CP-violating polarization asymmetry in charmless two-body decays of beauty baryons

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Several baryons containing a heavy b-quark, the b-baryons, have been discovered. The charmless two-body decays of b-baryons can provide a new platform for *CP* violating studies in a similar way provided by charmless two-body decays of B-meson. There are new *CP* violating observables related to baryon polarization in b-baryon decays. We show that in the flavor $SU(3)$ limit, there exists relations involving different combinations of the decay amplitudes compared with those in *CP* violating rate asymmetry. These new relations therefore provide interesting tests for the mechanism of *CP* violations in the standard model (SM) and flavor $SU(3)$ symmetry. Such tests could complement the b-meson decay studies which hint at a better flavor $SU(3)$ conservation in b-hadron decays than in kaon and hyperon decays. Future data from LHCb can provide new information about *CP* violation in the SM.

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I. INTRODUCTION

Several baryons containing a heavy b-quark, the beauty baryon (b-baryon) $\mathcal{B}$, have been discovered [1]. The study of heavy mesons containing a b-quark, the B mesons, provided crucial information [1] in establishing the standard model (SM) for *CP* violation, the Cabibbo-Kobayashi-Maskawa (CKM) model [2]. The decays of the $\mathcal{B}$ b-baryons can provide a new platform to further test the CKM model of *CP* violation [3–6]. It has been shown that $\mathcal{B}$ b-baryons decay into a light $SU(3)$ octet baryon $\mathcal{F}$ and a light $SU(3)$ octet pseudoscalar meson $\mathcal{M}$, the charmless two-body b-baryon decay $\mathcal{B} \to \mathcal{M} + \mathcal{F}$, can have sizeable *CP* violation [4]. There are also flavor $SU(3)$ [7] relations between *CP* violating rate asymmetries in some of these decays [6]. Similar relations for *CP* violating rate asymmetry have been obtained in B meson decays for two-body charmless B meson decays [8–11]. Experimental data have verified some of the relations to very good precision [1,6,11,12]. Therefore, it is interesting to see whether relations based on $SU(3)$ symmetry are accidental or are more universal for charmless two-body decays of hadrons containing a heavy b-quark. Experimental data from LHCb can provide crucial information when *CP* violation for two-body charmless b-baryon decays will be measured. In this paper, we study further possible new *CP* violating observables in $\mathcal{B} \to \mathcal{M} + \mathcal{F}$ decays associated with the polarization of baryon in the decays. We find that there exist relations in *CP* violating polarization asymmetries in some of the decay channels and can be tested at LHCb.

II. *CP* VIOLATING ASYMMETRY FOR b-BARYON DECAYS

The effective Hamiltonian inducing $\mathcal{B} \to \mathcal{M} + \mathcal{F}$ decays in the SM has both parity (P) conserving and violating parts given by

$$H_{\rm eff}(q) = \frac{4G_F}{\sqrt{2}}\left[V_{ub}V^*_{uq}(c_1O_1 + c_2O_2) - \sum_{i=3}^{12}(V_{ub}V^*_{uq}c_i^{uc} + V_{tb}V^*_{tq}c_i^{tc})O_i\right], \quad (1)$$

where q can be d or s. V_{ij} is the CKM matrix element. In the above the factor $V_{cb}V^*_{cq}$ has been eliminated using the unitarity property of the CKM matrix.The coefficients $c_{1,2}$ and $c_i^{jk} = c_i^j - c_i^k$, with j and k indicate the internal quark, are the Wilson coefficients (WC) which have been studied by several groups and can be found in Ref. [13]. The operators O_i are given by

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$$O_1 = (\bar{q}_i u_j)_{V-A}(\bar{u}_i b_j)_{V-A}, \qquad O_2 = (\bar{q}u)_{V-A}(\bar{u}b)_{V-A},$$
$$O_{3,5} = (\bar{q}b)_{V-A}\sum_{q'}(\bar{q}'q')_{V\mp A},$$
$$O_{4,6} = (\bar{q}_i b_j)_{V-A}\sum_{q'}(\bar{q}'_j q'_i)_{V\mp A},$$
$$O_{7,9} = \frac{3}{2}(\bar{q}b)_{V-A}\sum_{q'} e_{q'}(\bar{q}'q')_{V\pm A},$$
$$O_{8,10} = \frac{3}{2}(\bar{q}_i b_j)_{V-A}\sum_{q'} e_{q'}(\bar{q}'_j q'_i)_{V\pm A},$$
$$O_{11} = \frac{g_s}{16\pi^2}\bar{q}\sigma_{\mu\nu}G^{\mu\nu}(1+\gamma_5)b,$$
$$O_{12} = \frac{Q_b e}{16\pi^2}\bar{q}\sigma_{\mu\nu}F^{\mu\nu}(1+\gamma_5)b, \tag{2}$$

where $(\bar{a}b)_{V\pm A} = \bar{a}\gamma_\mu(1\pm\gamma_5)b$, $G^{\mu\nu}$ and $F^{\mu\nu}$ are the field strengths of the gluon and photon, respectively. $O_{1,2}$, $O_{3,4,5,6}$, and $O_{7,8,9,10}$ are the tree, penguin, and electroweak penguin operators. $O_{11,12}$ are the photonic and gluonic dipole penguin operators.

For later analysis, we need to know the flavor $SU(3)$ properties of the Hamiltonian. We summarize them here. The operator O_i contains $\bar{3}$, 6, $\overline{15}$ of flavor $SU(3)$ irreducible representations. Indicating these representations by matrices $H(\bar{3})$, $H(6)$, $H(\overline{15})$ [7,8]. The nonzero entries of the matrices $H(i)$ are given as follows [7,8].

