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Higher-derivative corrections to type II supergravity: Four Ramond-Ramond terms

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It is known that the sphere-level S-matrix element of four type II superstrings has one kinematic factor. At the low energy limit, this factor produces the kinematic factor of the corresponding Feynman amplitudes in the supergravity. It also produces higher-derivative couplings of four strings. In this paper, we explicitly calculate the kinematic factor of four Ramond-Ramond (RR) states in the supergravity. Using this factor, we then find the eight-derivative P-even and P-odd couplings of four RR fields, including the self-dual RR five-form field strength. We show that the P-even couplings are mapped to the standard $\bar{R}^4$ couplings by linear T-duality and S-duality transformations. We also confirm the P-even couplings with direct calculations in type II superstring theories.

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I. INTRODUCTION

Superstring theories at the low energy limit are described appropriately by supergravities which include only the massless modes and their interactions at the two-derivative level. These theories inherit many symmetries of the superstring theories such as string dualities [1–4]. For many purposes, it is enough to use only these effective theories, but there are situations for which one must go beyond the lowest order terms in the effective actions. The higher order terms must be corrections in α' and in the string coupling constant g_s. The main challenge thus is to implement the symmetries of the superstring theories to find an effective action that incorporates all such corrections, including nonperturbative effects [5].

Subleading terms in type II effective actions start at the eight-derivative level and were first calculated at the tree level from four-graviton scattering [6,7] as well as from the σ-model beta function [8–13]. They take the following form at tree level in the string frame:

$$S \supset \frac{\gamma}{3.2^7\kappa^2}\int d^{10}x e^{-2\phi}\sqrt{-G}\left(t_8t_8R^4+\frac{1}{4}\epsilon_8\epsilon_8R^4\right), \quad (1)$$

where $\gamma = \frac{\alpha'^3\zeta(3)}{2^5}$ and t_8 is a tensor which is antisymmetric within a pair of indices and is symmetric under the exchange of the pair of indices. The above expression, however, cannot be complete, as supersymmetry will necessarily bring in additional higher order terms built from the other fields in the supergravity multiplet. This includes the B-field and dilaton in the Neveu-Schwarz Neveu-Schwarz (NSNS) sector, the n-form field strengths in the Ramond-Ramond (RR) sector, and their corresponding fermionic superpartners. It would be desirable to obtain a supersymmetric invariant action at the eight-derivative level which is the completion of the above terms [14–20].

The bosonic couplings in the effective action (1) may also be found by constraining it to be consistent with the string dualities [20–25]. The couplings at weak field level, i.e., four-field couplings, may also be found more directly from the corresponding scattering amplitude of four vertex operators. They must be also consistent with linear string dualities. The sphere-level scattering amplitude of four strings has the following structure in the Einstein frame [6,7]:

$$\mathcal{A} = \left(\frac{\Gamma(-e^{-\phi_0/2}s/8)\Gamma(-e^{-\phi_0/2}t/8)\Gamma(-e^{-\phi_0/2}u/8)}{\Gamma(1+e^{-\phi_0/2}s/8)\Gamma(1+e^{-\phi_0/2}t/8)\Gamma(1+e^{-\phi_0/2}u/8)}\right)\mathcal{K}, \quad (2)$$

where $\mathcal{K}$ is the kinematic factor that depends on external states; ϕ_0 is the constant dilaton background; and s, t, u are the Mandelstam variables.[1] The low energy expansion of the Gamma functions is

$$\frac{\Gamma(-e^{-\phi_0/2}s/8)\Gamma(-e^{-\phi_0/2}t/8)\Gamma(-e^{-\phi_0/2}u/8)}{\Gamma(1+e^{-\phi_0/2}s/8)\Gamma(1+e^{-\phi_0/2}t/8)\Gamma(1+e^{-\phi_0/2}u/8)} = -\frac{2^9e^{3\phi_0/2}}{stu} - 2\zeta(3) + \cdots, \quad (3)$$

where dots refer to higher order contact terms. Thus, the kinematic factor plays two roles. It produces the Feynman amplitude of four massless strings in the supergravity [26].

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[1] The relation between the Einstein frame metric and the string frame metric is $G^E_{\mu\nu} = e^{-\phi/2}G^S_{\mu\nu}$.

On the other hand, it produces the couplings of four strings at order α'^3 [6,7].

The kinematic factor of RR states involves various traces over the ten-dimensional gamma matrices. Performing the traces, one expects that the amplitudes at the two-momentum level are reproduced by the corresponding Feynman amplitudes in the supergravity, and at the eight-momentum level, they reproduce the eight-derivative couplings in the action (1). Such a calculation for the scattering amplitude of two RR and two NSNS states has been done explicitly in Ref. [27]. It has been shown that the couplings at the eight-derivative level are related to four NSNS couplings found in Refs. [6,7,28] through the linear T-duality and S-duality [27].

In this paper, we are interested in the couplings of four RR fields in the effective action (1), including the RR five-form field strength which must be self-dual. We use the above double roles of the kinematic factor. That is, we first calculate the kinematic factor of four RR states in the type II supergravities, and then we use it to find the couplings of four RR states at order α'^3. The standard type IIB supergravity, however, is off by the fact that it does not include the self-duality of the RR five-form field strength. The self-duality must be imposed by hand on the equations of motion [29]. In this paper, we impose the self-duality of the RR five-form field strength by hand in the scattering amplitudes. The couplings we have found then have P-even and P-odd parts. We will confirm the P-even couplings by demonstrating that they are related to four NSNS couplings [6,7,28] through the linear T-duality and S-duality transformations. We will also confirm them by direct comparison with the kinematic factor in the type II superstring theories.

The paper is arranged as follows. In Sec. II, we use the type II supergravities to calculate various scattering amplitudes of four RR states and find their corresponding kinematic factors. We then transform these factors to spacetime and find various couplings of four RR field strengths at order α'^3. After imposing the self-duality on the RR five-form field strength, we find all P-even and P-odd couplings. In Sec. III, the S-duality and T-duality have been used as guiding principles to find the P-even couplings of four RR field strengths from the sphere-level couplings of four NSNS states. We find exact agreement with the P-even part of the above couplings. In Sec. IV, we confirm the P-even couplings directly in type II superstring theories by performing the traces in the corresponding kinematic factor of the S-matrix element of four RR vertex operators in the RNS and in the Pure spinor formalisms.

II. FIELD THEORY AMPLITUDE

In this section, we are going to calculate the S-matrix elements of four RR fields in supergravity. These amplitudes have the following structure in the Einstein frame:

$$A = \frac{\mathcal{K}_s}{s} + \frac{\mathcal{K}_t}{t} + \frac{\mathcal{K}_u}{u} = \frac{1}{stu}(tu\mathcal{K}_s + su\mathcal{K}_t + st\mathcal{K}_u), \tag{4}$$

where $\mathcal{K}_s$, $\mathcal{K}_t$, and $\mathcal{K}_u$ are the field-theory kinematic factors in the s-, t-, and u-channels, respectively. The Mandelstam variables are defined as $s = -4\alpha' k_1 \cdot k_2$, $u = -4\alpha' k_1 \cdot k_3$, $t = -4\alpha' k_2 \cdot k_3$, and they satisfy the on-shell condition $s + t + u = 0$. Comparing this amplitude with the leading term of the string-theory amplitude (2), one finds the following relation between the field-theory and the string-theory kinematic factors:

$$\mathcal{K} = -2^{-9} e^{-3\phi_0/2}(tu\mathcal{K}_s + su\mathcal{K}_t + st\mathcal{K}_u). \tag{5}$$

Multiplying this factor by $-2\zeta(3)$ and transforming it to the spacetime, one then finds the couplings of four RR fields at order α'^3.

The type II supergravities describe interactions of massless fields of type II superstring theories at the two-derivative level. The type IIA supergravity in the Einstein frame is given as (see, e.g., Ref. [30])

$$S_{IIA} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-G}\left(R - \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}e^{-\Phi}|H|^2 - \frac{1}{2}\sum_{n=2,4} e^{\frac{5-n}{2}\Phi}|\tilde{F}^{(n)}|^2\right) - \frac{1}{4\kappa^2}\int B\wedge dC^{(3)}\wedge dC^{(3)}, \tag{6}$$

where R is the scalar curvature, Φ is the dilaton field, and H is the B-field strength $H = dB$. The RR field strengths are $\tilde{F}^{(2)} = dC^{(1)}$ and $\tilde{F}^{(4)} = dC^{(3)} - H\wedge C^{(1)}$. The above action is the reduction of 11-dimensional supergravity on manifold $R^{1,9} \times S^1$.

