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Lepton mixing in gauge models

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We reexamine lepton mixing in gauge models by considering two theories within the type I seesaw mechanism, the extended Standard Model, i.e. $SU(2)_L \times U(1)_Y$ with singlet right-handed heavy neutrinos, and the left-right model, $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The former is often used as a simple heuristic approach to masses and mixing of light neutrinos and to leptogenesis, while we consider the latter as an introduction to other left-right symmetric gauge theories like $SO(10)$. We compare lepton mixing in both theories for general parameter space and discuss also some particular cases. In the electroweak broken phase, we study in parallel both models in the current basis (diagonal gauge interactions) and in the mass basis (diagonal mass matrices and mixing in the interaction), and perform the counting of CP-conserving and CP-violating parameters in both bases. We extend the analysis to the Pati-Salam model $SU(4)_C \times SU(2)_L \times SU(2)_R$ and to $SO(10)$. Although specifying the Higgs sector increases the predictive power, in the most general case one has the same parameter structure in the lepton sector for all the left-right symmetric gauge models. We make explicit the differences between the extended Standard Model and the left-right models, in particular CP-violating and lepton-number-violating new terms involving the W_R gauge bosons. As expected, at low energy, the differences in the light neutrino spectrum and mixing appear only beyond leading order in the ratio of the Dirac mass to the right-handed Majorana mass.

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I. INTRODUCTION

In the last years, an impressive experimental progress has been achieved on the neutrino spectrum and mixing. Using this information on the light neutrinos mass matrix m_L, one is tempted to use the inverse of the seesaw formula $M_R = -m_D^t m_L^{-1} m_D$, where m_D is the Dirac neutrino mass matrix, as a window on high-energy neutrino physics, i.e. on the heavy right-handed neutrino mass matrix M_R [1–5].

To use the inverse seesaw formula one needs information on the crucial Dirac mass matrix m_D. It has been often suggested that theoretical information on this matrix can be guessed within the $SO(10)$ grand unification gauge theory [6]. In order to study the whole structure of $SO(10)$ as far as lepton mixing is concerned, we have realized that it is convenient to begin by considering simpler theories that also exhibit left-right (LR) symmetry (for a review, see Ref. [7]).

The simplest gauge theory that has been builded to study lepton mixing is the one that we call the extended Standard Model (ESM), i.e. the Standard Model (SM) $SU(3) \times SU(2)_L \times U(1)_Y$ plus right-handed neutrinos N_R, one per generation, singlet under the SM gauge group. Although this scheme allows us to introduce heavy right-handed neutrinos, it does not exhibit LR symmetry like $SO(10)$.

One main aim of the present paper is to compare lepton mixing in the ESM, on the one hand, with lepton mixing in left-right models like $SO(10)$. Lepton mixing in the ESM has been thoroughly studied in the literature [8–11], especially in Ref. [10] on which the present paper heavily relies, together with the comprehensive review paper [12].

To compare the ESM with left-right gauge theories we have found convenient to consider next the left-right model (LRM) $SU(2)_L \times SU(2)_R \times U_{B-L}(1)$ [13,14], that exhibits a number of interesting new features concerning lepton mixing [15,16]. This gauge group has already an appreciable complexity that will be useful as an introduction for the study of larger LR gauge groups, like the Pati-Salam model $SU(4)_C \times SU(2)_L \times SU(2)_R$ [17], and the grand unified $SO(10)$ gauge group [6].

We will first consider completely general Dirac or Majorana mass matrices consistent with Lorentz invariance, that coincide with mass matrices arising from the most general Higgs structure. We then look for the parameters that can be rotated away, although in a different way in the ESM and the LRM. We will consider the "current basis," in which the interaction Lagrangian $\mathcal{L}_w$ is diagonal, and the "mass basis," in which the mass Lagrangian $\mathcal{L}_m$ is diagonal, and we check that, for a given model, the final number of independent parameters, angles and phases, is the same in both bases.

Some main results exposed below are already known. The purpose of this paper is in part didactic, and in part the understanding a number of particular points. We think it is worth to explain in detail the differences between the extended Standard model and the left-right gauge models as far as lepton mixing is concerned, specially the comparison

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of the interaction Lagrangians of both schemes in the mass basis.

Here below we expose briefly the fermion and gauge boson content of the ESM and LRM. In Secs. II and III we perform the counting of the lepton sector parameters of the ESM and LRM in the current and in the mass bases. For the mass basis, special care is given to the approximation $m_D \ll M_R$, as compared with exact results, and in Sec. IV we recall two different representations proposed in the literature for the Dirac mass matrix m_D. In Sec. V we briefly examine leptogenesis in the ESM and in the LRM. In Sec. VI we summarize the differences between both models for lepton mixing. Section VII is devoted to the extension of our results to other left-right theories, Pati-Salam and $SO(10)$, and in Sec. VIII we conclude. In the Appendix we present some details of the calculations.

A. Gauge boson and fermion content of the gauge models

We now expose the fermion and gauge boson content of the two gauge theories that we consider in detail, the extended Standard Model and the left-right model $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

1. Extended Standard Model

The extended Standard Model (ESM) is just the Standard Model (SM) $SU(3) \times SU(2)_L \times U(1)_Y$ with the addition of one Majorana fermion N_R per generation, singlet under the gauge group.

The fermion content of the model is for quarks

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim \left(\mathbf{3}, \mathbf{2}, \frac{1}{3}\right), \qquad u_R \sim \left(\mathbf{3}, \mathbf{1}, \frac{4}{3}\right), \qquad d_R \sim \left(\mathbf{3}, \mathbf{1}, -\frac{2}{3}\right) \tag{1}$$

and for leptons

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (\mathbf{1}, \mathbf{2}, -1), \quad e_R \sim (\mathbf{1}, \mathbf{1}, -2), \quad N_R \sim (\mathbf{1}, \mathbf{1}, 0) \tag{2}$$

with

$$Q = T_{3L} + \frac{Y}{2}. \tag{3}$$

The gauge bosons are the gluons $(\mathbf{8}, \mathbf{1}, 0)$, the W_L bosons $(\mathbf{1}, \mathbf{3}, 0)$ and the B boson $(\mathbf{1}, \mathbf{1}, 0)$.

The Higgs sector needed to achieve the Spontaneous Symmetry Breaking (SSB) and give masses to the fermions is the usual doublet $\phi \sim (\mathbf{1}, \mathbf{2}, -1)$. The novelty in the ESM with respect to the SM is just the presence of the Majorana N_R singlet. The right-handed fermion N_R can have a large mass, of a different scale than the SM, that can be originated from a Higgs boson singlet relative to the Standard Model $\Phi \sim (\mathbf{1}, \mathbf{1}, 0)$, or simply be a bare mass term,

$$(\mathbf{1}, \mathbf{1}, 0)_f \times (\mathbf{1}, \mathbf{1}, 0)_f = (\mathbf{1}, \mathbf{1}, 0), \tag{4}$$

that, together with the Dirac mass terms,

$$(\mathbf{1}, \mathbf{2}, -1)_f \times (\mathbf{1}, \mathbf{2}, 1)_{\bar{f}} \times (\mathbf{1}, \mathbf{2}, -1)_H = (\mathbf{1}, \mathbf{1}, 0) + \cdots \tag{5}$$

gives the general neutrino mass matrix,

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^t & M_R \end{pmatrix}, \tag{6}$$

where m_D and M_R are, respectively, general complex and complex symmetric matrices.

2. Left-right model

In the LRM model $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the classification of L and R fermions is for quarks

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim \left(\mathbf{3}, \mathbf{2}, \mathbf{1}, \frac{1}{3}\right), \qquad \begin{pmatrix} u_R \\ d_R \end{pmatrix} \sim \left(\mathbf{3}, \mathbf{1}, \mathbf{2}, \frac{1}{3}\right) \tag{7}$$

and for leptons

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (\mathbf{1}, \mathbf{2}, \mathbf{1}, -1), \qquad \begin{pmatrix} N_R \\ e_R \end{pmatrix} \sim (\mathbf{1}, \mathbf{1}, \mathbf{2}, -1) \tag{8}$$

with

$$Q = T_{3L} + T_{3R} + \frac{B - L}{2}. \tag{9}$$

The gauge bosons are the gluons $(\mathbf{8}, \mathbf{1}, \mathbf{1}, 0)$, the W_L bosons $(\mathbf{1}, \mathbf{3}, \mathbf{1}, 0)$, the W_R bosons $(\mathbf{1}, \mathbf{1}, \mathbf{3}, 0)$ and the $B - L$ singlet $(\mathbf{1}, \mathbf{1}, \mathbf{1}, 0)$.

The Higgs fields needed to achieve SSB and the seesaw mechanism are the bidoublet $\phi \sim (\mathbf{1}, \mathbf{2}, \mathbf{2}, 0)$ and the triplet $\Delta_R \sim (\mathbf{1}, \mathbf{1}, \mathbf{3}, 2)$.

The bidoublet, written as

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}, \tag{10}$$

breaks the SM group and gives masses to quarks and leptons through the Yukawa terms

$$\left(\mathbf{3},\mathbf{2},\mathbf{1},\frac{1}{3}\right)_f \times \left(\bar{\mathbf{3}},\mathbf{1},\mathbf{2},-\frac{1}{3}\right)_{\bar{f}} \times (\mathbf{1},\mathbf{2},\mathbf{2},0)_{H,\bar{H}}$$
$$= (\mathbf{1},\mathbf{1},\mathbf{1},0) + \cdots$$
$$(\mathbf{1},\mathbf{2},\mathbf{1},-1)_f \times (\mathbf{1},\mathbf{1},\mathbf{2},1)_{\bar{f}} \times (\mathbf{1},\mathbf{2},\mathbf{2},0)_{H,\bar{H}}$$
$$= (\mathbf{1},\mathbf{1},\mathbf{1},0) + \cdots, \quad (11)$$

with $H = \phi$ and $\bar{H} = \sigma_2 H^* \sigma_2$.

From the vacuum expectation values,

$$\langle \phi_1^0 \rangle = k_1, \qquad \langle \phi_2^0 \rangle = k_2, \quad (12)$$

which can be complex, the Yukawa couplings give the Dirac masses, as in the SM, but with a different pattern. Quark mass matrices m_u, m_d and the Dirac neutrino mass matrix m_D read

$$m_u = pk_1 + qk_2^*, \qquad m_d = pk_2 + qk_1^*$$
$$m_D = rk_1 + sk_2^*, \qquad m_e = rk_2 + sk_1^*, \quad (13)$$

where p, q, r and s are complex Yukawa coupling matrices.

The triplet $H = \Delta_R$ breaks the LR model to the SM and, at the same time, gives a Majorana mass to the right-handed neutrino N_R through the Yukawa term

$$(\mathbf{1},\mathbf{1},\mathbf{2},-1)_f \times (\mathbf{1},\mathbf{1},\mathbf{2},-1)_f \times (\mathbf{1},\mathbf{1},\mathbf{3},2)_H$$
$$= (\mathbf{1},\mathbf{1},\mathbf{1},0) + \cdots \quad (14)$$

$$\langle \Delta_R^0 \rangle = v_R, \qquad M_R = t v_R, \qquad M_R^t = M_R, \quad (15)$$

where t is a complex symmetric Yukawa coupling matrix.

The full neutrino mass matrix has the form

$$\mathcal{M} = \begin{pmatrix} 0 & rk_1 + sk_2^* \\ r^t k_1 + s^t k_2^* & t v_R \end{pmatrix}; \quad (16)$$

i.e., it has the general form (6).

We consider this minimal Higgs content that is necessary in the LRM, and we do not introduce a possible left-handed triplet $\Delta_L = (\mathbf{1},\mathbf{3},\mathbf{1},2)_H$ that could, in principle, contribute to the light neutrino masses.

II. CURRENT BASIS

In what follows, we consider the gauge models in the electroweak broken phase. We only make explicit the charged current terms in the interaction Lagragians of both gauge models.

