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Shifted focus point of the Higgs mass parameter from the minimal mixed mediation of supersymmetry breaking

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We employ both the minimal gravity- and the minimal gauge mediations of supersymmetry breaking at the grand unified theory (GUT) scale in a single supergravity framework, assuming the gaugino masses are generated dominantly by the minimal gauge mediation effects. In such a "minimal mixed mediation model," a "focus point" of the soft Higgs mass parameter, $m_{h_u}^2$ emerges at 3–4 TeV energy scale, which is exactly the stop mass scale needed for explaining the 126 GeV Higgs boson mass without the "A-term" at the three-loop level. As a result, $m_{h_u}^2$ can be quite insensitive to various trial stop masses at low energy, reducing the fine-tuning measures to be much smaller than 100 even for a 3–4 TeV low energy stop mass and $-0.5 < A_t/m_0 \lesssim +0.1$ at the GUT scale. The gluino mass is predicted to be about 1.7 TeV, which could readily be tested at LHC run2.

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Although the standard model (SM) has been extremely successful in the experimental side, it does not provide reasonable answers to some theoretical puzzles such as the naturalness of the electroweak (EW) scale and the Higgs boson mass. The main motivation of the low energy supersymmetry (SUSY) was to resolve the naturalness problem associated with the EW phase transition raised in the SM, since SUSY can protect the small Higgs mass against large quantum corrections [1,2]. Because of it, the minimal supersymmetric standard model (MSSM) has been believed to be the most promising theory beyond the SM, guiding the SM to a grand unified theory (GUT) or string theory. However, any evidence of the low energy SUSY has not been observed yet at the large hadron collider (LHC): the mass bounds on the SUSY particles have gradually increased, and now they seem to start threatening the traditional status of SUSY as a prominent solution to such a naturalness problem of the SM.

Actually, a barometer of the naturalness of the MSSM is the mass of the superpartner of the top quark ("stop"). Due to the large top quark Yukawa coupling (y_t), the top and stop make the dominant contributions to the radiative physical Higgs mass squared and also the renormalization of a soft mass squared of the Higgs ($m_{h_u}^2$) in the MSSM. The renormalization effect on $m_{h_u}^2$ would linearly be sensitive to the stop mass squared $\tilde{m}_t^2$ [1],

$$\Delta m_{h_u}^2 \approx \frac{3|y_t|^2}{8\pi^2}\tilde{m}_t^2 \log\left(\frac{\tilde{m}_t^2}{\Lambda^2}\right) + \cdots, \tag{1}$$

while it depends just logarithmically on a ultraviolet (UV) cutoff Λ. Since the Higgs mass parameters, $m_{h_u}^2$ and $m_{h_d}^2$ are related to the Z boson mass m_Z together with the "Higgsinos" (superpartners of the Higgs boson) mass μ [1],

$$\frac{1}{2}m_Z^2 = \frac{m_{h_d}^2 - m_{h_u}^2\tan^2\beta}{\tan^2\beta - 1} - |\mu|^2, \tag{2}$$

$\{m_{h_u}^2, m_{h_d}^2, |\mu|^2\}$ should be finely tuned to yield $m_Z^2 = (91\ \mathrm{GeV})^2$ for a given $\tan\beta$ [$\equiv\langle h_u\rangle/\langle h_d\rangle$, ratio of the vacuum expectation values (VEVs) of the two MSSM Higgs fields], if they are excessively large. According to the recent analysis based on the three-loop calculations, the stop mass required for explaining the 126 GeV Higgs boson mass [3] without any other help is about 3–4 TeV [4]. Thus, a fine-tuning of order 10^{-3} or smaller looks unavoidable in the MSSM for $\Lambda \sim 10^{16}$ GeV.

In order to more clearly see the UV dependence of $m_{h_u}^2$ and properly discuss this "little hierarchy problem," however, one should suppose a specific UV model and analyze its resulting full renormalization group (RG) equations. If the UV model is simple enough, addressing this problem successfully with SUSY, the low energy SUSY could still be regarded as an attractive solution to the naturalness problem.

