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Muon $g-2$ in focus point SUSY

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We point out that the anomaly of the muon $g-2$ can easily be explained in a focus point supersymmetry (SUSY) scenario, which realizes the seminatural SUSY. Among known focus point SUSY scenarios, we find that a model based on Higgs-gaugino mediation works with a mild fine-tuning $\Delta = 40$–80. We propose two new focus point SUSY scenarios where the anomaly of the muon $g-2$ is also explained. These scenarios are variants of the widely known focus point SUSY based on gravity mediation with universal scalar masses.

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I. INTRODUCTION

Low-energy supersymmetry (SUSY) has many attractive features and is a leading candidate for physics beyond the standard model (SM). In the minimal supersymmetric SM, three gauge coupling constants of SM gauge groups are unified at a high-energy scale around 10^{16} GeV. The electroweak symmetry breaking (EWSB) is induced via SUSY breaking, which was expected to solve the fine-tuning problem of the Higgs potential, namely, to explain the smallness of the EWSB breaking scale.

Another attractive and important feature of low-energy SUSY is that it has a potential of providing a solution to the long-standing puzzle, the anomaly of the muon anomalous magnetic moment ($g-2$). The experimental value of the muon anomalous magnetic moment is deviated from the SM prediction $(a_\mu)_{\rm SM}$ above the 3σ level [1,2]

$$(a_\mu)_{\rm EXP} - (a_\mu)_{\rm SM} = \left\{ \begin{array}{c} (26.1 \pm 8.0) \times 10^{-10} \\ (28.7 \pm 8.0) \times 10^{-10} \end{array} \right\}. \quad (1)$$

Here, $(a_\mu)_{\rm EXP}$ is the experimental value of the muon $(g-2)/2$ accurately measured at the Brookhaven E821 experiment [3]. In low-energy SUSY, smuons and chargino/neutralinos of $\mathcal{O}(100)$GeV give $\mathcal{O}(10^{-9})$ corrections to the muon $g-2$ and explain this discrepancy [4,5].

However, nonobservation of SUSY signals at the Large Hadron Collider (LHC) (see, e.g., Refs. [6]) and the relatively heavy Higgs boson of 125 GeV [7] push up the SUSY scale above TeV. Especially, the observed Higgs boson mass requires rather large radiative corrections from heavy stops [8]: it is suggested that the stop is as heavy as 3–5 TeV [9], including higher order corrections beyond the three-loop level. As a result, both the SUSY solution to the fine-tuning problem and the SUSY explanation of the muon $g-2$ anomaly seem to be difficult to work.

There are several attempts to attack these two difficulties, but separately. As a solution to the fine-tuning problem, the focus point SUSY now becomes more attractive [10] (see also [11,12]). In the focus point SUSY, a special relation among soft SUSY breaking parameters is assumed so that radiative corrections to the Higgs potential cancel each other. As a result, the EWSB scale becomes insensitive to the soft SUSY breaking mass scale. There are several focus point SUSY scenarios, based on gaugino mediation [13], Higgs-gaugino mediation [14], gravity mediation with nonuniversal gaugino masses [15], and gauge mediation [16].

On the other hand, light smuons and light chargino/neutralino are required to explain the muon $g-2$ anomaly, while the Higgs boson mass around 125 GeV requires rather heavy stops. In Refs. [17], it is shown that the Higgs boson mass and the muon $g-2$ anomaly are explained simultaneously by mass splitting among generations. Also, other possibilities are provided based on gauge mediation [18], gravity mediation [19], and gaugino mediation [20]: in these frameworks, colored and noncolored SUSY particles are split in their masses so that the SUSY contribution to the muon $g-2$ is enhanced.

In this paper, we show that the anomaly of the muon $g-2$ can easily be explained in a focus point SUSY scenario. In the next section, we review four known types of focus point scenarios and discuss whether the scenarios can explain the muon $g-2$ anomaly. We find that a model based on Higgs-gaugino mediation, which is recently proposed by the current authors [14], works. It is found that the discrepancy of the muon $g-2$ from the SM prediction is reduced to the 1σ level with a mild fine-tuning $\Delta = 40$–80. [See Eq. (15) for the definition of Δ.] We propose two new focus point scenarios which can explain the muon $g-2$ anomaly in Sec. IV. They are variants of the well-known focus point SUSY scenario proposed by Feng, Matchev, and Moroi [10].

II. FOCUS POINT FOR THE ELECTROWEAK SYMMETRY BREAKING

In focus point SUSY scenarios, the EWSB scale is relatively insensitive to the soft SUSY breaking mass scale.

This is achieved by introducing some fixed ratios between soft mass parameters at a high-energy scale. In this section, we review four known focus point scenarios and discuss whether they can explain the muon $g-2$ anomaly. We show that only one of them works.

The conditions for the EWSB are given by

$$\frac{g_1^2+g_2^2}{4}v^2 \simeq \left[-\mu^2 - \frac{(m_{H_u}^2 + \frac{1}{2v_u}\frac{\partial\Delta V}{\partial v_u})\tan^2\beta}{\tan^2\beta - 1} + \frac{m_{H_d}^2 + \frac{1}{2v_d}\frac{\partial\Delta V}{\partial v_d}}{\tan^2\beta-1}\right]\Bigg|_{M_{\rm IR}},$$

$$\frac{B\mu(\tan^2\beta+1)}{\tan\beta} \simeq \left[m_{H_u}^2 + \frac{1}{2v_u}\frac{\partial\Delta V}{\partial v_u} + m_{H_d}^2 + \frac{1}{2v_d}\frac{\partial\Delta V}{\partial v_d} + 2\mu^2\right]\Bigg|_{M_{\rm IR}}, \quad (2)$$

where g_1 and g_2 are gauge coupling constants of $U(1)_Y$ and $SU(2)_L$, v_u and v_d are the vacuum expectation values (VEV) of the up-type and down-type Higgs, $\tan\beta \equiv v_u/v_d$, $v \equiv \sqrt{v_u^2+v_d^2}$ is the EWSB scale, μ is the Dirac mass term of the Higgs doublets, m_{H_u} and m_{H_d} are soft masses for the up-type and down-type Higgses, $B\mu$ is the SUSY breaking holomorphic Higgs quadratic mass term, and ΔV is a one-loop contribution to the Higgs potential. $m_{H_u}^2$, $m_{H_d}^2$, and ΔV are evaluated at the geometric mean value of stop masses, $M_{\rm IR} = \sqrt{m_{Q_3}m_{\bar{U}_3}}$. For large $\tan\beta$, $m_{H_d}^2$ is relatively unimportant for the EWSB scale, since its effect is suppressed by $1/\tan^2\beta$.

