Interference phenomena in the dynamical Casimir effect for a single mirror with Robin conditions

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(Received 22 May 2015; published 27 July 2015)

In the literature, the interference phenomenon in the particle creation via the dynamical Casimir effect is investigated for cavities with two moving mirrors. Here, considering the Robin boundary condition (BC), we investigate the interference phenomenon produced by just a single moving mirror. Specifically, we consider a real massless scalar field in $1+1$ dimensions submitted to a Robin BC with a time-dependent Robin parameter at the instantaneous position of a moving mirror, and compute the expressions for the spectral distribution and the rate of created particles. These expressions, which include interference terms, generalize those found in the literature related to the isolated effects of a Robin BC with a time-dependent Robin parameter for a fixed mirror, or a Robin BC with a time-independent Robin parameter for a moving mirror. Differently from models where the problem of interference in the dynamical Casimir effect is considered for cavities with two Dirichlet moving mirrors, in the present model the spectrum is a continuum, and the interference pattern exhibits new features, in the sense that different regions of the spectrum can be affected in different manners by constructive or destructive effects. Furthermore, we also investigate interference in the context of superconducting circuits.

DOI: 10.1103/PhysRevD.92.025040 PACS numbers: 03.70.+k, 11.10.-z, 42.50.Lc

I. INTRODUCTION

The particle creation from a vacuum induced by the interaction of uncharged mirrors, whose position or material properties vary in time, with quantized fields is frequently called the dynamical Casimir effect (DCE), a denomination introduced by Yablonovitch [1] and reinforced by Schwinger [2]. This phenomenon was first investigated in the pioneering works of Moore [3], DeWitt [4], Fulling and Davies [5]. A large number of DCE theoretical predictions have been presented in the last 45 years (see Ref. [6] for more details). The first observation of the DCE was performed by Wilson and collaborators [7,8]. They considered a time-dependent magnetic flux applied in a superconducting circuit, which consists in a coplanar waveguide (transmission line) with a superconducting quantum interference device (SQUID) at one of the extremities, changing the inductance of the SQUID. This configuration simulates a moving mirror whose effective velocity is a non-neglected fraction of the speed of light, leading to a measurable particle creation rate. The second observation of the DCE was announced by Lähteenmäki *et al.* [9], and other experimental proposals for the observation of the DCE can be found in Refs. [10–13].

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In the theoretical model of the first observation of the DCE [8], the Robin boundary condition (BC) appears naturally. In such model, the electromagnetic field in the coplanar waveguide is described by a phase field operator represented by a scalar field $\phi(t,x)$ which obeys the wave equation in $1+1$ dimensions [8]. Appropriate Kirchhoff laws applied to the superconducting circuit leads to the BC

$$\phi(t,0) = \gamma(t)(\partial_x \phi)(t,0), \tag{1}$$

which corresponds to the Robin BC with a time-dependent Robin parameter $\gamma(t)$, for a fixed mirror. This BC was also considered in the context of the DCE in Refs. [14–17].

The DCE in the context of a Robin BC was first investigated in Refs. [18,19], where the authors considered a massless scalar field $\phi(t,x)$ submitted to a Robin BC with a time-independent Robin parameter γ_0 at the instantaneous position of a moving mirror, observed from the point of view of an inertial frame where the mirror is instantaneously at rest (called tangential frame), namely,

$$\phi'(t',0) = \gamma_0(\partial_{x'}\phi')(t',0), \tag{2}$$

where the prime superscripts means that the BC is taken in the tangential frame. For particular values of γ_0 it has been shown the occurrence of a drastic reduction in the particle creation rate, if compared to the similar problem with Dirichlet or Neumann BC, indicates a strong decoupling between mirror and field [18,19]. This inhibition of the DCE with Robin BC also occurs in $3+1$ dimensions [20].

Here, we investigate the DCE in the context of a single moving mirror, but with two sources of particle creation. Specifically, we consider a real massless scalar field in $1+1$ dimensions submitted to a Robin BC with a time-dependent Robin parameter at the instantaneous position of a moving mirror, and compute the spectral distribution and the rate of created particles. This situation is described by the following Robin BC,

$$\phi'(t',0) = \gamma'(t')(\partial_{x'}\phi')(t',0). \tag{3}$$

This Robin BC (3) is more general than those shown in Eqs. (1) and (2), in the sense that a moving mirror with properties varying in time acts as two simultaneous and distinct sources of particle creation, which can produce interference effects, generalizing the models considered in Refs. [14,19].

In the literature, interference phenomena in the DCE have been investigated for oscillating cavities with two moving mirrors [21–24]. In these articles, constructive and destructive interference in the spectral distribution of created particles are exhibited, depending on the relations among amplitude, oscillation frequency, and phase difference. Specifically, in Ref. [21], the authors considered a cavity with two oscillating Dirichlet mirrors, showing that the discrete spectrum N_k of created particles for the kth mode in an oscillating cavity is given by

$$N_k = N_k^L + N_k^R + (-1)^{s+1} 2\sqrt{N_k^L N_k^R}\cos\beta, \tag{4}$$

where N_k^L and N_k^R are, respectively, the spectrum generated by the left and the right mirror individually, s is an integer number (defined by the ratio between the oscillation frequency of the mirrors and the fundamental mode-frequency of the field within the cavity [21]), and β is the phase difference. We remark that, according to Eq. (4), the interference effect does not depend on k, thus the interference effect for a given created mode will be the same for any other created mode.

As mentioned in Ref. [21], except for the factor $(-1)^{s+1}$, the Eq. (4) for the created particles resembles the well-known formula for the wave intensity I in the case of the double-slit interference experiment, given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta, \tag{5}$$

where I_1 and I_2 are the intensities of the waves from each slit, and δ is the phase difference between the amplitudes. We can see this resemblance as a manifestation of the wave-particle duality.

In the present paper, we obtain the formula correspondent to Eq. (4) but for the case of a real massless scalar field submitted to the BC (3) at a single moving mirror. Differently from Eq. (4), in the model discussed here the spectrum is a continuum, and the interference pattern exhibits new features, in the sense that different regions of the created spectrum can be affected in different manners by constructive or destructive effects. We also discuss the problem of the interference in the context of the SQUID experiment [7,8], specifically considering the Eq. (1), but with $\gamma(t)$ associated with the presence of two sources of magnetic flux, different in phase and frequency.

