Geometric approach to modulus stabilization

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Modulus stabilization, a must for explaining the hierarchy problem in the context of Randall-Sundrum-like scenarios, is traditionally achieved through the introduction of an extra field with *ad hoc* couplings. We point out that the stabilization can, instead, be achieved in a purely geometrodynamical way, with plausible quantum corrections in the gravity sector playing the key role. The size of the corrections that lead to acceptable phenomenology is also delineated.

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Notwithstanding the recent discovery of the Higgs boson [1,2], the lack, so far, of any definitive signature of physics beyond the Standard Model (SM) is perplexing. Although the mass of the Higgs boson is such that new physics at a nearby scale is not demanded by considerations of triviality or vacuum stability, turning this around to imply that none exists until the Planck scale ($M_{\rm Pl}$) is, at the least, aesthetically repugnant. Indeed, the hierarchy problem of the SM continues to be a vexing issue, and, over the years, several mechanisms have been suggested to ameliorate this. While most of these scenarios also do promise explanations of some of the other puzzles that beset the SM, no direct evidence for any of the new states intrinsic to these theories have been seen so far. Furthermore, several of these have, associated with them, some form of a little hierarchy problem.

An interesting approach due to Randall and Sundrum (RS) to the hierarchy problem essentially does away with a fundamental weak scale, ascribing the apparent hierarchy to a geometrical origin [3]. Envisaging space-time to be a slice of AdS_5, the known world is confined to one of a pair of three-branes that sit atop the two fixed points of a S^1/Z_2 orbifold. The metric has the nonfactorizable form

$$ds^2 = e^{-2kr_c|y|}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 dy^2, \tag{1}$$

with $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$ being the Minkowski metric and $y \in [0,\pi]$ with $(x^\mu,-y) \equiv (x^\mu,y)$. On the (visible) brane at $y=\pi$, the natural mass scale is suppressed by a factor of $e^{-kr_c\pi}$ with respect to the fundamental scale, e.g., that operative at the (hidden) brane located at $y=0$. With $k \approx \mathcal{O}(M)$ arising naturally, having $kr_c \approx 11$ would "solve" the hierarchy problem. However, the modulus r_c is not determined by the dynamics. This can be cured by promoting r_c to a dynamical (radion) field $\mathcal{T}(x_\mu)$ and inventing a mechanism that forces it to settle to $\langle \mathcal{T}\rangle = r_c$.

To this end, Ref. [4] introduced a new scalar field ϕ in the bulk with a quadratic potential. Interacting, as it does, with $\mathcal{T}$ through the metric, integrating out ϕ would result in an effective potential $V_{\rm eff}(\mathcal{T})$. A suitable form for $V_{\rm eff}(\mathcal{T})$ can be arranged if $m_\phi \ll M_{\rm Pl}$ as well as if ϕ has brane localized potentials that ensure appropriate classical value on the branes, leading to

$$kr_c \simeq (k^2/m_\phi^2)\ln[\phi(y=0)/\phi(y=\pi)]. \tag{2}$$

The apparent success of this mechanism due to Goldberger and Wise (GW) [4] hinges on the *ad hoc* introduction of a new fundamental scalar, with masses and couplings being just so. A variation of this mechanism has been attempted in Ref. [5]. However it would be nice if the stabilization process could have emerged more naturally. To this end, we appeal to a geometric origin in the shape of corrections to the Einstein-Hilbert (EH) action itself. While this may seem an *ad hoc* measure as well, such corrections are liable to arise in any quantum theory of gravity. In the absence of a definitive theory, though, we are unable to determine the exact structure of such corrections and hence consider the effective action for gravity to constitute all possible structures consistent with diffeomorphism invariance as well as other imperatives. This, in general, allows for terms higher order in R, $R_{ab}R^{ab}$ and $R_{abcd}R^{abcd}$. Inclusion of the last two, generically, leads to instabilities, although particular linear combinations may escape this fate. On the other hand, the replacement $R \to f(R)$ is often free from such instabilities [6,7].

