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Cosmic distance-duality relation test using type Ia supernovae and the baryon acoustic oscillation

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A check of the validity of the distance-duality relation (DDR) is necessary since a violation of one of the assumptions underlying this relation might be possible. In this paper, we test the DDR by combining the Union2.1 type Ia supernovae (SNIa) and five angular diameter distance data from the baryonic acoustic oscillation (BAO) measurements. We find that the DDR is consistent with the observations at the 2σ confidence level (CL) for the case of the Hubble constant $h = 0.7$, and the consistency is improved to be 1σ CL when $h = 0.7$ is replaced by the latest constraint from the Planck satellite, i.e., $h = 0.678$, or h is marginalized. Our results show that the BAO measurement is a very powerful tool to test the DDR. With more and more BAO data being released in the future, we are expecting a better validity check of the DDR.

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I. INTRODUCTION

Based on two fundamental hypotheses that light travels always along null geodesics in the Riemanian geometry, and the number of photons is conserved, Etherington [1] proved a famous cosmic distance-duality relation (DDR), which connects the luminosity distance (LD) D_{L} and the angular diameter distance (ADD) D_{A} through the following identity

$$\frac{D_{\mathrm{L}}}{D_{\mathrm{A}}}(1+z)^{-2} = 1 \tag{1}$$

with z being the redshift. Since this relation is independent of the Einstein field equations and the nature of matter, it has been used in astronomical observations and modern cosmology without any doubt, and plays a fundamental role in them. However, a violation of one of the hypotheses leading to the DDR might be possible, which may be considered as a signal of exotic physics [2,3]. Thus, a validity check for this relation using astronomical observations is worthy and necessary.

To test the DDR against astronomical observations, one should in principle measure the LD and ADD of some objects at the same redshift. The LD can be estimated by means of the standard candle, such as type Ia supernovae (SNIa), and the values of ADD can be obtained from the observations, such as the X-ray plus Sunyaev-Zel'dovich effect (SZE) of galaxy clusters and the gas mass fraction measurement in a galaxy cluster. Since the redshifts of the observational LD and ADD usually do not match, some checks for the DDR are performed by comparing the observed values with the corresponding theoretical ones obtained from an assumed cosmological model. Based on the ΛCDM model, Uzan *et al.* [4] found that the DDR is consistent with the 18 ADD galaxy cluster samples [5] at the 1σ confidence level (CL). With a bigger galaxy cluster sample [6], De Bernardis Giusarma and Melchiorri [7] also obtained a nonviolation of the DDR. Using the SNIa standard candles and the standard rulers from the cosmic microwave background and baryon acoustic oscillation (BAO) measurements, Lazkoz *et al.* [8] discovered that the DDR is valid at the 2σ CL. From the galaxy cluster data from the elliptical and spherical β models (used to describe the galaxy clusters) [6,9], Holanda *et al.* [10] found that the elliptical and spherical β models are consistent with the validity of the DDR at the 1σ and 3σ CL, respectively. In addition, it was found that the current CMB observations imply that the reciprocity relation cannot be violated by more than 0.01% between decoupling and today [11]. Furthermore, the Gaussian process has been used to test the DDR [12,13].

Recently, Holanda and Lima [14] proposed a method to match the redshifts of D_{L} from the SNIa and D_{A} from the galaxy cluster. For each cluster data, they selected one SNIa whose redshift is the closest to the cluster's within the range $\Delta z = |z - z_{\mathrm{SNe}}| < 0.005$. From the Constitution SNIa data [15] and the galaxy cluster samples from the elliptical and spherical β models [6,9], they found that the elliptical β model is marginally compatible with the validity of the DDR at the 2σ CL, while, the spherical β model indicates a strong deviation from this relation. Using the Union 2 SNIa sample [16], Li *et al.* [17] also test the DDR, and find that it is accommodated at the 1σ CL for the elliptical model, and at the 3σ CL for the spherical model. Still some other authors used other observational data to test the DDR or proposed several new methods to match the redshifts of the observed SNIa with the galaxy cluster data [18–22], and obtained results similar to that in [14,17]. For the ADD values from the gas mass fraction of galaxy clusters, whose error bars are considerably smaller than those obtained from X-ray/SZE technique, it was found the DDR is consistent with them at the 1σ CL [23,24]. Using the Monte Carlo simulations, Goncalves *et al.* [25] obtained the number of galaxy clusters observations needed to check the validity of the DDR at a given confidence level. More recently, an improved method to test the DDR is discussed in [26] which overcomes the defect that the distance moduli of SNIa are dependent on a given cosmological model and Hubble constant. It is found that the DDR is consistent with galaxy clusters and SNIa at the 1σ CL.