One nice idea is the "focus point (FP) scenario" [5]. This scenario is based on the minimal gravity mediation (mGrM) of SUSY breaking. So the soft mass squareds such as $m_{h_{u,d}}^2$ and those of the left handed (LH) and right handed (RH) stops, $(m_{q_3}^2, m_{u_3^c}^2)$ as well as the gaugino (superpartners of the gauge fields) masses M_a ($a = 3, 2, 1$) are given to be *universal* at the GUT scale, $m_{h_u}^2 = m_{h_d}^2 = m_{q_3}^2 = m_{u_3^c}^2 = \cdots \equiv m_0^2$ and $M_3 = M_2 = M_1 \equiv m_{1/2}$. As pointed out in [5], if the holomorphic soft SUSY breaking terms ("A-terms") in the scalar potential are zero at the

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GUT scale and the unified gaugino mass $m_{1/2}$ is just a few hundred GeV, $m_{h_u}^2$ converges to a small negative value around the Z boson mass scale in this setup, *regardless of its initial values given by* m_0^2 *at the GUT scale* [5]: a FP of $m_{h_u}^2$ appears around the m_Z scale. In the RG solution of $m_{h_u}^2$ at the m_Z scale, namely,

$$m_{h_u}^2(Q = m_Z) = C_s m_0^2 - C_g m_{1/2}^2, \tag{3}$$

where the dimensionless numbers C_s, C_g (> 0) can numerically be estimated using RG equations, C_s happens to be quite small with the above universal soft masses, and the EW symmetry is broken dominantly by the C_g term. On the other hand, stop masses are quite sensitive to m_0^2. As a result, m_Z^2 could remain small enough even with a relatively heavy stop mass in the FP scenario in contrast to the naive expectation from Eq. (1).

However, the experimental bound on the gluino (superpartner of the gluon) mass M_3 has already exceeded 1.3 TeV [6]. As expected from Eqs. (2) and (3), a too large $m_{1/2}$ needed for $M_3 > 1.3$ TeV at low energy would require a fine-tuned large $|\mu|$ for m_Z of 91 GeV particularly for relatively light stop mass ($\lesssim$1 TeV) cases. When the stop mass is around 3–4 TeV, the stop should decouple from the RG equations below 3–4 TeV, which makes C_s *sizable* in Eq. (3) [7]. Then, a much larger $m_{1/2}$ is necessary for EW symmetry breaking. Since the RG running interval between 3–4 TeV and m_Z scale, to which modified RG equations should be applied, is too large, the FP behavior is seriously spoiled with such heavy SUSY particles.

The best way to rescue the FP idea is to somehow shift the FP up to the stop decoupling scale [7]: C_s needs to be made small enough before stops are decoupled. Then $m_{h_u}^2$ at the m_Z scale can be estimated using the Coleman-Weinberg potential [1,8]:

$$\begin{aligned} m_{h_u}^2(m_Z) \approx{}& m_{h_u}^2(\Lambda_T) + \frac{3|y_t|^2}{16\pi^2}\left[m_{q_3}^2\left\{\log\frac{m_{q_3}^2}{\Lambda_T^2} - 1\right\}\right. \\ &\left. + m_{u_3^c}^2\left\{\log\frac{m_{u_3^c}^2}{\Lambda_T^2} - 1\right\} - m_t^2\left\{\log\frac{m_t^2}{\Lambda_T^2} - 1\right\}\right] \\ \approx{}& m_{h_u}^2(\Lambda_T) - \frac{3|y_t|^2}{16\pi^2}(m_{q_3}^2 + m_{u_3^c}^2)|_{\Lambda_T}, \end{aligned} \tag{4}$$

where the cutoff Λ_T is set to the stop decoupling scale [$\approx (m_{q_3} m_{u_3^c})^{1/2}$], and the top quark mass (m_t) contributions are relatively suppressed. Since the m_0^2 dependence of stop masses would be loop-suppressed, $m_{h_u}^2$ needs to be well-focused around Λ_T. Due to the additional negative contribution to $m_{h_u}^2(m_Z)$ below Λ_T, a small positive $m_{h_u}^2(\Lambda_T)$ would be more desirable.

