

Magnetic moments of $J^P = \frac{3}{2}^+$ decuplet baryons using effective quark masses in a chiral constituent quark model

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The magnetic moments of $J^P = \frac{3}{2}^+$ decuplet baryons have been calculated in the chiral constituent quark model (χ CQM) with explicit results for the contribution coming from the valence quark polarizations, sea quark polarizations, and their orbital angular momentum. Since the $J^P = \frac{3}{2}^+$ decuplet baryons have short lifetimes, the experimental information about them is limited. The χ CQM has important implications for chiral symmetry breaking as well as SU(3) symmetry breaking since it works in the region between the QCD confinement scale and the chiral symmetry breaking scale. The predictions in the model not only give a satisfactory fit when compared with the experimental data but also show improvement over the other models. The effect of the confinement on quark masses has also been discussed in detail and the results of χ CQM are found to improve further with the inclusion of effective quark masses.

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I. INTRODUCTION

In order to understand the internal structure of the hadrons in the nonperturbative regime of quantum chromodynamics (QCD), one of the main challenges is the measurement of static and electromagnetic properties of hadrons like masses, magnetic moments, etc., both theoretically and experimentally. Ever since the proton polarized structure function measurements in the deep inelastic scattering (DIS) experiments [1–4] provided the first evidence that the valence quarks of a proton carry only a small fraction of its spin, the charge, current and spin structure of the nucleon has been extensively studied in experiments measuring the electromagnetic form factors from the elastic scattering of electrons.

The magnetic moments of the $J^P = \frac{1}{2}^+$ octet baryons have been accurately measured [5]. Our information about the $J^P = \frac{3}{2}^+$ decuplet baryons, however, is limited because of the difficulty in measuring their properties experimentally on account of their short lifetimes. Among all the decuplet baryons in the hadron spectrum, the Ω^- hyperon is unique, as the naive SU(6) quark model describes it as a state with three strange quarks in a totally symmetric flavor-spin space. As the strange quarks decay via the weak interaction, the Ω^- baryon is significantly more stable than other members of $J^P = \frac{3}{2}^+$ decuplet baryons, which have at least one light quark. The magnetic moment of $\Omega^- = -2.02 \pm 0.05\mu_N$ has been measured with high

precision [5]. In spite of the considerable progress made over the past few years to determine the magnetic moments of other decuplet baryons, there is hardly any consensus regarding the mechanisms that can contribute to it, and additional refined data are needed to bridge the gap.

The magnetic moments of $J^P = \frac{3}{2}^+$ decuplet baryons have been calculated theoretically using different approaches. The first calculation used a SU(6) symmetric quark model (NQM) [6]. This work was further improved by considering the individual contributions of the quark magnetic moments, the SU(3) symmetry breaking effects, sea quark contributions, quark orbital momentum effects, relativistic effects [7–14]. Typically, these models invoke the additivity hypothesis where a baryon magnetic moment is given by the sum of its constituent quark magnetic moments. The issue regarding the magnetic moments is difficult to understand since the magnetic moments of baryons receive contributions not only from the magnetic moments carried by the valence quarks but also from various complicated effects, such as relativistic and exchange current effects, pion cloud contributions, effect of the confinement on quark masses, etc. In the absence of any consistent way to calculate these effects simultaneously, it is very difficult to know their relative contributions. Recently, a number of theoretical and computational investigations involving the magnetic moment of decuplet baryons have been carried out using the relativistic quark model (RQM) [8], QCD-based quark model (QCDQM) [9], effective mass scheme (EMS) [14],

light cone QCD sum rule (LCQSR) [15], QCD sum rule (QCDSR) [16], Skyrme model [17], chiral quark soliton model (CQSM) [18,19], chiral perturbation theory (χ PT) [20], lattice QCD (LQCD) [21,22], bag model (BM) [23], large N_c [24], heavy baryon chiral perturbation theory approach (HB χ PT) [25], etc. In addition, the electromagnetic properties of the baryons have been extensively studied in a chiral quark model with exchange currents (χ QMEC) which are necessary for constructing a gauge-invariant current [26].

One of the important models which finds application in the nonperturbative regime of QCD is the chiral constituent quark model (χ CQM) [27–29]. The underlying idea is based on the possibility that chiral symmetry breaking takes place at a distance scale much smaller than the confinement scale. The χ CQM uses the effective interaction Lagrangian approach of the strong interactions, where the effective degrees of freedom are the valence quarks and the internal Goldstone bosons (GBs), which are coupled to the valence quarks [11,29–32]. The χ CQM with spin-spin generated configuration mixing [33–36] is able to give the satisfactory explanation for the spin and flavor distribution functions including the strangeness content of the nucleon [30,31], weak vector and axial-vector form factors [37], magnetic moments of octet baryons, their transitions and Coleman-Glashow sum rule [12], magnetic moments of octet baryon resonances [38], magnetic moments of Λ resonances [39], charge radii and quadrupole moment [40], etc. The model is successfully extended to predict the important role played by the small intrinsic charm content in the nucleon spin in the SU(4) χ CQM and to calculate the magnetic moment and charge radii of charm baryons including their radiative decays [41]. In view of the above developments in the χ CQM, it becomes desirable to extend the model to calculate the magnetic moment of the $J^P = \frac{3}{2}^+$ decuplet baryons as the knowledge of the magnetic moments of decuplet baryons undoubtedly provide vital clues to the spin structure and the nonperturbative aspects of QCD.

The purpose of the present paper is to formulate in detail the magnetic moment of the $J^P = \frac{3}{2}^+$ decuplet baryons in the SU(3) framework of χ CQM with explicit contributions coming from the valence spin polarization, quark sea polarization, and its orbital angular momentum. In order to understand the implications of nonvalence quarks and to make our analysis more responsive, it would also be interesting to examine the effects of chiral symmetry breaking and SU(3) symmetry breaking parameters on the magnetic moment. Further, we would also like to study the implications of variation of the quark masses, arising due to confinement of quarks on the magnetic moments [14,42].

