Lorentz breaking supersymmetry and Horava-Lifshitz-like models

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We present a Lorentz-breaking supersymmetric algebra characterized by a critical exponent z. Such construction requires a nontrivial modification of the supercharges and superderivatives. The improvement of renormalizability for supersymmetric scalar QED is shown, and the Kählerian effective potentials are calculated in different cases. We also show how the theory flows naturally to the Lorentz symmetric case at low energies.

I. INTRODUCTION

It is known that the addition of higher-derivative corrections improves the ultraviolet (UV) behavior of generic quantum field theories allowing in particular the construction of a renormalizable class of quantum gravity models [1]. However, usually the price to pay is the appearance of ghost excitations and the breaking of unitarity [2,3]. One possible way out of this problem consists of assuming scaling properties which are anisotropic between space and time, so that only high spatial derivatives are introduced, the so-called Horava-Lifshitz (HL) models (see [4–7]). Such anisotropy is characterized by a critical exponent z, related with the degree of the highest spatial derivative in such way that for $z = 1$, the isotropy between time and space holds and the theory is Lorentz invariant. For other values of z, Lorentz symmetry is broken although it is expected to be restored at low energies [8,9]. This procedure has been applied to models without or with gauge symmetry and also to supersymmetric theories.

Concerning the building of HL-like supersymmetric models, two approaches have been proposed; accordingly the superalgebra has the standard form [10] or was deformed to accommodate higher spatial derivatives [11]. In the first situation, modifications are made directly in the action leading to terms containing both time and spatial derivatives which break the simplicity of the original proposal. On the other hand, the inclusion of high spatial derivatives in the generators of the superalgebra has a drawback in the sense that they do not verify the Leibniz rule making it difficult to introduce self-interactions of chiral (or antichiral) superfields. We will show that, in spite of this difficulty, it is possible to formulate a theory free of pathologies. The crucial observation to take into account is that, although the self-interacting terms constructed as products of at least three chiral quantities are not chiral any longer, they do not

break the supersymmetry (SUSY) if they are integrated over the whole Grassmann space. Usually, for $z = 1$ theories, such a procedure leads to nonrenormalizable models but in the anisotropic situation $z > 1$, because of the ultraviolet improvement of the propagators, they may be allowed. In this work, we study a HL-like version of supersymmetric QED constructed along these lines, determine its Kählerian effective potential, and discuss the emergence of Lorentz symmetry at low energies.

This paper is organized as follows. In Sec. II we present a higher spatial derivative supersymmetric algebra and some considerations related with it. In Sec. III we supersymmetrize an anisotropic version of scalar QED and determine the superpropagators and the superficial degree of divergence of the theory. Section IV is devoted to one-loop calculations: the Kählerian effective potential, for $\mathcal{N} = 1$ SUSY and the analysis of the restoration of Lorentz symmetry at low energies. Finally, in Sec. V we present our conclusions. In the Appendix we collected some details of the calculations.

II. LORENTZ-VIOLATING SUSY

We are interested in the formulation of supersymmetric theories which behave anisotropically under a generic scaling, $x^i \to b x^i$, $t \to b^z t$. We recall that the usual Lorentz-invariant supersymmetric algebra is defined by the following anticommutation relation of the supercharges (we follow the notation of [12]):

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu, \tag{1}$$

where

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu, \tag{2}$$

$$\bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu. \tag{3}$$

In this case, the anticommutator $\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\}$ is proportional to $P_\mu = i\partial_\mu$ but, in the Lorentz-violating case, the

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Coleman-Mandula theorem is compatible with a larger class of superalgebras, allowing for a more general anti-commutation relation [11]:

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^{\mu'}_{\alpha\dot\alpha} P_\mu + 2\eta\sigma^i_{\alpha\dot\alpha} \mathcal{O}_i, \qquad (4)$$

where $\sigma^{0'} = \sigma^0$ and $\sigma^{i'} = c\sigma^i$, the dimension of the constant c is $[c] = z - 1$, and η is a free parameter measuring the breaking of the Lorentz invariance. The operator $\mathcal{O}_i$ will depend in general on the spatial part of the momentum (the Lorentz-violating SUSY in isotropic space was treated in [13]). For our purposes we are interested in the following operator:

$$\mathcal{O}_i = \Delta^{\frac{z-1}{2}}\partial_i, \qquad (5)$$

where $\Delta = -\partial_i\partial_i$. The Lorentz-breaking parameter is z, and when $z \to 1$ we recover the usual Lorentz-invariant algebra up to an appropriate redefinition of the constants. This new term in the superalgebra requires a modification of the supercharges (for more details see [11]),

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^{\mu'}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu - i\eta\sigma^i_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\Delta^{\frac{z-1}{2}}\partial_i, \qquad (6)$$

$$\bar{Q}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i\theta^\alpha\sigma^{\mu'}_{\alpha\dot\alpha}\partial_\mu + i\eta\sigma^i_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\Delta^{\frac{z-1}{2}}\partial_i, \qquad (7)$$

and superderivatives

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^{\mu'}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu + i\eta\sigma^i_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\Delta^{\frac{z-1}{2}}\partial_i, \qquad (8)$$

$$\bar{D}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\theta^\alpha\sigma^{\mu'}_{\alpha\dot\alpha}\partial_\mu - i\eta\sigma^i_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\Delta^{\frac{z-1}{2}}\partial_i. \qquad (9)$$

Note that the η parameter is dimensionless for this choice of $\mathcal{O}_i$ operator. Since for $z \neq 1$ the operators Q_α, $\bar{Q}_{\dot\alpha}$, and the corresponding superderivatives do not obey the Leibniz rule, we cannot define chiral superfields in the usual way. Indeed, the deformed supercharges (6) and (7) require the modification of the chiral superfields,

$$\tilde\Phi = \phi + i\theta\sigma^\mu\bar\theta\tilde\partial_\mu\phi + \frac{1}{4}\theta\theta\bar\theta\bar\theta\tilde\Box\phi + \sqrt{2}\theta\psi$$
$$- \frac{i}{\sqrt{2}}\theta\theta\tilde\partial_\mu\psi\sigma^\mu\bar\theta + \theta\theta F, \qquad (10)$$

where $\tilde\partial_0 = \partial_0, \tilde\partial_i = c\partial_i + \eta\Delta^{\frac{z-1}{2}}\partial_i$, and the modified D'Alembertian $\tilde\Box \equiv \tilde\partial_0^2 - \tilde\partial_i\tilde\partial_i = \partial_0^2 + c^2\Delta + 2c\eta\Delta^{\frac{z+1}{2}} + \eta^2\Delta^z$. The modified superfield satisfies the chirality condition $\bar{D}\Phi = 0$.