For $\Delta S = 0$,

$$H(\bar{3})^2 = 1, \qquad H(6)^{12}_1 = H(6)^{23}_3 = 1,$$
$$H(6)^{21}_1 = H(6)^{32}_3 = -1,$$
$$H(\overline{15})^{12}_1 = H(\overline{15})^{21}_1 = 3, \qquad H(\overline{15})^{22}_2 = -2,$$
$$H(\overline{15})^{32}_3 = H(\overline{15})^{23}_3 = -1, \tag{3}$$

and for $\Delta S = -1$,

$$H(\bar{3})^3 = 1, \qquad H(6)^{13}_1 = H(6)^{32}_2 = 1,$$
$$H(6)^{31}_1 = H(6)^{23}_2 = -1,$$
$$H(\overline{15})^{13}_1 = H(\overline{15})^{31}_1 = 3, \qquad H(\overline{15})^{33}_3 = -2,$$
$$H(\overline{15})^{32}_2 = H(\overline{15})^{23}_2 = -1. \tag{4}$$

The above effective Hamiltonian dictates that the b-baryon decay amplitude $\mathcal{A}$ have both parity conserving A_c and violating A_v amplitudes in the form

$$\mathcal{A} = \bar{\mathcal{F}}(A_v + iA_c\gamma_5)\mathcal{B} = \mathcal{S} + \mathcal{P}\sigma\cdot\vec{p}_c. \tag{5}$$

Here $|\vec{p}_c| = \sqrt{E^2_{\mathcal{F}} - m^2_{\mathcal{F}}}$ is size of the final baryon $\mathcal{F}$ momentum p_c. $m_{\mathcal{B}}$ and $m_{\mathcal{M}}, m_{\mathcal{F}}$ are masses of the initial baryon and the final meson and baryon particles. $E_{\mathcal{F}}$ is energy of the final baryon.

The amplitudes $\mathcal{S}$ and $\mathcal{P}$ are the so-called S-wave (parity $= -$) and P-wave (parity $= +$) amplitudes, respectively. Their relations with A_v and A_c are given by

$$\mathcal{S} = A_v\sqrt{\frac{(m_{\mathcal{B}} + m_{\mathcal{F}})^2 - m^2_{\mathcal{M}}}{16\pi m^2_{\mathcal{B}}}},$$
$$\mathcal{P} = A_c\sqrt{\frac{(m_{\mathcal{B}} - m_{\mathcal{F}})^2 - m^2_{\mathcal{M}}}{16\pi m^2_{\mathcal{B}}}}. \tag{6}$$

The anti-b-baryon decay amplitude $\bar{\mathcal{A}}$ is usually written as

$$\bar{\mathcal{A}} = -\bar{\mathcal{S}} + \bar{\mathcal{P}}\sigma\cdot\vec{p}_c. \tag{7}$$

In the limit of CP conservation, $\bar{\mathcal{S}} = \mathcal{S}$ and $\bar{\mathcal{P}} = \mathcal{P}$.

In the SM, there are tree and penguin contributions to $\mathcal{S}$ and $\mathcal{P}$ for $\Delta S = 0$ ($q = d$) and $\Delta S = -1$ ($q = s$). The $\mathcal{S}$ and $\mathcal{P}$ and their corresponding antiparticle decay amplitudes can be decomposed into tree T_i and penguin P_i amplitudes as

$$\mathcal{S}(q) = V_{ub}V^*_{uq}T(q)_0 + V_{tb}V^*_{tq}P(q)_0,$$
$$\mathcal{P}(q) = V_{ub}V^*_{uq}T(q)_1 + V_{tb}V^*_{tq}P(q)_1,$$
$$\bar{\mathcal{S}}(q) = V^*_{ub}V_{uq}T(q)_0 + V^*_{tb}V_{tq}P(q)_0,$$
$$\bar{\mathcal{P}}(q) = V^*_{ub}V_{uq}T(q)_1 + V^*_{tb}V_{tq}P(q)_1, \tag{8}$$

where the subindices 0,1 denote the S and P amplitudes.

The decay width is given by

$$\Gamma = 2p_c(|\mathcal{S}|^2 + |\mathcal{P}|^2). \tag{9}$$

If information on polarization of the decay is available, there are additional experimental observables in the decay angular distribution. In the rest frame of the initial b-baryon, the angular distribution can be expressed as [14]

$$\frac{4\pi}{\Gamma}\frac{d\Gamma}{d\Omega} = 1 + \alpha\vec{s}_{\mathcal{B}}\cdot\vec{n} + \vec{s}_{\mathcal{F}}\cdot[(\alpha + \vec{s}_{\mathcal{B}}\cdot\vec{n})\vec{n} + \beta\vec{s}_{\mathcal{B}}\times\vec{n} + \gamma(\vec{n}\times(\vec{s}_{\mathcal{B}}\times\vec{n}))], \tag{10}$$

where $\vec{s}_{\mathcal{B}}, \vec{s}_{\mathcal{F}}$ are the spins of initial b-baryon and final octet baryon, and $\vec{n} = \vec{p}_c/|p_c|$ is the direction of final baryon $\mathcal{F}$. The parameters α, β, and γ expressed in terms of $\mathcal{S}$ and $\mathcal{P}$ amplitudes are given by

$$\alpha = \frac{2\mathrm{Re}(\mathcal{S}^*\mathcal{P})}{|\mathcal{S}|^2 + |\mathcal{P}|^2}, \quad \beta = \frac{2\mathrm{Im}(\mathcal{S}^*\mathcal{P})}{|\mathcal{S}|^2 + |\mathcal{P}|^2}, \quad \gamma = \frac{|\mathcal{S}|^2 - |\mathcal{P}|^2}{|\mathcal{S}|^2 + |\mathcal{P}|^2} \tag{11}$$

only two of them are independent with $\alpha^2 + \beta^2 + \gamma^2 = 1$. With this constraint one can obtain

$$\beta = (1-\alpha^2)^{\frac{1}{2}}\sin\phi, \qquad \gamma = (1-\alpha^2)^{\frac{1}{2}}\cos\phi,$$
$$\phi = \tan^{-1}(\beta/\gamma). \tag{12}$$

Experimentally, to obtain α, one needs to measure initial baryon or final baryon polarization. The initial polarization may be difficult to obtain in the LHCb experiment. But the final baryon polarization can be measured by looking at the final baryon decays [15]. To obtain β and γ, one needs to have both the initial and final baryon polarization information. In the rest of the discussions, we will only discuss properties related to α which has chance to be measured.

