PHYSICAL REVIEW D **92,** 033011 (2015)

Examination of pairs in neutrino mixing matrix

Dianjing Liu[1] and Bo-Qiang Ma[1,2,3,*]

[1]*School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*

[2]*Collaborative Innovation Center of Quantum Matter, Beijing, China*

[3]*Center for High Energy Physics, Peking University, Beijing 100871, China*

(Received 1 June 2015; published 25 August 2015)

We examine the pairs of neutrino mixing matrix and suggest pairs that can be used in the construction of new mixing patterns, with "pair" denoting the equality of the modulus of a pair of matrix elements. The results show that the trimaximal mixing in ν_2 and the μ-τ interchange symmetry are good choices under current experimental results. The two cases of bipair mixing pattern depend on the mass hierarchy of neutrinos. We also derive constraints on the CP phase by the pairs. The results are compatible with the maximal CP violation in most cases that are both self-consistent and consistent with experimental results.

DOI: 10.1103/PhysRevD.92.033011 PACS numbers: 14.60.Pq, 11.30.Er, 12.15.Ff, 14.60.Lm

I. INTRODUCTION

It has been firmly established that neutrinos can transit from one flavor to another from various oscillation experiments. In the framework of three-generation neutrinos, the neutrino mass eigenstates are connected to flavor eigenstates by a unitary matrix, i.e., the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [1]. In the standard parametrization, i.e., the Chau-Keung (CK) scheme [2], the PMNS matrix is expressed as

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\alpha} & \\ & & e^{i\beta} \end{pmatrix}, \tag{1}$$

where s_{ij} denotes $\sin\theta_{ij}$ and c_{ij} denotes $\cos\theta_{ij}$ $(i,j = 1,2,3)$. The phase matrix $\mathrm{Diag}\{1, e^{i\alpha}, e^{i\beta}\}$ denotes the contribution from the Majorana type neutrinos and the two phases α and β do not manifest themselves in oscillations. Thus, there remain three mixing angles $(\theta_{12}, \theta_{13}, \theta_{23})$ and a CP phase δ for the description of neutrino oscillations. While experimental data on three mixing angles have been coming out continuously, there is no direct experimental measurement on δ. Nevertheless, some indirect analyses, including analysis of experiments on reactor and accelerator neutrinos [3] and global fit results [4], suggest that the CP phase is close to $-90°$ (assuming $\delta \in [-180°, 180°]$). This is in accord with the maximal CP violation, i.e., $\delta = \pm 90°$.

On the other hand, the search of mixing patterns of the PMNS matrix is a way to understand properties of neutrinos. In the search of mixing patterns, the concept of "pair" is often used, with a "pair" referring to the equality of the modulus of a pair of matrix elements. For example, the long discussed trimaximal mixing in ν_2 [5] can be expressed as three pairs

$$|U_{12}| = |U_{22}|, \tag{2}$$

$$|U_{12}| = |U_{32}|, \tag{3}$$

$$|U_{22}| = |U_{32}|, \tag{4}$$

where U_{ij} denotes the corresponding element (with row i and column j) of the PMNS matrix, i.e., Eq. (1). Similarly the μ-τ interchange symmetry [6,7] can be expressed as

$$|U_{21}| = |U_{31}|, \tag{5}$$

$$|U_{22}| = |U_{32}|, \tag{6}$$

$$|U_{23}| = |U_{33}|. \tag{7}$$

The so-called bipair mixing [8] assigns

$$|U_{12}| = |U_{32}|, \tag{8}$$

$$|U_{22}| = |U_{23}|, \tag{9}$$

as case (1), and

*mabq@pku.edu.cn

$$|U_{12}| = |U_{22}|, \tag{10}$$

$$|U_{32}| = |U_{33}|, \tag{11}$$

as case (2).

These phenomenological relations are included in many mixing patterns. An example is the extensively studied tri-bimaximal mixing pattern (TBM) [9]

$$U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{12}$$

which includes the trimaximal mixing and the μ-τ symmetry, as well as the assumption of $|U_{13}| = 0$. Another one is the bimaximal mixing pattern (BM) [10]

$$U_{\mathrm{BM}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{13}$$

which includes the μ-τ symmetry, the assumption of $|U_{13}| = 0$ and a pair $|U_{11}| = |U_{12}|$. The bipair mixing pattern [8], originally based on bipair mixing and $|U_{13}| = 0$, is described as

$$U_{\mathrm{BP}} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -t_{12}^2 & t_{12} & t_{12} \\ s_{12}t_{12} & -s_{12} & t_{12}/c_{12} \end{pmatrix}, \tag{14}$$

for case (1), and

$$U_{\mathrm{BP}} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}t_{12} & s_{12} & t_{12}/c_{12} \\ t_{12}^2 & -t_{12} & t_{12} \end{pmatrix}, \tag{15}$$

for case (2), with $s_{12}^2 = 1 - 1/\sqrt{2}$.

Although the hypothesis $|U_{13}| = 0$ contradicts the new data from the accelerator and reactor neutrino oscillation experiments [11], other relations based on the pairs may still hold. For example, a new mixing pattern [12] was proposed based on a nonzero θ_{13}, the μ-τ symmetry, the trimaximal mixing and the self-complementarity relation [13] (i.e., $\theta_1 + \theta_3 = 45°$ in another parametrization scheme). The new pattern has the form

$$|U| = \begin{pmatrix} \frac{\sqrt{2}+1}{3} & \frac{1}{\sqrt{3}} & \frac{\sqrt{2}-1}{3} \\ \frac{\sqrt{3-\sqrt{2}}}{3} & \frac{1}{\sqrt{3}} & \frac{\sqrt{3+\sqrt{2}}}{3} \\ \frac{\sqrt{3-\sqrt{2}}}{3} & \frac{1}{\sqrt{3}} & \frac{\sqrt{3+\sqrt{2}}}{3} \end{pmatrix}, \tag{16}$$

and it makes a prediction of maximal CP violation $\delta = \pm 90°$.

Therefore, the pairs would help in the search of new mixing patterns of the PMNS matrix. In order to know which pairs to choose in the construction of new mixing patterns, it is worthwhile to examine which of the pairs in the PMNS matrix are consistent with current experimental results, and whether they are consistent with each other.

What is more, the introduction of each pair produces a constraint on the four parameters of the PMNS matrix. Together with global fit results on three mixing angles, each pair would give a range of the CP phase (as is discussed in Sec. II, there are some exceptions in which the constraints do not include δ). Examinations of these ranges would give information about the consistency among pairs and the consistency between the pairs and the global fit results.

In Sec. II we consider constraints by each single pair separately. By comparing the pair constraints with the natural limit of $\cos\delta$, we evaluate their consistency with global fit results. In Sec. III we combine ranges of the pairs to give joint constraints, and discuss their self-consistency and consistency with global fit results. In Sec. IV we pick out cases that are self-consistent and consistent with experimental results, and compare their ranges to the maximal CP violation. Section V is served for conclusions.

II. SINGLE PAIR CONSTRAINTS

In our article all ranges of δ come from the ranges of $\cos\delta$. Thus we only discuss on the assumption that $\delta \in [0°, 180°]$. When extending to $[-180°, 180°]$, the results of $\delta \in [\delta_1, \delta_2]$ should also be extended to be $\delta \in [-\delta_2, -\delta_1]$ and $[\delta_1, \delta_2]$.

We consider a single pair, for example, $|U_{21}| = |U_{31}|$. It gives rise to a relation between $\cos\delta$ and mixing angles. In the case of $|U_{21}| = |U_{31}|$, it is

$$\cos\delta(\theta_{12}, \theta_{13}, \theta_{23}) = \frac{(\sin\theta_{12}\sin\theta_{23})^2 + (\sin\theta_{13}\cos\theta_{12}\cos\theta_{23})^2 - (\sin\theta_{12}\cos\theta_{23})^2 - (\sin\theta_{13}\sin\theta_{23}\cos\theta_{12})^2}{4\sin\theta_{12}\sin\theta_{13}\sin\theta_{23}\cos\theta_{12}\cos\theta_{23}}. \tag{17}$$

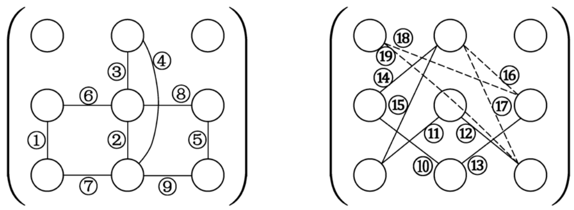

FIG. 1. The identification of possible pairs with numbers ①–⑲. Pairs ①–⑮ have overlap in 3σ range. Pairs ⑯–⑲ have overlap in 4σ range instead of 3σ. They are denoted by dash lines but also included in discussion.

By inserting the global fit results of mixing angles [4]

$$\theta_{12} = (33.48^{+0.78}_{-0.75})^\circ, \tag{18}$$

$$\theta_{13} = (8.50^{+0.20}_{-0.21})^\circ \oplus (8.51^{+0.20}_{-0.21})^\circ, \tag{19}$$

$$\theta_{23} = (42.3^{+3.0}_{-1.6})^\circ \oplus (49.5^{+1.5}_{-2.2})^\circ, \tag{20}$$

we obtain the central value and 1σ error of δ, which are

$$\cos\delta = -0.2009^{+0.2247}_{-0.1201} \tag{21}$$

for normal hierarchy (NH), and

$$\cos\delta = 0.3362^{+0.1148}_{-0.1677} \tag{22}$$

for inverted hierarchy (IH).

From global fit of 3σ ranges on the magnitude of matrix elements [4]

$$|U| = \begin{pmatrix} 0.801\to 0.845 & 0.514\to 0.580 & 0.137\to 0.158 \\ 0.225\to 0.517 & 0.441\to 0.699 & 0.614\to 0.793 \\ 0.246\to 0.529 & 0.464\to 0.713 & 0.590\to 0.776 \end{pmatrix}, \tag{23}$$

we pick out all pairs whose corresponding matrix elements have overlap in 3σ range, and number them as in Fig. 1. Pairs ①–⑮ have overlap in 3σ range. Pairs ⑯–⑲ have overlap in 4σ range instead of 3σ. Although pairs ⑯ ⑲ (or ⑰ ⑱) do not hold in 3σ range and therefore need not be discussed by principle, their constraints on mixing angles are the same as that of pair ⑩ (or ⑪). Thus, they are also included in the discussion.

Similar to previous calculations, each pair produces a constraint on $\cos\delta$, except pairs ⑤⑩ ⑪ ⑯ ⑰ ⑱ ⑲. Among them, pair ⑤ yields

$$\theta_{23} = 45^\circ, \tag{24}$$

⑩, ⑯, ⑲ yield the same relation

$$\theta_{12} = \theta_{23}, \tag{25}$$

and ⑪, ⑰, ⑱ all yield

$$\theta_{12} + \theta_{23} = 90^\circ. \tag{26}$$

Other results are shown in Fig. 2.