Besides that, we introduce a vector superfield

$$V = -\theta\sigma^\mu\bar\theta\tilde{A}_\mu + i\theta\theta\bar\theta\bar\lambda - i\bar\theta\bar\theta\theta\lambda + \frac{1}{2}\theta\theta\bar\theta\bar\theta D, \qquad (11)$$

where $\tilde{A}_0 = A_0, \tilde{A}_i = cA_i + \eta\Delta^{\frac{z-1}{2}}A_i$.

This is a natural modification of the gauge field preserving the gauge symmetry (a deformation only on the derivatives would lead to the breaking of gauge symmetry). Note that the deformed field strength $\tilde{F}_{\mu\nu} = \tilde\partial_\mu\tilde{A}_\nu - \tilde\partial_\nu\tilde{A}_\mu$ is invariant under the usual gauge transformation $A_\mu \to A_\mu + \partial_\mu\Lambda$.

Because of the fact that $\bar{D}_{\dot\alpha}(\Phi_1\Phi_2) \neq \bar{D}_{\dot\alpha}(\Phi_1)\Phi_2 + \Phi_1\bar{D}_{\dot\alpha}(\Phi_2)$, it is impossible to define chiral composed operators as products of chiral superfields. In this context, a question that naturally arises concerns the class of supersymmetric actions one may construct with this deformed algebra. Now, every D-term (obtained after integrating over all Grassmann space) is still supersymmetric. Let us analyze the F-terms. For a general supersymmetric variation $\delta = \epsilon^\alpha Q_\alpha + \bar\epsilon_{\dot\alpha}\bar{Q}^{\dot\alpha}$ and a general chiral F-term $\int d^2\theta W(\Phi)$, we have

$$\delta\int d^2\theta W(\Phi) = \int d^2\theta(\epsilon^\alpha Q_\alpha W(\Phi) + \bar\epsilon_{\dot\alpha}\bar{Q}^{\dot\alpha}W(\Phi)). \qquad (12)$$

The term proportional to ϵ^α vanishes because differentiation with respect to θ gives zero after the θ-integration,

$$\int d^2\theta\frac{\partial}{\partial\theta}(\text{Something}) = 0, \qquad (13)$$

and the term proportional to $\bar\theta$ is a total derivative vanishing after integration on the coordinate space,

$$\int d^4x\int d^2\theta(-i\sigma^{\mu'}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu - i\sigma^i_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\Delta^{\frac{z-1}{2}}\partial_i)(\text{Something}) = 0. \qquad (14)$$

Let us now analyze the term proportional to $\bar\epsilon_{\dot\alpha}$,

$$\delta|_{\bar\epsilon}\int d^2\theta W(\Phi) \equiv \bar\epsilon_{\dot\alpha}\int d^2\theta\bar{Q}^{\dot\alpha}W(\Phi)$$
$$= \bar\epsilon^{\dot\alpha}\int d^2\theta(-i\sigma^{\mu'}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu - i\sigma^i_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\Delta^{\frac{z-1}{2}}\partial_i)W(\Phi)$$
$$- \bar\epsilon^{\dot\alpha}\int d^2\theta\frac{\partial}{\partial\bar\theta^{\dot\alpha}}W(\Phi). \qquad (15)$$

The last term in the above relation in general is not a total derivative; therefore,

$$\delta|_{\bar{\epsilon}} \int d^2\theta W(\Phi) = \text{total derivative} - \bar{\epsilon}^{\dot{\alpha}} \int d^2\theta \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} W(\Phi).$$
(16)

For $W(\Phi) = \Phi^n$ we get schematically

$$\int d^2\theta \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} \Phi^n \propto (n-1)\phi^{n-2}\mathcal{O}_\mu\phi\psi_\alpha + \phi^{n-1}\mathcal{O}_\mu\psi_\alpha. \quad (17)$$

For $\mathcal{O}_\mu = \partial_\mu$, as in the Lorentz invariant situation, the above expression is a total derivative, and the SUSY is preserved. If $\mathcal{O}_\mu$ is a higher derivative operator (and this is our case), the above expression is not a total derivative for $n > 2$. The consequence of this observation, as it was pointed out by [11], is that our Lorentz-violating SUSY does not allow for F-terms of degree greater that 2; hence self-interacting chiral terms explicitly break the SUSY.

A. Deformed projection operators

Projection operators usually simplify superfield calculations in Lorentz-invariant supersymmetric theories. The ones we are interested in are the following:

$$\Pi_1 = \frac{D^2\bar{D}^2}{16\Box}, \qquad \Pi_2 = \frac{\bar{D}^2 D^2}{16\Box},$$

$$\Pi_0 = \frac{1}{16}\frac{\{D^2, \bar{D}^2\}}{\Box}, \qquad \Pi_{1/2} = -\frac{1}{8\Box}D^\alpha\bar{D}^2 D_\alpha. \quad (18)$$