Unlike the type IIA supergravity, there is a challenging feature in type IIB supergravity which is the self-duality of the five-form field strength. It is hard to formulate the action in a manifestly covariant form. One way to find the action is to first construct the supersymmetric equations of motion and then to write down an action that reproduces those equations when the self-duality condition is imposed by hand. The type IIB supergravity in the Einstein frame is given as (see, e.g., Ref. [30])

$$S_{IIB} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-G}\left(R - \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}e^{-\Phi}|H|^2 - \frac{1}{2\alpha}\sum_{n=1,3,5} e^{\frac{5-n}{2}\Phi}|\tilde{F}^{(n)}|^2\right) - \frac{1}{4\kappa^2}\int H\wedge dC^{(2)}\wedge C^{(4)}, \tag{7}$$

where $\alpha = 1$ for $n = 1, 3$ and $\alpha = 2$ for $n = 5$. The RR field strengths in this case are $\tilde{F}^{(1)} = dC^{(0)}$, $\tilde{F}^{(3)} = dC^{(2)} - HC^{(0)}$, and

$$\tilde{F}^{(5)} = dC^{(4)} - \frac{1}{2}C^{(2)}\wedge H + \frac{1}{2}B\wedge dC^{(2)}. \quad (8)$$

The self-duality condition that must be imposed in the equations of motion by hand is

$$\tilde{F}^{(5)} = \star\tilde{F}^{(5)}. \quad (9)$$

We will show that without the above self-duality condition the action (7) does not reproduce correctly the S-matrix element of string theory at low energy. However, imposing this constraint by hand on the S-matrix elements, we will find the consistency between field-theory and string-theory S-matrix elements.

Using the above supergravity actions, one can read various vertices and propagators and accordingly calculate the Feynman amplitude of four RR states. For this purpose, we assume the massless fields are small perturbations around the flat background, i.e.,

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}; \qquad B^{(2)} = 2\kappa b^{(2)};$$
$$\Phi = \phi_0 + \sqrt{2}\kappa\phi. \quad (10)$$

The explicit form of the propagators and the vertices that we need in this paper appears in the Appendix. The external states satisfy the on-shell relations $k^2 = 0$ and $k\cdot\varepsilon = 0$ where $\varepsilon^{\mu_1\mu_2\cdots}$ is the polarization of external RR states. Therefore, the couplings that we will find does not contain $\partial_\mu F^{\mu\mu_1\mu_2\cdots}$.

A. $\partial F^{(n)}\partial F^{(n)}\partial F^{(n)}\partial F^{(n)}$ couplings

There are five types of couplings in this section, i.e., $n = 1,2,3,4,5$. When the four RR forms have the same rank, the actions (6) and (7) dictate that for the cases $n = 1,2,3$ the Feynman amplitude in the s-channel is given by the following expression:

$$A_s = [\tilde{V}_{F_1^{(n)}F_2^{(n)}h}]^{\mu\nu}[\tilde{G}_h]_{\mu\nu,\lambda\rho}[\tilde{V}_{hF_3^{(n)}F_4^{(n)}}]^{\lambda\rho} + \tilde{V}_{F_1^{(n)}F_2^{(n)}\phi}\tilde{G}_\phi\tilde{V}_{\phi F_3^{(n)}F_4^{(n)}}, \quad (11)$$

where the vertices and propagators are given in the Appendix. The amplitude in the u-channel is the same as A_s in which the particle labels of the RR fields are interchanged, i.e., $A_u = A_s(2\leftrightarrow 3)$. Similarly, the amplitude in the t-channel is the same as A_u in which the particle labels of the external RR fields are interchanged, i.e., $A_t = A_u(3\leftrightarrow 4)$.

Replacing the vertices and propagators in (11), one can calculate the string-theory kinematic factor (5). To convert this factor to the couplings in the form of $(\partial F)^4$, we use the conservation of momentum, $\sum_{i=1}^4 k_i = 0$, and the on-shell relations on the external states to write the multiples of two Mandelstam variables which appear in (5) as

$$st = 8\alpha'^2(k_1\cdot k_3k_2\cdot k_4 - k_1\cdot k_2k_3\cdot k_4 - k_1\cdot k_4k_2\cdot k_3),$$
$$su = 8\alpha'^2(k_1\cdot k_4k_2\cdot k_3 - k_1\cdot k_2k_3\cdot k_4 - k_1\cdot k_3k_2\cdot k_4),$$
$$tu = 8\alpha'^2(k_1\cdot k_2k_3\cdot k_4 - k_1\cdot k_3k_2\cdot k_4 - k_1\cdot k_4k_2\cdot k_3), \quad (12)$$

where on the right-hand side each label appears once in each term. With the assistance of a field-theory inspired package for Mathematica, "xTras" [31], as well as a symbolic computer algebra system for field-theory problems known as "Cadabra" [32,33], we find the following couplings for $n = 1,2,3$ in the Einstein frame[2]:

$$\mathcal{K} = -\frac{\alpha'^3 e^{5\phi_0/2}}{2^9\kappa^2}[6F_{a,c}F_{b,d}F_{a,b}F_{c,d} - F_{a,c}F_{b,d}F_{a,c}F_{b,d}]$$
$$\mathcal{K} = \frac{\alpha'^3 e^{3\phi_0/2}}{2^{11}\kappa^2}[8F_{ab,e}F_{bc,f}F_{ad,f}F_{cd,e} - 2F_{ab,e}F_{ab,f}F_{cd,f}F_{cd,e} + F_{ab,e}F_{ab,e}F_{cd,f}F_{cd,f}]$$
$$\mathcal{K} = -\frac{\alpha'^3 e^{\phi_0/2}}{2^{11}3^2\kappa^2}[18F_{abc,g}F_{bcd,h}F_{aef,h}F_{def,g} - 2F_{abc,g}F_{abc,h}F_{def,h}F_{def,g} + 18F_{abc,g}F_{bcd,h}F_{aef,g}F_{def,h} - 18F_{abc,g}F_{bcd,g}F_{aef,h}F_{def,h} + F_{abc,g}F_{abc,g}F_{def,h}F_{def,h}]. \quad (13)$$

The antisymmetric properties of the RR field strengths have been taken into account to simplify the kinematic factors in above form. However, multiterm symmetry, i.e., the Bianchi identity for the RR field strength, $dF = 0$, which relates a sum of terms with different index distribution, has not yet been taken into account. This identity reduces the number of couplings to the minimal number.

To do this last step, we use the following algorithm. The general structure of each coupling in the momentum space contains four RR field strengths that each carry one momentum index. We first write it in terms of independent variables. This can be done by writing the RR field strengths in terms of RR potentials and using the conservation of momentum and the on-shell relations to rewrite the coupling in terms of independent variables, i.e., writing $k_4 = -k_3 - k_2 - k_1$ and $k_3\cdot\varepsilon_4 = -k_1\cdot\varepsilon_4 - k_2\cdot\varepsilon_4$. This imposes all symmetries, including the Bianchi identity. Then, we consider all possible contractions of four RR field strengths with unknown coefficients and rewrite them in terms of independent variables. By comparing these two results, one finds some algebraic equations between the unknown coefficients which can be solved to find the coefficients.

[2]All contracted indices have been written as subscripts for easier readability.

To find the minimum number of couplings, we set all unknown coefficients to zero except one of them and solve the equations. If there is a solution, then the coefficient of the minimum terms, which is 1 in this case, would be found. Otherwise, we have to repeat this procedure by setting all coefficients to zero except two of them. If there is a solution, then the coefficient of the minimum terms, which is 2 in this case, would be found. We continue this approach to find the minimal number of couplings.

Performing this calculation for the couplings (13), we simplify them to the following couplings in the string frame:

$$S \supset \frac{\gamma}{8\kappa^2}\int d^{10}x e^{2\phi_0}\sqrt{-G}\Big[2F_{a,b}F_{a,b}F_{c,d}F_{c,d} + 2F_{ad,e}F_{ab,c}F_{cf,d}F_{ef,b} + 2F_{ad,e}F_{ab,c}F_{cf,b}F_{ef,d} - \frac{1}{2}F_{ab,d}F_{ab,c}F_{ef,d}F_{ef,c} + F_{aef,d}F_{abc,d}F_{beg,h}F_{cfh,g} + F_{abe,f}F_{abc,d}F_{cfg,h}F_{deg,h} + F_{abe,f}F_{abc,d}F_{cgh,e}F_{fgh,d}\Big]. \tag{14}$$

While the above algorithm reduces the number of terms for $n = 1, 3$, it does not reduce the three couplings in the case of $n = 2$. However, the index distribution is changed. It means there are at least two different index distributions for the three terms that are identical up to the Bianchi identity. Note that to find the standard sphere-level dilaton factor $e^{-2\phi_0}$ in the string frame one has to normalize the RR potential C with $e^{\phi_0}C$. The normalization of the RR fields in the above action is consistent with the supergravities (6) and (7).

For the $n = 4$ case, there is another contribution to the scattering amplitude in the s-channel which is coming from the Chern–Simons term in (6). The Feynman amplitude in this case is given as

$$A_s = [\tilde{V}_{F_1^{(4)}F_2^{(4)}h}]^{\mu\nu}[\tilde{G}_h]_{\mu\nu,\lambda\rho}[\tilde{V}_{hF_3^{(4)}F_4^{(4)}}]^{\lambda\rho} + \tilde{V}_{F_1^{(4)}F_2^{(4)}\phi}\tilde{G}_\phi\tilde{V}_{\phi F_3^{(4)}F_4^{(4)}} + [\tilde{V}_{\epsilon_{10}F_1^{(4)}F_2^{(4)}b}]^{\mu\nu}[\tilde{G}_b]_{\mu\nu,\lambda\rho}[\tilde{V}_{bF_3^{(4)}F_4^{(4)}\epsilon_{10}}]^{\lambda\rho}.$$

The amplitude in the first line is the same as the amplitude (11). The term in the second line has two Levi-Civitá tensors which can be replaced by the generalized Kronecker delta according to the following expression:

$$\epsilon^{m_1\ldots m_d}\epsilon_{n_1\ldots n_d} = -\delta_{[n_1}{}^{m1}\ldots\delta_{n_d]}{}^{md}. \tag{15}$$

The massless pole in the u-channel is the same as A_s in which the particle labels of the external RR fields are interchanged, i.e., $A_u = A_s(2 \leftrightarrow 3)$. Similarly, the massless pole in the t-channel is the same as A_u in which the particle labels of the external RR fields are interchanged, i.e., $A_t = A_u(3 \leftrightarrow 4)$.