A. Extended Standard Model

The mass and interaction Lagrangians write, in an obvious compact notation,

$$\mathcal{L}_m = \bar{\nu}_L m_D N_R + \frac{1}{2}\overline{(N_R)^c} M_R N_R + \bar{e}_L m_e e_R + \text{H.c.}$$
$$\mathcal{L}_w = \bar{\nu}_L \gamma_\mu e_L W_L^\mu + \text{H.c.} \quad (17)$$

The matrices m_D and m_e are general complex, each has nine complex parameters, while M_R is general complex symmetric with six complex parameters.

The lepton number assignment $L(N_R) = -L((N_R)^c) = 1$ implies that the Majorana mass term is $|\Delta L| = 2$ while, like for the other fermions, while the Dirac mass term is $|\Delta L| = 0$.

From now on we adopt the following simplifying notation for the real parameters of an arbitrary square complex matrix M, that has $n(m)$ parameters, where n is the total number of real parameters, among which there are $m (m \leq n)$ are phases:

$$\boxed{M \text{ has n(m) real parameters} \leftrightarrow \text{n real parameters, m} \leq \text{n phases}} \quad (18)$$

In this example, m_D and m_e have 18(9) real parameters and M_R has 12(6) real parameters. Therefore, *a priori* one has in this model 30(15) real parameters.

Let us see now that we can reduce the number of independent parameters without modifying the interaction Lagrangian $\mathcal{L}_w$. Diagonalizing m_e and M_R by

$$m_e = V_{eL}^\dagger m_e^{\text{diag}} V_{eR}, \qquad M_R = U_R^t M_R^{\text{diag}} U_R \quad (19)$$

and redefining the fields

$$U_R N_R \to N_R, \quad V_{eR} e_R \to e_R, \quad \begin{pmatrix} V_{eL}\nu_L \\ V_{eL} e_L \end{pmatrix} \to \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad (20)$$

one gets

$$\mathcal{L}_m = \bar{\nu}_L V_{eL} m_D U_R^\dagger N_R + \frac{1}{2}\overline{(N_R)^c} M_R^{\text{diag}} N_R$$
$$+ \bar{e}_L m_e^{\text{diag}} e_R + \text{H.c.}$$
$$\mathcal{L}_w = \bar{\nu}_L \gamma_\mu e_L W_L^\mu + \text{H.c.} \quad (21)$$

The simultaneous transformation of ν_L and e_L in (20),(21) ensures the invariance of $\mathcal{L}_w$, but then V_{eL} appears in the Dirac mass term. Since m_D is a general complex symmetric matrix, so is $V_{eL} m_D U_R^\dagger$. Changing the notation

$$V_{eL} m_D U_R^\dagger \to m_D \quad (22)$$

one obtains

$$\mathcal{L}_m = \bar{\nu}_L m_D N_R + \frac{1}{2}\overline{(N_R)^c} M_R^{\text{diag}} N_R + \bar{e}_L m_e^{\text{diag}} e_R + \text{H.c.}$$
$$\mathcal{L}_w = \bar{\nu}_L \gamma_\mu e_L W_L^\mu + \text{H.c.} \quad (23)$$

We can redefine the doublet $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ and the singlet e_R by the same diagonal phase matrix P_e:

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \to \begin{pmatrix} P_e\nu_L \\ P_e e_L \end{pmatrix}, \qquad e_R \to P_e e_R \tag{24}$$

and one gets

$$\mathcal{L}_m = \bar{\nu}_L P_e^* m_D N_R + \frac{1}{2}\overline{(N_R)^c} M_R^{\rm diag} N_R + \bar{e}_L m_e^{\rm diag} e_R + \text{H.c.}$$
$$\mathcal{L}_w = \bar{\nu}_L \gamma_\mu e_L W_L^\mu + \text{H.c.} \tag{25}$$

Finally we can choose the phase matrix P_e to cancel three phases of m_D in $P_e^* m_D$,

$$\mathcal{L}_m = \bar{\nu}_L m_D N_R + \frac{1}{2}\overline{(N_R)^c} M_R^{\rm diag} N_R + \bar{e}_L m_e^{\rm diag} e_R + \text{H.c.}$$
$$\mathcal{L}_w = \bar{\nu}_L \gamma_\mu e_L W_L^\mu + \text{H.c.}, \tag{26}$$

where now the Dirac mass matrix m_D is *not* a general complex matrix, but has nine real parameters + six phases, i.e. 15(6) real parameters.

To summarize parameter counting, one is left in the current basis with 15(6)(from m_D) + 3(0) (from $m_e^{\rm diag}$) + 3(0) (from $M_R^{\rm diag}$) = 21(6) real parameters, i.e. among them 6 phases. This counting agrees with the one performed in Ref. [18].

B. Left-right model

In the LRM, the Lagrangian in the lepton sector reads

$$\mathcal{L}_m = \bar{\nu}_L m_D N_R + \frac{1}{2}\overline{(N_R)^c} M_R N_R + \bar{e}_L m_e e_R + \text{H.c.}$$
$$\mathcal{L}_w = \bar{\nu}_L \gamma_\mu e_L W_L^\mu + \bar{N}_R \gamma_\mu e_R W_R^\mu + \text{H.c.} \tag{27}$$

Notice that, to simplify the notation, possible $W_L - W_R$ mixing is for the moment neglected in the interaction term but will be considered later. The matrices m_D and m_e are *a priori* general complex with 18(9) parameters each, and M_R is a general complex symmetric matrix with 12(6) parameters.

An important remark is in order here. Parameter counting of the left-right model in the current basis means that we are assuming the whole interaction Lagrangian $\mathcal{L}_w$ in (27) to be diagonal, both in the left and the right sectors. For low-energy neutrino physics, it can seem academic to assume that the right-handed piece $\bar{N}_R \gamma_\mu e_R W_R^\mu + \text{H.c.}$ is kept diagonal, because it is an interaction term involving high-scale degrees of freedom. However, this natural assumption in any LR gauge theory is not only a formal point since, to keep this piece diagonal amounts to assuming that one assigns a lepton number to the N_R neutrinos, in just the same way as is done for the ν_L neutrinos in (27), and in consistency with the assignment $L(N_R) = -L((N_R)^c) = 1$ in the ESM. As we will see below, the diagonalization of the light neutrino mass matrix and of the right neutrino mass matrix will result in mixing matrices of the PMNS type for both the light and the heavy neutrinos.

Diagonalizing m_e by (19) and redefining the fields

$$\begin{pmatrix} V_{eL}\nu_L \\ V_{eL} e_L \end{pmatrix} \to \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \qquad \begin{pmatrix} V_{eR} N_R \\ V_{eR} e_R \end{pmatrix} \to \begin{pmatrix} N_R \\ e_R \end{pmatrix}, \tag{28}$$

one gets

$$\mathcal{L}_m = \bar{\nu}_L V_{eL} m_D V_{eR}^\dagger N_R + \frac{1}{2}\overline{(N_R)^c} V_{eR}^* M_R V_{eR}^\dagger N_R + \bar{e}_L m_e^{\rm diag} e_R + \text{H.c.}$$
$$\mathcal{L}_w = \bar{\nu}_L \gamma_\mu e_L W_L^\mu + \bar{N}_R \gamma_\mu e_R W_R^\mu + \text{H.c.} \tag{29}$$

Since m_D is general complex, so is $V_{eL} m_D V_{eR}^\dagger$, and M_R being complex symmetric, so is $V_{eR}^* M_R V_{eR}^\dagger$.

Changing the notation

$$V_{eL} m_D V_{eR}^\dagger \to m_D, \qquad V_{eR}^* M_R V_{eR}^\dagger \to M_R, \tag{30}$$

one obtains

$$\mathcal{L}_m = \bar{\nu}_L m_D N_R + \frac{1}{2}\overline{(N_R)^c} M_R N_R + \bar{e}_L m_e^{\rm diag} e_R + \text{H.c.}$$
$$\mathcal{L}_w = \bar{\nu}_L \gamma_\mu e_L W_L^\mu + \bar{N}_R \gamma_\mu e_R W_R^\mu + \text{H.c.} \tag{31}$$

We can redefine the doublets by the same diagonal phase matrix P_e,

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \to \begin{pmatrix} P_e\nu_L \\ P_e e_L \end{pmatrix}, \qquad \begin{pmatrix} N_R \\ e_R \end{pmatrix} \to \begin{pmatrix} P_e N_R \\ P_e e_R \end{pmatrix}, \tag{32}$$

and one gets

$$\mathcal{L}_m = \bar{\nu}_L P_e^* m_D P_e N_R + \frac{1}{2}\overline{(N_R)^c} P_e M_R P_e N_R + \bar{e}_L m_e^{\rm diag} e_R + \text{H.c.}$$
$$\mathcal{L}_w = \bar{\nu}_L \gamma_\mu e_L W_L^\mu + \bar{N}_R \gamma_\mu e_R W_R^\mu + \text{H.c.} \tag{33}$$

We can choose the phase matrix P_e to cancel three phases of m_D or three phases of M_R, but not both at the same time. We choose to absorb three phases in M_R. Changing the notation $P_e^* m_D P_e \to m_D$, one gets finally

$$\mathcal{L}_m = \bar{\nu}_L m_D N_R + \frac{1}{2}\overline{(N_R)^c} M_R N_R + \bar{e}_L m_e^{\rm diag} e_R + \text{H.c.}$$
$$\mathcal{L}_w = \bar{\nu}_L \gamma_\mu e_L W_L^\mu + \bar{N}_R \gamma_\mu e_R W_R^\mu + \text{H.c.}, \tag{34}$$

where m_D is an arbitrary complex matrix with 18(9) parameters and M_R is complex symmetric with 9(3) parameters.

To summarize, one gets finally in the LRM: 18(9) parameters from $m_D + 9(3)$ parameters from $M_R + 3$ eigenvalues in $m_e^{\rm diag} = 30(12)$ parameters.

Much more constrained models have been considered in the literature. For example, the *Minimal* LRM within supersymmetry with a Higgs content that implies $m_e = m_D, m_u = m_d$ (up-down unification) [19], that has a reduced number of parameters.

III. MASS BASIS

A. Extended Standard Model

For the diagonalization of the whole 6×6 neutrino mass matrix, we proceed step by step, and we begin with (26), where $m_e^{\rm diag}$ and $M_R^{\rm diag}$ are diagonal and the Dirac mass matrix m_D has 15(6) parameters. So we can rewrite

$$\mathcal{L}_m = \frac{1}{2}(\overline{\nu_L}, \overline{(N_R)^c})\mathcal{M}\begin{pmatrix}\nu_L^c \\ N_R\end{pmatrix} + \bar{e}_L m_e^{\rm diag} e_R + \text{H.c.}$$
$$\mathcal{L}_w = \bar{\nu}_L \gamma_\mu e_L W_L^\mu + \text{H.c.}, \tag{35}$$

where $\mathcal{M}$ has the form

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^t & M_R^{\rm diag}\end{pmatrix}. \tag{36}$$

This matrix has 18(6) parameters: 15(6) from m_D and 3(0) from $M_R^{\rm diag}$.