As we will see below we only need the multiplication rule for the last two of them. Those operators are idempotents, and their cross composition vanishes,

$$\Pi_i\Pi_j = \delta_{ij}\Pi_i, \qquad i,j = 0,1,2,1/2. \quad (19)$$

We want to show that when we deformed these operators (i.e. $\partial_\mu \to \tilde{\partial}_\mu$), these properties still hold. First we write the new projectors in terms of the deformed superderivatives and D'Alembertians,

$$\tilde{\Pi}_0 = \frac{1}{16}\frac{\{\tilde{D}^2, \bar{\tilde{D}}^2\}}{\tilde{\Box}}, \qquad \tilde{\Pi}_{1/2} = -\frac{1}{8\tilde{\Box}}\tilde{D}^\alpha\bar{\tilde{D}}^2\tilde{D}_\alpha, \quad (20)$$

but since $[\frac{\partial}{\partial\theta^\alpha}, \tilde{\partial}_\mu] = 0$ and $[\tilde{\partial}_\mu, \tilde{\partial}_\nu] = 0$, the relation for tilde operators is the same as for the usual ones (19),

$$\tilde{\Pi}_i\tilde{\Pi}_j = \delta_{ij}\tilde{\Pi}_i, \qquad i,j = 0,1,2,1/2. \quad (21)$$

From now on, we will omit tildes in the projectors and superderivatives, all quantities being understood to be the tilde ones. With this supersymmetric structure we are ready to do some calculations.

III. SUPERSYMMETRIC HL-LIKE ELECTRODYNAMICS

The model we are going to analyze is a Lorentz broken version of supersymmetric scalar QED. Without SUSY, variants of this model with different critical exponents and nonlocal operators were treated in the literature (see for example [14–18]).

The Lagrangian for an Abelian gauge theory with an arbitrary critical exponent z looks like

$$\mathcal{L} = \frac{1}{2}c^2 F_{0i}F^{0i} - \frac{1}{4}c^4 F_{ij}F^{ij} + \frac{1}{2}F_{0i}Q_1(\Delta;z)F^{0i}$$
$$- \frac{1}{4}c^4 F_{ij}Q_2(\Delta;z)F^{ij}. \quad (22)$$

The first two terms in the above expression correspond to the usual Abelian gauge theory with the appropriate anisotropic scaling. The other two terms constitute higher derivative corrections. The operators $Q_1(\Delta;z)$ and $Q_2(\Delta;z)$ are polynomials in Δ, with increasing degree in $z(\geq 1)$; we will fix them in order to obtain the supersymmetric extension. We can add a minimally coupled complex field ϕ,

$$\mathcal{L}_2 = -\bar{\phi}(\partial_0^2 - c^2\partial_i^2)\phi - \bar{\phi}Q_1(\Delta;z)\partial_i^2\phi$$
$$+ \text{gauge interactions}. \quad (23)$$

The gauge interactions will be introduced by promoting the usual interaction term $igA_\mu(\bar{\phi}\partial^\mu\phi - \partial^\mu\bar{\phi}\phi) + g^2 A_\mu A^\mu\bar{\phi}\phi$ to the deformed version, by replacing gauge fields and operators as defined in the text after Eqs. (10) and (11). We need also to add a gauge fixing term. A natural extension of the Feynman gauge to our anisotropic space has the form

$$\mathcal{L}_{gf} = -\frac{1}{2}(\partial_0 A_0 - c^2\partial_i A_i - Q_1(\Delta;z)\partial_i A_i)^2. \quad (24)$$

This gauge fixing eliminates the cross terms of the gauge fields, without the introduction of nonlocal terms in the action (see for example [16,17]). The free propagators of the theory are

$$\langle\phi\bar{\phi}\rangle = \frac{i}{(k_0^2 - c^2\bar{k}^2) - Q_1(\bar{k}^2;z)\bar{k}^2}, \quad (25)$$

$$\langle A_0 A_0\rangle = \frac{-i}{(k_0^2 - c^2\bar{k}^2) - Q_1(\bar{k}^2;z)\bar{k}^2}, \quad (26)$$

$$\langle A_i A_j\rangle = \frac{-i\delta_{ij}}{(k_0^2 - c^2\bar{k}^2) - Q_1(\bar{k}^2;z)k_0^2 - Q_2(\bar{k}^2;z)\bar{k}^2}. \quad (27)$$

Thus, if we choose the polynomials $Q_i(\bar{k}^2;1) = \alpha$, α being a dimensionless constant, then after a redefinition of

the variables to absorb the constants, the propagators have exactly the standard form. If, for a suitable choice of z, the degree of $Q_i(\bar{k}^2; z)$ in $\bar{k}^2$ is greater than one, then, at high energy, the propagators are dominated by this aniso-tropic term.

Our next step is to build the supersymmetric version of this family of models. The nice point of our "deformed" SUSY is that it provides a natural formulation for this class of theories. The supersymmetric version of the model (22) + (23) has the familiar form

$$\mathcal{L} = \frac{1}{4}\left(\int d^2\theta W^\alpha W_\alpha + \int d^2\bar{\theta}\bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}}\right) + \int d^2\theta d^2\bar{\theta}\,\bar{\Phi}\,e^{gV}\,\Phi, \tag{28}$$

where all the superfields are understood to be the ones corresponding to the modified superalgebra (10) and (11) such that the superfield strengths are

$$W_\alpha = \bar{D}^2 D_\alpha V. \tag{29}$$

For the gauge fixing action we choose

$$\mathcal{L}_{gf} = \frac{1}{16\lambda}\int d^2\theta d^2\bar{\theta}(\bar{D}^2 V)D^2 V. \tag{30}$$

If we expand in components, the bosonic sector (with $F = D = 0$) of this theory corresponds to (22) and (23) where

$$\begin{aligned} Q_1(\Delta; z) &= \mathcal{P}(\Delta; z)^2 - c^2, \\ Q_2(\Delta; z) &= \mathcal{P}(\Delta; z)^4 - c^4, \end{aligned} \tag{31}$$

being $\mathcal{P}(\Delta; z) = c + \eta\Delta^{\frac{z-1}{2}}$. Note that it is possible to change these higher derivative corrections in the super-symmetric theory by changing the superalgebra adding other spatial operators. Such a change modifies the poly-nomial $\mathcal{P}(\Delta; z)$ but the forms (31) of Q_1 and Q_2 remain unchanged.

