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Modulated bimaximal neutrino mixing

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The present article is an endeavor to look into some fruitful frameworks based on "bimaximal" (BM) neutrino mixing from a model-independent stand. The possibilities involving the correction or attenuation of the original BM mixing matrix followed by grand-unified-theory-inspired charged-lepton correction are invoked. The "symmetry basis," thus, constructed accentuates some interesting facets such as a modified quark lepton complementarity relation, $\theta_{12}+\theta_c \approx \frac{\pi}{4} - \theta_{13}\cos(n\pi - \delta_{CP})$, a possible linkup between neutrino and charged-lepton sectors, $\theta_{13}^{\nu} = \theta_{12}^{l} \sim \mathcal{O}(\theta_C)$, or that between neutrinos and quarks, $\theta_{13}^{\nu} = \theta_C$. The study vindicates the relevance of the bimaximal mixing as a first approximation.

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I. INTRODUCTION

In the past few years, certain significant descriptions are seen to be imprinted by the oscillation experiments [1–4], which compel one to rethink several enriched and fascinating frameworks developed in the last few decades [5–20]. Several models based on discrete flavor symmetries which posit a vanishing reactor angle, apparently sound less credible after the reactor angle is proclaimed to be large [21,22] and equivalent to the Cabibbo angle (θ_C) [23] in the quark sector. The bimaximal (BM) framework [24–36] is one such example that predicts a zero reactor angle and maximal solar angle, but both predictions are now null and void except the prediction of a maximal θ_{23} which is still consistent within a 1σ error. This indicates either to preclude the BM framework or to harmonize the underlying motivation a little. One such improved follow-up of the BM mechanism is the bilarge scenario [37–41], which assumes the solar and atmospheric angles are equal, nonmaximal, and large, in general, but leaves a scope to embrace a partial bimaximal scenario. Also, in this framework the reactor angle is assumed to be as large as the Cabibbo angle.

The neutrino mixing, in general, is characterized by six parameters—three mixing angles, θ_{12} (solar angle), θ_{13} (reactor angle), and θ_{23} (atmospheric angle)—followed by three phases—one Dirac-type (δ_{CP}) and two Majorana-type charge parity (CP)-violating phases (κ, γ). The neutrino oscillation experiments witness all the observational parameters mentioned above, except the two Majorana phases. All of these parameters are contained within U, the so-called Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix or lepton mixing matrix, which, under standard parametrization, appears as

$$U = R_{23}(\theta_{23})U_{13}(\theta_{13}:\delta)R_{12}(\theta_{12}).P, \quad (1)$$

where P is the diagonal matrix that shelters the two Majorana phases. In the *symmetry* basis, where both charged-lepton and neutrino mass matrices are considered as nondiagonal, one can identify the lepton mixing matrix $U = \mathcal{U}_{lL}^{\dagger}.U_{\nu}$ [42–48], where, $\mathcal{U}_{lL}$ is the left-handed charged-lepton diagonalizing matrix, and U_{ν} is the neutrino mixing matrix. If it is the *flavor* basis, only the charged-lepton mass matrix is diagonal and one sees $U = U_{\nu}$.

As per the BM ansatz, $U_{\rm BM} = R_{23}(\pi/4)R_{12}(\pi/4)$. The specific choice of the PMNS matrix $U = U_{\rm BM}$ highlights the working basis as flavor one, but once redirected to the symmetry basis, $U_{\rm BM}$ is open to further amendment from the charged-lepton sector, and $U = \mathcal{U}_{lL}^{\dagger}U_{\rm BM}$. At the same time, the choice of both $\mathcal{U}_{lL}$ and the charged-lepton mass matrix become significant. In this respect, we shall be headed by certain grand unified theory (GUT)-motivated phenomenology which highlights the possible kinship between "down-quark" and "charged-lepton" mass matrices, and we shall also briefly discuss the scenario where the tie-in is slacken. The first scenario leads to definite choices of $\mathcal{U}_{lL}$, whereas the second approach is a little conjectural and asks for a concrete conceptual rationale.

Here, we emphasize that the strides undertaken in the present approach differ from those where the original chassis is mended either from the charged-lepton or neutrino sector, and instead, we adopt the situation where both sectors share a part [49,50].

The present analysis endeavors to identify a predictive frame work based on a BM mixing mechanism and to probe

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those prospects that may lead the neutrino mixing towards the unified picture of flavors.

II. BM AND BM-DEVIATED SCENARIOS

A. Neutrino mixing matrix

We shall divide the discussion on the PMNS matrix deviated from BM mixing into three rostrums: scheme I, scheme II, and scheme III. First, we choose the flavor basis. Up to the Majorana phases, three versions of the neutrino mixing matrices are presented as follows:

(i) Scheme I

$$U_\nu = U_{\rm BM} = R_{23}\left(-\frac{\pi}{4}\right)R_{12}\left(\frac{\pi}{4}\right), \quad (2)$$

where the sign convention undertaken is in accordance with Ref. [5]. The present texture of U_ν just highlights the original BM proposition.

(ii) Scheme II

Multiplying the $U_{\rm BM}$ further with a boost $W_{12}^T(\epsilon)$ from the right, we obtain

$$U_\nu = U_{\rm BM}.W_{12}^T(\epsilon), \quad (3)$$

$$= R_{23}\left(-\frac{\pi}{4}\right)R_{12}\left(\frac{\pi}{4}-\epsilon\right), \quad (4)$$

where

$$W_{12} \approx \begin{bmatrix} 1-\frac{\epsilon^2}{2} & \epsilon & 0 \\ -\epsilon & 1-\frac{\epsilon^2}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (5)$$

where ϵ is an unknown parameter. The above texture emphasizes the possible modulation of BM mixing from the 1-2 sector.

(iii) Scheme III

Another speculation which associates a nonzero reactor angle $\theta_{13} \sim \mathcal{O}(\theta_C)$ with the original BM framework is presented as

$$U_\nu = R_{23}\left(\frac{\pi}{4}\right)U_{13}(\alpha\theta_C : \delta)R_{12}\left(\frac{\pi}{4}\right), \quad (6)$$

where α is an unknown $\mathcal{O}(1)$ coefficient. The above design is motivated in the bilarge mixing frameworks [37,40,41], and it finds some resemblance to the tribimaximal Cabibbo mixing matrix [51]. Unlike the original BM mixing scheme (scheme I), the present one permits the entry of a Dirac-*CP*-violating phase within the neutrino sector. The parameter δ is also free. With α being zero, scheme III coincides with scheme I.

The detailed textures of the U_ν's corresponding to schemes I–III are highlighted in Table I.

TABLE I. The description of schemes I and III in the flavor basis: $U = U_\nu$. Scheme I depicts the original BM mixing. Schemes II and III describe U_ν deviated from $U_{\rm BM}$ in terms of θ_{12} and θ_{13}, respectively.