In checking the validity of the DDR from observations directly in a model independent way, the galaxy cluster data are usually used to provide the observed ADD. The above discussions indicate that the impact of systematics related to the cluster geometry is quite strong, and it even leads to contrasting conclusions. In addition, due to the large systematic and statistic uncertainty of galaxy clusters data, a compelling accuracy from numerical results may be hard to be obtained. Thus, we need more accurate ADD data to test the validity of the DDR with more confidence. It is well known that the BAO measurement (see [27] for a review) is a very precise experiment and plays an important role in modern cosmology. The BAO data have been used to study the cosmic transparency in [28–30]. By combining measurements of the baryon acoustic peak and the Alcock-Paczynski distortion from galaxy clustering, the value of ADD can be determined. Recently, the WiggleZ Dark Energy Survey [31] and the Sloan Digital Sky Survey (SDSS) (including Data Release 7 (DR7 [32]) and Data Release 11 (DR11) [33]) released five ADD data points. In this paper, we plan to use these BAO ADD data and the Union2.1 SNIa data [34] to check the validity of the DDR.

II. METHOD AND DATA

In order to test the DDR from observations directly, we parametrize this relation as follows

$$\eta(z) = \frac{D_{\mathrm{L}}}{D_{\mathrm{A}}}(1+z)^{-2}, \tag{2}$$

and consider three different parametrizations

$$\eta(z, \eta_0) = 1 + \eta_0, \tag{3}$$

$$\eta(z, \eta_1) = 1 + \eta_1 z, \tag{4}$$

$$\eta(z, \eta_2) = 1 + \eta_2 \frac{z}{1+z}, \tag{5}$$

where η_0, η_1, and η_2 are constants. If the DDR is valid, the values of η_0, η_1, and η_2 stay close to zero. The constraints can be obtained by minimizing

$$\chi^2(\eta_j) = \sum_i \frac{[\eta(z, \eta_j) - \eta_{i,\mathrm{obs}}(z)]^2}{\sigma^2_{\eta_{i,\mathrm{obs}}}}, \tag{6}$$

where $j = 0, 1$, or 2, $\eta_{i,\mathrm{obs}}(z)$ is the value obtained from the observed LD and ADD, and $\sigma_{\eta_{i,\mathrm{obs}}}$ is the corresponding uncertainty. The probability distribution of parameter η_j is determined through $P = A\exp(-\chi^2(\eta_j)/2)$, where A is the normalization parameter. The best fit value is given by χ^2_{min}, which is the minimum of χ^2. The 1σ and 2σ CLs are determined through $\Delta\chi^2 = \chi^2 - \chi^2_{\mathrm{min}} \le 1$ and $\Delta\chi^2 \le 4$, respectively, when only one parameter is to be estimated. If two parameters are estimated, the values of $\Delta\chi^2$ to compare are 2.3 for the 1σ contour and 6.17 for the 2σ contour.

The observational ADD data come from the BAO measurements (see [27] for a review). In the early universe, the Thomson scattering leads to the coupling of photons and baryons. Due to the existence of a competition between radiation pressure and gravity, a system of standing sound waves within the plasma is created, which is the so-called baryon acoustic oscillations (BAOs). At recombination, the free electrons are quickly captured. Thus, the interaction between photons and baryons ends abruptly, which leads to a slight overdensity of baryons at the scale about 150 Mpc in today's universe. This scale has been measured in the clustering distribution of galaxies today and can be used as a standard ruler. Combining measurements of the baryon acoustic peak and the Alcock-Paczynski distortion from galaxy clusters, the ADD data can be obtained. As listed in Table I, there are five data points in the low redshift region

TABLE I. Summaries of the ADD measurement from the BAO, the binned D_L from Union 2.1 SNIa, and the obtained η_{obs}.

z	$D_A(z)$ (Mpc)	Survey	$\bar{D}_L$ (Mpc)	the binned SNIa number	η_{obs}
0.44	1205 ± 114	WiggleZ[31]	2488 ± 98	4	0.9957 ± 0.1020
0.6	1380 ± 95		3272 ± 196	3	0.9262 ± 0.0846
0.73	1534 ± 107		4523 ± 406	1	0.9851 ± 0.1119
0.35	1050 ± 38	SDSS DR7[32]	1887 ± 72	5	0.9858 ± 0.0527
0.57	1380 ± 23	SDSS DR11 CMASS [33]	3142 ± 194	4	0.9237 ± 0.0590

($z < 1$). Three of them are determined from the WiggleZ Dark Energy Survey, and the other two data points are from BOSS DR7 and DR11.

