Systematic study of Z_c^+ family from a multiquark color flux-tube model

Chengrong Deng,[1,2,†] Jialun Ping,[3,*] Hongxia Huang,[3,‡] and Fan Wang[4,§]

[1]*School of Science, Chongqing Jiaotong University, Chongqing 400074, People's Republic of China*
[2]*Department of Physics and Astronomy, University of California, Los Angeles, California 90095, USA*
[3]*Department of Physics, Nanjing Normal University, Nanjing 210097, People's Republic of China*
[4]*Department of Physics, Nanjing University, Nanjing 210093, People's Republic of China*

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Inspired by the present experimental results of charged charmonium-like states Z_c^+, we present a systematic study of the tetraquark states $[cu][\bar{c}\,\bar{d}]$ in a color flux-tube model with a multibody confinement potential. Our investigation indicates that charged charmonium-like states $Z_c^+(3900)$ or $Z_c^+(3885)$, $Z_c^+(3930)$, $Z_c^+(4020)$ or $Z_c^+(4025)$, $Z_1^+(4050)$, $Z_2^+(4250)$, and $Z_c^+(4200)$ can be described as a family of tetraquark $[cu][\bar{c}\,\bar{d}]$ states with the quantum numbers $n^{2S+1}L_J$ and J^P of 1^3S_1 and 1^+, 2^3S_1 and 1^+, 1^5S_2 and 2^+, 1^3P_1 and 1^-, 1^5D_1 and 1^+, and 1^3D_1 and 1^+, respectively. The predicted lowest mass charged tetraquark state $[cu][\bar{c}\,\bar{d}]$ with 0^+ and 1^1S_0 lies at 3780 ± 10 MeV/c^2 in the model. These tetraquark states have compact three-dimensional spatial configurations similar to a rugby ball with higher orbital angular momentum L between the diquark $[cu]$ and antidiquark $[\bar{c}\,\bar{d}]$ corresponding to a more prolate spatial distribution. The multibody color flux tube, a collective degree of freedom, plays an important role in the formation of those charged tetraquark states. However, the two heavier charged states $Z_c^+(4430)$ and $Z_c^+(4475)$ cannot be explained as tetraquark states $[cu][\bar{c}\,\bar{d}]$ in this model approach.

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I. INTRODUCTION

Quantum chromodynamics (QCD) has been widely accepted as the fundamental theory to describe the interactions among quarks and gluons and the structure of hadrons. Conventional hadrons are composed of either a valence quark q and an antiquark $\bar{q}$ (mesons) or three valence quarks qqq (baryons) on top of the sea of $q\bar{q}$ pairs and gluons. One of the long standing challenges in hadron physics is to establish and classify genuine multiquark states beyond the conventional hadron structure because multiquark states may contain more information about the low-energy QCD than that of conventional hadrons. In the past several years, a charged charmonium-like Z_c^+ family, including $Z_c^+(4430)$, $Z_1^+(4050)$, $Z_2^+(4250)$, $Z_c^+(3900)$, $Z_c^+(3885)$, $Z_c^+(3930)$, $Z_c^+(4020)$, $Z_c^+(4025)$, $Z_c^+(4475)$ and $Z_c^+(4200)$, has been successively observed by experimental collaborations [1–9]. Those charged charmonium-like states go beyond the conventional $c\bar{c}$-meson framework and are likely of tetraquark states with $c\bar{c}u\bar{d}$ constituents, which provides a unique system for testing various phenomenological models of hadron structure physics. A large amount of work has been devoted to describing the internal structure of these charged states, including meson-meson molecules [10,11], diquark-antidiquark states [12], hadrocharmonium or Born-Oppenheimer tetraquarks [13], coupled channel cusps [14], and kinematic effects [15].

A systematic understanding of the internal structure of these charged charmonium-like states may provide not only new insights into the strong interaction dynamics of multiquark systems and low-energy QCD, but also important information on future experimental searches for the missing higher orbital excitations in the Z_c^+ family. This is one of the goals of the present work. In our approach, a phenomenological model of a color flux tube with a multibody confinement potential, instead of the two-body one in the traditional quark model, is employed to explore properties of excited charged tetraquark states $c\bar{c}u\bar{d}$ systematically. The model has been successfully applied to the ground states of charged tetraquark states $[Qq][\bar{Q}'\bar{q}']$ ($Q, Q' = c, b$ and $q, q' = u, d, s$) in our previous work [16].

This paper is organized as follows: The color flux-tube model and the model parameters are given in Sec. II. The numerical results and discussions of the charged tetraquark states are presented in Sec. III. A brief summary is given in the last section.