The paper is organized as follows. In Sec. II, we use the scattering approach [25–27] in order to obtain the scattering matrix for the problem, which is used in the computation of the spectrum of the created particles. In Sec. III, we compute this spectrum considering typical functions for the moving mirror and time variation of the Robin parameter, and we also discuss the presence of constructive and destructive interference, comparing our results with those found in the literature. In Sec. IV, we investigate the problem of the interference in the context of the SQUID experiment. Final remarks are presented in Sec. V. Throughout this paper we consider $c = \hbar = 1$.

II. THE SCATTERING MATRIX

We start by considering a real and massless scalar field obeying the Klein-Gordon equation in $1+1$ dimensions,

$$(\partial_t^2 - \partial_x^2)\phi(t,x) = 0. \tag{6}$$

This field corresponds to the sum of two counterpropagating fields, namely,

$$\phi(t,x) = \tilde{\varphi}(t-x) + \tilde{\psi}(t+x). \tag{7}$$

Conveniently, these components of the field can be grouped in a column matrix

$$\tilde{\Phi}(t,x) = \begin{pmatrix} \tilde{\varphi}(t-x) \\ \tilde{\psi}(t+x) \end{pmatrix}. \tag{8}$$

For the case of a free field, the solution of Eq. (6) is given by

$$\phi(t,x) = \int \frac{\mathrm{d}\omega}{\sqrt{4\pi|\omega|}}[a(\omega)\mathrm{e}^{-i|\omega|t}\mathrm{e}^{i\omega x} + \mathrm{H.c.}], \tag{9}$$

where H.c. represents the Hermitian conjugate of the previous term, and $a(\omega)$ is the annihilation operator which obeys the relation $[a(\omega), a^\dagger(\omega')] = \delta(\omega - \omega')$. It is convenient to rewrite Eq. (9) as

$$\phi(t,x) = \int \frac{\mathrm{d}\omega}{\sqrt{2\pi}}[\varphi(\omega)\mathrm{e}^{-i\omega(t-x)} + \psi(\omega)\mathrm{e}^{-i\omega(t+x)}], \tag{10}$$

where the operators $\varphi(\omega)$ and $\psi(\omega)$ are given by

$$\varphi(\omega) = \frac{1}{\sqrt{2|\omega|}}[\Theta(\omega)a(\omega) + \Theta(-\omega)a^\dagger(-\omega)], \tag{11}$$

$$\psi(\omega) = \frac{1}{\sqrt{2|\omega|}}[\Theta(\omega)a(-\omega) + \Theta(-\omega)a^\dagger(\omega)], \quad (12)$$

and $\Theta(\omega)$ is the Heaviside step function. In the presence of a static mirror fixed at $x = 0$, the field can be rewritten as

$$\phi(t,x) = \Theta(x)\phi_R(t,x) + \Theta(-x)\phi_L(t,x), \quad (13)$$

where ϕ_R (ϕ_L) is the field in the right (left) side of the mirror. Both ϕ_R and ϕ_L obey Klein-Gordon equation and can be written as

$$\phi_R(t,x) = \int \frac{d\omega}{\sqrt{2\pi}}[\varphi_{out}(\omega)e^{i\omega x} + \psi_{in}(\omega)e^{-i\omega x}]e^{-i\omega t}, \quad (14)$$

$$\phi_L(t,x) = \int \frac{d\omega}{\sqrt{2\pi}}[\varphi_{in}(\omega)e^{i\omega x} + \psi_{out}(\omega)e^{-i\omega x}]e^{-i\omega t}, \quad (15)$$

where the subscripts *in* and *out* represent the incoming and the outgoing fields, respectively.

As required by causality, the incoming fields are the free fields described by Eqs. (11) and (12). The outgoing fields correspond to the incoming ones modified by the interaction with the mirror. In order to obtain the outgoing fields, following the Refs. [25–27], we consider the linear relation

$$\Phi_{out}(\omega) = S(\omega)\Phi_{in}(\omega), \quad (16)$$

where

$$\Phi_{out}(\omega) = \begin{pmatrix} \varphi_{out}(\omega) \\ \psi_{out}(\omega) \end{pmatrix}, \qquad \Phi_{in}(\omega) = \begin{pmatrix} \varphi_{in}(\omega) \\ \psi_{in}(\omega) \end{pmatrix}, \quad (17)$$

and $S(\omega)$ is a 2×2 unitary matrix denominated scattering matrix (or S-matrix). In this sense, the outgoing fields are completely described if the S-matrix is known. For a static and perfect reflecting mirror, the outgoing fields coincide with the incoming ones except for a phase factor, namely,

$$\varphi_{out}(\omega) = e^{i\theta(\omega)}\psi_{in}(\omega), \qquad \psi_{out}(\omega) = e^{-i\theta(\omega)}\varphi_{in}(\omega). \quad (18)$$

For this case the S-matrix is given by

$$S(\omega) = \begin{pmatrix} 0 & e^{i\theta(\omega)} \\ e^{-i\theta(\omega)} & 0 \end{pmatrix}. \quad (19)$$

The elements in the secondary diagonal of the S-matrix are the reflection coefficients of the mirror, which are unitary in modulus since the mirror is perfect reflecting. The main diagonal is null for the same reason.

Heretofore, the BC has not been specified and the reflecting coefficients [Eq. (19)] represent a generic perfect mirror. As a particular application, let us consider the Robin BC with a time-independent Robin parameter,

$$\phi(t,0) = \gamma_0(\partial_x\phi)(t,0). \quad (20)$$

In this case, it is straightforward to show that

$$e^{i\theta(\omega)} = -\frac{1 + i\gamma_0\omega}{1 - i\gamma_0\omega}. \quad (21)$$

As expected from the Robin BC, when $\gamma_0 \to 0$ the reflection coefficients equivalent to the Dirichlet BC are recovered [$\theta(\omega) = \pi$], and when $\gamma_0 \to \infty$ the recovered reflection coefficients are those equivalent to the Neumann BC [$\theta(\omega) = 0$].