The observational LD data are given by the Union2.1 SNIa compilation in our analysis. It is an update of the Union2 compilation and consists of 580 data points. All SNIa are fitted using the SALT2-1 lightcurve fitter and uniformly analyzed. The relation between the LD and the distance modulus μ of SNIa indicates that the observed LD can be obtained through $D_{\rm L} = 10^{\mu/5-5}$. In obtaining the distance modulus, the dimensionless Hubble constant h ($h = H_0/100\ {\rm km\,s^{-1}\,Mpc}$ with H_0 being Hubble constant) is taken to be $h = 0.7$. Since the present observations cannot determine precisely the value of h and a discrepancy of h from different observations exists even at the $1-2\sigma$ CL (see Fig. 1 in [35] for a summary of different H_0 measurements), setting $h = 0.7$ in calculating the distance modulus may lead to some bias for the test of the DDR and the effect due to the uncertainty of h needs to be considered. In this paper, besides the case of $h = 0.7$, we also test the DD relation by setting a different value of h, i.e., $h = 0.678$, from the latest Planck result [36]. And we analyze the effect of the uncertainty of h by using $D_{\rm L} = \frac{0.7}{h} 10^{\mu/5-5}$ since $D_{\rm L} \propto \frac{1}{h}$ and assuming h to be a free parameter. In this case, a marginalization over h can be obtained from $P(\eta_j) = \int_{-\infty}^{+\infty} P(\eta_j, h) dh$. If a prior distribution on h coming from observations is considered, the constraint on η_j can be obtained by calculating $P(\eta_j) = \int_{-\infty}^{+\infty} \bar{P}(h) * P(\eta_j, h) dh$, where a Gaussian distribution on h, i.e., $\bar{P}(h) = \exp(-\frac{1}{2}\frac{(h-h_{\rm obs})^2}{\sigma^2_{h_{\rm obs}}})$, is assumed.

To check the validity of the DDR, we must have the observed LD and ADD at the same redshift. To obtain the value of LD from SNIa at the redshifts of BAO data, instead of using the method proposed in [14] by adopting a selection criterion $\Delta z = |z_{\rm BAO} - z_{\rm SNIa}| < 0.005$ and choosing the nearest SNIa data, we neglect the SNIa-to-SNIa correlations and bin all SNIa data available in the range $\Delta z = |z_{\rm BAO} - z_{\rm SNIa}| < \Delta$, where constant Δ represents the redshift region binned. This method has the advantage of avoiding statistical errors resulting from merely one SNIa data point with all those available which meets the selection criterion, and has been used to discuss the DDR in [20] using the SNIa and the galaxy cluster data with $\Delta = 0.005$. Here, we also choose $\Delta = 0.005$. In the present paper, since the BAO data, whose accuracy is much better than that of the galaxy clusters, are considered, the uncertainties of both $D_{\rm L}$ and $D_{\rm A}$ affect the result. For all selected data, an inverse variance weighted average is employed. If $D_{{\rm L}i}$ denotes the ith appropriate SNIa luminosity distance data with $\sigma_{D_{{\rm L}i}}$ representing the corresponding observational uncertainty, one can straightforwardly obtain with the conventional data reduction techniques in [37] that

$$\bar{D}_{\rm L} = \frac{\sum (D_{{\rm L}i}/\sigma^2_{D_{{\rm L}i}})}{\sum 1/\sigma^2_{D_{{\rm L}i}}}, \tag{7}$$

$$\sigma^2_{\bar{D}_{\rm L}} = \frac{1}{\sum 1/\sigma^2_{D_{{\rm L}i}}}, \tag{8}$$

where $\bar{D}_{\rm L}$ represents the weighted mean luminosity distance at a given redshift, and $\sigma^2_{\bar{D}_{\rm L}}$ is its uncertainty. Then, the observed $\eta_{\rm obs}(z)$ at a given redshift can be obtained

$$\eta_{\rm obs}(z) = \frac{\bar{D}_{\rm L}}{D_{\rm A}}(1+z)^{-2}. \tag{9}$$

The corresponding $\sigma_{\eta_{\rm obs}}$ can be given through

$$\sigma^2_{\eta_{\rm obs}} = \frac{\sigma^2_{\bar{D}_{\rm L}}}{D^2_{\rm A}}(1+z)^{-4} + \frac{\bar{D}_{\rm L}{}^2}{D^4_{\rm A}}\sigma^2_{D_{\rm A}}(1+z)^{-4}. \tag{10}$$

In Table I, we summarize the obtained $\bar{D}_{\rm L}$ and $\eta_{\rm obs}$.

