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Hyperbolic geometry of cosmological attractors

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Cosmological α attractors give a natural explanation for the spectral index n_s of inflation as measured by Planck while predicting a range for the tensor-to-scalar ratio r, consistent with all observations, to be measured more precisely in future B-mode experiments. We highlight the crucial role of the hyperbolic geometry of the Poincaré disk or half plane in the supergravity construction. These geometries are isometric under Möbius transformations, which include the shift symmetry of the inflaton field. We introduce a new Kähler potential frame that explicitly preserves this symmetry, enabling the inflaton to be light. Moreover, we include higher-order curvature deformations, which can stabilize a direction orthogonal to the inflationary trajectory. We illustrate this new framework by stabilizing the single superfield α attractors.

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I. INTRODUCTION

Inflationary theory provides a simple explanation of the approximate homogeneity and isotropy of our world. For a broad set of initial conditions, the solutions of the equations of motion for the inflaton field and the geometry of space rapidly approach an inflationary attractor solution which describes an exponentially expanding nearly uniform universe. Moreover, inflation provides a physical mechanism to generate the deviations from smoothness due to quantum fluctuations. Cosmic microwave background (CMB) observations such as those made by Planck have tested and narrowed down the possibilities [1,2].

In this paper we discuss cosmological α attractors [3–5], which provide an excellent fit to the latest observational results for $\alpha \lesssim O(10)$; see Fig. 1. Similar to inflation itself, these attractors have the property that almost independent of the choice of the inflaton potential in these models, an inflationary model comes out that generates the right value of the spectral index n_s and a tensor-to-scalar ratio r determined by the geometry of the moduli space.

Thus, observational predictions of these models are to a large extent determined by geometry, as emphasized in [6], rather than by the potential. For decades there was an expectation that with more CMB data we would be able to reconstruct the inflationary potential. There is an ongoing change in the paradigm now: we attempt to reconstruct geometry of the moduli space, not the potential.

In order to highlight the role of geometry, following [6], we formulate these models in a way that makes the relevant symmetries of the moduli space manifest. Concretely, we use the freedom in the choice of the Kähler frame in supergravity to construct a Kähler potential that is invariant under the subgroup of the Möbius group that is relevant for this type of inflation. In this way, the shift of the inflaton is a symmetry of the Kahler potential during inflation, only slightly broken by the superpotential. This makes manifest a crucial feature of α-attractor models: *the inflaton is light*.

II. MÖBIUS TRANSFORMATIONS

First we describe the necessary mathematical background. The symmetry of the moduli space metric corresponds to the Möbius group, both in disk and in half-plane variables. The metric in half-plane variables reads

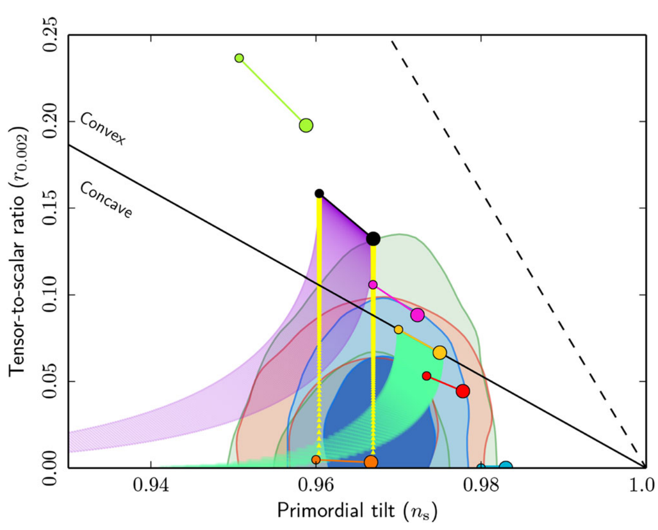

FIG. 1 (color online). The Planck/BICEP2/Keck 2015 constraints on n_s and r with the predictions of a number of models [1,2]. The yellow lines correspond to the simplest α-attractor models for a full range $0 < \alpha < \infty$ and $N = 50, 60$ [4]. These predictions nicely fit the latest cosmological data for the most natural choice of $\alpha \lesssim O(10)$.