From this point, we shall apply the same scattering procedure for the DCE. First, let us derive the S-matrix for the case of a mirror fixed at $x = 0$ imposing to the field the Robin BC with a time-dependent Robin parameter, namely,

$$\phi(t,0) = \gamma(t)(\partial_x\phi)(t,0). \quad (22)$$

For the sake of simplicity, we assume that $\gamma(t)$ slightly oscillates around a constant value γ_0 with a small amplitude $\epsilon\tilde{\gamma}_0$, thus

$$\gamma(t) = \gamma_0 + \epsilon\tilde{\gamma}_0 f(t), \quad (23)$$

with $|f(t)| \leq 1$ and $0 < \epsilon < 1$. Applying Eq. (22) in Eqs. (14) and (15) and neglecting terms $\mathcal{O}(\epsilon^2)$ we obtain the following expression in the Fourier domain,

$$\Phi_{out}(\omega) = S(\omega)\Phi_{in}(\omega) + \int \frac{d\Omega}{2\pi}\delta\mathcal{S}_\Gamma(\omega,\Omega)\Phi_{in}(\Omega), \quad (24)$$

where

$$\delta\mathcal{S}_\Gamma(\omega,\Omega) = -i\epsilon\tilde{\gamma}_0\Omega F(\omega - \Omega)[S(\omega)\eta_\omega - \eta_\omega S(\Omega)], \quad (25)$$

$$\eta_\omega = \frac{\eta_0 + i\gamma_0\omega}{1 + \gamma_0^2\omega^2}, \qquad \eta_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (26)$$

and $F(\omega)$ is the Fourier transform of $f(t)$.

We have calculated in Eq. (24) a first-order correction for the S-matrix due to the time-dependent Robin parameter, for a fixed mirror. Now, let us proceed to the case of a time-dependent Robin parameter, but for a moving mirror. This situation is described by the BC shown in Eq. (3). The movement of the mirror, represented by $q(t)$, is set as nonrelativistic ($|\dot{q}(t)| \ll 1$) and oscillatory with small amplitude $\epsilon\tilde{q}_0$, thus

$$q(t) = \epsilon\tilde{q}_0 g(t), \tag{27}$$

with $|g(t)| \leq 1$. In this approximation, we consider that the Lorentz transformation does not affect the Robin parameter, then $\gamma'(t') = \gamma(t)$. We assume that the relation between output and input fields in the tangential frame is also described by the Eq. (24), namely,

$$\Phi'_{\rm out}(\omega) = S'(\omega)\Phi'_{\rm in}(\omega) + \int \frac{d\Omega}{2\pi}\delta\mathcal{S}_\Gamma(\omega,\Omega)\Phi'_{\rm in}(\Omega). \tag{28}$$

Let us seek for the expression of $\Phi'_{\rm out}$ and $\Phi'_{\rm in}$ in the laboratory frame. In order to calculate the needed Lorentz transformation, we start from the relation

$$\tilde{\Phi}'(t',0) = \tilde{\Phi}(t, \epsilon\tilde{q}_0 g(t)), \tag{29}$$

which can be rewritten as

$$\tilde{\Phi}'(t',0) = [1 - \epsilon\tilde{q}_0 g(t)\eta_0\partial_t]\tilde{\Phi}(t,0) + \mathcal{O}(\epsilon^2), \tag{30}$$

and likewise

$$dt' = \sqrt{1 - \epsilon^2\tilde{q}_0^2\dot{g}^2(t)}dt = dt + \mathcal{O}(\epsilon^2). \tag{31}$$

Since the movement is nonrelativistic, the terms $\mathcal{O}(\epsilon^2)$ are negligible and, consequently, the time in laboratory frame coincides with the time in tangential frame. Thus, we shall rewrite the Eq. (30) replacing t' by t:

$$\tilde{\Phi}'(t,0) = [1 - \epsilon\tilde{q}_0 g(t)\eta_0\partial_t]\tilde{\Phi}(t,0). \tag{32}$$

In the Fourier domain the previous equation becomes

$$\Phi'(\omega) = \Phi(\omega) + \epsilon\int\frac{d\Omega}{2\pi}i\eta_0\Omega\tilde{q}_0 G(\omega - \Omega)\Phi(\Omega), \tag{33}$$

where $G(\omega)$ is the Fourier transform of $g(t)$, $\Phi(\omega)$, and $\Phi'(\omega)$ are short notations for $\Phi(\omega,0)$ and $\Phi'(\omega,0)$

Applying Eq. (33), properly indexed with "out" or "in," in Eq. (28) and neglecting terms $\mathcal{O}(\epsilon^2)$ we obtain

$$\Phi_{\rm out}(\omega) = S(\omega)\Phi_{\rm in}(\omega) + \int\frac{d\Omega}{2\pi}\delta\mathcal{S}_\Gamma(\omega,\Omega)\Phi_{\rm in}(\Omega) + \int\frac{d\Omega}{2\pi}\delta\mathcal{S}_Q(\omega,\Omega)\Phi_{\rm in}(\Omega)q, \tag{34}$$

where $\delta\mathcal{S}_\Gamma(\omega,\Omega)$ is given by Eq. (25) and

$$\delta\mathcal{S}_Q(\omega,\Omega) = i\epsilon\tilde{q}_0\Omega G(\omega - \Omega)[S(\omega)\eta_0 - \eta_0 S(\Omega)]. \tag{35}$$

The Eq. (34) contains two first-order corrections for the S-matrix, one due to the time-dependent properties of the mirror, other due to the movement. With the aid of this expression, we shall compute the spectrum of particles creation in next section.