In order to push up the FP to the desired stop mass scale 3–4 TeV, in this paper we suggest to combine the two representative SUSY breaking mediation scenarios, the mGrM and the minimal gauge mediation (mGgM) in a single supergravity (SUGRA) framework with a *common* SUSY breaking source. We will call it "minimal mixed mediation."

The chiral SUGRA Lagrangian is basically described in terms of the Kähler potential K, superpotential W, and gauge kinetic function. First, let us consider the minimal Kähler potential, and a superpotential where the observable and hidden sectors are separated as in the ordinary mGrM [1]:

$$K = \sum_{i,a}|z_i|^2 + |\phi_a|^2, \qquad W = W_H(z_i) + W_O(\phi_a), \tag{5}$$

where z_i [ϕ_a] denotes fields in the hidden [observable] sector, carrying hidden [SM or GUT] gauge quantum numbers. The kinetic terms of z_i and ϕ_a, thus, take the canonical form. We assume nonzero VEVs for z_is [2]:

$$\langle z_i\rangle = b_i M_P, \quad \langle\partial_{z_i} W_H\rangle = a_i^* m M_P, \quad \langle W_H\rangle = m M_P^2, \tag{6}$$

where a_i and b_i are dimensionless numbers, while M_P ($\approx 2.4\times 10^{18}$ GeV) denotes the reduced Planck mass. Then, $\langle W_H\rangle$ or m gives the gravitino mass, $m_{3/2} = e^{K/2M_P}\langle W\rangle/M_P^2 = e^{|b_i|^2/2} m$. The soft terms can read from the scalar potential of SUGRA:

$$V_F = e^{\frac{K}{M_P^2}}\left[|F_{z_i}|^2 + |F_{\phi_a}|^2 - \frac{3}{M_P^2}|W|^2\right], \tag{7}$$

where the "F-terms," F_i^* ($= D_i W = \partial_i W + \partial_i K W/M_P^2$) are given by

$$\begin{aligned} F_{z_i}^* &= \frac{\partial W_H}{\partial z_i} + z_i^*\frac{W}{M_P^2} = M_P\left[(a_i^* + b_i^*)m + b_i^*\frac{W_O}{M_P^2}\right], \\ F_{\phi_a}^* &= \frac{\partial W_O}{\partial\phi_a} + \phi_a^*\frac{W}{M_P^2} = \frac{\partial W_O}{\partial\phi_a} + \phi_a^*\left(m + \frac{W_O}{M_P^2}\right). \end{aligned} \tag{8}$$

The vanishing cosmological constant (C.C.) requires a fine-tuning between $\langle F_{z_i}\rangle$ and $\langle W_H\rangle$, i.e., from Eq. (7) $\sum_i\langle|F_{z_i}|^2\rangle = 3|\langle W_H\rangle|^2/M_P^2$, or $\sum_i|a_i + b_i|^2 = 3$. Neglecting the nonrenormalizable terms suppressed with $1/M_P^2$, Eq. (7) is rewritten as [2]

$$\begin{aligned} V_F \approx{}& |\partial_{\phi_a}\widetilde{W}_O|^2 + m_0^2|\phi_a|^2 \\ &+ m_0[\phi_a\partial_{\phi_a}\widetilde{W}_O + (A_\Sigma - 3)\widetilde{W}_O + \text{H.c.}], \end{aligned} \tag{9}$$