II. COLOR FLUX-TUBE MODEL AND PARAMETERS

Details of the color flux-tube model based on traditional quark models and the lattice QCD picture can be found in our previous paper [17]. Only prominent characteristics of the model are presented here. The model Hamiltonian for the $[cu][\bar{c}\,\bar{d}]$ state is given as follows,

*Corresponding author.
jlping@njnu.edu.cn
†crdeng@cqjtu.edu.cn
‡hxhuang@njnu.edu.cn
§fgwang@chenwang.nju.edu.cn

$$H_4 = \sum_{i=1}^{4}\left(m_i + \frac{\mathbf{p}_i^2}{2m_i}\right) - T_C + \sum_{i>j}^{4} V_{ij} + V_{\min}^{C} + V_{\min}^{C,LS},$$
$$V_{ij} = V_{ij}^{B} + V_{ij}^{B,SL} + V_{ij}^{\sigma} + V_{ij}^{\sigma,LS} + V_{ij}^{G} + V_{ij}^{G,LS}, \tag{1}$$

where T_c is the center-of-mass kinetic energy of the state, and $\mathbf{p}_i$ and m_i are the momentum and mass of the ith quark and antiquark, respectively. The codes of the quarks (antiquarks) c, u, $\bar{c}$ and $\bar{d}$ are assumed to be 1, 2, 3 and 4, respectively. Their positions are denoted as $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$ and $\mathbf{r}_4$.

The quadratic confinement potential, which is believed to be flavor independent, of the tetraquark state with a diquark-antidiquark structure has the following form,

$$V^C = K[(\mathbf{r}_1 - \mathbf{y}_{12})^2 + (\mathbf{r}_2 - \mathbf{y}_{12})^2 + (\mathbf{r}_3 - \mathbf{y}_{34})^2 + (\mathbf{r}_4 - \mathbf{y}_{34})^2 + \kappa_d(\mathbf{y}_{12} - \mathbf{y}_{34})^2]. \tag{2}$$

The positions $\mathbf{y}_{12}$ and $\mathbf{y}_{34}$ are the junctions of two Y-shaped color flux-tube structures. The parameter K is the stiffness of a three-dimensional flux tube, and $\kappa_d K$ is the compound color flux-tube stiffness. The relative stiffness parameter κ_d for the compound flux tube is [18]

$$\kappa_d = \frac{C_d}{C_3}, \tag{3}$$

where C_d is the eigenvalue of the Casimir operator associated with the $SU(3)$ color representation d at either end of the color flux tube, such as $C_3 = \frac{4}{3}$, $C_6 = \frac{10}{3}$, and $C_8 = 3$.

The minimum of the confinement potential $V_{\min}^C$ can be obtained by taking the variation of V^C with respect to $\mathbf{y}_{12}$ and $\mathbf{y}_{34}$, and it can be expressed as

$$V_{\min}^{C} = K\left(\mathbf{R}_1^2 + \mathbf{R}_2^2 + \frac{\kappa_d}{1+\kappa_d}\mathbf{R}_3^2\right). \tag{4}$$

The canonical coordinates $\mathbf{R}_i$ have the following forms,

$$\mathbf{R}_1 = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2), \qquad \mathbf{R}_2 = \frac{1}{\sqrt{2}}(\mathbf{r}_3 - \mathbf{r}_4),$$
$$\mathbf{R}_3 = \frac{1}{\sqrt{4}}(\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{r}_3 - \mathbf{r}_4),$$
$$\mathbf{R}_4 = \frac{1}{\sqrt{4}}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4). \tag{5}$$

The use of $V_{\min}^C$ can be understood here as the gluon field readjusting immediately to its minimal configuration. It should be noted that the confinement $V_{\min}^C$ arises from a multibody interaction in a multiquark state instead of the sum of many pairwise confinement interactions,

$$V^C = \sum_{i<j} \lambda_i^c \cdot \lambda_j^c r_{ij}^n, \tag{6}$$

in the Isgur-Karl quark model and the chiral quark model with $n = 1$ or 2 and $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$.

The central parts of one-boson exchange V_{ij}^B and σ-meson exchange V_{ij}^σ only occur between u and $\bar{d}$, and that of one-gluon exchange V_{ij}^G is universal. V_{ij}^B, V_{ij}^σ and V_{ij}^G take their standard forms and are listed in the following,

$$V_{ij}^B = V_{ij}^\pi \sum_{k=1}^{3} \mathbf{F}_i^k \mathbf{F}_j^k + V_{ij}^K \sum_{k=4}^{7} \mathbf{F}_i^k \mathbf{F}_j^k + V_{ij}^\eta (\mathbf{F}_i^8 \mathbf{F}_j^8 \cos\theta_P - \sin\theta_P), \tag{7}$$

$$V_{ij}^\chi = \frac{g_{ch}^2}{4\pi}\frac{m_\chi^3}{12 m_i m_j}\frac{\Lambda_\chi^2}{\Lambda_\chi^2 - m_\chi^2}\sigma_i\cdot\sigma_j \times \left(Y(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3}Y(\Lambda_\chi r_{ij})\right), \tag{8}$$