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$$ds^2 = 3\alpha \frac{dTd\overline{T}}{(T+\overline{T})^2} = 3\alpha \frac{d\tau d\overline{\tau}}{(2\,\mathrm{Im}\tau)^2}, \quad (1)$$

where $\tau = iT$. The full set of isometries of this geometry can be generated by the following four transformations:

(i) Translation of the imaginary part: $T \to T - ib$.
(ii) Dilatation of the entire plane: $T \to a^2 T$.
(iii) Inversion: $T \to 1/T$.
(iv) Reflection of the imaginary part: $T \to \overline{T}$.

The three holomorphic combinations of these, i.e. translations, dilatations and inversions, generate the following Möbius transformations:

$$\tau \to \frac{a\tau + b}{c\tau + d}, \qquad \Delta \equiv ad - bc \neq 0, \quad (2)$$

and a, b, c, d are real numbers. The Möbius group therefore corresponds to a transformation associated with an $GL(2,\mathbb{R})$ matrix

$$\mathcal{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in GL(2,\mathbb{R}). \quad (3)$$

The Poincaré line element above is invariant under any nonsingular transformation. However, when restricting to a particular half plane, this is only mapped onto itself when one takes the determinant Δ to be positive.

A general Möbius transformation can be conveniently parametrized via the Iwasawa decomposition,

$$\mathcal{M} = K \cdot A \cdot N, \\ = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix} \cdot \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}, \quad (4)$$

whose parameters are given by

$$r_1 = \sqrt{a^2 + c^2}, \qquad x = \frac{ab + cd}{a^2 + c^2}, \\ r_2 = \frac{\Delta}{\sqrt{a^2 + c^2}}, \qquad \cos\theta = \frac{a}{\sqrt{a^2 + c^2}}. \quad (5)$$

Here the $K \cdot A \cdot N$ subgroups parametrize the compact, Abelian and nilpotent transformations of the Möbius group, respectively. In the case that $\Delta = 1$ the symmetry is reduced to $SL(2,\mathbb{R})$.

III. A NEW KÄHLER FRAME

Now that we have phrased the Iwasawa decomposition, we can turn to explicit realizations of this geometry in terms of Kähler potentials, and a discussion of the physical significance as to which of the isometries they preserve.

First let us address the expression of T in canonical variables,

$$T = \exp\left(\sqrt{\frac{2}{3\alpha}}\varphi\right) + i\chi, \quad (6)$$

where the dilatonic field φ will be our inflaton, and χ our axion. This physical realization is determined crucially by the geometry we choose to employ. The $SL(2,\mathbb{R})$ symmetry of the kinetic terms of the axion-dilaton pair was first derived in the context of $\mathcal{N} = 4$ supergravity in [7]. The nilpotent subgroup N of the Iwasawa decomposition, relevant to the conventional Kähler potentials, acts as a shift on the axionic field:

$$\chi \to \chi + b. \quad (7)$$

In contrast, the Abelian dilatation shift symmetry, A, acts on both components,

$$\chi \to \frac{a}{d}\chi, \qquad \varphi \to \varphi + \sqrt{\frac{3\alpha}{2}}\log(a/d). \quad (8)$$

Note that this acts as a shift symmetry on the field φ, which will play the role of the inflaton in our context.

The conventional formulation of the Kähler potential in half-plane variables is

$$K = -3\alpha \log(T + \overline{T}). \quad (9)$$

While it is symmetric under the nilpotent subgroup, it is *not* invariant under the Abelian subgroup A corresponding to dilatations. The latter is particularly important as it corresponds to the shift symmetry of the inflaton, as we will show below. Therefore it would be valuable to highlight this shift symmetry in a Kähler potential, and only introduce a (small) shift symmetry breaking via the superpotential.

To this end we introduce a new Kähler potential, which in half-plane coordinates is defined by

$$K_{\mathbb{H}} = -3\alpha \log\left(\frac{T + \overline{T}}{2|T|}\right) = -\frac{3\alpha}{2}\log\left(\frac{(T + \overline{T})^2}{4T\overline{T}}\right), \quad (10)$$

where $|T| = (T\overline{T})^{1/2}$. It is related to the old Kähler potential by means of a Kähler transformation

$$K(T,\overline{T}) \to K(T,\overline{T}) + f(T) + \overline{f}(\overline{T}), \\ W(T) \to W(T)e^{-f(T)}, \quad (11)$$

with parameter $f(T) = \frac{3}{2}\alpha \log(T)$. Such a transformation preserves the scalar potential but changes the Kähler potential and superpotential. As a consequence, the symmetries of both Kähler potentials are different: the choices (9)–(10) are invariant under nilpotent and Abelian transformations, respectively, of which the latter correspond to the shift symmetry of the inflaton.