III. RESULTS

We first test the DDR relation with $h = 0.7$, which is used in obtaining the distance modulus of the Union2.1 SNIa, and $h = 0.678$ obtained from the latest Planck observations. For the first parametrization, $\eta(z) = 1 + \eta_0$, the likelihood distributions of η_0 are shown in Fig. 1, in which the solid and dashed lines correspond to $h = 0.7$ and 0.678, respectively. The corresponding numerical results are that at the 1σ CL $\eta_0 = -0.040 \pm 0.033$ for $h = 0.7$, and $\eta_0 = -0.009 \pm 0.033$ for $h = 0.678$. When $h = 0.7$, the DDR is consistent

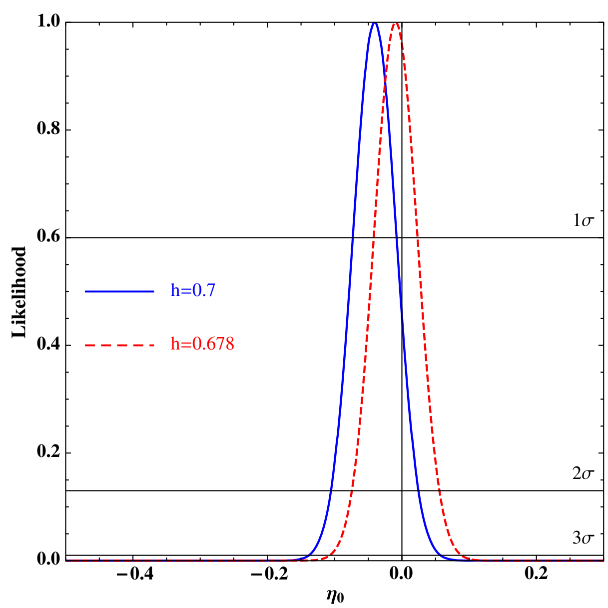

FIG. 1 (color online). The likelihood distributions of η_0 with different values of h.

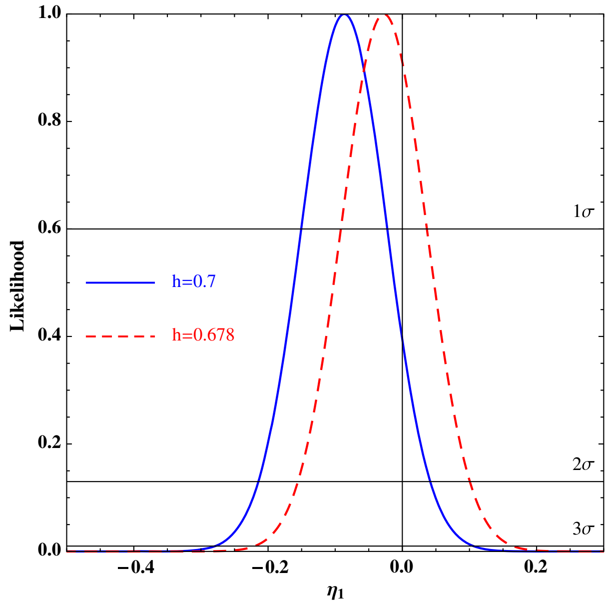

FIG. 2 (color online). The likelihood distributions of η_1 with different values of h.

with the observations only at the 2σ CL, while for $h = 0.678$ the consistency occurs at the 1σ CL. Thus, different values of h play an important role on the DDR test.

For the parametrizations, $\eta(z) = 1 + \eta_1 z$ and $\eta(z) = 1 + \eta_2 \frac{z}{1+z}$, the results are shown in Figs. 2, 3. We find that at the 1σ CL $\eta_1 = -0.086 \pm 0.064$ and $\eta_2 = -0.131 \pm 0.098$ for $h = 0.70$, and $\eta_1 = -0.027 \pm 0.064$ and $\eta_2 = -0.039 \pm 0.099$ for $h = 0.678$. These constraints are slightly weaker than that of η_0. But different parametrizations give almost the same results. For $h = 0.7$, the