In addition, pairs ⑫ and ⑮ produce the same relation between δ and mixing angles, as well as ⑬ and ⑭. That is

$$\cos\delta_{⑫}(\theta_{12},\theta_{13},\theta_{23}) = \cos\delta_{⑮}(\theta_{12},\theta_{13},\theta_{23}), \tag{27}$$

$$\cos\delta_{⑬}(\theta_{12},\theta_{13},\theta_{23}) = \cos\delta_{⑭}(\theta_{12},\theta_{13},\theta_{23}). \tag{28}$$

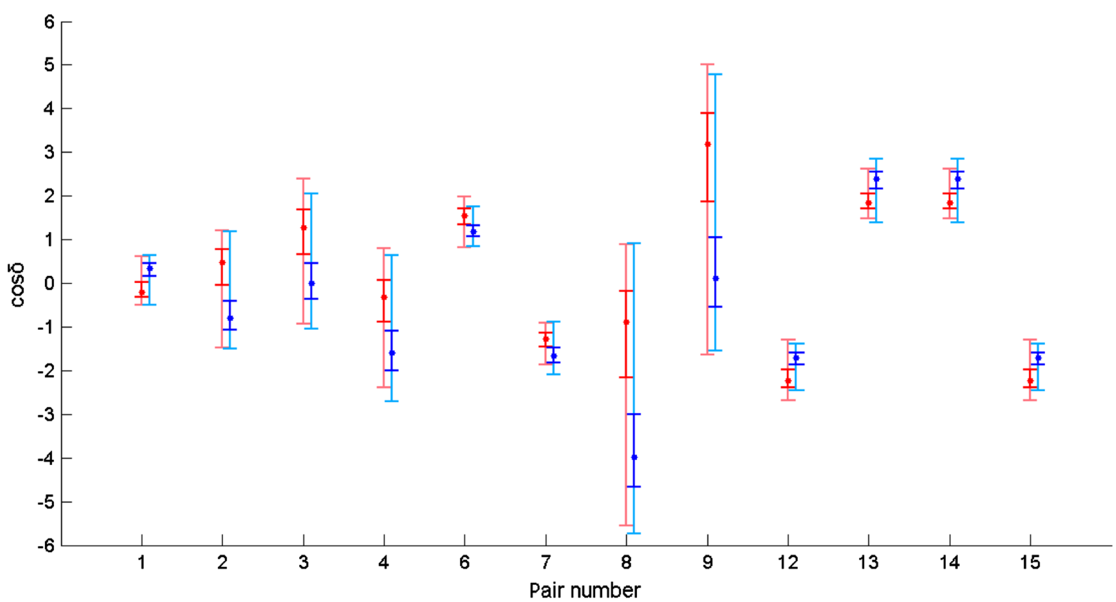

FIG. 2 (color online). Constraints on $\cos\delta$ produced by single pair. Each error bar shows 1σ and 3σ ranges of $\cos\delta$. The left and right error bars of each pair denote ranges in NH and IH, respectively.

TABLE I. The classification of pairs according to their ranks of consistency (marked with ☆) with the natural limit $\cos\delta \in [-1, 1]$. While pairs ⑤⑩ ⑪ ⑯ ⑰ ⑱ ⑲ have no constraints on $\cos\delta$, they are divided according to their constraints on mixing angles and their ranks are denoted by •.

Rank of consistency	Constraint compared with natural limit $\cos\delta \in [-1, 1]$	Pair: Normal hierarchy	Pair: Inverted hierarchy
5☆	$1\sigma, 3\sigma$ within $[-1, 1]$	①	①
4☆	1σ within $[-1, 1]$, 3σ partially beyond	②④	③
3☆	Central value within $[-1, 1]$, 1σ partially beyond	⑧	②⑨
2☆	Central value beyond $[-1, 1]$, 1σ partially within	③	...
1☆	1σ beyond $[-1, 1]$, 3σ partially within	⑥⑦⑨	④⑥⑦⑧
0☆	3σ beyond $[-1, 1]$	⑫ ⑬ ⑭ ⑮	⑫ ⑬ ⑭ ⑮
•	No constraint on $\cos\delta$, $\theta_{23} = 45°$	⑤	⑤
•	No constraint on $\cos\delta$, $\theta_{12} = \theta_{23}$	⑩ ⑯ ⑲	⑩ ⑯ ⑲
•	No constraint on $\cos\delta$, $\theta_{12} + \theta_{23} = 90°$	⑪ ⑰ ⑱	⑪ ⑰ ⑱

Therefore, ⑫ and ⑮ would produce the same constraint on δ, as well as ⑬ and ⑭.

While pair ① gives a strong constraint on $\cos\delta$ with the natural limit $\cos\delta \in [-1, 1]$ satisfied as well, it is necessary to point out that some constraints are completely not in agreement with the natural limit. Since the constraints are produced by pairs together with experimental results, it indicates that these pairs are not so consistent with current experimental results. Thus they might not be good choices when considering a new mixing pattern.

We divide the pairs into several classes according to the level of consistency between their constraints and the natural limit of $\cos\delta$. These classifications can be regarded as indications of their consistency with the experimental results. The results are shown in Table I.

Together with the natural limit, each of the pairs except ⑤⑩ ⑪ ⑯ ⑰ ⑱ ⑲ gives a constraint on δ. The ranges of δ on the assumption that $\delta \in [0°, 180°]$ are shown in Fig. 3.

III. COMBINED PAIR CONSTRAINTS

As is discussed in Sec. II, each pair produces a central value and an error of $\cos\delta$. If some of the pairs are supposed to hold simultaneously, their ranges should be regarded as measurements of the same Gaussian distribution of $\cos\delta$ and should be combined to give an estimation of the distribution.

When combining ranges from different pairs, we adopt standard weighted least-squares procedure advocated by the Particle Data Group [14]. The weighted average and error are

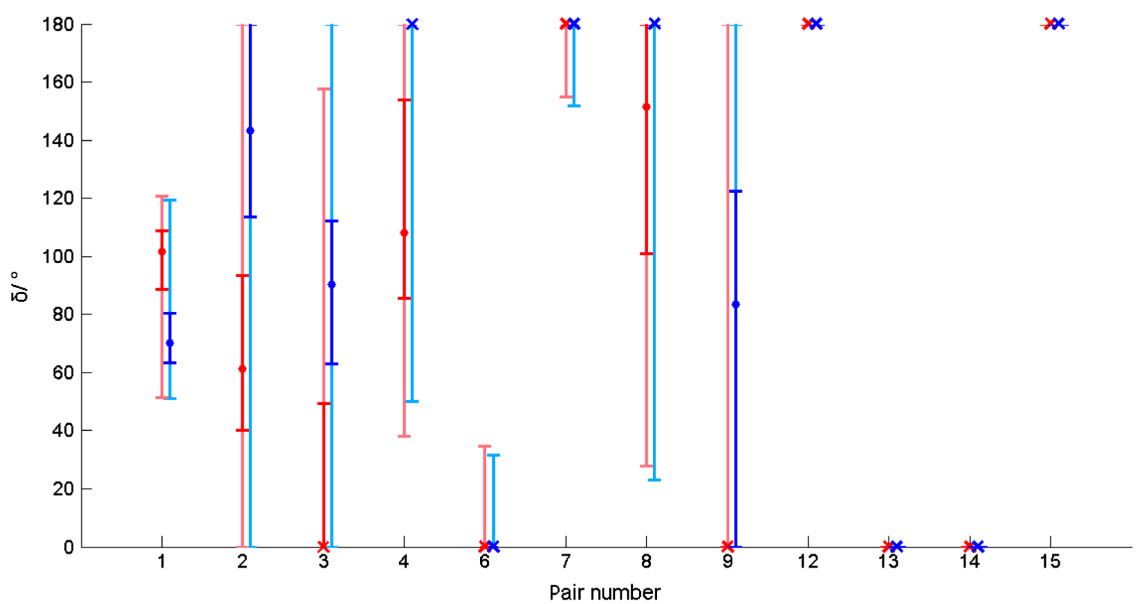

FIG. 3 (color online). Constraints on δ produced by single pair and the natural limit $\cos\delta \in [-1, 1]$. Each error bar shows 1σ and 3σ ranges of δ. The left and right error bars of each pair denote ranges in NH and IH, respectively. The × means that the central value is out of physical range.

$$\overline{\cos\delta} \pm \sigma_{\overline{\cos\delta}} = \frac{\Sigma_i \omega_i \cos\delta_i}{\Sigma_i \omega_i} \pm (\Sigma_i \omega_i)^{-\frac{1}{2}}, \tag{29}$$

where

$$\omega_i = \frac{1}{\sigma^2_{(\cos\delta_i)}}, \tag{30}$$

with i referring to pairs which are combined.

The scale factor is defined as

$$S = \sqrt{\frac{\chi^2}{N-1}}, \tag{31}$$

whose expectation value is 1 since the expectation value of χ^2 is $N-1$.

For cases with $S > 1$, we also calculate scaled output errors, which are

$$\sigma_{\text{scaled}} = S\sigma_{\text{unscaled}}. \tag{32}$$

The reason is as follows: the relatively large value of χ^2 is likely to be due to underestimation of errors in some of the measurements. Not knowing which of the input errors are underestimated, we assume they are all underestimated by the same factor S. If we scale up all the input errors by S, the χ^2 becomes $N-1$, and the output error scales up by the same factor.

What is more, the p-value of the combination is calculated (with unscaled input errors), and the corresponding confidence level serves to indicate exclusion level about self-consistency of the combination.

A. μ-τ symmetry and trimaximal mixing

First we combine pairs from the trimaximal mixing (i.e., pairs ②③④) to explore a joint constraint on $\cos\delta$ by this phenomenological relation. According to Eqs. (29)–(32), the range of $\cos\delta$ and confidence level of exclusion are calculated and listed in Table II (NH) and Table III (IH). In the table we also classify consistency between the results and the natural limit by the same labels used in Table I.

The same procedure is performed considering the μ-τ symmetry and the combination of the two relations. Here some explanation is necessary. Obviously, pair ⑤ has no influence on $\cos\delta$. When combined with other pairs, it simply contributes $\Delta\chi^2 = (\theta_{23} - 45°)^2/\sigma^2_{\theta_{23}} = 0.81$ (NH) or 4.18 (IH) to χ^2.

From Table II, we find that the three cases all give ranges compatible with the maximal CP violation ($\cos\delta = 0$) in 1σ errors, regardless of whether errors are scaled or not.

On the other hand, Table III shows that the μ-τ symmetry and the trimaximal mixing give ranges compatible with the maximal CP violation in 2σ(unscaled)/1σ(scaled) and 3σ(unscaled)/2σ(scaled) errors, respectively. The combination of the two relations fits the maximal CP violation well. However, the three cases in IH all lead to low p-values, indicating a low self-consistency of the combinations in IH.