where A_Σ is defined as $A_\Sigma \equiv \sum_i b_i^*(a_i + b_i)$. m_0 is identified with the gravitino mass $m_{3/2}$ ($= e^{|b_i|^2/2}m$) and $\widetilde{W}_O$ ($\equiv e^{|b_i|^2/2}W_O$) denotes the rescaled W_0. From now on, we will drop out the "tilde" for a simple notation. The first term of Eq. (9) corresponds to the F-term potential in global SUSY, the second term is the universal soft mass term, and the remaining terms are A-terms. The *universal* A-parameter here ($\equiv A_0 = A_t$) does not include Yukawa coupling constants, but it is proportional to m_0. If there is no quadratic term or higher powers of ϕ_a in W_O, one can get negative (positive) A-terms with $A_\Sigma < 2$ ($A_\Sigma > 2$). With the vanishing C.C. condition, the universal soft mass parameter, m_0 ($= e^{\langle K\rangle/2M_P^2}\langle W_H\rangle/M_P^2$) can be recast to $e^{\langle K\rangle/2M_P^2}(\sum_i|\langle F_{z_i}\rangle|^2)^{1/2}/\sqrt{3}M_P$, which is the conventional form in the mGrM scenario.

Next, let us introduce one pair of messenger superfields $\{\mathbf{5},\bar{\mathbf{5}}\}$, which are the SU(5) fundamental representations, protecting the gauge coupling unification. Through their coupling with a SUSY breaking source S, which is an MSSM singlet superfield,

$$W_m = y_S S\mathbf{5}\bar{\mathbf{5}}, \tag{10}$$

the soft masses of the MSSM gauginos and scalar superpartners are generated at one- and two-loop levels, respectively [1]:

$$M_a = \frac{g_a^2}{16\pi^2}\frac{\langle F_S\rangle}{\langle S\rangle}, \qquad m_i^2 = 2\sum_{a=1}^{3}\left[\frac{g_a^2}{16\pi^2}\frac{\langle F_S\rangle}{\langle S\rangle}\right]^2 C_a(i), \tag{11}$$

where $C_a(i)$ is the quadratic Casimir invariant for a superfield i, $(T^aT^a)_i^j = C_a(i)\delta_i^j$, and g_a ($a = 3,2,1$) denotes the MSSM gauge coupling constants. $\langle S\rangle$ and $\langle F_S\rangle$ are VEVs of the scalar and F-term components of the superfield S. Note that M_a and m_i^2 are almost independent of y_S only if $\langle F_S\rangle \lesssim y_S^2\langle S\rangle$ [1]. However, such mGgM effects would appear below the messenger scale, $y_S\langle S\rangle$. Here we assume that $\langle S\rangle$ has the same magnitude as the VEV of the SU(5) breaking Higgs v_G: $\langle \mathbf{24}_H\rangle = v_G\times \mathrm{diag}(2,2,2;-3,-3)/\sqrt{60}$. It is possible if a GUT breaking mechanism causes $\langle S\rangle$ [9]. Actually, the masses of "X" and "Y" gauge bosons induced by $\langle \mathbf{24}_H\rangle$, $M_X^2 = M_Y^2 = \frac{5}{24}g_G^2v_G^2$ [10], where g_G is the unified gauge coupling constant, can be identified with the MSSM gauge coupling unification scale, because the unified gauge interactions would become active above the $M_{X,Y}$ scale.

In addition to Eq. (5), the Kähler potential (and hidden local symmetries we do not specify here) can permit

$$K \supset f(z)S + \mathrm{H.c.}, \tag{12}$$

where $f(z)$ denotes a *holomorphic* monomial of hidden sector fields z_is with VEVs of order M_P in Eq. (6), and so it is of order $\mathcal{O}(M_P)$. Their kinetic terms still remain canonical. The $\mathrm{U}(1)_R$ symmetry forbids $M_Pf(z)S$ in the superpotential. Then, the resulting $\langle F_S\rangle$ can be

$$\langle F_S\rangle \approx m[\langle f(z)\rangle + \langle S^*\rangle] \tag{13}$$

by including the SUGRA corrections with $\langle W_H\rangle = mM_P^2$. Thus, the VEV of F_S is of order $\mathcal{O}(mM_P)$ like F_{z_i} in Eq. (8). They should be fine-tuned for the vanishing C.C.: a precise determination of $\langle F_S\rangle$ is indeed associated with the C.C. problem. Here we set $\langle F_S\rangle = m_0M_P$. F_{ϕ_a} is still given by Eq. (8), which induces the universal soft mass terms at tree level.