$$V_{ij}^G = \frac{\alpha_s}{4}\lambda_i^c \cdot \lambda_j^c\left(\frac{1}{r_{ij}} - \frac{2\pi\delta(\mathbf{r}_{ij})\sigma_i\cdot\sigma_j}{3m_i m_j}\right), \tag{9}$$

$$V_{ij}^\sigma = -\frac{g_{ch}^2}{4\pi}\frac{\Lambda_\sigma^2 m_\sigma}{\Lambda_\sigma^2 - m_\sigma^2}\left(Y(m_\sigma r_{ij}) - \frac{\Lambda_\sigma}{m_\sigma}Y(\Lambda_\sigma r_{ij})\right), \tag{10}$$

where χ stands for π, K and η, $Y(x) = e^{-x}/x$. The symbols $\mathbf{F}$, $\boldsymbol{\lambda}$ and $\boldsymbol{\sigma}$ are the flavor $SU(3)$, color SU(3) Gell-Mann and spin $SU(2)$ Pauli matrices, respectively. θ_P is the mixing angle between η_1 and η_8 to give the physical η meson. $g_{ch}^2/4\pi$ is the chiral coupling constant. α_s is the running strong coupling constant, and it takes the following form [19],

$$\alpha_s(\mu_{ij}) = \frac{\alpha_0}{\ln\left((\mu_{ij}^2 + \mu_0^2)/\Lambda_0^2\right)}, \tag{11}$$

where μ_{ij} is the reduced mass of two interacting particles q_i (or $\bar{q}_i$) and q_j (or $\bar{q}_j$). Λ_0, α_0 and μ_0 are model parameters. The function $\delta(\mathbf{r}_{ij})$ in V_{ij}^G should be regularized [20],

$$\delta(\mathbf{r}_{ij}) = \frac{1}{4\pi r_{ij} r_0^2(\mu_{ij})} e^{-r_{ij}/r_0(\mu_{ij})}, \tag{12}$$

where $r_0(\mu_{ij}) = \hat{r}_0/\mu_{ij}$, $\hat{r}_0$ is a model parameter.

The diquark $[cu]$ and antidiquark $[\bar{c}\,\bar{d}]$ can be considered as compound objects $\bar{Q}$ and Q with no internal orbital excitation, and the angular excitations L are assumed to occur only between Q and $\bar{Q}$ in the present work; the parity of the state $[cu][\bar{c}\,\bar{d}]$ is therefore related to L as $P = (-1)^L$. In this way, the state $[cu][\bar{c}\,\bar{d}]$ has lower energy than the states with additional internal orbital excitation in Q and $\bar{Q}$. In order to facilitate numerical calculations, the spin-orbit interactions are assumed to take place approximately between compound objects $\bar{Q}$ and Q, which is consistent with the work [21]. The spin-orbit-related interactions can be expressed as follows:

$$V_{\bar{Q}Q}^{G,LS} \approx \frac{\alpha_s}{4}\lambda_{\bar{Q}}^{\bar{c}} \cdot \lambda_Q^c \frac{1}{8M_{\bar{Q}}M_Q}\frac{3}{X^3}\mathbf{L}\cdot\mathbf{S}, \tag{13}$$

$$V_{\bar{Q}Q}^{\sigma,LS} \approx -\frac{g_{ch}^2}{4\pi}\frac{\Lambda_\sigma^2}{\Lambda_\sigma^2 - m_\sigma^2}\frac{m_\sigma^3}{2M_{\bar{Q}}M_Q}\mathbf{L}\cdot\mathbf{S} \times \left(G(m_\sigma X) - \frac{\Lambda_\sigma^3}{m_\sigma^3}G(\Lambda_\sigma X)\right), \tag{14}$$

$$V_{\bar{Q}Q}^{C,LS} \approx \frac{K}{8M_{\bar{Q}}M_Q}\frac{\kappa_d}{1+\kappa_d}\mathbf{L}\cdot\mathbf{S}, \tag{15}$$

where $M_{12} = M_{34} \approx m_c + m_{u,d}$, $G(x) = Y(x)(\frac{1}{x} + \frac{1}{x^2})$, and S stands for the total spin angular momentum of the tetraquark state $[cu][\bar{c}\,\bar{d}]$.

