Constraining higher derivative supergravity with scattering amplitudes

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We study supersymmetry constraints on higher derivative deformations of type IIB supergravity by consideration of superamplitudes. Combining constraints of on-shell supervertices and basic results from string perturbation theory, we give a simple argument for the nonrenormalization theorem of Green and Sethi, and some of its generalizations.

Supersymmetry constraints on higher derivative couplings in maximal supergravity theories have been investigated extensively in the past [1–7] and have led to remarkable exact results on the quantum effective action of string theory. The method of Refs. [3,8–10] in obtaining constraints on higher derivative terms in gauge and gravity theories with maximal supersymmetry was by explicitly analyzing supersymmetry variations of fields and the Lagrangian and their deformations, starting from the purely fermionic terms. In this paper we present a simple argument for such nonrenormalization theorems from scattering amplitudes, in the context of ten-dimensional type IIB supergravity and its deformations, largely inspired by the work of Refs. [11–13] on the classification of supergravity counterterms [14–16] using amplitudes.

To begin with, we recall the spinor helicity formulation of superamplitudes in type IIB supergravity [17,18]. A ten-dimensional null momentum p^m and the corresponding (constrained) spinor helicity variables λ_A^α are related by

$$p^m \delta_{AB} = \Gamma_{\alpha\beta}^m \lambda_A^\alpha \lambda_B^\beta, \qquad (1)$$

where α is a chiral spinor index of $SO(1,9)$ and A is a spinor index of the $SO(8)$ little group. The $2^8 = 256$ states in the supergraviton multiplet are built from monomials in a set of Grassmann variables η_A. The supermomentum is then defined as

$$q^\alpha = \lambda_A^\alpha \eta^A. \qquad (2)$$

A typical n-point superamplitude takes the form [19]

$$\mathcal{A} = \delta^{10}(P) \delta^{16}(Q) \mathcal{F}(\lambda_i, \eta_i), \qquad (3)$$

where $P = \sum_{i=1}^n p_i$, and the 32 supercharges that act on the n-particle asymptotic states can be expressed as

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$$Q^\alpha = \sum_{i=1}^n q_i^\alpha, \qquad \tilde{Q}^\alpha = \sum_{i=1}^n \lambda_i^{\alpha A} \frac{\partial}{\partial \eta_i^A}. \qquad (4)$$

They obey $\{Q^\alpha, \tilde{Q}^\beta\} = \frac{1}{2}\Gamma_m^{\alpha\beta} P^m$. The nontrivial supersymmetry Ward identities on $\mathcal{A}$ are

$$\delta^{10}(P)\delta^{16}(Q)\tilde{Q}^\alpha[\mathcal{F}(\lambda_i, \eta_i)] = 0. \qquad (5)$$

We can write the *CPT* conjugate of the amplitude $\mathcal{A}$ as

$$\bar{\mathcal{A}} = \delta^{10}(P)\tilde{Q}^{16}\mathcal{F}(\lambda_i, \partial/\partial\eta_i) \prod_{i=1}^n \eta_i^8. \qquad (6)$$

Evidently, if $\mathcal{A}$ obeys supersymmetry Ward identities, so does $\bar{\mathcal{A}}$.

Now let us focus on supervertices, namely superamplitudes with no poles in momenta. There are three basic types of supervertices we can write down. First, we can take $\mathcal{F}(\lambda_i, \eta_i)$ to be independent of η_i, namely

$$\mathcal{F}(\lambda_i, \eta_i) = f(s_{ij}), \qquad (7)$$

where $s_{ij} = -(p_i + p_j)^2 = -2p_i \cdot p_j$. The *CPT* conjugate of this construction gives another supervertex. We refer to these as F-term vertices [20]. A third type of supervertex (D-term) is given by

$$\delta^{10}(P)\delta^{16}(Q)\tilde{Q}^{16}h(\lambda_i, \eta_i). \qquad (8)$$

Here h is an arbitrary function of the spinor helicity variables. All supervertices we know of are of these three types. We conjecture that these are in fact the only supervertices that obey supersymmetry Ward identities, and we proceed with this assumption [23].