We start by postulating the five-dimensional pure gravity action, in the Jordan frame, to be

$$S_{\rm EH} = \int d^4x dy \sqrt{\tilde{g}}(2M^3 f(\tilde{R}) - 2\lambda M^5) - \int d^4x dy \sqrt{\tilde{g}}[\lambda_v \delta(y-\pi) + \lambda_h \delta(y)], \tag{3}$$

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where M is the fundamental mass scale and $\tilde{g}_{ab}$ the metric with $\tilde{g} = -\mathrm{Det}(\tilde{g}_{ab})$. While it could have been included in $f(\tilde{R})$ itself, we prefer to write the putative cosmological term explicitly, with $\lambda \lesssim \mathcal{O}(1)$. Similarly, $\lambda_{v,h}$ are the tensions associated, respectively, with the visible and the hidden brane.

The bulk action can be rewritten as

$$S_{\rm blk} = \int d^4x dy \sqrt{\tilde{g}}(2M^3\tilde{R}F - U - 2\lambda M^5), \qquad (4)$$

where $U = 2M^3[\tilde{R}F - f(\tilde{R})]$ and $F \equiv f'(\tilde{R})$. The non-minimal coupling above can be rotated away by a conformal transformation [6–8], *viz.*,

$$\tilde{g}_{ab} \to g_{ab} = \exp(2\omega(x^\mu, y))\tilde{g}_{ab}, \qquad (5)$$

with the actual form of $\omega(x^\mu, y)$ yet to be specified. The Ricci scalars in the two frames are related through

$$\tilde{R} = e^{2\omega}[R + 8\Box\omega - 12g^{ab}\partial_a\omega\partial_b\omega],$$

with $\Box$ representing the Laplacian operator appropriate for the Einstein frame (defined in terms of g_{ab}). Choosing a specific form of $\omega(x^\mu, y)$, *viz.*,

$$\omega = \frac{1}{3}\ln F \equiv \frac{\gamma\phi}{5}, \qquad \gamma \equiv \frac{5}{4\sqrt{3}M^{3/2}}, \qquad (6)$$

we have,

$$S = \int d^4x d\tilde{y}\, \sqrt{g}\Big[2M^3 R - \frac{1}{2}g^{ab}\partial_a\phi\partial_b\phi - V(\phi) - e^{-\gamma\phi}\{(\lambda_v + \mathcal{L}_{\rm SM})\delta(\tilde{y} - l) + \lambda_h\delta(\tilde{y})\}\Big], \qquad (7)$$

where we have introduced $\mathcal{L}_{\rm SM}$ for later use and

$$V(\phi) = [U(\phi) + 2\lambda M^5]\exp(-\gamma\phi). \qquad (8)$$

We have, thus, successfully traded the complex form of $f(\tilde{R})$ for the usual EH action, supplemented by a scalar field that essentially encapsulates the extra degree of freedom encoded in the higher powers of derivatives in $f(\tilde{R})$. As long as $V(\phi)$ is bounded from below, the system would be free from Ostrogradski instabilities.

The exact form of $V(\phi)$ would, of course, hinge on the form of $f(\tilde{R})$. Some features of the scenario, though, are ubiquitous. Unlike in the original RS scheme, the two brane tensions would, in general, be unequal in magnitude. This could have been anticipated in the Jordan frame as well, for the tensions were necessary to allow for the discontinuity in the logarithmic derivative of the metric at the orbifold fixed points; and for $f(\tilde{R}) \neq \tilde{R}$, the two junction conditions cannot be expected to be equivalent.[1] What may seem even more problematic is the existence of the bulk scalar field as we would now need to consider the coupled system (g_{ab}, ϕ) instead of the vacuum equations as examined in Ref. [3]. This can be done though, but usually results in a very complicated set of equations, which rarely is amenable to closed form analytic solutions [9,10]. Furthermore, there are several subtle issues that actually invalidate some of the approaches adopted in the literature. Rather than follow this path, and present a set of dense expressions and/or numerical solutions, we first appeal to a physically well-motivated approximation.

Consider the case where $V(\phi)$ has a minimum at $\phi = \phi_{\rm min}$. Given sufficient time, one would expect that ϕ would settle at $\phi_{\rm min}$ with $V(\phi_{\rm min})$ acting as the effective cosmological constant (i.e., it would assume the role of Λ in [3]). To the leading order, only small deviations about $\phi_{\rm min}$ should need be considered. If the mass of the "fluctuation field" is small, then so is the energy contained in it and neglecting the corresponding backreaction is justifiable and constitutes a very useful first approximation.