B. Bipair combination

In this part we consider all bipair combinations. That is, any two pairs in Sec. II are considered to examine their consistency and constraints on δ.

However, not all of the pairs are suitable for combination: similar to pair ⑤, ⑩ ⑪ ⑯ ⑰ ⑱ ⑲ would simply add to χ^2. Each pair of ⑩ ⑯ ⑲ contributes $\Delta\chi^2 = 24.55$ (NH) or 47.10 (IH), and each pair of ⑪ ⑰ ⑱ contributes $\Delta\chi^2 = 21.05$ (NH) or 17.24 (IH). Since we are combining no more than 3 pairs, $\chi^2 > 17.24$ leads to p-value < 0.00018 (over 3σ level of exclusion). Therefore, we do not include pairs ⑩ ⑯ ⑲ ⑪ ⑰ ⑱ in combination.

What is more, pairs ⑫ and ⑮ have the same constraint, as well as ⑬ and ⑭. Therefore we do not include ⑭, ⑮ for conciseness. The results are listed in Table IV (NH) and Table V (IH).

TABLE II. Three cases of pair combination in NH.

Case	Pairs included	$\overline{\cos\delta}$	$\sigma_{\overline{\cos\delta}}$ Unscaled	$\sigma_{\overline{\cos\delta}}$ Scaled	Self consistency χ^2	Self consistency p-value	Self consistency Exclusion level	Natural limit consistency Unscaled	Natural limit consistency Scaled
trimaximal mixing	②③④	0.257	0.263	0.420	5.0857	0.079	1-2σ	4 ☆	4 ☆
μ-τ symmetry	①②⑤	−0.134	0.168	0.179	2.2629	0.323	<1σ	5 ☆	5 ☆
μ-τ & trimaximal	①②③④⑤	−0.046	0.166	0.225	7.2975	0.121	1-2σ	5 ☆	5 ☆

TABLE III. Three cases of pair combination in IH.

Case	Pairs included	$\overline{\cos\delta}$	$\sigma_{\overline{\cos\delta}}$ Unscaled	$\sigma_{\overline{\cos\delta}}$ Scaled	Self consistency χ^2	Self consistency p-value	Self consistency Exclusion level	Natural limit consistency Unscaled	Natural limit consistency Scaled
trimaximal mixing	②③④	−0.657	0.228	0.416	6.6486	0.036	2-3σ	4 ☆	3 ☆
μ-τ symmetry	①②⑤	0.166	0.155	0.364	11.0783	0.0039	2-3σ	5 ☆	4 ☆
μ-τ & trimaximal	①②③④⑤	0.015	0.139	0.327	21.9753	0.0002	3-4σ	5 ☆	5 ☆

TABLE IV. Bipair combinations in NH.

		$\sigma_{\overline{\cos\delta}}$		Self-consistency			Natural limit consistency	
Pairs included	$\overline{\cos\delta}$	Unscaled	Scaled	χ^2	p-value	Exclusion level	Unscaled	Scaled
①②	−0.134	0.168	0.202	1.4529	2.28×10^{-1}	1-2σ	5 ☆	5 ☆
①③	−0.064	0.189	0.427	5.1242	2.36×10^{-2}	2-3σ	5 ☆	4 ☆
①④	−0.213	0.145	0.145	0.0548	8.15×10^{-1}	$<1\sigma$	5 ☆	5 ☆
①⑤	−0.201	0.157	0.157	0.8100	3.68×10^{-1}	$<1\sigma$	5 ☆	5 ☆
①⑥	0.738	0.153	0.875	32.5828	1.14×10^{-8}	$>5\sigma$	4 ☆	3 ☆
①⑦	−0.711	0.087	0.536	37.9554	7.24×10^{-10}	$>5\sigma$	5 ☆	3 ☆
①⑧	−0.230	0.145	0.145	0.9028	3.42×10^{-1}	$<1\sigma$	5 ☆	5 ☆
①⑨	−0.143	0.173	0.441	6.5201	1.07×10^{-2}	2-3σ	5 ☆	4 ☆
①⑫	−0.554	0.109	0.771	49.8493	1.66×10^{-12}	$>5\sigma$	5 ☆	3 ☆
①⑬	1.301	0.115	0.896	60.5831	7.11×10^{-15}	$>5\sigma$	1 ☆	2 ☆
②③	0.647	0.286	0.324	1.2847	2.57×10^{-1}	1-2σ	4 ☆	4 ☆
②④	−0.014	0.321	0.383	1.4264	2.32×10^{-1}	1-2σ	5 ☆	4 ☆
②⑤	0.479	0.373	0.373	0.8100	3.68×10^{-1}	$<1\sigma$	4 ☆	4 ☆
②⑥	1.178	0.169	0.512	9.1718	2.46×10^{-3}	3-4σ	1 ☆	2 ☆
②⑦	−1.176	0.127	0.404	10.1180	1.47×10^{-3}	3-4σ	1 ☆	2 ☆
②⑧	−0.023	0.417	0.655	2.4671	1.16×10^{-1}	1-2σ	4 ☆	4 ☆
②⑨	0.635	0.316	0.632	4.0004	4.55×10^{-2}	2-3σ	4 ☆	3 ☆
②⑫	−1.712	0.236	1.073	20.7529	5.23×10^{-6}	4-5σ	0 ☆	2 ☆
②⑬	1.591	0.121	0.522	18.4531	1.74×10^{-5}	4-5σ	0 ☆	1 ☆
③④	0.134	0.328	0.711	4.6908	3.03×10^{-2}	2-3σ	4 ☆	4 ☆
③⑤	1.269	0.490	0.490	0.8100	3.68×10^{-1}	$<1\sigma$	2 ☆	2 ☆
③⑥	1.511	0.168	0.168	0.3544	5.52×10^{-1}	$<1\sigma$	0 ☆	0 ☆
③⑦	−1.168	0.127	0.510	16.2066	5.68×10^{-5}	4-5σ	1 ☆	2 ☆
③⑧	0.306	0.459	1.068	5.4052	2.01×10^{-2}	2-3σ	4 ☆	3 ☆
③⑨	1.469	0.425	0.588	1.9126	1.67×10^{-1}	1-2σ	1 ☆	2 ☆
③⑫	−1.703	0.242	1.261	27.2497	1.79×10^{-7}	$>5\sigma$	1 ☆	2 ☆
③⑬	1.768	0.141	0.184	1.7137	1.91×10^{-1}	1-2σ	0 ☆	0 ☆
④⑤	−0.311	0.466	0.466	0.8100	3.68×10^{-1}	$<1\sigma$	4 ☆	4 ☆
④⑥	1.131	0.184	0.780	17.9223	2.30×10^{-5}	4-5σ	2 ☆	2 ☆
④⑦	−1.224	0.134	0.215	2.5580	1.10×10^{-1}	1-2σ	1 ☆	1 ☆
④⑧	−0.479	0.420	0.420	0.3792	5.38×10^{-1}	$<1\sigma$	4 ☆	4 ☆
④⑨	−0.010	0.387	0.984	6.4635	1.10×10^{-2}	2-3σ	4 ☆	4 ☆
④⑫	−1.916	0.239	0.718	8.9971	2.70×10^{-3}	2-3σ	0 ☆	1 ☆
④⑬	1.606	0.127	0.664	27.4527	1.61×10^{-7}	$>5\sigma$	0 ☆	2 ☆
⑤⑥	1.553	0.175	0.175	0.8100	3.68×10^{-1}	$<1\sigma$	0 ☆	0 ☆
⑤⑦	−1.275	0.146	0.146	0.8100	3.68×10^{-1}	$<1\sigma$	1 ☆	1 ☆
⑤⑧	−0.878	0.893	0.893	0.8100	3.68×10^{-1}	$<1\sigma$	3 ☆	3 ☆
⑤⑨	3.196	0.922	0.922	0.8100	3.68×10^{-1}	$<1\sigma$	1 ☆	1 ☆
⑤⑫	−2.238	0.202	0.202	0.8100	3.68×10^{-1}	$<1\sigma$	0 ☆	0 ☆
⑤⑬	1.836	0.167	0.167	0.8100	3.68×10^{-1}	$<1\sigma$	0 ☆	0 ☆
⑥⑦	−0.521	0.108	1.250	133.6315	0	$>5\sigma$	5 ☆	3 ☆
⑥⑧	1.346	0.200	0.678	11.4907	6.99×10^{-4}	3-4σ	1 ☆	2 ☆
⑥⑨	1.579	0.169	0.209	1.5269	2.17×10^{-1}	1-2σ	0 ☆	1 ☆
⑥⑫	0.077	0.164	1.848	127.5130	0	$>5\sigma$	5 ☆	3 ☆
⑥⑬	1.708	0.101	0.141	1.9665	1.61×10^{-1}	1-2σ	0 ☆	0 ☆
⑦⑧	−1.267	0.143	0.143	0.1509	6.98×10^{-1}	$<1\sigma$	1 ☆	1 ☆
⑦⑨	−1.226	0.138	0.464	11.3660	7.48×10^{-4}	3-4σ	1 ☆	2 ☆
⑦⑫	−1.565	0.144	0.442	9.4236	2.14×10^{-3}	3-4σ	0 ☆	1 ☆
⑦⑬	0.188	0.092	1.552	285.1556	0	$>5\sigma$	5 ☆	3 ☆
⑧⑨	−0.011	0.609	1.668	7.5110	6.13×10^{-3}	2-3σ	4 ☆	3 ☆
⑧⑫	−2.202	0.207	0.217	1.1014	2.94×10^{-1}	1-2σ	0 ☆	0 ☆
⑧⑬	1.727	0.137	0.533	15.0276	1.06×10^{-4}	3-4σ	0 ☆	1 ☆
⑨⑫	−2.060	0.239	0.968	16.4199	5.07×10^{-5}	4-5σ	0 ☆	1 ☆
⑨⑬	1.859	0.171	0.175	1.0452	3.07×10^{-1}	1-2σ	0 ☆	0 ☆
⑫⑬	0.992	0.119	1.651	191.2192	0	$>5\sigma$	3 ☆	3 ☆

TABLE V. Bipair combinations in IH.