The organization of the paper is as follows. To facilitate discussion, in Sec. II, chiral symmetry breaking and SU(3) symmetry breaking in the context of χ CQM are revisited

with an emphasis on the importance of the sea quarks. In Sec. III, we present the essential details of the spin structure to obtain the explicit contributions coming from the valence quark polarizations, sea quark polarizations, and their orbital angular momentum for the magnetic moments of the $J^P = \frac{3}{2}^+$ decuplet baryons. The input parameters, numerical results and their comparison with available data have been discussed in Sec. V.

II. CHIRAL CONSTITUENT QUARK MODEL (χ CQM)

The χ CQM was introduced by Weinberg and further developed by Manohar and Georgi [27] with the basic idea that the set of internal Goldstone bosons (GBs) couple directly to the valence quarks in the interior of the hadron but only at a distance scale where perturbative QCD is not applicable.

The dynamics of light quarks (u , d , and s) and gluons can be described by the QCD Lagrangian,

$$\mathcal{L} = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} + i\bar{\psi}_R \not{D}\psi_R + i\bar{\psi}_L \not{D}\psi_L - \bar{\psi}_R M \psi_L - \bar{\psi}_L M \psi_R, \quad (1)$$

where $G_{\mu\nu}^a$ is the gluonic gauge field strength tensor, D^μ is the gauge-covariant derivative, M is the quark mass matrix and ψ_L and ψ_R are the left- and right-handed quark fields, respectively,

$$\Psi_L \equiv \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} \quad \text{and} \quad \Psi_R \equiv \begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix}. \quad (2)$$

Since the mass terms change sign as $\psi_R \rightarrow \psi_R$ and $\psi_L \rightarrow -\psi_L$ under the chiral transformation ($\psi \rightarrow \gamma^5 \psi$), the Lagrangian in Eq. (1) no longer remains invariant. In case the mass terms in the QCD Lagrangian are neglected, the Lagrangian will have global chiral symmetry of the $SU(3)_L \times SU(3)_R$ group. Since the spectrum of hadrons in the known sector does not display parity doublets, the chiral symmetry is believed to be spontaneously broken around a scale of 1 GeV as

$$SU(3)_L \times SU(3)_R \rightarrow SU(3)_{L+R}. \quad (3)$$

As a consequence, there exists a set of massless GBs, identified with the observed (π , K , η) mesons. Within the region of the QCD confinement scale ($\Lambda_{\text{QCD}} \approx 0.1\text{--}0.3$ GeV) and the chiral symmetry breaking scale $\Lambda_{\chi SB}$, the constituent quarks, the octet of GBs (π , K , η mesons), and the *weakly* interacting gluons are the appropriate degrees of freedom.

The effective interaction Lagrangian in this region can be expressed as

$$\mathcal{L}_{\text{int}} = \bar{\psi}(i\mathcal{D} + \mathcal{M})\psi + ig_A \bar{\psi} \mathcal{A} \gamma^5 \psi + \dots, \quad (4)$$

where g_A is the axial-vector coupling constant. The gluonic degrees of freedom can be neglected owing to the small effect in the effective quark model at the low-energy scale. The vector and axial-vector currents V_μ and A_μ are defined as

$$\begin{pmatrix} V_\mu \\ A_\mu \end{pmatrix} = \frac{1}{2} (\xi^\dagger \partial_\mu \xi \pm \xi \partial_\mu \xi^\dagger), \quad (5)$$

where $\xi = \exp(2i\Phi/f_\pi)$, f_π is the pseudoscalar pion decay constant (≈ 93 MeV), and Φ is the field describing the dynamics of GBs as

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} & \pi^+ & \alpha K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} & \alpha K^0 \\ \alpha K^- & \alpha \bar{K}^0 & -\beta \frac{2\eta}{\sqrt{6}} \end{pmatrix}. \quad (6)$$

Expanding V_μ and A_μ in the powers of Φ/f_π , we get

$$V_\mu = 0 + O((\Phi/f_\pi)^2), \quad (7)$$

$$A_\mu = \frac{i}{f_\pi} \partial_\mu \Phi + O((\Phi/f_\pi)^2). \quad (8)$$

The effective interaction Lagrangian between GBs and quarks from Eq. (4) in the leading order can now be expressed as

$$\mathcal{L}_{\text{int}} = -\frac{g_A}{f_\pi} \bar{\psi} \partial_\mu \Phi \gamma^\mu \gamma^5 \psi, \quad (9)$$

which can be reduced to

$$\mathcal{L}_{\text{int}} \approx i \sum_{q=u,d,s} \frac{m_q + m_{q'}}{f_\pi} \bar{q}' \Phi \gamma^5 q = i \sum_{q=u,d,s} c_8 \bar{q}' \Phi \gamma^5 q, \quad (10)$$

using the Dirac equation $(i\gamma^\mu \partial_\mu - m_q)q = 0$. Here, $c_8 (= \frac{m_q + m_{q'}}{f_\pi})$ is the coupling constant for the octet of GBs and $m_q (m_{q'})$ is the quark mass parameter. The Lagrangian of the quark-GB interaction suppressing all the space-time structure to the lowest order can now be expressed as

$$\mathcal{L}_{\text{int}} = c_8 \bar{\psi} \Phi \psi. \quad (11)$$

The QCD Lagrangian is also invariant under the axial $U(1)$ symmetry, which would imply the existence of the ninth GB. This breaking symmetry picks the η' as the ninth GB. The effective Lagrangian describing interaction between quarks and a nonet of GBs, consisting of an octet and a singlet, can now be expressed as

$$\begin{aligned} \mathcal{L}_{\text{int}} &= c_8 \bar{\psi} \Phi \psi + c_1 \bar{\psi} \frac{\eta'}{\sqrt{3}} \psi = c_8 \bar{\psi} \left(\Phi + \zeta \frac{\eta'}{\sqrt{3}} I \right) \psi \\ &= c_8 \bar{\psi} (\Phi') \psi, \end{aligned} \quad (12)$$

where $\zeta = c_1/c_8$, c_1 is the coupling constant for the singlet GB and I is the 3×3 identity matrix.