III. INTERFERENCE FOR A MOVING MIRROR WITH A TIME-DEPENDENT ROBIN PARAMETER

The total number of created particles for the problem under investigation is

$$\mathcal{N} = \int_0^\infty\frac{d\omega}{2\pi}N(\omega), \tag{36}$$

where $N(\omega)$ is the spectral distribution of created particles, given by [25–27]

$$N(\omega) = 2\omega{\rm Tr}[\langle 0_{\rm in}|\Phi_{\rm out}(-\omega)\Phi^{\rm T}_{\rm out}(\omega)|0_{\rm in}\rangle], \tag{37}$$

and the input field is assumed to be in the vacuum state. Inserting Eq. (34) in Eq. (37) and considering the formula

$$\langle 0_{\rm in}|\Phi_{\rm in}(\omega)\Phi^{\rm T}_{\rm in}(\omega')|0_{\rm in}\rangle = \frac{\pi}{\omega}\delta(\omega+\omega')\Theta(\omega), \tag{38}$$

obtained from Eqs. (11) and (12), it is straightforward to show that [27],

$$N(\omega) = \int_0^\infty\frac{d\omega'}{2\pi}\frac{\omega}{\omega'}{\rm Tr}[\delta\mathcal{S}(\omega,-\omega')\delta\mathcal{S}^\dagger(\omega,-\omega')], \tag{39}$$

where $\delta\mathcal{S} = \delta\mathcal{S}_\Gamma + \delta\mathcal{S}_Q$.

The last equation leads to an expression for the particle creation spectrum composed by the sum of three different terms, namely,

$$N(\omega) = N_\gamma(\omega) + N_q(\omega) + N_{\rm int}(\omega). \tag{40}$$

The first one is given by

$$N_\gamma(\omega) = \frac{8\omega\epsilon^2\tilde{\gamma}_0^2}{1+\gamma_0^2\omega^2}\int_0^\infty\frac{d\Omega}{2\pi}\frac{\Omega}{1+\gamma_0^2\Omega^2}|F(\omega+\Omega)|^2, \tag{41}$$

which is equivalent to the spectrum of particles created by a fixed mirror with time-dependent properties that imposes the BC given by Eq. (1) to the field. It is in concordance with Ref. [14]. The second one,

$$N_q(\omega) = \frac{8\omega\epsilon^2\tilde{q}_0^2}{1+\gamma_0^2\omega^2}\int_0^\infty\frac{d\Omega}{2\pi}\frac{\Omega(1-\gamma_0^2\omega\Omega)^2}{1+\gamma_0^2\Omega^2}|G(\omega+\Omega)|^2, \tag{42}$$

is equivalent to the spectrum of particles created by a moving mirror that imposes the BC given by Eq. (2) to the field. This formula is in agreement with the results of Ref. [19]. The last term, $N_{\rm int}(\omega)$, is a new term which accomplishes the interference effect, given by

$$N_{\rm int}(\omega) = -\frac{8\omega\epsilon^2\tilde{\gamma}_0\tilde{q}_0}{1+\gamma_0^2\omega^2}\int_0^\infty \frac{d\Omega}{2\pi}\frac{\Omega(1-\gamma_0^2\omega\Omega)}{1+\gamma_0^2\Omega^2} \times 2{\rm Re}[G(\omega+\Omega)F^*(\omega+\Omega)]. \quad (43)$$

This term is originated by the fact that a moving mirror with time-dependent properties works as two distinct sources of particle creation. Notice that constructive or destructive interference patterns can be produced, depending on the movement and on the time-dependent Robin parameter.

In order to investigate the influence of the interference term in the spectrum of created particles, we shall consider the typical functions

$$f(t) = \cos(\omega_1 t)\exp(-|t|/\tau), \quad (44)$$

$$g(t) = \cos(\omega_2 t + \vartheta)\exp(-|t|/\tau), \quad (45)$$

where τ is the time for which the oscillation occurs effectively; ω_1 and ω_2 represent the characteristic frequencies; ϑ is a phase constant. The Fourier transforms of $f(t)$ and $g(t)$ are given, respectively, by

$$F(\omega) = \frac{2\tau[1+\tau^2(\omega^2+\omega_1^2)]}{[1+(\omega-\omega_1)^2\tau^2][1+(\omega+\omega_1)^2\tau^2]}, \quad (46)$$

$$G(\omega) = \frac{2\tau[1+\tau^2(\omega^2+\omega_2^2)]\cos\vartheta + 4i\omega\omega_2\tau^3\sin\vartheta}{[1+(\omega-\omega_2)^2\tau^2][1+(\omega+\omega_2)^2\tau^2]}. \quad (47)$$

Hereafter, using Eqs. (46) and (47), we analyze the behavior of $N_\gamma(\omega)/\tau$, $N_q(\omega)/\tau$ and $N_{\rm int}(\omega)/\tau$ in the monochromatic limit, $\omega_1\tau \gg 1$ and $\omega_2\tau \gg 1$ (see Appendix). Considering Eqs. (A1) and (A2), one gets

$$\frac{N_\gamma(\omega)}{\tau} = \frac{\epsilon^2}{\pi}\frac{\tilde{\gamma}_0^2\omega(\omega_1-\omega)\Theta(\omega_1-\omega)}{(1+\gamma_0^2\omega^2)(1+(\omega_1-\omega)^2\gamma_0^2)}, \quad (48)$$

which is in agreement with Ref. [14], and

$$\frac{N_q(\omega)}{\tau} = \frac{\epsilon^2}{\pi}\frac{\tilde{q}_0^2\omega(\omega_2-\omega)[1-\omega(\omega_2-\omega)\gamma_0^2]^2\Theta(\omega_2-\omega)}{(1+\gamma_0^2\omega^2)(1+(\omega_2-\omega)^2\gamma_0^2)}, \quad (49)$$

which is in accordance with Ref. [19]. The interference term requires a more careful analysis, because its behavior depends on the relation between ω_1 and ω_2. When $\omega_1 \neq \omega_2$, the terms involving ${\rm Re}[G(\omega)F^*(\omega)]/\tau$ behaves as shown in Eq. (A3), what leads to

$$N_{\rm int}(\omega)/\tau = 0, \qquad (\omega_1 \neq \omega_2). \quad (50)$$

On the other hand, when $\omega_1 = \omega_2 = \omega_0$ the terms involving ${\rm Re}[G(\omega)F^*(\omega)]/\tau$ behaves as shown in Eq. (A4), so that

$$\frac{N_{\rm int}(\omega)}{\tau} = \frac{-2\epsilon^2\tilde{\gamma}_0\tilde{q}_0}{\pi}\frac{\omega(\omega_0-\omega)[1-\omega(\omega_0-\omega)\gamma_0^2]}{(1+\gamma_0^2\omega^2)(1+(\omega_0-\omega)^2\gamma_0^2)} \times \cos\vartheta\,\Theta(\omega_0-\omega), \qquad (\omega_1=\omega_2=\omega_0). \quad (51)$$

Notice that the Eq. (51) can be rewritten in terms of $N_q(\omega)$ and $N_\gamma(\omega)$ as

$$N_{\rm int}(\omega) = -{\rm sgn}[I(\omega)]2\sqrt{N_q(\omega)N_\gamma(\omega)}\cos\vartheta, \quad (52)$$

where $I(\omega) \coloneqq 1-\omega(\omega_0-\omega)\gamma_0^2$. Next, we will discuss some features of this interference term.