Pairs included	$\overline{\cos\delta}$	$\sigma_{\overline{\cos\delta}}$ Unscaled	$\sigma_{\overline{\cos\delta}}$ Scaled	Self-consistency χ^2	Self-consistency p-value	Self-consistency Exclusion level	Natural limit consistency Unscaled	Natural limit consistency Scaled
①②	0.166	0.155	0.406	6.8944	8.65×10^{-3}	2-3σ	5 ☆	4 ☆
①③	0.305	0.136	0.136	0.5355	4.64×10^{-1}	$< 1\sigma$	5 ☆	5 ☆
①④	0.149	0.159	0.573	12.9107	3.27×10^{-4}	3-4σ	5 ☆	4 ☆
①⑤	0.336	0.136	0.279	4.1839	4.08×10^{-2}	2-3σ	5 ☆	4 ☆
①⑥	0.759	0.082	0.427	27.3863	1.67×10^{-7}	$> 5\sigma$	4 ☆	3 ☆
①⑦	−0.552	0.125	0.990	62.9428	2.11×10^{-15}	$> 5\sigma$	5 ☆	3 ☆
①⑧	0.233	0.154	0.661	18.4609	1.73×10^{-5}	4-5σ	5 ☆	4 ☆
①⑨	0.330	0.136	0.136	0.0751	7.84×10^{-1}	$< 1\sigma$	5 ☆	5 ☆
①⑫	−1.009	0.098	0.974	98.0825	0	$> 5\sigma$	2 ☆	2 ☆
①⑬	0.786	0.102	0.848	69.8745	1.11×10^{-16}	$> 5\sigma$	4 ☆	3 ☆
②③	−0.373	0.272	0.397	2.1345	1.44×10^{-1}	1-2σ	4 ☆	4 ☆
②④	−0.994	0.251	0.341	1.8378	1.75×10^{-1}	1-2σ	3 ☆	3 ☆
②⑤	−0.801	0.325	0.664	4.1839	4.08×10^{-2}	2-3σ	3 ☆	3 ☆
②⑥	1.036	0.111	0.534	22.9259	1.68×10^{-6}	4-5σ	2 ☆	2 ☆
②⑦	−1.384	0.154	0.398	6.6574	9.87×10^{-3}	2-3σ	1 ☆	2 ☆
②⑧	−1.036	0.270	0.833	9.5159	2.04×10^{-3}	3-4σ	2 ☆	2 ☆
②⑨	−0.581	0.319	0.391	1.5035	2.20×10^{-1}	1-2σ	4 ☆	4 ☆
②⑫	−1.564	0.111	0.339	9.3117	2.28×10^{-3}	3-4σ	0 ☆	1 ☆
②⑬	1.662	0.191	1.336	49.1950	2.32×10^{-12}	$> 5\sigma$	0 ☆	2 ☆
③④	−0.553	0.300	0.756	6.3592	1.17×10^{-2}	2-3σ	4 ☆	3 ☆
③⑤	−0.006	0.411	0.840	4.1839	4.08×10^{-2}	2-3σ	4 ☆	4 ☆
③⑥	1.116	0.115	0.289	6.3013	1.21×10^{-2}	2-3σ	1 ☆	2 ☆
③⑦	−1.321	0.167	0.664	15.8375	6.90×10^{-5}	3-4σ	1 ☆	2 ☆
③⑧	−0.490	0.347	1.302	14.0912	1.74×10^{-4}	3-4σ	4 ☆	3 ☆
③⑨	0.022	0.363	0.363	0.0185	8.92×10^{-1}	$< 1\sigma$	4 ☆	4 ☆
③⑫	−1.548	0.115	0.506	19.2342	1.16×10^{-5}	4-5σ	0 ☆	1 ☆
③⑬	1.954	0.196	0.921	22.0236	2.69×10^{-6}	4-5σ	0 ☆	1 ☆
④⑤	−1.597	0.453	0.927	4.1839	4.08×10^{-2}	2-3σ	1 ☆	2 ☆
④⑥	1.054	0.113	0.602	28.2693	1.06×10^{-7}	$> 5\sigma$	2 ☆	2 ☆
④⑦	−1.648	0.160	0.160	0.0148	9.03×10^{-1}	$< 1\sigma$	0 ☆	0 ☆
④⑧	−1.949	0.381	0.847	4.9330	2.63×10^{-2}	2-3σ	1 ☆	1 ☆
④⑨	−0.942	0.401	0.831	4.2837	3.85×10^{-2}	2-3σ	3 ☆	3 ☆
④⑫	−1.704	0.131	0.131	0.0638	8.01×10^{-1}	$< 1\sigma$	0 ☆	0 ☆
④⑬	1.780	0.200	1.431	51.4547	7.33×10^{-13}	$> 5\sigma$	0 ☆	2 ☆
⑤⑥	1.191	0.125	0.255	4.1839	4.08×10^{-2}	2-3σ	1 ☆	2 ☆
⑤⑦	−1.656	0.170	0.348	4.1839	4.08×10^{-2}	2-3σ	0 ☆	1 ☆
⑤⑧	−3.989	0.810	1.657	4.1839	4.08×10^{-2}	2-3σ	0 ☆	1 ☆
⑤⑨	0.111	0.769	1.572	4.1839	4.08×10^{-2}	2-3σ	4 ☆	3 ☆
⑤⑫	−1.714	0.139	0.283	4.1839	4.08×10^{-2}	2-3σ	0 ☆	1 ☆
⑤⑬	2.386	0.189	0.387	4.1839	4.08×10^{-2}	2-3σ	0 ☆	0 ☆
⑥⑦	0.399	0.099	1.275	167.4270	0	$> 5\sigma$	5 ☆	3 ☆
⑥⑧	1.117	0.118	0.612	26.7374	2.33×10^{-7}	$> 5\sigma$	2 ☆	2 ☆
⑥⑨	1.172	0.122	0.139	1.2834	2.57×10^{-1}	1-2σ	1 ☆	1 ☆
⑥⑫	−0.196	0.084	1.451	299.1174	0	$> 5\sigma$	5 ☆	3 ☆
⑥⑬	1.523	0.114	0.536	21.9778	2.76×10^{-6}	4-5σ	0 ☆	2 ☆
⑦⑧	−1.718	0.163	0.376	5.3578	2.06×10^{-2}	2-3σ	0 ☆	1 ☆
⑦⑨	−1.527	0.175	0.459	6.8914	8.66×10^{-3}	2-3σ	0 ☆	1 ☆
⑦⑫	−1.691	0.105	0.105	0.0736	7.86×10^{-1}	$< 1\sigma$	0 ☆	0 ☆
⑦⑬	0.068	0.142	1.999	199.5146	0	$> 5\sigma$	5 ☆	3 ☆
⑧⑨	−1.111	0.543	1.875	11.9311	5.52×10^{-4}	3-4σ	2 ☆	2 ☆
⑧⑫	−1.762	0.144	0.325	5.1237	2.36×10^{-2}	2-3σ	0 ☆	1 ☆
⑧⑬	2.097	0.212	1.326	39.2286	3.77×10^{-10}	$> 5\sigma$	0 ☆	2 ☆
⑨⑫	−1.645	0.126	0.349	7.6307	5.74×10^{-3}	2-3σ	0 ☆	1 ☆
⑨⑬	2.287	0.197	0.465	5.5475	1.85×10^{-2}	2-3σ	0 ☆	1 ☆
⑫⑬	−0.734	0.106	1.748	272.4518	0	$> 5\sigma$	4 ☆	3 ☆

TABLE VI. Tripair combinations in NH.

		$\sigma_{\overline{\cos\delta}}$		Self-consistency			Natural limit consistency	
Pairs included	$\overline{\cos\delta}$	Unscaled	Scaled	χ^2	p-value	Exclusion level	Unscaled	Scaled
①②③	0.046	0.195	0.333	5.8551	5.35×10^{-2}	1-2σ	5 ☆	4 ☆
①②④	−0.160	0.151	0.151	1.6025	4.49×10^{-1}	< 1σ	5 ☆	5 ☆
①②⑤	−0.134	0.168	0.179	2.2629	3.23×10^{-1}	< 1σ	5 ☆	5 ☆
①②⑧	−0.179	0.152	0.172	2.5660	2.77×10^{-1}	1-2σ	5 ☆	5 ☆
①②⑨	−0.047	0.188	0.366	7.5949	2.24×10^{-2}	2-3σ	5 ☆	4 ☆
①③④	−0.116	0.162	0.267	5.4425	6.58×10^{-2}	1-2σ	5 ☆	5 ☆
①③⑤	−0.064	0.189	0.325	5.9342	5.15×10^{-2}	1-2σ	5 ☆	4 ☆
①③⑧	−0.136	0.165	0.296	6.4678	3.94×10^{-2}	2-3σ	5 ☆	4 ☆
①③⑨	0.051	0.209	0.485	10.8220	4.47×10^{-3}	2-3σ	5 ☆	4 ☆
①④⑤	−0.213	0.145	0.145	0.8648	6.49×10^{-1}	< 1σ	5 ☆	5 ☆
①④⑧	−0.237	0.136	0.136	0.9291	6.28×10^{-1}	< 1σ	5 ☆	5 ☆
①④⑨	−0.168	0.155	0.282	6.6526	3.59×10^{-2}	2-3σ	5 ☆	4 ☆
①⑤⑧	−0.230	0.144	0.144	1.7128	4.25×10^{-1}	< 1σ	5 ☆	5 ☆
①⑤⑨	−0.143	0.173	0.331	7.3301	2.56×10^{-2}	2-3σ	5 ☆	4 ☆
①⑧⑨	−0.189	0.155	0.302	7.6024	2.23×10^{-2}	2-3σ	5 ☆	4 ☆
②③④	0.257	0.263	0.420	5.0857	7.86×10^{-2}	1-2σ	4 ☆	4 ☆
②③⑤	0.647	0.285	0.292	2.0947	3.51×10^{-1}	< 1σ	4 ☆	4 ☆
②③⑥	1.187	0.160	0.344	9.2009	1.00×10^{-2}	2-3σ	1 ☆	2 ☆
②③⑧	0.405	0.300	0.496	5.4863	6.44×10^{-2}	1-2σ	4 ☆	4 ☆
②③⑨	0.733	0.260	0.412	5.0196	8.13×10^{-2}	1-2σ	4 ☆	3 ☆
②④⑤	−0.014	0.320	0.339	2.2364	3.27×10^{-1}	< 1σ	5 ☆	4 ☆
②④⑦	−1.145	0.120	0.298	12.2685	2.17×10^{-3}	3-4σ	1 ☆	2 ☆
②④⑧	−0.168	0.304	0.349	2.6384	2.67×10^{-1}	1-2σ	4 ☆	4 ☆
②④⑨	0.165	0.291	0.554	7.2437	2.67×10^{-2}	2-3σ	4 ☆	4 ☆
②④⑫	−1.517	0.219	0.783	25.6687	2.67×10^{-6}	4-5σ	1 ☆	2 ☆
②④⑮	−1.517	0.219	0.783	25.6687	2.67×10^{-6}	4-5σ	1 ☆	2 ☆
②⑤⑥	1.178	0.169	0.377	9.9818	6.80×10^{-3}	2-3σ	1 ☆	2 ☆
②⑤⑦	−1.176	0.127	0.297	10.9280	4.24×10^{-3}	2-3σ	1 ☆	2 ☆
②⑤⑧	−0.023	0.417	0.534	3.2771	1.94×10^{-1}	1-2σ	4 ☆	4 ☆
②⑤⑨	0.635	0.316	0.490	4.8104	9.02×10^{-2}	1-2σ	4 ☆	3 ☆
②⑥⑨	1.210	0.168	0.401	11.4751	3.22×10^{-3}	2-3σ	1 ☆	2 ☆
②⑦⑧	−1.172	0.125	0.283	10.2145	6.05×10^{-3}	2-3σ	1 ☆	2 ☆
②⑦⑨	−1.144	0.122	0.397	21.0808	2.64×10^{-5}	4-5σ	1 ☆	2 ☆
②⑦⑫	−1.423	0.137	0.465	23.0357	9.95×10^{-6}	4-5σ	0 ☆	2 ☆
②⑦⑮	−1.423	0.137	0.465	23.0357	9.95×10^{-6}	4-5σ	0 ☆	2 ☆
②⑧⑨	0.320	0.346	0.690	7.9486	1.88×10^{-2}	2-3σ	4 ☆	3 ☆
②⑫⑮	−1.947	0.175	0.594	22.9756	1.03×10^{-5}	4-5σ	0 ☆	1 ☆
③④⑤	0.134	0.328	0.544	5.5008	6.39×10^{-2}	1-2σ	4 ☆	4 ☆
③④⑧	−0.037	0.306	0.548	6.4105	4.05×10^{-2}	2-3σ	5 ☆	4 ☆
③④⑨	0.312	0.319	0.704	9.7672	7.57×10^{-3}	2-3σ	4 ☆	3 ☆
③⑤⑥	1.511	0.168	0.168	1.1644	5.59×10^{-1}	< 1σ	0 ☆	0 ☆
③⑤⑧	0.306	0.459	0.810	6.2152	4.47×10^{-2}	2-3σ	4 ☆	3 ☆
③⑤⑨	1.469	0.425	0.496	2.7226	2.56×10^{-1}	1-2σ	1 ☆	2 ☆
③⑤⑬	1.768	0.141	0.158	2.5237	2.83×10^{-1}	1-2σ	0 ☆	0 ☆
③⑤⑭	1.768	0.141	0.158	2.5237	2.83×10^{-1}	1-2σ	0 ☆	0 ☆
③⑥⑧	1.334	0.185	0.443	11.5077	3.17×10^{-3}	2-3σ	1 ☆	2 ☆
③⑥⑨	1.538	0.163	0.163	1.9694	3.74×10^{-1}	< 1σ	0 ☆	0 ☆
③⑥⑬	1.687	0.098	0.121	3.0709	2.15×10^{-1}	1-2σ	0 ☆	0 ☆
③⑥⑭	1.687	0.098	0.121	3.0709	2.15×10^{-1}	1-2σ	0 ☆	0 ☆
③⑧⑨	0.619	0.434	0.955	9.6864	7.88×10^{-3}	2-3σ	3 ☆	3 ☆
③⑨⑬	1.782	0.143	0.171	2.8672	2.38×10^{-1}	1-2σ	0 ☆	0 ☆