The low-energy values of $m_{H_u}^2$ and $m_{H_d}^2$ are written in terms of gaugino masses and scalar masses at the high-energy scale,[1]

$$\begin{aligned} m_{H_u}^2(3\ \text{TeV}) &\simeq 0.009M_1^2 + 0.217M_2^2 - 1.168M_3^2 \\ &+ 0.005M_1M_2 - 0.109M_2M_3 - 0.016M_1M_3 \\ &+ 0.667m_{H_u}^2 + 0.026m_{H_d}^2 + 0.073m_L^2 - 0.074m_{\bar{E}}^2 \\ &- 0.385m_Q^2 - 0.163m_{\bar{U}}^2 - 0.070m_{\bar{D}}^2, \\ m_{H_d}^2(3\ \text{TeV}) &\simeq 0.030M_1^2 + 0.367M_2^2 - 0.120M_3^2 \\ &- 0.002M_1M_2 - 0.030M_2M_3 - 0.001M_1M_3 \\ &+ 0.019m_{H_u}^2 + 0.933m_{H_d}^2 - 0.088m_L^2 + 0.063m_{\bar{E}}^2 \\ &+ 0.044m_Q^2 - 0.145m_{\bar{U}}^2 + 0.043m_{\bar{D}}^2, \end{aligned} \quad (3)$$

for $M_{\rm IR} = 3$ TeV, $\tan\beta = 20$, $m_t = 173.34$ GeV, and $\alpha_s(m_Z) = 0.1185$. The soft SUSY breaking parameters in the right hand side of Eq. (3) are defined at $M_{\rm in} = 10^{16}$ GeV. Here, M_1, M_2, and M_3 are the bino, wino, and gluino masses, respectively, and m_Q, $m_{\bar{U}}$, $m_{\bar{D}}$, m_L, and $m_{\bar{E}}$ are generation-universal soft masses of left-handed squarks, right-handed up squarks, right-handed down squarks, left-handed sleptons, and right-handed sleptons, respectively. The above expressions are obtained by numerically solving two-loop renormalization group equations [21]. For this purpose, we use softSUSY 3.6.1 package [22].

In the focus point SUSY, $\tilde{m}_H^2 \equiv m_{H_u}^2 - (m_{H_d}^2 - m_{H_u}^2)/\tan^2\beta$ becomes insensitive to SUSY breaking parameters. This is achieved by introducing fixed ratio(s) among mass parameters at a high-energy scale, which we take as $M_{\rm in} = 10^{16}$ GeV. Currently, the following four focus point scenarios are known:

FPUS: universal scalar masses (m_0) and a fixed m_0/M_3.
FPGM: vanishing or small scalar masses and a fixed M_2/M_3.
FPHSG: high scale gauge mediation with a fixed messenger number, (N_2, N_3).
FPHGM: vanishing slepton and squark masses and a fixed m_{H_u}/M_3.

FPUS is based on gravity mediation, where universal scalar masses are assumed. In this case, their contributions to $m_{H_u}^2$ almost cancel each other. *FPGM* is based on gaugino mediation, where all soft scalar masses vanish at the high-energy scale $M_{\rm in}$. *FPHSG* is based on high scale gauge mediation, where scalar masses as well as gaugino masses are generated by messenger loops. Finally, *FPHGM* is based on Higgs-gaugino mediation motivated by the E_7 nonlinear sigma model [23], where squark and slepton masses vanish at the high-energy scale. More detailed descriptions are shown below.

Before discussing each focus point, we comment on nonuniversal gaugino masses. As we will see in the next section, nonuniversal gaugino masses are crucial in order to explain the muon $g-2$ anomaly and the observed Higgs boson mass around 125 GeV, simultaneously. Nonuniversal gaugino masses are naturally obtained if product group unification (PGU) is considered [24]. We note that PGU has an advantage over the minimal $SU(5)$ grand unification (GUT): PGU provides a solution to the doublet-triplet splitting problem [25,26]. The gauge coupling unification is still maintained approximately.

We briefly discuss how nonuniversal gaugino masses arise in the $SU(5)_{\rm SM} \times SU(3)_H \times U(1)_H$ PGU model [25], where the unification of quarks and leptons into $SU(5)$ multiplets is maintained. Gaugino masses are given by couplings between a SUSY breaking field Z and gauge multiplets,

[1] Here, we neglect the contribution from A terms, for simplicity. It does not change our conclusion qualitatively unless A terms are so large that they dominate the quantum corrections to $m_{H_u}^2$.

$$\mathcal{L} \supset \int d^2\theta \left[\left(\frac{1}{4g_5^2} - \frac{k_5 Z}{M_P} \right) W_5 W_5 + \left(\frac{1}{4g_{3H}^2} - \frac{k_{3H} Z}{M_P} \right) W_{3H} W_{3H} + \left(\frac{1}{4g_{1H}^2} - \frac{k_{1H} Z}{M_P} \right) W_{1H} W_{1H} \right] + \text{H.c.}, \quad (4)$$

where g_5, g_{3H}, and g_{1H} are the gauge coupling constants of $SU(5)_{\rm SM}$, $SU(3)_H$, and $U(1)_H$ gauge interactions, respectively. The field strength superfields of the gauge multiplets are denoted by W_5, W_{3H}, and W_{1H}, and k_5, k_{3H}, and k_{1H} are constants. After $SU(5)_{\rm SM} \times SU(3)_H \times U(1)_H$ is broken down to $SU(3)_C \times SU(2)_L \times U(1)_Y$, nonuniversal gaugino masses are generated at the GUT scale as

$$M_1/M_2 \simeq \frac{k_5 \mathcal{N} + k_{1H}}{k_5} \frac{1}{\mathcal{N}}, \qquad M_3/M_2 \simeq \frac{k_5 + k_{3H}}{k_5}, \quad (5)$$

where we take the strong coupling limit, $g_{1H}^2, g_{3H}^2 \gg g_5^2$. The constant $\mathcal{N}$ is determined by the $U(1)_H$ charge of GUT breaking Higgs fields, which break $SU(5)_{\rm SM} \times SU(3)_H \times U(1)_H$ down into the SM gauge group. In the strong coupling limit of $SU(3)_H$ and $U(1)_H$, the gauge coupling unification is still maintained approximately as $g_1^2 \simeq g_3^2 \simeq g_2^2 = g_5^2$ at the GUT scale. Here, g_1, g_2, and g_3 are gauge coupling constants of $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, respectively.

(i) *FPUS* The original focus point is proposed in a framework of gravity mediation. Surprisingly, if all the scalar masses are universal, their contributions to $\tilde{m}_H^2$ almost cancel each other at the low-energy scale [10];

$$\tilde{m}_H^2(3\ \text{TeV}) \simeq -1.170 M_3^2 + 0.072 m_0^2 + \cdots, \quad (6)$$

where $\cdots$ denotes other contributions containing M_1 or M_2. If the ratio m_0/M_3 is fixed to be 4–5, the low-energy value of $\tilde{m}_H^2$ becomes insensitive to the SUSY breaking mass scale [12].[2] [Because of the correction ΔV in Eq. (2), $\tilde{m}_H^2$ is not necessarily negative for the successful EWSB.]

In *FPUS* sleptons as well as squarks are as heavy as a few TeV to explain the observed Higgs mass; therefore the SUSY contribution to the muon $g-2$, Δa_μ, is suppressed.

(ii) *FPGM* In gaugino mediation models, we have a focus point with nonuniversal gaugino masses. Assuming that scalar masses vanish at the GUT scale, $\tilde{m}_H^2$ is given by

$$\tilde{m}_H^2(3\ \text{TeV}) \simeq -1.170 M_3^2 + 0.217 M_2^2 - 0.109 M_2 M_3, \quad (7)$$

where we have dropped negligible contributions depending on M_1. One can see that above $\tilde{m}_H^2$ nearly vanishes for $M_2/M_3 \simeq 2.6$ and -2.1 [13]. Universal scalar masses are introduced without much affecting the fine-tuning of the EWSB scale, as long as m_0 is not very large [15].