Replacing the vertices and propagators in above amplitude, one can evaluate the kinematic factor $\mathcal{K}$. Using the antisymmetry property of the RR field strength, we simplify the result to the following couplings in the string frame:

$$S \supset -\frac{\gamma}{2^9.3^2\kappa^2}\int d^{10}x e^{2\phi_0}\sqrt{-G} \times [72F_{abfg,e}F_{abcd,e}F_{cdrt,h}F_{fgrt,h} - 36F_{abfg,r}F_{abcd,e}F_{cdth,r}F_{fgth,e} - 64F_{abcf,g}F_{abcd,e}F_{drth,e}F_{frth,g} - F_{abcd,e}F_{abcd,e}F_{fgrt,h}F_{fgrt,h} + 6F_{abcd,f}F_{abcd,e}F_{grth,f}F_{grth,e}]. \tag{16}$$

In this case, we have tried to use the Bianchi identity to reduce the number of terms. However, we could not reduce the number to less than five terms. So the above terms are the minimum number of terms for the couplings with the structure $(\partial F^{(4)})^4$.

For $n = 5$ case, the supergravity action (7) dictates that there is only one contribution to the scattering amplitude in the s-channel. The Feynman amplitude in this case is given as

$$A_s = [\tilde{V}_{F_1^{(5)}F_2^{(5)}h}]^{\mu\nu}[\tilde{G}_h]_{\mu\nu,\lambda\rho}[\tilde{V}_{hF_3^{(5)}F_4^{(5)}}]^{\lambda\rho}$$

The massless pole in the u-channel is the same as A_s in which the particle labels of the external RR fields are interchanged, i.e., $A_u = A_s(2 \leftrightarrow 3)$. Similarly, the massless pole in the t-channel is the same as A_u in which the particle labels of the external RR fields are interchanged, i.e., $A_t = A_u(3 \leftrightarrow 4)$. These amplitudes produce the following coupling in the string frame:

$$-\frac{\gamma}{2^{11}.3^2.5\kappa^2}\int d^{10}x e^{2\phi_0}\sqrt{-G}[F_{abcde,k}F_{abcde,l}F_{fghij,k}F_{fghij,l} + F_{abcde,k}F_{abcde,l}F_{fghij,k}F_{fghij,l} - F_{abcde,k}F_{abcde,k}F_{fghij,l}F_{fghij,l} + 10F_{abcde,f}F_{abcdg,f}F_{ehijk,l}F_{ghijk,l} - 10F_{abcde,f}F_{abcdg,h}F_{eijkl,h}F_{gijkl,f} - 10F_{abcde,f}F_{abcdg,h}F_{eijkl,f}F_{gijkl,h}], \tag{17}$$

which are only P-even couplings. We have compared them with the corresponding scattering amplitude of four RR vertex operators in string theory and found disagreement. This indicates that the type IIB supergravity does not correctly describe the couplings of the RR five-form field strength. However, the supergravity (7) is expected to describe only the self-dual part of the RR five-form field strength after imposing the self-duality by hand. Therefore, we expect the above couplings to be physical only after imposing the following transformation by hand:

$$F_5 \to \frac{1}{2}(F_5 + \star F_5). \tag{18}$$

Then the couplings (17) are expected to be consistent with the α'^3 terms of the corresponding string-theory scattering amplitude.

We impose the above self-duality in the couplings (17) and use the identity (15) to rewrite the even number of Levi-Civitáa tensors in terms of the metric and the odd number of Levi-Civitáa tensors in terms of one Levi-Civitáa tensor. Using Cadabra [32,33], we have found the following result:

$$\begin{aligned} S \supset -\frac{\gamma}{2^{17}.3^3.5^2\kappa^2}\int d^{10}x e^{2\phi_0}\sqrt{-G}[240(&F_{bcdef,a}F_{bcdef,a}F_{hijkl,g}F_{hijkl,g} \\ &+ 180F_{bcdef,a}F_{bghij,a}F_{cdghl,k}F_{efijl,k} - 360F_{bcdef,a}F_{bcghi,a}F_{degkl,j}F_{fhikl,j} \\ &+ 160F_{bcdef,a}F_{bcdgh,a}F_{egjkl,i}F_{fhjkl,i} - 40F_{bcdef,a}F_{ghijk,a}F_{bcdgh,l}F_{efijk,l} \\ &+ 10F_{bcdef,a}F_{ghijk,a}F_{bcdeg,l}F_{fhijk,l} + 2F_{bcdef,a}F_{ghijk,a}F_{bcdef,l}F_{ghijk,l} \\ &- 30F_{bcdef,a}F_{bghij,a}F_{cdefl,k}F_{ghijl,k} + 40F_{bcdef,a}F_{bcghi,a}F_{defkl,j}F_{ghikl,j} \\ &+ 40F_{bcdef,a}F_{bcdgh,a}F_{efjkl,i}F_{ghjkl,i} - 30F_{bcdef,a}F_{bcdeg,a}F_{fijkl,h}F_{gijkl,h}) \\ &- \epsilon_{abcdefghij}(360F_{abcde,r}F_{fgknp,r}F_{hlmnp,q}F_{ijklm,q} \\ &- 180F_{abcde,r}F_{fgknp,q}F_{hlmnp,r}F_{ijklm,q} - 160F_{abcde,r}F_{fghkp,r}F_{ilmnp,q}F_{jklmn,q} \\ &+ 20F_{abcde,r}F_{fghip,r}F_{jklmn,q}F_{klmnp,q} - 50F_{abcde,r}F_{fghnp,r}F_{ijklm,q}F_{klmnp,q} \\ &+ 55F_{abcde,q}F_{fghip,r}F_{jklmn,q}F_{klmnp,r} + 50F_{abcde,r}F_{fghnp,q}F_{ijklm,q}F_{klmnp,r} \\ &- 50F_{abcde,q}F_{fghnp,r}F_{ijklm,q}F_{klmnp,r} + F_{abcde,q}F_{fghij,r}F_{klmnp,q}F_{klmnp,r})]. \end{aligned} \tag{19}$$

The P-odd couplings above are then produced only by the self-duality transformation (18). The above couplings must be invariant under the transformation $F^{(5)} \to \star F^{(5)}$. So they describe the couplings of four self-dual five-forms at order α'^3. Note that there are no P-odd couplings in (16), so the above action must produce no couplings with the structure $\epsilon_{10}(\partial F^{(4)})^4$ under the T-duality. It is easy to verify it by noticing that it is impossible to have four RR five-form field strengths in the P-odd part that each carry one Killing index.

Since the couplings with the structure $(\partial F^{(5)})^4$ have too many indices, it is hard to find all such couplings with unknown coefficients with the xTras package [31]. So we could not apply the algorithm given above equation (14) to reduce the couplings to the minimum number.

B. $\partial F^{(n)}\partial F^{(n)}\partial F^{(n-2)}\partial F^{(n-2)}$ couplings

Since the maximum rank of the RR field strength is 5, there are three types of couplings in this section, i.e., $n = 3, 4, 5$. The scattering amplitude in the s-channel for the cases $n = 3, 4$ is given by

$$\begin{aligned} A_s = {}& [\tilde{V}_{F_1^{(n)}F_2^{(n)}h}]^{\mu\nu}[\tilde{G}_h]_{\mu\nu,\lambda\rho}[\tilde{V}_{hF_3^{(n-2)}F_4^{(n-2)}}]^{\lambda\rho} \\ &+ \tilde{V}_{F_1^{(n)}F_2^{(n)}\phi}\tilde{G}_\phi\tilde{V}_{\phi F_3^{(n-2)}F_4^{(n-2)}}. \end{aligned}$$

The scattering amplitude in the u-channel is given as

$$A_u = [\tilde{V}_{F_1^{(n)}F_3^{(n-2)}b}]^{\mu\nu}[\tilde{G}_b]_{\mu\nu,\lambda\rho}[\tilde{V}_{bF_2^{(n)}F_4^{(n-2)}}]^{\lambda\rho}.$$

And the Feynman amplitude in the t-channel is the same as A_u in which the particle labels of the external RR fields are interchanged, i.e., $A_t = A_u(3 \leftrightarrow 4)$.

Replacing the vertices and propagators in the above amplitudes, one finds the following couplings for $n = 3, 4$ in the string frame:

$$S \supset \frac{\gamma}{4\kappa^2}\int d^{10}x e^{2\phi_0}\sqrt{-G}\Big[2F_{aef,c}F_{def,b}F_{a,b}F_{c,d} + \frac{1}{3}F_{def,c}F_{def,b}F_{a,c}F_{a,b} - \frac{1}{6}F_{cde,f}F_{cde,f}F_{a,b}F_{a,b} + F_{acgh,f}F_{bdgh,e}F_{ab,c}F_{de,f} - F_{adfg,h}F_{bfgh,e}F_{ab,c}F_{de,c} + F_{abgh,d}F_{cefg,h}F_{ab,c}F_{de,f} + F_{afgh,d}F_{cegh,b}F_{ab,c}F_{de,f} + \frac{2}{3}F_{bfgh,d}F_{cfgh,e}F_{ad,e}F_{ab,c}\Big], \tag{20}$$

where we have also used the algorithm given above Eq. (14) to reduce the couplings to the minimum number.