(Table continued)

TABLE VI. (*Continued*)

		$\sigma_{\overline{\cos\delta}}$		Self-consistency			Natural limit consistency	
Pairs included	$\overline{\cos\delta}$	Unscaled	Scaled	χ^2	p-value	Exclusion level	Unscaled	Scaled
③⑨⑭	1.782	0.143	0.171	2.8672	2.38×10^{-1}	1-2σ	0 ☆	0 ☆
③⑬⑭	1.796	0.107	0.107	1.8123	4.04×10^{-1}	$<1\sigma$	0 ☆	0 ☆
④⑤⑦	−1.224	0.134	0.174	3.3680	1.86×10^{-1}	1-2σ	1 ☆	1 ☆
④⑤⑧	−0.479	0.420	0.420	1.1892	5.52×10^{-1}	$<1\sigma$	4 ☆	4 ☆
④⑤⑨	−0.009	0.387	0.738	7.2735	2.63×10^{-2}	2-3σ	4 ☆	4 ☆
④⑤⑫	−1.916	0.239	0.530	9.8071	7.42×10^{-3}	2-3σ	0 ☆	1 ☆
④⑤⑮	−1.916	0.239	0.530	9.8071	7.42×10^{-3}	2-3σ	0 ☆	1 ☆
④⑦⑧	−1.218	0.132	0.153	2.6786	2.62×10^{-1}	1-2σ	1 ☆	1 ☆
④⑦⑨	−1.186	0.128	0.336	13.7154	1.05×10^{-3}	3-4σ	1 ☆	2 ☆
④⑦⑫	−1.493	0.140	0.366	13.7339	1.04×10^{-3}	3-4σ	0 ☆	1 ☆
④⑦⑮	−1.493	0.140	0.366	13.7339	1.04×10^{-3}	3-4σ	0 ☆	1 ☆
④⑧⑨	−0.205	0.360	0.704	7.6597	2.17×10^{-2}	2-3σ	4 ☆	4 ☆
④⑧⑫	−1.876	0.235	0.518	9.7241	7.73×10^{-3}	2-3σ	0 ☆	1 ☆
④⑧⑮	−1.876	0.235	0.518	9.7241	7.73×10^{-3}	2-3σ	0 ☆	1 ☆
④⑨⑫	−1.753	0.236	0.808	23.5452	7.71×10^{-6}	4-5σ	0 ☆	2 ☆
④⑨⑮	−1.753	0.236	0.808	23.5452	7.71×10^{-6}	4-5σ	0 ☆	2 ☆
④⑫⑮	−2.095	0.160	0.357	9.9977	6.75×10^{-3}	2-3σ	0 ☆	0 ☆
⑤⑥⑨	1.579	0.169	0.182	2.3369	3.11×10^{-1}	1-2σ	0 ☆	0 ☆
⑤⑥⑬	1.708	0.101	0.118	2.7765	2.50×10^{-1}	1-2σ	0 ☆	0 ☆
⑤⑥⑭	1.708	0.101	0.118	2.7765	2.50×10^{-1}	1-2σ	0 ☆	0 ☆
⑤⑦⑧	−1.267	0.143	0.143	0.9609	6.18×10^{-1}	$<1\sigma$	1 ☆	1 ☆
⑤⑦⑨	−1.226	0.137	0.339	12.1760	2.27×10^{-3}	3-4σ	1 ☆	2 ☆
⑤⑦⑫	−1.565	0.144	0.326	10.2336	6.00×10^{-3}	2-3σ	0 ☆	1 ☆
⑤⑦⑮	−1.565	0.144	0.326	10.2336	6.00×10^{-3}	2-3σ	0 ☆	1 ☆
⑤⑧⑨	−0.011	0.609	1.241	8.3210	1.56×10^{-2}	2-3σ	4 ☆	3 ☆
⑤⑧⑫	−2.202	0.207	0.207	1.9114	3.85×10^{-1}	$<1\sigma$	0 ☆	0 ☆
⑤⑧⑮	−2.202	0.207	0.207	1.9114	3.85×10^{-1}	$<1\sigma$	0 ☆	0 ☆
⑤⑨⑫	−2.060	0.239	0.701	17.2299	1.81×10^{-4}	3-4σ	0 ☆	1 ☆
⑤⑨⑬	1.859	0.171	0.171	1.8552	3.96×10^{-1}	$<1\sigma$	0 ☆	0 ☆
⑤⑨⑭	1.859	0.171	0.171	1.8552	3.96×10^{-1}	$<1\sigma$	0 ☆	0 ☆
⑤⑨⑮	−2.060	0.239	0.701	17.2299	1.81×10^{-4}	3-4σ	0 ☆	1 ☆
⑤⑫⑮	−2.238	0.142	0.142	0.8100	6.67×10^{-1}	$<1\sigma$	0 ☆	0 ☆
⑤⑬⑭	1.836	0.118	0.118	0.8100	6.67×10^{-1}	$<1\sigma$	0 ☆	0 ☆
⑥⑨⑬	1.715	0.101	0.128	3.2120	2.01×10^{-1}	1-2σ	0 ☆	0 ☆
⑥⑨⑭	1.715	0.101	0.128	3.2120	2.01×10^{-1}	1-2σ	0 ☆	0 ☆
⑥⑬⑭	1.746	0.085	0.093	2.4398	2.95×10^{-1}	1-2σ	0 ☆	0 ☆
⑦⑧⑨	−1.220	0.135	0.324	11.4877	3.20×10^{-3}	2-3σ	1 ☆	2 ☆
⑦⑧⑫	−1.553	0.143	0.316	9.8089	7.41×10^{-3}	2-3σ	0 ☆	1 ☆
⑦⑧⑮	−1.553	0.143	0.316	9.8089	7.41×10^{-3}	2-3σ	0 ☆	1 ☆
⑦⑨⑫	−1.509	0.143	0.478	22.3006	1.44×10^{-5}	4-5σ	0 ☆	1 ☆
⑦⑨⑮	−1.509	0.143	0.478	22.3006	1.44×10^{-5}	4-5σ	0 ☆	1 ☆
⑦⑫⑮	−1.720	0.126	0.340	14.4854	7.15×10^{-4}	3-4σ	0 ☆	1 ☆
⑧⑫⑮	−2.220	0.144	0.144	1.1166	5.72×10^{-1}	$<1\sigma$	0 ☆	0 ☆
⑨⑫⑮	−2.164	0.153	0.444	16.7466	2.31×10^{-4}	3-4σ	0 ☆	1 ☆
⑨⑬⑭	1.847	0.119	0.119	1.0543	5.90×10^{-1}	$<1\sigma$	0 ☆	0 ☆

TABLE VII. Tripair combinations in IH.