Let us inspect a particularly simple set of $n = (4 + k)$-point F-term vertices, with $\mathcal{F}(\lambda_i, \eta_i) = 1$,

$$\delta^{10}(P)\delta^{16}(Q). \qquad (9)$$

In component fields, we expand the axion-dilaton field as $\tau = \tau_0 + \varphi$, where τ_0 is the background value. Such a vertex then corresponds to an independent set of couplings in the Lagrangian of the form [2,4]

$$\varphi^k R^4 + \cdots. \tag{10}$$

Similarly, the conjugate vertex

$$\delta^{10}(P)\tilde{Q}^{16} \prod_{i=1}^{4+k} \eta_i^8 \tag{11}$$

corresponds to the coupling $\bar{\varphi}^k R^4 + \cdots$. Note that in the $k = 0$ case, $\delta^{16}(Q) = \tilde{Q}^{16}\prod_{i=1}^4 \eta_i^8$ is self-conjugate, and corresponds to the R^4 vertex [24]. In particular, we see that there are *no independent* supervertices of the form $\varphi^k\bar{\varphi}^\ell R^4 + \cdots$ with $k, \ell \geq 1$. In other words, the supersymmetry completion of such couplings must be a super-amplitude nonlocal in momenta.

Note that in a superamplitude, two $SO(8)$ little group invariant monomials in η_i^A, namely 1 and η_i^8, correspond to the ith external particle being φ and $\bar{\varphi}$, respectively. The nonlinearly realized $SL(2, \mathbb{R})$ of type IIB supergravity is broken by the expectation value of τ to a $U(1)$ [25], which acts on the amplitude by $\sum_i(\frac{1}{4}\eta_i \frac{\partial}{\partial\eta_i} - 1)$ and assigns opposite charges to φ and $\bar{\varphi}$. This $SL(2, \mathbb{R})$ is generally broken explicitly by the higher derivative supervertices of consideration here.

Now, we would like to constrain the coupling

$$f(\tau, \bar{\tau})R^4 + \cdots \tag{12}$$

by type IIB supersymmetry. In a vacuum in which τ acquires a constant expectation value τ_0, expanding $\tau = \tau_0 + \varphi$, we obtain a series of operators,

$$f(\tau_0, \bar{\tau}_0)R^4 + \partial_\tau f(\tau_0, \bar{\tau}_0)\varphi R^4 + \partial_{\bar{\tau}}f(\tau_0, \bar{\tau}_0)\bar{\varphi}R^4$$
$$+ \partial_\tau\partial_{\bar{\tau}}f(\tau_0, \bar{\tau}_0)\varphi\bar{\varphi}R^4 + \cdots. \tag{13}$$

Since there are independent φR^4 and $\bar{\varphi}R^4$ supervertices, $\partial_\tau f$ and $\partial_{\bar{\tau}}f$ can take an arbitrary value at $\tau = \tau_0$. This reflects a freedom in adjusting $f(\tau, \bar{\tau})$ by a holomorphic and an antiholomorphic function of τ. On the other hand, $\partial_\tau\partial_{\bar{\tau}}f$ at $\tau = \tau_0$ is not independent, because there is no independent $\varphi\bar{\varphi}R^4$ vertex. This 6-point coupling, therefore, must be constrained in terms of the R^4 coefficient, namely $f(\tau_0, \bar{\tau}_0)$, by supersymmetry.