Even though much of what follows can be applied to a wide class[2] of $f(\tilde{R})$, we choose to work with a series expansion in $\tilde{R}/M^2$, retaining only a few terms so as to facilitate an immediate examination of each step in the analysis, *viz.*

$$f(\tilde{R}) = \tilde{R} + aM^{-2}\tilde{R}^2 + bM^{-4}\tilde{R}^3, \qquad (9)$$

where a, b are dimensionless free parameters with each, presumably, $\lesssim \mathcal{O}(1)$. As would be expected, we need $b > 0$ for both obtaining a sufficiently negative $V(\phi_{\rm min})$ as also the desirability of a small second derivative at the minimum. On the other hand, a can assume either sign or even vanish. It is interesting to note that $f(\tilde{R}) = \tilde{R}^\beta$, typically, fails the twin test, unless β is a certain very specific fraction. The corresponding potential has the form

$$V = 2M^{-5}F^{-5/3}[\lambda + aR^2(F) + 2bR^3(F)], \qquad (10)$$

where (for phenomenological reasons) we confine ourselves to a specific branch, namely

$$R(F) = \frac{-a - \sqrt{a^2 - 3b(1-F)}}{3b}. \qquad (11)$$

[1] This would also have been forced upon us in the GW-like scenario were backreaction taken into account.

[2] While it has been argued that $f(R)$ models and generalized scalar-tensor theories are equivalent, the mapping is a very non-trivial one [7]. For example, the geometrical dual of the GW scenario requires an extremely complicated, and ill-motivated, form for $f(R)$. On the contrary, we restrict ourselves to some of the leading quantum corrections to the EH action [see Eq. (9)].

We look for a situation whereby, in the Einstein frame, the only nontrivial dependence of the metric is on the coordinate $\tilde{y} \equiv r_c y$, namely

$$ds^2 = e^{-2\sigma(\tilde{y})}\eta_{\mu\nu}dx^\mu dx^\nu + d\tilde{y}^2. \tag{12}$$

The Einstein's equations reduce to

$$\begin{aligned} 6\sigma'^2 &= \frac{1}{4M^3}\left[\frac{1}{2}\phi'^2 - V\right], \\ 3\sigma'' &= \frac{1}{4M^3}[\phi'^2 + e^{-\gamma\phi}(\lambda_h\delta(y) + \lambda_v\delta(y-l))], \end{aligned} \tag{13}$$

whereas the scalar field satisfies

$$\phi'' - 4\sigma'\phi' - \frac{dV}{d\phi} + \gamma e^{-\gamma\phi}[\lambda_h\delta(y) + \lambda_v\delta(y-l)] = 0. \tag{14}$$

Canonical quantization of the SM fields requires a field redefinition factor of $\exp[\gamma\phi(\pi)/2 + kr_c\pi]$ and the corresponding masses are scaled as $m_i \to m_i \exp[-kr_c\pi]$.

Although the system can be solved numerically, it is instructive to consider an approximation so as to allow for closed form analytic expressions. Expanding around $\phi = \phi_a \sim \phi_{\rm min}$, we write

$$\frac{V}{M^5} = V_0 + \left(\frac{V_1}{M^{7/2}}\right)\xi + \left(\frac{V_2}{M^2}\right)\xi^2, \tag{15}$$

where $\xi(\tilde{y}) = M^{-3/2}(\phi - \phi_a)$ and V_i are constants. It might seem counterintuitive to consider $\phi_a \neq \phi_{\rm min}$; this, however, is useful to enhance the applicability of the approximation, which we require to be better than ~10% over the range of interest (see Fig. 1).

