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Nonstandard supersymmetry breaking and Dirac gaugino masses without supersoftness

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I consider models in which nonstandard supersymmetry-breaking terms, including Dirac gaugino masses, arise from F-term breaking mediated by operators with a $1/M^3$ suppression. In these models, the supersoft properties found in the case of D-term breaking are absent in general, but can be obtained as a special case that is a fixed point of the renormalization group equations. The μ term is replaced by three distinct supersymmetry-breaking parameters, decoupling the Higgs scalar potential from the Higgsino masses. Both holomorphic and nonholomorphic scalar cubic interactions with minimal flavor violation are induced in the supersymmetric Standard Model Lagrangian.

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I. INTRODUCTION

In the minimal supersymmetric Standard Model (MSSM) the gaugino partners of the gauge bosons can only have Majorana masses. However, by enlarging the particle content of the model to include chiral superfields in the adjoint representation, it is possible to instead have Dirac gaugino masses [1–3]. This amounts to promoting the gauge sector particle content of the theory to that of $N = 2$ supersymmetry. In Ref. [4], Fox *et al.* proposed a particularly compelling and predictive way to incorporate Dirac gaugino masses, called supersoft supersymmetry breaking. In this framework, supersymmetry is broken by a D-term vacuum expectation value (VEV), leading directly to Dirac gaugino masses together with specific nonholomorphic scalar cubic couplings. The MSSM squarks and sleptons remain massless at tree level, and do not receive ultraviolet (UV) divergent or renormalization group (RG) corrections. Earlier, Jack and Jones [5,6] had noted the existence of the corresponding RG trajectory in the context of a general theory with "nonstandard" supersymmetry breaking: nonholomorphic scalar cubic interactions and supersymmetry-breaking chiral fermion masses in addition to Dirac gaugino masses.

Supersymmetric models with Dirac gaugino masses from supersoft breaking have unique phenomenological properties. As noted in Ref. [4], the real scalar part of the adjoint chiral superfield receives a mass at tree level, but the imaginary part (in an appropriate phase convention) is massless at tree level, and another Lagrangian term that can be added to the theory threatens to make one or the other of them tachyonic. After integrating out the heavy real scalar adjoint field, the resulting effective theory does not include the MSSM scalar quartic interactions that usually follow from integrating out the D-term auxiliary fields of the Standard Model gauge groups. This makes it somewhat problematic to stabilize the Higgs potential sufficiently to accommodate the observed Higgs mass of $M_h = 125$ GeV. Solving these problems requires some interesting and nontrivial model building. Dirac gaugino masses together with an approximate R symmetry, or an exact R symmetry together with an extension of the Higgs sector, provide a strong natural suppression of flavor- and CP-violating effects in low-energy experiments, even if flavor and CP symmetries are not respected at all in the squark and slepton mass sectors [7]. Given the present lack of evidence for superpartner production at the Large Hadron Collider (LHC), another attractive feature of supersoft models is that they predict [8,9] a significant weakening of the limits that can be obtained for any given beam energy. This is partly because gluinos are predicted to be much heavier than squarks, and partly because of the suppression of squark pair production due to the Dirac nature of the gluino. Recent years have seen other important studies on the phenomenological implications of Dirac gaugino mass models for colliders [10–15] and dark matter [16–20]. Dirac gaugino models have been further developed in Refs. [21–60] in a variety of interesting directions.

In this paper, I consider models with Dirac gaugino masses arising from an F-term VEV, rather than the D-term VEV in supersoft models. In these models, the supersoft property is lost in general, but appears as a special case, a fixed point of the RG equations. The adjoint scalars can naturally be made heavy. The μ problem is solved in a way that decouples the naturalness of the electroweak breaking scale from the Higgsino masses, similar to that proposed in the supersoft case in Ref. [56].

II. DIRAC GAUGINO MASSES FROM F-TERM VEVS

In this paper, the MSSM gauginos will be denoted λ^a, where a is an index that runs over the adjoint representation of the gauge group with gauge coupling g_a. The usual

Majorana gaugino masses then can be written in two-component notation as[1]

$$\mathcal{L} = -\frac{1}{2} M_a \lambda^a \lambda^a + \text{c.c.} \tag{2.1}$$

In general, to obtain Dirac gaugino masses in the low-energy effective theory, one introduces new chiral superfields A^a with complex scalar component ϕ^a and two-component fermion component ψ^a. Then one can have Dirac gaugino masses by coupling the gauginos to the adjoint chiral fermions:

$$\mathcal{L} = -m_{Da} \psi^a \lambda^a + \text{c.c.} \tag{2.2}$$

It is also possible to have a Majorana mass term for the chiral adjoint fermions:

$$\mathcal{L} = -\frac{1}{2} \mu_a \psi^a \psi^a + \text{c.c.} \tag{2.3}$$

A completely general theory would have all three terms.

In supersoft models [4], it is assumed that the main source of supersymmetry breaking in the MSSM can be written as

$$\mathcal{L} = \frac{k_a}{M} \int d^2\theta \mathcal{W}'^{\alpha} \mathcal{W}^a_{\alpha} A^a + \text{c.c.}, \tag{2.4}$$

where M is a scale associated with the communication between the supersymmetry-breaking sector and the MSSM, k_a are dimensionless parameters, $\mathcal{W}^a_\alpha = \lambda^a_\alpha + \cdots$ are the MSSM gauge group field strength superfields, $\mathcal{W}'^\alpha = \langle D \rangle \theta^\alpha$ is an Abelian superfield strength with a D-term spurion component, and α is a Weyl spinor index. As a convention, $\langle D \rangle$ is chosen to be positive. In terms of the component fields, the result is Dirac gaugino masses accompanied by specific scalar interactions:

$$\begin{aligned} \mathcal{L} = &-m_{Da}(\psi^a \lambda^a + \text{c.c.}) + \sqrt{2} m_{Da} D^a (\phi^a + \phi^{a*}) \\ &+ g_a D^a (\phi_i^\dagger t^a \phi_i) + \frac{1}{2} (D^a)^2 \end{aligned} \tag{2.5}$$

where the indices a and i are implicitly summed over, with i labeling the scalar field flavors in the theory, t^a are the generators of the gauge group Lie algebra, and the Dirac gaugino masses are

$$m_{Da} = k_a \langle D \rangle / \sqrt{2} M. \tag{2.6}$$

The last two terms in Eq. (2.5) come from the kinetic terms of the chiral and gauge superfields, respectively. After integrating out the MSSM gauge group auxiliary fields D^a, one finds [4] that the canonically normalized real scalar adjoint field, $R_a = (\phi^a + \phi^{a*})/\sqrt{2}$, has a squared mass equal to $4m_{Da}^2$ and a nonholomorphic supersymmetry-breaking interaction with the other scalars that is also fixed in terms of the Dirac gaugino mass, while the imaginary scalar adjoint field $I_a = i(\phi^{*a} - \phi^a)/\sqrt{2}$ remains massless and free of supersymmetry-breaking interactions:

$$\begin{aligned} \mathcal{L} = &-m_{Da}(\psi^a \lambda^a + \text{c.c.}) - 2m_{Da}^2 R_a^2 - 2g_a m_{Da} R_a (\phi_i^\dagger t^a \phi_i) \\ &- \frac{1}{2} g_a^2 (\phi_i^\dagger t^a \phi_i)^2. \end{aligned} \tag{2.7}$$