The model parameters are determined as follows. The mass parameters m_π, m_K and m_η in the interaction V_{ij}^B take their experimental values, namely, $m_\pi = 0.7\ \text{fm}^{-1}$, $m_K = 2.51\ \text{fm}^{-1}$ and $m_\eta = 2.77\ \text{fm}^{-1}$. The cutoff parameters take the values $\Lambda_\pi = \Lambda_\sigma = 4.20\,\text{fm}^{-1}$ and $\Lambda_\eta = \Lambda_K = 5.20\ \text{fm}^{-1}$, and the mixing angle $\theta_P = -15°$ [19]. The mass parameter m_σ in the interaction V_{ij}^σ can be determined through the PCAC relation $m_\sigma^2 \approx m_\pi^2 + 4m_{u,d}^2$ [22], $m_{u,d} = 280$ MeV and $m_\sigma = 2.92\ \text{fm}^{-1}$. The chiral coupling constant g_{ch} can be obtained from the πNN coupling constant through

$$\frac{g_{ch}^2}{4\pi} = \left(\frac{3}{5}\right)^2 \frac{g_{\pi NN}^2}{4\pi}\frac{m_{u,d}^2}{m_N^2} = 0.43. \tag{16}$$

The other adjustable parameters and their errors can be determined by fitting the masses of the ground states of mesons using the Minuit program, and they are shown in Table I. The mass spectrum of the ground states of mesons, which is listed in Table II, can be obtained by solving the two-body Schrödinger equation

$$(H_2 - E_2)\Phi_{IJ}^{\text{meson}} = 0. \tag{17}$$

TABLE I. Adjustable model parameters (units: m_s, m_c, m_b, μ_0, Λ_0, MeV; K, MeV. fm^{-2}; r_0, MeV. fm; α_0, dimensionless).

Parameters	x_i	Δx_i	Parameters	x_i	Δx_i
m_s	511.78	0.228	α_0	4.554	0.018
m_c	1601.7	0.441	Λ_0	9.173	0.175
m_b	4936.2	0.451	μ_0	0.0004	0.540
K	217.50	0.230	r_0	35.06	0.156

TABLE II. Ground-state meson spectra, in MeV.

States	E_2	ΔE_2	PDG	States	E_2	ΔE_2	PDG
π	142	26	139	η_c	2912	5	2980
K	492	20	496	J/Ψ	3102	4	3097
ρ	826	4	775	B^0	5259	5	5280
ω	780	4	783	B^*	5301	4	5325
K^*	974	4	892	B_s^0	5377	5	5366
ϕ	1112	4	1020	B_s^*	5430	4	5416
$D^\pm$	1867	8	1880	B_c	6261	7	6277
D^*	2002	4	2007	B_c^*	6357	4	$\cdots$
$D_s^\pm$	1972	9	1968	η_b	9441	8	9391
D_s^*	2140	4	2112	$\Upsilon(1S)$	9546	5	9460

The mass error of mesons ΔE_2 introduced by the parameter uncertainty Δx_i can be calculated by the formula of error propagation,

$$\Delta H_2 = \sum_{i=1}^{8}\left|\frac{\partial H_2}{\partial x_i}\right|\Delta x_i, \tag{18}$$

$$\Delta E_2 \approx \langle \Phi_{IJ}^{\text{meson}}|\Delta H_2|\Phi_{IJ}^{\text{meson}}\rangle, \tag{19}$$

where x_i and Δx_i represent the ith adjustable parameter and its error, respectively, which are listed in Table I.

III. NUMERICAL RESULTS AND DISCUSSIONS

Within the framework of the diquark-antidiquark configuration, the wave function of the state $[cu][\bar{c}\,\bar{d}]$ can be written as a sum of the following direct products of color χ_c, isospin η_I, spin χ_s and spatial ϕ terms,

$$\Phi_{IM_IJM_J}^{[cu][\bar{c}\,\bar{d}]} = \sum_\alpha \xi_\alpha \left[\left[[\phi_{l_am_a}^G(\mathbf{r})\chi_{s_a}]_{j_a}^{[cu]}[\phi_{l_bm_b}^G(\mathbf{R}) \times \chi_{s_b}]_{j_b}^{[\bar{c}\,\bar{d}]}\right]_{J_{ab}}^{[cu][\bar{c}\,\bar{d}]} F_{LM}(\mathbf{X})\right]_{JM_J}^{[cu][\bar{c}\,\bar{d}]} \times \left[\eta_{I_a}^{[cu]}\eta_{I_b}^{[\bar{c}\,\bar{d}]}\right]_{IM_I}^{[cu][\bar{c}\,\bar{d}]}\left[\chi_{c_a}^{[cu]}\chi_{c_b}^{[\bar{c}\,\bar{d}]}\right]_{CW_C}^{[cu][\bar{c}\,\bar{d}]}, \tag{20}$$

in which the subscripts a and b represent the diquark $[cu]$ and antidiquark $[\bar{c}\,\bar{d}]$, respectively. $\mathbf{R}$ and $\mathbf{X}$ are relative spatial coordinates,

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \qquad \mathbf{R} = \mathbf{r}_3 - \mathbf{r}_4$$
$$\mathbf{X} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1+m_2} - \frac{m_3\mathbf{r}_3 + m_4\mathbf{r}_4}{m_3+m_4}. \tag{21}$$