Neglecting the backreaction altogether would reduce the system to the standard GW scenario with the corresponding warping $\sigma_{(0)}(\tilde{y})$ being linear in $|\tilde{y}|$ with the coefficient determined by V_0 which corresponds to the *effective* bulk cosmological constant in this scenario. However, doing so is not really justified as it can be as much as 10% or larger. Hence, we effect an inclusion by solving the bulk equations iteratively. To the lowest order, the function $\sigma(|\tilde{y}|)$ is linear, with the degree of warping being controlled by a constant k defined through

$$k^2 \equiv \frac{1}{24M^3}\left[\frac{V_1^2}{4V_2} - V_0\right]. \tag{16}$$

Note that even the definition of k differs somewhat from the original RS form [3] on account of the linear term V_1 in Eq. (15). Introducing the notation

$$\nu \equiv \sqrt{4 + 2V_2/k^2}, \qquad \alpha_{1,2} \equiv (2 \pm \nu)k, \tag{17}$$

the first order solution to the warping is

$$\sigma_{(1)} = k|\tilde{y}| + \frac{1}{18M^3}\left[c_1^2 e^{2\alpha_1|\tilde{y}|} + c_2^2 e^{2\alpha_2|\tilde{y}|} - \frac{c_1c_2V_2}{k^2}e^{4k|\tilde{y}|}\right]. \tag{18}$$

The dimensionless constants $c_{1,2}$ can be determined by matching the discontinuities, leading to

$$\begin{aligned} 0 &= \frac{c_1}{2\nu - 21} - \frac{c_2}{2\nu + 21} + \left(20\sqrt{3} + 25\frac{V_1}{V_2}\right), \\ 0 &= \frac{c_1 e^{(2+\nu)kl}}{2\nu - 21} - \frac{c_2 e^{(2-\nu)kl}}{2\nu + 21} + \left(20\sqrt{3} + 25\frac{V_1}{V_2}\right), \\ l &\equiv r_c\pi. \end{aligned} \tag{19}$$

For large kl (applicable since we need $kl \approx 35$ to explain the hierarchy), one obtains

$$\begin{aligned} c_1 &\simeq \left(20\sqrt{3} + 25\frac{V_1}{V_2}\right)\frac{e^{-2\nu kl} - e^{-(2+\nu)kl}}{2\nu - 21}, \\ c_2 &\simeq \left(20\sqrt{3} + 25\frac{V_1}{V_2}\right)\frac{1 + e^{-2\nu kl} - e^{-(2+\nu)kl}}{2\nu + 21}. \end{aligned} \tag{20}$$

The nonlinear terms in Eq. (18) account for the leading backreaction due to the scalar field, which, to this order, is given by

$$\xi_{(1)}(\tilde{y}) = \frac{-V_1}{2V_2} + M^{3/2}[c_1 e^{\alpha_1|\tilde{y}|} + c_2 e^{\alpha_2|\tilde{y}|}]. \tag{21}$$

One could extend this to even higher orders, with the additional corrections being given in terms of confluent hypergeometric functions (also see Ref. [9]).

Substituting Eq. (21) in the action and integrating over $\tilde{y}$, the effective potential for the modulus field is obtained to be

$$\frac{V_{\rm eff}}{M^3 k} = [d_0 + d_1(e^{-2\nu kl} - 2e^{-(2+\nu)kl}) + d_2 e^{-4kl}] \tag{22}$$

where

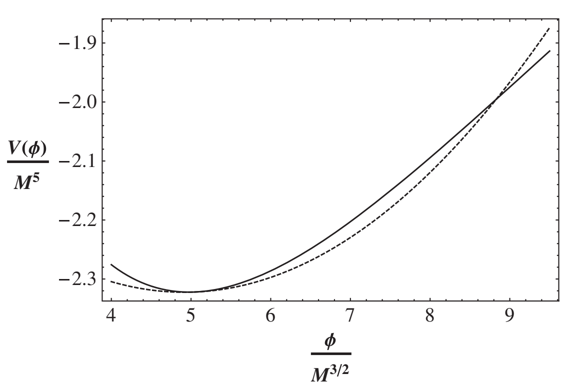

FIG. 1. The solid line denotes the potential for $a = 0.01$, $b = 0.01, \lambda = -0.6$, while the dashed line denotes a typical approximation for $\phi_a = 5.05M^{3/2}$.