The last term is the usual supersymmetric D-term-induced scalar quartic interaction. The other terms in Eq. (2.7) form the specific combination of supersymmetry-breaking couplings that was recognized as a RG invariant trajectory in [6]. The reason for this becomes apparent by writing it in terms of a (nonrenormalized) superpotential spurion term as in Eq. (2.4).

The last three terms in Eq. (2.7) are proportional to the square of $g_a(\phi_i^\dagger t^a \phi_i) + 2M_{Da} R_a$. Therefore, this quantity is set equal to 0 by the equations of motion upon integrating out the heavy field R_a, eliminating [4] the scalar quartic terms that are usually present in the low-energy effective theory. These include the quartic terms responsible for stabilizing the Higgs scalar boson potential, so the absence of such terms increases the difficulty of obtaining $M_h = 125$ GeV.

A term that could be expected to accompany Eq. (2.4) is the so-called "lemon-twist" term

$$\begin{aligned} \mathcal{L} &= \frac{k_a^{LT}}{M^2} \int d^2\theta \mathcal{W}'^\alpha \mathcal{W}'_\alpha A^a A^a + \text{c.c.} \\ &= k_a^{LT} \frac{\langle D \rangle^2}{M^2} (\phi^a \phi^a + \text{c.c.}) \end{aligned} \tag{2.8}$$

$$= -k_a^{LT} \frac{\langle D \rangle^2}{M^2} (I_a^2 - R_a^2), \tag{2.9}$$

where k_a^{LT} are dimensionless parameters, taken to be real here. If $k_a^{LT} < 0$, then this holomorphic scalar squared mass term makes the imaginary scalar adjoint I_a tachyonic, unless there are other positive contributions to the squared mass. On the other hand, if $k_a^{LT} > k_a^2$, we see by comparing with Eq. (2.7) that then R_a will be tachyonic at tree level. In simple UV completions of the supersoft Lagrangian, k_a^{LT} is indeed found to be larger in magnitude than k_a^2, posing a tachyonic adjoint problem [4,28,45] in the absence of fine-tuning or contrivance. Some proposals to deal with this issue are given in Refs. [4,28,45,56,59,60].

In this paper, I will consider the possibility that Dirac gaugino masses instead come from an F-term VEV spurion $X = \theta\theta\langle F \rangle$, via the Lagrangian term [62]:

[1]The spinor and superspace conventions used here are as in Ref. [61].

$$\mathcal{L} = -\frac{c_a^{(1)}}{\sqrt{2}M^3} \int d^4\theta X^* X \mathcal{W}^{a\alpha} \nabla_\alpha A^a = -m_{Da} \psi^a \lambda^a \quad (2.10)$$

where $\langle F \rangle$ is chosen real as a convention and $c_a^{(1)}$ is a dimensionless parameter for each of $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$, and now instead of Eq. (2.6),

$$m_{Da} = c_a^{(1)} \langle F \rangle^2 / M^3. \quad (2.11)$$

Note that $D_\alpha \Phi$ is not supergauge covariant if Φ is a nonsinglet chiral superfield. Here

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} - i(\sigma^\mu \theta^\dagger)_\alpha \partial_\mu \quad (2.12)$$

is the usual chiral covariant superderivative, with the "covariant" here traditionally referring to supersymmetry transformations, rather than supergauge transformations. Therefore, Eq. (2.10) instead uses a "gauge-covariant chiral covariant superderivative," whose action on a chiral superfield Φ is defined by

$$\nabla_\alpha \Phi = e^{-V} D_\alpha (e^V \Phi) \quad (2.13)$$

where $V = 2g_a V^a t^a$, with t^a the matrix generator for the rep of Φ and V^a is the MSSM vector superfield for the index a. However, in Wess-Zumino gauge, the e^V and e^{-V} factors have no practical effect on the component-level expressions here or below when spurions $X^* X = \theta^\dagger \theta^\dagger \theta\theta \langle F \rangle^2$ are present.

Equation (2.10) is a nonholomorphic source for the Dirac gaugino mass. Therefore, the Dirac gaugino masses are not accompanied by the supersoft scalar couplings, in general.

III. OTHER LAGRANGIAN TERMS AND MODEL-BUILDING CRITERIA

A. Terms with $1/M^3$ suppression

The Dirac gaugino mass with F-term spurion origin given by Eq. (2.10) can be accompanied by other supersymmetry-breaking Lagrangian terms in the low-energy effective theory. Since it is suppressed by $1/M^3$, it is not at all clear whether it can be the dominant source of supersymmetry breaking in the MSSM sector.

In particular, even if X carries a conserved charge, this term is allowed:

$$\mathcal{L} = -\frac{k_{\Phi_i^* \Phi_j}}{M^2} \int d^4\theta X^* X \Phi_i^* e^V \Phi_j \quad (3.1)$$

where Φ_i are the chiral superfields of the theory, including the quarks, leptons and Higgs fields of the MSSM and the adjoint chiral superfields. If present, this term can give nonholomorphic squared masses to the MSSM Higgs, squarks and sleptons with a mass scale of order $\langle F \rangle / M$, which should be much larger than the Dirac gaugino masses, unless the dimensionless parameters $k_{\Phi_i^* \Phi_j}$ are very small, or $\langle F \rangle$ is comparable to M^2. There are also terms

$$\mathcal{L} = -\frac{1}{M^2} \int d^4\theta X^* X (k_{AA} A^a A^a + k_{H_u H_d} H_u H_d) \quad (3.2)$$

that can give holomorphic squared mass terms to the scalar adjoints and the Higgs fields.