The superpropagators are then

$$\langle\Phi(\xi_1)\bar{\Phi}(\xi_2)\rangle = i\frac{\bar{D}^2 D^2}{16\tilde{\square}}\delta^8(\xi_1 - \xi_2), \tag{32}$$

$$\langle V(\xi_1)V(\xi_1)\rangle = -\frac{i}{\tilde{\square}}\left(\lambda\frac{\{D^2, \bar{D}^2\}}{16\tilde{\square}} - \frac{D^\alpha\bar{D}^2 D_\alpha}{8\tilde{\square}}\right)\delta^8(\xi_1 - \xi_2), \tag{33}$$

where $\xi = (x, \theta, \bar{\theta})$.

All the observations we made for the propagators also hold for the superpropagators. It will also be useful to write (32) and (33) in terms of the modified projection operators (Sec. II A)

$$\langle\Phi(\xi_1)\bar{\Phi}(\xi_2)\rangle = i\Pi_1\delta^8(\xi_1 - \xi_2), \tag{34}$$

$$\langle V(\xi_1)V(\xi_2)\rangle = -i\frac{1}{\tilde{\square}}(\Pi_{1/2} + \lambda\Pi_0)\delta^8(\xi_1 - \xi_2). \tag{35}$$

A. Superficial degree of divergence

With the explicit expressions of the superpropagators we may determine the superficial degree of divergence (SDD). Because of the anisotropic scaling between time and space coordinates, the dimensions of the derivatives are $[\partial_0] = z$ and $[\partial_i] = 1$, and therefore, in d spatial dimensions we have

$$[A_0] = \frac{d+z}{2} - 1, \qquad [A_i] = \frac{d-z}{2}, \tag{36}$$

and

$$[\phi] = [\bar{\phi}] = \frac{d-z}{2}, \qquad [g] = \frac{z-d}{2} + 1. \tag{37}$$

However, the deformed operators in the supersymmetric algebra are such that the dimensions of space and time components are equal and the anisotropy is compensated (since $[c] = z - 1$),

$$[\tilde{\partial}_0] = [\tilde{\partial}_i] = z, \qquad [\tilde{A}_0] = [\tilde{A}_i] = \frac{d-z}{2}. \tag{38}$$

Let us consider a supergraph with L loops and P internal propagators. From the integration over the internal momenta, each loop contributes to the SDD with a factor $(d + z)$, so that the total contribution will be $(d + z)L$. For the power counting, it is convenient to associate the D^2 and $\bar{D}^2$ factors in the scalar propagator with the vertices joined by the corresponding line. Thus, each propagator, $\langle VV\rangle$ or $\langle\Phi\bar{\Phi}\rangle$ contributes with a factor $-2z$, and to the SDD with $-2zP$. From each vertex, and due to the factors D^2 and $\bar{D}^2$, we get an extra contribution $2zV$, unless the vertex is connected to a external chiral or antichiral line, which gives an extra factor $-zE_c$. Besides that, taking into account the following fundamental relation:

$$\delta_{12}D^2\bar{D}^2\delta_{12} = 16\delta_{12}, \tag{39}$$

we have to subtract $2z$ for every loop, and thus $-2zL$ for the SDD. Putting all together, the SDD is given by

$$\omega = (d + z)L - 2zP - 2zL + 2zV - zE_c. \tag{40}$$

Finally, the use of the topological relation $L + V - P = 1$ yields

$$\omega = (d - z) + (d - 3z)(P - V) - zE_c, \tag{41}$$

so that, for $d = 3$,

$$\omega = (3 - z) + 3(1 - z)(P - V) - zE_c. \tag{42}$$

Note that for $z = 1$ we recover the SDD of the Lorentz invariant theory. The model is superrenormalizable when $z > 1$. As expected, the behavior of the theory is improved by the critical exponent z. For example, in the relativistic theory $(z = 1)$ tadpole diagrams with one external leg diverge linearly, but in the LV theory for $z > 3/2$ they converge; also, in the relativistic context graphs with two external legs diverge logarithmically, but in the LV theory with $z > 1$ such diagrams are convergent.

IV. ONE-LOOP CALCULATIONS AND THE KÄHLER POTENTIAL

With the superpropagators written in terms of the projection operators we can determine the Kähler potential. For details in the computation of Kähler potentials see [19–21]. Our calculation follows closely the one presented in [22], but with the modified SUSY in anisotropic space-time. First we need to sum over all one-loop diagrams that have $n - \bar{\Phi}\Phi$ external legs and contain only internal gauge propagators, as shown in Fig. 1, which corresponds to the following series:

$$i\mathcal{K}_a = \int d^8\xi \sum_{n=1}^{\infty} \frac{(-1)^n}{2n} \left(\frac{g^2\bar{\Phi}\Phi}{\tilde{\Box}}(\Pi_{1/2} + \lambda\Pi_0)\right)^n \delta^8(\xi_1 - \xi_2). \tag{43}$$

By carrying out the Fourier transformation, performing a Wick rotation, and taking into account the algebra (21) of the projection operators which satisfy the relations $\tilde{\Box}\Pi_0\delta^8(\xi_1 - \xi_2)|_{\theta_1 = \theta_2} = 2$ and $\tilde{\Box}\Pi_{1/2}\delta^8(\xi_1 - \xi_2)|_{\theta_1 = \theta_2} = -2$, we get