Let us now diagonalize $\mathcal{M}$ with the unitary matrix V [9–11]

$$\mathcal{M} = V\mathcal{M}^{\rm diag}V^t, \tag{37}$$

where

$$\mathcal{M}^{\rm diag} = \begin{pmatrix} m_L^{\rm diag} & 0 \\ 0 & M_R^{\rm diag}\end{pmatrix} \tag{38}$$

$$V = \begin{pmatrix} K & R \\ S & T\end{pmatrix}. \tag{39}$$

Notice that since $\mathcal{M}^{\rm diag}$ has 6 eigenvalues, and $\mathcal{M}$ has 18 (6) parameters, the 6×6 unitary matrix V will have $18(6) - 6(0) = 12(6)$ parameters. Rewriting (35) under the form

$$\mathcal{L}_m = \frac{1}{2}(\overline{\nu_L}, \overline{(N_R)^c})V\mathcal{M}^{\rm diag}V^t\begin{pmatrix}\nu_L^c \\ N_R\end{pmatrix} + \bar{e}_L m_e^{\rm diag} e_R + \text{H.c.}$$
$$\mathcal{L}_w = (\overline{\nu_L}, \overline{(N_R)^c})\gamma_\mu \begin{pmatrix}1 & 0 \\ 0 & 0\end{pmatrix}\begin{pmatrix}e_L \\ e_L\end{pmatrix} W_L^\mu + \text{H.c.} \tag{40}$$

and redefining

$$V^t\begin{pmatrix}\nu_L^c \\ N_R\end{pmatrix} \to \begin{pmatrix}\nu_L^c \\ N_R\end{pmatrix},$$
$$(\bar{\nu}, \overline{(N_R)^c})V \to (\bar{\nu}, \overline{(N_R)^c}), \tag{41}$$

one gets

$$\mathcal{L}_m = \frac{1}{2}(\overline{\nu_L}, \overline{(N_R)^c})\mathcal{M}^{\rm diag}\begin{pmatrix}\nu_L^c \\ N_R\end{pmatrix} + \bar{e}_L m_e^{\rm diag} e_R + \text{H.c.}$$
$$\mathcal{L}_w = (\overline{\nu_L}, \overline{(N_R)^c})\gamma_\mu V^\dagger \begin{pmatrix}1 & 0 \\ 0 & 0\end{pmatrix}\begin{pmatrix}e_L \\ e_L\end{pmatrix} W_L^\mu + \text{H.c.} \tag{42}$$

or

$$\mathcal{L}_m = \frac{1}{2}\overline{\nu_L} m_L^{\rm diag}(\nu_L)^c + \frac{1}{2}\overline{(N_R)^c}M_R^{\rm diag}N_R + \bar{e}_L m_e^{\rm diag} e_R + \text{H.c.}$$
$$\mathcal{L}_w = (\overline{\nu_L}K^\dagger e_L + \overline{(N_R)^c}R^\dagger e_L)\gamma_\mu W_L^\mu + \text{H.c.} \tag{43}$$

The first term in $\mathcal{L}_w$ describes the Standard Model $\Delta L = 0$ decay,

$$W_L \to e_L \bar{\nu}_L, \tag{44}$$

while the second term corresponds to the well-known $\Delta L = 2$ process,

$$(N_R)^c \to e_L W_L, \tag{45}$$

$(L(N_R) = -L((N_R)^c) = L(e_L))$. The notation $(N_R)^c$ for the heavy neutrino makes explicit also the chirality conservation of the $V - A$ interaction.

Notice that only the 3×3 complex matrices K and R from the 6×6 unitary matrix (39) are involved in the formula (43). Let us now count the parameters of these matrices. From the zero in the matrix $\mathcal{M}$ (36) and the definitions (37)–(39) one finds (see Eq. (A8) of the Appendix for $m_L = 0$),

$$Km_L^{\rm diag}K^t + RM_R^{\rm diag}R^t = 0. \tag{46}$$

Using the unitarity of the matrix V (39) one has

$$KK^\dagger + RR^\dagger = 1. \tag{47}$$

Equations (46) and (47) are identities between 3×3 matrices involving only the mixing matrices K and R and not the whole matrix (39). Due to these relations, *the matrices K and R are correlated.*

The conditions (46) and (47) reduce the number of independent parameters. Equation (46) is self-transposed, and gives 12(6) constraints, while (47) is Hermitian, giving 9(3) constraints. This reduces the number of parameters of the two complex matrices K and R from 36(18) down to 15(9).

Finally, redefining the charged lepton fields by a diagonal 3×3 phase matrix Q_e

$$e_L \to Q_e^\dagger e_L, \qquad e_R \to Q_e^\dagger e_R \tag{48}$$

one gets, from (43),

$$\mathcal{L}_m = \frac{1}{2}\overline{\nu_L} m_L^{\rm diag} (\nu_L)^c + \frac{1}{2}\overline{(N_R)^c} M_R^{\rm diag} N_R + \bar{e}_L m_e^{\rm diag} e_R + {\rm H.c.}$$

$$\mathcal{L}_w = (\overline{\nu_L}(Q_e K)^\dagger + \overline{(N_R)^c}(Q_e R)^\dagger)\gamma_\mu e_L W_L^\mu + {\rm H.c.} \tag{49}$$

On the other hand, multiplying (46) on the left by Q_e and on the right by Q_e^t, and (47) on the left by Q_e and on the right by $Q_e^\dagger$, these equations become

$$(Q_e K) m_L^{\rm diag} (Q_e K)^t + (Q_e R) M_R^{\rm diag} (Q_e R)^t = 0 \tag{50}$$

$$(Q_e K)(Q_e K)^\dagger + (Q_e R)(Q_e R)^\dagger = 1, \tag{51}$$

and we can absorb three phases of one of the matrices K or R, but not of both matrices at the same time.

In summary, the matrices K and R have *together* 12(6) parameters, and adding the 9(0) parameters from $m_e^{\rm diag}$, $m_L^{\rm diag}$ and $M_R^{\rm diag}$ one obtains a total of 21(6) parameters, the same number as in the current basis. In the ESM the matrices K and R are *decoupled* from S and T of (39), and obey relations (46) and (47).

We can now go somewhat further by considering first the whole matrix (39), and assuming $m_D \ll M_R$.

1. The matrices K, R, S, T in the extended Standard Model

Starting from the Lagrangian in the current basis (26), m_D has now 15(6) parameters. Particularizing formulas (A8)–(A10) of the Appendix to the present case, we have

$$K m_L^{\rm diag} K^t + R M_R^{\rm diag} R^t = 0 \tag{52}$$

$$S m_L^{\rm diag} S^t + T M_R^{\rm diag} T^t = M_R^{\rm diag} \tag{53}$$

$$K m_L^{\rm diag} S^t + R M_R^{\rm diag} T^t = m_D. \tag{54}$$

Considering for the moment the unitarity of the matrix (39), the number of independent parameters in the lhs will be 36(21) from (K, R, S, T) + 3(0) from $m_L^{\rm diag}$ + 3(0) from $M_R^{\rm diag} = 42(21)$.

The complex symmetric matrix equation (52) gives 12(6) constraints. On the other hand, $M_R^{\rm diag}$ appears already in the rhs of (53), and this equation implies $12(6) - 3(0) = 9(6)$ constraints. Since m_D has now 15(6) free parameters, Eq. (54) gives 3(3) constraints, giving a total of $12(6) + 9(6) + 3(3) = 24(15)$ constraints. Therefore the number of independent parameters is $42(21) - 24(15) = 18(6)$ parameters. Adding the 3(0) eigenvalues of $m_e^{\rm diag}$ one gets $18(6) + 3(0) = 21(6)$ parameters, the same result as in the current basis.

Moreover, subtracting from this total of 21(6) parameters the 9(0) mass eigenvalues $m_e^{\rm diag}$, $m_L^{\rm diag}$ and $M_R^{\rm diag}$, the set of matrices (K, R, S, T) has 12(6) parameters, the same number that we have found for K and R, so that S and T are not independent.

Exact relations between the matrices K, R, S, T.—On the other hand, from (36)–(39) one has

$$\begin{pmatrix} 0 & m_D \\ m_D^t & M_R^{\rm diag} \end{pmatrix} \begin{pmatrix} K^* & R^* \\ S^* & T^* \end{pmatrix} = \begin{pmatrix} K & R \\ S & T \end{pmatrix} \begin{pmatrix} m_L^{\rm diag} & 0 \\ 0 & M_R^{\rm diag} \end{pmatrix}, \tag{55}$$

hence,

$$\begin{pmatrix} m_D S^* & m_D T^* \\ m_D^t K^* + M_R^{\rm diag} S^* & m_D^t R^* + M_R^{\rm diag} T^* \end{pmatrix} = \begin{pmatrix} K m_L^{\rm diag} & R M_R^{\rm diag} \\ S m_L^{\rm diag} & T M_R^{\rm diag} \end{pmatrix}, \tag{56}$$

and, therefore, one obtains the following exact expressions of the matrices R, S in terms of K, T, m_D and the mass eigenvalues:

$$R = m_D T^* (M_R^{\rm diag})^{-1} \tag{57}$$

$$S = (m_D^*)^{-1} K^* m_L^{\rm diag}. \tag{58}$$

From inspection of the precedent equations, one sees that (57) and (58) are relations between the mass basis quantities $(K, R, S, T, m_L^{\rm diag}, M_R^{\rm diag})$ and the current basis matrices $m_D, M_R^{\rm diag}$, since M_R is diagonalized and appears in both bases. Eliminating m_D, one finds an exact relation between quantities in the mass basis

$$M_R^{\rm diag} T^{-1} S = (R^*)^{-1} K^* m_L^{\rm diag}, \tag{59}$$

The matrices (K, R, S, T) for $m_D \ll M_R$.—If $m_D \ll M_R$, one has the order of magnitude

$$R \sim S \sim O\left(\frac{m_D}{M_R}\right). \tag{60}$$

Neglecting in Eqs. (A2)–(A7) of the Appendix the terms of $O(\frac{m_D^2}{M_R^2})$, one gets the approximate unitarity conditions

$$K K^\dagger \simeq K^\dagger K \simeq 1 \tag{61}$$

$$T T^\dagger \simeq T^\dagger T \simeq 1. \tag{62}$$

Moreover, from (61) and (62), both Eqs. (A4) and (A7) imply the same approximate relation between R and S:

$$R \simeq -KS^\dagger T. \tag{63}$$

In conclusion, in the present approximation one gets two unitary matrices K and T (61) and (62) and the matrix R given in terms of (K, T, S) by (63).

On the other hand, neglecting terms of $O(\frac{m_D^2}{M_R^2})$ in (52)–(54), one gets

$$K m_L^{\rm diag} K^t + R M_R^{\rm diag} R^t = 0 \tag{64}$$

$$T M_R^{\rm diag} T^t \simeq M_R^{\rm diag} \tag{65}$$

$$R M_R^{\rm diag} T^t \simeq m_D. \tag{66}$$

Equation (65) implies

$$T \simeq 1. \tag{67}$$

Notice that (66) is identical to the relation (57) obtained above. On the other hand, combining (63) with the exact relation (59) one consistently obtains (64).

One can see that (63) gives just the seesaw formula. From (57), (58), and (67), Eq. (63) implies, after some algebra,

$$K m_L^{\rm diag} K^t \simeq -m_D (M_R^{\rm diag})^{-1} m_D^t, \tag{68}$$

and from the general complex symmetric matrix m_L,

$$m_L = K m_L^{\rm diag} K^t, \tag{69}$$

one gets the seesaw formula in the ESM:

$$m_L \simeq -m_D (M_R^{\rm diag})^{-1} m_D^t. \tag{70}$$

We see that K is the mixing matrix for light neutrinos that appears in (43) in the basis in which m_e is diagonal.

On the other hand, relation (57) or (66), together with (67), implies

$$R \simeq m_D (M_R^{\rm diag})^{-1}, \tag{71}$$

and using the seesaw formula (70), relation (58) becomes

$$S = -(M_R^{\rm diag})^{-1} m_D^\dagger K \tag{72}$$

consistent with (63).

The whole set K, R, S, T has 12(6) parameters, implying from (67) that K, R and S have 12(6) independent parameters. Since according to (72) the matrix S is not independent, the matrices K, R that appear in the interaction Lagrangian (43), have together 12(6) parameters. From (71) and the 15(6) number of parameters of m_D, we see that R will have 12(6) parameters. Since K is unitary in the present approximation, we can choose 6(3) independent parameters *within* R to provide the unitary matrix K with 6(3) parameters, the physically relevant PMNS structure. Then R will have other extra 6(3) parameters. However, other solutions are allowed, since K is unitary, not necessarily of the PMNS type.

2. Summary of the parameter counting in the mass basis

In the mass basis, parameter counting in the physically relevant case is 12(6) parameters from *both* the complex matrices K, R (among these, 6(3) parameters from the PMNS-like matrix K) $+3(0)$ parameters from $M_R^{\rm diag}$ $+3(0)$ parameters from $m_L^{\rm diag}$$+3(0)$ parameters from $m_e^{\rm diag} =$ $21(6)$, the same counting as in the current basis.