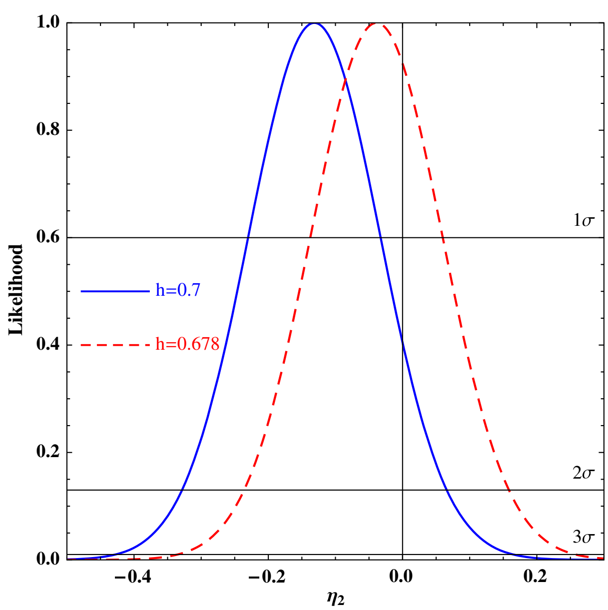

FIG. 3 (color online). The likelihood distributions of η_2 with different values of h.

DDR is consistent with the SNIa and BAO observations at the 2σ CL, which is improved to be 1σ when $h = 0.678$ is used.

Now, we analyze the effect of the uncertainty of h on the test of the DDR by allowing h to be a free parameter rather than a concrete fixed value. Two parametrizations, $\eta(z) = 1 + \eta_1 z$ and $\eta(z) = 1 + \eta_2 \frac{z}{1+z}$, are considered. The contour diagrams on $h - \eta_1$ and $h - \eta_2$ planes are shown in the left panels of Fig. 4 with the best fit values occurring at $\{h = 0.724, \eta_1 = -0.147\}$ and $\{h = 0.753, \eta_2 = -0.334\}$, respectively. It is easy to see that there is a strong anticorrelation between h and η_1 (or η_2). If h is a constant, a value of h slightly less than 0.7 can improve the consistency between the DDR and the SNIa + BAO observations, which agrees with our previous results since for $h = 0.678$ the consistency between the DDR and the SNIa + BAOs is better than that for $h = 0.70$. In the right panels of this figure, we plot the likelihood distributions of η_1 and η_2 with a marginalization over h. We find that at the 1σ CL $\eta_1 = -0.174^{+0.253}_{-0.199}$ and $\eta_2 = -0.409^{+0.529}_{-0.381}$, which means that the DDR is consistent with the observations at the 1σ CL.

Figure 4 shows that the constraint on h is very weak when h is allowed to be a free parameter and it dramatically affects the results on η_1 (η_2). Currently, the CMB measurements from Planck satellite have given a strong constraint on h: $h = 0.678 \pm 0.009$ at the 1σ CL [36]. Thus, we can add this information by assuming a prior distribution on h. Considering a Gaussian distribution on h and taking a marginalization over it, we obtain the likelihood distributions of η_1 and η_2, which are shown in Fig. 5, and we find that $\eta_1 = -0.029^{+0.071}_{-0.070}$ and $\eta_2 = -0.041^{+0.110}_{-0.108}$ at the 1σ CL. It is easy to see that more stringent constraints on η_1 and η_2 are obtained, and the DDR is accommodated at the 1σ CL.

In Table II and Fig. 6, we give a comparison for the constraints on η_1 and η_2 from different data sets. Except for our analysis in the present paper, all other works use the galaxy cluster data or the gas matter function data from galaxy clusters to provide the observed ADD. As is discussed in the Introduction, the inconsistency among different galaxy cluster data samples is obvious. For the galaxy cluster data, the tightest constraint comes from the 91 measurements of the gas mass fraction of galaxy clusters recently reported by the Atacama Cosmology Telescope survey along with the Union2.1 SNIa compilation [24], which supports the DDR at the 1σ CL. But, the constraint on η_1 is still weaker than our results obtained from the Union2.1 SNIa + BAOs with h being a constant or having a Gaussian distribution coming from Planck measurements although the data number of BAO (five data points) is much less than that of the gas mass fraction of galaxy clusters. The BAO data improve the accuracy of η_1 about 30% at the 1σ CL. When h is a constant, the results from SNIa + BAOs are also slightly better than that from the most recent $f_{X\text{-ray}}$ and $f_{\rm SZE}$ data with a different method [22].