The presence of the frequency-dependent term $I(\omega)$ produces an interesting effect: different regions of the spectrum can be affected in different manners by constructive or destructive interference. For $\gamma_0\omega_0 \le 2$, the function $I(\omega)$ presents the same sign for any ω and, therefore, the interference effect presents the same nature for any frequency: destructive if $0 \le \vartheta < \pi/2$ and constructive if $\pi/2 < \vartheta \le \pi$ (see dotted, asterisk and dash-dotted lines in Fig. 1). On the other hand, for $\gamma_0\omega_0 > 2$ the term $I(\omega)$ presents two real roots (symmetrical with respect to $\omega_0/2$) given by

$$\omega_\pm = \frac{\gamma_0^2\omega_0 \pm \sqrt{\gamma_0^4\omega_0^2 - 4\gamma_0^2}}{2\gamma_0^2}. \quad (53)$$

If $0 \le \vartheta < \pi/2$ the interference is destructive for $\omega < \omega_-$ and $\omega > \omega_+$, and constructive for $\omega_- < \omega < \omega_+$ (see, for instance, the long-dashed line in Fig. 1); if $\pi/2 < \vartheta \le \pi$, the opposite behavior occurs. The limiting case $\gamma_0\omega_0 \gg 2$ leads to $\omega_- \to 0$ and $\omega_+ \to \omega_0$ (see space-dashed and solid lines in Fig. 1), so that the nature of the interference effect

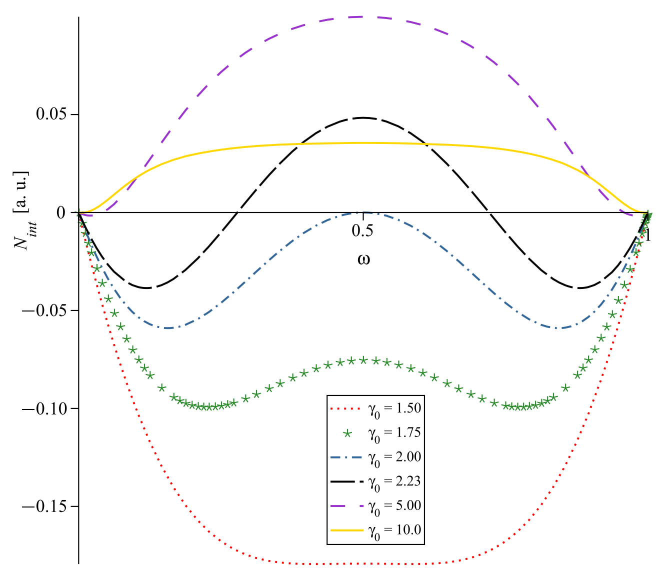

FIG. 1 (color online). $N_{\rm int}(\omega)/(2\epsilon^2\tau\tilde{\gamma}_0\tilde{q}_0\pi^{-1})$ as a function of ω for some values of γ_0, considering $\vartheta = 0$ and $\omega_0 = 1$.

tends to become the same for any frequency again (constructive if $0 \le \vartheta < \pi/2$ or destructive if $\pi/2 < \vartheta \le \pi$).

The case of null interference for different frequencies, shown in Eq. (50), has similarity with the result found in the problem of a cavity with two oscillating Dirichlet mirrors, in which different frequencies of oscillation of the mirrors also leads to a null interference [21].

We see from Eqs. (48), (49), and (51) that there are no created particles with frequency $\omega > \omega_0$ due to interference, and the interference spectrum is symmetrical with respect to $\omega = \omega_0/2$, since it is invariant under the change $\omega \to \omega_0 - \omega$ for any value of the parameters ϵ, τ, ϑ, γ_0, $\tilde{\gamma}_0$, and $\tilde{q}_0$. These properties are also found in the other terms N_q and N_γ.

Now, let us investigate how the rate of created particles ($\mathcal{N}/\tau$) is affected by the interference. Inserting Eq. (40) in (36), we obtain the formula for the total number of created particles given by $\mathcal{N} = \mathcal{N}_q + \mathcal{N}_\gamma + \mathcal{N}_{\rm int}$. The noninterference terms $\mathcal{N}_q$ and $\mathcal{N}_\gamma$ are respectively given by

$$\mathcal{N}_q(\omega_0, \gamma_0, \tilde{q}_0) = (\epsilon^2 \tau \tilde{q}_0^2 \omega_0^3/\pi)\mathcal{F}(\gamma_0\omega_0), \tag{54}$$

$$\mathcal{N}_\gamma(\omega_0, \gamma_0, \tilde{\gamma}_0) = (\epsilon^2 \tau \tilde{\gamma}_0^2 \omega_0^3/\pi)\mathcal{G}(\gamma_0\omega_0), \tag{55}$$

where

$$\mathcal{F}(\alpha) = \frac{\alpha(\alpha^3 + 4\alpha + 12\arctan\alpha) - 6(2+\alpha^2)\ln(1+\alpha^2)}{6\alpha^2(\alpha^2+4)}, \tag{56}$$

$$\mathcal{G}(\alpha) = \frac{(\alpha^2+2)\ln(1+\alpha^2) - 2\alpha\arctan\alpha}{\alpha^4(\alpha^2+4)}. \tag{57}$$

These equations are in agreement with the results of Refs. [14,19], respectively. The contribution due to the interference is given by

$$\mathcal{N}_{\rm int}(\omega_0, \gamma_0, \tilde{\gamma}_0, \tilde{q}_0, \vartheta) = -\frac{2\epsilon^2 \tau \tilde{\gamma}_0 \tilde{q}_0 \omega_0^3 \cos\vartheta}{\pi}\mathcal{I}(\gamma_0\omega_0), \tag{58}$$

where

$$\mathcal{I}(\alpha) = \frac{\alpha[\ln(1+\alpha^2) - 4 - \alpha^2] + 2(2+\alpha^2)\arctan\alpha}{\alpha^2(\alpha^2+4)}. \tag{59}$$