In principle, one can ask for the most general 6-point superamplitude that obeys supersymmetry Ward identities and factorization through lower point amplitudes by unitarity. By dimensional analysis, the 6-point φ-$\bar{\varphi}$-R^4 super-amplitude could only factorize through a single R^4 supervertex and supergravity vertices (Fig. 1). The $\varphi\bar{\varphi}R^4$
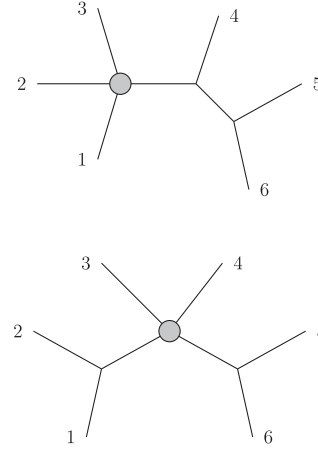

FIG. 1. Factorization of the 6-point amplitude through one $D^n R^4$ vertex (shaded blob) and a pair of supergravity cubic vertices.

coupling itself can then be recovered by taking the soft limit on a pair of φ and $\bar{\varphi}$ scalar lines [26].

We do not know a systematic way of building higher point superamplitudes with the R^4 on-shell supervertex [27]. However, from unitarity, we know that such a relation must exist, and is linear in this case, namely

$$(\mathrm{Im}\tau_0)^2\partial_\tau\partial_{\bar{\tau}}f(\tau_0, \bar{\tau}_0) \propto f(\tau_0, \bar{\tau}_0), \tag{14}$$

where the $(\mathrm{Im}\tau_0)^2$ factor comes from the normalization of the dilaton-axion kinetic term. To determine the relative coefficient, it suffices to find *any* set of such couplings that solve the supersymmetry and unitarity constraints. String perturbation theory already gives such a solution. Since the tree-level effective action of type IIB string theory contains an R^4 coupling at α'^3 order, it suffices to examine this coupling in the Einstein frame, which takes the form

$$\tau_2^{3/2}R^4, \tag{15}$$

where τ_2 is the imaginary part of τ.

Since $\partial_\tau\partial_{\bar{\tau}}\tau_2^{3/2} = \frac{3}{16}\tau_2^{-1/2}$, we immediately obtain the relation

$$4(\mathrm{Im}\tau_0)^2\partial_\tau\partial_{\bar{\tau}}f(\tau_0, \bar{\tau}_0) = \frac{3}{4}f(\tau_0, \bar{\tau}_0), \tag{16}$$

which must then hold for the general $f(\tau, \bar{\tau})$ at all values of τ_0. This is the nonrenormalization theorem of Green and Sethi [3]. Below, we write $f_n(\tau, \bar{\tau})$ for the coefficient of $D^n R^4$, and so $f(\tau, \bar{\tau})$ is denoted $f_0(\tau, \bar{\tau})$.

Note that there is no independent $D^2 R^4$ supervertex, as the corresponding superamplitude must be proportional to $\delta^{16}(Q)(s + t + u) = 0$. We next apply the argument to the $f_4(\tau, \bar{\tau})D^4 R^4$ coupling. Once again, the holomorphic and antiholomorphic parts of $f_4(\tau, \bar{\tau})$ are unconstrained by supersymmetry, as there are independent $\varphi^k R^4$ and $\bar{\varphi}^k R^4$
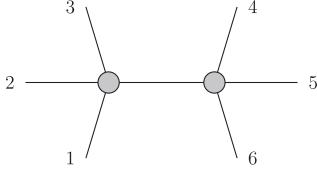
FIG. 2. Factorization of the 6-point amplitude through a pair of R^4 vertices.

supervertices. On the other hand, $\partial_\tau \partial_{\bar{\tau}} f_4$ must obey a linear relation with $\tau_2^{-2} f_4(\tau, \bar{\tau})$, due to the factorization of the 6-point superamplitude. Note that the 6-point amplitude at this order in the momentum expansion does not factorize through two R^4 vertices (Fig. 2), as the latter can only contribute to the 6-point amplitude at $D^6 R^4$ order [30].