	Flavor basis
I	$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{bmatrix}$
II	$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} 1-\frac{\epsilon^2}{2} & \epsilon & 0 \\ -\epsilon & 1-\frac{\epsilon^2}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$
III	$\begin{bmatrix} \frac{1}{\sqrt{2}}(1-\frac{\alpha^2\lambda^2}{2}) & \frac{1}{\sqrt{2}}(1-\frac{\alpha^2\lambda^2}{2}) & \alpha\lambda e^{-i\delta} \\ -\frac{1}{2}(1-\alpha\lambda e^{i\delta}) & \frac{1}{2}(1+\alpha\lambda e^{i\delta}) & -\frac{1}{\sqrt{2}}(1-\frac{\alpha^2\lambda^2}{2}) \\ -\frac{1}{2}(1+\alpha\lambda e^{i\delta}) & \frac{1}{2}(1-\alpha\lambda e^{i\delta}) & \frac{1}{\sqrt{2}}(1-\frac{\alpha^2\lambda^2}{2}) \end{bmatrix}$

B. Neutrino mass matrix

It is pertinent to trace out the textures of the neutrino mass matrices in schemes II and III, which are necessary to understand the mechanism of symmetry and the breakdown of the same as well.

(i) We know that scheme I shows an S_3 invariant texture,

$$M_{\rm BM} = \begin{bmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{bmatrix}. \quad (7)$$

The above texture follows 2-3 interchange symmetry (also called μ-τ symmetry).

(ii) If it is scheme II, the texture in Eq. (7) is muddled, but the 2-3 symmetry is fortified. We express its texture as

$$M'_{\mu\tau} \approx M_{\rm BM} + \sqrt{2}y\epsilon \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} - 2y\epsilon^2 \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} - \frac{1}{\sqrt{2}}y\epsilon^3 \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}. \quad (8)$$

(iii) However, in scheme III, the μ-τ symmetry is breached. However, it is interesting to note that the neutrino mass matrix as a whole reflects a blend of μ-τ and anti-μ-τ symmetric textures. For simplicity, when $\delta = 0$, one can enunciate

$$M_\nu = \begin{bmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{bmatrix} + \sqrt{2}a\lambda \begin{bmatrix} 0 & x-z & z-x \\ x-z & y & 0 \\ z-x & 0 & -y \end{bmatrix} + a^2\lambda^2 \begin{bmatrix} 2(z-x) & -\frac{y}{2} & -\frac{y}{2} \\ -\frac{y}{2} & x-z & z-x \\ -\frac{y}{2} & z-x & x-z \end{bmatrix} + \frac{1}{\sqrt{2}}a^3\lambda^3 \begin{bmatrix} 0 & z-x & x-z \\ z-x & 0 & 0 \\ x-z & 0 & 0 \end{bmatrix} + \frac{a^4\lambda^4}{8} \begin{bmatrix} 2x & 0 & 0 \\ 0 & 2z-x & x-2z \\ 0 & x-2z & 2z-x \end{bmatrix} + \mathcal{O}(\lambda^5). \quad (9)$$

It is interesting to note that the textures associated with the even powers of λ are μ-τ symmetric and those for the odd powers are anti-μ-τ symmetric. Although schemes II and III encompass the possible amendments either in terms of θ_{12} or θ_{13}, they fail to describe a complete picture. Hence, we need to redefine the schemes in the *symmetry* basis, but before that, we investigate the possible forms of the $\mathcal{U}_{lL}$'s inspired in GUTs.

III. TEXTURES OF $\mathcal{U}_{lL}$ FROM SU(5) GUT

Though the lepton and quark sectors differ a lot from the mixing point of view, the SO(10) and SU(5) GUTs reflect rational possibilities to link the two sectors to a certain extent. In GUT, a single joint operator can engender the elements of both quark and lepton Yukawa matrices. This signifies a possible link-up between the Yukawa matrices for "down-type" quarks (Y_d) and "charged" leptons (Y_l) in terms of certain "GUT"-motivated relations.

For example, the Pati-Salam models posit $Y_l \simeq Y_d$ [52–56]. If Y_e is exactly equal to Y_d, then one can directly equate θ^l_{12} to $\theta^d_{12}(\simeq\theta_C)$. This, at the same time, says $\mathcal{U}_{lL} = U_d \simeq V_{\rm CKM}$.

On the other hand, SU(5) models reveal $Y_l \simeq Y_d^T$ [57,58]. Following the road maps of Refs. [59–63], we develop certain SU(5)-inspired textures of Y_l, which describe

$$\sin\theta^l_{12} = \beta\lambda, \quad (10)$$

where $\lambda = \sin\theta_C \simeq \theta_C$, and β encompasses the possibilities of both $\beta \gtrsim 1$ and $\beta \lesssim 1$ (see Table II). We summarize the steps undertaken. According to SU(5) GUT, if

$$Y_d = \begin{bmatrix} d & b & 0 \\ a & c & 0 \\ 0 & 0 & f \end{bmatrix}, \quad \text{then} \quad (11)$$

$$Y_l = \begin{bmatrix} \mathcal{A}_{11}d & \mathcal{A}_{12}b & 0 \\ \mathcal{A}_{21}a & \mathcal{A}_{22}c & 0 \\ 0 & 0 & \mathcal{A}_{33}f \end{bmatrix}^T. \quad (12)$$

The $\mathcal{A}_{ij}$ coefficients are driven by the common joint operator from where Y_d and Y_l emerge. With the dimension-five operator, the $\mathcal{A}_{ij}$'s have the following choices:

TABLE II. Different possibilities for $(Y_e)_{12}$ are illustrated based on SU(5) GUT models. The coefficients appearing in all the matrices are allowed by a dimension-five operator. The above textures respect the GUT-motivated relation: $y_\mu : y_s \approx 6$. The fermion mass ratios are the important parameters in appraising the validity of the above textures. The important parameter $(y_\mu/y_s)(y_d/y_e)$ must lie within $10.7^{+1.8}_{-0.8}$. The above textures highlight different possibilities to parametrize the 1-2 rotation of U_{lL}. One can see that all the possibilities including $\beta \gtrsim 1$ or $\beta \lesssim 1$ are allowed, where $\beta = \sin\theta^l_{12}/\sin\theta_C$. In the above textures, the input parameter c is chosen as unity.