The fluctuation process describing the effective Lagrangian is

$$q^\pm \rightarrow \text{GB} + q'^\mp \rightarrow (q\bar{q}') + q'^\mp, \quad (13)$$

where $q\bar{q}' + q'$ constitute the sea quarks [11,29,31]. The GB field can be expressed in terms of the GBs and their transition probabilities as

$$\Phi' = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} & \pi^+ & \alpha K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} & \alpha K^0 \\ \alpha K^- & \alpha \bar{K}^0 & -\beta \frac{2\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} \end{pmatrix}. \quad (14)$$

The transition probability of chiral fluctuation $u(d) \rightarrow d(u) + \pi^{+(-)}$, given in terms of the coupling constant for the octet GBs $|c_8|^2$, is defined as a and is introduced by considering nondegenerate quark masses $M_s > M_{u,d}$. In terms of a , the probabilities of transitions of $u(d) \rightarrow s + K^{+(0)}$, $u(d, s) \rightarrow u(d, s) + \eta$, and $u(d, s) \rightarrow u(d, s) + \eta'$ are given as $\alpha^2 a$, $\beta^2 a$ and $\zeta^2 a$, respectively [11,29]. The parameters α and β are introduced by considering nondegenerate GB masses $M_K, M_\eta > M_\pi$ and the parameter ζ is introduced by considering $M_{\eta'} > M_K, M_\eta$.

III. MAGNETIC MOMENTS

The magnetic moment of a given baryon in the χ CQM receives contribution from the spin of the valence quarks, spin of the sea quarks and the orbital angular motion of the sea quarks. The total magnetic moment is expressed as

$$\mu(B)_{\text{total}} = \mu(B)_V + \mu(B)_S + \mu(B)_O, \quad (15)$$

where $\mu(B)_V$ and $\mu(B)_S$ are the magnetic moment contributions of the valence quarks and the sea quarks, respectively, coming from their spin polarizations, whereas $\mu(B)_O$ is the magnetic moment contribution due to the rotational motion of the two bodies constituting the sea quarks (q') and GB and referred to as the orbital angular momentum contribution of the quark sea [29].

In terms of quark magnetic moments and spin polarizations, the valence spin ($\mu(B)_V$), sea spin ($\mu(B)_S$), and sea orbital ($\mu(B)_O$) contributions can be defined as

$$\mu(B)_V = \sum_{q=u,d,s} \Delta q_{\text{val}} \mu_q, \quad (16)$$

$$\mu(B)_S = \sum_{q=u,d,s} \Delta q_{\text{sea}} \mu_q, \quad (17)$$

$$\mu(B)_O = \sum_{q=u,d,s} \Delta q_{\text{val}} \mu(q_+ \rightarrow), \quad (18)$$

where $\mu_q = \frac{e_q}{2M_q}$ ($q = u, d, s$) is the quark magnetic moment in the units of μ_N (nuclear magneton), Δq_{val} and Δq_{sea} are the valence and sea quark spin polarizations, respectively, $\mu(q_+ \rightarrow)$ is the orbital moment for any chiral fluctuation, and e_q and M_q are the electric charge and the mass, respectively, for the quark q .

The valence and sea quark spin contributions for a given baryon can be calculated from the spin structure of the baryons. Following references [11,29,31], the quark spin polarization can be defined as

$$\Delta q = q^+ - q^-, \quad (19)$$

where q^\pm can be calculated from the spin structure of a baryon,

$$\hat{B} \equiv \langle B | \mathcal{N} | B \rangle = \langle B | q^+ q^- | B \rangle. \quad (20)$$

Here $|B\rangle$ is the baryon wave function and $\mathcal{N} = q^+ q^-$ is the number operator measuring the sum of the quark numbers with spin up or down, for example,

$$\begin{aligned} q^+ q^- &= \sum_{q=u,d,s} (n_{q^+} q^+ + n_{q^-} q^-) \\ &= n_{u^+} u^+ + n_{u^-} u^- + n_{d^+} d^+ + n_{d^-} d^- + n_{s^+} s^+ \\ &\quad + n_{s^-} s^-, \end{aligned} \quad (21)$$

with the coefficients of the q^\pm giving the number of q^\pm quarks.

The valence quark spin polarizations ($\Delta q_{\text{val}} = n_{q^+} - n_{q^-}$) for a given baryon can be calculated using the SU(6) spin-flavor wave functions $|B\rangle$ in Eq. (20).

The quark sea spin polarizations (Δq_{sea}) coming from the fluctuation process in Eq. (13) can be calculated by

substituting for every valence quark

$$q^\pm \rightarrow \sum P_q q^\pm + |\psi(q^\pm)|^2, \quad (22)$$

where the transition probability of the emission of a GB from any of the q quark ($\sum P_q$) and the transition probability of the q^\pm quark ($|\psi(q^\pm)|^2$) can be calculated from the Lagrangian. They are expressed as

$$\sum P_u = -a \left(\frac{9 + \beta^2 + 2\zeta^2}{6} + \alpha^2 \right) \quad \text{and}$$

$$|\psi(u^\pm)|^2 = \frac{a}{6} (3 + \beta^2 + 2\zeta^2) u^\mp + a d^\mp + a \alpha^2 s^\mp,$$

$$\sum P_d = -a \left(\frac{9 + \beta^2 + 2\zeta^2}{6} + \alpha^2 \right) \quad \text{and}$$

$$|\psi(d^\pm)|^2 = a u^\mp + \frac{a}{6} (3 + \beta^2 + 2\zeta^2) d^\mp + a \alpha^2 s^\mp,$$

$$\sum P_s = -a \left(\frac{2\beta^2 + \zeta^2}{3} + 2\alpha^2 \right) \quad \text{and}$$

$$|\psi(s^\pm)|^2 = a \alpha^2 u^\mp + a \alpha^2 d^\mp + \frac{a}{3} (2\beta^2 + \zeta^2) s^\mp.$$