Since M_2 is large, left-handed sleptons become inevitably heavy. The low-energy value of m_L^2 is given by

$$\begin{aligned} m_L^2&(3\ \text{TeV}) \\ &\simeq 0.391 M_2^2 + 0.033 M_1^2 + (\text{smaller terms}) \\ &\simeq 2.643 M_3^2 + 0.033 M_1^2 + (\text{smaller terms}), \end{aligned} \quad (8)$$

where we take $M_2 = 2.6 M_3$ in the second line. Consequently, *FPGM* cannot explain $\Delta a_\mu \gtrsim 10^{-9}$.

(iii) *FPHSG* It has been shown in Refs. [16] that a focus point exists in high scale gauge mediation models.[3] In *FPHSG*, the number of $SU(2)_L$ doublet messengers (N_L) and $SU(3)_C$ triplet messengers (N_D) are not equal: for $N_L \gg N_D$, the EWSB scale becomes insensitive to the fundamental SUSY breaking parameter, $m_{\rm mess}$ (see the Appendix for details).

$$\begin{aligned} \tilde{m}_H^2(3\ \text{TeV}) \simeq \frac{1}{N_D^2} [&0.217 N_L^2 - 0.116 N_D N_L \\ &+ 0.589 N_L - 1.175 N_D^2 \\ &- 1.640 N_D] M_3^2, \end{aligned} \quad (9)$$

where $M_3 \simeq (\alpha_{\rm GUT}/(4\pi)) N_D m_{\rm mess}$. For instance, $(N_L, N_D) = (29, 11)$ gives

$$\tilde{m}_H^2(3\ \text{TeV}) \simeq 0.017 M_3^2. \quad (10)$$

However, the masses of the wino and the mass squared of the left-handed slepton are proportional to N_L, and it is impossible to explain the discrepancy of the muon $g-2$.

(iv) *FPHGM* We have a focus point in Higgs-gaugino mediation (FPHGM) motivated by the E_7 nonlinear sigma model [23]. In Higgs-gaugino mediation, soft masses for squarks and sleptons vanish at $M_{\rm in}$, while those for the Higgs doublets are as large as gaugino masses. This is consistent with nonobservation of flavor-violating processes. The low-energy $\tilde{m}_H$ is

[2] Originally, it is assumed that $M_3 \ll m_0$ [10]. However, for the original scenario, the observed Higgs mass now pushes up the fine-tuning measure to $\Delta \sim 200$–500.

[3] Although the gravitino mass $m_{3/2}$ is as large as $m_{3/2} \sim F_{\rm mess}/M_P$, it is assumed that the contribution from gravity mediation is suppressed.

$$\tilde{m}_H^2(3\ \mathrm{TeV}) \simeq -1.167 M_3^2 + 0.693 m_H^2 + \cdots. \quad (11)$$

Here, we assume that $m_{H_d}^2 = m_{H_u}^2 \equiv m_H^2$ at the high-energy scale, for simplicity. The ratio $m_H/M_3 \simeq 5/4 - 4/3$ leads to a small $\tilde{m}_H^2$ [14]. In this model, sleptons as well as the wino can be light. As is shown in the next section, it is possible to obtain $\Delta a_\mu \gtrsim 10^{-9}$.

As we have shown, among four focus point scenarios, only *FPHGM* can explain the muon $g-2$ anomaly. In the next section, we give a more detailed explanation for this point.

III. THE MUON $g-2$ IN THE FOCUS POINT SUSY

The SUSY contribution to the muon $g-2$ is enhanced when gaugino(s) and smuon(s) are light. There are two dominant SUSY contributions to the muon $g-2$: wino-Higgsino-(muon sneutrino) diagram and bino-(L-smuon)-(R-smuon) diagram. (Here, L and R denote left handed and right handed, respectively.) To enhance these contributions, at least, the left-handed slepton needs to be light. Clearly, *FPUS* cannot explain the discrepancy of the muon $g-2$, since all the sleptons as well as squarks are heavy as a few TeV. Also, L-smuon is too heavy to obtain $\Delta a_\mu \gtrsim 10^{-9}$ in *FPGM* and *FPHSG*. Therefore, the only remaining possibility is *FPHGM*.

The wino-Higgsino-(muon sneutrino) contribution to $(\Delta a_\mu)_{\mathrm{SUSY}}$ is given by [5]

$$(a_\mu)_{\tilde{W}-\tilde{H}-\tilde{\nu}} \simeq (1-\delta_{2L})\frac{\alpha_2}{4\pi}\frac{m_\mu^2 \tilde{M}_2 \mu}{m_{\tilde{\nu}}^4}\tan\beta \cdot F_C\left(\frac{\mu^2}{m_{\tilde{\nu}}^2}, \frac{\tilde{M}_2^2}{m_{\tilde{\nu}}^2}\right)$$
$$\simeq 18.2 \times 10^{-10}\left(\frac{500\ \mathrm{GeV}}{m_{\tilde{\nu}}}\right)^2 \frac{\tan\beta}{25}, \quad (12)$$

where we take $\mu = (1/2)m_{\tilde{\nu}}$ and $\tilde{M}_2 = m_{\tilde{\nu}}$ in the second line. Here, $\tilde{M}_2$ is the wino mass at the soft mass scale. The leading two-loop contribution δ_{2L} comes from large QED logarithms [27,28],

$$\delta_{2L} = \frac{4\alpha}{\pi}\ln\frac{m_{\tilde{\nu}}}{m_\mu}. \quad (13)$$

To explain Δa_μ by this contribution, the masses of the wino and L-smuon should be around 500 GeV. Obviously, the wino or L-smuon are too heavy to obtain $(a_\mu)_{\tilde{W}-\tilde{H}-\tilde{\nu}} \gtrsim 10^{-9}$ in *FPUS*, *FPGM*, and *FPHSG*. In *FPHGM*, on the other hand, the wino mass is unimportant for the focus point, and hence can be small enough to explain the anomaly of the muon $g-2$. As we will see, the L-smuon is also light enough.

The bino-(L-smuon)-(R-smuon) contribution is found to be [5]

$$(a_\mu)_{\tilde{B}-\tilde{\mu}_L-\tilde{\mu}_R} \simeq (1-\delta_{2L})\frac{3}{5}\frac{\alpha_1}{4\pi}\frac{m_\mu^2 \mu}{\tilde{M}_1^3}\tan\beta \cdot F_N\left(\frac{m_{\tilde{\mu}_L}^2}{\tilde{M}_1^2}, \frac{m_{\tilde{\mu}_R}^2}{\tilde{M}_1^2}\right)$$
$$\simeq 21.7 \times 10^{-10}\frac{\mu}{640\ \mathrm{GeV}}\frac{\tan\beta}{40}\left(\frac{110\ \mathrm{GeV}}{\tilde{M}_1}\right)^3, \quad (14)$$

where we take $m_{\tilde{\mu}_L} = 3\tilde{M}_1$ and $m_{\tilde{\mu}_R} = 2\tilde{M}_1$ in the second line. From the requirement of the small fine-tuning ($\Delta < 100$), there is an upper bound on μ: $\mu \lesssim 650$ GeV. It can be seen that $(a_\mu)_{\tilde{B}-\tilde{\mu}_L-\tilde{\mu}_R}$ is sufficiently large only when the bino and smuons are very light as 200–300 GeV, and $\tan\beta$ is larger than 40. Although Eq. (14) does not contain $\tilde{M}_2$, it implicitly depends on $\tilde{M}_2$ through the renormalization group running from M_{in} to M_{IR}: large M_2 thus $\tilde{M}_2$ leads to large L-slepton masses through the radiative corrections. Therefore, L-smuon becomes too heavy in *FPGM* and *FPHSG* [see Eq. (8)]. Moreover, with large $\tan\beta \sim 40$, the tau Yukawa coupling becomes large and the stau mass becomes easily tachyonic. Because of these reasons, it is difficult to obtain $(a_\mu)_{\tilde{B}-\tilde{\mu}_L-\tilde{\mu}_R} \gtrsim 10^{-9}$ in the known focus point SUSY scenarios.