	$(Y_l)_{12}$	$\{d,a,b\}$	β	$\lvert\frac{y_e}{y_\mu}\rvert$	$\lvert\frac{y_s}{y_d}\rvert$	$\lvert V_{us}\rvert$	$\lvert\frac{y_\mu}{y_s}\frac{y_d}{y_e}\rvert$
(a)	$\begin{bmatrix} 9d & \frac{3}{2}b \\ 3a & 6c \end{bmatrix}^T$	$\{0.0016, 0.24, 0.244\}$	0.527075	0.004832	19.56	0.2257	10.57
(b)	$\begin{bmatrix} 0 & -\frac{2}{3}b \\ \frac{9}{2}a & 6c \end{bmatrix}^T$	$\{0, 0.24, 0.244\}$	0.785059	0.004723	19.56	0.2257	10.82
(c)	$\begin{bmatrix} \frac{1}{6}d & -\frac{1}{2}b \\ 6a & 6c \end{bmatrix}^T$	$\{-0.003, 0.22, 0.243\}$	0.952475	0.004168	19.56	0.2258	12.26
(d)	$\begin{bmatrix} -\frac{2}{3}d & -\frac{1}{2}b \\ 6a & 6c \end{bmatrix}^T$	$\{0.001, 0.24, 0.244\}$	1.03452	0.004507	19.35	0.2256	11.46
(e)	$\begin{bmatrix} 1d & -\frac{1}{2}b \\ 6a & 6c \end{bmatrix}^T$	$\{-0.0002, 0.251, -0.245\}$	1.07894	0.004850	18.26	0.2253	11.28
(f)	$\begin{bmatrix} \frac{3}{2}d & -\frac{1}{2}b \\ 9a & 6c \end{bmatrix}^T$	$\{-0.005, 0.20, 0.241\}$	1.27392	0.004379	20.59	0.2254	11.09

$$\mathcal{A}_{ij} = \left\{\frac{1}{6}, -\frac{1}{2}, -\frac{2}{3}, 1, \pm\frac{3}{2}, -3, \frac{9}{2}, 6, 9, -18\right\}. \quad (13)$$

The $\mathcal{U}_{lL}$ is the matrix that diagonalizes $Y_l.Y_l^\dagger$, and for the above texture, it assumes the form

$$\mathcal{U}_{lL} \simeq \begin{bmatrix} 1-\frac{\beta\lambda^2}{2} & \beta\lambda & 0 \\ -\beta\lambda & 1-\frac{\beta\lambda^2}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (14)$$

under the assumption that $\{a, b, c, d\}$ are all real. The parametrization of these quantities and selection of $\mathcal{A}_{ij}$'s must accompany a consistent prediction of fermion mass ratios and $|V_{us}|$ [63–65]:

$$\left|\frac{y_\mu}{y_s}\right| \approx 9/2, 6, \qquad \left|\frac{y_\tau}{y_b}\right| \approx 3/2, \quad (15)$$

$$\left|\frac{y_\mu y_d}{y_s y_e}\right| \approx 10.7^{+1.8}_{-0.8}, \qquad |V_{us}| \approx 0.2255. \quad (16)$$

To illustrate, let

$$Y_d \sim \begin{bmatrix} d & b & 0 \\ a & c & 0 \\ 0 & 0 & f \end{bmatrix}, \quad (17)$$

$$Y_l \sim \begin{bmatrix} 0 & -\frac{2}{3}b & 0 \\ \frac{9}{2}a & 6c & 0 \\ 0 & 0 & -\frac{3}{2}f \end{bmatrix}^T, \quad (18)$$

and with $\{a, b, c\} = \{0.24, 0.244, 1\}$, it predicts

$$\sin\theta_{12}^l \approx 0.785\lambda \quad (19)$$

and satisfies all the necessary conditions. For more details, we refer to Table II where five other possibilities are also highlighted. In the present context, we are interested in the GUT-motivated relation $|y_\mu/y_s| \approx 6$, and the textures of the Y_l's are designed accordingly. A more rigorous treatment of this issue is available in Ref. [60], but the present discussion includes those possibilities like $\mathcal{A}_{ij} = 1/6, -2/3$ which were predicted later in Ref. [63] and are unfounded in the former.

The model independent parameter β appears in several ways. For example, in Georgi-Jarlskog mechanism, it takes a value 1/3, [66]. In Ref. [61], we obtain $\beta = 1$. Also several supersymmetry breaking schemes like minimal gauge mediated supersymmetry breaking and constrained minimal supersymmetric standard model assign β certain fractional values 1/6 and 2/9 respectively [59]. Another possibility is found in Ref. [67], where certain operators generating fermion masses may lead to $\beta = 3/2$.

A. CKM-like texture

The above discussion contributes a lot to delineate the texture of $\mathcal{U}_{lL}$. The $\mathcal{U}_{lL}$ being a 3×3 unitary matrix requires six phases ϕ_{ij} in addition to three angular parameters to parametrize the same. This motivates one to define a generalized "CKM-like" texture with

$$\theta_{12}^l \approxeq \beta\lambda, \qquad \theta_{23}^l \approx A\lambda^2, \qquad \theta_{13}^l \sim \lambda^3$$

as shown by

$$\mathcal{U}_{lL} \approx U_{12}(\beta\lambda:\phi_{12}).U_{13}(A\lambda^3:\phi_{13})U_{12}(A\lambda^2:\phi_{23}). \quad (20)$$

Here, λ, A, ρ, and η are the standard Wolfenstein parameters [68]. Out of six phases, only three appear in the texture of CKM-like $\mathcal{U}_{lL}$ and the rest are absorbed with a redefinition of the right-handed charged lepton fields. If the U_ν does not contain any CP phase other than two Majorana-CP-violating phases [tri-bimaximal (TB) and BM scenarios], then such a CKM-like $\mathcal{U}_{lL}$ that shelters arbitrary phases helps to construct an all-embracing PMNS matrix.

B. Close-to-CKM texture

Once we portray $\mathcal{U}_{lL}$ as V_{CKM} or close to V_{CKM}, besides the similarity of angles, the similarity of the phases becomes important. The subsequent choices of U_{eL} respect this stand. We put $\phi_{12} = 0$ and let $\phi_{23} = 0$ or π,

$$\mathcal{U}_{lL} \approx R_{12}(\beta\lambda).U_{13}(A\lambda^3:\phi_{13})R_{12}(\pm A\lambda^2). \quad (21)$$

On neglecting the small Cabibbo-Kobayashi-Maskawa (CKM) type 1-2 and 2-3 rotational effects, the above texture coincides exactly with the original SU(5) texture of $\mathcal{U}_{lL}$ in Eq. (14).

C. Exact-CKM texture

Additionally, if $\beta = 1$, which suggests $Y_e = Y_d$, we encounter $\mathcal{U}_{lL} = V_{\text{CKM}}$,

$$\mathcal{U}_{lL} \approx R_{12}(\lambda).U_{13}(A\lambda^3:\phi_{13})R_{12}(\pm A\lambda^2). \quad (22)$$

In principle, the contribution of $U_{13}(A\lambda^3:\phi_{13})$ can be neglected, but its presence may highlight the small CKM-like CP contribution towards the lepton mixing matrix in terms of ϕ_{13}, where

$$\phi_{13} = -\tan^{-1}\frac{\eta}{\rho}. \quad (23)$$

Table III contains the details of the above textures.

IV. SYMMETRY BASIS

Now we shall redefine schemes I–III in the symmetry basis. For all numerical analyses or comparisons, we shall adhere to Ref. [22].

TABLE III. The different choices of U_{lL}'s with CKM-like, close-to-CKM, and exact-CKM textures depicted. In the second and third textures, both possibilities of $\phi_{23}=0$ and $\phi_{23}=\pi$ are considered.