		$\sigma_{\overline{\cos\delta}}$		Self-consistency			Natural limit consistency	
Pairs included	$\overline{\cos\delta}$	Unscaled	Scaled	χ^2	p-value	Exclusion level	Unscaled	Scaled
①②③	0.146	0.145	0.273	7.0365	2.97×10^{-2}	2-3σ	5 ☆	5 ☆
①②④	0.018	0.148	0.441	17.7885	1.37×10^{-4}	3-4σ	5 ☆	4 ☆
①②⑤	0.166	0.155	0.364	11.0783	3.93×10^{-3}	2-3σ	5 ☆	4 ☆
①②⑧	0.068	0.153	0.528	23.9351	6.35×10^{-6}	4-5σ	5 ☆	4 ☆
①②⑨	0.164	0.152	0.282	6.8992	3.18×10^{-2}	2-3σ	5 ☆	4 ☆
①③④	0.130	0.149	0.381	13.0255	1.48×10^{-3}	3-4σ	5 ☆	4 ☆
①③⑤	0.305	0.135	0.208	4.7194	9.44×10^{-2}	1-2σ	5 ☆	5 ☆
①③⑧	0.196	0.150	0.460	18.6676	8.84×10^{-5}	3-4σ	5 ☆	4 ☆
①③⑨	0.299	0.134	0.134	0.5921	7.44×10^{-1}	$<1\sigma$	5 ☆	5 ☆
①④⑤	0.149	0.159	0.466	17.0946	1.94×10^{-4}	3-4σ	5 ☆	4 ☆
①④⑧	0.045	0.157	0.607	29.7848	3.41×10^{-7}	$>5\sigma$	5 ☆	4 ☆
①④⑨	0.147	0.156	0.397	12.9129	1.57×10^{-3}	3-4σ	5 ☆	4 ☆
①⑤⑧	0.233	0.154	0.517	22.6448	1.21×10^{-5}	4-5σ	5 ☆	4 ☆
①⑤⑨	0.330	0.136	0.198	4.2590	1.19×10^{-1}	1-2σ	5 ☆	5 ☆
①⑧⑨	0.227	0.152	0.462	18.4770	9.72×10^{-5}	3-4σ	5 ☆	4 ☆
②③④	−0.657	0.228	0.416	6.6486	3.60×10^{-2}	2-3σ	4 ☆	3 ☆
②③⑤	−0.373	0.272	0.483	6.3183	4.25×10^{-2}	2-3σ	4 ☆	4 ☆
②③⑥	0.978	0.108	0.403	27.7377	9.48×10^{-7}	4-5σ	3 ☆	3 ☆
②③⑧	−0.642	0.248	0.667	14.4852	7.15×10^{-4}	3-4σ	4 ☆	3 ☆
②③⑨	−0.313	0.255	0.287	2.5383	2.81×10^{-1}	1-2σ	4 ☆	4 ☆
②④⑤	−0.994	0.251	0.436	6.0217	4.92×10^{-2}	1-2σ	3 ☆	3 ☆
②④⑦	−1.404	0.147	0.271	6.8407	3.27×10^{-2}	2-3σ	1 ☆	1 ☆
②④⑧	−1.148	0.234	0.537	10.5627	5.09×10^{-3}	2-3σ	2 ☆	2 ☆
②④⑨	−0.855	0.248	0.366	4.3594	1.13×10^{-1}	1-2σ	3 ☆	3 ☆
②④⑫	−1.565	0.108	0.233	9.3169	9.48×10^{-3}	2-3σ	0 ☆	1 ☆
②④⑮	−1.565	0.108	0.233	9.3169	9.48×10^{-3}	2-3σ	0 ☆	1 ☆
②⑤⑥	1.036	0.111	0.410	27.1098	1.30×10^{-6}	4-5σ	2 ☆	2 ☆
②⑤⑦	−1.383	0.154	0.359	10.8413	4.42×10^{-3}	2-3σ	1 ☆	1 ☆
②⑤⑧	−1.036	0.270	0.707	13.6998	1.06×10^{-3}	3-4σ	2 ☆	2 ☆
②⑤⑨	−0.581	0.319	0.537	5.6874	5.82×10^{-2}	1-2σ	4 ☆	3 ☆
②⑥⑨	1.023	0.111	0.382	23.8825	6.52×10^{-6}	4-5σ	2 ☆	2 ☆
②⑦⑧	−1.445	0.152	0.394	13.3598	1.26×10^{-3}	3-4σ	1 ☆	1 ☆
②⑦⑨	−1.303	0.150	0.363	11.6918	2.89×10^{-3}	2-3σ	1 ☆	2 ☆
②⑦⑫	−1.590	0.094	0.205	9.5075	8.62×10^{-3}	2-3σ	0 ☆	1 ☆
②⑦⑮	−1.590	0.094	0.205	9.5075	8.62×10^{-3}	2-3σ	0 ☆	1 ☆
②⑧⑨	−0.878	0.269	0.665	12.1759	2.27×10^{-3}	3-4σ	3 ☆	3 ☆
②⑫⑮	−1.626	0.085	0.190	10.0760	6.49×10^{-3}	2-3σ	0 ☆	0 ☆
③④⑤	−0.553	0.300	0.688	10.5431	5.14×10^{-3}	2-3σ	4 ☆	3 ☆
③④⑧	−0.839	0.287	0.844	17.3020	1.75×10^{-4}	3-4σ	3 ☆	3 ☆
③④⑨	−0.441	0.273	0.518	7.1859	2.75×10^{-2}	2-3σ	4 ☆	4 ☆
③⑤⑥	1.116	0.115	0.264	10.4852	5.29×10^{-3}	2-3σ	1 ☆	2 ☆
③⑤⑧	−0.490	0.347	1.049	18.2751	1.08×10^{-4}	3-4σ	4 ☆	3 ☆
③⑤⑨	0.022	0.362	0.525	4.2024	1.22×10^{-1}	1-2σ	4 ☆	4 ☆
③⑤⑬	1.954	0.196	0.710	26.2075	2.04×10^{-6}	4-5σ	0 ☆	1 ☆
③⑤⑭	1.954	0.196	0.710	26.2075	2.04×10^{-6}	4-5σ	0 ☆	1 ☆
③⑥⑧	1.055	0.112	0.450	32.3693	9.36×10^{-8}	$>5\sigma$	2 ☆	2 ☆
③⑥⑨	1.102	0.114	0.219	7.4249	2.44×10^{-2}	2-3σ	2 ☆	2 ☆
③⑥⑬	1.435	0.111	0.446	32.3245	9.57×10^{-8}	$>5\sigma$	0 ☆	2 ☆
③⑥⑭	1.435	0.111	0.446	32.3245	9.57×10^{-8}	$>5\sigma$	0 ☆	2 ☆
③⑧⑨	−0.367	0.310	0.839	14.7075	6.40×10^{-4}	3-4σ	4 ☆	3 ☆
③⑨⑬	1.878	0.192	0.688	25.6714	2.66×10^{-6}	4-5σ	0 ☆	1 ☆

(Table continued)

TABLE VII. (*Continued*)

		$\sigma_{\overline{\cos\delta}}$		Self-consistency			Natural limit consistency	
Pairs included	$\overline{\cos\delta}$	Unscaled	Scaled	χ^2	p-value	Exclusion level	Unscaled	Scaled
③⑨⑭	1.878	0.192	0.688	25.6714	2.66×10^{-6}	4-5σ	0 ☆	1 ☆
③⑬⑭	2.149	0.145	0.506	24.2126	5.52×10^{-6}	4-5σ	0 ☆	1 ☆
④⑤⑦	−1.648	0.160	0.231	4.1987	1.23×10^{-1}	1-2σ	0 ☆	1 ☆
④⑤⑧	−1.949	0.381	0.814	9.1169	1.05×10^{-2}	2-3σ	1 ☆	1 ☆
④⑤⑨	−0.942	0.401	0.826	8.4675	1.45×10^{-2}	2-3σ	3 ☆	3 ☆
④⑤⑫	−1.704	0.131	0.191	4.2477	1.20×10^{-1}	1-2σ	0 ☆	0 ☆
④⑤⑮	−1.704	0.131	0.191	4.2477	1.20×10^{-1}	1-2σ	0 ☆	0 ☆
④⑦⑧	−1.704	0.153	0.253	5.4225	6.65×10^{-2}	1-2σ	0 ☆	1 ☆
④⑦⑨	−1.537	0.163	0.303	6.9161	3.15×10^{-2}	2-3σ	0 ☆	1 ☆
④⑦⑫	−1.686	0.102	0.102	0.1161	9.44×10^{-1}	$< 1\sigma$	0 ☆	0 ☆
④⑦⑮	−1.686	0.102	0.102	0.1161	9.44×10^{-1}	$< 1\sigma$	0 ☆	0 ☆
④⑧⑨	−1.385	0.358	0.892	12.3840	2.05×10^{-3}	3-4σ	1 ☆	2 ☆
④⑧⑫	−1.744	0.135	0.218	5.2530	7.23×10^{-2}	1-2σ	0 ☆	0 ☆
④⑧⑮	−1.744	0.135	0.218	5.2530	7.23×10^{-2}	1-2σ	0 ☆	0 ☆
④⑨⑫	−1.642	0.121	0.237	7.6432	2.19×10^{-2}	2-3σ	0 ☆	1 ☆
④⑨⑮	−1.642	0.121	0.237	7.6432	2.19×10^{-2}	2-3σ	0 ☆	1 ☆
④⑫⑮	−1.709	0.095	0.095	0.0669	9.67×10^{-1}	$< 1\sigma$	0 ☆	0 ☆
⑤⑥⑨	1.172	0.122	0.202	5.4673	6.50×10^{-2}	1-2σ	1 ☆	2 ☆
⑤⑥⑬	1.523	0.114	0.413	26.1617	2.08×10^{-6}	4-5σ	0 ☆	1 ☆
⑤⑥⑭	1.523	0.114	0.413	26.1617	2.08×10^{-6}	4-5σ	0 ☆	1 ☆
⑤⑦⑧	−1.718	0.163	0.355	9.5417	8.47×10^{-3}	2-3σ	0 ☆	1 ☆
⑤⑦⑨	−1.527	0.175	0.412	11.0753	3.94×10^{-3}	2-3σ	0 ☆	1 ☆
⑤⑦⑫	−1.691	0.105	0.153	4.2574	1.19×10^{-1}	1-2σ	0 ☆	0 ☆
⑤⑦⑮	−1.691	0.105	0.153	4.2574	1.19×10^{-1}	1-2σ	0 ☆	0 ☆
⑤⑧⑨	−1.111	0.543	1.541	16.1149	3.17×10^{-4}	3-4σ	2 ☆	2 ☆
⑤⑧⑫	−1.761	0.144	0.310	9.3076	9.53×10^{-3}	2-3σ	0 ☆	1 ☆
⑤⑧⑮	−1.761	0.144	0.310	9.3076	9.53×10^{-3}	2-3σ	0 ☆	1 ☆
⑤⑨⑫	−1.645	0.126	0.307	11.8146	2.72×10^{-3}	2-3σ	0 ☆	1 ☆
⑤⑨⑬	2.287	0.197	0.435	9.7314	7.71×10^{-3}	2-3σ	0 ☆	1 ☆
⑤⑨⑭	2.287	0.197	0.435	9.7314	7.71×10^{-3}	2-3σ	0 ☆	1 ☆
⑤⑨⑮	−1.645	0.126	0.307	11.8146	2.72×10^{-3}	2-3σ	0 ☆	1 ☆
⑤⑫⑮	−1.714	0.098	0.142	4.1839	1.23×10^{-1}	1-2σ	0 ☆	0 ☆
⑤⑬⑭	2.386	0.134	0.193	4.1839	1.23×10^{-1}	1-2σ	0 ☆	0 ☆
⑥⑨⑬	1.503	0.113	0.395	24.1799	5.62×10^{-6}	4-5σ	0 ☆	1 ☆
⑥⑨⑭	1.503	0.113	0.395	24.1799	5.62×10^{-6}	4-5σ	0 ☆	1 ☆
⑥⑬⑭	1.711	0.101	0.419	34.3870	3.41×10^{-8}	$> 5\sigma$	0 ☆	1 ☆
⑦⑧⑨	−1.604	0.166	0.422	12.8434	1.63×10^{-3}	3-4σ	0 ☆	1 ☆
⑦⑧⑫	−1.716	0.106	0.173	5.3578	6.86×10^{-2}	1-2σ	0 ☆	0 ☆
⑦⑧⑮	−1.716	0.106	0.173	5.3578	6.86×10^{-2}	1-2σ	0 ☆	0 ☆
⑦⑨⑫	−1.648	0.102	0.199	7.6314	2.20×10^{-2}	2-3σ	0 ☆	0 ☆
⑦⑨⑮	−1.648	0.102	0.199	7.6314	2.20×10^{-2}	2-3σ	0 ☆	0 ☆
⑦⑫⑮	−1.699	0.083	0.083	0.0919	9.55×10^{-1}	$< 1\sigma$	0 ☆	0 ☆
⑧⑫⑮	−1.737	0.100	0.161	5.1804	7.50×10^{-2}	1-2σ	0 ☆	0 ☆
⑨⑫⑮	−1.676	0.093	0.184	7.7679	2.06×10^{-2}	2-3σ	0 ☆	0 ☆
⑨⑬⑭	2.339	0.137	0.230	5.6794	5.84×10^{-2}	1-2σ	0 ☆	0 ☆