$$\mathcal{K}_a = \int d^8\xi \int \frac{d^4 k_E}{(2\pi)^4} \frac{1}{\tilde{k}_E^2} \left(\ln\left(1 + \frac{g^2\bar{\Phi}\Phi}{\tilde{k}_E^2}\right) - \ln\left(1 + \frac{\lambda g^2\bar{\Phi}\Phi}{\tilde{k}_E^2}\right)\right), \tag{44}$$
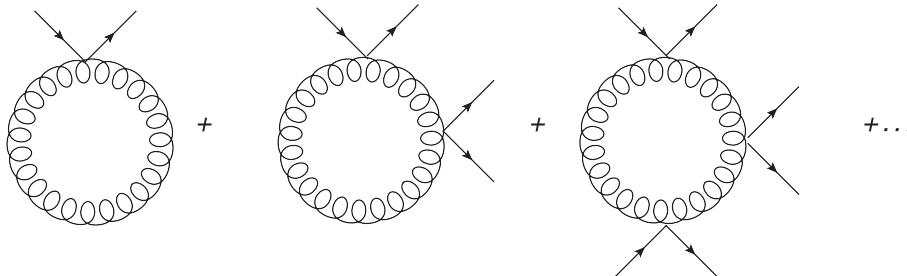
where $\tilde{k}_E^2 = k_0^2 + c^2\mathbf{k}^2 + 2c\eta(\mathbf{k}^2)^{\frac{z+1}{2}} + \eta^2(\mathbf{k}^2)^z$. At the one-loop level, there is yet another family of diagrams contributing to the Kähler potential involving both V and Φ propagators, but, before considering that, we need to add all possible insertions of $\bar{\Phi}\Phi$ external legs in all V-propagators. We can perform this sum by introducing the "dressed" propagator [17], consisting of the VV-propagator with all possible insertions of $\bar{\Phi}\Phi$ external legs (Fig. 2).

The dressed propagator has the following expression:

$$\langle V(\xi_1)V(\xi_2)\rangle_D$$

$$= \langle VV\rangle \sum_{n=0}^{\infty} (g^2\bar{\Phi}\Phi\langle VV\rangle)^n \delta^8(\xi_1 - \xi_2)$$

$$= -\sum_{n=0}^{\infty} (g^2\bar{\Phi}\Phi)^n \left(\frac{1}{\tilde{\Box}}\right)^{n+1} (\Pi_{1/2} + \lambda^{n+1}\Pi_0)\delta^8(\xi_1 - \xi_2)$$

$$= -\left(\frac{1}{\tilde{\Box} + g^2\bar{\Phi}\Phi}\Pi_{1/2} + \frac{\lambda}{\tilde{\Box} + \lambda g^2\bar{\Phi}\Phi}\Pi_0\right)\delta^8(\xi_1 - \xi_2). \tag{45}$$

Now, if we insert the dressed propagator in the one-loop diagrams involving also Φ-propagators, we have the series shown in Fig. 3, which gives

$$i\mathcal{K}_b = \int d^8\xi_1 \sum_{n=1}^{\infty} \frac{1}{2n} (g^2\bar{\Phi}\Phi\Pi_0\langle VV\rangle_D)^n \delta^8(\xi_1 - \xi_2)|_{\theta_1 = \theta_2}, \tag{46}$$

where $\langle VV\rangle_D$ is the dressed propagator excluding the δ factor. By using properties of the projection operators, after performing a Fourier transformation and Wick rotation, we arrive at

$$\mathcal{K}_b = \int d^8\xi \int \frac{d^4 k_E}{(2\pi)^4} \frac{1}{\tilde{k}_E^2} \ln\left(1 + \frac{\lambda g^2\bar{\Phi}\Phi}{\tilde{k}_E^2 + \lambda g^2\bar{\Phi}\Phi}\right). \tag{47}$$

The total Kähler potential is the sum of both contributions, $\mathcal{K}^{\text{one-loop}}(z, \lambda) = \mathcal{K}_a^{\text{one-loop}} + \mathcal{K}_b^{\text{one-loop}}$. For the sake of simplicity, we will work in the Landau gauge, $\lambda = 0$, where the potential results in

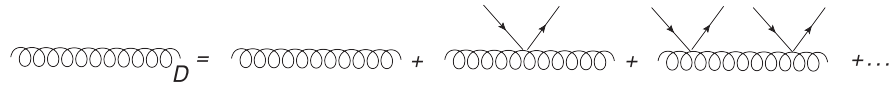FIG. 1.   One-loop gauge field contribution: $\mathcal{K}_a$.

FIG. 2.   Dressed gauge propagator.

$$\mathcal{K} = \int d^8\xi \int \frac{d^4k_E}{(2\pi)^4} \frac{1}{\tilde{k}_E^2} \ln\left(1 + \frac{\mu^2}{\tilde{k}_E^2}\right), \qquad (48)$$

where we defined $\mu^2 = g^2\bar{\Phi}\Phi$. Up to a μ independent constant, by carrying out the integrations on the angles and in k_0, we arrive at

$$\mathcal{K} = \int d^8\xi \left[\frac{1}{2\pi^2}\int_0^\infty dk \frac{k^2}{\Delta} \ln\left(\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{\mu^2}{\Delta^2}}\right)\right], \quad (49)$$

where $\Delta = ck + \eta k^z$. Unless for $z = 1$, the k integral does not have a simple analytic expression. However, by straightforward power counting it is easy to see that it is convergent if $z > 1$ but logarithmically divergent if $z = 1$. Actually, for $z = 1$, using dimensional regularization, we get

$$\mathcal{K}(z = 1, \lambda = 0) = -\frac{1}{16\pi^2(c+\eta)^3} \int d^8\xi \left[\frac{2g^2\Phi\bar{\Phi}}{d-3}\right.$$
$$\left. + g^2\bar{\Phi}\Phi \log\left(\frac{g^2\Phi\bar{\Phi}}{4\pi(c+\eta)^2 e^{2-\gamma}}\right)\right], \quad (50)$$

where, as usual, the divergence manifests itself as a pole in $d = 3$. A mass renormalization is necessary to eliminate the pole $1/(d-3)$ and define the potential.