U_{lL}	β	Texture
CKM-like	$\beta\neq 1$	$\begin{bmatrix} 1-\frac{\beta^2\lambda^2}{2} & \beta\lambda e^{-i\phi_{12}} & A\lambda^3(\rho-i\eta) \\ -\beta\lambda e^{i\phi_{12}} & 1-\frac{\beta^2\lambda^2}{2} & A\lambda^2 e^{-i\phi_{23}} \\ A\lambda^3(\beta e^{i(\phi_{12}+\phi_{23})}-\rho-i\eta) & -A\lambda^2 e^{i\phi_{23}} & 1 \end{bmatrix}$
Close to CKM	$\beta\neq 1$	$\begin{bmatrix} 1-\frac{\beta^2\lambda^2}{2} & \beta\lambda & A\lambda^3(\rho-i\eta) \\ -\beta\lambda & 1-\frac{\beta^2\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(\beta-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}, \begin{bmatrix} 1-\frac{\beta^2\lambda^2}{2} & \beta\lambda & A\lambda^3(\rho-i\eta) \\ -\beta\lambda & 1-\frac{\beta^2\lambda^2}{2} & -A\lambda^2 \\ -A\lambda^3 & (\beta+\rho+i\eta)A\lambda^2 & 1 \end{bmatrix}$
Exact CKM texture	$\beta=1$	$\begin{bmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}, \begin{bmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & -A\lambda^2 \\ -A\lambda^3(1+\rho+i\eta) & A\lambda^2 & 1 \end{bmatrix}$

A. Scheme I and *CP* conservation

In scheme I [see Eq. (2)] the association of the *close-to-CKM*-type $\mathcal{U}_{lL}$ [see Eq. (21)] with the existing U_ν brings about

$$\theta_{12}\approx\frac{\pi}{4}-\frac{\beta\lambda}{\sqrt{2}}+\frac{A\beta\lambda^3}{\sqrt{2}}-\frac{\beta^3\lambda^3}{3\sqrt{2}}-\frac{A\lambda^3\rho}{\sqrt{2}}, \tag{24}$$

$$\theta_{13}\approx\frac{\beta\lambda}{\sqrt{2}}, \tag{25}$$

$$\theta_{23}\approx\frac{\pi}{4}+\lambda^2\left(A-\frac{\beta^2}{4}\right), \tag{26}$$

$$\delta_{CP}\approx n\pi+\frac{A\eta\lambda^2}{\beta}. \tag{27}$$

We see that $\theta_{12}\approx\frac{\pi}{4}-\theta_{13}$, so if θ_{13} rises, then θ_{12} will go down, but θ_{13} is not free and is dominated by the model-dependent parameter β. To obtain the best results for θ_{12}, $\beta>1$, and that for θ_{13} requires $\beta<1$. So in this scheme, the best possibility is to choose the limiting condition $\beta=1$, which reveals

$$\theta_{12}\approx 36^0(2\sigma),\qquad \theta_{13}\approx 9.17^0(1\sigma), \tag{28}$$

$$\theta_{23}\approx 46.64^0(1\sigma),\qquad \delta_{CP}\approx\pi. \tag{29}$$

As another possibility, we associate CKM-like $\mathcal{U}_{lL}$ [see Eq. (20)] with scheme I. With this modified setup, the oscillation observables appear as follows:

$$\theta_{12}\approx\frac{\pi}{4}-\frac{\beta\lambda}{\sqrt{2}}\cos\phi_{12}+\frac{A\beta\lambda^3}{\sqrt{2}}\cos(\phi_{12}+\phi_{23}) -\frac{\beta^3\lambda^3}{3\sqrt{2}}\cos^3\phi_{12}-\frac{A\lambda^3\rho}{\sqrt{2}}, \tag{30}$$

$$\theta_{13}\approx\frac{\beta\lambda}{\sqrt{2}}, \tag{31}$$

$$\theta_{23}\approx\frac{\pi}{4}+\lambda^2\left(A\cos\phi_{23}-\frac{\beta^2}{4}\right), \tag{32}$$

$$\delta_{CP}\approx n\pi-\phi_{12}+\frac{A\lambda^2}{\beta}(\beta\sin\phi_{23}-\eta\cos\phi_{12}+\rho\sin\phi_{12}). \tag{33}$$

In contrast to the previous scenario, the predictions now involve two angular parameters $\{\phi_{12},\phi_{23}\}$ which are constrained within 0 and π. However, the parametrization of both unknowns with respect to the observables is difficult. Let us choose $\beta=1.03452$ (see Table II) and apply a condition $\phi_{12}+\phi_{23}=90^0$ so that θ_{12} is depleted maximally from 45^0. Let $\phi_{12}=0$, and one sees that θ_{12} reaches

$$\theta_{12}\approx 35.32^0(1\sigma)<\sin^{-1}\left(\frac{1}{\sqrt{3}}\right). \tag{34}$$

The other observables are predicted as follows:

$$\theta_{13}\approx 9.49^0(2\sigma),\qquad \theta_{23}\approx 44.22^0(1\sigma),\qquad \delta_{CP}\approx 0.99\pi. \tag{35}$$

The similar treatment, if conducted with $\beta=1$, leads to

$$\theta_{12}\approx 35.65^0(2\sigma),\qquad \theta_{13}\approx 9.17^0(1\sigma), \tag{36}$$

$$\theta_{23}\approx 44.27^0(1\sigma),\qquad \delta_{CP}\approx 0.99\pi. \tag{37}$$

We conclude that scheme I depicts only the *CP*-suppressed scenarios and highlights both possibilities: $\theta_{23}>45^0$ and $\theta_{23}<45^0$. At a time, either θ_{12} or θ_{13}

can be predicted more precisely than the other. Scheme I is simple and hardly uses any observational parameters as input. It is also interesting to note that with an appropriate choice of β, the solar angle can be lowered even from the tri-bimaximal prediction.

B. Scheme II and the modified quark lepton complementarity relation

The scheme II θ_{12} is subjugated mostly from the neutrino sector with an unknown parameter ϵ, and an extension of scheme II in light of CKM-like $\mathcal{U}_{lL}$ leads to the following sum rules:

$$\theta_{12} \approx \frac{\pi}{4} - \frac{\beta\lambda}{\sqrt{2}}\cos\phi_{12} + \frac{A\beta^3\lambda^3}{\sqrt{2}}\cos(\phi_{12}+\phi_{23}) - \frac{A\lambda^3\rho}{\sqrt{2}} - \frac{\beta^3\lambda^3}{3\sqrt{2}}\cos^3\phi_{12} - \epsilon(\beta^2\lambda^2\cos^2\phi_{12} - \beta^2\lambda^2 + 1),$$

$$\theta_{13} \approx \frac{\beta\lambda}{\sqrt{2}}, \tag{38}$$

$$\theta_{23} \approx \frac{\pi}{4} + \lambda^2\left(A\cos\phi_{23} - \frac{\beta^2}{4}\right), \tag{39}$$

$$\delta_{CP} \approx n\pi - \phi_{12} + \frac{A\lambda^2}{\beta}(\sin\phi_{23} - \eta\cos\phi_{12} + \rho\sin\phi_{12}). \tag{40}$$