For CP violation studies, one can define new quantities related to polarization parameters which vanish in the CP conservation limit. To this end, we study the particle and antiparticle polarization difference as a new CP violating observable

$$\Delta_\alpha(q) = \Gamma(q)\alpha(q) + \bar{\Gamma}(q)\bar{\alpha}(q). \tag{13}$$

Express Δ_α in terms of the $\mathcal{S}$ and $\mathcal{P}$ amplitudes, we have

$$\begin{aligned}\Delta_\alpha &= 4p_c(\mathrm{Re}(\mathcal{S}^*\mathcal{P}) - \mathrm{Re}(\bar{\mathcal{S}}^*\bar{\mathcal{P}})),\\ &= p_c\mathrm{Im}(V_{ub}V_{uq}^*V_{tb}^*V_{tq})[\mathrm{Im}(T(q)_0^*P(q)_1)\\ &\quad - \mathrm{Im}(P(q)_0^*T(q)_1)].\end{aligned} \tag{14}$$

In the flavor $SU(3)$ limit, U-spin (d and s exchange) related decay modes have the same tree and penguin amplitudes, that is $T(d)_j = T(s)_j$ and $P(d)_j = P(s)_j$. Using the well-known relation of the CKM matrix, $\mathrm{Im}(V_{ub}V_{ud}^*V_{tb}^*V_{td}) = -\mathrm{Im}(V_{ub}V_{us}^*V_{tb}^*V_{ts})$, one therefore has

$$\Delta_\alpha(d) = -\Delta_\alpha(s). \tag{15}$$

For the weighted polarization asymmetry, $\mathcal{A}_\alpha = (\Gamma\alpha + \bar{\Gamma}\bar{\alpha})/(\Gamma + \bar{\Gamma})$, we have

$$\frac{\mathcal{A}_\alpha(\mathcal{B}_a \to \mathcal{M}\mathcal{F})_{\Delta S=0}}{\mathcal{A}_\alpha(\mathcal{B}_b \to \mathcal{M}\mathcal{F})_{\Delta S=-1}} = -\frac{\mathrm{Br}(\mathcal{B}_b \to \mathcal{M}\mathcal{F})_{\Delta S=-1}}{\mathrm{Br}(\mathcal{B}_a \to \mathcal{M}\mathcal{F})_{\Delta S=0}} \cdot \frac{\tau_{\mathcal{B}_a}}{\tau_{\mathcal{B}_b}}, \tag{16}$$

where $\tau_{\mathcal{B}_{a,b}}$ indicate the lifetimes of b-baryons $\mathcal{B}_{a,b}$, and Br indicates branching ratio. A similar CP violating observable $(\Gamma\alpha + \bar{\Gamma}\bar{\alpha})/(\Gamma\alpha - \bar{\Gamma}\bar{\alpha})$ had been discussed in Ref. [14] for hyperon decays. In the definition of A_α above, the denominator is simply $\Gamma + \bar{\Gamma}$ allowing simple relations shown above among different decays to hold.

Let us compare a U-spin relation for rate asymmetry obtained in Ref. [6],

$$\frac{A_{CP}(\mathcal{B}_a \to \mathcal{M}\mathcal{F})_{\Delta S=0}}{A_{CP}(\mathcal{B}_b \to \mathcal{M}\mathcal{F})_{\Delta S=-1}} = -\frac{\mathrm{Br}(\mathcal{B}_b \to \mathcal{M}\mathcal{F})_{\Delta S=-1}}{\mathrm{Br}(\mathcal{B}_a \to \mathcal{M}\mathcal{F})_{\Delta S=0}} \cdot \frac{\tau_{\mathcal{B}_a}}{\tau_{\mathcal{B}_b}}, \tag{17}$$

where $A_{CP}(q) = \Delta(q)/(\Gamma(q) + \bar{\Gamma}(q))$ with

$$\begin{aligned}\Delta(q) &= \Gamma(q) - \bar{\Gamma}(q) = 2p_c(|\mathcal{S}|^2 + |\mathcal{P}|^2 - (|\bar{\mathcal{S}}|^2 + |\bar{\mathcal{P}}|^2)),\\ &= -8p_c\mathrm{Im}(V_{ub}V_{uq}^*V_{tb}^*V_{tq})\mathrm{Im}[T(q)_0P(q)_0^*\\ &\quad + T(q)_1P(q)_1^*],\end{aligned} \tag{18}$$

and in the $SU(3)$ limit, the U-spin related states have

$$\Delta(d) = -\Delta(s). \tag{19}$$

It is clear that the relations in Eq. (16) and Eq. (17) test different aspects of CP violating properties in the decay amplitudes. The rate asymmetry A_{CP} probes CP violation due to interference between tree and penguin amplitudes of the same parity, while the polarization asymmetry $\mathcal{A}_\alpha$ probes CP violation due to interference between tree and penguin amplitudes of opposite parities. The relation associated with rate asymmetry has been discussed. The relation associated with polarization asymmetry is new and is our emphasis in this paper.

To test the above relations, one needs to make sure that indeed there are processes which are related by U-spin as described in the above. We show next that indeed such relations exist in two-body charmless decays of both antitriplet and sextet b-baryons.