The more constrained condition $m_D \ll M_R$ provides a particular case: R has 12(6) parameters, among which one has to choose the 6(3) parameters of the PMNS matrix K.

B. Left-right model

Let us start from the Lagrangian (34) of the LRM. At this stage M_R is complex symmetric with 9(3) parameters. We rewrite (34) under the form

$$\mathcal{L}_m = \frac{1}{2}(\overline{\nu_L}, \overline{(N_R)^c})\mathcal{M}\begin{pmatrix}(\nu_L)^c \\ N_R\end{pmatrix} + \bar{e}_L m_e^{\rm diag} e_R + \text{H.c.}$$
$$\mathcal{L}_w = (\overline{\nu_L}, \overline{(N_R)^c})\gamma_\mu \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}e_L \\ e_L\end{pmatrix} W_L^\mu$$
$$+ (\overline{N_R}, \overline{(\nu_L)^c})\gamma_\mu \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}e_R \\ e_R\end{pmatrix} W_R^\mu + \text{H.c.}, \tag{73}$$

where

$$\mathcal{M} = \begin{pmatrix}0 & m_D \\ m_D^t & M_R\end{pmatrix}. \tag{74}$$

Unlike the case of the ESM, the complex symmetric block M_R is not diagonalized, it has 9(3) parameters since three phases have been rotated away.

Using the unitary matrix V (36)–(39),

$$(\overline{\nu_L}, \overline{(N_R)^c}) \to (\overline{\nu_L}, \overline{(N_R)^c})V^\dagger$$
$$(\overline{N_R}, \overline{(\nu_L)^c}) \to (\overline{N_R}, \overline{(\nu_L)^c})\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} V^t \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \tag{75}$$

we obtain the following Lagrangian in the mass basis:

$$\mathcal{L}_m = \frac{1}{2}\overline{\nu_L} m_L^{\rm diag}(\nu_L)^c + \frac{1}{2}\overline{(N_R)^c} M_R^{\rm diag} N_R + \bar{e}_L m_e^{\rm diag} e_R + \text{H.c.}$$
$$\mathcal{L}_w = (\overline{\nu_L} K^\dagger + \overline{(N_R)^c} R^\dagger)\gamma_\mu e_L W_L^\mu$$
$$+ (\overline{N_R} T^t + \overline{(\nu_L)^c} S^t)\gamma_\mu e_R W_R^\mu + \text{H.c.}. \tag{76}$$

The 3×3 matrices K and R enter in the left sector, while T and S enter in the right sector, in a symmetric way.

A formula of similar structure to Eq. (76) follows from the results of Ref. [16], but uses a quite different notation.

It is important to point out that the terms dependent on K and T are lepton number conserving, $\Delta L = 0$, while those that depend on R and S are lepton number violating, $\Delta L = 2$.

1. The matrices K, R, S, T in the left-right model

Particularizing (A8)–(A10) of the Appendix to the case (74), we have

$$Km_L^{\rm diag}K^t + RM_R^{\rm diag}R^t = 0 \tag{77}$$

$$Sm_L^{\rm diag}S^t + TM_R^{\rm diag}T^t = M_R \tag{78}$$

$$Km_L^{\rm diag}S^t + RM_R^{\rm diag}T^t = m_D. \tag{79}$$

Considering for the moment only the unitarity of the full matrix V (39), that has 36(21) parameters, the number of independent parameters in the lhs of the precedent equations will be 36(21) from $(K,R,S,T) + 3(0)$ from $m_L^{\rm diag} + 3(0)$ from $M_R^{\rm diag} = 42(21)$ parameters.

The complex symmetric matrix equation (77) gives 12(6) constraints. On the other hand, M_R in the rhs of (78) has 9(3) free parameters, and this equation implies $12(6) - 9(3) = 3(3)$ constraints. Finally, since m_D is a general complex matrix, with 18(9) free parameters, Eq. (79) does not give any constraint. This gives a total of $12(6) + 3(3) = 15(9)$ constraints. Therefore one has $42(21) - 15(9) = 27(12)$ independent parameters. Adding the 3(0) eigenvalues of $m_e^{\rm diag}$, not counted up to now, one gets $27(12) + 3(0) = 30(12)$ parameters, the same result as in the current basis.

Moreover, subtracting from this total number of 30(12) parameters the 9(0) mass eigenvalues $m_e^{\rm diag}$, $m_L^{\rm diag}$ and $M_R^{\rm diag}$, we see that the set of matrices (K,R,S,T), that appear in the interaction term (76), have a total of 21(12) parameters.

In the $SU(2)_L \times SU(2)_R \times U(1)$ Model one obtains also the exact relations between the matrices K, R, S, T given above by Eqs. (55)–(59).

The matrices (K,R,S,T) for $m_D \ll M_R$.—
The relations given above within the approximation $m_D \ll M_R$ (61)–(63) for the ESM also hold in the LR model.

Let us rewrite Eqs. (77)–(79) neglecting terms of $O(\frac{m_D^2}{M_R^2})$:

$$Km_L^{\rm diag}K^t + RM_R^{\rm diag}R^t = 0 \tag{80}$$

$$TM_R^{\rm diag}T^t \simeq M_R \tag{81}$$

$$RM_R^{\rm diag}T^t \simeq m_D \tag{82}$$

Equation (82) is the above obtained exact relation (57) if one neglects in the latter higher-order terms. This means that in (79) the first term of the lhs, that is of $O(m_D^3/M_R^2)$, is compensated by higher-order terms in the second term $RM_R^{\rm diag}T^t$. On the other hand, combining (63) with the exact relation (59), one consistently obtains the exact relation (77).

According to (62) and (81), the matrix T is the unitary mixing matrix of right-handed neutrinos, for which we can take 6(3) parameters, i.e. a matrix of the PMNS type. Equation (61) holds also in the LRM, and K is the unitary mixing matrix of light left-handed neutrinos.

Since the whole set K, R, S and T has 21(12) parameters and the matrices K, T have 6(3) parameters each, this implies that R and S can have together 9(6) extra independent parameters.

In the LR model, from relations (63) and (81) one obtains

$$Km_L^{\rm diag}K^t \simeq -m_D T^* (M_R^{\rm diag})^{-1} T^\dagger m_D^t, \tag{83}$$

i.e. the seesaw formula

$$m_L \simeq -m_D M_R^{-1} m_D^t, \tag{84}$$

where M_R is not diagonalized, to be compared with the seesaw formula (70) in the case of the ESM.

Notice the important point that in Sec. I we have disregarded the possibility in the LRM of a Higgs triplet Δ_L that, in principle, could also contribute to the mass of the light neutrinos (see for example [3,20]), so that formula (84) is only correct in the LRM if one neglects this type II seesaw contribution.

Equation (82) implies, using the approximate unitarity of T,

$$R \simeq m_D T^* (M_R^{\rm diag})^{-1}, \tag{85}$$

to be distinguished from (71), that holds in the ESM case. We see that in the LR case the PMNS matrix T of the heavy neutrinos T enters in the matrix R and, on the other hand, the matrix S satisfies relation (72) that we found in the ESM.

2. Summary of the parameter counting in the mass basis

We have seen that the set of matrices K, R, S and T have together 21(12) parameters. Unlike the case of the ESM, in the LR model we have enough parameter space to accomodate two different PMNS matrices for K and T, with 6(3) parameters each. Then, R and S can have together extra 9 (6) parameters. However, this situation is not compulsory: there can be overlap between the parameters of all the four matrices K, R, S and T.

In conclusion, the parameter counting in the physically interesting solution is as follows: 6(3) parameters from the PMNS-like unitary matrix $K + 6(3)$ parameters from the PMNS-like matrix $T + 9(6)$ extra parameters from the complex matrices R, $S + 3(0)$ from $M_R^{\rm diag} + 3(0)$ from

$m_L^{\rm diag} + 3(0)$ from $m_e^{\rm diag} = 30(12)$ parameters, the same number as in the current basis.

3. Possible observables in the left-right model

The gauge bosons W_L and W_R are mixed in the left-right model,

$$W_L = \cos\zeta W_1 - \sin\zeta W_2,$$
$$W_R = e^{i\omega}(\sin\zeta W_1 + \cos\zeta W_2), \quad (86)$$

where W_1 and W_2 are mass eigenstates, and the mixing angle ζ, in terms of the vacuum expectation values (12) and (15), is of the order [15]

$$\zeta \simeq \pm \frac{g_L}{g_R}\frac{2|k_1 k_2|}{|v_R|^2}. \quad (87)$$

From (76), it is interesting to write down the lightest mass vector boson W_1 couplings to leptons,

$$\mathcal{L}_w^{W_1} = [\cos\zeta(\overline{\nu_L}K^\dagger + \overline{(N_R)^c}R^\dagger)\gamma_\mu e_L$$
$$+ e^{i\omega}\sin\zeta(\overline{N_R}T^t + \overline{(\nu_L)^c}S^t)\gamma_\mu e_R]W_1^\mu + \text{H.c.} \quad (88)$$

Besides the $\sim\cos\zeta$ term that describes the processes $\Delta L = 0$ (44) and $\Delta L = 2$ (45) as in the ESM case, the subleading term $\sim\sin\zeta$ describes the $\Delta L = 0$ process,

$$N_R \to e_R W_1, \quad (89)$$

and the lepton-number-violating decay $\Delta L = 2$ of the gauge boson

$$W_1 \to \bar{e}_R(\nu_L)^c, \quad (90)$$

$[L(\bar{e}_R) = L((\nu_L)^c) = -L(e_R) = -L(\nu_L)]$. However, the amplitude for this latter decay is very small, as we will see below.

On the other hand, the heavier vector boson W_2 couplings to leptons read

$$\mathcal{L}_w^{W_2} = [-\sin\zeta(\overline{\nu_L}K^\dagger + \overline{(N_R)^c}R^\dagger)\gamma_\mu e_L$$
$$+ e^{i\omega}\cos\zeta(\overline{N_R}T^t + \overline{(\nu_L)^c}S^t)\gamma_\mu e_R]W_2^\mu + \text{H.c.} \quad (91)$$

Here, the subleading $\sim\sin\zeta$ term describes the $\Delta L = 0$ process,

$$W_2 \to \bar{e}_L\nu_L, \quad (92)$$

and the $\Delta L = 2$ transition, assuming the mass of W_2 is heavier that the one of N_R,

$$W_2 \to \bar{e}_L(N_R)^c \quad (93)$$

On the other hand, the leading $\sim\cos\zeta$ term describes the process $\Delta L = 0$,

$$W_2 \to \bar{e}_R N_R, \quad (94)$$

and the $\Delta L = 2$ involving light leptons,

$$W_2 \to \bar{e}_R(\nu_L)^c. \quad (95)$$

Of course, the phenomenological relevance of the $\Delta L = 2$ decay involving the W_R gauge boson depends on its mass scale.

Concerning the possibility of physics of the LRM at relatively low energies, with observables at LHC scales, one should remember that there are severe constraints on such a low-energy LRM. This point has been carefully studied in a detailed paper by Deshpande, Gunion, Kayser and Olness [21], who have examined the relevant constraints: structure of the vacuum, limits on flavor-changing neutral currents, etc. The conclusion is that, although such a low-energy LRM is not excluded, it is not natural in a straightforward way and can only be formulated through some degree of fine-tuning.

If one assumes that the mass scale of the LRM is low, it makes sense to look at the LHC for lepton-number-violation processes through the search of $pp \to \ell\ell jj$ topologies, where the two leptons are of the same charge (see for example the recent Refs. [22–24]).

Indeed, using (91) there is the possibility of the $\Delta L = 2$ process,

$$W_2^+ \sim W_R^+ \to e_R^+ N_R \to e_R^+ e_L^+ W_L^- \to e_R^+ e_L^+ jj, \quad (96)$$

where W_L^- decays into two hadronic jets, the subscripts in e_R and e_L mean the couplings to W_R and W_L, and we use the notation $(e_R)^c = e_R^+, (e_L)^c = e_L^+$. The decay chain (96) is the very interesting Keung-Senjanović process proposed long time ago [25] that tests, at the same time, the decay of the gauge boson W_R and the Majorana character of the right-handed neutrino N_R.