Unlike scheme I, the prediction of θ_{12} depends a little on θ_{13}. Here, we find three free parameters: $\{\epsilon, \phi_{12}, \phi_{23}\}$. To illustrate, let $\beta = 1$. For simplicity, we assume the maximal deviation of θ_{23} from 45^0 which implies $\phi_{23} = (2n+1)\pi/2$, so one sees either

$$\theta_{23} \approx 41.9^0(1\sigma) \quad \text{or} \quad \theta_{23} \approx 46.64^0(1\sigma). \tag{41}$$

The related sum rule is approximated as follows:

$$\theta_{23} \pm A\lambda^2 \approx \frac{\pi}{4} - \theta_{13}^2. \tag{42}$$

Let us visualize the situation when CP violation is maximum. It reveals $\phi_{12} \approx 0.5\pi$, and on choosing $\epsilon \approx \theta_C$, one sees

$$\theta_{12} \approx 32.62^0(3\sigma) < \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \tag{43}$$

followed by a sum rule up to $\mathcal{O}(\lambda^2)$,

$$\theta_{12} + \theta_C \approx \frac{\pi}{4}, \tag{44}$$

which is the original quark lepton complementarity (QLC) relation [67,69–72]. However, in order to acquire a precise θ_{12}, we deviate a little from the condition of maximal CP violation. On choosing $\phi_{12} \approx 0.567\pi$, we obtain

$$\theta_{12} \approx 34.62^0 \text{ (central value)}, \qquad \delta_{CP} \approx 1.43\pi(1\sigma), \tag{45}$$

which one can relate with a new version of the QLC relation [obtained from Eqs. (38)–(40)] as highlighted below,

$$\theta_{12} + \theta_c \approx \frac{\pi}{4} - \theta_{13}\cos(n\pi - \delta_{CP}). \tag{46}$$

It is to be noted that Eq. (44) is obtainable only when $\delta_{CP} = (2n+1)\pi/2$. The reactor angle depends only on β. and for this special case, when $\beta = 1$, it is predicted as

$$\theta_{13} \approx 9.17^0(1\sigma). \tag{47}$$

With a different choice of β, further lowering of the same is possible. The details of the present scheme are sorted in Table IV.

Let us summarize the possibilities with scheme II.

(i) In contrast to scheme I, the prediction of the results is more precise and are consistent within the 1σ range. The strife between θ_{12} and θ_{13} is tamed.
(ii) The parametrization concerns three free parameters. The observable θ_{12} is chosen as input, and ϕ_{23} is fixed either at 0 or π.
(iii) The interesting feature of scheme II is that it hoists the QLC relation in revised form, and the original form is reinstated if CP violation is maximum. In view of this, the choice of the free parameter ϵ as θ_C is relevant.
(iv) Scheme II does not advocate for CP-suppressed cases, but in order to obtain precise θ_{12}, it depicts a CP violation shifted a little from maximality.

C. Scheme III and large $\theta_{13}^\nu \sim \mathcal{O}(\theta_c)$

For scheme II, the prediction of observable CP violation is solely dependent on the charged-lepton sector, but if it is scheme III, this dependency is subdued. In the present scheme, we shall concentrate mostly on the parametrization, where the neutrino sector leads the CP violation.

TABLE IV. The predictions of scheme II (with $\epsilon = \theta_C$) for the observable parameters θ_{13}, θ_{23}, and δ_{CP} are highlighted. Here, θ_{12} is taken as an input parameter: $\sin^2\theta_{12} = 0.323$. The CKM-like charged-lepton corrections are employed, where θ_{12}^l is fixed by Table II.

β	$\frac{\phi_{12}}{\pi}$	$\theta_{13}(^0)$	$\theta_{23}(^0)$	$\frac{\delta_{CP}}{\pi}$
0.52707	0.6642	4.82(−)	42.43–47.17 (1σ)	1.33(1σ)
0.78505	0.5975	7.19(−)	42.18–46.92 (1σ)	1.39(1σ)
0.95247	0.5730	8.73(1σ)	41.95–46.71 (1σ)	1.42(1σ)
1	0.5674	9.17(1σ)	41.90–46.64 (1σ)	1.43(1σ)
1.03452	0.5636	9.49(2σ)	41.85–46.59 (1σ)	1.43(1σ)
1.07894	0.5589	9.90(3σ)	41.78–46.52 (1σ)	1.44(1σ)
1.27392	0.5417	11.71(−)	41.44–46.19 (1σ)	1.46(1σ)

We first concentrate on the generalized extension of scheme III where both the charged and neutrino sectors contribute toward observable CP violation. This framework embraces a CKM-like $\mathcal{U}_{lL}$ [see Eq. (20)], and one sees that the concerned observational parameters are headed by four unknown parameters: (α, δ) from the neutrino sector and (ϕ_{12}, ϕ_{23}) from the charged-lepton sector. We see the sum rules appear as shown in the following:

$$\theta_{12} \approx \frac{\pi}{4} - \frac{\beta\lambda\cos\phi_{12}}{\sqrt{2}} + \lambda^3\left(\frac{1}{2}\alpha\beta^2\cos\delta - a\beta^2\cos\phi_{12}\cos(\phi_{12}-\delta) + \frac{Ab\cos(\phi_{12}+\phi_{23})}{\sqrt{2}} - \frac{A\rho}{\sqrt{2}} - \frac{\beta^3\cos^3\phi_{12}}{3\sqrt{2}} - \frac{\alpha^2\beta\cos\phi_{12}}{2\sqrt{2}}\right), \tag{48}$$

$$\theta_{13} \approx \theta_c\sqrt{\alpha^2 + \sqrt{2}\alpha\beta\cos(\delta-\phi_{12}) + \frac{\beta^2}{2}} + \frac{1}{6}\lambda^3\left(\alpha^2 + \sqrt{2}\alpha\beta\cos(\delta-\phi_{12}) + \frac{\beta^2}{2}\right)^{3/2}, \tag{49}$$

$$\theta_{23} \approx \frac{\pi}{4} + \lambda^2\left(A\cos\phi_{23} - \frac{\alpha\beta\cos(\delta-\phi_{12})}{\sqrt{2}} - \frac{\beta^2}{4}\right), \tag{50}$$

$$\delta_{CP} \approx n\pi - \tan^{-1}\left(\frac{2\alpha\sin\delta + \sqrt{2}\beta\sin\phi_{12}}{2\alpha\cos\delta + \sqrt{2}\beta\cos\phi_{12}}\right) - \frac{\lambda^2}{2(2\alpha^2 + 2\sqrt{2}\alpha\beta\cos(\phi_{12}-\delta) + \beta^2)} \times \{-\sqrt{2}\alpha^3\beta\sin(\phi_{12}-\delta) + \sqrt{2}\alpha(2A(\beta\sin(\phi_{12}+\phi_{23}-\delta) - \eta\cos\delta + \rho\sin\delta)) + \beta^3\sqrt{2}\alpha\sin(\phi_{12}-\delta) - 2A\beta\eta\cos\phi_{12} + 2A\beta(\beta\sin\phi_{23} + \rho\sin\phi_{12})\}. \tag{51}$$