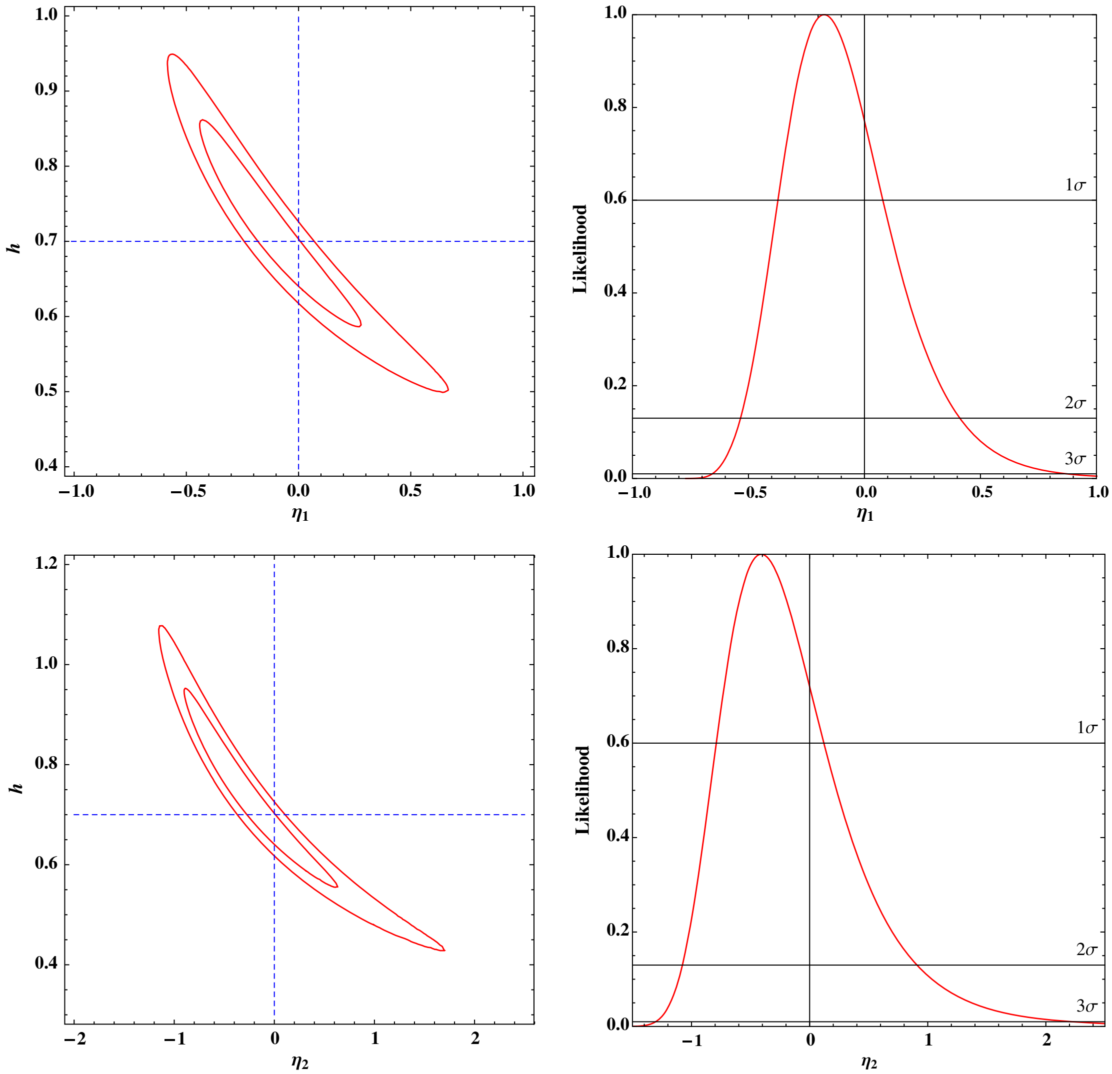

FIG. 4 (color online). The contour plots of $h - \eta_1$ and $h - \eta_2$ in the 68% and 95% CLs. The right panels show the likelihood distributions of η_1 and η_2 with a marginalization over h.