The PMNS mixing matrix T of the heavy right-handed neutrinos N_R controls the decay $W_R^+ \to e_R^+ N_R$. On the other hand, we see from formula (76) that the secondary decay $N_R \to e_L^+ W_L^-$ is controlled by the matrix $R \simeq m_D T^*(M_R^{\rm diag})^{-1}$ [cf. (85)]. Therefore, this latter decay is controlled by the Dirac mass [25] in the basis in which M_R is diagonalized, $m_D' = m_D T^*$ (see below the leptogenesis part).

The decay chain (96) through $W_R^+ \to e_R^+ N_R \to e_R^+ e_L^+ W_L^-$ depends on both matrices T and R. Let us suppose that, through the kinematics of the two jets in the decay $W_L^- \to jj$, one can reconstruct the W_L^- boson. Then, the angular distribution of the three body decay $W_R^+ \to e_R^+ e_L^+ W_L^-$ will give information on the matrices T and R.

The discussion of the observables in these decay chains depends on the assumed N_R spectrum. One usually assumes that the gauge boson W_R has a mass bigger or of the order of the heaviest N_R, that would correspond to a Yukawa coupling of $O(1)$, in analogy with the top quark. However, to simplify what follows, let us assume that all $N_{R_i}(i = 1, 2, 3)$ are lighter than the W_R.

From relation (85) one can see that, in the limit of degenerate heavy neutrinos, summing over the three N_{R_i}, the amplitude for the process $W_R^+ \to e_R^+ e_L^+ W_L^-$ depends on the product $m_D^* T M_R^{-1}$, where M_R is the *nondiagonalized* right-handed neutrino mass.

One relevant question is to ask whether one can measure the PMNS mixing matrix T. Although quite difficult, as said above, the W_L could, in principle, be reconstructed through its decays into two jets $W_L \to jj$, and the different N_{R_i} could be reconstructed as well through the decays $N_{R_i} \to e_L^+ W_L^-$. Our starting point was the mass Lagrangian where the charged lepton part is diagonalized (73), and the final output was the interaction Lagrangian (76), where the decays $W_R^+ \to e_R^+ N_{R_i}(i = 1, 2, 3)$ depend on the PMNS matrix T. Considering the possibility of the three leptons $e_i(i = 1, 2, 3)$ of the Standard Model e, μ, τ, we see that through the rates of these decays, the moduli of all the matrix elements T_{ij} are, in principle, accessible to experiment.

IV. REPRESENTATIONS OF THE DIRAC MASS MATRIX

The Dirac mass matrix m_D is a crucial input in neutrino physics, making the link between high and low energy. We review now some useful representations of m_D.

A. Triangular parametrization

An interesting representation of the Dirac mass matrix m_D has been proposed by Branco *et al.* [10],

$$m_D = U m_\Delta, \tag{97}$$

where U is a unitary matrix with 6(3) parameters of the PMNS form, although not identical to it, and m_Δ is a triangular matrix, with three vanishing off-diagonal elements, three real diagonal elements and three complex off-diagonal elements.

The factorization formula (97) is usually called in mathematics "QR decomposition" of a complex matrix M. In MATHEMATICA notation [26] QRDecomposition[M] gives the decomposition of a numerical complex matrix M in terms of a unitary matrix U and an upper triangular matrix m_Δ, while [9,10] refer to a lower triangular matrix, although this is not an essential point. This decomposition can be numerically very useful for texture models of the matrix m_D, since it isolates m_Δ and, hence, the parameters that are relevant for leptogenesis.

The counting of parameters for m_D holds in (97): 15(6) parameters of $m_D = 6(3)$ parameters of $U + 9(3)$ parameters from the triangular matrix m_Δ. Relation (97) also holds if m_D is general complex and U a general unitary matrix: 18(9) parameters of $m_D = 9(6)$ parameters of $U + 9(3)$ parameters from the triangular matrix m_Δ. In the same way that three phases of m_D can be rotated away by the transformation (24)–(26), and one can consistently rotate away three phases of the general unitary matrix U [10].

Relation (97) is nontrivial. Indeed, because of the unitarity of U, we see that $m_D^\dagger m_D$ is given by

$$m_D^\dagger m_D = m_\Delta^\dagger m_\Delta, \tag{98}$$

and, therefore, the three CP phases of m_Δ control the amount of leptogenesis at high energies in the one-flavor approximation.

1. Extended Standard Model

With (97), Eq. (71), obtained within the seesaw, reads

$$R \simeq U m_\Delta (M_R^{\mathrm{diag}})^{-1}. \tag{99}$$

We have seen above that if we decide that K is of the PMNS type with 6(3) parameters, then the parameters of K have to be chosen among the ones of R. A solution satisfying this criterium is a Dirac mass matrix given by [9]

$$m_D = K m_\Delta, \qquad R \simeq K m_\Delta (M_R^{\mathrm{diag}})^{-1}. \tag{100}$$

Besides its historical interest, this solution has the very nice feature of factorization of the Dirac mass matrix into two pieces, a low-energy PMNS mixing matrix K with 6(3) parameters, and a high-energy mass matrix m_Δ, that has 9(3) parameters and controls leptogenesis.

Another extreme case would be to assume that $U = 1$ [27,28] that implies

$$m_D = m_\Delta, \qquad R \simeq m_\Delta (M_R^{\mathrm{diag}})^{-1}. \tag{101}$$

This ansatz relates directly the CP-violating phase in leptogenesis and CP violation at low energy in neutrino oscillations.

However, there are many other solutions, since in all generality one can choose the parameters of K among the ones of the product $m_D = U m_\Delta$.

2. Left-right model

Equation (85) reads

$$R = U m_\Delta T^* (M_R^{\mathrm{diag}})^{-1}, \tag{102}$$

where we see that the matrix T, unlike the case of the ESM (99), enters in the definition of the matrix R, that controls leptogenesis.

B. The orthogonal parametrization

Another useful parametrization of m_D has been proposed by Casas and Ibarra [29].

1. Extended Standard Model

Starting from the seesaw formula (70) and diagonalizing m_L by the PMNS matrix K (69),

$$m_L^{\text{diag}} = -K^\dagger m_D (M_R^{\text{diag}})^{-1} m_D^t K^* \tag{103}$$

As pointed out in [29], this relation implies,

$$-(m_L^{\text{diag}})^{-1/2} K^\dagger m_D (M_R^{\text{diag}})^{-1/2} (M_R^{\text{diag}})^{-1/2} m_D^t K^* (m_L^{\text{diag}})^{-1/2} = 1 \tag{104}$$

and, therefore, the matrix $i(m_L^{\text{diag}})^{-1/2} K^\dagger m_D (M_R^{\text{diag}})^{-1/2}$ is an orthogonal complex matrix O

$$O = i(m_L^{\text{diag}})^{-1/2} K^\dagger m_D (M_R^{\text{diag}})^{-1/2}, \tag{105}$$

i.e. $OO^t = 1$. One finds the general expression for m_D in terms of the matrix O

$$m_D = -iK(m_L^{\text{diag}})^{1/2} O (M_R^{\text{diag}})^{1/2}. \tag{106}$$

One can check from this expression that $m_D = K m_\Delta$ (100) is not the most general form for m_D because O, being a general complex orthogonal matrix, the combination $-i(m_L^{\text{diag}})^{1/2} O (M_R^{\text{diag}})^{1/2}$ is not triangular in general.

The parametrization (106) is very useful to analyze leptogenesis CP asymmetries when taking flavor into account.

2. Left-right model

From Eq. (83) one gets, instead of (104),

$$-(m_L^{\text{diag}})^{-1/2} K^\dagger m_D T^* (M_R^{\text{diag}})^{-1/2} (M_R^{\text{diag}})^{-1/2} T^\dagger m_D^t K^* (m_L^{\text{diag}})^{-1/2} = 1, \tag{107}$$

which defines the orthogonal matrix

$$O' = i(m_L^{\text{diag}})^{-1/2} K^\dagger m_D T^* (M_R^{\text{diag}})^{-1/2}, \tag{108}$$

and m_D is now in the LRM,

$$m_D = -iK(m_L^{\text{diag}})^{1/2} O' (M_R^{\text{diag}})^{1/2} T^t, \tag{109}$$

which includes the PMNS mixing matrix T of right-handed neutrinos.

C. Relation between the triangular and orthogonal forms

The orthogonal parametrization of the Dirac mass matrix m_D appears to be powerful because it explicitly includes low-energy quantities, the light neutrino eigenvalues m_L^{diag} and the PMNS mixing matrix K and, on the other hand, high-energy quantities, the heavy right-handed neutrino eigenvalues M_R^{diag} and an unknown orthogonal complex matrix O. One can write down the relation between both representations.

In the ESM, from relation (106) one can write the QR decomposition of the matrix

$$-i(m_L^{\text{diag}})^{1/2} O (M_R^{\text{diag}})^{1/2} = V m_\Delta, \tag{110}$$

where V is another unitary matrix, and m_Δ a triangular matrix. We see therefore that the matrix m_D has the form of the triangular parametrization (97) $m_D = U m_\Delta$, with the PMNS matrix K being a factorizable part of the unitary matrix U, namely $U = KV$. Therefore, although one can set $U = 1$, i.e. $V = K^{-1}$, and then the low-energy phases are part of m_Δ and hence of leptogenesis, the natural solution seems to be that the PMNS matrix K is a unitary factor of the matrix U, i.e. $U = KV$, V being a unitary matrix.

V. LEPTOGENESIS

The gauge models that we consider conserve $B - L$. As nicely pointed out by Strumia [30], the mere existence of sphalerons, that violate $B + L$ in the Standard Model at high temperature, suggests that baryogenesis can proceed via leptogenesis [31,32]. From (43) or (76), we see that lepton number is violated by the decays of heavy right-handed neutrinos, giving rise to a lepton asymmetry that is partially converted into a baryon asymmetry by the sphalerons. The out-of-equilibrium CP-violating decays of heavy Majorana neutrinos, supplemented by sphaleron interactions, satisfy the three Sakharov criteria [33] to obtain baryogenesis.

In this section we consider leptogenesis in the electroweak broken phase, coming from the CP-violating $\Delta L = 2$ decay $(N_R)^c \to e_L W_L$ in the Lagrangians (43) of the ESM and (76) of the LRM.

The actual leptogenesis occurs at very high temperature, in the electroweak unbroken phase. The connection between cosmological CP violation in the unbroken phase [34] with a single massless Higgs doublet and in the broken phase has been underlined by Branco *et al.* [10]. In the case of the left-right model, this connection is not clear *a priori* because the massless Higgs fields in the unbroken case belong to the bidoublet (10). As we emphasize below, this relation is worth to be investigated. For the moment, we are interested here in the possible differences between the ESM and the LRM in the broken phase, where the interaction Lagrangians (43) and (76) apply.

A. One-flavor approximation

1. Extended Standard Model

In this part on the ESM we reproduce the results of Ref. [10], with the aim of comparing below with the LRM. The lepton number asymmetry from the decay of the 1st, lightest heavy Majorana neutrino, in the broken electroweak phase and in the one-flavor approximation is given by

$$\epsilon_1 = \frac{g^2}{M_W^2}\frac{1}{16\pi}\frac{1}{(R^\dagger R)_{11}}\sum_{k\neq 1} F(x_k) M_k^2 \mathrm{Im}[(R^\dagger R)_{1k}]^2 \tag{111}$$

since, from (43), the matrix R is responsible for the transition $(N_R)^c \to e_L W_L$ or, equivalently, the decay $(N_R)^c \to e_L H$ above the phase transition. In Eq. (111) the function $F(x_k)$ reads

$$F(x_k) = \sqrt{x_k}\left[1 + (1+x_k)\ln\left(\frac{x_k}{1+x_k}\right) + \frac{1}{1-x_k}\right] \left(x_k = \frac{M_k^2}{M_1^2}\right). \tag{112}$$

As pointed out in Ref. [10], from (71) $R \simeq m_D (M_R^{\mathrm{diag}})^{-1}$, which holds in the ESM for $m_D \ll M_R$, one gets the lepton number asymmetry in terms of the Dirac mass or, equivalently, in terms of the Yukawa couplings $\frac{m_D}{v}$ in the unbroken phase:

$$\epsilon_1 = \frac{g^2}{M_W^2}\frac{1}{16\pi}\frac{1}{(m_D^\dagger m_D)_{11}}\sum_{k\neq 1} F(x_k)\mathrm{Im}[(m_D^\dagger m_D)_{1k}]^2. \tag{113}$$

While the expression of the lepton number asymmetry (111) depends only on quantities of the mass basis, namely on the matrices R, M_R^{diag}, expression (113) depends only on quantities of the current basis, since the matrix M_R is diagonalized from the beginning in both bases. Notice that, as exposed in [10], expression (113) has a well-defined limit for the SM vacuum expectation value limit $v \to 0$, given in terms of Yukawa couplings corresponding to the decay in the unbroken electroweak phase $(N_R)^c \to e_L H$ [34].