Let $\{\phi_{12}, \phi_{23}, \delta\} \neq 0$. For the present parametrization, $\beta = 3/2$ [67] is found to be the most suitable one. Here the number of free parameters is equal to that of the observational ones. On assigning the angular parameters to their central values $\theta_{12} = 34.63^0$, $\theta_{13} = 8.87^0$, $\theta_{23} = 48.85^0$, and $\delta_{CP} \approx 1.32\pi$, one obtains $\phi_{12} \approx 0.23\pi$, $\phi_{23} \approx 0.032\pi$, $\delta = -0.96\pi$, and $a \approx 1.23$. One can see that the parametrization reflects an inherent CP-suppressed scenario ($\delta \sim -\pi$) and substantiates a large 1-3 mixing ($\theta_{13}^\nu \sim a\lambda$),

$$\theta_{13}^\nu \approx 15.6^0 > \theta_C, \tag{52}$$

within the neutrino sector. The present parametrization involves as many free parameters as the observable ones and is less predictive.

Next, we shall focus on the parametrization of scheme III with the following possibilities:

(i) $\phi_{12} = 0$, that is, a charged-lepton sector contributes the least towards observable CP violation, and we expect $\mathcal{U}_{lL}$ to assume a close-to-CKM texture. This involves only two free parameters α and δ;

(ii) in addition, as we expect $\theta_{13}^\nu \sim \mathcal{O}(\theta_C)$ to be similar to that for θ_{12}^l, one can further make the parametrization more predictive with a rational ansatz $\theta_{13}^\nu = \theta_{12}^l \sim \mathcal{O}(\theta_C)$, which implies $\alpha = \beta$. This indicates the involvement of the single free parameter δ.

To address the first possibility, we consider the observable parameters θ_{13} and δ_{CP} as input parameters and θ_{13}^ν and treat the internal CP phase δ, θ_{12}, and θ_{23} as the predictions. To illustrate, let $\sin^2\theta_{13} = 0.023$ and $\delta_{CP} = 1.34\pi$ (central values). Say, $\beta = 1.07894$. Adopting either of the possibilities, $\phi_{23} = 0$ or π, one sees

$$\begin{aligned} \theta_{13}^\nu &= -9.18^0(-9.94^0), \\ \delta &= 0.0925\pi(0.09571\pi), \\ \theta_{12} &= 35.33^0(34.51^0)[1\sigma], \\ \theta_{23} &= 47.55^0(42.71^0)[1\sigma]. \end{aligned} \tag{53}$$

Here, we see that $|\theta_{13}^\nu| \lesssim \theta_C$, but with the same environment, another possibility $|\theta_{13}^\nu| \gtrsim \theta_C$ along with a precise prediction of other observable parameters are also obtainable, as shown in the following:

$$\begin{aligned} \theta_{13}^\nu &= -16.05^0(-15.67^0), \\ \delta &= -0.0508\pi(-0.0545\pi), \\ \theta_{12} &= 35.33^0(34.51^0)[1\sigma], \\ \theta_{23} &= 48.96^0(44.11^0)[1\sigma]. \end{aligned} \tag{54}$$

One can show that if the model-dependent parameter $\beta < 1$, one hardly obtains $|\theta_{13}^\nu| > \theta_C$. For a detailed analysis, we refer to Tables V and VI.

The same treatment, when applied to another possibility $\delta_{CP} = 1.48\pi$ (another central value), results in

TABLE V. Scheme III is tested along with the corrections introduced from different close-to-CKM-like U_{lL}'s (see Table II) with inputs; $\sin^2\theta_{13} \approx 0.023$, $\delta_{CP} \approx 1.34\pi$ (central values). The predictions of θ^{ν}_{13}, δ (the internal CP phase of U_ν), the observable parameters θ_{12} and θ_{23} are made in pairs. The first column of each pair corresponds to $\phi_{23} = 0$ and the rest follows $\phi_{23} = \pi$. This table is associated with $\theta^{\nu}_{13} \lesssim \theta_C$.

β	$\theta^{\nu}_{13}(^0)$	$\theta^{\nu}_{13}(^0)$	$\frac{\delta}{\pi}$	$\frac{\delta}{\pi}$	$\theta_{12}(^0)$	$\theta_{12}(^0)$	$\theta_{23}(^0)$	$\theta_{23}(^0)$
0.527075	7.67	7.48	−0.1681	−0.1642	40.27(−)	39.88(−)	47.22(1σ)	42.46(1σ)
0.785059	8.32	−7.92	0.4381	0.7605	37.96(−)	37.37(3σ)	47.31(1σ)	42.52(1σ)
0.952475	−9.1	−8.56	0.1032	0.1068	36.46(2σ)	35.74(2σ)	47.43(1σ)	42.61(1σ)
1	−9.36	−8.78	0.099	0.1024	36.04(2σ)	35.28(1σ)	47.47(1σ)	42.64(1σ)
1.03452	−9.57	−8.96	0.0961	0.0994	35.73(2σ)	34.95(1σ)	47.5(1σ)	42.67(1σ)
1.07894	−9.84	−9.19	0.0925	0.09571	35.33(1σ)	34.51(1σ)	47.55(1σ)	42.71(1σ)
1.27392	−11.19	−10.38	0.079	0.08141	33.57(2σ)	32.61(3σ)	47.8(1σ)	42.91(1σ)

TABLE VI. The same description as Table V but highlights the scenarios when $\theta^{\nu}_{13} \gtrsim \theta_C$. One sees that for $\beta < 1$, the $\theta^{\nu}_{13} \gtrsim \theta_C$ predictions are unfounded.

β	$\theta^{\nu}_{13}(^0)$	$\theta^{\nu}_{13}(^0)$	$\frac{\delta}{\pi}$	$\frac{\delta}{\pi}$	$\theta_{12}(^0)$	$\theta_{12}(^0)$	$\theta_{23}(^0)$	$\theta_{23}(^0)$
1	−15.41	−15.06	−0.0518	−0.0555	36.04(2σ)	35.28(1σ)	48.78(1σ)	43.95(1σ)
1.03452	−15.69	−15.32	−0.0508	−0.0545	35.73(2σ)	34.95(1σ)	48.85(1σ)	44.02(1σ)
1.07894	−16.05	−15.67	−0.0495	−0.0532	35.33(1σ)	34.51(1σ)	48.96(1σ)	44.11(1σ)
1.27392	−17.69	−17.21	−0.0443	−0.0481	33.57(2σ)	32.61(3σ)	49.44(1σ)	44.55(1σ)

$$\theta^{\nu}_{13} = -13.01^0(-12.40^0),$$
$$\delta = 0.074\pi(0.078\pi),$$
$$\theta_{12} = 35.33^0(34.51^0)[1\sigma],$$
$$\theta_{23} = 48.18^0(43.31^0)[1\sigma]. \tag{55}$$

It is interesting to note that a condition $|\theta^{\nu}_{13}| \approxeq \theta_C$ is reached. In contrast to the previous situation, when input parameter $\delta_{CP} = 1.34\pi$, the present scenario highlights either of the two possibilities: $|\theta^{\nu}_{13}| \geq \theta_C$ or $|\theta^{\nu}_{13}| \leq \theta_C$, and never two at a time. The details of the parametrization are highlighted in Table VII.