In detail, from the full set of Möbius transformations (3), the new Kähler potential is preserved by the following transformations:

$$\mathcal{M} = \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}: \; T \to \frac{aT}{d} \qquad \text{dilatation},$$
$$\mathcal{M} = \begin{pmatrix} 0 & b \\ c & 0 \end{pmatrix}: \; T \to -\frac{b}{cT} \qquad \text{inversion}. \qquad (12)$$

Note that in the Iwasawa decomposition the latter can be seen as an arbitrary dilatation, $r_1 = c$, $r_2 = -b$, followed by a discrete inversion with $\theta = 90°$ in (4).

The new Kähler potential has a symmetry under the shift of the inflaton, accompanied by the rescaling of the inflaton partner. This symmetry leads to the following feature of the new Kähler potential: during inflation, in these models the inflaton partner $T - \overline{T}$ vanishes and $K = 0$, as will be explained in the "universal stabilization" section. This is obviously invariant under the inflaton shift. This inflaton shift symmetry is only slightly broken by the superpotential, resulting in a naturally light inflaton.

The difference between both Kähler potentials is reminiscent of the mechanism of the inflaton shift symmetry for a flat moduli space [8]. There the Kähler potential does not depend on the inflaton direction, which one can take as the real part of the chiral multiplet Φ, and depends only on its partner: $K = \frac{1}{2}(\Phi - \overline{\Phi})^2$. While this is related by a Kähler transformation to the canonical case $K = \Phi\overline{\Phi}$, only the former has a shift symmetry for the inflaton. Again, since only the superpotential breaks this symmetry of the Kähler potential, the inflaton can be naturally light during inflation in the supergravity model of the quadratic chaotic inflation [8]. Moreover, this construction can be generalized by including a generic function in the superpotential. This results in a broad class of chaotic inflation model in supergravity with nearly arbitrary inflaton potentials proposed in [9].

Our new Kähler frame can be seen as the curved analog of the flat Kähler potential with a shift symmetry. In the limit $\alpha \to \infty$ where the curvature tends to zero, the new Kähler potential (10) goes to $K = \frac{1}{2}(\Phi - \overline{\Phi})^2$ after the identification $T = \exp(2\Phi/\sqrt{3\alpha})$, as used in [10]. A peculiar property of both is that K as well as K_Φ vanish on the inflationary trajectory $\Phi = \overline{\Phi}$.

IV. DISK VARIABLES

In disk variables, related to the half-plane variables by the Cayley transform,

$$Z = \frac{T-1}{T+1}, \qquad T = \frac{1+Z}{1-Z}, \qquad (13)$$

the conventional Kähler potential is

$$K = -3\alpha \log(1 - Z\overline{Z}), \qquad Z = \frac{e^{\sqrt{\frac{2}{3\alpha}}\varphi} + i\chi - 1}{e^{\sqrt{\frac{2}{3\alpha}}\varphi} + i\chi + 1}. \qquad (14)$$

Note that an interesting and cosmologically important feature of a disk variable is that at $\chi = 0$

$$Z|_{\chi=0} = \tanh\left(\frac{\varphi}{\sqrt{6\alpha}}\right). \qquad (15)$$

This Kähler potential parametrizes the same geometry: it is related to the half-plane Kähler potentials (9)–(10) by a Cayley and a Kähler transformation. A consequence of the latter is that the symmetries of the Kähler potential change. As is clear from the formulation in disk variables, this form of the Kähler potential has a rotational symmetry, corresponding to the compact group K.