Estimating naively, if $m_{Da} \sim \langle F \rangle^2 / M^3$ is to be of order $m_{\tilde{g}} \sim 1$ TeV, then if $k_{\Phi_i^* \Phi_j}$ is of order 1, the squark mass scale $\langle F \rangle / M$ should be of order $m_{\tilde{Q}} \sim \sqrt{M m_{\tilde{g}}}$. This can be up to an intermediate scale 10^{11} GeV if M is the reduced Planck mass, but could be much smaller if M is low. For large M, one can have a version of supersymmetry with Dirac gaugino masses and hierarchically heavier squarks and sleptons (sometimes called "PeV-scale" or "split" or "semisplit" supersymmetry, depending on the extent of the hierarchy). While such possibilities should not be dismissed immediately and can have some intriguing properties [63–65], this goes against the main motivation for supersymmetry, the solution to the hierarchy problem associated with the electroweak scale. Therefore, for the rest of this paper I instead prefer to pursue the possibility that the operators in Eqs. (3.1) and (3.2) are absent or sufficiently suppressed, and ask what happens if the Dirac gaugino masses are among the largest manifestations of supersymmetry breaking in the visible sector.

There is no obvious symmetry that would allow the Dirac gaugino mass operator of Eq. (2.10) while forbidding Eq. (3.1). Indeed, realizations of Dirac gaugino masses using F-term VEVs in gauge mediation evidently do [25,26,28] generically have scalar masses of the type given in Eq. (3.1). The Dirac gaugino masses can be comparable to, but somewhat smaller than, these scalar squared masses, but this requires a low M. This has the drawback that it appears to force one to view the apparent gauge coupling unification as a mere accident, as the combined presence of light adjoint and light messenger chiral superfields will cause the Standard Model gauge couplings to become nonperturbatively strong in the UV before they unify. Perhaps a more palatable approach is that in models of deconstructed gaugino mediation [66,67], it is possible to highly suppress ("screen") the nonholomorphic scalar squared masses compared to the Dirac gaugino masses [32], even though the former are not forbidden by symmetry.

Rather than commit to a particular type of UV completion, I will instead consider a set of model-building criteria that are designed to allow F-term generated Dirac gaugino masses to dominate over, or be comparable to, other sources of supersymmetry breaking. First, I assume that X carries some conserved charge, so that parametrically larger Majorana gaugino masses arising from

$$-\frac{1}{M}\int d^2\theta X \mathcal{W}^{a\alpha}\mathcal{W}^a_\alpha, \qquad (3.3)$$

as well as holomorphic scalar interactions from superpotential terms involving X, are forbidden. Second, suppose that all interactions between the spurions X, X^* and the MSSM sector are suppressed by $1/M^3$, where M is a characteristic large mediation mass scale, with terms of order $1/M^2$ either forbidden or suppressed. This appeal to dimensional analysis (which perhaps could have a geographical or dynamical origin, as in [32]), rather than symmetry, would eliminate from contention Eqs. (3.1) and (3.2). Third, suppose that the spurion interactions respect the approximate flavor symmetries of the Standard Model; this assumption is technically natural, and effectively bans squark and slepton chiral superfields from appearing in the spurion terms. Finally, if one wants the Dirac gaugino masses and other supersymmetry-breaking interactions discussed below to be larger than the effects of anomaly-mediated supersymmetry breaking (AMSB) [68], one must have $\langle F\rangle\beta/M_{\rm Planck} \lesssim \langle F\rangle^2/M^3$, where β schematically represents the beta function or anomalous dimension suppression inherent in AMSB. This can hold if M is not larger than about 10^{13} GeV, so the scenario below apparently requires supersymmetry breaking to occur and to be communicated at a scale well below the Planck mass. I admit to not knowing of any UV completion that guarantees all of these criteria as stated, and it is conceivable that none exists. Nevertheless, without further apology, I will proceed to consider their consequences.

Besides the Dirac gaugino masses of Eq. (2.10), one has the following set of Lagrangian terms (and their complex conjugates) allowed by the above criteria:

$$\frac{c_a^{(2)}}{\sqrt{2}M^3}\int d^4\theta X^* X A^a \nabla_\alpha \mathcal{W}^{a\alpha}, \qquad (3.4)$$

$$-\frac{c_a^{(3)}}{2M^3}\int d^4\theta X^* X \mathcal{W}^{a\alpha}\mathcal{W}^a_\alpha, \qquad (3.5)$$

$$-\frac{c_a^{(4)}}{4M^3}\int d^4\theta X^* X \nabla^\alpha A^a \nabla_\alpha A^a, \qquad (3.6)$$

$$-\frac{c_a^{(5)}}{4M^3}\int d^4\theta X^* X A^a \nabla^\alpha \nabla_\alpha A^a, \qquad (3.7)$$

$$-\frac{c_a^{(6)}}{4M^3}\int d^4\theta X^* X A^{a*}(e^V \nabla^\alpha \nabla_\alpha A)^a, \qquad (3.8)$$

$$-\frac{c^{(7)}}{2M^3}\int d^4\theta X^* X \nabla^\alpha H_u \nabla_\alpha H_d, \qquad (3.9)$$

$$-\frac{c^{(8)}}{4M^3}\int d^4\theta X^* X H_u \nabla^\alpha \nabla_\alpha H_d, \qquad (3.10)$$

$$-\frac{c^{(9)}}{4M^3}\int d^4\theta X^* X H_d \nabla^\alpha \nabla_\alpha H_u, \qquad (3.11)$$

$$-\frac{c^{(10)}}{4M^3}\int d^4\theta X^* X H_u^* e^V \nabla^\alpha \nabla_\alpha H_u, \qquad (3.12)$$

$$-\frac{c^{(11)}}{4M^3}\int d^4\theta X^* X H_d^* e^V \nabla^\alpha \nabla_\alpha H_d, \qquad (3.13)$$

where the $c^{(i)}$ are dimensionless parameters, and $\nabla^\alpha\nabla_\alpha\Phi = e^{-V}D^\alpha D_\alpha(e^V\Phi)$ for a chiral superfield Φ. I do not impose an exact $U(1)$ R symmetry; otherwise all but $c_a^{(1)}$ and $c_a^{(2)}$ would vanish, and it would be necessary to introduce an extra pair of Higgs doublet chiral superfields, as in [7]. Also, for simplicity I do not consider terms of the form $\frac{1}{M^3}\int d^4\theta X^* X\Phi^3 + \text{c.c.}$ and $\frac{1}{M^3}\int d^4\theta X^* X\Phi^2\Phi^* + \text{c.c.}$ where Φ^3 and $\Phi^2\Phi^*$ represent different gauge-invariant combinations of adjoint and Higgs chiral superfields. These can contribute scalar cubic interactions of the same magnitude as the Dirac gaugino masses. I also neglect the effects of any superpotential terms that do not involve the MSSM quark and lepton superfields. Thus there is no supersymmetric μ term and any superpotential couplings of the adjoints are taken to be small. Now let us consider the component field form of each of the terms in Eqs. (3.4)–(3.13) in turn.