The contribution of the angular momentum of the sea quarks to the magnetic moment of a given quark is

$$\mu(q^+ \rightarrow q'^-) = \frac{e_{q'}}{2M_q} \langle l_q \rangle + \frac{e_q - e_{q'}}{2M_{\text{GB}}} \langle l_{\text{GB}} \rangle, \quad (23)$$

where

$$\langle l_q \rangle = \frac{M_{\text{GB}}}{M_q + M_{\text{GB}}} \quad \text{and} \quad \langle l_{\text{GB}} \rangle = \frac{M_q}{M_q + M_{\text{GB}}}, \quad (24)$$

$\langle l_q, l_{\text{GB}} \rangle$ and (M_q, M_{GB}) are the orbital angular momenta and masses of quark and GB, respectively. The orbital moment of each process is then multiplied by the probability for such a process to take place to yield the magnetic moment due to all the transitions starting with a given valence quark, for example,

$$[\mu(u^\pm(d^\pm) \rightarrow)] = \pm a \left[\mu(u^+(d^+) \rightarrow d^-(u^-)) + \alpha^2 \mu(u^+(d^+) \rightarrow s^-) + \left(\frac{1}{2} + \frac{1}{6} \beta^2 + \frac{1}{3} \zeta^2 \right) \mu(u^+(d^+) \rightarrow u^-(d^-)) \right], \quad (25)$$

$$[\mu(s^\pm \rightarrow)] = \pm a \left[\alpha^2 \mu(s^+ \rightarrow u^-) + \alpha^2 \mu(s^+ \rightarrow d^-) + \left(\frac{2}{3} \beta^2 + \frac{1}{3} \zeta^2 \right) \mu(s^+ \rightarrow s^-) \right]. \quad (26)$$

The above equations can easily be generalized by including the coupling breaking and mass breaking terms, for example, in terms of the coupling breaking parameters a, α, β and ζ as well as the masses of GBs M_π, M_K and M_η . The orbital moments of u, d and s quarks, respectively, are

$$[\mu(u^+ \rightarrow)] = a \left[\frac{3M_u^2}{2M_\pi(M_u + M_\pi)} - \frac{\alpha^2(M_K^2 - 3M_u^2)}{2M_K(M_u + M_K)} + \frac{\beta^2 M_\eta}{6(M_u + M_\eta)} + \frac{\zeta^2 M_{\eta'}}{3(M_u + M_{\eta'})} \right] \mu_u, \quad (27)$$

$$[\mu(d^+ \rightarrow)] = -2a \left[\frac{3(M_\pi^2 - 2M_d^2)}{4M_\pi(M_d + M_\pi)} - \frac{\alpha^2 M_K}{2(M_d + M_K)} - \frac{\beta^2 M_\eta}{12(M_d + M_\eta)} - \frac{\zeta^2 M_{\eta'}}{6(M_d + M_{\eta'})} \right] \mu_d, \quad (28)$$

$$[\mu(s^+ \rightarrow)] = -2a \left[\frac{\alpha^2(M_K^2 - 3M_s^2)}{2M_K(M_s + M_K)} - \frac{\beta^2 M_\eta}{3(M_s + M_\eta)} - \frac{\zeta^2 M_{\eta'}}{6(M_s + M_{\eta'})} \right] \mu_s. \quad (29)$$

The orbital contribution to the magnetic moment of the baryon of the type $B(q_1 q_2 q_3)$ is given as

$$\mu(B)_O = \Delta q_{1\text{val}}[\mu(q_1^+ \rightarrow)] + \Delta q_{2\text{val}}[\mu(q_2^+ \rightarrow)] + \Delta q_{3\text{val}}[\mu(q_3^+ \rightarrow)]. \quad (30)$$

We now discuss in detail the valence quark spin, sea quark spin and sea quark orbital contributions to the magnetic moment of $J^P = \frac{3}{2}^+$ decuplet baryons (B^*). To calculate $\mu(B^*)_V$, we need to calculate the valence spin polarizations $\Delta q_{\text{val}}^{B^*}$ from the valence spin structure in a totally symmetric flavor-spin space from Eq. (20),

$$\widehat{B}^* \equiv \left\langle B^* \left(J^P = \frac{3}{2}^+ \right) | \mathcal{N} | B^* \left(J^P = \frac{3}{2}^+ \right) \right\rangle. \quad (31)$$

The spin structure for the $J^P = \frac{3}{2}^+$ decuplet baryons is expressed as

$$\begin{aligned} \Delta^{++} &= 3u^+ + 0u^- + 0d^+ + 0d^- + 0s^+ + 0s^-, \\ \Delta^+ &= 2u^+ + 0u^- + 1d^+ + 0d^- + 0s^+ + 0s^-, \\ \Delta^0 &= 1u^+ + 0u^- + 2d^+ + 0d^- + 0s^+ + 0s^-, \\ \Delta^- &= 0u^+ + 0u^- + 3d^+ + 0d^- + 0s^+ + 0s^-, \\ \Sigma^{*+} &= 2u^+ + 0u^- + 0d^+ + 0d^- + 1s^+ + 0s^-, \\ \Sigma^{*0} &= 1u^+ + 0u^- + 1d^+ + 0d^- + 1s^+ + 0s^-, \\ \Sigma^{*-} &= 0u^+ + 0u^- + 2d^+ + 0d^- + 1s^+ + 0s^-, \\ \Xi^{*0} &= 1u^+ + 0u^- + 0d^+ + 0d^- + 2s^+ + 0s^-, \\ \Xi^{*-} &= 0u^+ + 0u^- + 1d^+ + 0d^- + 2s^+ + 0s^-, \\ \Omega^- &= 0u^+ + 0u^- + 0d^+ + 0d^- + 3s^+ + 0s^-. \end{aligned} \quad (32)$$

The resulting spin polarizations give the valence contribution to the magnetic moment obtained by substituting these in Eq. (16). The results have been presented in Table I.