From Eq. (58), one can see that the nature of the interference effect depends only on the values of $\gamma_0\omega_0$ and ϑ. If $0 \le \vartheta < \pi/2$, the interference is destructive for $0 < \gamma_0\omega_0 < 2.23$ and constructive for $\gamma_0\omega_0 > 2.23$ (see the solid line in Fig. 2). The opposite behavior occurs if $\pi/2 < \vartheta \le \pi$. For the specific value $\gamma_0\omega_0 \approx 2.23$, there is no net interference effect in the total number of created particles ($\mathcal{N}_{\rm int} = 0$). However this does not mean that the interference effect is suppressed if $\gamma_0\omega_0 \approx 2.23$: the spectral distribution of created particles is rearranged (see the long-dashed line in Fig. 1). For the specific value $\vartheta = \pi/2$, the interference effect is completely cancelled.

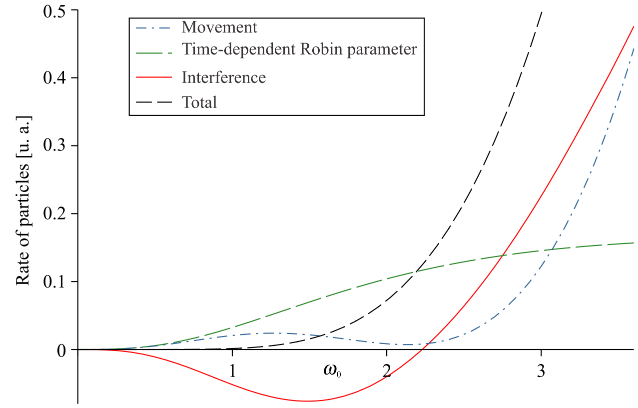

FIG. 2 (color online). The rate of created particles (short-dashed line) and its individual parts as functions of ω_0, considering $\gamma_0 = 1$, $\vartheta = 0$, and $\tilde{q}_0 = \tilde{\gamma}_0 = 1$. The dot-dashed line corresponds to $[\pi/(\epsilon^2\tau)]\mathcal{N}_q$. The long-dashed line corresponds to $[\pi/(\epsilon^2\tau)]\mathcal{N}_\gamma$, whereas the solid line corresponds to $[\pi/(\epsilon^2\tau)]\mathcal{N}_{\rm int}$. The value $\gamma_0\omega_0 \approx 2.23$ is the border between constructive and destructive effects.

It is worthwhile to remark that, in the literature, the value $\gamma_0\omega_0 \approx 2.23$ has already been linked to interesting situations concerning the DCE with Robin BC. Mintz *et al.* [18] reported a drastic inhibition of the particle creation rate when this value is achieved. In addition, the inhibition of the DCE with Robin BC in $3+1$ dimensions is more expressive for the same particular value [20].

IV. INTERFERENCE IN THE CONTEXT OF THE SQUID EXPERIMENT

Now, we use some formulas considered in the previous sections and in Appendix to investigate the interference phenomenon in the context of the SQUID experiment [7,8]. This experiment uses a superconducting coplanar waveguide with capacitance and inductance per unit of length, respectively, C_0 and L_0, and a SQUID at one of the extremities of the waveguide. In this context, a time-dependent magnetic flux is applied to the SQUID, changing its effective inductance, resulting in a time-dependent BC. Due to the presence of Josephson junctions in the system, a phase field operator $\phi(t,x)$ associated to the electromagnetic field in the waveguide obeys the wave equation and the Robin BC shown in Eq. (1), where the parameter $\gamma(t)$ is given by

$$\gamma(t) = -\Phi_0^2[(2\pi)^2 E_J(t) L_0]^{-1}. \tag{60}$$

In the previous formula, Φ_0 is the magnetic fundamental quantum flux, L_0 is the inductance per unit length of the waveguide, and $E_J(t)$ is the effective Josephson energy. The time-dependent magnetic flux drives a small-amplitude harmonic variation in the Josephson energy, namely, $E_J(t) = E_J^0[1 + \epsilon f(t)]$, which leads to the following time-dependent Robin parameter

$$\gamma(t) \approx \gamma_0[1 - \epsilon f(t)], \tag{61}$$

where $\gamma_0 = -\Phi_0^2[(2\pi)^2 E_J^0 L_0]^{-1}$. The resulting spectral distribution of this system is given by Eq. (48), with $\tilde{\gamma}_0 = \gamma_0$.

Now, we consider two independent sources of magnetic flux, which can present different phases and frequencies, both sources driving harmonic variations in the Josephson energy of the SQUID, leading to the following time-dependent Robin parameter,

$$\gamma(t) \approx \gamma_0[1 - \epsilon_1 f_1(t) - \epsilon_2 f_2(t)], \tag{62}$$

where

$$f_1(t) = \cos(\omega_1 t) e^{-|t|/\tau}, \tag{63}$$

$$f_2(t) = \cos(\omega_2 t + \vartheta) e^{-|t|/\tau}. \tag{64}$$

From Eqs. (A1), (A2), and (A3), we have that, if $\omega_1 \neq \omega_2$, the spectrum of created particles is the direct sum of the spectrum generated by each one of the sources of magnetic flux, with a null interference term:

$$N_\gamma(\omega) = N_\gamma^{(1)}(\omega) + N_\gamma^{(2)}(\omega), \tag{65}$$

where,

$$N_\gamma^{(1)}(\omega) = \frac{\epsilon_1^2 \tau}{\pi} \frac{\gamma_0^2 \omega(\omega_1 - \omega)\Theta(\omega_1 - \omega)}{(1 + \gamma_0^2\omega^2)(1 + (\omega_1 - \omega)^2\gamma_0^2)}, \tag{66}$$

$$N_\gamma^{(2)}(\omega) = \frac{\epsilon_2^2 \tau}{\pi} \frac{\gamma_0^2 \omega(\omega_2 - \omega)\Theta(\omega_2 - \omega)}{(1 + \gamma_0^2\omega^2)(1 + (\omega_2 - \omega)^2\gamma_0^2)}. \tag{67}$$

Equation (65) predicts, for different frequencies, a simple addition of the spectra, what generates asymmetrical final shapes, as, for instance, that shown in Fig. 3.