TABLE VIII. Deviations from the maximal CP violation in NH.

Pairs included	Exclusion level of self-consistency	Natural limit consistency		Deviation from the maximal CP violation	
		Unscaled	Scaled	Unscaled	Scaled
②③④	1-2σ	4 ☆	4 ☆	< 1σ	< 1σ
①②⑤	< 1σ	5 ☆	5 ☆	< 1σ	< 1σ
①②③④⑤	1-2σ	5☆	5☆	< 1σ	< 1σ
①	⋯	5 ☆	⋯	< 1σ	⋯
②	⋯	4 ☆	⋯	< 1σ	⋯
④	⋯	4 ☆	⋯	< 1σ	⋯
⑧	⋯	3 ☆	⋯	1-2σ	⋯
①②	1-2σ	5 ☆	5 ☆	< 1σ	< 1σ
①③	2-3σ	5 ☆	4 ☆	< 1σ	< 1σ
①④	< 1σ	5 ☆	5 ☆	1-2σ	1-2σ
①⑤	< 1σ	5 ☆	5 ☆	1-2σ	1-2σ
①⑧	< 1σ	5 ☆	5 ☆	1-2σ	1-2σ
①⑨	2-3σ	5 ☆	4 ☆	< 1σ	< 1σ
②③	1-2σ	4 ☆	4 ☆	2-3σ	2-3σ
②④	1-2σ	5 ☆	4 ☆	< 1σ	< 1σ
②⑤	< 1σ	4 ☆	4 ☆	1-2σ	1-2σ
②⑧	1-2σ	4 ☆	4 ☆	< 1σ	< 1σ
②⑨	2-3σ	4 ☆	3 ☆	2-3σ	1-2σ
③④	2-3σ	4 ☆	4 ☆	< 1σ	< 1σ
③⑧	2-3σ	4 ☆	3 ☆	< 1σ	< 1σ
④⑤	< 1σ	4 ☆	4 ☆	< 1σ	< 1σ
④⑧	< 1σ	4 ☆	4 ☆	1-2σ	1-2σ
④⑨	2-3σ	4 ☆	4 ☆	< 1σ	< 1σ
⑤⑧	< 1σ	3 ☆	3 ☆	< 1σ	< 1σ
⑧⑨	2-3σ	4 ☆	3 ☆	< 1σ	< 1σ
①②③	1-2σ	5 ☆	4 ☆	< 1σ	< 1σ
①②④	< 1σ	5 ☆	5 ☆	1-2σ	1-2σ
①②⑧	1-2σ	5 ☆	5 ☆	1-2σ	1-2σ
①②⑨	2-3σ	5 ☆	4 ☆	< 1σ	< 1σ
①③④	1-2σ	5 ☆	5 ☆	< 1σ	< 1σ
①③⑤	1-2σ	5 ☆	4 ☆	< 1σ	< 1σ
①③⑧	2-3σ	5 ☆	4 ☆	< 1σ	< 1σ
①③⑨	2-3σ	5 ☆	4 ☆	< 1σ	< 1σ
①④⑤	< 1σ	5 ☆	5 ☆	1-2σ	1-2σ
①④⑧	< 1σ	5 ☆	5 ☆	1-2σ	1-2σ
①④⑨	2-3σ	5 ☆	4 ☆	1-2σ	< 1σ
①⑤⑧	< 1σ	5 ☆	5 ☆	1-2σ	1-2σ
①⑤⑨	2-3σ	5 ☆	4 ☆	< 1σ	< 1σ
①⑧⑨	2-3σ	5 ☆	4 ☆	1-2σ	< 1σ
②③⑤	< 1σ	4 ☆	4 ☆	2-3σ	2-3σ
②③⑧	1-2σ	4 ☆	4 ☆	1-2σ	< 1σ
②③⑨	1-2σ	4 ☆	3 ☆	2-3σ	1-2σ
②④⑤	< 1σ	5 ☆	4 ☆	< 1σ	< 1σ
②④⑧	1-2σ	4 ☆	4 ☆	< 1σ	< 1σ
②④⑨	2-3σ	4 ☆	4 ☆	< 1σ	< 1σ
②⑤⑧	1-2σ	4 ☆	4 ☆	< 1σ	< 1σ
②⑤⑨	1-2σ	4 ☆	3 ☆	2-3σ	1-2σ
②⑧⑨	2-3σ	4 ☆	3 ☆	< 1σ	< 1σ
③④⑤	1-2σ	4 ☆	4 ☆	< 1σ	< 1σ
③④⑧	2-3σ	5 ☆	4 ☆	< 1σ	< 1σ
③④⑨	2-3σ	4 ☆	3 ☆	< 1σ	< 1σ
③⑤⑧	2-3σ	4 ☆	3 ☆	< 1σ	< 1σ
③⑧⑨	2-3σ	3 ☆	3 ☆	1-2σ	< 1σ

(Table continued)

TABLE VIII. (*Continued*)

Pairs included	Exclusion level of self-consistency	Natural limit consistency		Deviation from the maximal *CP* violation	
		Unscaled	Scaled	Unscaled	Scaled
④⑤⑧	$< 1\sigma$	4 ☆	4 ☆	1-2σ	1-2σ
④⑤⑨	2-3σ	4 ☆	4 ☆	$< 1\sigma$	$< 1\sigma$
④⑧⑨	2-3σ	4 ☆	4 ☆	$< 1\sigma$	$< 1\sigma$
⑤⑧⑨	2-3σ	4 ☆	3 ☆	$< 1\sigma$	$< 1\sigma$

TABLE IX. Deviations from the maximal *CP* violation in IH.

Pairs included	Exclusion level of self-consistency	Natural limit consistency		Deviation from the maximal *CP* violation	
		Unscaled	Scaled	Unscaled	Scaled
②③④	2-3σ	4 ☆	3 ☆	2-3σ	1-2σ
①②⑤	2-3σ	5 ☆	4 ☆	1-2σ	$< 1\sigma$
①	...	5 ☆	...	$2-3\sigma$	...
②	...	3 ☆	...	2-3σ	...
③	...	4 ☆	...	$< 1\sigma$	...
⑨	...	3 ☆	...	$< 1\sigma$	...
①②	2-3σ	5 ☆	4 ☆	1-2σ	$< 1\sigma$
①③	$< 1\sigma$	5 ☆	5 ☆	2-3σ	2-3σ
①⑤	2-3σ	5 ☆	4 ☆	2-3σ	1-2σ
①⑨	$< 1\sigma$	5 ☆	5 ☆	2-3σ	2-3σ
②③	1-2σ	4 ☆	4 ☆	1-2σ	$< 1\sigma$
②④	1-2σ	3 ☆	3 ☆	3-4σ	2-3σ
②⑤	2-3σ	3 ☆	3 ☆	2-3σ	1-2σ
②⑨	1-2σ	4 ☆	4 ☆	1-2σ	1-2σ
③④	2-3σ	4 ☆	3 ☆	1-2σ	$< 1\sigma$
③⑤	2-3σ	4 ☆	4 ☆	$< 1\sigma$	$< 1\sigma$
③⑨	$< 1\sigma$	4 ☆	4 ☆	$< 1\sigma$	$< 1\sigma$
④⑨	2-3σ	3 ☆	3 ☆	2-3σ	1-2σ
⑤⑨	2-3σ	4 ☆	3 ☆	$< 1\sigma$	$< 1\sigma$
①②③	2-3σ	5 ☆	5 ☆	1-2σ	$< 1\sigma$
①②⑨	2-3σ	5 ☆	4 ☆	1-2σ	$< 1\sigma$
①③⑤	1-2σ	5 ☆	5 ☆	2-3σ	1-2σ
①③⑨	$< 1\sigma$	5 ☆	5 ☆	2-3σ	2-3σ
①⑤⑨	1-2σ	5 ☆	5 ☆	2-3σ	1-2σ
②③⑤	2-3σ	4 ☆	4 ☆	1-2σ	$< 1\sigma$
②③⑨	1-2σ	4 ☆	4 ☆	1-2σ	1-2σ
②④⑤	1-2σ	3 ☆	3 ☆	3-4σ	2-3σ
②④⑨	1-2σ	3 ☆	3 ☆	3-4σ	2-3σ
②⑤⑨	1-2σ	4 ☆	3 ☆	1-2σ	1-2σ
③④⑤	2-3σ	4 ☆	3 ☆	1-2σ	$< 1\sigma$
③④⑨	2-3σ	4 ☆	4 ☆	1-2σ	$< 1\sigma$
③⑤⑨	1-2σ	4 ☆	4 ☆	$< 1\sigma$	$< 1\sigma$
④⑤⑨	2-3σ	3 ☆	3 ☆	2-3σ	1-2σ

It is worthwhile to point out that in the search for new mixing patterns, two pairs may automatically imply a third one. For example, pairs ②③ contain the same matrix element U_{22}, and it leads to another pair $|U_{12}| = |U_{32}|$, which is ④. Another example is that any two of the pairs in the μ-τ symmetry would lead to the third one due to the unitarity of the mixing matrix. However, the combined results of ②③ are different from that of ④, according to Tables IV, V and Fig. 2.