III. AMPLITUDE RELATIONS IN TWO-BODY CHARMLESS b-BARYON DECAYS

We now study the decay amplitudes of charmless two-body decays of low-lying b-baryon. The low-lying $\frac{1}{2}^+$ $\mathcal{B}$ b-baryons are made up of a b quark and two light quarks. Here the light quark q is one of u, d, or s quarks. Under the flavor $SU(3)$ symmetry, the b quark is a singlet and the light quark q is a member in the fundamental representation 3. The b-baryons then have representations under flavor $SU(3)$ as $1 \times 3 \times 3 = \bar{3} + 6$, that is, the b-baryons contain an antitriplet and a sextet in $SU(3)$ flavor space [16]. The antitriplet $\mathcal{B}$ and sextet $\mathcal{C}$ b-baryons will be indicated by

$$(\mathcal{B}_{ij}) = \begin{pmatrix} 0 & \Lambda_b^0 & \Xi_b^0 \\ -\Lambda_b^0 & 0 & \Xi_b^- \\ -\Xi_b^0 & -\Xi_b^- & 0 \end{pmatrix},$$

$$\mathcal{C}_{ij} = \begin{pmatrix} \Sigma_b^+ & \frac{\Sigma_b^0}{\sqrt{2}} & \frac{\Xi_b'^0}{\sqrt{2}} \\ \frac{\Sigma_b^0}{\sqrt{2}} & \Sigma_b^- & \frac{\Xi_b'^-}{\sqrt{2}} \\ \frac{\Xi_b'^0}{\sqrt{2}} & \frac{\Xi_b'^-}{\sqrt{2}} & \Omega_b^- \end{pmatrix}. \tag{20}$$

Their quark compositions are the following [16]

$$\begin{aligned}&\Lambda_b^0 = \frac{1}{\sqrt{2}}(ud - du)b, \qquad \Xi_b^0 = \frac{1}{\sqrt{2}}(us - su)b,\\ &\Xi_b^- = \frac{1}{\sqrt{2}}(ds - sd)b,\\ &\Sigma_b^+ = uub, \qquad \Sigma_b^0 = \frac{1}{\sqrt{2}}(ud + du)b, \qquad \Sigma_b^- = ddb,\\ &\Xi_b'^0 = \frac{1}{\sqrt{2}}(us + su)b, \qquad \Xi_b'^- = \frac{1}{\sqrt{2}}(ds + sd)b,\\ &\Omega_b^- = ssb.\end{aligned} \tag{21}$$

The two charmless states in the final state of $\mathcal{B}$ decay are the $\frac{1}{2}^+$ octet baryons $\mathcal{F}$ and the pseudoscalar octet mesons $\mathcal{M}$, respectively. They are

$$(\mathcal{M}_{ij}) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix},$$

$$(\mathcal{F}_{ij}) = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda^0}{\sqrt{6}} \end{pmatrix}. \qquad (22)$$

At the hadron level, the decay amplitude can be generically written as

$$\mathcal{A} = \langle \mathcal{F}\mathcal{M} | H_{\mathrm{eff}}(q) | \mathcal{B} \rangle = V_{ub} V^*_{uq} T(q) + V_{tb} V^*_{tq} P(q). \qquad (23)$$

We now organize the decay amplitudes according to flavor $SU(3)$ symmetry using detailed $SU(3)$ tensor properties of the effective Hamiltonian given in Eqs. (3) and (4).

To obtain the $SU(3)$ invariant decay amplitude for a b-baryon, one first uses the Hamiltonian to annihilate the b-quark in $\mathcal{B}$ and then contract $SU(3)$ indices in an appropriate way with final states $\mathcal{F}$ and $\mathcal{M}$. As far as $SU(3)$ properties are concerned, the $\mathcal{S}$ and $\mathcal{P}$ amplitudes will have various $SU(3)$ irreducible amplitudes. Taking the antitriplet tree amplitude $T_t(q)_0$ and sextet tree amplitude $T_s(q)_0$ in $\mathcal{S}$ as examples, we have [6]