The new Kähler potential reads

$$K_{\mathbb{D}} = -\frac{3\alpha}{2}\log\left[\frac{(1 - Z\overline{Z})^2}{(1 - Z^2)(1 - \overline{Z}^2)}\right], \qquad (16)$$

which is related to $K_{\mathbb{H}}$ by means of a Cayley transformation, without the need of any supplementing Kähler transformations. Therefore it has the same explicit symmetries. In disk variables, the dilatation symmetry acts as

$$(1 \pm Z) \to (1 \pm Z)\frac{\beta \pm \gamma}{\beta + Z\gamma} \qquad (17)$$

with real parameters $\beta = (a + d)$ and $\gamma = (a - d)$. In addition, the inversion symmetry takes the form

$$(1 \pm Z) \to (1 \mp Z)\frac{\tilde{\beta} \mp \tilde{\gamma}}{\tilde{\beta} + Z\tilde{\gamma}} \qquad (18)$$

again for real parameters $\tilde{\beta} = (b - c)$ and $\tilde{\gamma} = -(b + c)$. A particular case of this is $Z \to -Z$ and corresponds to a $\theta = 90°$ rotation in the Iwasawa decomposition.

Noting that dilatation takes

$$(Z\overline{Z} - 1) \to (Z\overline{Z} - 1)\frac{(\beta - \gamma)(\beta + \gamma)}{(\beta + \gamma Z)(\gamma\overline{Z} + \beta)}, \qquad (19)$$

it is apparent that the dilatation operation leaves invariant the quantity

$$\mathcal{I} = \frac{Z\overline{Z} - 1}{(1 - Z)(1 + \overline{Z})}.$$

This object is also special under the inversion operation, which simply takes the conjugation, swapping between $\mathcal{I} \leftrightarrow \overline{\mathcal{I}}$. As such, the disk Kähler potential makes these symmetries manifest when the argument of the logarithm is written as $\mathcal{I}\overline{\mathcal{I}}$.

V. UNIVERSAL STABILIZATION

The crucial issue of the stabilization of the inflaton partner during inflation as well as the stabilization of both the inflaton and its partner at the minimum has been studied in detail over the years [11,12]. In particular, in the case of a single superfield, the average mass of its two components reads [11,13] [see Eqs. (2.20) and (A.1) in [13]]:

$$m^2 = K^{\Phi\overline{\Phi}}\nabla_\Phi\nabla_{\overline{\Phi}}V = 3\left(R+\frac{2}{3}\right)m_{3/2}^2 + RV, \quad (20)$$

for a critical point with $V' = 0$, $DW \neq 0$, and where R is the Ricci scalar, $V = e^K(|DW|^2 - 3|W|^2)$ and $DW = \partial_\Phi W + K_\Phi W$. The above is only a condition on the average of the two masses, of the inflaton and its partner, and does not address stability in each of the two directions separately. Moreover, this condition is derived for $V' = 0$. However, during inflation the potential is rather flat: the inflaton mass squared is much smaller than V due to slow roll conditions. Therefore a positive average $m^2 = \mathcal{O}(V)$ implies that the inflaton partner is stable during inflation. Hence $R > -2/3$ is a necessary condition and $R \geq 0$ is a sufficient condition for stabilization during inflation.

We demonstrate that the crucial role played by the Kähler curvature allows one to stabilize the inflaton partner in a universal way. To this end, we deform the maximally symmetric Kähler manifold by adding quadratic and quartic terms:

$$K = \frac{-3\alpha}{1+2c_2}\log\left(\frac{T+\overline{T}}{2|T|}\left[1 + c_2\left(\frac{T-\overline{T}}{T+\overline{T}}\right)^2 + c_4\left(\frac{T-\overline{T}}{T+\overline{T}}\right)^4\right]\right) \quad (21)$$

where we redefine the overall coefficient 3α to include a set of α-attractor Kähler potentials which have been studied in [5,14]. Both of the new terms contribute to the curvature and can be used to stabilize the imaginary direction. Importantly, the higher-order terms preserve all symmetries of this Kähler potential, both the dilatations and the inversions. We therefore retain the crucial inflaton shift symmetry, while breaking the axionic shift symmetry even stronger.