As an application of this result, let us consider the value for γ_0 considered in the context of the SQUID experiment, relabeled conveniently as $\gamma_{0\,\mathrm{exp}}$, given by $\gamma_{0\,\mathrm{exp}} = -0.44 \times 10^{-3}$ m. We also consider the following values for the other relevant quantities for the SQUID experiment: $\omega_1 = 2\pi \times 10.30$ GHz, $\epsilon_1 = 0.25$ and $v = 1.2 \times 10^8$ m/s [7,8], where $v = 1/\sqrt{C_0 L_0}$ is the speed of light in the waveguide. In addition, we also consider the presence of a second source of magnetic flux, for which $\omega_2 = \sigma\omega_1$ (with $0 < \sigma < 1$) and $\epsilon_2 = \epsilon_1$. In the presence of both sources, the modified spectrum of created particles can present one or two maximum points. For $\sigma \leq 1/3$ there is just one peak at $\omega = \omega_1/2$. Setting $1/3 < \sigma < 1/2$ one gets two peaks: one at $\omega \approx \omega_1(\sigma + 1)/4$ and other at $\omega = \omega_1/2$. Specifically, if $\sigma \approx 0.41$, the two peaks have approximately the same height. If $1/2 \leq \sigma < 1$ one gets only one peak at $\omega \approx \omega_1(\sigma + 1)/4$. For instance, the case for which $\sigma = 1/2$ is shown in Fig. 3, where we can see that, in comparison with the usual spectrum based on a single source (dashed line), one gets a modified spectrum (continuous line) presenting a larger number of created particles and also a maximum point deviated from $\omega = \omega_1/2$ to $\omega \approx 3\omega_1/8$. In summary: for the modification of the spectrum with the generation of two peaks, one needs to set up $1/3 < \sigma < 1/2$; to get only one peak, but displaced, we have to set up $1/2 \leq \sigma < 1$, being the case $\sigma = 1/2$ that producing the largest displacement from $\omega_1/2$.

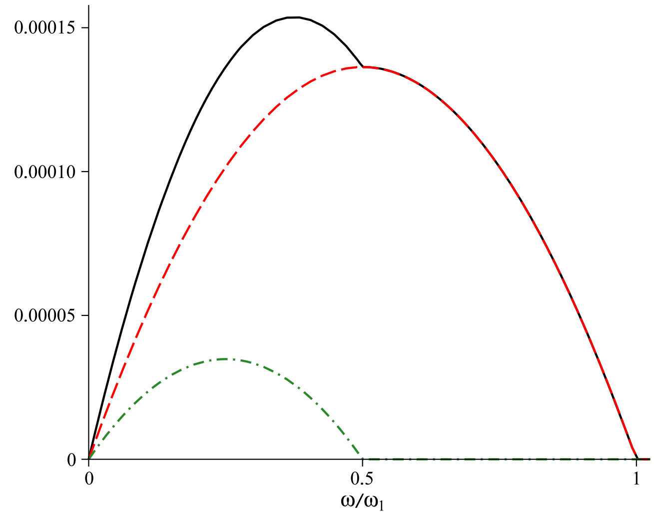

FIG. 3 (color online). Spectral distribution for the case of one source of magnetic flux with frequency ω_1 (dashed line), for the case of one source with frequency $\omega_1/2$ (dot-dashed line), and when both sources are considered simultaneously (solid line).

For the case $\omega_1 = \omega_2 = \omega_0$, interference effects appear, so that

$$N_\gamma(\omega) = N_\gamma^{(1)}(\omega) + N_\gamma^{(2)}(\omega) + 2\sqrt{N_\gamma^{(1)}(\omega) N_\gamma^{(2)}(\omega)} \cos\vartheta. \tag{68}$$

In this case, from Eqs. (66), (67), and (68), one can show that

$$N_\gamma(\omega) = \frac{\epsilon(\vartheta)^2 \tau}{\pi} \frac{\gamma_0^2 \omega(\omega_0 - \omega)\Theta(\omega_0 - \omega)}{(1 + \gamma_0^2\omega^2)(1 + (\omega_0 - \omega)^2\gamma_0^2)}, \tag{69}$$

where

$$\epsilon(\vartheta)^2 = \epsilon_1^2 + \epsilon_2^2 + 2\epsilon_1\epsilon_2\cos\vartheta. \tag{70}$$

The presence of the interference term leads to a final spectrum enhanced or reduced, if compared to the case of a single source of magnetic flux, but it does not change the original shape of the spectrum and, consequently preserving the symmetry with respect to $\omega = \omega_0/2$.

V. FINAL REMARKS

Differently from models where the problem of interference in the DCE were considered for cavities, in the present paper we discussed models with just a single mirror associated with two independent sources of particle creation. One of these models consisted of a field submitted to a Robin BC with a time-dependent Robin parameter $\gamma(t)$ for a moving mirror [see Eq. (3)], which generalizes the previous boundary conditions given by Eqs. (1) [14] and (2) [18,19]. The other model investigated here considered a Robin BC with $\gamma(t)$, for a static mirror, but with the presence of two sources of field perturbation [see Eqs. (62), (63), and (64)].

For the model with the Robin BC (3), we obtained the spectrum given by (41) (in agreement with [14]) plus (42) (in agreement with [19]), added by an interference term (43). For the typical time-variation of the parameters given by Eqs. (44) and (45), we got a null value for the interference term if both sources of field perturbation have different frequencies, whereas, for equal frequencies, the interference term is given by Eq. (52), which exhibits an interesting feature: different regions of the spectrum can be affected in different manners by constructive or destructive effects. For the total number of created particles related to the interference term, another interesting feature occurs: besides the case for which the sources have different frequencies, the net interference is also null when both frequencies (equal to ω_0) are so that $\gamma_0\omega_0 \approx 2.23$. This is the same value that produces a strong decoupling between field and mirror, minimizing $\mathcal{N}_q$ in $1+1$ [18,19] and in $3+1$ dimensions [20]. Moreover, $\gamma_0\omega_0 \approx 2.23$ is the border between constructive and destructive effects (see Fig. 2).