$$
\begin{aligned}
d_0 &= 24 - \frac{12V_1^2}{V_1^2 - 4V_0V_2} + \frac{\nu - 2}{4(2\nu + 21)^2}\left(40\sqrt{3} + 25\frac{V_1}{V_2}\right)^2 \\
&\quad + \frac{25}{16(2\nu - 21)^2}\left(\frac{5V_1}{V_2} + 8\sqrt{3}\right) \\
&\quad \times \left(8\sqrt{3}(67 - 4\nu) + 125\frac{V_1}{V_2}\right), \\
d_1 &= \frac{250\nu}{4\nu^2 - 441}\left(\frac{5V_1}{V_2} + 8\sqrt{3}\right)\left[(\nu^2 + 21)\frac{V_1}{V_2} + 210\sqrt{3}\right], \\
d_2 &= \frac{48(21\nu - \nu^2 + 46)}{(2\nu - 21)^2} + \frac{250\sqrt{3}(\nu + 2)}{(2\nu - 21)^2}\frac{V_1}{V_2} \\
&\quad + \frac{48V_0V_2}{V_1^2 - 4V_0V_2} + \frac{625(4\nu - 17)}{16(2\nu - 21)^2}\frac{V_1^2}{V_2^2}.
\end{aligned} \tag{23}
$$

The consequent extrema are given by

$$
e^{(2-\nu)kl} = \frac{(2+\nu) \pm \sqrt{(2+\nu)^2 - 8\nu d_2/d_1}}{2\nu}. \tag{24}
$$

A minimum is present only for the solution corresponding to the negative sign above. Approximating $\nu \simeq 2 + \epsilon$ with $\epsilon = V_2/2k^2$, its position is given by

$$
kl \simeq -\epsilon^{-1} \ln n_0, \tag{25}
$$

where

$$
n_0 = 1 - \frac{17}{8}\left[\frac{768V_0V_2^3 + V_1^2V_0V_2 - 25V_1^4}{(4V_0V_2 - V_1^2)(1008V_2^2 + 250\sqrt{3}V_1V_2 + 25V_1^2)}\right]^{1/2}. \tag{26}
$$

At this stage, it is useful to recall the relationship chain between the parameters of the theory and the hierarchy. The expansion coefficients V_i [see Eq. (15)] are determined in terms of the parameters a, b, λ [see Eq. (9) or Eq. (10)]. A particular combination of the V_i, *viz.* k is the primary driver of the hierarchy, vide Eq. (16). Along with V_2, the quantity k determines the warping function $\sigma_{(1)}(\tilde{y})$, with the constants c_i determining the corrections to the brane tensions $\lambda_{v,h}$ (once the radius has been stabilized). Other combinations of the V_i, determine, in turn, the coefficients d_i's [Eq. (23)] appearing in $V_{\rm eff}$ (the effective potential for the radion) and, hence, the equilibrium configuration [see Eqs. (25) and (26)]. The entire relationship can be summarized numerically in terms of isohierarchy contours in the parameter space (see Fig. 2).

To understand the figure, note that for $V_1 \to 0$, $n_0 \to 0.0726$ leading to a direct correspondence between the hierarchy and $V_2/2k^2$. Since V_2 has only a very weak dependence on b, this implies $\lambda \propto 1/\sqrt{b}$ for a given kl, a relation exhibited to a very large degree by the curves in Fig. 2. This clearly rules out the possibility of $b = 0$ and indicates the importance of the R^3 term. On the other hand, $a = 0$ is clearly admissible. While the relationship between a and λ (Fig. 3) is more complicated, it is interesting to note that the isohierarchy curves tend to a fixed point in this plane with the location depending on the value of b.

It is important to note that, our scalar field ϕ is of a geometrical origin and we are not allowed to introduce brane-localized potentials (as in the GW scenario) unless accompanied by a corresponding change in the geometrodynamics [e.g., the introduction of additional and arbitrary brane-localized $f(R)$ terms]. Appealing to Occam's razor, we deliberately eschew this. In other words, the values of ϕ at the two branes are fixed by the warp factor $\sigma(\tilde{y})$. This, in turn, fixes the value of the brane tensions $\lambda_{v,h}$ which are no longer equal and opposite, but differ slightly in magnitude [to the same extent as $\sigma(\tilde{y})$ differs from linearity] so that the