Let us concentrate on the second stand, which encompasses the provision $\alpha = \beta$. This parametrization is the most predictive in the sense that it uses only one variable δ, and in order to parametrize it, we fix the observable parameter $\sin^2\theta_{13} = 0.023$ as an input. Say, if $\beta = 1.07894$, one sees for $\phi_{23} = 0, \pi$,

$$\theta^{\nu}_{13} = 13.94^0,$$
$$\delta = 0.25\pi,$$
$$\theta_{12} = 35.33^0(34.51^0)[1\sigma],$$
$$\theta_{23} = 48.40^0(43.66^0)[1\sigma],$$
$$\delta_{CP} = 1.47\pi, (1.44\pi)[1\sigma]. \tag{56}$$

The other possibilities are shown in Table VIII. The present parametrization gives better results for $\beta > 1$.

Scheme III has characteristic features which we highlight in the following:

TABLE VII. The depiction of scheme III along with the corrections introduced from close-to-CKM-like U_{lL}'s (see Table II), with inputs; $\sin^2\theta_{13} \approx 0.023$, $\delta_{CP} \approx 1.48\pi$ (central values). With these, we predict θ^{ν}_{13}, δ (the internal CP phase of U_ν), the observable parameters θ_{12} and θ_{23}. Prediction of each parameter appears in two columns. The first column of corresponds to $\phi_{23} = 0$, and the rest is applicable to $\phi_{23} = \pi$. This table contains all the scenarios $\theta^{\nu}_{13} \lesssim \theta_C$ and $\theta^{\nu}_{13} \gtrsim \theta_C$.

β	$\theta^{\nu}_{13}(^0)$	$\theta^{\nu}_{13}(^0)$	$\frac{\delta}{\pi}$	$\frac{\delta}{\pi}$	$\theta_{12}(^0)$	$\theta_{12}(^0)$	$\theta_{23}(^0)$	$\theta_{23}(^0)$
0.527075	−9.71	−9.51	−0.5247	−0.521	40.26(−)	39.86(−)	47.53(1σ)	42.76(1σ)
0.785059	−11.05	−10.66	0.0915	0.0956	37.96(−)	37.37(3σ)	47.77(1σ)	42.96(1σ)
0.952475	−12.86	−11.6	−0.0747	0.085	36.46(2σ)	35.74(2σ)	48.15(1σ)	43.14(1σ)
1	−12.45	−11.89	0.0782	0.0823	36.04(2σ)	35.28(1σ)	48.06(1σ)	43.2(1σ)
1.03452	−12.69	−12.11	0.0763	0.0804	35.73(2σ)	34.95(1σ)	48.11(1σ)	43.25(1σ)
1.07894	−13.01	−12.4	0.074	0.078	35.33(1σ)	34.51(1σ)	48.18(1σ)	43.31(1σ)
1.27392	−14.51	−13.74	0.065	0.0688	33.57(2σ)	32.61(2σ)	48.54(1σ)	43.62(1σ)

TABLE VIII. In scheme III, one sees $\theta_{13}^{\nu} \simeq \theta_C$, similar to the 1-2 rotation angle θ_{12}^{l}, which is also $\theta_{12}^{l} \simeq \theta_C$. This motivates one to look into those possibilities where $\theta_{13}^{\nu} = \theta_{12}^{l} \simeq \theta_C$, which says $\alpha = \beta$. It is found that these possibilities are more relevant for the cases $\beta \geq 1$.

$\beta = \alpha$	θ_{13}^{ν}	$\frac{\delta}{\pi}$	$\frac{\delta_{CP}}{\pi}$	$\frac{\delta_{CP}}{\pi}$	θ_{12}	θ_{12}	θ_{23}	θ_{23}
0.527075	6.8	0.149	1.72	1.71	40.26(−)	39.86(−)	47.11(1σ)	42.37(1σ)
0.785059	10.14	0.217	1.57	1.55	37.96(−)	37.37(3σ)	47.6(1σ)	42.87(1σ)
0.952475	12.3	0.2388	1.51	1.48	36.46(3σ)	35.74(1σ)	48.03(1σ)	43.29(1σ)
1	12.92	0.2436	1.49	1.47	36.04(2σ)	35.28(1σ)	48.16(1σ)	43.42(1σ)
1.03452	13.36	0.2469	1.48	1.46	35.73(1σ)	34.95(1σ)	48.26(1σ)	43.53(1σ)
1.07894	13.94	0.25	1.47	1.44	35.33(1σ)	34.51(1σ)	48.4(1σ)	43.66(1σ)
1.27392	16.45	0.2647	1.42	1.38	33.57(2σ)	32.61(3σ)	49.07(1σ)	44.33(1σ)

(i) The present scheme vindicates the assumption of $\theta_{12} = \theta_{23} = 45^0$ (maximal mixing) as a first approximation.

(ii) We emphasize that θ_{13}^{ν} is the output of the parametrization. Interestingly, we see that the inherent 1-3 angle within the neutrino sector can be larger $\theta_{13}^{\nu} \sim \pi/10, \pi/20$. In addition, one can see $\theta_{13}^{\nu} = \theta_C$, and in a certain occasion, $\theta_{13}^{\nu} = \theta_{12}^{l}$ also. These two features sound relevant in the context of a unified theory of flavors.

(iii) In fact, the observable CP violation in the lepton sector may share the contribution both from the charged-lepton and neutrino sectors in terms of ϕ_{12} and δ, respectively, as is evident from the approximated generalized expression

$$\delta_{CP} \approx 2\pi - \tan^{-1}\left(\frac{2\alpha\sin\delta + \sqrt{2}\beta\sin\phi_{12}}{2\alpha\cos\delta + \sqrt{2}\beta\cos\phi_{12}}\right). \quad (57)$$

However, once we choose the $\mathcal{U}_{lL}$'s with close-to-CKM texture and adhere to the β's described in Table II, we are more close to the original description of the $\mathcal{U}_{lL}$'s [see Eq. (14)] motivated in SU(5) GUT. The description negates the presence of ϕ_{12}. Hence, in this respect, the internal CP phase δ from the neutrino sector plays a promising role. In the present parametrization as $\theta_{13}^{\nu} \neq 0$, this feature is more prominent and seems logical in contrast to those model-independent possibilities discussed in Refs. [41,50].