The other details of the construction of the wave function can be found in our previous work [16]. Subsequently, the converged numerical results can be obtained by solving the four-body Schrödinger equation,

TABLE III. The energy $E_4 \pm \Delta E_4$ and rms $\langle \mathbf{r}^2 \rangle^{\frac{1}{2}}$, $\langle \mathbf{R}^2 \rangle^{\frac{1}{2}}$ and $\langle \mathbf{X}^2 \rangle^{\frac{1}{2}}$ of charged tetraquark states $[cu][\bar{c}\,\bar{d}]$ with J^P and $n^{2S+1}L_J$ (unit of energy: MeV; unit of rms: fm).

J^P	0^+	0^-	0^+	1^+	1^+	1^-	1^-	1^-	1^+	1^+
$n^{2S+1}L_J$	1^1S_0	1^3P_0	1^5D_0	1^3S_1	2^3S_1	1^1P_1	1^3P_1	1^5P_1	1^3D_1	1^5D_1
$E_4 \pm \Delta E_4$	3782 ± 12	4097 ± 8	4274 ± 7	3858 ± 10	3950 ± 10	4075 ± 8	4097 ± 8	4153 ± 7	4235 ± 7	4273 ± 7
$\langle \mathbf{r}^2 \rangle^{\frac{1}{2}}$	0.85	0.96	1.01	0.90	0.92	0.94	0.96	1.00	0.98	1.01
$\langle \mathbf{R}^2 \rangle^{\frac{1}{2}}$	0.85	0.96	1.01	0.90	0.92	0.94	0.96	1.00	0.98	1.01
$\langle \mathbf{X}^2 \rangle^{\frac{1}{2}}$	0.42	0.85	1.12	0.48	0.66	0.85	0.85	0.92	1.10	1.12
J^P	1^-	2^+	2^-	2^-	2^+	2^+	2^+	2^-	2^-	3^-
$n^{2S+1}L_J$	1^5F_1	1^5S_2	1^3P_2	1^5P_2	1^1D_2	1^3D_2	1^5D_2	1^3F_2	1^5F_2	1^5P_3
$E_4 \pm \Delta E_4$	4387 ± 7	4001 ± 7	4096 ± 8	4152 ± 7	4212 ± 8	4235 ± 7	4273 ± 7	4354 ± 7	4387 ± 7	4150 ± 7
$\langle \mathbf{r}^2 \rangle^{\frac{1}{2}}$	1.02	1.03	0.96	1.00	0.95	0.98	1.01	0.99	1.02	1.00
$\langle \mathbf{R}^2 \rangle^{\frac{1}{2}}$	1.02	1.03	0.96	1.00	0.95	0.98	1.01	0.99	1.02	1.00
$\langle \mathbf{X}^2 \rangle^{\frac{1}{2}}$	1.30	0.57	0.85	0.92	1.09	1.10	1.12	1.30	1.30	0.92
J^P	3^+	3^+	3^-	3^-	3^-	4^+	4^-	4^-	5^-	
$n^{2S+1}L_J$	1^3D_3	1^5D_3	1^1F_3	1^3F_3	1^5F_3	1^5D_4	1^3F_4	1^5F_4	1^5F_5	
$E_4 \pm \Delta E_4$	4234 ± 7	4272 ± 7	4332 ± 8	4353 ± 7	4386 ± 7	4274 ± 7	4353 ± 7	4387 ± 7	4387 ± 7	
$\langle \mathbf{r}^2 \rangle^{\frac{1}{2}}$	0.98	1.01	0.96	0.99	1.02	1.01	0.99	1.02	1.02	
$\langle \mathbf{R}^2 \rangle^{\frac{1}{2}}$	0.98	1.01	0.96	0.99	1.02	1.01	0.99	1.02	1.02	
$\langle \mathbf{X}^2 \rangle^{\frac{1}{2}}$	1.10	1.12	1.27	1.30	1.30	1.12	1.30	1.30	1.30	

$$(H_4 - E_4)\Phi^{[cu][\bar{c}\,\bar{d}]}_{IM_IJM_J} = 0, \tag{22}$$

with the Rayleigh-Ritz variational principle.