In the following, we discuss *FPHGM* in detail. We assume $M_1 = M_3$, for simplicity.

A. Focus point in Higgs-gaugino mediation

We consider the FPHGM and estimate the fine-tuning of the EWSB scale in this model. For this purpose, we employ the following fine-tuning measure [29]:

$$\Delta = \max_a\{|\Delta_a|\},$$
$$\Delta_a = \left\{\frac{\partial \ln v}{\partial \ln \mu}\bigg|_{v_{\mathrm{obs}}}, \frac{\partial \ln v}{\partial \ln M_3}\bigg|_{v_{\mathrm{obs}}}, \frac{\partial \ln v}{\partial \ln M_2}\bigg|_{v_{\mathrm{obs}}}, \frac{\partial \ln v}{\partial \ln B_0}\bigg|_{v_{\mathrm{obs}}}\right\}, \quad (15)$$

where $v_{\mathrm{obs}} \simeq 174.1$ GeV. The fundamental mass parameters in Δ_a are defined at $M_{\mathrm{in}} = 10^{16}$ GeV. As shown in Eq. (2), The VEV v in Δ_a is determined by the Higgs potential including one-loop radiative corrections, which are in fact non-negligible. It is very interesting if there is a small Δ region where the observed Higgs boson mass and the muon anomaly $g-2$ are simultaneously explained.

In our numerical calculations, the Higgs boson mass is calculated using FeynHiggs 2.10.3 [30] and the SUSY mass spectra as well as Δ is evaluated utilizing softSUSY 3.6.1 [22]. The strong coupling constant and the top pole mass are taken as $\alpha_s(M_Z) = 0.1185$ and $m_t = 173.34$ GeV.

We show the contours for the Higgs boson mass and Δ in Fig. 1. In the orange (yellow) region, the SUSY contribution Δa_μ reduces the discrepancy of the muon $g-2$ from the SM prediction to 1σ (2σ). For the SM prediction of the muon $g-2$, we use $(a_\mu)_{\mathrm{EXP}} - (a_\mu)_{\mathrm{SM}} = (26.1 \pm 8.0) \times 10^{-10}$ [see Eq. (1)]. The gray regions are excluded since the stau

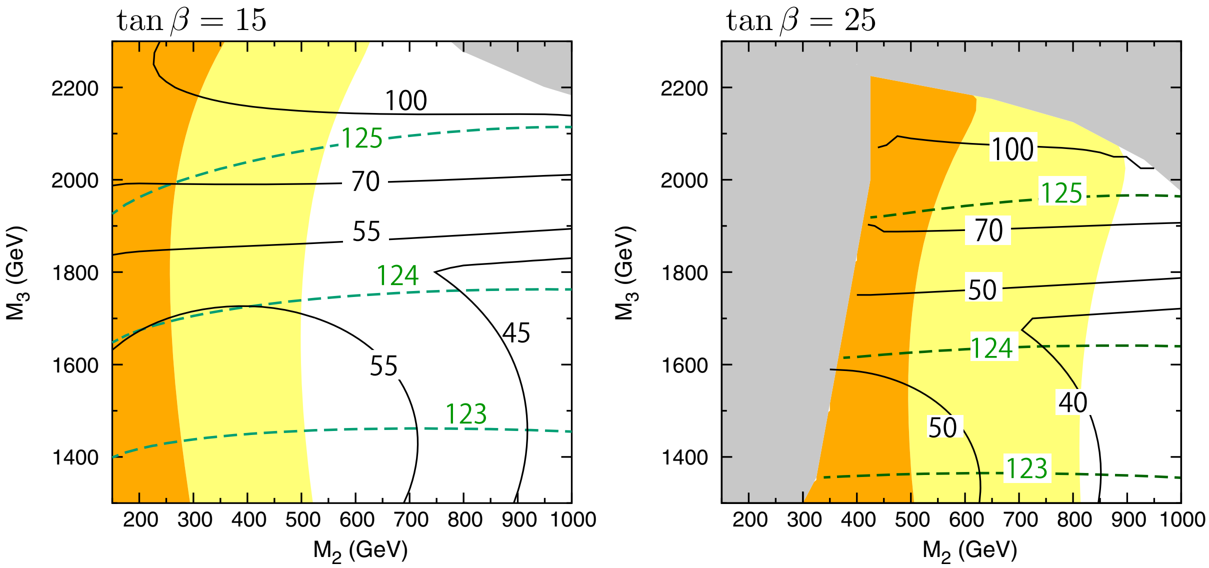

FIG. 1 (color online). The contours of Δ (black solid line) and m_h (green dashed line) in *FPHGM*. The Higgs mass m_h is shown in the unit of GeV. We take $m_H/M_3 = 4/3$. In the orange (yellow) region the SUSY contribution to the muon $g-2$ reduces the discrepancy to 1σ (2σ). The gray regions are excluded since the stau becomes too light (left part, $m_{\tilde{\tau}_1}$ or $m_{\tilde{\nu}_\tau} < 100$ GeV) or the EWSB does not occur (upper part).

becomes too light (left part) or the EWSB does not occur (upper part). For $\tan\beta = 15$ (25), M_2 smaller than 300 (500) GeV can reduce the discrepancy of the muon $g-2$ to the 1σ level. Here, $M_2 = (300, 500)$ GeV corresponds to the wino mass around (200, 370) GeV at the stop mass scale $M_{\rm IR}$. The observed Higgs boson mass around 125 GeV is also consistently explained with $\Delta = 40$–100.

Also, we show the maximum value of Δa_μ in Fig. 2 for different parameter sets (A, B, C). We vary $\tan\beta$ within a range [10:60] in each parameter set such that Δa_μ is maximized. We require that $m_{\tilde{\tau}_1}, m_{\tilde{\nu}_\tau} \gtrsim 100$ GeV; therefore the region with too small m_L or too large $\tan\beta$ is not allowed. (The allowed range of $\tan\beta$ is up to $\sim$30 in the parameter region preferred for the muon $g-2$.) The maximum value of Δa_μ easily exceeds 1.8×10^{-9} in the mild fine-tuning region. For C, M_2 smaller than 750 GeV (580 GeV at $M_{\rm IR}$) is allowed to explain the anomaly of the muon $g-2$. In this case, the level of the fine-tuning is still as low as $\Delta < 40$.