The sea quark polarizations which contribute to the magnetic moment of $J^P = \frac{3}{2}^+$ decuplet baryons can be calculated by substituting Eq. (22) for every valence quark in Eq. (32). Consequently, the magnetic moment contributions of the sea quarks $\mu(B^*)_S$ can be calculated from Eq. (17) and the results have been presented in Table II.

The orbital contribution to the total magnetic moment of the $J^P = \frac{3}{2}^+$ decuplet baryons, as given by Eq. (30), is expressed as

$$\begin{aligned} \mu(\Delta^{++})_O &= 3\mu(u^+ \rightarrow), \\ \mu(\Delta^+)_O &= 2\mu(u^+ \rightarrow) + \mu(d^+ \rightarrow), \\ \mu(\Delta^0)_O &= \mu(u^+ \rightarrow) + 2\mu(d^+ \rightarrow), \\ \mu(\Delta^-)_O &= 3\mu(d^+ \rightarrow), \\ \mu(\Sigma^{*+})_O &= 2\mu(u^+ \rightarrow) + \mu(s^+ \rightarrow), \\ \mu(\Sigma^{*0})_O &= \mu(u^+ \rightarrow) + \mu(d^+ \rightarrow) + \mu(s^+ \rightarrow), \\ \mu(\Sigma^{*-})_O &= 2\mu(d^+ \rightarrow) + \mu(s^+ \rightarrow), \\ \mu(\Xi^{*0})_O &= \mu(u^+ \rightarrow) + 2\mu(s^+ \rightarrow), \\ \mu(\Xi^{*-})_O &= \mu(d^+ \rightarrow) + 2\mu(s^+ \rightarrow), \\ \mu(\Omega^-)_O &= 3\mu(s^+ \rightarrow). \end{aligned} \quad (33)$$

Using Eqs. (27), (28) and (29), the orbital contribution to the total magnetic moment of the $J^P = \frac{3}{2}^+$ decuplet baryons can be calculated.

TABLE I. Valence contribution to the magnetic moment $\mu(B^*)_V$ for the $J^P = \frac{3}{2}^+$ hyperons.

$B^*(J^P = \frac{3}{2}^+)$	$\mu(B^*)_V$
$\Delta^{++}(uuu)$	$3\mu_u$
$\Delta^+(uud)$	$2\mu_u + \mu_d$
$\Delta^0(udd)$	$\mu_u + 2\mu_d$
$\Delta^-(ddd)$	$3\mu_d$
$\Sigma^{*+}(uus)$	$2\mu_u + \mu_s$
$\Sigma^{*0}(uds)$	$\mu_u + \mu_d + \mu_s$
$\Sigma^{*-}(dds)$	$2\mu_d + \mu_s$
$\Xi^{*0}(uss)$	$\mu_u + 2\mu_s$
$\Xi^{*-}(dss)$	$\mu_d + 2\mu_s$
$\Omega^-(sss)$	$3\mu_s$

TABLE II. Sea contribution to the magnetic moment $\mu(B^*)_S$ for the $J^P = \frac{3}{2}^+$ decuplet baryons.

$B^*(J^P = \frac{3}{2}^+)$	$\mu(B^*)_S$
$\Delta^{++}(uuu)$	$-a[(6 + 3\alpha^2 + \beta^2 + 2\zeta^2)\mu_u + 3\mu_d + 3\alpha^2\mu_s]$
$\Delta^+(uud)$	$-a[(5 + 2\alpha^2 + \frac{2}{3}\beta^2 + \frac{4}{3}\zeta^2)\mu_u + (4 + \alpha^2 + \frac{1}{3}\beta^2 + \frac{2}{3}\zeta^2)\mu_d + 3\alpha^2\mu_s]$
$\Delta^o(udd)$	$-a[(4 + \alpha^2 + \frac{1}{3}\beta^2 + \frac{2}{3}\zeta^2)\mu_u + (5 + 2\alpha^2 + \frac{2}{3}\beta^2 + \frac{4}{3}\zeta^2)\mu_d + 3\alpha^2\mu_s]$
$\Delta^-(ddd)$	$-a[3\mu_u + (6 + 3\alpha^2 + \beta^2 + 2\zeta^2)\mu_d + 3\alpha^2\mu_s]$
$\Sigma^{*+}(uus)$	$-a[(4 + 3\alpha^2 + \frac{2}{3}\beta^2 + \frac{4}{3}\zeta^2)\mu_u + (\alpha^2 + 2)\mu_d + 2(2\alpha^2 + \frac{2}{3}\beta^2 + \frac{1}{3}\zeta^2)\mu_s]$
$\Sigma^{*o}(uds)$	$-a[(3 + 2\alpha^2 + \frac{1}{3}\beta^2 + \frac{2}{3}\zeta^2)\mu_u + (3 + 2\alpha^2 + \frac{1}{3}\beta^2 + \frac{2}{3}\zeta^2)\mu_d + 2(2\alpha^2 + \frac{2}{3}\beta^2 + \frac{1}{3}\zeta^2)\mu_s]$
$\Sigma^{*-}(dds)$	$-a[(\alpha^2 + 2)\mu_u + (4 + 3\alpha^2 + \frac{2}{3}\beta^2 + \frac{4}{3}\zeta^2)\mu_d + 2(2\alpha^2 + \frac{2}{3}\beta^2 + \frac{1}{3}\zeta^2)\mu_s]$
$\Xi^{*o}(uss)$	$-a[(2 + 3\alpha^2 + \frac{1}{3}\beta^2 + \frac{2}{3}\zeta^2)\mu_u + (2\alpha^2 + 1)\mu_d + (5\alpha^2 + \frac{8}{3}\beta^2 + \frac{4}{3}\zeta^2)\mu_s]$
$\Xi^{*-}(dss)$	$-a[(2\alpha^2 + 1)\mu_u + (2 + 3\alpha^2 + \frac{1}{3}\beta^2 + \frac{2}{3}\zeta^2)\mu_d + (5\alpha^2 + \frac{8}{3}\beta^2 + \frac{4}{3}\zeta^2)\mu_s]$
$\Omega^-(sss)$	$-3a[\alpha^2\mu_u + \alpha^2\mu_d + 2(\alpha^2 + \frac{2}{3}\beta^2 + \frac{1}{3}\zeta^2)\mu_s]$