A. Adding more interactions

Despite the fact that this anisotropic SUSY does not allow for chiral interactions, i.e. terms of the form $\int d^2\theta \mathcal{P}(\Phi) + \text{H.c.}$ ($\mathcal{P}$ being a polynomial of degree greater than 2), it is possible to introduce derivative interactions as D-terms by integrating over the whole Grassmann space. We may add for example a quartic interaction of the form $\int d^2\theta d^2\bar{\theta} D^\alpha\Phi D_\alpha\Phi\bar{D}_{\dot{\beta}}\bar{\Phi}\bar{D}^{\dot{\beta}}\bar{\Phi}$ or $\int d^2\theta d^2\bar{\theta}\,\bar{\Phi}\,\Phi\bar{\Phi}\Phi$. These kind of terms were introduced in [23–26] in the context of supersymmetric galileons, SUSY ghost condensates, higher derivative supergravity, and SUSY Skyrme models, respectively. Of course, they are derivative interactions of at
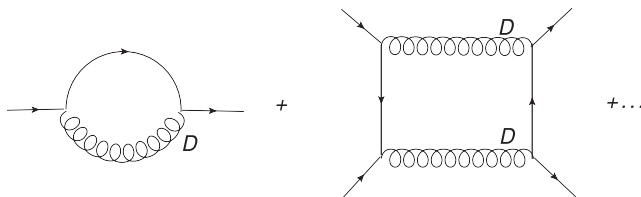
least fourth order, since the integration over the full Grassmann space always generates derivatives. To determine the effective Kähler potential for the quartic self-interaction, we need to add to the effective Kähler potential calculated before one extra family of diagrams consisting of one loop on Φ and n $\bar{\Phi}\Phi$ external legs, corresponding to Fig. 4.

These graphs contribute with

$$i\int d^8\xi \int \frac{d^dk_E}{(2\pi)^d} \frac{1}{\tilde{k}_E^2}\ln(1 + \lambda\bar{\Phi}\Phi), \qquad (51)$$

which gives zero in the context of dimensional regularization. To determine the contribution corresponding to the remaining diagrams, Eqs. (44) and (46), we introduce the dressed $\Phi\bar{\Phi}$-propagator (see Fig. 5),

$$\langle\Phi(\xi_1)\bar{\Phi}(\xi_2)\rangle_D = \frac{1}{1-\lambda\bar{\Phi}\Phi}\Pi_0\delta^8(\xi_1 - \xi_2). \qquad (52)$$

By repeating a calculation similar to the one we did before we find finally

$$\mathcal{K}_a = \int d^8\xi \int \frac{d^4k_E}{(2\pi)^4} \frac{1}{\tilde{k}_E^2}\left(\ln\left(1 + \frac{g^2\bar{\Phi}\Phi}{\tilde{k}_E^2}\right)\right.$$
$$\left. - \ln\left(1 + \frac{\lambda g^2\bar{\Phi}\Phi}{\tilde{k}_E^2}\right)\right), \qquad (53)$$

$$\mathcal{K}_b = \int d^8\xi \int \frac{d^4k_E}{(2\pi)^4} \frac{1}{\tilde{k}_E^2}\ln\left(1 + \frac{\lambda g^2\bar{\Phi}\Phi}{(1-\lambda\bar{\Phi}\Phi)(\tilde{k}_E^2 + \lambda g^2\bar{\Phi}\Phi)}\right). \tag{54}$$

In this case, the effective potential is deformed by the factor $\frac{1}{1-\lambda\bar{\Phi}\Phi}$, but in the gauge where $\lambda \to 0$, the extra interaction $\int d^2\theta d^2\bar{\theta}\,\bar{\Phi}\,\Phi\bar{\Phi}\Phi$ does not affect the one-loop

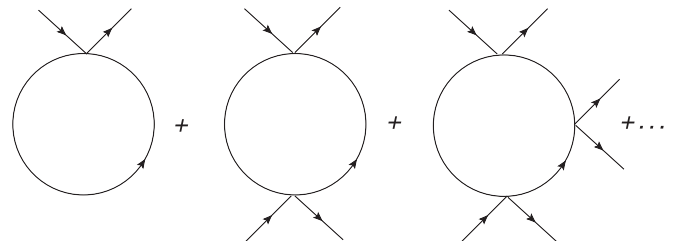FIG. 3.   One-loop gauge-boson contribution: $\mathcal{K}_b$.

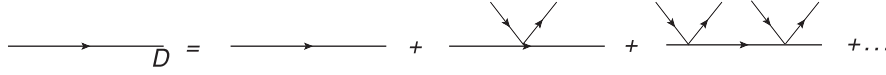FIG. 4.   One-loop self-interacting contribution.

FIG. 5.   Dressed boson propagator.

Kähler potential. The analysis of this situation is completely analog to the model we studied before.

B. Two-point functions: Resaturation of the Lorentz symmetry at low energy and improvement of the UV behavior

In this section we study the one-loop two-point functions of the scalar and vector superfields. We will see that at low energy, the standard Lorentz invariant SUSY Maxwell action is restored. For $z > 1$ the two-point function of the superfields are finite, improving the behavior of the Lorentz invariant case. For simplicity, we will work in the super-Fermi-Feynman gauge, $\lambda = 1$. In this gauge the propagator of the vector superfield can be written as

$$\langle V(\xi_1)V(\xi_2)\rangle = \frac{1}{\tilde{\square}}\delta^8(\xi_1 - \xi_2). \tag{55}$$

(Remember that the tilde notation refers to the deformed quantities.) We start with the one-loop correction to the two-point function of the scalar superfield Φ shown in Fig. 6.

The contribution of the tadpole diagram can be written as follows:

$$\Sigma_2^{\text{tadpole}}(p)$$
$$= -g^2 \int d^4\theta_1 d^4\theta_2 \int \frac{d^4k}{(2\pi)^4} \Phi(-\tilde{p},\theta_1)\bar{\Phi}(\tilde{p},\theta_2)\frac{1}{\tilde{k}^2}\delta_{12}\delta_{12}, \tag{56}$$

which is zero since $\delta_{12}\delta_{12} = 0$, while for the "fish" graph we have

$$\Sigma_2^{\text{fish}}(p) = -g^2 \int d^4\theta \int \frac{d^4k}{(2\pi)^4} \Phi(-\tilde{p},\theta)\bar{\Phi}(\tilde{p},\theta)A(\tilde{p},\tilde{k}), \tag{57}$$

where $A(\tilde{p},\tilde{k}) = \frac{1}{\tilde{k}^2}\frac{1}{(\tilde{k}+\tilde{p})^2}$. The superfields in the expressions above were defined in Eqs. (10), and therefore we can expand them in momentum and truncate the expansion at second order. This means that we are disregarding high
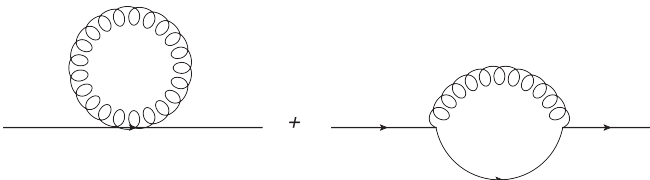
FIG. 6.   One-loop two-point function, $\bar{\Phi}(-p)\Phi(p)$.