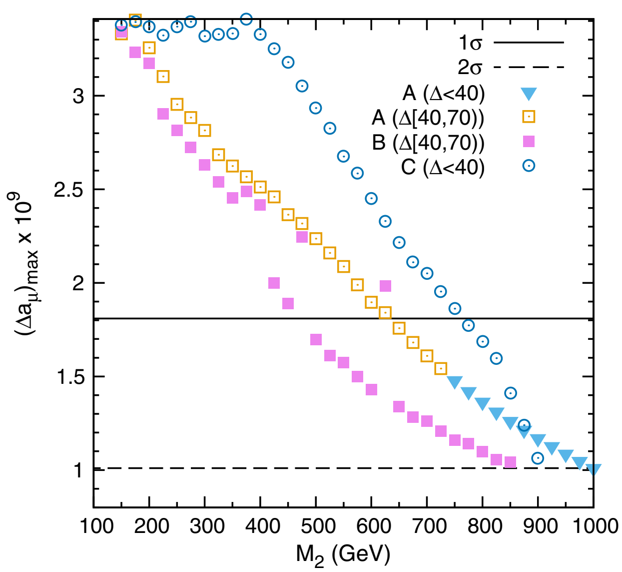

FIG. 2 (color online). The maximum value of $\Delta a_\mu \times 10^9$ in *FPHGM* for different parameter sets. Here, A:$(M_3, m_H/M_3) = (1500, 4/3)$; B:$(1900, 4/3)$; and C: (1400,1.37). In each point, $\tan\beta$ is varied within a range [10:60], requiring $m_{\tilde{\tau}_1}$, $m_{\tilde{\nu}_\tau} > 100$ GeV.

Interestingly, in this *FPHGM* the muon $g-2$ anomaly is easily explained. This is due to the smallness of scalar masses at $M_{\rm in}$, which gives small radiative corrections to the staus during the renormalization group equation running: the lighter L-smuon and larger $\tan\beta$ are allowed compared to models which will be discussed in the next section.

Let us present some sample mass spectra and Δ in Table I. One can see that the discrepancy of the muon $g-2$ is, in fact, explained in the region $\Delta \sim 40$–80. The calculated Higgs boson mass is consistent with the observed value. Note that the tau sneutrino is the lightest SUSY particle (LSP) in these model points, and one may need to pay attention to it.

B. Sneutrino LSP

Before closing this section, let us comment on the (tau) sneutrino LSP from viewpoints of the cosmology and collider searches, since the tau sneutrino tends to be the LSP in the parameter region of our interest (apart from the region where the wino mass is around 100 GeV). If the sneutrino LSP is absolutely stable, it is easily excluded by direct detection experiments due to a large scattering cross

TABLE I. Model points of *FPHGM*. Here, $M_1 = M_3$ at $M_{\rm in}(= 10^{16}$ GeV) is assumed.

P1	
M_3	2000 GeV
M_2	400 GeV
m_H/M_3	4/3
$\tan\beta$	20
μ	353
Δ	82
$m_{\rm gluino}$	4.14 TeV
$m_{\tilde{q}}$	3.55–3.56 TeV
$m_{\tilde{t}_{1,2}}$	2.82, 3.16 TeV
$m_{\tilde{\mu}_L}$, $m_{\tilde{\mu}_R}$	403 GeV, 739 GeV
$m_{\tilde{\tau}_1}$, $m_{\tilde{\nu}_\tau}$	271 GeV, 261 GeV
$m_{\chi_1^0}$, $m_{\chi_2^0}$	267, 366 GeV
$m_{\chi_3^0}$, $m_{\chi_4^0}$	395, 876 GeV
$m_{\chi_1^\pm}$, $m_{\chi_2^\pm}$	269, 401 GeV
m_h	125.2 GeV
Δa_μ	19.2×10^{-10}
P2	
M_3	1650 GeV
M_2	495 GeV
m_H/M_3	4/3
$\tan\beta$	27
μ	433
Δ	45
$m_{\rm gluino}$	3.46 TeV
$m_{\tilde{q}}$	2.98–3.00 TeV
$m_{\tilde{t}_{1,2}}$	2.35, 2.64 TeV
$m_{\tilde{\mu}_L}$, $m_{\tilde{\mu}_R}$	408 GeV, 611 GeV
$m_{\tilde{\tau}_1}$, $m_{\tilde{\nu}_\tau}$	211 GeV, 208 GeV
$m_{\chi_1^0}$, $m_{\chi_2^0}$	350, 444 GeV
$m_{\chi_3^0}$, $m_{\chi_4^0}$	474, 721 GeV
$m_{\chi_1^\pm}$, $m_{\chi_2^\pm}$	352, 480 GeV
m_h	124.1 GeV
Δa_μ	19.9×10^{-10}

section with nuclei [31]. However, the sneutrino LSP can easily decay into SM particles with a lifetime less than 0.1–1 s, if there is a tiny R-parity violation (e.g., $W = LL\bar{E}, LQ\bar{D}$). Therefore, the sneutrino LSP conflicts with neither the direct detection experiments nor standard cosmology.

The sneutrino LSP may behave as a stable particle inside the detector. In this case, the sneutrino can be searched for at the LHC through the production of chargino-neutralino, which eventually decay into multileptons with a missing transverse momentum. It may be distinguishable from an ordinary neutralino LSP case, since the flavors of the final state leptons are uncorrelated for the sneutrino LSP [32].

IV. VARIANTS OF *FPUS*

So far, among known focus point SUSY scenarios, only *FPHGM* can explain the muon $g-2$ anomaly. In this section, we discuss possible modifications of other focus point SUSY scenarios.

In *FPGM* and *FPHSG*, the heavy wino is crucial for realizing seminatural SUSY; therefore, it is very difficult to modify these scenarios to be consistent with the muon $g-2$ experiment. On the other hand, the modification may be possible for *FPUS* by relaxing the condition of universal scalar masses and taking slepton masses to be small. Although the fine-tuning is rather insensitive to the slepton masses, this modification is not very easy. This is because radiative corrections induce negative squared masses for staus. Staus become very light or tachyonic via radiative corrections for $\tan\beta = \mathcal{O}(10)$.[4] We have found, however, two possible modifications of *FPUS*, which we refer to as *FPNUS1* and *FPNUS2*. Here, *FPNUS1* respects the $SU(5)$ unification while *FPNUS2* does not.[5] In *FPNUS1*, the discrepancy of the muon $g-2$ from the SM prediction is reduced to 1σ level for the wino as light as ~100 GeV when $m_{H_u} \sim m_{H_d}$ at $M_{\rm in}$, and for the wino as light as ~400 GeV when $m_{H_d} \ll m_{H_u}$ at $M_{\rm in}$. There is a larger parameter space in *FPNUS2*.

(v) *FPNUS1* Scalar contributions to $m_{H_u}^2$ can be canceled, even if scalar masses are not completely universal. Similar to *FPUS*, we take $m_Q = m_{\bar{U}} = m_H = m_{\bar{E}}$. Then, we have

$$\tilde{m}_H^2(3\ {\rm TeV}) \simeq -1.170 M_3^2 + 0.069 m_Q^2 + 0.074 m_L^2 - 0.070 m_{\bar{D}}^2 + \cdots . \quad (16)$$

Note that the relation $m_Q = m_{\bar{U}} = m_{\bar{E}}$ is consistent with the $SU(5)$ unification. In the $SU(5)$ unification, the relation $m_{\bar{D}} = m_L$ is imposed, which we take as a free parameter independent of m_Q. Assuming that $m_Q/M_3 \sim 4$–5, we obtain the focus point.