$$\begin{aligned}
T_t(q)_0 = {}& a_t(\bar{3})\langle \mathcal{F}^k_l \mathcal{M}^l_k | H(\bar{3})^i | \mathcal{B}_{i'i''}\rangle \epsilon^{ii'i''} + b_t(\bar{3})_1 \langle \mathcal{F}^k_j \mathcal{M}^i_k | H(\bar{3})^j | \mathcal{B}_{i'i''}\rangle \epsilon^{ii'i''} + b_t(\bar{3})_2 \langle \mathcal{F}^i_k \mathcal{M}^k_j | H(\bar{3})^j | \mathcal{B}_{i'i''}\rangle \epsilon^{ii'i''} \\
& + a_t(6)_1 \langle \mathcal{F}^k_l \mathcal{M}^l_j | H(6)^{ij}_k | \mathcal{B}_{i'i''}\rangle \epsilon^{ii'i''} + a_t(6)_2 \langle \mathcal{F}^l_j \mathcal{M}^k_l | H(6)^{ij}_k | \mathcal{B}_{i'i''}\rangle \epsilon^{ii'i''} + b_t(6)_1 \langle \mathcal{F}^l_k \mathcal{M}^i_j | H(6)^{jk}_l | \mathcal{B}_{i'i''}\rangle \epsilon^{ii'i''} \\
& + b_t(6)_2 \langle \mathcal{F}^i_j \mathcal{M}^l_k | H(6)^{jk}_l | \mathcal{B}_{i'i''}\rangle \epsilon^{ii'i''} + a_t(\overline{15})_1 \langle \mathcal{F}^k_l \mathcal{M}^l_j | H(\overline{15})^{ij}_k | \mathcal{B}_{i'i''}\rangle \epsilon^{ii'i''} + a_t(\overline{15})_2 \langle \mathcal{F}^l_j \mathcal{M}^k_l | H(\overline{15})^{ij}_k | \mathcal{B}_{i'i''}\rangle \epsilon^{ii'i''} \\
& + b_t(\overline{15})_1 \langle \mathcal{F}^l_k \mathcal{M}^i_j | H(\overline{15})^{jk}_l | \mathcal{B}_{i'i''}\rangle \epsilon^{ii'i''} + b_t(\overline{15})_2 \langle \mathcal{F}^i_j \mathcal{M}^l_k | H(\overline{15})^{jk}_l | \mathcal{B}_{i'i''}\rangle \epsilon^{ii'i''} + c_t(\bar{3}) \langle \mathcal{M}^i_j \mathcal{F}^{i'}_{j'} | H(\bar{3})^{i''} | \mathcal{B}_{jj'}\rangle \epsilon_{ii'i''} \\
& + d_t(\bar{3})_1 \langle \mathcal{M}^i_j \mathcal{F}^{i'}_{j'} | H(\bar{3})^j | \mathcal{B}_{i''j'}\rangle \epsilon_{ii'i''} + d_t(\bar{3})_2 \langle \mathcal{F}^i_j \mathcal{M}^{i'}_{j'} | H(\bar{3})^j | \mathcal{B}_{i''j'}\rangle \epsilon_{ii'i''} + e_t(\bar{3})_1 \langle \mathcal{M}^i_{j'} \mathcal{F}^{i'}_j | H(\bar{3})^j | \mathcal{B}_{i''j'}\rangle \epsilon_{ii'i''} \\
& + e_t(\bar{3})_2 \langle \mathcal{F}^i_{j'} \mathcal{M}^{i'}_j | H(\bar{3})^j | \mathcal{B}_{i''j'}\rangle \epsilon_{ii'i''} + c_t(6) \langle \mathcal{M}^i_j \mathcal{F}^{i'}_{j'} | H(6)^{jj'}_k | \mathcal{B}_{i''k}\rangle \epsilon_{ii'i''} + d_t(6)_1 \langle \mathcal{M}^i_j \mathcal{F}^{i'}_{j'} | H(6)^{i''j}_k | \mathcal{B}_{j'k}\rangle \epsilon_{ii'i''} \\
& + d_t(6)_2 \langle \mathcal{F}^i_j \mathcal{M}^{i'}_{j'} | H(6)^{i''j}_k | \mathcal{B}_{j'k}\rangle \epsilon_{ii'i''} + e_t(6)_1 \langle \mathcal{M}^i_j \mathcal{F}^{i'}_{j'} | H(6)^{i''j'}_k | \mathcal{B}_{jk}\rangle \epsilon_{ii'i''} + e_t(6)_2 \langle \mathcal{F}^i_j \mathcal{M}^{i'}_{j'} | H(6)^{i''j'}_k | \mathcal{B}_{jk}\rangle \epsilon_{ii'i''} \\
& + f_t(6) \langle \mathcal{M}^i_j \mathcal{F}^k_{j'} | H(6)^{i'i''}_k | \mathcal{B}_{jj'}\rangle \epsilon_{ii'i''} + g_t(6) \langle \mathcal{M}^k_j \mathcal{F}^i_{j'} | H(6)^{i'i''}_k | \mathcal{B}_{jj'}\rangle \epsilon_{ii'i''} + m_t(6) \langle \mathcal{M}^k_j \mathcal{F}^j_k | H(6)^{ii'}_l | \mathcal{B}_{i''l}\rangle \epsilon_{ii'i''} \\
& + n_t(6)_1 \langle \mathcal{M}^k_j \mathcal{F}^j_l | H(6)^{ii'}_k | \mathcal{B}_{i''l}\rangle \epsilon_{ii'i''} + n_t(6)_2 \langle \mathcal{F}^k_j \mathcal{M}^j_l | H(6)^{ii'}_k | \mathcal{B}_{i''l}\rangle \epsilon_{ii'i''} + c_t(\overline{15}) \langle \mathcal{M}^i_j \mathcal{F}^{i'}_{j'} | H(\overline{15})^{jj'}_k | \mathcal{B}_{i''k}\rangle \epsilon_{ii'i''} \\
& + d_t(\overline{15})_1 \langle \mathcal{M}^i_j \mathcal{F}^{i'}_{j'} | H(\overline{15})^{i''j}_k | \mathcal{B}_{j'k}\rangle \epsilon_{ii'i''} + d_t(\overline{15})_2 \langle \mathcal{F}^i_j \mathcal{M}^{i'}_{j'} | H(\overline{15})^{i''j}_k | \mathcal{B}_{j'k}\rangle \epsilon_{ii'i''} + e_t(\overline{15})_1 \langle \mathcal{M}^i_j \mathcal{F}^{i'}_{j'} | H(\overline{15})^{i''j'}_k | \mathcal{B}_{jk}\rangle \epsilon_{ii'i''} \\
& + e_t(\overline{15})_2 \langle \mathcal{F}^i_j \mathcal{M}^{i'}_{j'} | H(\overline{15})^{i''j'}_k | \mathcal{B}_{jk}\rangle \epsilon_{ii'i''} \qquad (24)
\end{aligned}$$