The energies $E_4 \pm \Delta E_4$ of the charged states $[cu][\bar{c}\,\bar{d}]$ with $n^{2S+1}L_J$ and J^P under the assumptions of $S = 0, \ldots, 2$ and $L = 0, \ldots, 3$ are systematically calculated and presented in Table III. The mass error of the states ΔE_4 can be calculated as ΔE_2, which are around several MeV except for the state of 1^1S_0. The spin-orbit interactions are extremely weak, less than 2 MeV. Therefore, the energies for excited states with the same L and S but different J are almost degenerate. The energies of the excited states with 1^5D_0, 1^5D_1 1^5D_2, 1^5D_3 and 1^5D_4 are listed in Table III, and the result is consistent with the conclusion of the work [23]. Other spin-related interactions are stronger and contribute a larger energy difference than that of spin-orbital interactions, especially for the ground states with 1^1S_0, 1^3S_1 and 1^5S_2. The energy difference among excited states mainly comes from the kinetic energy and confinement potential, which are proportional to the relative orbital excitation L. However, the relative kinetic energy between two clusters $[cu]$ and $[\bar{c}\,\bar{d}]$ is inversely proportional to $\langle \mathbf{X}^2 \rangle$, while the confinement potential is proportional to $\langle \mathbf{X}^2 \rangle$ so that they compete with each other to reach an optimum balance.

The rms $\langle \mathbf{r}^2 \rangle^{\frac{1}{2}}$, $\langle \mathbf{R}^2 \rangle^{\frac{1}{2}}$ and $\langle \mathbf{X}^2 \rangle^{\frac{1}{2}}$ stand for the size of the diquark $[cu]$, the antidiquark $[\bar{c}\,\bar{d}]$ and the distance between the two clusters, respectively, which have also been calculated and listed in Table III. The diquark $[cu]$ and antidiquark $[\bar{c}\,\bar{d}]$ are found to share the same size in every Z_c^+ state. The sizes of the diquark $[cu]$ and antidiquark $[\bar{c}\,\bar{d}]$ ($\langle \mathbf{r}^2 \rangle^{\frac{1}{2}}$ and $\langle \mathbf{R}^2 \rangle^{\frac{1}{2}}$) are mainly determined by the total spin S. The relative orbital excitation L of the states has a minor effect on their sizes. However, the sizes do not vary greatly with the total spin S, especially for higher orbital excited states. So the diquark $[cu]$ and antidiquark $[\bar{c}\,\bar{d}]$ are rather rigid against the rotation. For example, the sizes of the two groups $1^1S_0 - 1^3S_1 - 1^5S_2$ and $1^1F_3 - 1^3F_3 - 1^5F_3$ change gradually with the total spin S, 0.85-0.90–1.03 fm and 0.96-0.99–1.02 fm, respectively. And the sizes of the two groups $1^1S_0 - 1^1P_1 - 1^1D_2 - 1^1F_3$ and $1^3S_1 - 1^3P_1 - 1^3D_1 - 1^3F_2$ vary slightly with relative orbital excitation L, 0.85-0.94-0.95–0.96 fm and 0.90-0.96-0.98–0.99 fm, respectively. The distance between the diquark $[cu]$ and antidiquark $[\bar{c}\,\bar{d}]$ ($\langle \mathbf{X}^2 \rangle^{\frac{1}{2}}$), on the other hand, changes remarkably with the relative orbital excitation L between the two clusters and is irrelevant to the total spin of the system, as shown in Table III for the sizes of $1^3S_1 - 1^3P_1 - 1^3D_1 - 1^3F_2$ and $1^1S_0 - 1^3S_1 - 1^5S_2$. The sizes of the diquark $[cu]$, antidiquark $[\bar{c}\,\bar{d}]$ and the distance between the two clusters provide valuable insight into understanding the trend of changing energies for charged states Z_c^+ with quantum numbers S and L.

In order to illustrate the spatial configuration of charged states $[cu][\bar{c}\,\bar{d}]$, the distances in four states between any two constituents are given in Table IV. The ground state (1^1S_0 and 1^+) of charged tetraquark $[cu][\bar{c}\,\bar{d}]$ possesses a three-dimensional spatial configuration due to the competition of the confinement and the kinetic energy of the systems [16], which is similar to a rugby ball. The diquark $[cu]$ and antidiquark $[\bar{c}\,\bar{d}]$ pairs in the ground state have a large overlap because of the small $\langle \mathbf{X}^2 \rangle^{\frac{1}{2}}$, so the picture of the

TABLE IV. The average distances $\langle \mathbf{r}_{ij}^2 \rangle^{\frac{1}{2}}$ of the states $[cu][\bar{c}\,\bar{d}]$ with 1^1S_0, 1^1P_1, 1^1D_2, and 1^1F_3, $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ (units in fm).