(vi) *FPNUS2* There is another focus point once the $SU(5)$ unification is abandoned. For $m_Q = m_{\bar{U}} = m_{\bar{D}} = m_H$ with a fixed ratio of m_Q/M_3, small $\tilde{m}_H^2$ compared to M_3^2 can be obtained as well, although this condition is not consistent with the $SU(5)$ unification.

[4] Focus point with light sleptons is discussed in Ref. [33]. There, slepton masses are not determined by renormalization group equations from a high scale down to the weak scale, but are simply put by hand at a weak scale.

[5] Here, "$SU(5)$ unification" means the unification quarks and leptons into $SU(5)$ multiplets. The GUT gauge group itself is not assumed to be a single $SU(5)$. See the comment on product groups in Sec. II.

A. FPNUS1

Let us evaluate the fine-tuning Δ, the Higgs boson mass, and Δa_μ in *FPNUS1*. Here, we consider the case of $m_Q = m_U = m_E = m_H$ and the fixed ratio m_Q/M_3. Also, $m_L = m_{\bar{D}}$ is assumed so that quarks and leptons are unified into $SU(5)$ multiplets. The fine-tuning of this model can be estimated by the following measure:

$$\Delta = \max_a\{|\Delta_a|\},$$
$$\Delta_a = \left\{ \frac{\partial \ln v}{\partial \ln \mu}\bigg|_{v_{\rm obs}}, \frac{\partial \ln v}{\partial \ln M_3}\bigg|_{v_{\rm obs}}, \frac{\partial \ln v}{\partial \ln M_2}\bigg|_{v_{\rm obs}}, \frac{\partial \ln v}{\partial \ln m_L}\bigg|_{v_{\rm obs}}, \frac{\partial \ln v}{\partial \ln B_0}\bigg|_{v_{\rm obs}} \right\}. \quad (17)$$

In Fig. 3, the Higgs boson mass m_h and Δ are shown for different M_3. Here, r_Q is the ratio of the squark mass to the gluino mass, m_Q/M_3. The gluino mass at $M_{\rm in}$ is taken as $M_3 = (800, 900, 1000, 1100, 1200)$ GeV. As r_Q increases, Δ is minimized at a certain point. Above the vertical line, the EWSB no longer occurs. In small Δ region, the calculated Higgs boson mass of $m_h \simeq (123.5, 124.5, 125)$ GeV is obtained for $M_3 = (800, 900, 1000)$ GeV and $\tan\beta = 25$, while larger M_3 is required for $\tan\beta = 15$.

Next, we see whether we can explain the muon $g-2$ anomaly in *FPNUS1*. In Fig. 4, the maximum value of Δa_μ in the region with mild fine-tuning is shown. We take different parameter sets denoted by A, B, C, D, E, and F as shown in the caption. We vary $\tan\beta$ within a range [10:60]

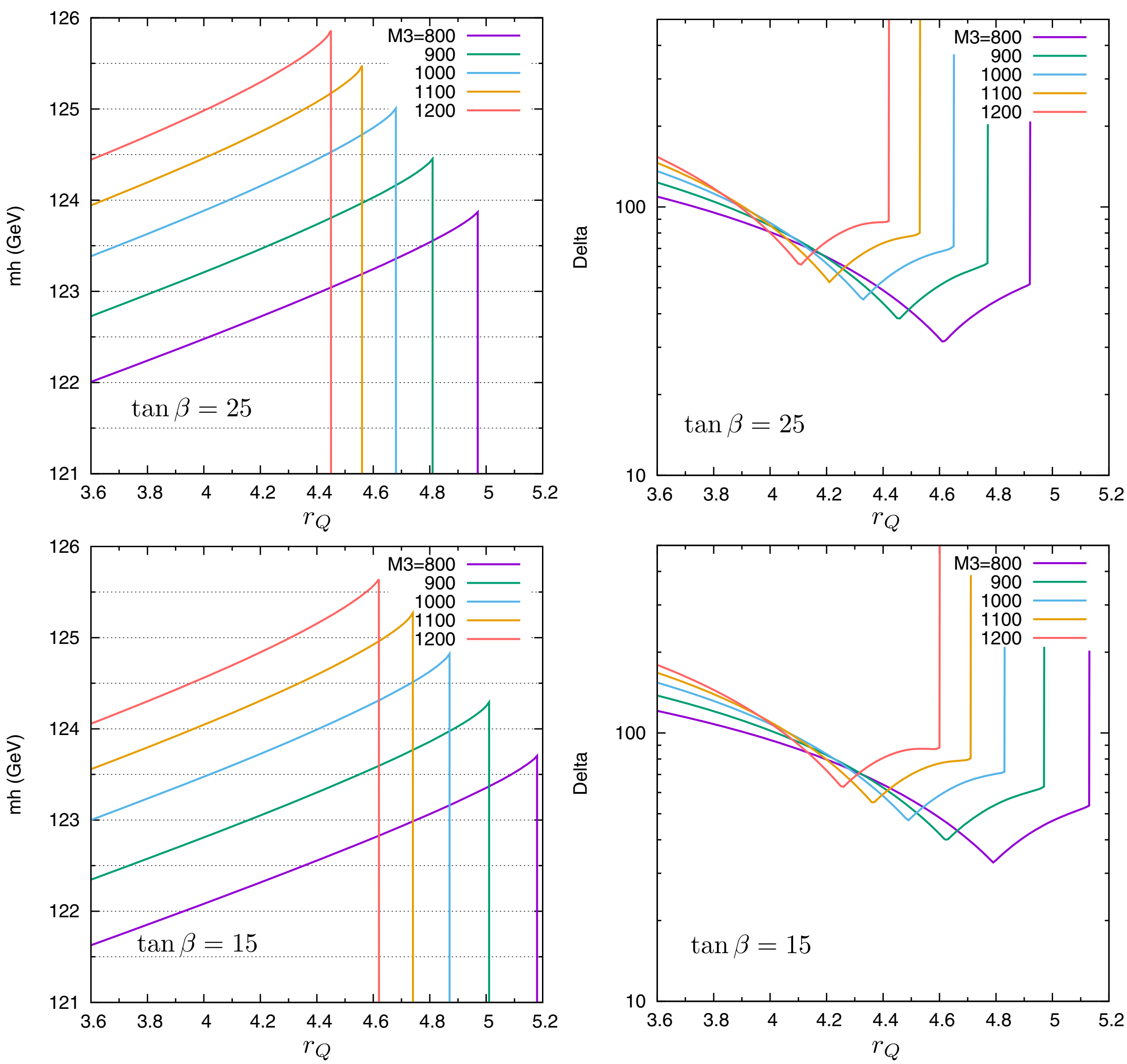

FIG. 3 (color online). The Higgs boson mass and Δ in *FPNUS1*, with parameter sets $(M_3, M_2, m_L) = (800–900, 500, 1000)$, (1000–1200,500,1200) GeV. In the upper (lower) panel, $\tan\beta = 25$ (15). Here, $r_Q \equiv m_Q/M_3$.