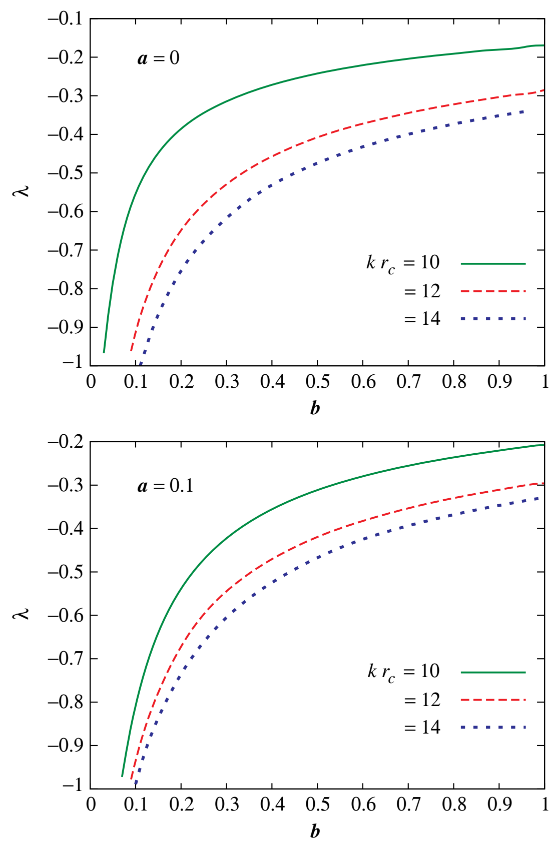

FIG. 2 (color online). Isohierarchy (kr_c) contours in the (b, λ) plane for $a = 0$ (top) and $a = 0.1$ (bottom).

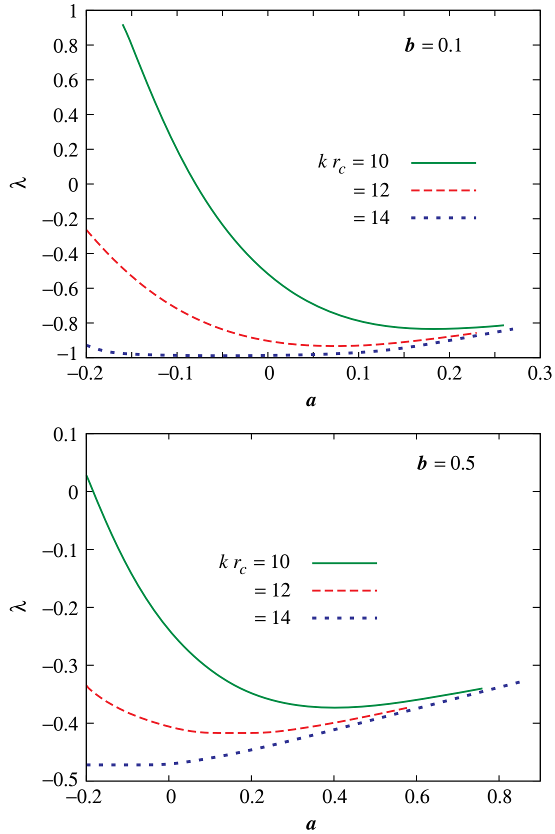

FIG. 3 (color online). Isohierarchy (kr_c) contours in the (a, λ) plane for $b = 0.1$ (top) and $b = 0.5$ (bottom).

entire solution is a self-consistent one. This is analogous to the scenario discussed in Ref. [9] where the effects of the backreaction of the bulk scalar field are taken into consideration. As mentioned in the beginning, this is but a consequence of incorporating the backreaction (in the Einstein frame) or, equivalently, the nonlinear form of $f(\tilde{R})$ (in the Jordan frame). Note that this small difference between λ_v and λ_h does not imply an additional fine-tuning. Indeed, even within the GW scenario, such a difference between the brane tensions (as opposed to the values of the brane-localized expectation values of the stabilizing field) would have been required if the backreaction on gravity (due to the GW scalar) is taken into account, as it should have been. In fact a similar consequence would be forced upon us for any variant of the original RS scenario as soon as the nongravity fields are allowed to intrude into the bulk. It is also interesting to note that if the brane tensions were fixed externally, it would amount to fixing the combination kl (and, thus, the hierarchy) by matching the discontinuities in the derivatives of the warp-function $\sigma(\tilde{y})$. However, it should be borne in mind that such a scenario may need one to consider higher order corrections to $\sigma(\tilde{y})$ [i.e., going beyond $\sigma_{(1)}(\tilde{y})$], and, in severe cases, may even invalidate the approximation method that we have adopted. Once again, such an eventuality might befall any generic warping scenario with a nontrivial backreaction. Only very special potentials (such as those in Ref. [9]) may escape this.