Now taking the IIB string tree-level effective action, and expanding to α'^5 order, we find in the Einstein frame the coupling

$$\tau_2^{5/2}(s^2 + t^2 + u^2)R^4. \tag{17}$$

By comparison, we then immediately obtain the relation

$$4\tau_2^2 \partial_\tau \partial_{\bar{\tau}} f_4(\tau, \bar{\tau}) = \frac{15}{4} f_4(\tau, \bar{\tau}). \tag{18}$$

At $f_6(\tau, \bar{\tau}) D^6 R^4$ order, we encounter a novelty: as already mentioned, the 6-point amplitude at this order in the momentum expansion admits a factorization into a pair of R^4 supervertices. Thus, we expect the coefficient $f_6(\tau, \bar{\tau})$ to obey a relation of the form

$$\tau_2^2 \partial_\tau \partial_{\bar{\tau}} f_6 = a f_6(\tau, \bar{\tau}) + b f_0(\tau, \bar{\tau})^2, \tag{19}$$

where a and b are two constants. More precisely, we define $f_6(\tau, \bar{\tau})$ to be the coefficient of $(s^3 + t^3 + u^3)R^4 = 3stuR^4$. Inspecting the well-known string tree-level massless 4-point amplitude,

$$\delta^{16}(Q) \frac{\Gamma(-\frac{\alpha' s}{4})\Gamma(-\frac{\alpha' t}{4})\Gamma(-\frac{\alpha' u}{4})}{\Gamma(1+\frac{\alpha' s}{4})\Gamma(1+\frac{\alpha' t}{4})\Gamma(1+\frac{\alpha' u}{4})}$$
$$= \delta^{16}(Q)\left[-\frac{64}{\alpha'^3 stu} - 2\zeta(3) - \frac{\zeta(5)}{16}\alpha'^2(s^2 + t^2 + u^2) \right.$$
$$\left. -\frac{\zeta(3)^2}{96}\alpha'^3(s^3 + t^3 + u^3) + \cdots \right], \tag{20}$$

we can identify the following couplings in the Einstein frame [31],

$$-2\zeta(3)\tau_2^{3/2}\alpha'^3 R^4 - \frac{\zeta(5)}{16}\alpha'^5 \tau_2^{5/2}(s^2 + t^2 + u^2)R^4$$
$$-\frac{\zeta(3)^2}{96}\alpha'^6 \tau_2^3(s^3 + t^3 + u^3)R^4 + \cdots \tag{21}$$

Comparing to Eq. (19), with $f_0 \propto \tau_2^{3/2}$ and $f_6 \propto \tau_2^3$, we immediately obtain a linear relation between a and b. Another relation between a and b may be extracted from the string 1-loop effective action. The perturbative contribution to f_0 and f_6 can be expanded in the form [6]

$$f_n(\tau, \bar{\tau}) = f_n^{\text{tree}} + f_n^{\text{1-loop}} + f_n^{\text{2-loop}} + f_n^{\text{3-loop}} + \cdots. \tag{22}$$

In particular, at 1-loop order, we expect

$$\tau_2^2 \partial_\tau \partial_{\bar{\tau}} f_6^{\text{1-loop}} = a f_6^{\text{1-loop}}(\tau, \bar{\tau}) + 2b f_0^{\text{tree}}(\tau, \bar{\tau}) f_0^{\text{1-loop}}(\tau, \bar{\tau}). \tag{23}$$

The 4-point massless genus one string amplitude amplitude has analytic as well as nonanalytic terms in the momentum expansion. The R^4 term, with coefficient $f_0^{\text{1-loop}} \propto \tau_2^{-1/2}$, and the $D^6 R^4$ term, with coefficient $f_6^{\text{1-loop}} \propto \tau_2$, are analytic, and were computed in Ref. [35]. They give an independent linear relation which then fixes a and b, as in (5.39) of Ref. [6]. In the end, one finds

$$4\tau_2^2 \partial_\tau \partial_{\bar{\tau}} f_6 = 12 f_6(\tau, \bar{\tau}) - 6 f_0(\tau, \bar{\tau})^2. \tag{24}$$

As was pointed out in Ref. [6], the string 3-loop contribution $f_6^{\text{3-loop}}$ [6,36–39], proportional to τ_2^{-3}, is what solves the homogeneous version of the constraining equation (namely, it is annihilated by $4\tau_2^2 \partial_\tau \partial_{\bar{\tau}} - 12$).