In terms of the matrix m_Δ one gets

$$\epsilon_1 = \frac{g^2}{M_W^2}\frac{1}{16\pi}\frac{1}{(m_\Delta^\dagger m_\Delta)_{11}}\sum_{k\neq 1} F(x_k)\mathrm{Im}[(m_\Delta^\dagger m_\Delta)_{1k}]^2, \tag{114}$$

which depends only on the three phases of m_Δ.

On the other hand, in terms of the orthogonal matrix O defined in (105) the CP asymmetry is given by

$$\epsilon_1 = \frac{g^2}{M_W^2}\frac{1}{16\pi}\frac{1}{M_1 \sum_i m_i |O_{i1}|^2} \times \sum_{k\neq 1} F(x_k) M_1 M_k \mathrm{Im}\left[\sum_j (m_j O_{j1})^2\right]. \tag{115}$$

2. Left-right model

In the LR model one has, in principle, two types of contributions to the light neutrino masses, through type I seesaw and type II seesaw, the latter arising from triplet Higgs exchange (see, for example, Refs. [3,20,21]). As pointed out above, in this paper we consider only the contribution of the type I seesaw.

In the LR case we have seen that the matrix responsible for the transitions $(N_R)^c \to e_L W_L$ is the matrix called also R in the mass basis Lagrangian (76). Then, the lepton number asymmetry from the decay of the first heavy Majorana neutrino, in the single-flavor approximation, is given by the same formulas (111),(112).

In the LR model we have now R given by (85), that yields the lepton number asymmetry in terms of the Dirac mass and the mixing matrix T of the heavy neutrinos:

$$\epsilon_1 = \frac{g^2}{M_W^2}\frac{1}{16\pi}\frac{1}{(T^t m_D^\dagger m_D T^*)_{11}}\sum_{k\neq 1} F(x_k)\mathrm{Im}[(T^t m_D^\dagger m_D T^*)_{1k}]^2. \tag{116}$$

In the LR model the lepton number asymmetry depends on the current basis matrix m_D in (34) and also on the PMNS matrix T of the heavy neutrinos. Consistently, the presence of the matrix T appears in (116) because, to compute the decay rates $(N_1)^c \to e_L W_L$, one needs first to diagonalize the mass matrix $M_R = t v_R$ (15).

In other terms, the matrix $m_D T^* = m'_D$ is the Dirac mass matrix in the basis in which M_R in (27) is diagonalized. In this basis the left-handed term of the interaction Lagrangian $\bar{\nu}_L e_L W_L$ remains diagonal, but the right-handed term $\bar{N}_R e_R W_R$ is not anymore.

Expression (116) for the CP asymmetry in the electroweak broken phase follows from the R term in the interaction Lagrangian (76), responsible for the decay $(N_R)^c \to e_L W_L$. This is the expression that has been used precisely to compute the leptogenesis CP asymmetry within LRM (see, for example, Refs. [20,35]).

However, in the LRM the broken electroweak phase is more involved than in the ESM because there are two vacuum expectation values k_1 and k_2 (12) that contribute to m_D and to M_W, besides the possibility of a vacuum expectation value v_L (not considered in subsection I A 2) that could also contribute to the W_L mass.

In the unbroken electroweak phase, the Higgs bidoublet (10) would be massless, and one should consider both contributions $N_1 \to e\varphi_{1,2}$ to the leptogenesis asymmetry, with both Higgses $\varphi_{1,2}$ contributing to the loops needed to interfere with the tree diagram to obtain CP violation. This situation reminds the one of the Standard Model with several Higgs doublets [36]. The relation between the CP asymmetries in the broken and unbroken phases of the LRM deserves further investigation.

Since the matrix m_D is general complex, so is $m_D T^*$ and we can write a decomposition in terms of another general unitary matrix U' and another triangular matrix m'_Δ:

$$m'_D = m_D T^* = U' m'_\Delta. \tag{117}$$

The lepton asymmetry reads

$$\epsilon_1 = \frac{g^2}{M_W^2} \frac{1}{16\pi} \frac{1}{(m'^\dagger_\Delta m'_\Delta)_{11}} \sum_{k\neq 1} F(x_k) \mathrm{Im}[(m'^\dagger_\Delta m'_\Delta)_{1k}]^2, \tag{118}$$

which now depends on the three CP phases of m'_Δ.

On the other hand, notice that the interaction Lagrangian (76) contains also the $\Delta L = 2$ term $\overline{(\nu_L)^c} S^t e_R W_R$ that could give a contribution to the lepton asymmetry through the decay

$$W_R \to \bar{e}_R (\nu_L)^c. \tag{119}$$

The masses M_{W_R} and M_i are both generated by the same Higgs triplet, and since one usually assumes that the Yukawa coupling of the heaviest neutrino N_3 is of $O(1)$, then $M_{W_R} \gg M_1$, assuming a hierarchical spectrum for the heavy neutrinos. Hence, the lepton asymmetry generated by the decay of W_R could be washed out and only the one due to the N_1 decays would survive. However, one should keep in mind in model building the possibility of leptogenesis through the decay (119).

B. Flavored leptogenesis

1. Extended Standard Model

A crucial progress in leptogenesis has been achieved by taking into account flavor [37–40]. At high temperatures $T \geq 10^{12}$ GeV, all three τ, μ and e are out of equilibrium because their Yukawa couplings are weak compared to the temperature. In this regime, the one-flavor approximation can be applied since the different lepton flavors are undistinguishable.

However, for “realistic” temperatures $T \simeq M_1$ such that $10^9 \leq T \leq 10^{12}$ GeV, the τ lepton doublet Yukawa coupling is large enough to be in thermal equilibrium, while the μ and e doublets are out of equilibrium. The net result is that the leptogenesis CP violation splits into two pieces, ϵ_τ and $\epsilon_2 = \epsilon_\mu + \epsilon_e$, since the flavors μ and e remain undistinguishable. Then, in the range $10^9 \leq T \leq 10^{12}$ GeV, the final baryon asymmetry Y_B is the sum of two contributions, given by the lepton CP asymmetries ϵ_τ and ϵ_2 affected by different wash-out factors η_τ and η_2: $Y_B \propto \epsilon_\tau \eta_\tau + \epsilon_2 \eta_2$. A recent updated flavor covariant description of flavor effects in leptogenesis can be found in Ref. [41].

The CP-violating asymmetry for each flavor is given by the expression (see for example [5]):

$$\begin{aligned}\epsilon_{1\ell} = {} & \frac{g^2}{M_W^2} \frac{1}{16\pi} \frac{1}{(m_D^\dagger m_D)_{11}} \\ & \times \sum_{k\neq 1} F(x_k) \mathrm{Im}[(m_D^\dagger)_{1\ell} (m_D)_{\ell k} (m_D^\dagger m_D)_{1k}]^2 \\ & + \frac{g^2}{M_W^2} \frac{1}{16\pi} \frac{1}{(m_D^\dagger m_D)_{11}} \\ & \times \sum_{k\neq 1} G(x_k) \mathrm{Im}[(m_D^\dagger)_{1\ell} (m_D)_{\ell k} (m_D^\dagger m_D)_{k1}]^2,\end{aligned} \tag{120}$$

where the second term corresponds to the lepton-flavor-violating but lepton-number-conserving self-energy diagram [39]. The function $F(x_k)$ is given by (112), and

$$G(x_k) = \frac{1}{1 - x_k}. \tag{121}$$

The second term in (120) vanishes when summing over ℓ, while the first term gives the one-flavor approximation expression (113), because $\sum_\ell \epsilon_{1\ell} = \epsilon_1$. On the other hand, the second term in (120) is subleading if one assumes $M_1 \ll M_2, M_3$.

The flavored wash-out factors read [40]

$$\eta_\ell = \eta \frac{(m_D^\dagger)_{1\ell} m_{D\ell 1}}{(m_D^\dagger m_D)_{11}}, \tag{122}$$

where η is the wash-out factor in the single-flavor approximation.

Concerning the link between low-energy CP violation in the PMNS mixing matrix and leptogenesis CP violation, the situation is quite different if flavor is taken into account [40]. As an illustration, let us write the CP asymmetry $\epsilon_{1\ell}$, where the subindex 1 means decay of the lightest heavy Majorana neutrino N_1, by using the orthogonal parametrization (106). The flavor CP asymmetries $\epsilon_{1\ell}$ depend then on the low-energy parameters, i.e. the light neutrino masses and the PMNS mixing matrix K. Assuming $M_1 \ll M_2 < M_3$, one finds from (106) and (120) the leptonic CP violation parameter $\epsilon_{1\ell}$ [40]:

$$\epsilon_{1\ell} \simeq -\frac{3}{32\pi} \frac{g^2}{M_W^2} \frac{\mathrm{Im}(\sum_{k,j} m_j m_k^{3/2} K^*_{\ell j} K_{\ell k} O^*_{j1} O^*_{k1})}{\sum_i m_i |O_{i1}|^2}. \tag{123}$$

2. Left-right model

As we have seen in the LRM in the one-flavor approximation [formula (116)], m_D is replaced by $m_D T^*$, and the formula for the lepton asymmetry in this approximation is the same as in the extended Standard model with the replacement $m_D \to m'_D = m_D T^*$ where m'_D is the Dirac mass matrix in the basis in which the mass matrix M_R is diagonalized.

Because of (108), formulas for the CP asymmetry (120) and the wash-out factor (122) remain correct for the left-right model, with the replacement $m_D \to m'_D = m_D T^*$, where m_D is given by (109), that has a complete left-right symmetry in the dependence on the mass eigenvalues $m_L^{\rm diag}, M_R^{\rm diag}$ as well as on the mixing matrices K, T. Then, the flavor asymmetry has the same form (123), with the replacement $O \to O'$.

VI. COMPARISON BETWEEN THE EXTENDED STANDARD MODEL AND THE LEFT-RIGHT MODEL

We now summarize the comparison between the ESM and the LRM, as far as lepton mixing is concerned.

(1) In the current basis both models differ in the following way.

In the ESM the Dirac matrix m_D has 15(6) parameters because one can rotated away 3 phases and one can diagonalize the right-handed mass matrix M_R. One has finally a total of 21(6) parameters.

In the LRM one cannot diagonalize M_R without changing the interaction Lagrangian. On the other hand, one cannot rotate away phases in both m_D and in M_R, but only three phases in one of these matrices, that we have chosen to be M_R. Then, one is left with a general complex m_D with 18(9) parameters and a complex symmetric M_R with 9(3) parameters. With the m_e mass eigenvalues, this gives a total of 30(12) parameters.

However, if in the LRM one diagonalizes M_R from the start, the left-handed interaction term $\bar{\nu}_L \gamma_\mu e_L W_L^\mu$ remains diagonal, while the right-handed term $\bar{N}_R \gamma_\mu e_R W_R^\mu$ is modified. Also m_D is modified to another Dirac mass term, that would eventually control leptogenesis. Therefore, as far as one considers the mass terms and the W_L interation, one has the same number of parameters as in the ESM. For physics at low energy and also for leptogenesis, if the latter is attributed to the decays of the lightest right-handed heavy neutrino N_1, one can disregard the W_R interaction term, that involves heavier degrees of freedom.