Given that $m_h \approx 125$ GeV, the fairly sensitive nature of the hierarchy contour, in principle, allows us a determination of the $f(R)$ parameters. Note, however, that the exact hierarchy in any such warped scenario is determined in terms of the value of the Higgs mass on the Planck brane, $m_h(\mathrm{Pl})$. The latter, though, is unprotected by any symmetry, and consequently, its natural value is close to the cutoff of the theory m_0, which, nominally should be the fundamental scale M itself. For the RS model, it has been argued [11], under the assumption of the bulk curvature k being sufficiently small compared to M ensuring the validity of the classical solution [12], the nonobservation of the Kaluza-Klein (KK) gravitons at the LHC [13] indicates that m_0 must be at least 2 orders of magnitude lower than M. This little hierarchy would be further exacerbated if the gravitons continue to evade discovery. In contrast, the scenario we discuss here provides a concrete threshold in the shape of m, the mass of the scalar. With the latter being of geometrodynamical origin, its value is determined by the same quantum corrections that determine the hierarchy and, indeed, there is a nonzero correlation between the two. For example, if one were to start with a six-dimensional doubly warped scenario [14] (which has been shown to evade this tension [15]), a nontrivial $f(\tilde{R})$ would be generated once the smaller of the two warped directions is integrated out.

The quantum fluctuations of ϕ about the classical configuration $\phi_{\mathrm{cl}}(\tilde{y})$ are endowed with a bulk mass two orders below M_{Pl}, and, thus, play no role in observable physics except for, presumably, late time cosmology. The mass (m_{rad}) of the radion fluctuation $\hat{\mathcal{T}}(\equiv \mathcal{T} - r_c)$ is determined by V_{eff} and turns out to be

$$m_{\mathrm{rad}}^2 \simeq 3 d_0 \epsilon e^{-2kr_c\pi} k^2. \tag{27}$$

Now, to $\mathcal{O}(\epsilon^0)$,

$$d_0 = \frac{4032}{289} + \frac{1000\sqrt{3}V_1}{289V_2} - \frac{5625V_1^2}{4624V_2^2} + \frac{48V_0V_2}{V_1^2 - 4V_0V_2},$$

and for a vanishing V_1, we have $d_0 \to 2$, or, in other words,

$$m_{\mathrm{rad}}^2 \to 3V_2 e^{-2kr_c\pi}.$$

The exponential factor brings down the radion mass from the Planck scale to the TeV scale without the need for any additional fine-tuning of parameters. The parametric dependence, in this limit, is encapsulated in the various panels of Fig. 4. The information in the figure has to be interpreted carefully. Note that the middle panel ($kr_c = 11$) is the one that corresponds to the correct value of the hierarchy. It is worthwhile to notice that the radion, in the present scheme, tends to be somewhat heavier than in the GW case. A smaller value of kr_c (upper panel) would