$n^{2S+1}L_J$	$\langle \mathbf{r}_{12}^2 \rangle^{\frac{1}{2}}$	$\langle \mathbf{r}_{34}^2 \rangle^{\frac{1}{2}}$	$\langle \mathbf{r}_{24}^2 \rangle^{\frac{1}{2}}$	$\langle \mathbf{r}_{13}^2 \rangle^{\frac{1}{2}}$	$\langle \mathbf{r}_{14}^2 \rangle^{\frac{1}{2}}$	$\langle \mathbf{r}_{23}^2 \rangle^{\frac{1}{2}}$	$\langle \mathbf{X}^2 \rangle^{\frac{1}{2}}$
1^1S_0	0.85	0.85	1.11	0.46	0.85	0.85	0.42
1^1P_1	0.94	0.94	1.41	0.87	1.17	1.17	0.85
1^1D_2	0.95	0.95	1.59	1.11	1.37	1.37	1.09
1^1F_3	0.96	0.96	1.72	1.28	1.52	1.52	1.27

diquark or antidiquark is not particularly distinctive. However, all distances except for the sizes of the diquark and antidiquark ($\langle \mathbf{r}_{12}^2 \rangle^{\frac{1}{2}}$ and $\langle \mathbf{r}_{34}^2 \rangle^{\frac{1}{2}}$) increase with the orbital angular momentum L in the excited states, as shown in Table IV. Our result indicates that the picture of the diquark or antidiquark becomes more distinctive with the increase of the orbital angular momentum L. The spatial configuration of the excited states is still similar to a rugby ball; the higher the orbital angular momentum L, the more prolate the shape of the excited states. The multibody color flux tube based on the lattice QCD picture (a collective degree of freedom) plays an important role in the formation of these charged tetraquark states. Such a flux-tube interaction picture may provide a dynamical mechanism for the formation of the tetraquark states.

Next, we discuss the properties of the charged states Z_c^+ observed in experiments and their possible candidates in the color flux-tube model, which are presented in Table V. The spin and parity of the $Z_c^+(3900)$ have not been established yet. The $Z_c^+(3900)$ may correspond to the same state as the $Z_c^+(3885)$ with 1^+ [3]. The charged state $[cu][\bar{c}\,\bar{d}]$ with 1^+ and 1^3S_1 has a mass of 3858 ± 10 MeV in the color flux-tube model, which is very close to those of the two charged states $Z_c^+(3885)$ and $Z_c^+(3900)$. It cannot be excluded that the main component of $Z_c^+(3885)$ and $Z_c^+(3900)$ is the state $[cu][\bar{c}\,\bar{d}]$ with 1^+ and 1^3S_1, which is supported by many theoretical works [12]. The radial excited state 2^3S_1 has a mass of 3950 ± 10 MeV, which is extremely close to that of $Z_c^+(3930)$. It is possible to identify $Z_c^+(3930)$ as the tetraquark state $[cu][\bar{c}\,\bar{d}]$ with 1^+ and 2^3S_1. In other words, the $Z_c^+(3930)$ is the first radial excited state of the $Z_c^+(3900)$ in the color flux-tube model. The pair $Z_c^+(4020)$ and $Z_c^+(4025)$ show up with a similar mass (slightly above $D^*\bar{D}^*$ threshold). They might therefore be the same resonance; their spin and parity are unclear. The QCD sum rule identified the $Z_c^+(4020)$ and $Z_c^+(4025)$ as a tetraquark state $[cu][\bar{c}\,\bar{d}]$ with 1^+ [24], the same approach also favored a tetraquark state but with different quantum numbers 2^+ and ${}^5S_2$ [25]. In our calculations, the nearest tetraquark state $[cu][\bar{c}\,\bar{d}]$ to the $Z_c^+(4020)$ or $Z_c^+(4025)$ occupies quantum numbers 2^+ and 1^5S_2. The tetraquark states $[cu][\bar{c}\,\bar{d}]$ with 1^- and 1^1P_1 and 1^+ and 1^5D_1 have the energies of 4075 ± 8 MeV and 4273 ± 7 MeV, respectively, which are consistent with those of $Z_1^+(4050)$ and $Z_2^+(4250)$. So the two states may be assigned as the tetraquark states $[cu][\bar{c}\,\bar{d}]$ with 1^- and 1^1P_1 and 1^+ and 1^5D_1, respectively, in the color flux-tube model. The newly observed $Z_c^+(4200)$ prefers 1^+, which can be described as the tetraquark state $[cu][\bar{c}\,\bar{d}]$ with 1^+ and 1^3D_1 in the color flux-tube model. The study of the three-point function sum rules on this state supports the tetraquark interpretation [26]. Of course, it seems difficult to rule out two other possibilities of 2^+ and 1^1D_2 versus 2^+ and 1^3D_2 in the model. The $Z_c^+(4430)$ is the first charged state, the J^P of the state is determined unambiguously to be 1^+, and the $Z_c^+(4475)$ favors the spin-parity 1^+ over other hypotheses [8]. Because of the heavy mass of the diquark and antidiquark, the energy of the radial excitation between the diquark and the antidiquark is too small to make the tetraquark state $[cu][\bar{c}\,\bar{d}]$ above the energy of 4400 MeV. Internal excited states of the diquark and/or antidiquark are needed to account for the heavy charged states, whose details are to be addressed in the future. Alternatively, a meson-meson molecular state configuration for the two states has been suggested by several theoretical methods as well [11].