(2) In the mass basis in the ESM without approximations one has two mixing matrices K and R in the left sector, that have together 12(6) parameters. For $m_D \ll M_R$ one has *a priori* 12(6) parameters for the set of matrices K, R (mixing in the left sector), and S, T (mixing in the right sector). The mixing matrix of the left-handed neutrinos is approximately unitary and can be chosen to be of the PMNS type, with 6(3) parameters. The model constrains the mixing matrix of the right-handed neutrinos to be $T \simeq 1$, the matrix R (71) has a total of 12(6) parameters and S is not independent because of relation (72). The parameters of the PMNS mixing matrix for light neutrinos K have to chosen among the ones of R. Adding the mass eigenvalues $m_L^{\rm diag}, M_R^{\rm diag}, m_e^{\rm diag}$ one has a total of 21(6) parameters.

In the LRM in the mass basis one has more symmetry: two mixing matrices K, R in the left sector and two S, T in the right sector. These four matrices have *together* 21(12) parameters, that added to the mass eigenvalues $m_L^{\rm diag}, M_R^{\rm diag}, m_e^{\rm diag}$ gives again a total of 30(12) parameters. In the approximation $m_D \ll M_R$, the mixing matrices K (left sector) and T (right sector) are unitary, and both can be chosen to be of the PMNS type, with 6(3) parameters each. This is different from the ESM for the right sector, where T is trivial. This feature of the ESM seems unnatural, since physically one should expect a full PMNS matrix for the heavy right-handed neutrinos as well.

(3) Adopting the decomposition $m_D = U m_\Delta$ (U unitary and m_Δ triangular complex), in the ESM the matrix U has 6(3) parameters and m_Δ 9(3) parameters, corresponding to the 15(6) parameters of m_D. The natural solution is that the PMNS matrix K is a unitary factor of the matrix U, namely $U = KV$, V being also unitary. In the LRM the situation is somewhat different: m_D is a general complex matrix with 18(9) parameters, U is a *general* unitary matrix with 9(6) parameters and m_Δ has also 9(3) parameters. The Dirac mass matrix in the basis in which M_R is diagonal (117) $m'_D = m_D T^*$ can be decomposed in the same way: $m'_D = U' m'_\Delta$.

(4) Concerning the lepton asymmetry relevant for leptogenesis, we find the following situation in both models.

In the ESM, in the one-flavor approximation, the asymmetry is dependent on matrix elements of the matrices $R^\dagger R$ or $m_D^\dagger m_D$ or $m_\Delta^\dagger m_\Delta$, i.e. dependent on the three CP phases of m_Δ. In the flavored case, the asymmetry (120) depends on the PMNS matrix K and the three high-energy phases of the orthogonal matrix O (105).

In the LRM, in the one-flavor approximation, the lepton asymmetry is dependent on $R^\dagger R$ or $T^t m_D^\dagger m_D T^*$. Writing the product $m_D T^*$ as in (117), the asymmetry depends on the three CP phases of the triangular matrix m'_Δ through $m'^\dagger_\Delta m'_\Delta$. In the flavored case, the asymmetry depends on the three phases of the PMNS mixing matrix K and on the three phases of O' (108).

As far as model building is concerned, the situation is different in both schemes. As an example, imagine that one has a model for the Yukawas with some ansatz for m_D and M_R. In the ESM, M_R is diagonalized and m_D is enough to compute the lepton asymmetry. In the LRM one needs to compute the matrix T that diagonalizes M_R, in order to get m'_D.

(5) A possible identification between low-energy phases and leptogenesis phases is not possible in general. In the ESM one could imagine models in which the three CP phases of the light neutrinos mixing matrix K are the same as the three phases of the triangular matrix m_Δ, since one has to choose the parameters of K among the ones of the matrix R in the lepton asymmetry formula (111). In the LRM one could choose the three phases of K to be the same as the ones of m'_Δ (117).

As to whether, in general, the leptogenesis CP asymmetry could depend on the low-energy phases, in the flavored regime the usual argument that $\epsilon_{1\ell}$ in the ESM depends on the PMNS matrix K and on the matrix O (105) extends to the LRM with another orthogonal matrix O' (108).

(6) Relative to the ESM, we have found that the LRM has some interesting new features:
   (a) The nontrivial PMNS mixing matrix T of the heavy neutrinos enters in the quantitative estimation of decay branching ratios of heavy neutrinos N_{R_i} to various final states.
   (b) On the other hand, in the calculation of the leptogenesis CP asymmetries, the matrix T is unobservable because the Dirac matrix that plays a role is now (117) $m'_D = m_D T^*$, the Dirac matrix in the basis in which M_R is diagonal.
   (c) The term $\overline{(\nu_L)^c} S^t e_R W_R$ in (76) could give a contribution to the cosmological lepton asymmetry through the $\Delta L = 2$ lepton-number-violating decay to light leptons $W_R \to \bar{e}_R (\nu_L)^c$. As we have indicated above, this latter possibility seems unlikely in reasonable left-right models because W_R is heavier than the lightest neutrino N_1. However, one should keep in mind this possibility in model building.
   (d) Considering the W_1, W_2 basis, i.e. without neglecting $W_L - W_R$ mixing, we have seen in Sec. III B that there is a term involving the lighter W_1 boson $\sim \sin\zeta \overline{(\nu_L)^c} S^t \gamma_\mu e_R W_1^\mu$ that allows for the subleading $\Delta L = 2$ lepton-number-violating decay to light leptons $W_1 \to \bar{e}_R (\nu_L)^c$.

VII. EXTENSION TO PATI-SALAM AND $SO(10)$

One can extend the precedent considerations to other left-right gauge models like the Pati-Salam gauge theory $SU(4)_C \times SU(2)_L \times SU(2)_R$ [17] or $SO(10)$ [6].

We can consider first each of these models in the current basis, with general mass terms determined only by the Dirac or Majorana character of the fermions, and perform the counting of the CP-conserving and CP-violating free parameters. In a second step, one can diagonalize the mass matrices and obtain mixing in the interaction terms and, in a third step, *switch on the Higgs sector* of each theory and see how, according to the different hypothesis on this sector, the predictive power of each scheme is improved. Of course, with the most general Higgs structure for each model, one populates the general parameter space of the mass terms obtained by imposing only Lorentz invariance.

Moreover, since in these theories leptons are related to quarks, lepton mixing in the Dirac mass term will be related to quark mixing, at least for some Higgs structures. This feature is interesting in view of increasing the predictive power of $SO(10)$ for leptogenesis, and has been used more or less quantitatively in the literature.

Let us give some details for the Pati-Salam model and for $SO(10)$. Consider first the general mass Lagrangian consistent with Lorentz invariance of Dirac and Majorana mass terms

$$\mathcal{L}_m = \bar{\nu}_L m_D N_R + \frac{1}{2}\overline{(N_R)^c} M_R N_R + \bar{e}_L m_e e_R + \bar{u}_L m_u u_R + \bar{d}_L m_d d_R + \text{H.c.} \tag{124}$$

For the moment the matrices m_D, m_e, m_u and m_d are general complex with 18(9) parameters each and M_R is a general complex symmetric matrix with 12(6) parameters. This gives *a priori* a total of 84(42) parameters, while in the lepton sector one has $18(9)(\text{from } m_D) + 18(9)(\text{from } m_e) + 12(6)(\text{from } M_R) = 48(24)$ parameters.

In the Pati-Salam model and in $SO(10)$, the interaction Lagrangian has the general form

$$\mathcal{L}_{\text{int}} = \mathcal{L}_w + \mathcal{L}_x, \tag{125}$$

where one has in both models, keeping only the interesting flavor-changing terms:

$$\mathcal{L}_w = \bar{e}_L \gamma_\mu \nu_L W_L^\mu + \bar{e}_R \gamma_\mu N_R W_R^\mu + \bar{d}_L \gamma_\mu u_L W_L^\mu + \bar{d}_R \gamma_\mu u_R W_R^\mu + \text{H.c.} \tag{126}$$

The extra interaction term in the Pati-Salam model reads

$$\mathcal{L}_x^{PS} = \bar{e}_L \gamma_\mu d_L X_L^\mu + \bar{e}_R \gamma_\mu d_R X_R^\mu + \bar{\nu}_L \gamma_\mu u_L X_L^\mu + \bar{N}_R \gamma_\mu u_R X_R^\mu + \text{H.c.}, \tag{127}$$

where the colored gauge bosons have charges $|Q(X_L)| = |Q(X_R)| = \frac{2}{3}$.

In $SO(10)$ one has [42,43]

$$\begin{aligned}\mathcal{L}_x^{SO(10)} &= [\epsilon^{ijk}\overline{(u_R^i)^c}\gamma_\mu u_L^j + \overline{d_L^k}\gamma_\mu(e_R)^c - \overline{e_L}\gamma_\mu(d_R^k)^c]X^{k\mu} \\ &+ [\epsilon^{ijk}\overline{(u_R^i)^c}\gamma_\mu d_L^j + \overline{\nu_L}\gamma_\mu(d_R^k)^c - \overline{u_L^k}\gamma_\mu(e_R)^c]Y^{k\mu} \\ &+ [\epsilon^{ijk}\overline{(d_R^i)^c}\gamma_\mu u_L^j + \overline{e_L}\gamma_\mu(u_R^k)^c - \overline{d_L^k}\gamma_\mu(N_R)^c]Y'^{k\mu} \\ &+ [\epsilon^{ijk}\overline{(d_R^i)^c}\gamma_\mu d_L^j + \overline{\nu_L}\gamma_\mu(u_R^k)^c - \overline{u_L^k}\gamma_\mu(N_R)^c]X_D^{k\mu} \\ &+ [\overline{\nu_L}\gamma_\mu u_L^k + \overline{e_L}\gamma_\mu d_L^k - \overline{(d_R^k)^c}\gamma_\mu(e_R)^c \\ &- \overline{(u_R^k)^c}\gamma_\mu(N_R)^c]S^{k\mu} + \text{H.c.}, \end{aligned} \tag{128}$$

where i, j, k are color indices and the colored gauge bosons X, Y, Y', X_D, S have the charges: $|Q(X)| = \frac{4}{3}, |Q(Y)| = |Q(Y')| = \frac{1}{3}, |Q(X_D)| = |Q(S)| = \frac{2}{3}$.

Let us see how many parameters can be rotated away in both models. Analogously to the LRM, one can diagonalize m_e and absorb three phases in M_R in (124) while keeping $\mathcal{L}_w$ (126) invariant. However, as is obvious from (127) and (128), $\mathcal{L}_x$ is changed under these transformations. In the pure lepton sector, leaving aside the quark-lepton terms in $\mathcal{L}_x$, the starting point for the diagonalization of the mass terms is the same as in the LRM (34), with 30(12) parameters in m_D, M_R and m_e^{diag}. Diagonalizing (124) one gets the flavor-changing mixing in the interaction Lagrangian $\mathcal{L}_w + \mathcal{L}_x$.

In the pure lepton sector our conclusions are the following. The diagonalization has the same form for $SU(2)_L \times SU(2)_R \times U(1)$, Pati-Salam and $SO(10)$ models. Separately, the 3×3 matrices K and R enter in the left sector, while the 3×3 matrices T and S enter in the right sector, like in the LRM, Eq. (76). In $SU(2)_L \times SU(2)_R \times U(1)$, Pati-Salam and $SO(10)$ models we have in the lepton sector the same counting of free parameters, i.e. 30 real parameters, among them 12 CP-violating phases.