Now let us consider $D^8 R^4$ terms. There is again one independent 4-point supervertex one can write down [40],

$$\delta^{16}(Q)(s^4 + t^4 + u^4). \tag{25}$$

This is, in fact, proportional to the D-term vertex

$$\delta^{16}(Q)\tilde{Q}^{16}\left[\sum_{i<j}^4 \eta_i^8 \eta_j^8\right]. \tag{26}$$

To understand the constraints on $f_8(\tau, \bar{\tau})$, let us inspect $(n = 4 + k)$-point supervertices of the form

$$\delta^{16}(Q)\tilde{Q}^{16} F(\eta_i^8), \tag{27}$$

where $F(\eta_i^8)$ is a polynomial in the little group invariants η_i^8, of total degree $8m$ in the η's, for some integer $m \geq 2$. This then corresponds to a coupling of the form $\varphi^{k-m+2}\bar{\varphi}^{m-2} D^8 R^4$. Since these D-term vertices by construction obey supersymmetry Ward identities, there are no constraints on the coefficients of $\varphi^{k-m+2}\bar{\varphi}^{m-2} D^8 R^4$, thus, no constraint on $f_8(\tau, \bar{\tau})$ from supersymmetry alone.

At order $D^{10} R^4$, there is again just one independent 4-point supervertex $\delta^{16}(Q)(s^5 + t^5 + u^5)$. This is proportional to the D-term vertex $\delta^{16}(Q)\tilde{Q}^{16}[\sum_{i<j}^n s_{ij}\eta_i^8 \eta_j^8]$ [41]. As in the $D^8 R^4$ case, there are no supersymmetry

constraints on the coefficient $f_{10}(\tau, \bar{\tau})$. In other words, the differential constraint proposed in Ref. [42] should be a consequence of additional properties of IIB string theory.

In conclusion, the formulation of higher derivative couplings in maximally supersymmetric gravity theories in terms of on-shell supervertices gives a simple classification of independent couplings allowed by supersymmetry. When combined with solutions to supersymmetry Ward identities provided by string perturbation theory, the consideration of supervertices then leads to a derivation of type IIB supersymmetry constraints on the F-term $f(\tau, \bar{\tau})D^n R^4$ coupling ($n = 0, 4, 6$). The result is nonetheless a consequence of maximal supersymmetry on higher derivative supergravity theories, and no longer depends on string theory. Clearly, this strategy generalizes straightforwardly to maximal supergravity theories in other dimensions as well [43]. We shall leave this to a future publication.

Finally, let us comment on the role of $SL(2, \mathbb{R})$ symmetry of type IIB supergravity, which, as already mentioned, is explicitly broken by these higher derivative terms. A coupling of the form $f_n(\tau, \bar{\tau})D^n R^4$ violates $SL(2, \mathbb{R})$ unless f_n is a constant, but the latter is incompatible with the supersymmetry constraints (a nontrivial second order

differential equation in $\tau, \bar{\tau}$) for F-term vertices. From this perspective, a role of the nonlinearly realized $SL(2, \mathbb{R})$ symmetry of type IIB supergravity is to rule out F-terms as potential counterterms. Indeed, the UV divergence in type IIB supergravity first arises at 2-loop order, corresponding to an $SL(2, \mathbb{R})$-invariant D-term counterterm of the form $D^{10}R^4$. One may expect that the $E_{7(7)}$ symmetry of four-dimensional maximal supergravity plays a similar role in that it rules out F-terms as counterterms, but there appear to be plenty of D-term supervertices compatible with $E_{7(7)}$ that could serve as counterterms [44–52].

ACKNOWLEDGMENTS

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