IV. EFFECTIVE QUARK MASSES

The basic assumptions of χ CQM suggest that the constituent quarks are supposed to have only Dirac magnetic moments governed by the respective quark masses. Since we do not have any definite guidelines for the constituent quark masses, we can use the most widely accepted values of quark masses in hadron spectroscopy. In order to study the effect of quark confinement on the magnetic moments, the effective quark masses can be considered [14,42]. In conformity with additivity assumption, the simplest way to incorporate this adjustment is to first express M_q in the magnetic moment operator in terms of M_B , the mass of the baryon obtained additively from the quark masses, which then is replaced by $M_B + \Delta M$, with ΔM being the mass difference between the experimental value and M_B . This leads to the following modification in the quark magnetic moments:

$$\begin{aligned}\mu_u^{\text{eff}} &= 2[1 - (\Delta M/M_B)]\mu_N, \\ \mu_d^{\text{eff}} &= -[1 - (\Delta M/M_B)]\mu_N, \\ \mu_s^{\text{eff}} &= -M_u/M_s[1 - (\Delta M/M_B)]\mu_N.\end{aligned}\quad (34)$$

In addition to this, various sum rules derived from the spin-spin interactions for different baryons [34–36] can be used to fix M_s , for example, $(\Sigma^* - \Sigma)/(\Delta - N) = M_u/M_s$ and $(\Xi^* - \Xi)/(\Delta - N) = M_u/M_s$. The baryon magnetic moments calculated after incorporating this effect would be referred to as baryon magnetic moments with effective quark masses.

V. RESULTS AND DISCUSSION

The explicit numerical values of the valence, sea, and orbital contributions to the magnetic moment of $J^P = \frac{3}{2}^+$ decuplet baryons in χ CQM can be calculated. The magnetic moment calculations in χ CQM with SU(3) broken symmetry involve the symmetry breaking parameters a , $a\alpha^2$,

$a\beta^2$, $a\zeta^2$, representing, respectively, the probabilities of fluctuations of a constituent quark into pions, K , η , η' . These parameters provide the basis to understand the extent to which the sea quarks and their orbital angular momentum contribute to the structure of the baryon. The hierarchy for the probabilities, which scale as $\frac{1}{M_q^2}$, can be obtained as

$$a > a\alpha^2 \geq a\beta^2 > a\zeta^2. \quad (35)$$

Since the parameters cannot be fixed independently, therefore, we have carried out a broader analysis to find the ranges of the χ CQM parameters from experimentally well-known quantities pertaining to the spin polarization functions and quark distribution functions. The range of the coupling breaking parameter a can be easily found by considering the expressions of the quark spin polarization functions by giving the full variation of parameters α , β and ζ [31]. We obtain $0.10 \lesssim a \lesssim 0.14$. The range of the parameter ζ can be found from the latest experimental measurement of \bar{u}/\bar{d} [43] which involves only β and ζ . Using the possible range of β , i.e. $0 < \beta < 1$ one finds $-0.70 \lesssim \zeta \lesssim -0.10$. The range of β can be found by using the $\bar{u} - \bar{d}$ asymmetry representing the violation of Gottfried sum rule [44] and expressed as $\bar{u} - \bar{d} = \frac{a}{3}(2\zeta + \beta - 3)$. Using the above found ranges of a and ζ as well as the latest measurement of $\bar{u} - \bar{d}$ asymmetry [43], β falls in the range $0.2 \lesssim \beta \lesssim 0.7$. Similarly, the range of α can be found by considering the flavor nonsinglet component $\Delta_3 (= \Delta u - \Delta d)$, and it comes out to be $0.2 \lesssim \alpha \lesssim 0.5$.

In addition to the parameters of χ CQM as discussed above, the orbital angular momentum contributions are characterized by the quark and GB masses. For evaluating their contribution, we have used their on-shell mass values in accordance with several other similar calculations. In particular, for the constituent quark masses u , d , and s , we have used their widely accepted values in hadron spectroscopy $M_u = M_d = 330$ MeV, $M_s = 510$ MeV

TABLE III. Magnetic moments in units of μ_N for the $J^P = \frac{3}{2}^+$ decuplet baryons.