and

$$\begin{aligned}
T_s(q)_0 = {}& a_s(\bar{3}) \langle \mathcal{M}^i_j \mathcal{F}^{i'}_{j'} | H(\bar{3})^{i''} | \mathcal{C}_{jj'}\rangle \epsilon_{ii'i''} + b_s(\bar{3})_1 \langle \mathcal{M}^i_j \mathcal{F}^{i'}_{j'} | H(\bar{3})^j | \mathcal{C}_{i''j'}\rangle \epsilon_{ii'i''} + b_s(\bar{3})_2 \langle \mathcal{F}^i_j \mathcal{M}^{i'}_{j'} | H(\bar{3})^j | \mathcal{C}_{i''j'}\rangle \epsilon_{ii'i''} \\
& + c_s(\bar{3})_1 \langle \mathcal{M}^i_{j'} \mathcal{F}^{i'}_j | H(\bar{3})^j | \mathcal{C}_{i''j'}\rangle \epsilon_{ii'i''} + c_s(\bar{3})_2 \langle \mathcal{F}^i_{j'} \mathcal{M}^{i'}_j | H(\bar{3})^j | \mathcal{C}_{i''j'}\rangle \epsilon_{ii'i''} + a_s(6) \langle \mathcal{M}^i_j \mathcal{F}^{i'}_{j'} | H(6)^{jj'}_k | \mathcal{C}_{i''k}\rangle \epsilon_{ii'i''} \\
& + b_s(6)_1 \langle \mathcal{M}^i_j \mathcal{F}^{i'}_{j'} | H(6)^{i''j}_k | \mathcal{C}_{j'k}\rangle \epsilon_{ii'i''} + b_s(6)_2 \langle \mathcal{F}^i_j \mathcal{M}^{i'}_{j'} | H(6)^{i''j}_k | \mathcal{C}_{j'k}\rangle \epsilon_{ii'i''} + c_s(6)_1 \langle \mathcal{M}^i_j \mathcal{F}^{i'}_{j'} | H(6)^{i''j'}_k | \mathcal{C}_{jk}\rangle \epsilon_{ii'i''} \\
& + c_s(6)_2 \langle \mathcal{F}^i_j \mathcal{M}^{i'}_{j'} | H(6)^{i''j'}_k | \mathcal{C}_{jk}\rangle \epsilon_{ii'i''} + d_s(6) \langle \mathcal{M}^i_j \mathcal{F}^k_{j'} | H(6)^{i'i''}_k | \mathcal{C}_{jj'}\rangle \epsilon_{ii'i''} + e_s(6) \langle \mathcal{M}^k_j \mathcal{F}^i_{j'} | H(6)^{i'i''}_k | \mathcal{C}_{jj'}\rangle \epsilon_{ii'i''} \\
& + f_s(6) \langle \mathcal{M}^k_j \mathcal{F}^j_k | H(6)^{ii'}_l | \mathcal{C}_{i''l}\rangle \epsilon_{ii'i''} + g_s(6)_1 \langle \mathcal{M}^k_j \mathcal{F}^j_l | H(6)^{ii'}_k | \mathcal{C}_{i''l}\rangle \epsilon_{ii'i''} + g_s(6)_2 \langle \mathcal{F}^k_j \mathcal{M}^j_l | H(6)^{ii'}_k | \mathcal{C}_{i''l}\rangle \epsilon_{ii'i''} \\
& + a_s(\overline{15}) \langle \mathcal{M}^i_j \mathcal{F}^{i'}_{j'} | H(\overline{15})^{jj'}_k | \mathcal{C}_{i''k}\rangle \epsilon_{ii'i''} + b_s(\overline{15})_1 \langle \mathcal{M}^i_j \mathcal{F}^{i'}_{j'} | H(\overline{15})^{i''j}_k | \mathcal{C}_{j'k}\rangle \epsilon_{ii'i''} + b_s(\overline{15})_2 \langle \mathcal{F}^i_j \mathcal{M}^{i'}_{j'} | H(\overline{15})^{i''j}_k | \mathcal{C}_{j'k}\rangle \epsilon_{ii'i''} \\
& + c_s(\overline{15})_1 \langle \mathcal{M}^i_j \mathcal{F}^{i'}_{j'} | H(\overline{15})^{i''j'}_k | \mathcal{C}_{jk}\rangle \epsilon_{ii'i''} + c_s(\overline{15})_2 \langle \mathcal{F}^i_j \mathcal{M}^{i'}_{j'} | H(\overline{15})^{i''j'}_k | \mathcal{C}_{jk}\rangle \epsilon_{ii'i''}. \qquad (25)
\end{aligned}$$

The penguin amplitudes P have the same structure and can be obtained to replace the expressions for T by P. Also the P-wave amplitudes can be similarly constructed.

Expanding the above amplitudes, one can obtain the individual decay amplitude. Due to mixing between η_8 and η_1, the decay modes with η_8 in the final sates are not as clean as those with π and K in the final state to study. We will not consider processes involving η_8 further. We find the U-spin related amplitudes ($\Delta S = 0$ and $\Delta S = -1$) for antitriplet satisfy the following relations

$$\begin{aligned}
&T_t(\Xi_b^- \to K^- n) = T_t(\Xi_b^- \to \pi^- \Xi^0), \qquad T_t(\Xi_b^0 \to \bar{K}^0 n) = -T_t(\Lambda_b^0 \to K^0 \Xi^0),\\
&T_t(\Xi_b^- \to K^0 \Xi^-) = T_t(\Xi_b^- \to \bar{K}^0 \Sigma^-), \qquad T_t(\Xi_b^0 \to K^0 \Xi^0) = -T_t(\Lambda_b^0 \to \bar{K}^0 n),\\
&T_t(\Xi_b^0 \to \pi^- \Sigma^+) = -T_t(\Lambda_b^0 \to K^- p), \qquad T_t(\Lambda_b^0 \to \pi^- p) = -T_t(\Xi_b^0 \to K^- \Sigma^+),\\
&T_t(\Xi_b^0 \to \pi^+ \Sigma^-) = -T_t(\Lambda_b^0 \to K^+ \Xi^-), \qquad T_t(\Lambda_b^0 \to K^+ \Sigma^-) = -T_t(\Xi_b^0 \to \pi^+ \Xi^-),\\
&T_t(\Xi_b^0 \to K^- p) = -T_t(\Lambda_b^0 \to \pi^- \Sigma^+), \qquad T_t(\Xi_b^0 \to K^+ \Xi^-) = -T_t(\Lambda_b^0 \to \pi^+ \Sigma^-).
\end{aligned} \tag{26}$$