B. Optional supersoft interactions

The Lagrangian contribution from the term in Eq. (3.4) together with its complex conjugate can be written as

$$\mathcal{L} = m_{R_a} D^a(\phi^a + \phi^{a*})/\sqrt{2} = m_{R_a} D^a R_a, \qquad (3.14)$$

where

$$m_{R_a} = 2c_a^{(2)}\langle F\rangle^2/M^3. \qquad (3.15)$$

After combining this with the rest of the Lagrangian involving the D^a auxiliary field, and integrating it out, one obtains

$$\mathcal{L} = -\frac{1}{2}(m_{R_a} R_a + g_a \phi_i^\dagger t^a \phi_i)^2. \qquad (3.16)$$

This is recognized as the scalar part (only) of the supersoft interaction, but with a parameter m_{R_a} that is independent of the Dirac gaugino mass parameter $m_{Da} = c_a^{(1)}\langle F\rangle^2/M^3$. A specific linear combination of Eqs. (2.10) and (3.4), namely $c_a^{(1)} = c_a^{(2)}$ so that $m_{Ra} = 2m_{Da}$, gives a combination proportional to the complete supersoft interaction. The reason for this can be seen by noting that [taking $c_a^{(1)} = c_a^{(2)} = 1$] integration by parts in superspace yields

$$\frac{1}{\sqrt{2}M^3}\int d^4\theta X^* X D_\alpha(A^a \mathcal{W}^{a\alpha}) = \frac{1}{4\sqrt{2}M^3}\int d^2\theta D^\dagger D^\dagger D_\alpha(X^* X)A^a\mathcal{W}^{a\alpha}, \quad (3.17)$$

so that the chiral superfield $\frac{1}{M^3}D^\dagger D^\dagger D_\alpha(X^* X)$ now plays the role of the D-term spurion $\frac{1}{M}\mathcal{W}'^\alpha$ in the supersoft Lagrangian Eq. (2.4). Previous papers that discuss Dirac gaugino masses in the context of F-term spurions have used this supersoft form; see for example Refs. [25,27,32]. However, with F-term breaking, that specific linear combination is not preferred in general, except that it is a fixed point of the RG running, with mixed stability properties to be discussed below. Therefore it is possible to assume that $|c_a^{(2)}|$ is smaller than $|c_a^{(1)}|$, so that the Dirac gaugino mass parameter dominates over the scalar adjoint interactions. This will avoid the problem of the missing scalar quartic couplings in the low-energy MSSM effective theory that can occur in the supersoft case.

C. General gaugino masses

The terms in Eqs. (3.5) and (3.6), together with their complex conjugates, provide Majorana masses for the gaugino and the adjoint chiral fermion, respectively, with

$$\mathcal{L} = -\frac{1}{2}M_a\lambda^a\lambda^a - \frac{1}{2}\mu_a\psi^a\psi^a + \text{c.c.}, \quad (3.18)$$

where

$$M_a = c_a^{(3)}\langle F\rangle^2/M^3, \quad (3.19)$$

$$\mu_a = c_a^{(4)}\langle F\rangle^2/M^3. \quad (3.20)$$

These terms, and the Dirac gaugino mass m_{Da} from Eqs. (2.10)–(2.11), are all parametrically of the same order, so the gaugino mass can be the most general allowed by gauge invariance. In the basis (λ^a, ψ^a), the gaugino mass matrix is

$$\begin{pmatrix} M_a & m_{Da} \\ m_{Da} & \mu_a \end{pmatrix}, \quad (3.21)$$

The gluinos will be Dirac-like if $|c_a^{(3)}|$ and $|c_a^{(4)}|$ are both much less than $|c_a^{(1)}|$, or Majorana-like if at least one of $|c_a^{(3)}|$ and $|c_a^{(4)}|$ is much greater than $|c_a^{(1)}|$, or could have a mixed Dirac/Majorana character. This provides a continuous set of possibilities for gluino couplings to quark-squark in the MSSM, following from the mixing. For the electroweak gauginos, there is of course a further complication due to mixing with the Higgsinos.

D. Scalar adjoint masses

The Lagrangian term of Eq. (3.7) and its complex conjugate give a common positive-definite squared mass to both the real and imaginary parts of the adjoint scalar:

$$\mathcal{L} = m_{Sa}\phi^a F_a + \text{c.c.} \to -|m_{Sa}|^2|\phi^a|^2 = -\frac{1}{2}|m_{Sa}|^2(R_a^2 + I_a^2), \quad (3.22)$$

where the $\to$ indicates the effect of integrating out the chiral adjoint auxiliary field F_a in this term together with its kinetic term contribution $|F_a|^2$, and

$$m_{Sa} = c_a^{(5)}\langle F\rangle^2/M^3. \quad (3.23)$$

This mass scale is again parametrically the same order as the Dirac gaugino mass. Unlike the minimal version of the supersoft model, the adjoint scalar R_a and pseudoscalar I_a therefore can naturally have a common positive squared mass at tree level, in addition to the positive squared mass for R_a if $c_a^{(2)}$ does not vanish.

Note that the particular linear combination $c_a^{(4)} = c_a^{(5)}$ would give a supersymmetric mass to the chiral adjoint superfield, with $m_{Sa} = \mu_a$. The reason for this is that the corresponding Lagrangian term is [for $c_a^{(4)} = c_a^{(5)} = 1$]

$$-\frac{1}{8M^3}\int d^4\theta X^* X DD(A^a A^a), \quad (3.24)$$

which, upon integration by parts twice, can be written as a superpotential term:

$$\frac{1}{32M^3}\int d^2\theta D^\dagger D^\dagger DD(X^* X)A^a A^a = \frac{\langle F\rangle^2}{2M^3}\int d^2\theta A^a A^a. \quad (3.25)$$

In fact, this term has precisely the same effect as the one proposed by Nelson and Roy in Ref. [56] in the supersoft case with D-term breaking. However, again in the present context there is no reason in general to prefer this specific linear combination.