Baryons	Data	χ CQM				χ CQM with μ_q^{eff}			
		$\mu(B^*)_{\text{V}}$	$\mu(B^*)_{\text{S}}$	$\mu(B^*)_{\text{O}}$	$\mu(B^*)$	$\mu(B^*)_{\text{V}}$	$\mu(B^*)_{\text{S}}$	$\mu(B^*)_{\text{O}}$	$\mu(B^*)$
$\mu(\Delta^{++})$	$3.7 < \mu(\Delta^{++}) < 7.5$ [5]	6	-1.32 ± 0.15	1.07 ± 0.12	5.82 ± 0.08	6.06	-1.31 ± 0.15	1.08 ± 0.12	5.82 ± 0.08
$\mu(\Delta^+)$	$2.7^{+1.0}_{-1.3} \pm 1.5 \pm 3$ [47]	3	-0.83 ± 0.09	0.42 ± 0.06	2.63 ± 0.06	3.03	-0.82 ± 0.09	0.42 ± 0.06	2.63 ± 0.06
$\mu(\Delta^0)$...	0	-0.33 ± 0.04	-0.23 ± 0.03	-0.56 ± 0.09	0	-0.33 ± 0.05	-0.23 ± 0.03	-0.55 ± 0.09
$\mu(\Delta^-)$...	-3	0.16 ± 0.05	-0.85 ± 0.11	-3.75 ± 0.08	-3.03	0.16 ± 0.05	-0.87 ± 0.09	-3.75 ± 0.08
$\mu(\Sigma^{*+})$...	3.37	-0.85 ± 0.08	0.68 ± 0.06	3.25 ± 0.05	3.24	-0.81 ± 0.06	0.658 ± 0.07	3.09 ± 0.03
$\mu(\Sigma^{*0})$...	0.37	-0.35 ± 0.03	0.018 ± 0.004	0.05 ± 0.03	0.33	-0.33 ± 0.03	0.018 ± 0.004	0.018 ± 0.03
$\mu(\Sigma^{*-})$...	-2.63	0.14 ± 0.04	-0.62 ± 0.07	-3.14 ± 0.06	-2.58	0.14 ± 0.04	-0.62 ± 0.07	-3.07 ± 0.06
$\mu(\Xi^{*0})$...	0.74	-0.36 ± 0.04	0.26 ± 0.04	0.65 ± 0.03	0.52	-0.32 ± 0.04	0.26 ± 0.04	0.46 ± 0.03
$\mu(\Xi^{*-})$...	-2.26	0.13 ± 0.04	-0.38 ± 0.06	-2.53 ± 0.06	-2.30	0.13 ± 0.05	-0.38 ± 0.06	-2.55 ± 0.05
$\mu(\Omega^-)$	-2.02 ± 0.05 [5]	-1.89	0.12 ± 0.04	-0.14 ± 0.07	-1.92 ± 0.07	-1.98	0.13 ± 0.05	-0.14 ± 0.07	-2.08 ± 0.08

[29,34,45]. In addition, the effect of the confinement on quark masses can be added by taking the effective quark magnetic moments μ_q^{eff} as discussed in the previous section.

This model has already been applied to calculate the magnetic moments of the octet baryons [12] where experimental data are available for all the cases. It is interesting to observe that our results for the magnetic moments of p , Σ^+ , Ξ^0 , and Λ give a perfect fit to the experimental values [5] whereas for all other octet baryons our predictions are within 10% of the observed values. Besides this, we have also been able to get an excellent fit to the violation of the Coleman-Glashow sum rule [46]. The fit becomes all the more impressive when it is realized that none of the magnetic moments are used as inputs, and the violation of Coleman-Glashow sum rule can be described without resorting to additional parameters. In all the cases, the contribution of the quark sea and its orbital angular momentum is quite significant when compared with the valence contribution.

Using the inputs discussed above and performing a full scan of the χ CQM parameters at 1σ C.L., in Table III we have presented the χ CQM results of the $J^P = \frac{3}{2}^+$ decuplet baryons magnetic moments. In order to study closely the role of confinement on quark masses on the magnetic moments, in the table we have also presented the χ CQM results with the effective quark magnetic moments μ_q^{eff} . One can immediately see that the total contribution to the magnetic moment is coming from different sources with similar and opposite signs; for example, the orbital is contributing with the same sign as the valence part, whereas the sea is contributing with opposite sign. For example, in the case of $\mu(\Delta^-)$ and $\mu(\Sigma^-)$, because the orbital part dominates over the sea quark polarization, the magnetic moments are higher as compared to just the valence contributions. On the other hand, in the case of $\mu(\Delta^+)$ and $\mu(\Sigma^+)$, the sea quark polarization dominates over the orbital part as a consequence of which the magnetic moment contribution is more or less the same as that of

the valence contribution. In general, one can find that whenever there is an excess of d quarks, the orbital part dominates, whereas when we have an excess of u quarks, the sea quark polarization dominates. A measurement of these magnetic moments, therefore, would have important implications for the χ CQM. The sea and orbital contributions, in a very interesting manner, add on to the valence contributions leading to better agreement with data. This clearly suggests that the sea quarks and their orbital contributions could perhaps provide the dominant dynamics of the constituents in the nonperturbative regime of QCD on which further corrections could be evaluated.

Since the experimental data are available for Δ^{++} , Δ^+ and Ω^- , it would be interesting to compare these with the χ CQM results as well as with the results of other models. From the table, it is clearly evident that a very good agreement pertaining to the case of $\mu(\Delta^+)$ is obtained. The result for $\mu(\Delta^{++})$ also lies very well within the available range. In both the cases, the sea and orbital contributions are quite significant. They cancel in the right direction and with the right magnitude to give the total magnetic moments. The magnetic moment of Ω^- agrees with the experimentally observed value -2.02 ± 0.05 [5]. Since there is an excess of strange quarks in the valence structure of Ω^- , the contribution of the quark sea and its orbital angular momentum is almost negligible as compared to the valence contribution. This is due to the fact that the strange contribution to the magnetic moment is almost an order of magnitude smaller than the up and down quarks thus leading to a very small contribution from the heavy quarks when compared with the contribution coming from the light quarks. This becomes more clear when we study the implications of SU(3) symmetry breaking. When we carry out the calculations in SU(3) symmetry with $\alpha = \beta = -\zeta = 1$, we obtain $\mu(\Omega^-)_{\text{V}} = -1.94$, $\mu(\Omega^-)_{\text{S}} = 0.60$ and $\mu(\Omega^-)_{\text{O}} = -0.20$ giving $\mu(\Omega^-)_{\text{total}} = -1.54$ which is even worse than just the valence contribution. The result with SU(3) symmetry is also clearly in disagreement with the experimentally observed value -2.02 ± 0.05 [5].

TABLE IV. Phenomenological results of some other theoretical approaches for magnetic moments of the $J^P = \frac{3}{2}^+$ decuplet baryons.