While the U-spin related amplitudes ($\Delta S = 0$ and $\Delta S = -1$) for the sextet satisfy

$$\begin{aligned}
&T_s(\Sigma_b^+ \to n\pi^+) = -T_s(\Sigma_b^+ \to \Xi^0 K^+), \qquad T_s(\Sigma_b^+ \to \Sigma^+ K^0) = -T_s(\Sigma_b^+ \to p\bar{K}^0),\\
&T_s(\Sigma_b^- \to n\pi^-) = -T_s(\Omega_b^- \to \Xi^0 K^-), \qquad T_s(\Sigma_b^- \to \Sigma^- K^0) = -T_s(\Omega_b^- \to \Xi^- \bar{K}^0),\\
&T_s(\Omega_b^- \to \Xi^0 \pi^-) = -T_s(\Sigma_b^- \to n K^-), \qquad T_s(\Omega_b^- \to \Sigma^- \bar{K}^0) = -T_s(\Sigma_b^- \to \Xi^- K^0),\\
&T_s(\Sigma_b^0 \to \Sigma^- K^+) = -T_s(\Xi_b'^0 \to \Xi^- \pi^+), \qquad T_s(\Sigma_b^0 \to p\pi^-) = -T_s(\Xi_b'^0 \to \Sigma^+ K^-),\\
&T_s(\Xi_b'^0 \to \Xi^- K^+) = -T_s(\Sigma_b^0 \to \Sigma^- \pi^+), \qquad T_s(\Xi_b'^0 \to \Sigma^- \pi^+) = -T_s(\Sigma_b^0 \to \Xi^- K^+),\\
&T_s(\Xi_b'^0 \to p K^-) = -T_s(\Sigma_b^0 \to \Sigma^+ \pi^-), \qquad T_s(\Xi_b'^0 \to \Sigma^+ \pi^-) = -T_s(\Sigma_b^0 \to p K^-),\\
&T_s(\Xi_b'^0 \to \Xi^0 K^0) = -T_s(\Sigma_b^0 \to n\bar{K}^0), \qquad T_s(\Xi_b'^0 \to n\bar{K}^0) = -T_s(\Sigma_b^0 \to \Xi^0 K^0),\\
&T_s(\Xi_b'^- \to n K^-) = -T_s(\Xi_b'^- \to \Xi^0 \pi^-), \qquad T_s(\Xi_b'^- \to \Xi^- K^0) = -T_s(\Xi_b'^- \to \Sigma^- \bar{K}^0).
\end{aligned} \tag{27}$$

IV. RESULTS AND DISCUSSIONS

For the processes in each of the pairs in Eqs. (26) and (27), there are relations in the form given by Eqs. (16) and (17). Some of them may be able to be tested at LHCb. Experimentally, it is difficult to measure a neutral state P which does not decay in the detector with high energy. It is therefore difficult to carry out tests using decay modes with a high energy P particle in the final states, which may be the case for a P from b-baryon decay. A P from secondary decay of a light octet may have a better chance to be studied because it has lower energy. Also for relations associated with polarization asymmetries, one needs to measure the polarizations of initial or final baryon polarization. It is difficult to measure the initial baryon polarization at LHCb, because the proton beams are not polarized. Information on polarization can be extracted by the secondary decay of the final baryon. Therefore, the final baryon which does not decay will not be useful for testing the relations associated with polarization asymmetries.

Relations associated with CP violating rate asymmetries for antitriplet b-baryon decays have been studied recently. Practical tests for the relation

$$\frac{A_{CP}(\mathcal{B}_a \to \mathcal{MF})_{\Delta S=0}}{A_{CP}(\mathcal{B}_b \to \mathcal{MF})_{\Delta S=-1}} = -\frac{\mathrm{Br}(\mathcal{B}_b \to \mathcal{MF})_{\Delta S=-1}}{\mathrm{Br}(\mathcal{B}_a \to \mathcal{MF})_{\Delta S=0}} \cdot \frac{\tau_{\mathcal{B}_a}}{\tau_{\mathcal{B}_b}} \tag{28}$$

can be carried out using the following pairs

$$\begin{aligned}
&(\Xi_b^- \to K^0 \Xi^-, \Xi_b^- \to \bar{K}^0 \Sigma^-),\\
&(\Xi_b^0 \to \pi^- \Sigma^+, \Lambda_b^0 \to K^- p), \quad (\Lambda_b^0 \to \pi^- p, \Xi_b^0 \to K^- \Sigma^+),\\
&(\Xi_b^0 \to \pi^+ \Sigma^-, \Lambda_b^0 \to K^+ \Xi^-), \quad (\Lambda_b^0 \to K^+ \Sigma^-, \Xi_b^0 \to \pi^+ \Xi^-),\\
&(\Xi_b^0 \to K^- p, \Lambda_b^0 \to \pi^- \Sigma^+), \quad (\Xi_b^0 \to K^+ \Xi^-, \Lambda_b^0 \to \pi^+ \Sigma^-).
\end{aligned} \tag{29}$$

In the above, we have not listed the ones having a neutron in the final state because the neutron from b-baryon decay has high energy and may be difficult to measure.

For polarization asymmetry measurement, the final baryon should decay into other particles to provide polarization information. In the above, three of the processes have the stable proton in final states, therefore, these decay modes are not useful for testing relations of polarization asymmetry. Only the following four pairs

$$\begin{aligned}
&(\Xi_b^- \to K^0 \Xi^-, \Xi_b^- \to \bar{K}^0 \Sigma^-), \quad (\Xi_b^0 \to \pi^+ \Sigma^-, \Lambda_b^0 \to K^+ \Xi^-),\\
&(\Lambda_b^0 \to K^+ \Sigma^-, \Xi_b^0 \to \pi^+ \Xi^-), \quad (\Xi_b^0 \to K^+ \Xi^-, \Lambda_b^0 \to \pi^+ \Sigma^-),
\end{aligned} \tag{30}$$

may be useful in testing the polarization asymmetry relation,

$$\frac{\mathcal{A}_\alpha(\mathcal{B}_a\to\mathcal{MF})_{\Delta S=0}}{\mathcal{A}_\alpha(\mathcal{B}_b\to\mathcal{MF})_{\Delta S=-1}} = -\frac{\mathrm{Br}(\mathcal{B}_b\to\mathcal{MF})_{\Delta S=-1}}{\mathrm{Br}(\mathcal{B}_a\to\mathcal{MF})_{\Delta S=0}}\cdot\frac{\tau_{\mathcal{B}_a}}{\tau_{\mathcal{B}_b}}. \tag{31}$$