If we also include the term Eq. (3.8), then Eq. (3.22) is generalized to

$$\mathcal{L} = (m_{Sa}\phi_a + m'_{Sa}\phi_a^*)F_a + \text{c.c.}, \quad (3.26)$$

where

$$m'_{Sa} = c_a^{(6)}\langle F\rangle^2/M^3, \quad (3.27)$$

so that after integrating out F_a we get

$$\mathcal{L} = -(|m_{Sa}|^2 + |m'_{Sa}|^2)|\phi_a|^2 - (m_{Sa}m'^*_{Sa}\phi_a^2 + \text{c.c.}). \quad (3.28)$$

This still always provides positive semidefinite squared masses for both of the adjoint scalar degrees of freedom, but splits them apart. The squared mass eigenvalues are $(|m_{Sa}| \pm |m'_{Sa}|)^2$.

E. Solution to the μ problem

The three Lagrangian terms in Eqs. (3.9)–(3.11) provide a novel solution to the μ problem. First, Eq. (3.9) and its complex conjugate yield a mass for the Higgsinos only:

$$\mathcal{L} = -\tilde{\mu}\tilde{H}_u\tilde{H}_d + \text{c.c.} \tag{3.29}$$

where

$$\tilde{\mu} = c^{(7)}\langle F\rangle^2/M^3. \tag{3.30}$$

Equations (3.10) and (3.11) and their complex conjugates provide terms:

$$\mathcal{L} = \mu_u H_u F_{H_d} + \text{c.c.} \to -|\mu_u|^2|H_u|^2 + \cdots, \tag{3.31}$$

$$\mathcal{L} = \mu_d H_d F_{H_u} + \text{c.c.} \to -|\mu_d|^2|H_d|^2 + \cdots, \tag{3.32}$$

where

$$\mu_u = c^{(8)}\langle F\rangle^2/M^3, \qquad \mu_d = c^{(9)}\langle F\rangle^2/M^3. \tag{3.33}$$

The $\to$ in Eqs. (3.31) and (3.32) corresponds to the effect of integrating out the auxiliary fields F_{H_d} and F_{H_u} when their kinetic terms $|F_{H_d}|^2$ and $|F_{H_u}|^2$ are included. The ellipses in Eqs. (3.31) and (3.32) refer to nonholomorphic scalar cubic couplings, which are

$$\begin{aligned}\mathcal{L} = {} & y_t\mu_d\tilde{t}_R(\tilde{t}_L^*H_d^0 + \tilde{b}_L^*H_d^-)\\ & + y_b\mu_u\tilde{b}_R(\tilde{b}_L^*H_u^0 + \tilde{t}_L^*H_u^+)\\ & + y_\tau\mu_u\tilde{\tau}_R(\tilde{\tau}_L^*H_u^0 + \tilde{\nu}_\tau^*H_u^+) + \text{c.c.}\end{aligned} \tag{3.34}$$

in the approximation that the only Yukawa couplings are y_t, y_b, and y_τ. These have the same form as the scalar cubic terms that occur in the supersymmetric part of the MSSM Lagrangian. However, here these terms are supersymmetry violating in general, because μ_u, μ_d, and $\tilde{\mu}$ are different.

Thus, there are really three μ terms, all parametrically of the same order but otherwise distinct: $\tilde{\mu}$ for the Higgsinos, μ_u for the up-type Higgs scalars, and μ_d for the down-type Higgs scalars. There is a special choice with $c^{(7)} = c^{(8)} = c^{(9)}$ that yields a supersymmetric relation $\tilde{\mu} = \mu_u = \mu_d$, but in general this specific linear combination is not preferred. This means that the Higgsino mass $\tilde{\mu}$ is independent of the Higgs scalar potential sector, effectively decoupling the Higgsinos from electroweak-scale naturalness issues. A quite similar mechanism[2] has been proposed in Ref. [56] in the supersoft context, where there can be two distinct μ terms, one shared by the Higgsinos and the H_u scalars, and the other common to the Higgsinos and the H_d scalars. In fact, the two Nelson-Roy Higgs μ terms are obtained in the present context by restricting to the special parameter subspace with $2c^{(7)} = c^{(8)} + c^{(9)}$.

The holomorphic scalar squared mass term $\mathcal{L} = -bH_uH_d + \text{c.c.}$ will arise by RG evolution from $\tilde{\mu}$. While this is loop suppressed, one can obtain a sufficiently large b if $|\tilde{\mu}|$ is not too small, with no naturalness concerns since it is not tied to $|\mu_u|$ in this model. Therefore, naturalness of electroweak symmetry breaking might actually prefer a relatively heavier Higgsino, in contradiction with popular argument. However, there is another, probably better, way to get the b-term, discussed in the next subsection.

[2]Some other intriguing ways of decoupling the Higgsino mass from the naturalness of the Higgs potential are proposed in Refs. [69–72].

F. MSSM a-term and b-term (holomorphic scalar) couplings

Finally, consider including the terms in Eqs. (3.12) and (3.13) and their complex conjugates, in conjunction with the terms in Eqs. (3.10) and (3.11) just considered. Their effect is to modify Eqs. (3.31) and (3.32) to give a total:

$$\mathcal{L} = (\mu_u H_u + \mu'_d H_d^*)F_{H_d} + (\mu'_u H_u^* + \mu_d H_d)F_{H_u} + \text{c.c.}, \tag{3.35}$$

where

$$\mu'_u = c^{(10)}\langle F\rangle^2/M^3, \qquad \mu'_d = c^{(11)}\langle F\rangle^2/M^3. \tag{3.36}$$

Now, adding in the $|F_{H_u}|^2$ and $|F_{H_d}|^2$ kinetic terms and integrating out the auxiliary fields one obtains, in addition to the nonholomorphic scalar cubic couplings of Eq. (3.34), terms that have exactly the same form as the usual MSSM soft scalar interactions:

$$\begin{aligned}\mathcal{L} = {} & -(H_u\tilde{\bar{u}}\mathbf{a_u}\tilde{Q} - H_d\tilde{\bar{d}}\mathbf{a_d}\tilde{Q} - H_d\tilde{\bar{e}}\mathbf{a_e}\tilde{L} + bH_uH_d + \text{c.c.})\\ & - |\mathcal{M}_u|^2|H_u|^2 - |\mathcal{M}_d|^2|H_d|^2.\end{aligned} \tag{3.37}$$

Here the Higgs scalar squared mass parameters are now

$$|\mathcal{M}_u|^2 = |\mu_u|^2 + |\mu'_u|^2, \tag{3.38}$$

$$|\mathcal{M}_d|^2 = |\mu_d|^2 + |\mu'_d|^2, \tag{3.39}$$

$$b = \mu_u\mu_d'^* + \mu_d\mu_u'^*, \tag{3.40}$$

and the a-terms are, in terms of the corresponding superpotential Yukawa coupling matrices $\mathbf{y_u}$, $\mathbf{y_d}$, and $\mathbf{y_e}$,

$$\mathbf{a_u} = \mu_u'^*\mathbf{y_u}, \tag{3.41}$$

$$\mathbf{a_d} = \mu_d'^*\mathbf{y_d}, \qquad \mathbf{a_e} = \mu_d'^*\mathbf{y_e}. \tag{3.42}$$

In this way, one obtains minimal flavor-violating a-terms, including the Higgs-top-squark-anti-top-squark coupling a_t which is useful in obtaining one-loop contributions that help give a Higgs mass as high as 125 GeV. The magnitude of a_t is related at tree level to a lower bound on $|\mathcal{M}_u|$, as seen from comparing Eqs. (3.38) and (3.41). Note that all of these terms are parametrically related to the mass scale $\langle F\rangle^2/M^3$.