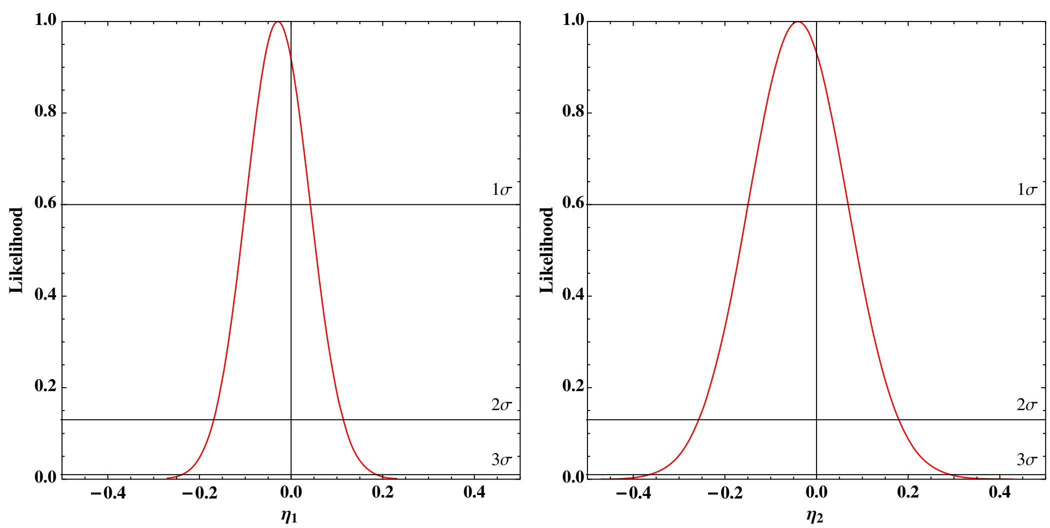

FIG. 5 (color online). The likelihood distributions of η_1 and η_2 with a Gaussian distribution on h from the Planck estimation.

TABLE II. Summary of the constraints on η_1 and η_2. Mh^a: a marginalization over h with h being a free parameter. Mh^b: a marginalization over h where h satisfies a Gaussian distribution coming from the Planck estimation. Galaxy^a: the elliptical β model. Galaxy^b: the spherical β model.

	η_1	η_2
Union2.1 + BAOs($h = 0.700$)	$-0.086 \pm 0.064(1\sigma)$	$-0.131 \pm 0.098(1\sigma)$
Union2.1 + BAOs ($h = 0.678$)	$-0.027 \pm 0.064(1\sigma)$	$-0.039 \pm 0.099(1\sigma)$
Union2.1 + BAOs (Mh^a)	$-0.174^{+0.253}_{-0.199}(1\sigma)$	$-0.409^{+0.529}_{-0.381}(1\sigma)$
Union2.1 + BAOs (Mh^b)	$-0.029^{+0.071}_{-0.070}(1\sigma)$	$-0.041^{+0.110}_{-0.108}(1\sigma)$
Constitution + Galaxy^a[17]	$-0.37 \pm 0.18(1\sigma)$	$-0.56 \pm 0.25(1\sigma)$
Constitution + Galaxy^b[17]	$-0.30 \pm 0.11(1\sigma)$	$-0.46 \pm 0.17(1\sigma)$
Union2 + Galaxy^a[17]	$-0.07 \pm 0.19(1\sigma)$	$-0.11 \pm 0.26(1\sigma)$
Union2 + Galaxy^b[17]	$-0.22 \pm 0.11(1\sigma)$	$-0.33 \pm 0.16(1\sigma)$
SDSS + Galaxy^a[18]	$-0.28 \pm 0.21(1\sigma)$	$-0.43 \pm 0.29(1\sigma)$
SDSS + Galaxy^b[18]	$-0.39 \pm 0.11(1\sigma)$	$-0.61 \pm 0.16(1\sigma)$
Union2 + Galaxy^b[21]	$-0.232 \pm 0.232(2\sigma)$	$-0.351 \pm 0.368(2\sigma)$
38 galaxy clusters [22]	$-0.15 \pm 0.07(1\sigma)$	$-0.22 \pm 0.10(1\sigma)$
29 galaxy clusters [22]	$-0.06 \pm 0.07(1\sigma)$	$-0.07 \pm 0.12(1\sigma)$
Union2 + 38 f_{gas}[23]	$-0.03^{+1.03}_{-0.65}(2\sigma)$	$-0.08^{+2.28}_{-1.22}(2\sigma)$
Union2.1 + 91 f_{gas} [24]	$-0.08^{+0.11}_{-0.10}(1\sigma)$	

Apparently, all data favor negative best fit values for η_1 and η_2. In addition, our results are consistent with the one obtained in [8] where it has been found that the DDR is valid at the 2σ CL by using the SNIa standard candles and the standard rulers from the cosmic microwave background and BAO to obtain the evolutionary curves of $\eta(z)$.