The reason is that the implication of a third pair is based on the condition that the two pairs hold precisely, i.e., the modulus of elements of each pair are precisely equal to each other. But for experimental results, pairs hold with errors. Therefore, pairs ②③ do not necessarily imply the third pair ④. Another example can be seen from Eq. (23): while pairs ⑥⑧ both hold in 3σ errors, the corresponding "third pair," i.e., $|U_{21}| = |U_{23}|$ does not hold in 3σ error. Therefore we do not consider correlations among pairs in our discussion due to the presence of experimental errors.

While a total number of 55 cases are listed in Tables IV and V, not all of them are self-consistent. When regarding 3σ as a dividing line of self-consistency (i.e., regarding p-value > 0.0027 as self-consistent), there are 39 cases in NH and 33 cases in IH that are self-consistent. The overall self-consistency in NH exceeds that in IH slightly.

Moreover, many cases give constraints on $\cos\delta$ that are not consistent with natural limit $\cos\delta \in [-1, 1]$. Number of cases with central value $\overline{\cos\delta} \in [-1, 1]$—namely cases over 3☆ level—are less than half of the total. The number in NH is 25, and in IH is 23.

The numbers of cases both self-consistent and consistent with natural limit are 18 in NH and 13 in IH. If we have a close look at these cases, we would find them consistent with the maximal *CP* violation in 3σ error range, except for ②④ in IH and with an unscaled error. Especially, within all 18 cases in NH, 11 of them are compatible with the maximal *CP* violation within 1σ range, whether the errors are scaled or not. The detailed results are listed in Table VIII (NH) and Table IX (IH) in Sec. IV.

C. Tripair combination

Similarly we consider all tripair combinations among ①–⑨ & ⑫ – ⑮. For there are too many combination cases (286 in total) but most of them are not self-consistent (over 3σ exclusion level), we do not list combinations over 3σ exclusion in both NH and IH. The results are listed in Table VI (NH) and Table VII (IH).

Self-consistent cases are not necessarily the majority, and the numbers of them are only 77 in NH and 61 in IH. As is the same with bipair combination, the number in NH exceeds that in IH slightly. On the other hand, cases over 3☆ level have a number of 130 in NH and 146 in IH, nearly half of the total. There are 39 (NH) and 16 (IH) cases satisfying the two conditions simultaneously.

Similarly, most of the cases satisfying the two conditions are compatible with the maximal *CP* violation in 3σ range. Exceptions are ②④⑤ and ②④⑨ in IH and with unscaled error. Especially, with unscaled errors, 21 of all 39 cases in NH are compatible with the maximal *CP* violation within 1σ range. When errors are scaled, the number increases to 25. The detailed results are listed in Table VIII (NH) and Table IX (IH) in Sec. IV.

IV. CHECKING THE MAXIMAL *CP* VIOLATION

In this section we compare constraints by the pairs to the maximal *CP* violation. We include all cases that are self-consistent and over 3☆ level and discuss separately in NH and IH.

For each case, we list the deviation from the maximal *CP* violation for both unscaled and scaled errors. The results are listed in Table VIII (NH) and Table IX (IH).

As is mentioned in the previous section, combinations in NH tend to be slightly more self-consistent. All of the cases in NH are compatible with the maximal *CP* violation in 3σ range, while 3 exceptions are found in IH with errors unscaled. What is more, cases in NH seem to be more consistent with the maximal *CP* violation, since the majority of the cases (36 in 57 cases) deviate from the maximal *CP* violation within 1σ range, even when errors are not scaled. However, in IH only 6 in all 33 cases are compatible with the maximal *CP* violation in 1σ with unscaled errors. When errors are scaled, the number increases to 15.

V. CONCLUSIONS

From the examination of the pairs, we find that some of the pairs are not so consistent with the experimental results, and that some of the pairs are not consistent with each other when combined together. When seeking for new mixing patterns, these cases are not good choices. On the other hand, some of the cases agree with current experimental results and are self-consistent as well, such as the trimaximal mixing case and the μ-τ symmetry. They can act as a starting point when constructing a new mixing pattern. While the first case of the bipair mixing (i.e., ④⑧) is a good choice only in NH and the second case ③⑨ is good only in IH.

In addition, the examination provides information about the constraint on the *CP* phase by pairs. Especially, among cases that are both self-consistent and consistent with the natural limit, a majority of them are compatible with the maximal *CP* violation. It is necessary to point out that when deriving the range of the *CP* phase in Fig. 3, we adopt the assumption of $\delta \in [0°, 180°]$. When extending to $[-180°, 180°]$, the results of $\delta \in [\delta_1, \delta_2]$ should also be extended to be $\delta \in [-\delta_2, -\delta_1]$ and $[\delta_1, \delta_2]$. Therefore, the results agree with the hint of the maximal *CP* violation with $\delta \sim -90°$ from analysis in Ref. [3] and global fit results.

ACKNOWLEDGMENTS

This work is supported by the Principal Fund for Undergraduate Research at Peking University. It is also supported by the National Natural Science Foundation of China (Grants No. 11035003 and No. 11120101004) and by the National Fund for Fostering Talents of Basic Science (Grants No. J1103205 and No. J1103206).

[1] B. Pontecorvo, Sov. Phys. JETP **26**, 984 (1968); Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

[2] L. L. Chau and W. Y. Keung, Phys. Rev. Lett. **53**, 1802 (1984).

[3] K. Abe *et al.* (T2K Collaboration), Phys. Rev. Lett. **112**, 061802 (2014).

[4] M. C. Gonzalez-Garcia, M. Maltoni, and T. Schwetz, J. High Energy Phys. 11 (2014) 052.

[5] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. B **349**, 137 (1995); **396**, 186 (1997); **458**, 79 (1999); James D. Bjorken, P. F. Harrison, and W. G. Scott, Phys. Rev. D **74**, 073012 (2006).

[6] T. Fukuyama and H. Nishiura, arXiv:hep-ph/9702253; R. N. Mohapatra and S. Nussinov, Phys. Rev. D **60**, 013002 (1999); E. Ma and M. Raidal, Phys. Rev. Lett. **87**, 011802 (2001); **87**, 159901(E) (2001); K. R. S. Balaji, W. Grimus, and T. Schwetz, Phys. Lett. B **508**, 301 (2001); C. S. Lam, Phys. Lett. B **507**, 214 (2001); Phys. Rev. D **71**, 093001 (2005); W. Grimus and L. Lavoura, J. High Energy Phys. 07 (2001) 045; Acta Phys. Pol. B **32**, 3719 (2001); E. Ma, Phys. Rev. D **66**, 117301 (2002); A. Ghosal, Mod. Phys. Lett. A **19**, 2579 (2004); S. Choubey and W. Rodejohann, Eur. Phys. J. C **40**, 259 (2005); T. Kitabayashi and M. Yasue, Phys. Lett. B **621**, 133 (2005); R. N. Mohapatra and W. Rodejohann, Phys. Rev. D **72**, 053001 (2005); I. Aizawa, T. Kitabayashi, and M. Yasue, Nucl. Phys. **B728**, 220 (2005); I. Aizawa and M. Yasue, Phys. Rev. D **73**, 015002 (2006); K. Fuki and M. Yasue, Phys. Rev. D **73**, 055014 (2006); N. Haba and W. Rodejohann, Phys. Rev. D **74**, 017701 (2006); Z.-z. Xing, H. Zhang, and S. Zhou, Phys. Lett. B **641**, 189 (2006); K. Fuki and M. Yasue, Nucl. Phys. **B783**, 31 (2007); S. Gupta, A. S. Joshipura, and K. M. Patel, J. High Energy Phys. 09 (2013) 035.

[7] P. F. Harrison and W. G. Scott, Phys. Lett. B **547**, 219 (2002); W. Grimus and L. Lavoura, Phys. Lett. B **579**, 113 (2004); A. S. Joshipura, B. P. Kodrani, and K. M. Patel, Phys. Rev. D **79**, 115017 (2009); S. Gupta, A. S. Joshipura, and K. M. Patel, Phys. Rev. D **85**, 031903 (2012); R. N. Mohapatra and C. C. Nishi, Phys. Rev. D **86**, 073007 (2012); F. Feruglio, C. Hagedorn, and R. Ziegler, J. High Energy Phys. 07 (2013) 027; M. Holthausen, M. Lindner, and M. A. Schmidt, J. High Energy Phys. 04 (2013) 122.

[8] T. Kitabayashi and M. Yasuè, Phys. Lett. B **696**, 478 (2011).

[9] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. B **458**, 79 (1999); **530**, 167 (2002); Z. Z. Xing, Phys. Lett. B **533**, 85 (2002); P. F. Harrison and W. G. Scott, Phys. Lett. B **557**, 76 (2003); X.-G. He and A. Zee, Phys. Lett. B **560**, 87 (2003); N. Li and B.-Q. Ma, Phys. Rev. D **71**, 017302 (2005); see also L. Wolfenstein, Phys. Rev. D **18**, 958 (1978); Y. Yamanaka, H. Sugawara, and S. Pakvasa, Phys. Rev. D **25**, 1895 (1982); **29**, 2135(E) (1984).

[10] F. Vissani, arXiv:hep-ph/9708483; V. D. Barger, S. Pakvasa, T. J. Weiler, and K. Whisnant, Phys. Lett. B **437**, 107 (1998); A. J. Baltz, A. S. Goldhaber, and M. Goldhaber, Phys. Rev. Lett. **81**, 5730 (1998); I. Stancu and D. V. Ahluwalia, Phys. Lett. B **460**, 431 (1999); H. Georgi and S. L. Glashow, Phys. Rev. D **61**, 097301 (2000); N. Li and B.-Q. Ma, Phys. Lett. B **600**, 248 (2004).

[11] K. Abe *et al.* (T2K Collaboration), Phys. Rev. Lett. **107**, 041801 (2011); P. Adamson *et al.* (MINOS Collaboration), Phys. Rev. Lett. **107**, 181802 (2011); Y. Abe *et al.* (Double Chooz Collaboration), Phys. Rev. Lett. **108**, 131801 (2012); F. P. An *et al.* (DAYA BAY Collaboration), Phys. Rev. Lett. **108**, 171803 (2012); J. K. Ahn *et al.* (RENO Collaboration), Phys. Rev. Lett. **108**, 191802 (2012).

[12] H. Qu and B.-Q. Ma, Phys. Rev. D **88**, 037301 (2013).

[13] Y.-j. Zheng and B.-Q. Ma, Eur. Phys. J. Plus **127**, 7 (2012); X. Zhang and B.-Q. Ma, Phys. Lett. B **710**, 630 (2012); Y. Zhang, X. Zhang, and B.-Q. Ma, Phys. Rev. D **86**, 093019 (2012).

[14] K. A. Olive *et al.* (Particle Data Group), Chin. Phys. C **38**, 090001 (2014).