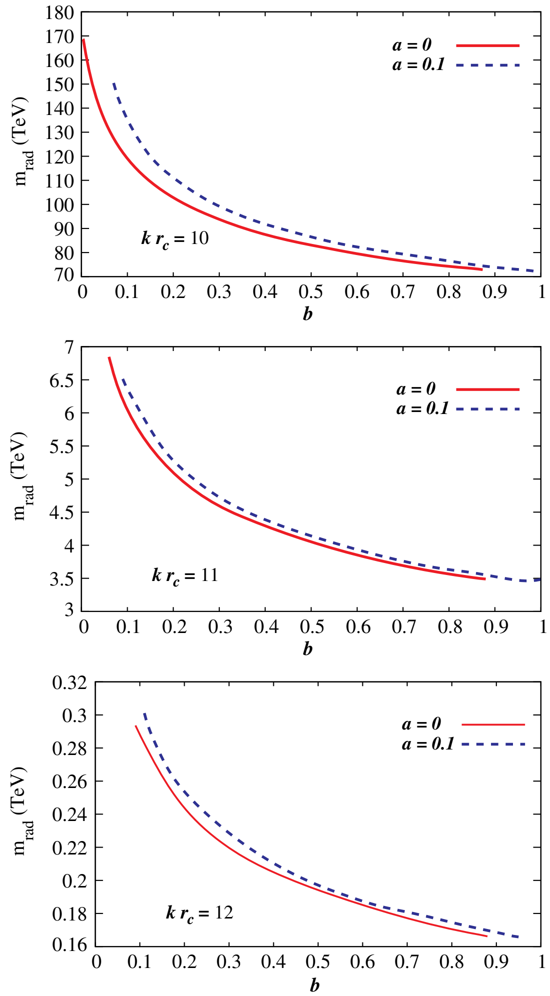

FIG. 4 (color online). The mass of the radion as a function of b for fixed values of the parameter a and the hierarchy kr_c. (The value of λ is automatically determined vide Figs. 2.) The figures have been made for $V_1 = 0$ and $m_{\rm rad}$ calculated to $\mathcal{O}(\epsilon^{1/2})$. Relaxing these would lead to thickening of the curves into bands.

lead to a little hierarchy problem and it is not surprising that the associated radion would be in the $\mathcal{O}(100\ \mathrm{TeV})$ range. The lower panel, on the other hand, corresponds to a somewhat unphysical situation (of oversuppression of the electroweak scale), and has been included only to facilitate comparisons. To the leading order, the radion couples to the SM fields through the trace of the energy-momentum tensor with the difference from the GW case [16] being generated through the heavy field $\hat{\phi}$. And while radion-Higgs mixing does take place, it is too small to be of any consequence.

Before we end, it is worthwhile to reexamine some theoretical aspects pertaining to this paper. The model proposed here is a generalization of RS-like warped geometry where, in addition to the Einstein-Hilbert and five-dimensional cosmological constant terms, certain higher curvature terms are also present. The latter are presumably the lowest order quantum corrections to the classical action, arising from an as yet undefined UV completion. It may be argued, though, that in a specific such completion wherein compactification can be obtained dynamically, not only would typically more terms appear in the low-energy gravity action, but other fields, including scalars, one of which could be an analogue of the Goldberger-Wise field, would appear naturally too. For example, starting with an $\mathrm{AdS}_5 \times S^5$ model in type IIB string theory, in the presence of multiple D-branes and three-form fluxes, it has been shown [17,18] that one can obtain a metric with a large warping similar to the RS model. The warp factor depends on the flux integers and the three-form fluxes play a crucial role in stabilizing the radion.

Rather than appeal to such a specific UV completion, our work demonstrates that in an effective field theoretic description within the gravity framework alone, the existence of higher curvature terms can lead to a natural and dynamic stabilization of the radion. In other words, just as the original RS model captures the essence of warped solution of a more UV complete theory, a bulk with higher-curvature terms not only admits such a solution, but also stabilizes it. In the spirit of effective field theories, it thus captures the essence of UV-complete theories (such as the ones referred to above), without tying itself to a particularly restrictive class.

To summarize, we have shown that the modulus field in the RS scenario can be stabilized in a purely geometrical way. Appealing to plausible quantum corrections to the Einstein-Hilbert action, we trade the higher derivatives of the metric tensor for an equivalent scalar field with a complicated potential form and a nonminimal coupling to gravity. On going over to the Einstein frame (characterized by a nonminimal coupling), the corresponding potential is seen to have a local minimum leading to an negative effective bulk cosmological constant, and a fluctuation field with a naturally small mass. The resulting framework leads to the stabilization of the modulus without the need to appeal to boundary localized interactions or neglecting the backreaction. The correct hierarchy is obtained for a wide range of parameters. Moreover, the mechanism offers a natural way out of the tension between the theoretical expectations for the KK-graviton masses and the strong bounds obtained at the LHC.

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