To measure both polarization asymmetries in the above pairs, one needs to measure the polarizations of Ξ^- and Σ^-. Since Ξ^- decays mainly into $\Lambda\pi^-$, the polarization of Ξ^- can be relatively easily determined. At low energy, Σ^- polarization can be determined through $\Sigma^-\to n\pi^-$. In the LHCb environment, the neutron can have high energy in the laboratory frame; it may be difficult to measure. One needs to isolate events with low energy of Σ^- which may reduce event numbers. Σ^- also has a small decay branching ratio $[(5.73\pm0.27)\times10^{-5}]$ into $\Lambda e^-\bar{\nu}_e$ which may provide some additional information, but still have a neutral neutrino in the final states. Therefore it is very challenging to test the full pair relations to good precision.

The initial particles in the pair $\Xi_b^-\to K^0\Xi^-$ and $\Xi_b^-\to\bar{K}^0\Sigma^-$ decays are the same making the above corresponding equation simpler than other pairs above since no lifetime information is needed. These may be the best modes to carry out analysis for processes involving the antitriplet b-baryons experimentally.

For the sextet, it is possible to test the rate asymmetry relation of the type in Eq. (30) using the pairs below

$$\begin{aligned}
&(\Sigma_b^+\to\Sigma^+K^0,\Sigma_b^+\to p\bar{K}^0),\quad(\Sigma_b^-\to\Sigma^-K^0,\Omega_b^-\to\Xi^-\bar{K}^0),\\
&(\Omega_b^-\to\Sigma^-\bar{K}^0,\Sigma_b^-\to\Xi^-K^0),\quad(\Sigma_b^0\to\Sigma^-K^+,\Xi_b'^0\to\Xi^-\pi^+),\\
&(\Sigma_b^0\to p\pi^-,\Xi_b'^0\to\Sigma^+K^-),\quad(\Xi_b'^0\to\Xi^-K^+,\Sigma_b^0\to\Sigma^-\pi^+),\\
&(\Xi_b'^0\to\Sigma^-\pi^+,\Sigma_b^0\to\Xi^-K^+),\quad(\Xi_b'^0\to pK^-,\Sigma_b^0\to\Sigma^+\pi^-),\\
&(\Xi_b'^0\to\Sigma^+\pi^-,\Sigma_b^0\to pK^-),\quad(\Xi_b'^-\to\Xi^-K^0,\Xi_b'^-\to\Sigma^-\bar{K}^0).
\end{aligned} \tag{32}$$

For the polarization asymmetry relation test, one again has to remove the ones with a stable proton in final states. Therefore, the following pairs are good ones for the polarization asymmetry relation test,

$$\begin{aligned}
&(\Sigma_b^-\to\Sigma^-K^0,\Omega_b^-\to\Xi^-\bar{K}^0),\quad(\Omega_b^-\to\Sigma^-\bar{K}^0,\Sigma_b^-\to\Xi^-K^0),\\
&(\Sigma_b^0\to\Sigma^-K^+,\Xi_b'^0\to\Xi^-\pi^+),\quad(\Xi_b'^0\to\Xi^-K^+,\Sigma_b^0\to\Sigma^-\pi^+),\\
&(\Xi_b'^0\to\Sigma^-\pi^+,\Sigma_b^0\to\Xi^-K^+),\quad(\Xi_b'^-\to\Xi^-K^0,\Xi_b'^-\to\Sigma^-\bar{K}^0).
\end{aligned} \tag{33}$$

For the sextet, the best pair to carry out analysis may be the last pair above, $(\Xi_b'^-\to\Xi^-K^0,\Xi_b'^-\to\Sigma^-\bar{K}^0)$, because the initial particles are the same. Again it is very challenging to test the pair relation in polarization asymmetry to good precision because of the difficulty in measuring the polarization of Σ^-.

To actually test the relations discussed here, it is desirable that the CP violating rate and polarization asymmetries are reasonably large. Some theoretical estimates for some of the decays have been carried out [4]. For example, for the rate asymmetries for $\Lambda_b\to p\pi^-, pK^-$ are of order a few percent [4] which are reasonably sizeable for testing the relations studied here.

$SU(3)$ breaking effects are known to exist to some degree. In kaon and hyperon decays, the breaking effects are at the order of 20 to 30 percent. One may expect that the breaking effects are at a similar level and therefore the relations studied hold also at that level. However, the CP violating rate asymmetry relations similar to Eq. (17) for the pair $\bar{B}_s^0\to K^+\pi^-$ and $\bar{B}^0\to K^-\pi^+$ seems to hold at a better level. If one defines a measure of $SU(3)$ breaking by $r_c=-[A_{CP}(\bar{B}_s^0\to K^+\pi^-)/A_{CP}(\bar{B}^0\to K^-\pi^+)]/[\mathrm{Br}(\bar{B}^0\to K^-\pi^+)\tau_{\bar{B}_s^0}/\mathrm{Br}(\bar{B}_s^0\to K^+\pi^-)\tau_{\bar{B}^0}]$. In the $SU(3)$ limit, $r_c=1$. Experimentally [1,6,11] $r_c=0.96\pm0.19$. The central value is about 5% away from 1. The 1σ level error bar is about 20%. This is an indication that $SU(3)$ may work better in systems with a b quark than that of kaon and hyperon systems. Whether this is an accident or $SU(3)$ works better for B decays needs to be understood. It is therefore important to test relations discussed here experimentally. Experimental data obtained will provide crucial information to understand the dynamics of b-hadron decays. We eagerly urge our experimental colleagues to carry out related tests discussed here.

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