TABLE V. Z_c^+ states observed in experiments and their possible candidates in the color flux-tube model.

	Experiment		Model		
State	Mass, MeV	J^P	Mass, MeV	J^P	$n^{2S+1}L_J$
$Z_1^+(4050)$ [1]	4051^{+14+20}_{-14-41}	??	4075 ± 8	1^-	1^1P_1
$Z_2^+(4250)$ [1]	$4248^{+44+180}_{-29-35}$	??	4273 ± 7	1^+	1^5D_1
$Z_c^+(3900)$ [2]	$3899.0^{+3.6+4.9}_{-3.6-4.9}$	??	3858 ± 10	1^+	1^3S_1
$Z_c^+(3885)$ [3]	$3883.9^{+1.5+4.2}_{-1.5-4.2}$	1^+	3858 ± 10	1^+	1^3S_1
$Z_c^+(3930)$ [4]	3929^{+5+2}_{-5-2}	1^+	3950 ± 10	1^+	2^3S_1
$Z_c^+(4025)$ [5]	$4026.3^{+2.6+3.7}_{-2.6-3.7}$	??	4001 ± 7	2^+	1^5S_2
$Z_c^+(4020)$ [6]	$4022.9^{+0.8+2.7}_{-0.8-2.7}$	??	4001 ± 7	2^+	1^5S_2
$Z_c^+(4200)$ [7]	4196^{+36+17}_{-29-13}	??	4235 ± 7	1^+	1^3D_1
$Z_c^+(4475)$ [8]	4475^{+22+28}_{-22-11}	1^+	…	…	…
$Z_c^+(4430)$ [9]	4433^{+2+4}_{-2-4}	1^+	…	…	…

From the above analysis and Table III, we can see that most of the low energy theoretical states can be matched with the experimental ones. One of the exceptions is the state with 0^+ and 1^1S_0, which has a mass of 3780 ± 10 MeV. The experimental search of the η_c-like charged state will give a crucial test of the present approach. Our calculation also suggests that there are two negative parity states around 4100 MeV. More experimental information on states around this energy will shed more light on the structure of these hadrons.

The model assignments of the Z_c^+ states are completed just in terms of the proximity to the experimental masses; the more stringent check of the assignment is to study the decay properties of the states. These charged states should eventually decay into several color singlet mesons due to their high energy. In the course of the decay, the color flux-tube structure should break down first, which leads to the collapses of the three-dimensional spatial configuration, and then through the recombination of the color flux tubes, the particles of decay products form. The decay widths of the charged states $[cu][\bar{c}\,\bar{d}]$ are determined by the transition probability of the breakdown and recombination of color flux tubes. The calculations are in progress. This decay mechanism is similar to compound nucleus decay and therefore should induce a resonance, which we previously called a "color confined, multiquark resonance" state [27].

IV. SUMMARY

The charged tetraquark states $[cu][\bar{c}\,\bar{d}]$ are systematically studied using the framework of the color flux-tube model with a four-body confinement potential. Our model calculation demonstrates that the charged charmonium-like states $Z_c^+(3900)$ or $Z_c^+(3885)$, $Z_c^+(3930)$, $Z_c^+(4020)$ or $Z_c^+(4025)$, $Z_1^+(4050)$, $Z_2^+(4250)$, and $Z_c^+(4200)$ can be uniquely identified as tetraquark states $[cu][\bar{c}\,\bar{d}]$ with the quantum numbers 1^3S_1 and 1^+, 2^3S_1 and 1^+, 1^5S_2 and 2^+, 1^3P_1 and 1^-, 1^5D_1 and 1^+, and 1^3D_1 and 1^+, respectively. The predicted lowest charged tetraquark state $[cu][\bar{c}\,\bar{d}]$ with 0^+ and 1^1S_0 has a mass of 3780 ± 10 MeV in the color flux-tube model. The model predictions would shed light on other possible charmonium-like charged states in the future at the BESIII, LHCb and Belle-II. Our calculation favors three-dimensional spatial structures, which is similar to a rugby ball: the higher the orbital angular momentum L, the more prolate the shape of the states. Those charged charmonium-like states may be the so-called "color confined, multiquark resonance." However, the two heavier charged states $Z_c^+(4430)$ and $Z_c^+(4475)$ cannot be described as tetraquark states $[cu][\bar{c}\,\bar{d}]$ in the current color flux-tube model.

The multibody color flux tube employs collective degrees of freedom whose dynamics play an important role in the formation and decay of those compact states. Like the colorful organic world because of chemical bonds, the multiquark hadron world may be equally diverse and rich due to the color flux-tube structure. The recently discovered charged state $Z_c^+(3900)$ and dibaryon d^* resonance state have given us a stimulating glance into the abundant multiquark hadronic world.

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