Investigating the model with the BC given by (1), (62), (63), and (64), we showed how the interference effects could be observed with some small changes in the context of a SQUID experiment [8]. For instance, considering two sources of magnetic flux acting on the SQUID, our results showed that any interference effect just occurs when the magnetic fluxes oscillate with the same frequency. Magnetic fluxes with different frequencies will produce a final spectrum which is the direct sum of that produced independently by each source, leading to a final spectrum with shapes different from the usual predict by just one source, breaking the symmetry with respect to $\omega = \omega_1/2$ (see Fig. 3). Considering experimental parameters, one can get a deviation of the maximum point $\omega = \omega_1/2$. These results indicate new signatures of the DCE that could be detected. Moreover, the presence of the interference term leads to a final spectrum enhanced or reduced, if compared to the case of a single source of magnetic flux, but it does not change the original shape of the spectrum and, consequently, preserving the symmetry with respect to $\omega = \omega_0/2$.

ACKNOWLEDGMENTS

This work was partially supported by CNPq and CAPES—Brazil.

APPENDIX: THE MONOCHROMATIC LIMIT

Considering the monochromatic limit, $\omega_1\tau \gg 1$ and $\omega_2\tau \gg 1$, both $|F(\omega)|^2/\tau$ and $|G(\omega)|^2/\tau$ present very sharp peaks around the values $\omega = \pm\omega_1$ and $\omega = \pm\omega_2$, respectively, and the heights of these peaks go to infinity (see Fig. 4), so that these functions exhibit sifting properties around $\omega = \pm\omega_i$ $(i = 1, 2)$, namely,

$$\lim_{\tau\to\infty}\int_{-\infty}^{+\infty}\frac{|F(\omega)|^2}{\tau}P(\omega)\mathrm{d}\omega = \frac{\pi}{2}[P(-\omega_1) + P(\omega_1)], \tag{A1}$$

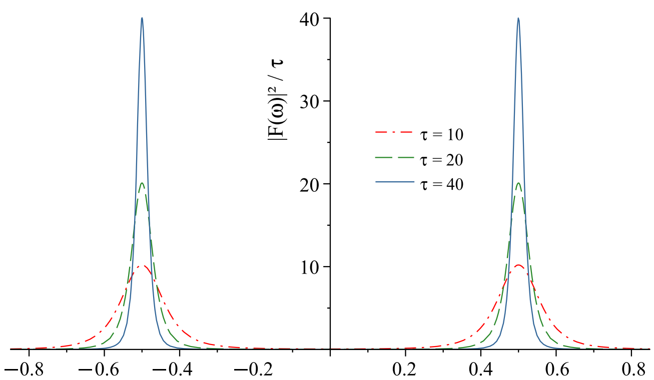

FIG. 4 (color online). $|F(\omega)|^2/\tau$ considering $\omega_1 = 0.5$ and some values for τ ($|G(\omega)|^2/\tau$ behaves the same way).

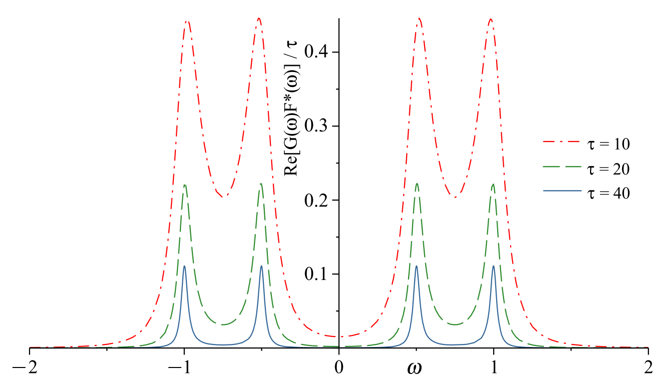

FIG. 5 (color online). Plot of $\mathrm{Re}[G(\omega)F^*(\omega)]/\tau$ considering $\omega_1 \neq \omega_2$ with $\omega_1 = 0.5$ and $\omega_2 = 1.0$ and some values for τ.

$$\lim_{\tau\to\infty}\int_{-\infty}^{+\infty}\frac{|G(\omega)|^2}{\tau}P(\omega)\mathrm{d}\omega=\frac{\pi}{2}[P(-\omega_2)+P(\omega_2)],\tag{A2}$$

where $P(\omega)$ is any continuous function at $\omega=\pm\omega_i$.

We also have performed an approximation for the cross term $\mathrm{Re}[G(\omega)F^*(\omega)]$ in the previous sections in order to compute the interference spectrum. The mentioned term must be analyzed in two different situations: the first one when $\omega_1\neq\omega_2$, and the second one for $\omega_1=\omega_2$. If $\omega_1\neq\omega_2$ there are sharp peaks around the values $\omega=\pm\omega_1$ and $\omega=\pm\omega_2$, with their heights going to zero in the monochromatic limit (see Fig. 5), so that

$$\lim_{\tau\to\infty}\int_{-\infty}^{+\infty}\frac{\mathrm{Re}[G(\omega)F^*(\omega)]}{\tau}P(\omega)\mathrm{d}\omega=0.\tag{A3}$$

However, if $\omega_1=\omega_2=\omega_0$ one can see that the cross term $\mathrm{Re}[G(\omega)F^*(\omega)]$ becomes identical to $|F(\omega)|^2$ (except by a factor of $\cos\vartheta$). Therefore, in the monochromatic limit, this term is also a representation of the Dirac delta function, namely,

$$\lim_{\tau\to\infty}\int_{-\infty}^{+\infty}\frac{\mathrm{Re}[G(\omega)F^*(\omega)]}{\pi\tau}P(\omega)\mathrm{d}\omega = [P(\omega_0)+P(-\omega_0)]\times\frac{\cos\vartheta}{2}.\tag{A4}$$

Summarizing, $|F(\omega)|^2$ and $|G(\omega)|^2$ represent Dirac deltas in the monochromatic limit. The cross term $\mathrm{Re}[G(\omega)F^*(\omega)]$ only represents a Dirac delta if $\omega_1=\omega_2$, vanishing if $\omega_1\neq\omega_2$.

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