The terms in the effective Lagrangian listed above include nonstandard supersymmetry breaking operators, including those claimed to be hard breaking in the classification of Ref. [73]. Here, they have been shown to arise from a consistent spurion analysis, but one might still worry about destabilizing divergences associated with tadpoles in the case of a gauge singlet chiral superfield [74]. One way to avoid this is to only include Dirac gauginos for the $SU(2)_L$ and $SU(3)_c$ gauginos. Alternatively, one may assume that at very high energies the gauge singlet chiral superfields are actually in a nonsinglet representation of an extended gauge group.

IV. RENORMALIZATION GROUP RUNNING EFFECTS

In the previous section, it was found that the supersymmetry breaking from an F-term spurion VEV and mediated by operators suppressed by $1/M^3$ can produce all types of supersymmetry breaking with positive mass dimension, including the "nonstandard" terms: Dirac gaugino masses, chiral fermion masses, and nonholomorphic scalar cubic interactions. Note that the Higgs-related terms discussed here are actually independent of the Dirac gaugino mass issue. One can delete any or all of the adjoint chiral superfields from the theory, and the same mechanism will work to provide three independent μ terms, in a theory with F-term breaking and suppression of communication of supersymmetry breaking by $1/M^3$.

If the adjoint chiral superfields and Dirac gaugino masses are included, with a mass scale of order TeV, then gauge-coupling unification can be achieved by also adding in vectorlike chiral superfields in the leptonlike representations

$$L+\bar{L}+2\times[e+\bar{e}] = (\mathbf{1},\mathbf{2},-1/2)+(\mathbf{1},\mathbf{2},+1/2) + 2\times[(\mathbf{1},\mathbf{1},-1)+(\mathbf{1},\mathbf{1},+1)] \tag{4.1}$$

of $SU(3)_c\times SU(2)_L\times U(1)_Y$. The resulting two-loop running of gauge couplings is shown in Fig. 1, using a simplified supersymmetric threshold at 2 TeV. Although the $SU(3)_c$ gauge coupling would not run in the one-loop approximation, it actually becomes significantly stronger in the UV due to two-loop effects, with $\alpha_3(M_{\mathrm{GUT}})/\alpha_3(2\ \mathrm{TeV}) = 1.3$.

The complete two-loop RG equations for a general theory of this type have already been given in [5,6].

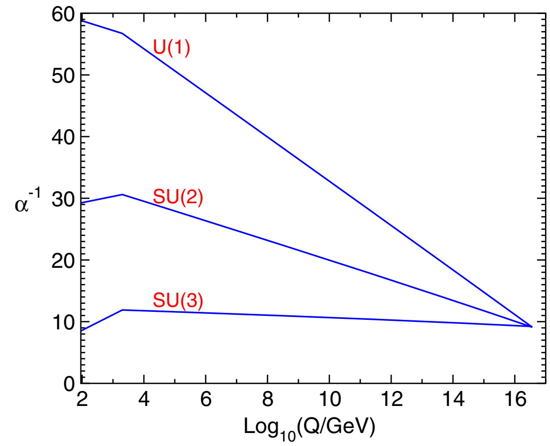

FIG. 1 (color online). The two-loop running of the $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$ inverse gauge couplings α_a^{-1}, as a function of the renormalization scale Q, with the MSSM particle content plus adjoint chiral superfields and the vectorlike chiral superfields in the representations of Eq. (4.1). For simplicity, the masses of all particles that are beyond the Standard Model are put at a single threshold at 2 TeV.

The specialization to the MSSM (plus chiral adjoint superfields) will not be given here, as this can now be done easily by symbolic manipulation, for example using modern tools such as Ref. [38]. The case discussed here is different than e.g. in Ref. [37,51], because here the supersoft scalar interactions have been decoupled from the Dirac gaugino masses.

Because the supersoft case is a fixed point of the more general case, it is interesting to consider whether that fixed-point solution is attractive (stable) in the infrared (IR). To investigate this, without taking on the most general case, consider the following supersymmetry-breaking Lagrangian terms that involve the gauginos and the chiral adjoint fields:

$$\begin{aligned}\mathcal{L} = -\Big[&\frac{1}{2}M_a\lambda^a\lambda^a + \frac{1}{2}\mu_a\psi^a\psi^a + m_{Da}\psi^a\lambda^a \\ &+ \sqrt{2}g_a m_{Da}N_a\phi^a(\phi_i^\dagger t^a\phi_i) \\ &+ \frac{1}{2}b_a(\phi^a)^2 + \text{c.c.}\Big] - m_a^2|\phi_a|^2.\end{aligned} \tag{4.2}$$

Here I have assumed that the scalar cubic couplings of adjoints to MSSM fields labeled by i are actually independent of i. This condition is preserved by one-loop RG running if it is true at any scale, and it is a feature of Eq. (3.16), which may serve as a boundary condition on the running. These couplings are also normalized to the gauge coupling g_a and the Dirac gaugino mass m_{Da}, so that they are represented by three dimensionless running parameters N_a, one for each of the gauge groups $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$. The one-loop beta functions of the gauge couplings and the gaugino/adjoint fermion masses and the N_a are found from Ref. [6]:

$$16\pi^2\beta_{g_a} = g_a^3[T_a(R_F) - 2C(G_a)], \tag{4.3}$$

$$16\pi^2\beta_{M_a} = g_a^2 M_a[2T_a(R_F) - 4C(G_a)], \tag{4.4}$$

$$16\pi^2\beta_{\mu_a} = g_a^2\mu_a[-4C(G_a)], \tag{4.5}$$

$$16\pi^2\beta_{m_{Da}} = g_a^2 m_{Da}[T_a(R_F) - 4C(G_a)], \tag{4.6}$$

$$16\pi^2\beta_{N_a} = 4g_a^2 C(G_a)(N_a - 1), \tag{4.7}$$

where $C(G_a)$ is the quadratic Casimir of the adjoint representation of the gauge group, and $T_a(R_F)$ is the Dynkin index of the chiral superfields that are in the fundamental representation (i.e., not including the adjoint representation chiral superfields). For $SU(3)_c$, one has $C(G_a) = 3$ and $T_a(R_F) = 6$. For $SU(2)_L$, one has $C(G_a) = 2$ and $T_a(R_F) = 7 + n_{L+\bar{L}}$. For $U(1)_Y$, one has $C(G_a) = 0$ and $T_a(R_F) = (33 + 3n_{L+\bar{L}} + 6n_{e+\bar{e}})/5$ in a GUT normalization (so using $g_1 = \sqrt{5/3}g'$). For the minimal MSSM with Dirac gaugino masses, $n_{L+\bar{L}} = n_{e+\bar{e}} = 0$, and for the model that unifies gauge couplings with Eq. (4.1), $n_{L+\bar{L}} = 1$, $n_{e+\bar{e}} = 2$. I will use the latter in the numerical results and fixed-point analysis below.

Also found from Ref. [6] are the beta functions for the nonholomorphic and holomorphic adjoint scalar masses, respectively:

$$16\pi^2\beta_{m_a^2} = g_a^2[4T_a(R_f)|N_a|^2|m_{Da}|^2 - C(G_a)(8|M_a|^2 + 8|\mu_a|^2 + 16|m_{Da}|^2)], \tag{4.8}$$

$$16\pi^2\beta_{b_a} = g_a^2[4T_a(R_f)N_a^2 m_{Da}^2 + C(G_a) \times (8M_a\mu_a - 8m_{Da}^2 - 4b_a)]. \tag{4.9}$$

Now, for illustrative purposes, let us specialize to the case that M_a and μ_a can be neglected in comparison to m_{Da}, and normalize the adjoint scalar squared masses to the latter:

$$m_a^2 = 2E_a|m_{Da}|^2, \tag{4.10}$$

$$b_a = 2B_a m_{Da}^2. \tag{4.11}$$

This defines, for each gauge group, two dimensionless running parameters E_a and B_a, in terms of which the adjoint scalar tree-level squared mass eigenvalues are $2m_{Da}^2(E_a \pm |B_a|)$. Note that N_a, E_a, and B_a are each 1 in the supersoft case. From Eqs. (4.8) and (4.9), the beta functions for the last two are

$$16\pi^2\beta_{E_a} = g_a^2[2T_a(R_F)(N_a^2 - E_a) + 8C(G_a)(E_a - 1)], \tag{4.12}$$

$$16\pi^2\beta_{B_a} = g_a^2[2T_a(R_F)(N_a^2 - B_a) + 4C(G_a)(B_a - 1)]. \tag{4.13}$$

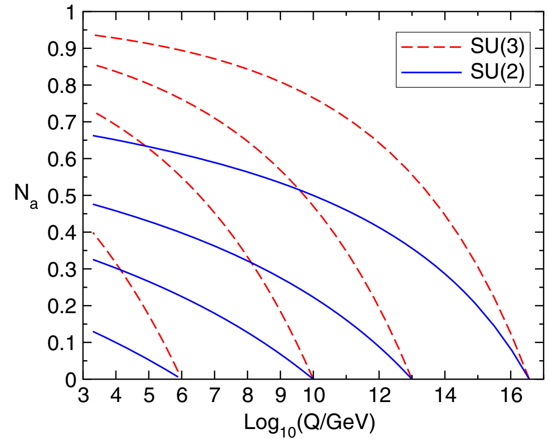

FIG. 2 (color online). Four examples of the one-loop running of the scalar cubic coupling parameters N_2 and N_3 [for $SU(2)_L$ and $SU(3)_c$ respectively] as defined by Eq. (4.2). The parameter N_1 does not run at one-loop order. The boundary conditions are $N_2 = N_3 = 0$ at input scales $M = 10^6$ and 10^{10} and 10^{13} GeV and the gauge coupling unification scale. The vectorlike chiral superfields of Eq. (4.1) are included to provide gauge coupling unification.

It is clear from Eqs. (4.7), (4.12), and (4.13) that the supersoft trajectory $B_a = E_a = N_a = 1$ is indeed a fixed point, as originally observed by Ref. [6]. However, if $c_a^{(1)}$ and $c_a^{(2)}$ in Eqs. (2.10) and (3.4) are nonzero but different from each other, then one will have $B_a = E_a = N_a \neq 1$ initially. The subsequent RG running will then make them all different. The $U(1)_Y$ scalar cubic parameter[3] N_1 does not run at all at one-loop order, and the $E_1 = N_1^2$ and $B_1 = N_1^2$ fixed points are actually unstable in the IR. From Eq. (4.7), we see that the fixed points for $N_3 = 1$ and $N_2 = 1$ are stable in the IR, but while the $E_3 = 1$ fixed point is formally stable, in practice that stability is never realized in the running even if the input scale is very high. The fixed points $B_3 = 1$ and $E_2 = 1$ are not even formally stable in the IR at one-loop order, while the fixed point $B_2 = 1$ is definitely unstable in the IR.

If one assumes that at the input scale M the starting boundary condition is $N_2 = N_3 = 0$, the resulting running for N_2 and N_3 [for $SU(2)_L$ and $SU(3)_c$ respectively] is shown in Fig. 2. In this graph, four different choices for the input scale are shown: $M = 10^6$ and 10^{10} and 10^{13} GeV and the gauge coupling unification scale. (However, as noted above, the input scale M probably should be less than roughly 10^{13} GeV, if one wants AMSB contributions to the gaugino mass to be not larger than the Dirac gaugino masses.) We see that the attractive fixed point at $N_3 = 1$ is

[3]Gauge invariance dictates that couplings with different indices a corresponding to the same simple or Abelian gauge group component are degenerate. Therefore, as a slight abuse of notation, in the following 1, 2, 3 are used for the index a to label the $U(1)_Y$, $SU(2)_L$, and $SU(3)_c$ components respectively.

not actually approached unless the input scale M is very high, while the fixed point $N_2 = 1$ is quite weakly attractive, due to the smaller Casimir invariant and smaller gauge coupling below the unification scale.