For the case $n = 5$, the type IIB supergravity (7) gives the following Feynman amplitudes in the s-channel and u-channel:

$$A_s = [\tilde{V}_{F_1^{(5)}F_2^{(5)}h}]^{\mu\nu}[\tilde{G}_h]_{\mu\nu,\lambda\rho}[\tilde{V}_{hF_3^{(3)}F_4^{(3)}}]^{\lambda\rho}$$
$$A_u = [\tilde{V}_{F_1^{(5)}F_3^{(3)}b} + \tilde{V}_{\epsilon_{10}F_1^{(5)}F_3^{(3)}b}]^{\mu\nu}[\tilde{G}_b]_{\mu\nu,\lambda\rho}[\tilde{V}_{bF_2^{(5)}F_4^{(3)}} + \tilde{V}_{bF_2^{(5)}F_4^{(3)}\epsilon_{10}}]^{\lambda\rho}.$$

The amplitude in the t-channel is the same as A_u in which the particle labels of the external RR fields are interchanged. Replacing the vertices and propagators in the above amplitudes, one finds the following couplings in the string frame:

$$\frac{\gamma}{2^8.5.3^3\kappa^2}\int d^{10}x e^{2\phi_0}\sqrt{-G}[120F_{abc,d}F_{efg,d}F_{abchi,j}F_{efghi,j} - 180F_{abc,d}F_{aef,d}F_{bcghi,j}F_{efghi,j} - 90F_{abc,d}F_{abe,d}F_{cfghi,j}F_{efghi,j} + 12F_{abc,d}F_{abc,d}F_{efghi,j}F_{efghi,j} + 180F_{abc,d}F_{aef,g}F_{bchij,g}F_{efhij,d} - 180F_{abc,d}F_{aef,g}F_{bchij,d}F_{efhij,g} + 90F_{abc,d}F_{abe,f}F_{cghij,f}F_{eghij,d} + 270F_{abc,d}F_{abe,f}F_{cghij,d}F_{eghij,f} - 36F_{abc,d}F_{abc,e}F_{fghij,d}F_{fghij,e} + \epsilon_{abcdefgklm}(F_{hij,p}F_{klm,n}F_{abcde,n}F_{fghij,p} - F_{hij,n}F_{klm,p}F_{abcde,n}F_{fghij,p} - F_{hij,p}F_{klm,p}F_{abcde,n}F_{fghij,n})],$$

which has P-even and P-odd parts.

Since the above couplings involve the RR five-form field strength, we have to impose the transformation (18) by hand to produce correct couplings. We have found the following couplings for the self-dual RR five-form:

$$S \supset \frac{\gamma}{2^9.5.3^3\kappa^2}\int d^{10}x e^{2\phi_0}\sqrt{-G}[240F_{abc,d}F_{efg,d}F_{abchi,j}F_{efghi,j} - 360F_{abc,d}F_{aef,d}F_{bcghi,j}F_{efghi,j} + 6F_{abc,d}F_{abc,d}F_{efghi,j}F_{efghi,j} + 360F_{abc,d}F_{aef,g}F_{bchij,g}F_{efhij,d} - 360F_{abc,d}F_{aef,g}F_{bchij,d}F_{efhij,g} + 360F_{abc,d}F_{abe,f}F_{cghij,d}F_{eghij,f} - 36F_{abc,d}F_{abc,e}F_{fghij,d}F_{fghij,e} + \epsilon_{abcdefghij}(3F_{abm,p}F_{klm,n}F_{cdekl,p}F_{fghij,n} - 3F_{abm,n}F_{klm,p}F_{cdekl,p}F_{fghij,n} - 3F_{abm,p}F_{klm,p}F_{cdekl,n}F_{fghij,n} + 3F_{alm,n}F_{klm,p}F_{bcdek,p}F_{fghij,n} + 2F_{abc,p}F_{klm,p}F_{deklm,n}F_{fghij,n})]. \tag{21}$$

The above couplings satisfy the self-duality condition $F^{(5)} = \star F^{(5)}$. Since the number of indices is too many, we could not find all contractions of the structure $(\partial F^{(5)})^2(\partial F^{(3)})^2$. So we could not perform the algorithm given above Eq. (14) to reduce the number of couplings to the minimum number.

Note that, under dimensional reduction on a circle, the P-odd couplings in (21) produce no term in which each RR field strength carries one Killing index. As a result, the P-odd couplings in the above equation produce no couplings with the structure $\epsilon_{10}(F^{(4)})^2(F^{(2)})^2$ under T-duality. This is consistent with the fact that there is no such coupling in (20).

C. $\partial F^{(n)}\partial F^{(n)}\partial F^{(n-4)}\partial F^{(n-4)}$ couplings

Since the minimum rank of the RR field strength is 1, there is only one type of coupling in this section, i.e., $n = 5$. The effective action (7) produces the following s-channel amplitude:

$$A_s = [\tilde{V}_{F_1^{(5)}F_2^{(5)}h}]^{\mu\nu}[\tilde{G}_b]_{\mu\nu,\lambda\rho}[\tilde{V}_{hF_3^{(1)}F_4^{(1)}}]^{\lambda\rho}.$$

In this case, one can easily observe that there is no amplitude in u- and t-channels. Therefore, the total amplitude comes from the s-channel which produces the following couplings in the string frame:

$$\frac{\gamma}{2^8.5.3^2\kappa^2}\int d^{10}xe^{2\phi_0}\sqrt{-G}[240F_{a,b}F_{c,d}F_{aefgh,c}F_{befgh,d}],$$

where we have also used the algorithm given above Eq. (14) to reduce the number of couplings to the minimum number. By imposing the self-duality transformation (18) on the above coupling, we obtain the following couplings for the self-dual RR form:

$$\begin{aligned}S\supset\frac{\gamma}{2^9.5.3^2\kappa^2}\int d^{10}xe^{2\phi_0}\sqrt{-G}\\ \times[240F_{a,b}F_{c,d}F_{aefgh,c}F_{befgh,d}\\ -\epsilon_{abcdefghim}(F_{j,l}F_{k,m}F_{abcde,l}F_{fghij,k}\\ +F_{j,l}F_{k,m}F_{abcde,k}F_{fghij,l}-F_{k,j}F_{k,m}F_{abcde,l}F_{fghij,l})],\end{aligned} \tag{22}$$

where we have also reduced the P-even couplings to the minimum number.

D. $\partial F^{(n)}\partial F^{(n-2)}\partial F^{(n-2)}\partial F^{(n-4)}$ couplings

In this case also, there is only one type of coupling, i.e., $n=5$. There is no Feynman amplitude in the s-channel. The amplitude in the u-channel is given as

$$\begin{aligned}A_u=[\tilde{V}_{F_1^{(5)}F_3^{(3)}b}]^{\mu\nu}[\tilde{G}_b]_{\mu\nu,\lambda\rho}[\tilde{V}_{bF_2^{(3)}F_4^{(1)}}]^{\lambda\rho}\\ +[\tilde{V}_{\epsilon_{10}F_1^{(5)}F_3^{(3)}b}]^{\mu\nu}[\tilde{G}_b]_{\mu\nu,\lambda\rho}[\tilde{V}_{bF_2^{(3)}F_4^{(1)}}]^{\lambda\rho}.\end{aligned}$$

The t-channel amplitude is the same as A_u in which the particle labels of the external RR fields are interchanged. Summing these two contributions, one finds the amplitude produces the following P-even and P-odd couplings:

$$\begin{aligned}S\supset-\frac{\gamma}{2^6.5.3^2\kappa^2}\int d^{10}xe^{2\phi_0}\sqrt{-G}[120F_{cdefg,h}F_{a,b}F_{acd,b}F_{efg,h}-120F_{cdfgh,b}F_{a,b}F_{acd,e}F_{fgh,e}\\ -120F_{cdfgh,e}F_{a,b}F_{acd,e}F_{fgh,b}+\epsilon_{abcdeghijk}(F_{abcde,l}F_{f,m}F_{fgh,l}F_{ijk,m}\\ +F_{abcde,l}F_{f,l}F_{fgh,m}F_{ijk,m}-F_{abcde,l}F_{f,m}F_{fgh,m}F_{ijk,l})].\end{aligned} \tag{23}$$

Applying the self-duality condition (18) on the above couplings and using the identity (15), we have found they are invariant. We have also reduced the P-even couplings above to the minimum number.

One may consider couplings with the structure $\partial F^{(n)}\partial F^{(n-4)}\partial F^{(n-4)}\partial F^{(n-2)}$. In this case, there is one possibility, i.e., $n=5$. However, the type IIB supergravity indicates that the vertices in the Feynman amplitudes are zero. So there is no such coupling at order α'^3.