momentum contributions, and hence the product or superfield $\bar{\Phi}(-p)\Phi(p)$ becomes Lorentz invariant. Let us call the truncated Lorentz invariant superfield $\Phi_{l.i.}(p)$, and then

$$\bar{\Phi}(-\tilde{p})\Phi(\tilde{p}) = \bar{\Phi}_{\text{l.i.}}(-p)\Phi_{\text{l.i.}}(p) + \mathcal{O}(p^z). \tag{58}$$

We will show that the function $A(\tilde{p},\tilde{k})$ is also Lorentz invariant for low p. First of all, we use the Feynman parametrization to write the integral in the following way:

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{\tilde{k}^2} \frac{1}{(\tilde{k}+\tilde{p})^2}$$
$$= \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{1}{((\tilde{k}+x\tilde{p})^2 - x^2\tilde{p}^2 + x\tilde{p}^2)^2}. \tag{59}$$

We then shift the momentum $\tilde{k}$ as follows:

$$k_0 \to k_0 - x\tilde{p}_0 \tag{60}$$

$$k_i\left[c + \eta\left(\sum_{j=1}^3 k_i^2\right)^{\frac{z-1}{2}}\right] \to k_i - x\tilde{p}_i. \tag{61}$$

Note that $\tilde{k}_i = k_i[c + \eta(\sum_{j=1}^3 k_i^2)^{\frac{z-1}{2}}]$. We arrive finally at

$$\int \frac{d^4k}{(2\pi)^4}A(\tilde{p},\tilde{k}) = \int_0^1 dx \int \frac{d^4k}{(2\pi)^4}|J^{-1}|\frac{1}{(k^2 - x^2\tilde{p}^2 + x\tilde{p}^2)^2}, \tag{62}$$

where $|J^{-1}|$ is the inverse Jacobian

$$|J| = c^3 + c^2(2+z)\eta|\bar{k}^2|^{\frac{z-1}{2}} + c(1+2z)|\bar{k}^2|^{\frac{z-1}{2}} + z\eta^3|\bar{k}^2|^{\frac{3(z-1)}{2}}. \tag{63}$$

Now, at low external momentum, after disregarding higher powers in p, we have $\tilde{p}^2 = p^2 + \mathcal{O}(p^z)$, being p^2 a Lorentz scalar. We arrive finally at

$$\Sigma_2(p) = -g^2 \int d^4\theta \bar{\Phi}(-p,\theta)_{\text{l.i.}}\Phi(p,\theta)_{\text{l.i.}}$$
$$\times \int_0^1 dx \int \frac{d^4k}{(2\pi)^4}|J^{-1}|\frac{1}{(k^2 - x^2p^2 + xp^2)^2}$$
$$+ (\text{high momentum}). \tag{64}$$

We observe that the two-point function for Φ is Lorentz invariant at one-loop order and therefore, in the low energy limit the usual quadratic Lorentz invariant action for the chiral superfield is generated. Moreover, for $z > 1$ the
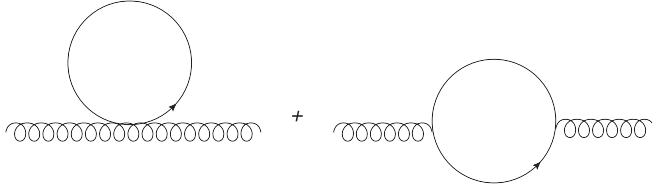
FIG. 7.   One-loop two-point function, $V(-p)V(p)$.

integral in k is convergent and the one-loop two-point function is finite. We can follow the same procedure to obtain the two-point function of the vector superfield V.

The diagrams which contribute at one-loop order are shown in Fig. 7.

The contribution of the tadpole diagram can be written as follows:

$$\Omega_2^{\text{tadpole}}(p) = g^2 \int d^4\theta_1 d^4\theta_2 \frac{d^4k}{(2\pi)^4} V(-p,\theta_1)\delta_{12}$$
$$\times \frac{D^2\bar{D}^2}{16\tilde{k}^2}\delta_{12}V(p,\theta_2), \tag{65}$$

and for the fish diagram,

$$\Omega_2^{\text{fish}}(p) = \frac{g^2}{2} \int d^4\theta_1 d^4\theta_2 \frac{d^4k_E}{(2\pi)^4} V(-p,\theta_1)$$
$$\times \frac{\bar{D}^2 D^2}{16\tilde{k}^2}\delta_{12} \frac{D^2\bar{D}^2}{16(\tilde{k}+\tilde{p})^2}\delta_{12}V(p,\theta_2). \tag{66}$$

Again we expand the vector superfield in powers of the momentum p and disregard terms with powers greater than two. After that, the expansion of the term $V(-p,\theta)V(p,\theta)$ becomes the Lorentz invariant one (we call the truncated superfield $V_{1.i.}$). After integration by parts (see for example [21]) the tadpole contribution is compensated by part of (66). The remaining contribution can be written as follows:

$$\Omega_2(p) = \frac{g^2}{2} \int d^4\theta V_{1.i.}(-p,\theta)D^\alpha \bar{D}^2 D_\alpha V_{1.i.}(p,\theta)$$
$$\times \int_0^1 dx \int \frac{d^4k}{(2\pi)^4}|J^{-1}| \frac{1}{(k^2 - x^2p^2 + xp^2)^2}$$
$$+ (\text{high momentum}), \tag{67}$$

and it is, of course, Lorentz invariant. Therefore we have shown that in the low energy limit the Lorentz invariance is restored and the usual supersymmetric Maxwell action is generated.