Other models	$\mu(\Delta^{++})$	$\mu(\Delta^+)$	$\mu(\Delta^0)$	$\mu(\Delta^-)$	$\mu(\Sigma^{*+})$	$\mu(\Sigma^{*0})$	$\mu(\Sigma^{*-})$	$\mu(\Xi^{*0})$	$\mu(\Xi^{*-})$	$\mu(\Omega^-)$
NQM [6]	5.56	2.73	-0.09	-2.92	3.09	0.27	-2.56	-0.63	-2.2	-1.84
RQM [8]	4.76	2.38	0.00	-2.38	1.82	-0.27	-2.36	-0.60	-2.41	-2.48
QCDQM [9]	5.689	2.778	-0.134	-3.045	2.933	0.137	-2.659	0.424	-2.307	-1.970
EMS [10]	3.67±0.07	1.83±0.04	0±0	-1.83±0.04	1.89±0.04	0±0	-1.89±0.04	0±0	-1.95±0.05	-2.02±0.05
χ QM [11]	5.30	2.58	-0.13	-2.85	2.88	0.17	-2.55	0.47	-2.25	-1.95
EMS [14]	4.56	2.28	0	-2.28	2.56	0.23	-2.10	0.48	-1.90	-1.67
LCQSR [15]	4.4±0.8	2.2±0.4	0.0	-2.2±0.4	2.7±0.6	0.20±0.05	-2.28±0.5	0.40±0.08	-2.0±0.4	-1.65±0.35
QCDSR [16]	4.39±1.00	2.19±0.50	0.00	-2.19±0.50	2.13±0.82	0.32±0.15	-1.66±0.73	-0.69±0.29	-1.51±0.52	-1.49±0.45
CQSM [18]	4.85	2.35	-0.14	-2.63	2.47	-0.02	-2.52	0.09	-2.40	-2.29
CQSM [19]	4.73	2.19	-0.35	-2.90	2.52	-0.08	-2.69	0.19	-2.48	-2.27
χ PT [20]	5.390	2.383	-0.625	-3.632	2.519	-0.303	-3.126	0.149	-2.596	-2.042
LQCD [21]	4.91±0.61	2.46±0.31	0.00	-2.46±0.31	2.55±0.26	0.27±0.05	-2.02±0.18	0.46±0.07	-1.68±0.12	-1.40±0.10
LQCD [22]	5.24(18)	0.97(65)	-0.035(2)	-2.98(19)	1.27(6)	0.33(5)	-1.88(4)	0.16(4)	-0.62(1)	...
CBM [23]	4.52	2.12	-0.29	-2.69	2.63	0.08	-2.48	0.44	-2.27	-2.06
Large N_c [24]	5.9(4)	2.9(2)	...	-2.9(2)	3.3(2)	0.3(1)	-2.8(3)	0.65(20)	-2.30(15)	-1.94
HB χ PT [25]	4.0(4)	2.1(2)	-0.17(4)	-2.25(19)	2.0(2)	-0.07(2)	-2.2(2)	0.10(4)	-2.0(2)	-1.94
χ QMEC [26]	6.93	3.47	0.00	-3.47	4.12	0.53	-3.06	1.10	-2.61	-2.13
This work: χ CQM	5.82±0.08	2.63±0.06	-0.56±0.09	-3.75±0.08	3.25±0.05	0.05±0.03	-3.14±0.06	0.65±0.03	-2.53±0.06	-1.92±0.07
χ CQM with μ_q^{eff}	5.82±0.08	2.63±0.06	-0.55±0.09	-3.75±0.08	3.09±0.03	0.018±0.03	-3.07±0.06	0.46±0.03	-2.55±0.05	-2.09±0.08

Thus, SU(3) symmetry breaking plays an important role in obtaining the fit. Another interesting observation for the case of Ω^- is that the central value of $\mu(\Omega^-)$ is not equal to the sum of central values of $\mu(B^*)_V$, $\mu(B^*)_S$ and $\mu(B^*)_O$ for the χ CQM results with the effective quark magnetic moments. This is primarily because of the valence structure of Ω^- which has three strange quarks. The orbital contribution of Ω^- which comes purely from the orbital moment of s quarks plays a very crucial role here. On having a closer look at the expression of the orbital moment of s quarks from Eq. (29), we find that, unlike the orbital moments of u and d quarks [Eqs. (27) and (28)] where the major numerical contribution comes from the mass terms without α^2 , β^2 and ζ^2 , the orbital moment of s quark has no such term. This yields a rather large value of error ($\approx 50\%$) in the result of the orbital contribution given in Eq. (33). This is not observed in any other baryon. For the sake of comparison, we have presented the results of other available phenomenological and theoretical models in Table IV. Our model predictions for all the decuplet baryons are more or less in agreement with the predictions of other models existing in literature. In the case of Δ^{++} , Δ^+ and Ω^- where data are available, our results are even better than most of the predictions of other models.

To summarize, in a very interesting manner, the chiral constituent quark model (χ CQM) is able to phenomenologically estimate the orbital and sea contributions to the magnetic moment of the $J^P = \frac{3}{2}^+$ decuplet baryons. These results along with the valence contributions lead to a better agreement with data. The magnetic moments have implications for chiral symmetry breaking and SU(3) symmetry breaking and provide a basis to understand the significance of sea quarks in the baryon structure. The results also suggest that the effect of the confinement on quark masses included by taking effective quark masses plays a positive role and is found to improve the results further. The present χ CQM results are able to give a qualitative and quantitative description of the results and further endorse the earlier conclusion that constituent quarks and weakly interacting Goldstone bosons provide the appropriate degree of freedom in the nonperturbative regime of QCD. The future experiments for other $J^P = \frac{3}{2}^+$ baryons will not only provide a direct method to determine the sea quark contributions to the magnetic moments but also impose important constraints on the χ CQM parameters.

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