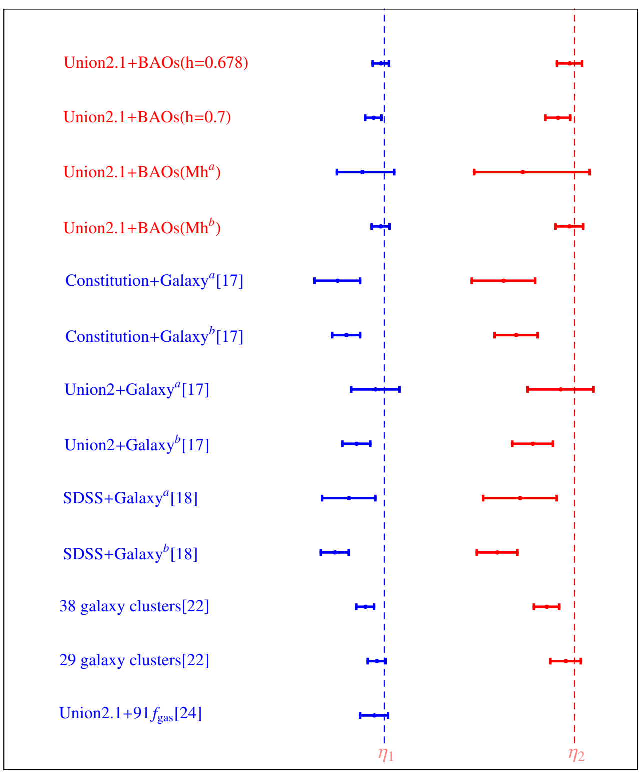

FIG. 6 (color online). A comparison of η_1 and η_2 from different observations. The dashed lines show $\eta_1 = 0$ and $\eta_2 = 0$, respectively. Mh^a: a marginalization over h with h being a free parameter. Mh^b: a marginalization over h where h satisfies a Gaussian distribution coming from the Planck observation. Galaxy^a: the elliptical β model. Galaxy^b: the spherical β model.

IV. CONCLUSIONS

The DDR, which plays an important role in astronomical observations and modern cosmology, needs to be checked by the observations since a violation of one of the assumptions underlying this relation might be possible. The current tests on the DDR with a model-independent method are usually based on the SNIa and galaxy cluster data, which provide the observed LD and ADD, respectively. Due to the ambiguity in the cluster geometry and the large uncertainty of galaxy clusters data, different data sets of galaxy clusters may give different and even contrasting results. Thus, a more reliable check of the validity of the DDR requires more accurate ADD data.

The BAO observation can provide precise ADD data by combining the measurements of the baryon acoustic peak and Alcock-Paczynski distortion from galaxy clustering. Currently, five ADD data from BAO measurements have been released by the WiggleZ Dark Energy Survey and the SDSS. In this paper, we test the DDR using these BAO data and the Union2.1 SNIa one. In order to obtain the observed LD at the redshifts of BAO data, we bin all SNIa data points whose redshifts satisfy a selecting criteria $\Delta z = |z_{\mathrm{BAO}} - z_{\mathrm{SNIa}}| < 0.005$. Our results show that with $h = 0.7$ the DDR is consistent with the observations only at the 2σ confidence, while this consistency is improved to be 1σ if $h = 0.7$ is changed to be 0.678, which is given by the latest CMB measurements from the Planck satellite.

Since the value of the distance modulus of Union2.1 SNIa is determined by taking the dimensionless Hubble constant h to be 0.7, we analyze the effect of the uncertainty of h on the DDR test by letting h be a free parameter. We find that in this case there is a strong anticorrelation between h and η_1 (η_2) and the value of h affects the result very remarkably. For a constant h, the consistency between the DDR and observations can be significantly improved by a value of h slightly less than 0.7. Since the constraint on h is very weak when h is allowed to be a free parameter and it dramatically affects the results on η_1 (η_2), we also consider a Gaussian distribution on h with the observed value given by the Planck satellite. After a marginalization over h with h being a free parameter or satisfying a Gaussian distribution, we find that the DDR is consistent with the observations at the 1σ CL. Our results show that the BAO measurement is a very powerful tool to test the DDR. With the measuring accuracy of h increasing and more and more accurate data being released from observations in the future, we can obtain a better check for the validity of the DDR. For example, future observation like EUCLID [38] will give about 15 BAO data points and enlarge the SNIa data sample about several times, and these are expected to improve the test accuracy remarkably.

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