(iv) The present parametrization is predictive. It uses only one (θ_{13}) or two (θ_{13} and δ_{CP}) observational parameters as input and predicts the rest and also the two unphysical parameters $\{\theta_{13}^{\nu}, \delta\}$.

V. WHEN $Y_l \sim Y_d$

The discussion so far focuses on the possible patterns of the symmetry basis believing Y_l and Y_d are originated from a single joint operator, whereas another possibility that reinforces a different origin of Y_l and Y_d is also relevant [73]. We add a small extension in this line. We assume that the neutrino sector follows BM mixing and there is no modulation, and the charged-lepton sector alone is responsible for all the observable deviation. With this motivation, we put forward the following texture zero Yukawa matrix (Y_l) up to $\mathcal{O}(\lambda^7)$ as the following:

$$Y_l \simeq \begin{bmatrix} 2\lambda^6 & \frac{\lambda^3}{\sqrt{2}}\left(1 - i\frac{1}{\sqrt{2}}\right) & -\frac{\lambda}{\sqrt{2}}(1 + i2\lambda) \\ 0 & \lambda^2 & \frac{\lambda^2}{3}(1+i) \\ 0 & -\lambda^3\left(1 + i\frac{\lambda}{3}\right) & 1 \end{bmatrix}. \quad (58)$$

This Y_l can be diagonalized with a left-handed diagonalizing matrix $\mathcal{U}_{lL}$ of which the information is supplied and shown as

$$|\mathcal{U}_{lL}| \approx \begin{bmatrix} 0.969 & 0.176 & 0.172 \\ 0.175 & 0.984 & 0.023 \\ 0.173 & 0.013 & 0.984 \end{bmatrix}, \quad (59)$$

$$\arg[\mathcal{U}_{lL}] \approx \begin{bmatrix} 0.131 & 0.383 & -0.865 \\ -0.608 & 0.640 & 0.255 \\ 0 & 0 & 0 \end{bmatrix}\pi. \quad (60)$$

The right-handed diagonalizing matrix of the above Y_l is $V_{lR} \approx \mathrm{diag}\{i, 1, 1\}$. Also, $|y_e/y_\mu|$ is predicted as 0.00494. The PMNS matrix constructed $U = \mathcal{U}_{lL}^{\dagger}.R_{23}^{\nu}(\pi/4)R_{12}^{\nu}\times(\pi/4)$ in this background leads to

$$\theta_{13} = 8.17^0[1\sigma], \quad (61)$$

$$\theta_{12} = 33.52^0[2\sigma], \quad (62)$$

$$\theta_{23} = 44.35^0[1\sigma], \quad (63)$$

$$\delta_{CP} = 1.69\pi[1\sigma]. \quad (64)$$

But more important is to trace out the framework where the texture in Eq. (58) may emerge. Interestingly, we see this texture is encouraged in Refs. [74,75].

VI. DISCUSSION AND SUMMARY

All the $\mathcal{U}_{lL}$'s and the related Y_l's discussed in the present article [except Eq. (58)] are motivated in SU(5) GUT. Similar to the charged-lepton sector, it would have been a good exercise to work out the first principle supporting the model-independent textures of both U_ν and M_ν highlighted in schemes II and III, but this is beyond the scope of the present article. In short, we wish to discuss the possible linkups that may help the model builders to think in this line.

In the neutrino mass matrix under scheme II [see Eq. (8)], the parameter ϵ is responsible for deviating M_ν from the BM mixing scenario within the μ-τ symmetric regime. This phenomenon is somehow akin to the flavor twisting effect which is motivated in the extra-dimension-inspired frameworks [76].

Also, the parameter ϵ in scheme II leaves a scope to achieve the original QLC relation with little modification [see Table II and Eq. (46)] by tuning the former to θ_C. Perhaps this is not just a mere numerical coincidence, and one finds the related discussion in the "Cabibbo-haze"-based theories [77,78].

In scheme III, the neutrino mass matrix, M_ν in Eq. (9) is approximated as

$$M_{\rm BM} + c_1\lambda \underbrace{\begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}}_{\delta m^{\rm typeI}} + c_2\lambda \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{\delta m^{\rm typeII}}, \qquad (65)$$

where $\delta m^{\rm typeI}$ and $\delta m^{\rm typeII}$ resemble the first-order perturbation to $M_{\rm BM}$, and, possibly, the type-I and type-II seesaw mechanisms in the S_4 symmetric background may generate these deviation matrices in their respective order [79]. Scheme III describes one possibility, which, in addition, to μ-τ symmetry breaking, requires charged-lepton correction also. We see that this methodology is motivated in grand unified theories [80,81].

In scheme III, the situation which highlights $\theta_{13}^\nu \sim 18^0$ is motivated in Refs. [82,83].

Also, the scheme III scenarios $\sin\theta_{13}^\nu \simeq \lambda$, $\sin\theta_{12}^\nu = \sin\theta_{23}^\nu \simeq 3.13\lambda (= 1/\sqrt{2})$ are inspired in Ref. [38]. Perhaps the former pattern is derivable in the bilarge-based frameworks based on $U(1)\times Z_m \times Z_n$ symmetry, with m and n having different parities.

The present model-independent analysis aspires us to refine the BM-based framework and tries to relate the same to the unified theory of flavors. References [84–87] discuss the possibilities to amend the BM framework following other alternatives. The present work finds some similarity with Ref. [50], but the motivations in either case differ. The latter concerns θ_{13}^ν as input and assigns preferred values to it, and the charged-lepton diagonalizing matrices considered therein are arbitrary. In contrast, the present work considers θ_{13}^ν as a prediction of a certain parametrization (scheme III) and encounters several interesting possibilities like $\theta_{13}^\nu = \theta_C$ and even $\theta_{13}^\nu = \theta_{12}^l \sim \mathcal{O}(\theta_C)$ (we hope these relations are important in the context of GUT), and in addition to those, $\theta_{13}^\nu \sim \pi/10$, $\theta_{13}^\nu = \pi/20$, etc. Also, the charged-lepton corrections adopted in the present analysis are not arbitrary and inspired in SU(5) GUT, and the present work uses one or two observational parameters as input and sounds more predictive.

To summarize, we have highlighted the new possibilities of $\mathcal{U}_{lL}$'s motivated in SU(5) GUT and have tried to reinstate the BM mixing scheme in terms of modulation, either in 1-2 rotation or 1-3 rotation in light of charged-lepton correction. The parametrization is predictive and hoists a revised QLC relation of which the original one appears as a special case. This scenario, however, supports a little deviation from maximal *CP* violation. In addition, it spotlights the BM scenarios with the 1-3 angle as large as the Cabibbo angle, lesser and even larger than the same and also accents the scenarios like $\theta_{13}^\nu = \theta_{12}^l$. In conclusion, one may infer that the BM mixing, which is less attractive in light of present experimental data, sounds tenable as a first approximation if the original motivation is tuned a little.

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