E. $\partial F^{(n)}\partial F^{(n)}\partial F^{(n)}\partial F^{(n-2)}$ couplings

There are three possibilities in this case, i.e., $n=3,4,5$. However, the type IIB supergravity (7) indicates that the Feynman amplitudes in the s-, t-, and u-channels are zero for $n=3,5$. For the case $n=4$, the type IIA supergravity (6) indicates that the amplitude in the s-channel is given as

$$A_s=[\tilde{V}_{\epsilon_{10}F_1^{(4)}F_2^{(4)}b}]^{\mu\nu}[\tilde{G}_b]_{\mu\nu,\lambda\rho}[\tilde{V}_{bF_3^{(4)}F_4^{(2)}}]^{\lambda\rho}.$$

The amplitude in the u-channel is the same as A_s in which the particle labels of the external RR fields are interchanged, i.e., $A_u=A_s(2\leftrightarrow 3)$. Similarly, the amplitude in the t-channel is the same as A_u in which the particle labels of the external RR fields are interchanged, i.e., $A_t=A_u(1\leftrightarrow 2)$. Replacing the appropriate vertices and propagators in the amplitudes, one finds the kinematic factors which produce the following couplings in the string frame:

$$\begin{aligned}S\supset\frac{\gamma}{2^9.3^2\kappa^2}\int d^{10}xe^{2\phi_0}\sqrt{-G}\epsilon_{abcdefghij}\\ \times[2F_{ijmn,k}F_{efgh,l}F_{abcd,k}F_{mn,l}\\ -F_{ijmn,k}F_{efgh,l}F_{abcd,l}F_{mn,k}],\end{aligned} \tag{24}$$

which has only P-odd couplings. Note that under dimensional reduction on a circle the above couplings produce no term in which the RR four-forms each carry one Killing index and the RR two-form carries no Killing index. As a result, they produce no couplings with the structure $\epsilon_{10}(F^{(3)})^4$ under T-duality. This is consistent with the fact that there are no such couplings in (14).

One may consider couplings with the structure $\partial F^{(n)}\partial F^{(n)}\partial F^{(n)}\partial F^{(n-4)}$. In this case, there is one possibility, i.e., $n=5$. However, the type IIB supergravity indicates that the vertices in the Feynman amplitudes are zero. So there is no such coupling at order α'^3. It is also consistent with the T-duality of the couplings in (24). In fact, the RR two-form in (24) does not contract with the Levi-Civitá tensor, so under the dimensional reduction, there is no coupling in which the RR four-forms carry no Killing index and the RR two-form carries one Killing index. As a result, the couplings (24) produce no terms with the structure $\partial F^{(5)}\partial F^{(5)}\partial F^{(5)}\partial F^{(1)}$.

III. CONSISTENCY WITH DUALITIES

We have found all different couplings of four RR states at order α'^3 in the previous section. In this section, we would like to show that these couplings are related to the standard four NSNS couplings under S-duality and T-duality transformations. We begin with the couplings in Sec. II A. Our starting point in this case is the coupling $F^{(3)}F^{(3)}F^{(3)}F^{(3)}$ which can be found by making the H^4 couplings to be S-duality invariant. The H^4 couplings on the other hand can be derived from the coupling $t_8t_8R^4$ in (1) by extending the Riemann curvature to the generalized Riemann curvature [28], i.e.,

$$R_{ab}{}^{cd} \to \bar{R}_{ab}{}^{cd} = R_{ab}{}^{cd} - \frac{\kappa}{\sqrt{2}}\eta_{[a}{}^{[c}\phi_{;b]}{}^{d]} + 2e^{-\phi_0/2}H_{ab}{}^{[c;d]}, \tag{25}$$

where the bracket notation is defined as $H_{ab}{}^{[c;d]} = \frac{1}{2}(H_{ab}{}^{c;d} - H_{ab}{}^{d;c})$ and ϕ_0 is the constant dilaton background and the semicolon symbol denotes the covariant derivative. The resulting action has the following form in the Einstein frame [28]:

$$S \supset \frac{\gamma}{\kappa^2}\int d^{10}x\sqrt{-G}e^{-3\phi_0/2}\Big[\bar{R}_{hkmn}\bar{R}_{krnp}\bar{R}_{rsqm}\bar{R}_{shpq} + \frac{1}{2}\bar{R}_{hkmn}\bar{R}_{krnp}\bar{R}_{rspq}\bar{R}_{shqm} - \frac{1}{2}\bar{R}_{hkmn}\bar{R}_{krmn}\bar{R}_{rspq}\bar{R}_{shpq} - \frac{1}{4}\bar{R}_{hkmn}\bar{R}_{krpq}\bar{R}_{rsmn}\bar{R}_{shpq} + \frac{1}{16}\bar{R}_{hkmn}\bar{R}_{khpq}\bar{R}_{rsmn}\bar{R}_{srpq} + \frac{1}{32}\bar{R}_{hkmn}\bar{R}_{khmn}\bar{R}_{rspq}\bar{R}_{srpq}\Big]. \tag{26}$$

The couplings of four H can be read from the above action. Invariance of this action under S-duality transformations in type IIB theory requires the couplings of four $F^{(3)}$ to be the same as the couplings of four H; i.e., the couplings in the string frame are

$$S \supset \frac{16\gamma}{\kappa^2}\int d^{10}x\sqrt{-G}e^{2\phi_0}\Big[F_{hk[m;n]}F_{kr[n;p]}F_{rs[q;m]}F_{sh[p;q]} + \frac{1}{2}F_{hk[m;n]}F_{kr[n;p]}F_{rs[p;q]}F_{sh[q;m]} - \frac{1}{2}F_{hk[m;n]}F_{kr[m;n]}F_{rs[p;q]}F_{sh[p;q]} - \frac{1}{4}F_{hk[m;n]}F_{kr[p;q]}F_{rs[m;n]}F_{sh[p;q]} + \frac{1}{16}F_{hk[m;n]}F_{kh[p;q]}F_{rs[m;n]}F_{sr[p;q]} + \frac{1}{32}F_{hk[m;n]}F_{kh[m;n]}F_{rs[p;q]}F_{sr[p;q]}\Big]. \tag{27}$$

Writing the above couplings and the couplings $(F^{(3)})^4$ in (14) in terms of independent variables, we have found that they are exactly identical.[3]

We use the following steps on the couplings (27) to find the couplings with the structure $(F^{(n)})^4$ for $n = 2, 1$: We first use the dimensional reduction on the couplings (27) and keep the terms with the structure $(F_y^{(2)})^4$ where the index y is the Killing index. Under the linear T-duality transformations, the RR field strength $F_y^{(n)}$ transforms to $F^{(n-1)}$ with no Killing index. Therefore, under the T-duality transformation, the above couplings transform to the couplings with the structure $(F^{(2)})^4$ in type IIA theory. Performing the same steps once more, we have found the couplings with the structure $(F^{(1)})^4$ in type IIB theory. We have checked that these couplings are exactly equal to the corresponding couplings in (14) when we write them in terms of independent variables.

Now, consider the couplings in the dimensional reduction of (27) in which the RR three-forms carry no Killing index. Under the T-dality, they transform to the couplings with the structure $(F_y^{(4)})^4$ in type IIA theory. We compare these couplings with the couplings with the structure $(F_y^{(4)})^4$ in the dimensional reduction of the couplings (16). Writing both sets of couplings in terms of independent variables, we have found exact agreement.

To show that the coupling with the structure $(F^{(5)})^4$ in (19) is consistent with dualities, we note that RR five-form field strength is invariant under the S-duality. We already pointed out that the P-odd couplings in (19) are consistent with T-duality. To verify that the P-even couplings in (19) are consistent with T-duality, we consider the couplings in

[3]Note that the normalization of H in Ref. [28] is twice the normalization of H in the supergravities (6) and (7). As a result, the normalization of RR field strengths in the actions in this section is twice the normalization of $F^{(n)}$ in the previous section.

the dimensional reduction of (19) in which the RR five-forms each carry one Killing index, i.e., $(F_y^{(5)})^4$. Under the T-duality, they transform to the couplings with the structure $(F^{(4)})^4$. We compare them with the couplings in (16). Writing both sets of couplings in terms of independent variables, we have found exact agreement.

We now compare the couplings in Sec. II B with dualities. Our starting point in this case is the couplings with the structure $F^{(5)}F^{(5)}F^{(3)}F^{(3)}$. Using the consistency of the couplings (26) with S-duality and T-duality, the couplings with the structure $F^{(5)}F^{(5)}HH$ have been found in Ref. [23]. Under the S-duality, the RR five-form is invariant, and the B-field strength H transforms to the RR three-form field strength. So the consistency of the couplings found in Ref. [23] with S-duality requires the following couplings in the string frame:

$$S \supset \frac{\gamma}{\kappa^2}\int d^{10}xe^{2\phi_0}\sqrt{-G}\Big[-\frac{2}{3}F_{hrstu,n}F_{qrstu,m}F_{knp,h}F_{mpq,k}+\frac{2}{3}F_{nqstu,h}F_{pqstu,m}F_{knr,h}F_{mpr,k}-\frac{1}{90}F_{nqstu,h}F_{nqstu,k}F_{mpr,h}F_{mpr,k}$$
$$+\frac{1}{6}F_{hqstu,n}F_{kqstu,m}F_{mpr,k}F_{npr,h}+\frac{1}{6}F_{hqstu,m}F_{kqstu,n}F_{mpr,k}F_{npr,h}+\frac{4}{9}F_{hkmnu,t}F_{pqrtu,n}F_{hkm,s}F_{pqr,s}$$
$$+\frac{1}{3}F_{nqrtu,h}F_{npstu,k}F_{hqr,m}F_{kps,m}-\frac{1}{3}F_{nqrtu,h}F_{mrstu,k}F_{npq,k}F_{mps,h}-\frac{4}{3}F_{mnptu,k}F_{nrstu,h}F_{kpq,m}F_{hqs,r}$$
$$+4F_{nqrtu,h}F_{mpstu,k}F_{hkr,n}F_{pqs,m}+\frac{1}{3}F_{mnptu,h}F_{mqstu,h}F_{npr,k}F_{qrs,k}\Big]. \quad (28)$$

To compare them with the couplings in (21), we have to impose the self-duality transformation (18) on the above couplings. Using the identity (15) to write the multiple of two Levi-Civitáa tensors in terms of the metric, we find two types of terms. One type has terms with no Levi-Civitá tensor which is the same as (28) up to the overall factor of $\frac{1}{2}$. The other type has terms with one Levi-Civitáa tensor. This part, in the momentum space, produces terms with zero or one Mandelstam variable which are not consistent with the superstring-theory amplitudes [21,34]. This indicates that the couplings (28) must have some P-odd couplings which are not related to the couplings (26) by the string dualities. In fact, the corresponding Feynman amplitude in Sec. II B has P-odd couplings even before imposing the self-duality transformation.

