matsufuru, and J.-I. Noaki, Phys. Rev. D **87**, 114514 (2013); **88**, 019901(E) (2013).
[20] M. Buchoff *et al.*, Phys. Rev. D **89**, 054514 (2014).
[21] S. Sharma *et al.*, *Proc. Sci.*, Lattice2013 (2013) 164 [arXiv:1311.3943].
[22] T. Banks and A. Zaks, Nucl. Phys. **B196**, 189 (1982).
[23] T. Kalaydzhyan, Nucl. Phys. **A913**, 243 (2013).
[24] K. Yamawaki, M. Bando, and K.-i. Matumoto, Phys. Rev. Lett. **56**, 1335 (1986).
[25] W. Appelquist, D. Karabali, and L. C. R. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986).
[26] V. Dick, F. Karsch, E. Laermann, S. Mukherjee, and S. Sharma, Phys. Rev. D **91**, 094504 (2015).