Let us now make some remarks on masses and mixing in some particular cases in the interesting $SO(10)$ case. Let us look at the product

$$\mathbf{16} \times \mathbf{16} = \mathbf{10}_S + \overline{\mathbf{126}}_S + \mathbf{120}_A, \tag{129}$$

where $\mathbf{10} + \overline{\mathbf{126}}$ is the symmetric part and $\mathbf{120}$ the antisymmetric part. The representations $\mathbf{10}$ and $\mathbf{120}$ are real, $\mathbf{126}$ is complex, and the Yukawa terms that can give mass to the fermions are

$$\mathbf{16}_f \times \mathbf{16}_f \times \mathbf{10}_H = \mathbf{1} + \cdots \tag{130}$$

$$\mathbf{16}_f \times \mathbf{16}_f \times \overline{\mathbf{126}}_H = \mathbf{1} + \cdots \tag{131}$$

$$\mathbf{16}_f \times \mathbf{16}_f \times \mathbf{120}_H = \mathbf{1} + \cdots. \tag{132}$$

The Yukawa part of the Lagrangian reads

$$\mathcal{L}_Y = \mathbf{16}_f(Y_{10}\mathbf{10}_H + Y_{126}\overline{\mathbf{126}}_H + Y_{120}\mathbf{120}_H)\mathbf{16}_f, \tag{133}$$

where a possible sum over Higgs representations and Yukawa coupling matrices in family space is implicit. After spontaneous symmetry breaking, one gets the mass Lagrangian (see for example [44]),

$$\begin{aligned} m_d &= v_{10}^d Y_{10} + v_{126}^d Y_{126} + v_{120}^d Y_{120} \\ m_u &= v_{10}^u Y_{10} + v_{126}^u Y_{126} + v_{120}^u Y_{120} \\ m_e &= v_{10}^d Y_{10} - 3v_{126}^d Y_{126} + v_{120}^e Y_{120} \\ m_D &= v_{10}^u Y_{10} - 3v_{126}^u Y_{126} + v_{120}^D Y_{120} \\ M_R &= v_{126}^R Y_{126}, \end{aligned} \tag{134}$$

where the Yukawa matrices Y_{10} and Y_{126} are complex symmetric, Y_{120} is complex antisymmetric, and the v's are Higgs vacuum expectation values. From the term (130) alone we obtain the well-known relations $m_e = m_d$ and $m_D = m_u$, while the term (131) alone would give the relations $m_e = -3m_d$ and $m_D = -3m_u$, and *no relation* from the term (132).

The VEVs in (134) are, in all generality, complex numbers if we assume that CP can be spontaneously broken (soft CP violation). If CP is not spontaneously broken, the VEVs are real and all CP violation comes from the Yukawa couplings (hard CP violation).

One could wonder how within SO(10) one can get the most general counting of parameters done above, i.e. 84 (42) parameters for the whole mass sector (124), with 48 (24) parameters in the lepton sector. As said above, this is simply achieved if all the representations $\mathbf{10}_H, \overline{\mathbf{126}}_H, \mathbf{120}_H$ in (134) are present and are different for each mass matrix, that becomes then completely general.

An interesting particular case is to consider only the $\mathbf{10}$ and $\overline{\mathbf{126}}$ representations in (134), with $\mathbf{120}$ absent:

$$\begin{aligned} m_d &= m_{10}^d + m_{126}^d \\ m_u &= m_{10}^u + m_{126}^u \\ m_e &= m_{10}^d - 3m_{126}^d \\ m_D &= m_{10}^u - 3m_{126}^u \\ M_R &= m_{126}^R. \end{aligned} \tag{135}$$

In this situation, all mass matrices m_u, m_d, m_D, m_e and M_R are complex symmetric.

Let us count again the number of parameters under this hypothesis. The complex symmetric matrices $m_{10}^d, m_{126}^d, m_{10}^u, m_{126}^u, m_{126}^R$, have 12(6) parameters each, that gives a total number of 60(30) parameters, a reduction relative to the 84(42) total number of parameters of the general case. One can diagonalize the complex symmetric matrices $m_d, \ldots M_R$ with unitary matrices $V_d, \ldots V_R$. Because of relations (135), the unitary matrices V_e, V_D, V_R are, in principle, given in terms of V_u and V_d and mass eigenvalues. Notice that, as discussed in the mass basis for the pure lepton sector, we can adopt without loss of generality the basis in which $m_e = m_e^{\rm diag}$. However, these relations give complicated equations between the elements of mixing matrices. Within this case of considering both **10** and $\overline{\bf 126}$, it seems hard to find relations between the mixing matrices in the quark and the lepton sector, at least in a model-independent way.

Let us consider two limiting cases: while the $\overline{\bf 126}$ contributes to M_R, only the **10** or only the $\overline{\bf 126}$ contribute to m_d, m_u, m_e and m_D.

From (135) we see that in both cases one has quark-lepton symmetry in the mixing matrices, i.e. a relation between the left-handed neutrino Dirac mixing matrix V_L, where $m_D = V_L^\dagger m_D^{\rm diag} V_R$, and the CKM quark matrix

$$V_L = V_u V_d^\dagger = V_{\rm CKM}. \tag{136}$$

This relation has been often used in a number of phenomenological schemes [2,4,5]. However, as is well known, one needs both representations **10** and $\overline{\bf 126}$ to describe fermion masses in $SO(10)$ [45,46] and, therefore, we must conclude that there is a clash between a good description of fermion masses and the one of obtaining quark-lepton symmetry in mixing.

Although the point of view of obtaining useful theoretical hints from $SO(10)$ on the eigenvalues and mixing of the Dirac neutrino mass matrix has been advanced in a number of works [1–5], it is worth to point out that there could be an alternative philosophy concerning the Dirac mass matrix. Within the left-right model, if the W_R gauge boson and the lightest heavy neutrino N_R are light enough, there is the interesting possibility of a complete determination of the Dirac mass matrix from the experimental study of W_R and N_R decays [47].

VIII. CONCLUSIONS

We have examined the parameter counting and structure of CP-conserving and CP-violating lepton mixing in two gauge models in the electroweak broken phase, the extended Standard Model, i.e. the Standard Model plus one right-handed heavy neutrino per generation, and the left-right Model $SU(2)_L \times SU(2)_R \times U_{B-L}(1)$. We have used both the current basis, in which the gauge interactions are diagonal, and the mass basis, where the mass matrices are diagonal and mixing appears in the charged current gauge-fermion part of the Lagrangian. On the other hand, we have distinguished between results that are exact and results that hold within the approximation of Dirac masses that are small relative to right-handed neutrino masses, $m_D \ll M_R$.

We think that it is worth to compare these two models. One reason is that, for simplicity, in the literature people usually discuss lepton mixing within the simple ESM, while actually have in mind left-right grand unified theories like $SO(10)$, that naturally include heavy right-handed neutrinos. The simplest LR model that we study in this paper is a kind of prototype for these more involved LR theories.

Although the outline of the parameter counting and structure of lepton mixing is rather close in both schemes, there are differences between the two models. In particular, the extended Standard Model can accommodate a PMNS mixing matrix K for light neutrinos, but there is no room in parameter space for a mixing matrix T for the heavy neutrinos, the mixing matrix being close to the identity. On the other hand, as one could naturally expect, the left-right model is consistent with PMNS mixing matrices for both light and heavy neutrinos. The lepton asymmetry relevant for leptogenesis depends not only on the Dirac mass m_D but also on the matrix T, which is nontrivial. But the lepton asymmetry is given in terms of the Dirac mass in the basis in which the right-handed heavy neutrino mass matrix is diagonal, while the interaction term in the right-handed sector is not diagonal anymore.

In the case of the LR model, the connection between the lepton CP asymmetry in the electroweak broken phase, coming from the decay $(N_R)^c \to W_L e_L$ and its CP conjugate, and the one in the unbroken phase coming from the decay above the phase transition $N_R \to e\varphi$, where φ is the Higgs bidoublet, is an open problem worth to be investigated.

Mixing in the LRM contains new terms that involve $\Delta L = 2$ CP-violating interactions involving the W_R gauge bosons. Considering the $W_L - W_R$ mixing, there are interesting new possible $\Delta L = 2$ processes with light leptons in the final state: the subleading decay $W_1 \to \bar{e}_R(\nu_L)^c$ and the leading one $W_2 \to \bar{e}_R(\nu_L)^c$. As emphasized above, it is worth to keep in mind, in model building, the possibility of the latter as a contribution to leptogenesis.

We have extended these results to other LR theories, namely the Pati-Salam model $SU(4)_C \times SU(2)_L \times SU(2)_R$ and the grand unified model SO(10), for which we find that the structure of mixing in the lepton sector is, in the most general case, the same as in the left-right model $SU(2)_L \times SU(2)_R \times U_{B-L}(1)$. The specification of the Higgs sector provides schemes that have more predictive power.

If one assumes both symmetric **10** and **126** Higgs representations, necessary to describe the quark mass spectrum, we emphasize that there is a clash between the description of this spectrum and the assumption that the left-handed Dirac mixing matrix is approximately given by the quark CKM matrix, as sometimes it is assumed in phenomenological models arguing naive quark-lepton symmetry.

Phenomenological analyses are usually done within these gauge models as $SO(10)$ supplemented by simplifying hypotheses that give tractable schemes. But one should keep in mind that the general parameter space can yield other possibilities concerning the description of the interesting observables.

Concerning low-energy observables, there are no differences between the extended Standard Model and the minimal left-right model at leading order in m_D/M_R. The cosmological baryon asymmetry via leptogenesis above the electroweak phase transition deserves however further investigation within the left-right model.

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APPENDIX: A GENERAL DIGRESSION ON THE MATRICES *K*, *R*, *S*, *T*

To count the number of independent parameters in each scheme, it is useful to consider the general case of diagonalization of a 6×6 complex symmetric matrix,

$$\mathcal{M} = \begin{pmatrix} m_L & m_D \\ m_D^t & M_R \end{pmatrix} \tag{A1}$$

where m_L and M_R are 3×3 complex symmetric. In general, a 6×6 complex symmetric matrix has 42(21) real parameters.

Let us now diagonalize $\mathcal{M}$ with the unitary matrix V (37)–(39). The unitarity condition $VV^\dagger = 1$ is an Hermitian relation that implies 36(15) constraints. A general complex 6×6 matrix has 72(36) parameters. Therefore, because of these constraints, V must have $72(36) - 36(15) = 36(21)$ parameters, consistent with the number of $\frac{n(n-1)}{2}$ angles and $\frac{n(n+1)}{2}$ phases of a $n\times n$ unitary matrix. Since $\mathcal{M}^{\text{diag}}$ has 6(0) parameters, the rhs of (37) has $36(21)(\text{from}\, V) + 6(0) = 42(21)$, in consistency with the counting of parameters of the matrix $\mathcal{M}$ (A1).

The unitarity of the matrix V (39) implies [9,10]

$$KK^\dagger + RR^\dagger = 1 \tag{A2}$$

$$SS^\dagger + TT^\dagger = 1 \tag{A3}$$

$$KS^\dagger + RT^\dagger = 0 \tag{A4}$$

$$K^\dagger K + S^\dagger S = 1 \tag{A5}$$

$$R^\dagger R + T^\dagger T = 1 \tag{A6}$$

$$K^\dagger R + S^\dagger T = 0. \tag{A7}$$

Let us do the exercise of counting the number of parameters of the matrices (K, R, S, T). If each of them were general complex, we would have 18(9) parameters for each, which gives for (K, R, S, T) a total of 72(36) parameters. Relations (A2) and (A3) are Hermitian, giving each 9(3) constraints, while (A4) is general complex, giving 18(9) constraints. In total, we have again $9(3) + 9(3) + 18(9) = 36(15)$ constraints, and therefore, the set (K, R, S, T) has $72(36)(-)36(15) = 36(21)$ independent parameters, in agreement with the counting of independent parameters of the unitary matrix V.

On the other hand, the diagonalization of (A1) reads

$$K m_L^{\text{diag}} K^t + R M_R^{\text{diag}} R^t = m_L \tag{A8}$$

$$S m_L^{\text{diag}} S^t + T M_R^{\text{diag}} T^t = M_R \tag{A9}$$

$$K m_L^{\text{diag}} S^t + R M_R^{\text{diag}} T^t = m_D. \tag{A10}$$

Verifying again the counting of parameters, we have for the rhs of (A8)–(A10), $12(6) + 12(6) + 18(9)$ parameters from, respectively, m_L, M_R and m_D. This gives a total of 42(21) independent parameters for the rhs, which is equal to the number of parameters of the lhs, $36(21) + 3(0) + 3(0)$ from, respectively, (K, R, S, T), m_L^{diag} and M_R^{diag}.

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