C. Higher anisotropic space

There is another interesting situation depending on the value of the critical exponent z. Let us analyze the behavior of the effective Kähler potential as z increases. The intermediate cutoff tends to c, and therefore the low energy contribution to the potential is always convergent, while the high energy contribution goes to 0 [this can be deduced directly from expression (48)]. In the limit $z \to \infty$ we get

$$\lim_{z\to\infty}\mathcal{K}(z,0) = \frac{(2c^2 + \mu^2)\log\left(\sqrt{\frac{c^2}{\mu^2}+1}+\frac{c}{\mu}\right) + c(c\log(\mu^2) - \sqrt{c^2 + \mu^2})}{8\pi^2 c^3}, \tag{68}$$

and hence in this limiting case, the Kähler potential can be calculated exactly and behaves as the low energy Lorentz invariant one. Moreover, we find a similar behavior for the two-point functions of scalar and vector superfields. In this case, the integration over the internal momenta in the loop is performed over the compact region $[0, c]$, the propagators are the Lorentz invariant ones (out of this region the contribution to the two-point function is zero), and as a consequence all divergences disappear. In general all n-amplitudes or the $z = \infty$ theory is determined by the usual Lorentz invariant propagators, such that every integration over internal momenta must be performed in the region $[0, c]$.

V. SUMMARY

In this work we studied a Lorentz-violating superalgebra constructed with higher spatial derivative operators. With this modified structure, we supersymmetrized a Lorentz-violating SUSY scalar QED theory. After the modification of the SUSY algebra we were able to construct terms of the form $F_{0i}F_{0i} - F_{ij}\mathcal{O}F_{ij}$ (being $\mathcal{O}$ some function of the Laplacian operator), although the superderivatives and supercharges do not verify the Leibniz rule. We showed how the UV behavior is improved, and we analyzed it with the explicit calculation of the superficial degree of divergence.

We also obtained the one-loop Kähler potential for arbitrary critical exponent z. We analyzed the UV behavior of the theory as well as the renormalization improvement and proved that for $z > 1$ the effective Kähler potential is free of divergences. We studied different limits and showed how the theory naturally flows to the Lorentz invariant situation as z approaches one. We furthermore explore the possibility of more general interactions despite the fact that self-interactions of chiral and antichiral fields must be

integrated in the whole Grassmann space and therefore contains derivative interactions in the component fields.

We analyzed the two-point functions of the chiral and vector superfields and showed that, in the low energy limit, a Lorentz invariant supersymmetric QED is generated, for $z > 1$ the two-point functions of chiral and vector superfields being free of divergences.

ACKNOWLEDGMENTS

APPENDIX: TWO-POINT SUPERFIELD GREEN'S FUNCTIONS

The quadratic action involving chiral superfields and chiral sources has the following form:

$$S_{J,\text{chiral}}[\tilde{\Phi}, \bar{\tilde{\Phi}}, J, \bar{J}] = \int d^8\xi \tilde{\Phi} \bar{\tilde{\Phi}} + \left(\int d^6\xi \tilde{\Phi} J + \text{H.c.} \right),$$
(A1)

where all quantities are understood to be the tilde ones with derivatives and superfields defined in Sec. II. We can write the previous action in terms of the measure $d^6\xi$ by using the identity

$$\int d^8\xi \tilde{\Phi} \bar{\tilde{\Phi}} = \left(\int d^6\xi \frac{1}{2} \tilde{\Phi} \left(-\frac{\bar{\tilde{D}}^2}{4} \right) \bar{\tilde{\Phi}} + \text{H.c.} \right),$$
(A2)

which also holds for the usual Lorentz invariant SUSY. Therefore,

$$S_{J,\text{chiral}}[\tilde{\Phi}, \bar{\tilde{\Phi}}, J, \bar{J}] = \int d^6\xi_1 d^6\xi_2 \begin{bmatrix} \tilde{\Phi}(\xi_1) & \bar{\tilde{\Phi}}(\xi_1) \end{bmatrix} \Delta \delta \begin{bmatrix} \tilde{\Phi}(\xi_2) \\ \bar{\tilde{\Phi}}(\xi_2) \end{bmatrix},$$
(A3)

where

$$\Delta = \begin{pmatrix} 0 & -\frac{1}{4}\bar{\tilde{D}}^2 \\ -\frac{1}{4}\tilde{D}^2 & 0 \end{pmatrix}$$
(A4)

and

$$\delta = \begin{pmatrix} -\frac{1}{4}\tilde{D}^2 \delta^8(\xi_1 - \xi_2) & 0 \\ 0 & -\frac{1}{4}\bar{\tilde{D}}^2 \delta^8(\xi_1 - \xi_2) \end{pmatrix}.$$
(A5)

Notice that everywhere appear the modified superderivatives. After integration over the chiral superfields we obtain the following partition function:

$$Z[J, \bar{J}] = \det^{-1/2}\Delta \exp\left\{ -\frac{i}{2} \int d^6\xi_1 d^6\xi_2 \right.$$
$$\left. \times \begin{bmatrix} J(\xi_1) & \bar{J}(\xi_1) \end{bmatrix} \Delta^{-1}\delta \begin{bmatrix} J(\xi_2) \\ \bar{J}(\xi_2) \end{bmatrix} \right\},$$
(A6)

which allow us to determine the two-point Green function,

$$G(\xi_1, \xi_2) = \frac{1}{i^2} \frac{\delta^2 Z[J, \bar{J}]}{\delta J(\xi_1)\delta\bar{J}(\xi_2)}$$
$$= -i \left(-\frac{1}{4} \right)^2 \frac{\bar{\tilde{D}}_1^2 \bar{\tilde{D}}_2^2}{\tilde{\Box}} \delta^8(\xi_1 - \xi_2).$$
(A7)

By replacing the tilde quantities by the usual one, this expression coincides with the known Lorentz invariant expression. Similarly, we may obtain the vector propagators corresponding to (33).

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