The curvature of the geometry based on (21) is only constant in the case with $c_2 = c_4 = 0$. However, the curvature corrections are constant along the inflationary trajectory $T = \overline{T}$, leading to

$$R = -\frac{2(1 + 8c_2 + 6{c_2}^2 - 12c_4)}{3\alpha(1+2c_2)}. \quad (22)$$

Note that the mass of the inflaton partner χ can be made large, for example by taking c_2 small and $c_4 \gg 1$. In this case $R \approx \frac{8c_4}{\alpha}$ and the total mass of the inflaton and its partner in Eq. (12) is $m^2 \approx \frac{8c_4}{\alpha}(3m_{3/2}^2 + V) + 2m_{3/2}^2$. The inflaton being light, we see that the inflaton partner near $\chi = 0$ is heavy and tends to reach the minimum quickly. We perform a more detailed study of the likelihood of the configuration with $\chi = 0$ in the case of the two-superfield model in [15]. There we find that the configuration with $\chi = 0$ naturally emerges as a result of the cosmological evolution, and it is stable. Thus the above conditions on stability during inflation, either the necessary one $R > -2/3$ or the sufficient one $R \geq 0$, can always be achieved, for any α, by tuning c_2 and c_4.

Single-superfield models have seen a lot of progress recently; general potentials were constructed in [16,17], one of the first inflationary models in supergravity was unearthed again [18,19] and the first examples of α attractors were constructed [10,20]. The difference between the latter two resides in the choice of Kähler potential. The first used the case $c_2 = c_4 = 0$ leading to the following curvature and necessary stability condition [10]:

$$R = -\frac{2}{3\alpha} > -\frac{2}{3} \Leftrightarrow \alpha > 1. \quad (23)$$

Instead, the second used the Kähler potential introduced in [5] with $2c_2 = 1 - \alpha$ and $c_4 = 0$, leading to

$$R = -\frac{2}{3\alpha} - 1 + \frac{1}{\alpha^2} > -\frac{2}{3} \Leftrightarrow \alpha < 1. \quad (24)$$

As a consequence, their regimes of stability turn out to be complementary. From the above it follows that these stability constraints are mere consequences of the particular choice of higher-order terms. With the results of this article, however, one can achieve stability for any configuration of α by including general quadratic and quartic terms [21].

A similar analysis of the stability cosmological attractors based on two superfields, in case the second superfield is nilpotent [22], has the following features. We consider models of the kind

$$K = K(\Phi, \overline{\Phi}, S\overline{S}), \qquad W = Sf(\Phi), \quad (25)$$

where Φ can be either half-plane or disk coordinates. Due to the nilpotency of S, we are only interested in stabilizing the inflaton partner during inflation. The average mass formula is, again up to slow roll corrections [9],

$$m^2 = (1 + R_{\mathrm{bs}})V, \qquad R_{\mathrm{bs}} = K^{\Phi\overline{\Phi}}K^{S\overline{S}}R_{\Phi\overline{\Phi}S\overline{S}}. \quad (26)$$

In this case it is therefore the bisectional curvature that determines stability. An example of this is provided by the Kähler potential

$$K = -3\alpha \log\left(\frac{T+\overline{T}}{2|T|} - \frac{S\overline{S}}{2|T|}\left[1 - c_{\rm bs}\left(\frac{T-\overline{T}}{T+\overline{T}}\right)^2\right]\right), \tag{27}$$

which leads to a bisectional curvature given by $R_{\rm bs} = -(1+2c_{\rm bs})/(3\alpha)$. Without the stabilization term this model is therefore unstable for $\alpha < 1/3$ [4]. However, it follows from this general discussion of the cosmological attractor models based on two superfields (where the second superfield is nilpotent) that there is a universal geometric mechanism of stabilization of the inflaton partner during inflation, based on the bisectional curvature. The details will be described separately in [15].

VI. CONCLUSIONS

We have proposed a new Kähler potential for the hyperbolic geometry which preserves the shift symmetry of the inflaton φ. In terms of the Iwasawa decomposition into $K \cdot A \cdot N$ subgroups, the new Kähler frame exactly picks out the relevant Abelian subgroup A of the full Möbius group. Higher-order corrections can be used to stabilize the orthogonal directions while retaining the inflaton shift symmetry.

This improvement of the Kähler potential is especially suitable for the investigation of cosmological attractors, which make cosmological predictions determined almost solely by the geometry of the moduli space rather than by the details of the inflaton potential. Our construction explains the naturally flat inflaton potential by having a shift symmetry in the Kähler potential that is only weakly broken by the superpotential; moreover, due to the robustness of the cosmological attractors, the details of this breaking do not affect the cosmological predictions.

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