The P-even couplings (28) and their P-odd partners must be consistent with T-duality before or after imposing the self-duality transformation because they are not produced by type IIB supergravity which is off for the RR five-form field strength. In particular, under the dimensional reduction of (28), the couplings with the structure $(F_y^{(5)})^2(F_y^{(3)})^2$ transform under T-duality to the couplings with the structure $(F^{(4)})^2(F^{(2)})^2$ in (20). This indicates that the self-duality transformation of the action (28) and its P-odd partner should produce the same couplings as (28). The transformation of (28) under the self-duality (18) produces the same couplings with the overall factor $\frac{1}{2}$ and some P-odd couplings. The other factor of $\frac{1}{2}$ must then be reproduced by a self-duality transformation of the P-odd terms.

One may try to find the P-odd partner of (28) by considering all contractions with the structure $\epsilon^{(10)}(F^{(5)})^2(F^{(3)})^2$ with unknown coefficients and fix them by requiring them to produce the above factor of $\frac{1}{2}$ under the self-duality transformation and requiring them to produce no term with zero or one Mandelstam variable in the momentum space [21,34]. However, there are too many such contractions, so we do not try to find the P-odd partner of the couplings (28) in this paper. We have written the couplings (28) and the P-even part of the couplings (21) in terms of independent variables and found exact agreement.

Using the dimensional reduction on the couplings (28) in type IIB theory, one can find the couplings with the structure $(F_y^{(5)})^2(F_y^{(3)})^2$. Under T-duality, they transform to the couplings with the structure $(F^{(4)})^2(F^{(2)})^2$ in type IIA theory. Repeating these steps on the couplings $(F^{(4)})^2(F^{(2)})^2$, one can find the couplings with the structure $(F^{(3)})^2(F^{(1)})^2$ in type IIB theory. Writing these couplings and the corresponding couplings in (20) in terms of independent variables, we have again found exact agreement between the two sets of couplings.

We now compare the couplings in Sec. II C with dualities. There is only one coupling in this section, i.e., the couplings with the structure $(F^{(5)})^2(F^{(1)})^2$. Such couplings have been found in Ref. [27] by imposing the S-duality on the couplings of two RR five-form and two dilatons (see Eq. (69) in Ref. [27]). Alternatively, the couplings with the structure $(F^{(5)})^2(F^{(1)})^2$ can be found by imposing the dualities on the couplings (28). To this end, we use the dimensional reduction on the couplings with the structure $(F^{(4)})^2(F^{(2)})^2$ that we have found in the above paragraph and consider terms with the structure $(F^{(4)})^2(F_y^{(2)})^2$. Then, under T-duality, they transform to the couplings with the structure $(F_y^{(5)})^2(F^{(1)})^2$. Converting the Killing index to a complete spacetime index and taking the symmetry factors into account, one finds the couplings with the structure $(F^{(5)})^2(F^{(1)})^2$ without any ambiguity because it is impossible to have couplings in which the RR five-forms do not contract with each other. These couplings are the same as the couplings found in Ref. [27]; i.e, in the string frame they are

$$S \supset \frac{2}{3}\frac{\gamma}{\kappa^2}\int d^{10}x\sqrt{-G}e^{2\phi_0}[F_{aefgh,c}F_{befgh,d}F_{a,b}F_{c,d}]. \quad (29)$$

Transforming the above couplings under the self-duality (18), we have found couplings which are identical to the couplings in (22) after writing both sets in terms of independent variables. This indicates that there is no P-odd coupling in the above action. As in the field theory, Sec. II C, the P-odd part in the self-dual action comes only from the self-duality transformation.

To compare the couplings in Sec. II D with dualities, we consider the couplings with the structure $F^{(5)}F^{(1)}HH$ which have been found in Ref. [27] by imposing dualities on the couplings in (27). The S-duality invariant of these couplings produces among other things the couplings with the structure $F^{(5)}F^{(1)}F^{(3)}F^{(3)}$, i.e.,

$$\begin{aligned} S \supset -\frac{\gamma}{\kappa^2}\int d^{10}x\sqrt{-G}e^{2\phi_0}[&8F_{h,k}F_{mnpqr,s}F_{hpq,m}F_{krs,n} \\ &+4F_{h,k}F_{kmnpq,r}F_{mns,h}F_{pqr,s} \\ &-2F_{h,k}F_{kmnpq,h}F_{mns,r}F_{pqr,s}]. \end{aligned} \quad (30)$$

Note that these couplings are only P even. Transforming them under the self-duality (18), one finds P-even and P-odd couplings. Writing them in terms of independent variables, we have found they are exactly equal to the couplings in (23). It is important to note that the field-theory couplings in Sec. II D have a P-odd part even before imposing the self-duality transformation, whereas the above couplings have no P-odd part. This indicates the field-theory couplings (23) are consistent with the duality transformations of (26), i.e., (30), only after imposing the self-duality transformation on (30).

Finally, the P-odd couplings in Sec. II D should be related to the P-odd couplings in (24) under T-duality. Under the dimensional reduction, the latter couplings produce, among other things, the couplings with the structure $\epsilon_y^{(10)}F^{(4)}(F_y^{(4)})^2F_y^{(2)}$. Under T-duality, they transform to the couplings with the structure $\epsilon_y^{(10)}F_y^{(5)}(F^{(3)})^2F^{(1)}$. Completing the Killing index to the full spacetime index, one finds the couplings with the structure $\epsilon_{10}F^{(5)}(F^{(3)})^2F^{(1)}$. Writing the resulting couplings and the P-odd couplings in (23) in terms of independent variables, we have found exact agreement.

IV. CONSISTENCY WITH STRING AMPLITUDES

In this section, we are going to calculate the kinematic factor $\mathcal{K}$ directly in the type II superstring theory and compare it with the couplings found in Sec. II.

The tree-level scattering amplitude of four RR states in the RNS formalism [35] is given by the correlation function of their corresponding vertex operators on the sphere world sheet. Since the background superghost charge of the sphere is $Q_\phi = 2$, one has to choose the vertex operators in the appropriate pictures to produce the compensating charge $Q_\phi = -2$. We choose the RR vertex operators in the $(-1/2,-1/2)$ picture. The amplitude is given by the following correlation function [35]:

$$\mathcal{A} \sim \int \prod_{i=1}^{4} d^2z_i \langle \prod_{j=1}^{4} V_{RR}^{(-1/2,-1/2)}(z_j,\bar{z}_j)\rangle, \quad (31)$$

where the vertex operators are

$$\begin{aligned} V_{RR}^{(-1/2,-1/2)}(z_j,\bar{z}_j) = (P_{\mp}\Gamma_{j(n)})^{AB} :&e^{-\phi(z_j)/2}S_A(z_j)e^{ik_j\cdot X(z_j)}: \\ &\times e^{-\tilde{\phi}(\bar{z}_j)/2}\tilde{S}_B(\bar{z}_j)e^{ik_j\cdot\tilde{X}(\bar{z}_j)}:, \end{aligned} \quad (32)$$

where $j = 1, \ldots 4$ and the indices $A, B, \ldots$ are the Dirac spinor indices and $P_{\mp} = \frac{1}{2}(1\mp\gamma_{11})$ is the chiral projection operators which make the calculation of the gamma matrices to be with the full 32×32 Dirac matrices of the ten dimensions. The RR field strength appears in the definition of $\Gamma_{i(n)}$ as

$$\Gamma_{i(n)} = \frac{a_n}{n!}F_{i\mu_1\cdots\mu_n}\gamma^{\mu_1\cdots\mu_n}, \quad (33)$$

where the factor $a_n = -1$ in the type IIA theory and $a_n = i$ in the type IIB theory [36]. There is ambiguity in choosing the chiral projection operator in the vertex operator (32), e.g., P_- or P_+. As we will see, this makes it difficult to confirm the P-odd couplings in Sec. II with the string-theory scattering amplitudes. The normalization of the amplitude (31) in which we are not interested in this section may be fixed after fixing the conformal symmetry of the integrand.