The one-loop order beta functions for the MSSM scalar squared masses are (including the effects of possible Majorana gaugino masses M_a)

$$16\pi^2\beta_{(m^2)_i^j} = 8g_a^2 C_a(i)\delta_i^j[(|N_a|^2 - 1)|m_{Da}|^2 - |M_a|^2] + \cdots \tag{4.14}$$

where $C_a(i)$ are the quadratic Casimir invariants [4/3 for squarks for $SU(3)_c$, 3/4 for doublets for $SU(2)_L$, and $3Y_i^2/5$ for scalars with weak hypercharge Y_i], and the ellipses represent the usual Yukawa and a-term contributions from the MSSM. In the supersoft case, $N_a = 1$ and $M_a = 0$, so there is no positive gaugino mass contribution to squark and slepton squared masses from running. In the scenario of the present paper, there is such a contribution even neglecting M_a, since N_a is not at its fixed-point value. This contribution will be positive definite from running into the IR as long as $|N_a| < 1$. In practice, this will always be the case if N_a starts from 0 at M, as was seen in Fig. 2.

In Fig. 3, the squark and the two scalar color adjoint (sgluon) mass eigenvalues are shown for the case that the Dirac gluino mass $c_a^{(1)}$ dominates at the input scale $M_{\rm input}$, so that $N_3 = E_3 = B_3 = 0$ there and both the Majorana gluino mass M_3 and the supersymmetry-breaking color adjoint fermion mass μ_3 are neglected. The results are expressed as ratios of the scalar masses to the gluino Dirac mass at the renormalization scale $Q = 2$ TeV, as a function of the input scale $M_{\rm input}$. Only one-loop QCD-enhanced effects are included. A realistic model probably must have $M_{\rm input}$ at least as large as 10^4 GeV, but the results are shown for $M_{\rm input}$ all the way down to 2 TeV, to illustrate the expected behavior that if there is no RG running then squarks and sgluons are massless at tree level.

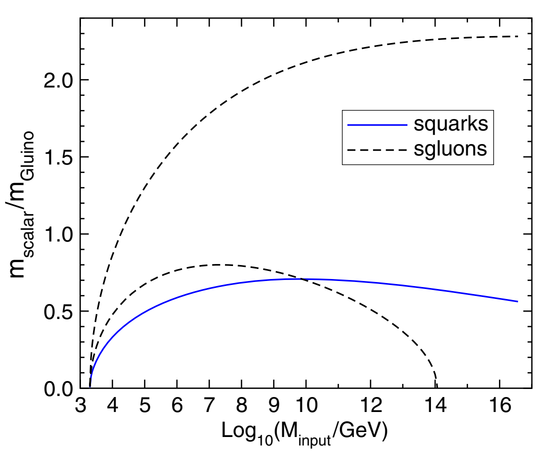

FIG. 3 (color online). The masses of squarks (solid line) and the two color adjoint scalar sgluons (dashed lines) expressed as tree-level ratios $m_{\rm scalar}/m_{D3}$ at the scale $Q = 2$ TeV. Results are shown as a function of the input scale $M_{\rm input}$ at which the boundary condition $N_3 = E_3 = B_3 = 0$ is applied. Only one-loop QCD-enhanced RG contributions due to the Dirac gluino masses m_{D3} are included.

Clearly, even one decade of RG running is enough to generate sufficient squark and sgluon masses. Figure 3 shows that for $M_{\rm input} > 100$ TeV, the (tree-level) first- and second-generation squark masses are between about 0.5 and 0.7 of the gluino Dirac mass; this is in comparison to a factor of 0.1 to 0.2 for the corresponding ratio of pole masses in supersoft models. Of course, additional model parameter-dependent contributions to the gluino mass matrix Eq. (3.21) can strongly modify this prediction in either direction, but it shows that the RG contributions to sfermion squared masses due to Dirac gaugino masses are generically significant and positive. Also we see that both sgluons have positive squared masses, provided that the input scale $M_{\rm input}$ is smaller than 10^{14} GeV, even without using the contributions from the mechanism of Sec. III D. For $M_{\rm input}$ larger than about 10^{14} GeV, the lighter sgluon is tachyonic, breaking color, but as mentioned previously the AMSB contribution to gaugino masses should dominate in that case anyway. One of the sgluons is heavier than the Dirac gluino provided that $M_{\rm input} > 20$ TeV, and one is lighter. Of course, finite one-loop corrections and two-loop RG corrections, as well as electroweak and Yukawa effects for the squarks, should also be taken into account in order to get more precise estimates. Moreover, nonzero values of $c_a^{(2)}$, $c_a^{(3)}$, $c_a^{(4)}$, $c_a^{(5)}$, and $c_a^{(6)}$ can all disrupt these simple predictions in calculable ways.

V. OUTLOOK

In this paper, I have considered a spurion operator analysis of a scenario in which supersymmetry breaking appears in the MSSM sector via operators with F-term VEVs that are suppressed by $1/M^3$ where M is a mediation mass scale. The result of this is that one can obtain all soft terms, including Dirac gaugino masses and nonholomorphic scalar cubic interactions, with a common mass scale $\langle F\rangle^2/M^3$. The supersymmetric μ term of the MSSM is replaced by three independent supersymmetry-breaking parameters, decoupling the Higgsino mass from the Higgs scalar potential. This illustrates that although it is traditional to think of μ as a superpotential parameter, it might be more sensible, depending on the mechanism for supersymmetry breaking, to instead regard it as a part of the soft supersymmetry-breaking Lagrangian.

In general, Dirac gaugino mass parameters need not be accompanied by supersoft scalar interactions. This has both good and bad implications. The adjoint scalars are naturally both massive, and there is no problem in maintaining the electroweak scalar quartic interactions that provide for a large Higgs mass. The squarks and sleptons of the MSSM get positive RG corrections to their masses from gauginos, unlike in the supersoft case. However, the supersoft mechanisms for safety from flavor- and CP-violating effects, and for explaining the lack of detection by the last run of the LHC, are diminished. The gaugino masses can in principle be of the most general mixed Majorana/Dirac form, with consequences for phenomenology that have already been explored in Refs. [8–15]. One interesting possibility is that the gluino can be mostly Dirac and accompanied by the (approximate) scalar supersoft interactions, as this is an IR quasistable fixed point of the RG equations, while the electroweak gauginos could be either purely Majorana with no adjoint chiral superfields, or else very far from the supersoft fixed-point trajectory, which is not attractive in the IR for $SU(2)_L$ or $U(1)_Y$. Alternatively, one can simply discard all of the adjoint chiral superfields, as the mechanisms for nonstandard supersymmetry breaking and three distinct μ parameters will still go through.

An obvious important remaining question is whether the model-building criteria assumed here can be realized (at least approximately) in a full UV completion. If so, it would be interesting to outline the requirements for doing so, and any relationships between couplings that might be implied.

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