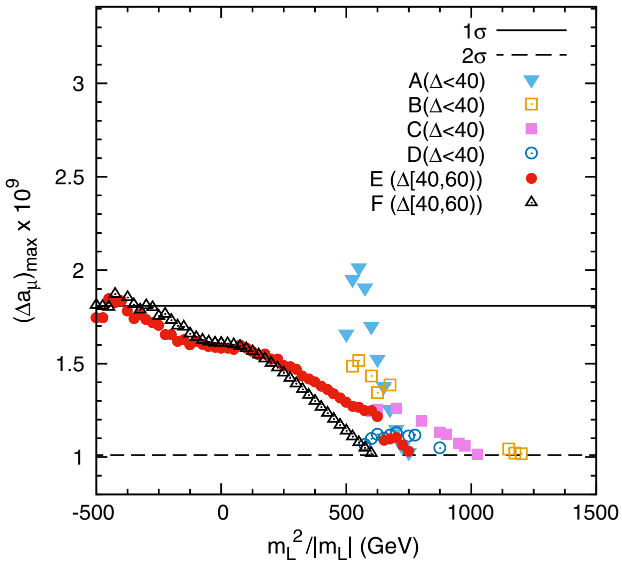

FIG. 4 (color online). The maximum value of $\Delta a_\mu \times 10^9$ in *FPNUS1* for different parameter sets. A: $(M_3, M_2, m_Q/M_3, m_{H_d}/m_Q) = (750, 150, 5.5, 1.0)$; B: (750,400,5.1,1.0); C: (900,150,4.7,1.0); D: (900,400,4.6,1.0); E: (900,150,5.6,0.3); and F: (750,400,6.0,0.2). In each point, $\tan\beta$ is varied within a range [10:60], requiring $m_{\tilde{\tau}_1}, m_{\tilde{\nu}_\tau} > 100$ GeV. The condition $m_{H_d} = m_{H_u}$ is relaxed for E and F.

in each parameter set such that Δa_μ is maximized. (The allowed range of $\tan\beta$ is up to ~20.) We see that, in the very light wino case A, the discrepancy of the muon $g-2$ from the SM prediction can be reduced to the 1σ level, while in the heavier wino case B the discrepancy is reduced to 1.5σ. In E and F, the condition $m_{H_u} = m_{H_d}$ at $M_{\rm in}$ is relaxed, and there is a region where the discrepancy is reduced to the 1σ level for $m_L^2(M_{\rm in}) < 0$. Note that $m_{H_u} \neq m_{H_d}$ is consistent with the $SU(5)$ unification.

Let us present a sample mass spectrum and Δ in Table II (*P3*). One can see the discrepancy of the muon $g-2$ is reduced around 1σ if the winolike chargino is as light as ~100 GeV.

B. FPNUS2

Once we abandon the $SU(5)$ unification, we have another focus point (*FPNU2*). Here, we consider the case for $m_Q = m_{\bar{U}} = m_{\bar{D}} = m_H$ with the fixed ratio of $m_Q/M_3 \equiv r_Q$. Although this model is not consistent with the $SU(5)$ unification, a larger parameter space with $\Delta a_\mu \gtrsim 1.8 \times 10^{-9}$ exists. The fine-tuning measure Δ is slightly changed from *FPNU1* as[6]

[6]Unless m_L or $m_{\bar{E}}$ is very large, Δ is dominated by Δ_μ or Δ_{M_3} so far.

TABLE II. Model points of *FPNUS1* (*P3*) and *FPNUS2* (*P4*) are shown. Here, $M_1 = M_3$ and $m_{H_d} = m_{H_u}$ at $M_{\rm in}(= 10^{16}$ GeV) is assumed.

P3	
M_3	800 GeV
M_2	200 GeV
...	
$m_L = m_{\bar{D}}$	560 GeV
m_Q/M_3	5.3
$\tan\beta$	13
μ	221
Δ	40
$m_{\rm gluino}$	1.89 TeV
$m_{\tilde{q}}$	1.46–4.46 TeV
$m_{\tilde{t}_{1,2}}$	2.85, 3.71 TeV
$m_{\tilde{\mu}_L}$, $m_{\tilde{\mu}_R}$	435 GeV, 4251 GeV
$m_{\tilde{\tau}_1}$, $m_{\tilde{\nu}_\tau}$	160 GeV, 139 GeV
$m_{\chi_1^0}$, $m_{\chi_2^0}$	126, 236 GeV
$m_{\chi_3^0}$, $m_{\chi_4^0}$	254, 364 GeV
$m_{\chi_1^\pm}$, $m_{\chi_2^\pm}$	129, 269 GeV
m_h	123.8 GeV
Δa_μ	17.5×10^{-10}
P4	
M_3	1000 GeV
M_2	350 GeV
$m_{\bar{E}}$	1000 GeV
$m_{\bar{L}}$	560 GeV
m_Q/M_3	4.9
$\tan\beta$	19
μ	168
Δ	62
$m_{\rm gluino}$	2.37 TeV
$m_{\tilde{q}}$	5.16–5.18 TeV
$m_{\tilde{t}_{1,2}}$	3.34, 4.25 TeV
$m_{\tilde{\mu}_L}$, $m_{\tilde{\mu}_R}$	515 GeV, 984 GeV
$m_{\tilde{\tau}_1}$, $m_{\tilde{\nu}_\tau}$	143 GeV, 119 GeV
$m_{\chi_1^0}$, $m_{\chi_2^0}$	146, 181 GeV
$m_{\chi_3^0}$, $m_{\chi_4^0}$	310, 445 GeV
$m_{\chi_1^\pm}$, $m_{\chi_2^\pm}$	154, 314 GeV
m_h	125.0 GeV
Δa_μ	18.6×10^{-10}

$$\Delta = \max_a\{|\Delta_a|\},$$
$$\Delta_a = \left\{ \frac{\partial \ln v}{\partial \ln \mu}\bigg|_{v_{\rm obs}}, \frac{\partial \ln v}{\partial \ln M_3}\bigg|_{v_{\rm obs}}, \frac{\partial \ln v}{\partial \ln M_2}\bigg|_{v_{\rm obs}}, \frac{\partial \ln v}{\partial \ln m_L}\bigg|_{v_{\rm obs}}, \frac{\partial \ln v}{\partial \ln m_{\bar{E}}}\bigg|_{v_{\rm obs}}, \frac{\partial \ln v}{\partial \ln B_0}\bigg|_{v_{\rm obs}} \right\}. \quad (18)$$

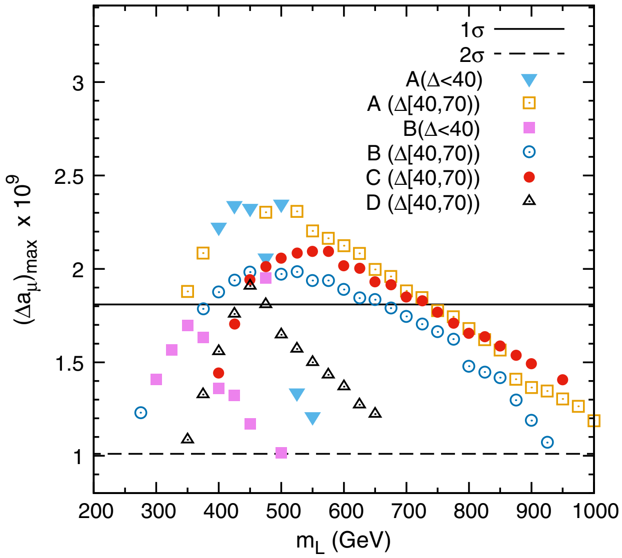

FIG. 5 (color online). The maximum value of $\Delta a_\mu \times 10^9$ in *FPNUS2* for different parameter sets. A: $(M_3, M_2, m_Q/M_3, m_{\bar{E}}/m_L) = (750, 150, 5.1, 2.0)$; B: (750,400,5.1,2.0); C: (900, 150,5.0,2.0); and D: (900,400,5.1,2.0). In each point, $\tan\beta$ is varied within a range [10:60].