Substituting the vertex operators (32) into (31), and using the fact that there is no correlation between holomorphic and antiholomorphic for the sphere world sheet, one can separate the amplitude to the holomorphic and the antiholomorphic parts as

$$\begin{aligned} \mathcal{A} \sim\ & (P_-\Gamma_{1(n)})^{AB}(P_-\Gamma_{2(m)})^{CD}(P_-\Gamma_{3(p)})^{EF}(P_-\Gamma_{4(q)})^{GH} \\ &\times \int \prod_{i=1}^{4} d^2z_i I_{ACEG}\otimes\tilde{I}_{BDFH}, \end{aligned} \quad (34)$$

where the holomorphic part is

$$I^{\mu\alpha}_{AC} = \langle :e^{-\phi(z_1)/2}:e^{-\phi(z_2)/2}:e^{-\phi(z_3)/2}:e^{-\phi(z_4)/2}:\rangle \times \langle :e^{ik_1\cdot X(z_1)}:e^{ik_2\cdot X(z_2)}:e^{ik_3\cdot X(z_3)}:e^{ik_4\cdot X(z_4)}:\rangle \times \langle :S_A(z_1):S_C(z_2):S_E(z_3):S_G(z_4):\rangle \quad (35)$$

and the antiholomorphic part $\tilde{I}_{BDFH}$ is given by a similar expression.

Using the standard propagators and the correlation function of four spin operators [35], one can perform the correlators in (34). Using the on-shell relations and the conservation of momentum, one can check that the integrand of the amplitude is invariant under $SL(2,R)\times SL(2,R)$ transformations which is the conformal symmetry of the z-plane. Fixing this symmetry by setting $z_1=0, z_2\equiv z, z_3=1$, and $z_4=\infty$ and normalizing the amplitude, one can write it in the string frame form of (2) in which the kinematic factor is

$$\mathcal{K} = (P_\mp\Gamma_{1(n)})^{AB}(P_\mp\Gamma_{2(m)})^{CD}(P_\mp\Gamma_{3(p)})^{EF} \times (P_\mp\Gamma_{4(q)})^{GH} K_{ACEG}\otimes \tilde{K}_{BDFH}, \quad (36)$$

where the kinematic factor in the holomorphic part is

$$K_{ACEG} = -\frac{1}{8}[t(\gamma^\mu C^{-1})_{AC}(\gamma_\mu C^{-1})_{EG} - s(\gamma^\mu C^{-1})_{AG}(\gamma_\mu C^{-1})_{CE}] \quad (37)$$

and the kinematic factor in the antiholomorphic part is similar to the above expression.

One may use the Kawai-Lewellen-Tye (KLT) prescription [37] to calculate the sphere-level scattering amplitude of closed string states from the corresponding disk-level scattering amplitude of open string states. According to the KLT prescription, the sphere-level amplitude of four closed string states is given by

$$\mathcal{A} \sim \sin(\alpha'\pi k_2\cdot k_3/2)A_{\rm open}(s/8,t/8)\otimes \tilde{A}_{\rm open}(t/8,u/8), \quad (38)$$

where $A_{\rm open}(s/8,t/8)$ is the disk-level scattering amplitude of four open string states in the $s-t$-channel which has been calculated in Ref. [6],

$$A_{\rm open}(s/8,t/8) \sim \frac{\Gamma(-s/8)\Gamma(-t/8)}{\Gamma(1+u/8)}K, \quad (39)$$

where the Mandelstam variables are the same as in the closed string amplitude. The open string kinematic factor K depends on the momentum and the polarization of the external states [6].

To find the sphere-level scattering amplitude of four RR states, one has to consider the disk-level scattering amplitude of four R states. The kinematic factor for this case is [6]

$$K(u_1,u_2,u_3,u_4) = -\frac{1}{8}[t\bar{u}_1^A(\gamma^\mu C^{-1})_{AC}u_2^C\bar{u}_3^E(\gamma_\mu C^{-1})_{EG}u_4^G - s\bar{u}_1^A(\gamma^\mu C^{-1})_{AG}u_4^G\bar{u}_2^C(\gamma_\mu C^{-1})_{CE}u_3^E], \quad (40)$$

where u_i with $i=1,\ldots,4$ are the spinor polarizations. They satisfy the following on-shell relations:

$$k_i^2 = 0, \qquad (\gamma\cdot k_i C^{-1})_{AB}u_i^B = 0. \quad (41)$$

Using these relations, one can write the open string kinematic factor (40) in terms of the holomorphic kinematic factor (37) as

$$K(u_1,u_2,u_3,u_4) = -4i\sqrt{2}u_1^A u_2^C u_3^E u_4^G K_{ACEG}. \quad (42)$$

Similarly, for the antiholomorphic part, i.e.,

$$\tilde{K}(\tilde{u}_1,\tilde{u}_2,\tilde{u}_3,\tilde{u}_4) = -4i\sqrt{2}\tilde{u}_1^B\tilde{u}_2^D\tilde{u}_3^F\tilde{u}_4^H\tilde{K}_{BDFH}.$$

Using the above relations and $\Gamma(x)\Gamma(1-x)=\pi/\sin(\pi x)$, and substituting the following relations in (38),

$$u_1^A\otimes\tilde{u}_1^B \to (P_\mp\Gamma_{1(n)})^{AB}, \quad u_2^C\otimes\tilde{u}_2^D \to (P_\mp\Gamma_{2(m)})^{CD}, \quad u_3^E\otimes\tilde{u}_3^F \to (P_\mp\Gamma_{1(n)})^{EF}, \quad u_4^G\otimes\tilde{u}_4^H \to (P_\mp\Gamma_{2(m)})^{GH}, \quad (43)$$

one can write it as the string frame form of (2) with the kinematic factor (36), as expected. While the open string kinematic factor (40) is the final result for the S-matrix element of four open string spinors, the closed string kinematic factor (36) is not yet the final result. The Dirac matrices in the kinematic factor appear in trace operators which should then be evaluated explicitly to find the final kinematic factor of the closed string amplitude.

The closed string kinematic factor (36) has four different terms; each one has one of the following factors:

$$\begin{aligned}
T_1 &= (P_\mp\Gamma_{1(n)})^{AB}(P_\mp\Gamma_{2(m)})^{CD}(P_\mp\Gamma_{3(p)})^{EF}(P_\mp\Gamma_{4(q)})^{GH} \times (\gamma^\mu C^{-1})_{AC}(\gamma_\mu C^{-1})_{EG}(\gamma^\nu C^{-1})_{BD}(\gamma_\nu C^{-1})_{FH},\\
T_2 &= (P_\mp\Gamma_{1(n)})^{AB}(P_\mp\Gamma_{2(m)})^{CD}(P_\mp\Gamma_{3(p)})^{EF}(P_\mp\Gamma_{4(q)})^{GH} \times (\gamma^\mu C^{-1})_{AG}(\gamma_\mu C^{-1})_{CE}(\gamma^\nu C^{-1})_{BD}(\gamma_\nu C^{-1})_{FH},\\
T_3 &= (P_\mp\Gamma_{1(n)})^{AB}(P_\mp\Gamma_{2(m)})^{CD}(P_\mp\Gamma_{3(p)})^{EF}(P_\mp\Gamma_{4(q)})^{GH} \times (\gamma^\mu C^{-1})_{AC}(\gamma_\mu C^{-1})_{EG}(\gamma^\nu C^{-1})_{BH}(\gamma_\nu C^{-1})_{DF},\\
T_4 &= (P_\mp\Gamma_{1(n)})^{AB}(P_\mp\Gamma_{2(m)})^{CD}(P_\mp\Gamma_{3(p)})^{EF}(P_\mp\Gamma_{4(q)})^{GH} \times (\gamma^\mu C^{-1})_{AG}(\gamma_\mu C^{-1})_{CE}(\gamma^\nu C^{-1})_{BH}(\gamma_\nu C^{-1})_{DF}. \quad (44)
\end{aligned}$$

Using the properties of the charge conjugation matrix and the Dirac matrices (see, e.g., Appendix B in Ref. [36]), one can write the tensors $T_1, \ldots, T_4$ in terms of the RR field strengths and the trace of the gamma matrices as

$$
\begin{aligned}
T_1 &= \frac{(-1)^{\frac{1}{2}[m(m+1)+q(q+1)]}a_n a_m a_p a_q}{n!m!p!q!} F_{1\mu_1\ldots\mu_n} F_{2\nu_1\ldots\nu_m} F_{3\alpha_1\ldots\alpha_p} F_{4\beta_1\ldots\beta_q} \mathrm{Tr}(P_\pm \gamma^\mu \gamma^{\mu_1\ldots\mu_n}\gamma^\nu\gamma^{\nu_1\ldots\nu_m})\mathrm{Tr}(P_\pm\gamma_\mu\gamma^{\alpha_1\ldots\alpha_p}\gamma_\nu\gamma^{\beta_1\ldots\beta_q}),\\
T_2 &= \frac{(-1)^{\frac{1}{2}[m(m+1)+q(q+1)]}a_n a_m a_p a_q}{n!m!p!q!} F_{1\mu_1\ldots\mu_n} F_{2\nu_1\ldots\nu_m} F_{3\alpha_1\ldots\alpha_p} F_{4\beta_1\ldots\beta_q} \mathrm{Tr}(P_\pm \gamma^\nu \gamma^{\nu_1\ldots\nu_m}\gamma^\mu\gamma^{\alpha_1\ldots\alpha_p}\gamma_\nu\gamma^{\beta_1\ldots\beta_q}\gamma_\mu\gamma^{\mu_1\ldots\mu_n}),\\
T_3 &= \frac{(-1)^{\frac{1}{2}[n(n+1)+p(p+1)]}a_n a_m a_p a_q}{n!m!p!q!} F_{1\mu_1\ldots\mu_n} F_{2\nu_1\ldots\nu_m} F_{3\alpha_1\ldots\alpha_p} F_{4\beta_1\ldots\beta_q} \mathrm{Tr}(P_\pm \gamma^\nu \gamma^{\nu_1\ldots\nu_m}\gamma^\mu\gamma^{\alpha_1\ldots\alpha_p}\gamma_\nu\gamma^{\beta_1\ldots\beta_q}\gamma_\mu\gamma^{\mu_1\ldots\mu_n}),\\
T_4 &= \frac{(-1)^{\frac{1}{2}[m(m+1)+q(q+1)]}a_n a_m a_p a_q}{n!m!p!q!} F_{1\mu_1\ldots\mu_n} F_{2\nu_1\ldots\nu_m} F_{3\alpha_1\ldots\alpha_p} F_{4\beta_1\ldots\beta_q} \mathrm{Tr}(P_\pm \gamma^\mu \gamma^{\mu_1\ldots\mu_n}\gamma^\nu\gamma^{\beta_1\ldots\beta_q})\mathrm{Tr}(P_\pm\gamma_\mu\gamma^{\alpha_1\ldots\alpha_p}\gamma_\nu\gamma^{\nu_1\ldots\nu_m}).
\end{aligned}
\tag{45}
$$

Using the above factors, the closed string kinematic factor (36) can be written as

$$
\mathcal{K} = \frac{1}{64}[t^2 T_1 - stT_2 - stT_3 + s^2 T_4], \tag{46}
$$

Performing the traces, one finds how the four RR field strengths contract among themselves. Writing $t^2 = 16\alpha'^2 k_2\cdot k_3 k_1\cdot k_4$, $s^2 = 16\alpha'^2 k_1\cdot k_2 k_3\cdot k_4$ and using the first relation in (12), one may write the kinematic factor (46) in the form of $(\partial F)^4$ in the spacetime which can then be compared with the couplings in Sec. II.

The scattering amplitude of four RR states in type II superstring theories have been also calculated in the Pure spinor formalism in Ref. [38]. The couplings of four RR field strengths at order α'^3 have been found to be

$$
S \supset \sum_{M,N,P,Q} v^{a_1\ldots a_M;b_1\ldots b_N;c_1\ldots c_P;d_1\ldots d_Q}\partial_i\partial_j F_{a_1\ldots a_M}\partial^i\partial^j F_{b_1\ldots b_N}F_{c_1\ldots c_P}F_{d_1\ldots d_Q}, \tag{47}
$$

where the sum over $M, \ldots, Q$, runs over even integers from zero to 4 for type IIA supergravity and over odd integers from 1 to 5 for type IIB. The tensor v is defined in terms of the trace of the gamma matrices as follows:

$$
\begin{aligned}
v^{a_1\ldots a_M;b_1\ldots b_N;c_1\ldots c_P;d_1\ldots d_Q} = \frac{32}{9}\frac{c_M c_N c_P c_Q}{M!N!P!Q!}&[\mathrm{Tr}(P_\mp\gamma^{a_1\ldots a_M}\gamma_q\gamma^{b_1\ldots b_N}\gamma_n\gamma^{c_1\ldots c_P}\gamma^q\gamma^{d_1\ldots d_Q}\gamma^n)\varepsilon_N\varepsilon_Q\\
&- \mathrm{Tr}(P_\mp\gamma^{a_1\ldots a_M}\gamma_q\gamma^{b_1\ldots b_N}\gamma_n)\mathrm{Tr}(P_\mp\gamma^{c_1\ldots c_P}\gamma^q\gamma^{d_1\ldots d_Q}\gamma^n)\varepsilon_N\varepsilon_Q\\
&- 5\mathrm{Tr}(P_\mp\gamma^{a_1\ldots a_M}\gamma_q\gamma^{c_1\ldots c_P}\gamma_n\gamma^{b_1\ldots b_N}\gamma^q\gamma^{d_1\ldots d_Q}\gamma^n)\varepsilon_P\varepsilon_Q\\
&+ 4\mathrm{Tr}(P_\mp\gamma^{a_1\ldots a_M}\gamma_q\gamma^{c_1\ldots c_P}\gamma_n)\mathrm{Tr}(P_\mp\gamma^{b_1\ldots b_N}\gamma^q\gamma^{d_1\ldots d_Q}\gamma^n)\varepsilon_P\varepsilon_Q\\
&+ \mathrm{Tr}(P_\mp\gamma^{a_1\ldots a_M}\gamma_q\gamma^{c_1\ldots c_P}\gamma_n\gamma^{d_1\ldots d_Q}\gamma^q\gamma^{b_1\ldots b_N}\gamma^n)\varepsilon_N\varepsilon_Q],
\end{aligned}
\tag{48}
$$

where $c_p^2 = (-1)^{p+1}/16\sqrt{2}$ and $\varepsilon_N = (-1)^{\frac{1}{2}N(N-1)}$. We have included the chiral projection operators in the traces because the RR vertex operators that have been considered in Ref. [38] have no chiral projection operator. Using the RR vertex operator (32) instead, one has to consider $P_\mp$ inside the traces.

The γ_{11} in the chiral projection operators has one Levi-Civitá tensor. As a result, the kinematic factor (46) and tensor v in (48) have terms with zero, one, and two Levi-Civitá tensors. Since there is ambiguity in the chiral projection operator in the vertex (32), the signs of P-odd terms in T_1, T_2, T_3, T_4, and in tensor v are ambitious. Therefore, we consider only the P-even terms in the kinematic factor (46) and in tensor v. Moreover, we use the identity (15) to write the two Levi-Civitá tensors in them in terms of the metric. Using the symbolic program for the manipulation the gamma matrices [39], we have performed the traces in the kinematic factor (46) and in the tensor v.

Using on-shell relations, we have found that the P-even terms in RNS and in the Pure spinor formalisms at order α'^3 are exactly identical. We have also found these couplings are identical to various P-even couplings in Sec. II.

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APPENDIX: THREE-POINT VERTICES AND PROPAGATORS

Using the supergravities (6) and (7), one can read the propagators of the NSNS fields and the three-point vertices for two on-shell RR states and one off-shell NSNS state that we need in evaluating the Feynman amplitudes in Sec. II. The propagators are:

(i) Graviton propagator:

$$[\tilde{G}_h]_{\mu\nu,\lambda\rho} = -\frac{i}{2k^2}\left(\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda} - \frac{1}{4}\eta_{\mu\nu}\eta_{\lambda\rho}\right). \tag{A1}$$

(ii) B-field propagator:

$$[\tilde{G}_b]_{\mu\nu,\lambda\rho} = -\frac{ie^{\phi_0}}{2k^2}(\eta_{\mu\lambda}\eta_{\nu\rho} - \eta_{\mu\rho}\eta_{\nu\lambda}). \tag{A2}$$

(iii) Dilaton propagator:

$$\tilde{G}_\phi = -\frac{i}{k^2}. \tag{A3}$$

The vertices are the following:

(i) Two RR and one graviton[4]:

$$[\tilde{V}_{F_1^{(n)}F_2^{(n)}h}]^{\lambda\rho} = \frac{ie^{\frac{(5-n)}{2}\phi_0}}{4\kappa\alpha n!}(2nF_1{}^{(\lambda}{}_{\nu_1\ldots\nu_{n-1}}F_2^{\rho)\nu_1\ldots\nu_{n-1}} - \eta^{\lambda\rho}F_{1\nu_1\ldots\nu_n}F_2^{\nu_1\ldots\nu_n}). \tag{A4}$$

(ii) Two RR and one B-field:

$$[\tilde{V}_{F_1^{(n)}F_2^{(n-2)}b}]^{\lambda\rho} = -\frac{ie^{\frac{(5-n)}{2}\phi_0}}{4\kappa\alpha(n-2)!}F_1{}^{\lambda\rho}{}_{\nu_1\ldots\nu_{n-2}}F_2^{\nu_1\ldots\nu_{n-2}}. \tag{A5}$$

(iv) Two RR and one dilaton:

$$\tilde{V}_{F_1^{(n)}F_2^{(n)}\phi} = -\frac{ie^{\frac{(5-n)}{2}\phi_0}}{4\sqrt{2}\kappa n!}(5-n)F_{1\nu_1\ldots\nu_n}F_2^{\nu_1\ldots\nu_n}. \tag{A6}$$

(v) Two RR four-forms, one B-field, and one Levi-Civitá tensor:

$$[\tilde{V}_{\epsilon_{10}F_1^{(4)}F_2^{(4)}b}]^{\alpha\beta} = -\frac{i}{1152\kappa}\epsilon^{\alpha\beta\gamma\delta\epsilon\zeta\eta\lambda\mu\nu}F_{1\gamma\delta\epsilon\zeta}F_{2\eta\lambda\mu\nu}. \tag{A7}$$

(vi) One RR five-form, one RR three-form, one B-field, and one Levi-Civitá tensor:

$$[\tilde{V}_{\epsilon_{10}F_1^{(5)}F_2^{(3)}b}]^{\alpha\beta} = -\frac{i}{2880\kappa}\epsilon^{\alpha\beta\gamma\delta\epsilon\zeta\eta\lambda\mu\nu}F_{1\zeta\eta\lambda\mu\nu}F_{2\gamma\delta\epsilon}. \tag{A8}$$

[4]The parentheses notation over indices means symmetrization with a factor $\frac{1}{2}$.

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