In Fig. 5, the maximum values of Δa_μ for different parameter sets are shown. Here, we only consider the mild fine-tuning region. (The Higgs boson mass is similar to the one in *FPNUS1*.) As in the case of *FPNUS1*, $\tan\beta$ is varied within a range [10:60] to find a maximum value of Δa_μ. One can see that A and B can reduce the discrepancy of the muon $g-2$ to the 1σ level with $\Delta < 40$. If the required upper bound on Δ is relaxed to $\Delta < 70$, all parameter sets (A, B, C, D) shown in the figure can reduce the discrepancy of the muon $g-2$ to the 1σ level: the Higgs boson mass and the muon $g-2$ anomaly are explained relatively easily in *FPNUS2* compared to *FPNUS1*. This is because the stau is heavier for the same L-slepton mass at $M_{\rm IR}$ and $\tan\beta$. This allows larger $\tan\beta$ and smaller $m_{\tilde{L}}$, avoiding the too light stau.

Finally, let us present a sample mass spectrum and Δ in Table II (*P4*). Although this model is not consistent with the $SU(5)$ unification, the anomaly of the muon $g-2$ is, in fact, explained in the region with $\Delta \simeq 60$.

V. CONCLUSIONS

The focus point SUSY scenario is very attractive, since it explains seminaturally the observed electroweak breaking scale $v \simeq 174.1$ GeV even when masses of squarks and gluinos are in several TeV region. One interesting prediction of the focus point SUSY breaking scenario is the light Higgsino with a mass of several hundred GeV. This relatively light Higgsino provides a possibility of explaining the anomaly of the muon $g-2$. In fact, if the wino and the left-handed smuon are also light, the anomaly of the muon $g-2$ is explained.

In this paper, we have found that, among the known focus point SUSY scenarios, a scenario based on the Higgs-gaugino mediation can explain the observed value of the $g-2$ with mild fine-tuning measures $\Delta = 40$–80. This scenario is proposed recently by the current authors motivated by the E_7 nonlinear sigma model, which may explain why the family number is three. There, the wino mass is unimportant for the focus point and hence can be light enough. The mass of the left-handed smuon is mainly given by the quantum correction from the wino loop and is small.

The tau-sneutrino is likely to be the LSP in the parameter region of our interest, which gives a distinctive collider signal as described in Sec. III B. Therefore, this intriguing possibility may be tested and distinguished from other SUSY scenarios at the LHC.

Also, we propose two new focus point SUSY scenarios based on gravity mediation, which are variants of the well known focus point SUSY scenario. Unlike the original one, the scalar masses are no longer universal and the left-handed sleptons are light. We have shown that the muon $g-2$ anomaly is explained.

In this paper, we have mainly discussed the anomaly of the muon $g-2$ in focus point SUSY scenarios. The focus point SUSY needs some relations among relevant mass parameters. We hope that those relations may be given by more fundamental physics (see, e.g., [12,34]). It is, however, beyond the scope of this paper.

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APPENDIX: HIGH SCALE GAUGE MEDIATION

We consider a high scale gauge mediation model with N_L pairs of $SU(2)_L$ doublet messengers and N_D pairs of $SU(3)_C$ triplet messengers. The SUSY breaking mass and SUSY invariant mass of the messenger superfield are denoted by $F_{\rm mess}$ and $M_{\rm mess}$, respectively. It is assumed that $F_{\rm mess}$ and $M_{\rm mess}$ are common for all the messenger fields.

In this setup, the gaugino masses are given by

$$M_1 = \frac{\alpha_1}{4\pi} m_{\rm mess}\left(\frac{3}{5}N_L + \frac{2}{5}N_D\right), \quad M_2 = \frac{\alpha_2}{4\pi} m_{\rm mess} N_L,$$
$$M_3 = \frac{\alpha_3}{4\pi} m_{\rm mess} N_D, \tag{A1}$$

where $m_{\rm mess} = F_{\rm mess}/M_{\rm mess}$. The scalar masses are

$$m_Q^2 = \left[\frac{8}{3}\left(\frac{\alpha_3}{4\pi}\right)^2 N_D + \frac{3}{2}\left(\frac{\alpha_2}{4\pi}\right)^2 N_L + \frac{1}{30}\left(\frac{\alpha_1}{4\pi}\right)^2\left(\frac{3}{5}N_L + \frac{2}{5}N_D\right)\right] m_{\rm mess}^2,$$
$$m_{\bar{U}}^2 = \left[\frac{8}{3}\left(\frac{\alpha_3}{4\pi}\right)^2 N_D + \frac{8}{15}\left(\frac{\alpha_1}{4\pi}\right)^2\left(\frac{3}{5}N_L + \frac{2}{5}N_D\right)\right] m_{\rm mess}^2, \qquad m_{\bar{D}}^2 = \left[\frac{8}{3}\left(\frac{\alpha_3}{4\pi}\right)^2 N_D + \frac{2}{15}\left(\frac{\alpha_1}{4\pi}\right)^2\left(\frac{3}{5}N_L + \frac{2}{5}N_D\right)\right] m_{\rm mess}^2,$$
$$m_L^2 = \left[\frac{3}{2}\left(\frac{\alpha_2}{4\pi}\right)^2 N_L + \frac{3}{10}\left(\frac{\alpha_1}{4\pi}\right)^2\left(\frac{3}{5}N_L + \frac{2}{5}N_D\right)\right] m_{\rm mess}^2, \qquad m_{\bar{E}}^2 = \left[\frac{6}{5}\left(\frac{\alpha_1}{4\pi}\right)^2\left(\frac{3}{5}N_L + \frac{2}{5}N_D\right)\right] m_{\rm mess}^2,$$
$$m_{H_u}^2 = m_{H_d^2} = m_L^2. \tag{A2}$$

If we take $M_{\rm mess} = M_{\rm GUT}$, the low-energy value of $m_{H_u}^2 - (m_{H_d}^2 - m_{H_u}^2)/\tan^2\beta(\equiv \tilde{m}_H^2)$ is written by

$$\tilde{m}_H^2(3\ {\rm TeV}) \simeq \frac{1}{N_D^2}[0.216N_L^2 - 0.116N_D N_L + 0.587N_L - 1.172N_D^2 - 1.636N_D]M_3^2, \tag{A3}$$

where we take $\tan\beta = 20$.

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