Thus, the typical size of mGgM effects is estimated as

$$\frac{\langle F_S\rangle}{16\pi^2\langle S\rangle} = \frac{m_0M_P}{16\pi^2M_X}\sqrt{\frac{5}{24}}g_G \approx 0.36\times m_0. \tag{14}$$

Here we set the unified gauge coupling at the GUT scale [$\approx(1.3\pm 0.4)\times 10^{16}$ GeV] to $g_G^2/4\pi \approx 1/26$ due to relatively heavy colored superpartners ($\gtrsim$3 TeV). Even for $|y_S| \ll 1$, we will keep this value, since it is fixed by a UV model.

The fact that the mGgM effects by Eq. (11) are proportional to m_0 or m_0^2 are important. Moreover, A-terms from Eq. (9) are also proportional to m_0. In this setup, thus, an (extrapolated) FP of $m_{h_u}^2$ must still exist at a higher energy scale [9]. As C_g is converted to a member of C_s in Eq. (3), the naturalness of $m_{h_u}^2$ and m_Z^2 becomes gradually improved, making C_s smaller and smaller, until the FP reaches the stop decoupling scale.

For $|y_S| \lesssim 1$ in Eq. (10), the messenger scale Q_M drops down below $M_{X,Y}$. Since X and Y gauge sectors have already been decoupled below the messenger scale, the soft masses generated by the mGgM in Eq. (11) become *nonuniversal* for $Q_M < M_{X,Y}$. Of course, the beta function coefficients of the MSSM fields should be modified above the scale of $y_S\langle S\rangle$ by the messenger fields $\{\mathbf{5},\bar{\mathbf{5}}\}$. Thus, the RG equations of the MSSM gauge couplings and gaugino masses are

$$8\pi^2\frac{dg_a^2}{dt} = b_ag_a^4, \qquad 8\pi^2\frac{dM_a}{dt} = b_ag_a^2M_a, \tag{15}$$

where $t \equiv \log[Q/\mathrm{GeV}]$, and $b_a = (-2,2,\frac{38}{5})$ for $Q > Q_M$ while $b_a = (-3,1,\frac{33}{5})$ for $Q < Q_M$. For the RG equations of the Yukawa couplings of the third generation of quarks and leptons (y_t, y_b, y_τ) and other soft parameters, refer to the Appendix of Ref. [7].

The boundary conditions at the GUT scale are given by the universal form as seen in Eq. (9). Unlike the case of the mGrM, we have *additional* nonuniversal contributions by Eq. (11). They should be imposed at a given messenger scale, and so affect the RG evolutions of MSSM parameters for $Q \le Q_M$. To see clearly how the original FP scenario is modified by the additional mGgM effects, in this paper we do not consider the superheavy RH neutrinos in the RG analysis as in [5], assuming their couplings are small enough, even if they are helpful for improving the naturalness [7,11].

We also suppose that the gaugino masses from the mGrM are relatively suppressed. In fact, the gaugino mass term in SUGRA is associated with the first derivative of the gauge kinetic function [2], and so a constant gauge kinetic function at tree level ($= \delta_{ab}$) can realize it. In fact, it is the simplest case, yielding the canonical gauge kinetic terms in the Lagrangian. Accordingly, the gaugino masses by Eq. (11) dominates over them in this case. Then Eqs. (11), (14), and (15) admit a simple analytic expression for the gaugino masses at the stop mass scale:

$$M_a(t_T) \approx 0.36 \times m_0 \times g_a^2(t_T), \tag{16}$$

It does *not* depend on messenger scales.

Figure 1 displays RG evolutions of $m_{h_u}^2$ for various trial m_0^2s. The solid [dotted] lines correspond to the case of $t_M \approx 37$ (or $Q_M \approx 1.3 \times 10^{16}$ GeV, "Case A") [$t_M \approx 23$ (or $Q_M = 1.0 \times 10^{10}$ GeV, "Case B")]. The discontinuities of the lines by additional boundary conditions arise at the messenger scales. As seen in Fig. 1, a FP of $m_{h_u}^2$ appears always at $t = t_T \approx 8.2$ (or $Q_T \approx 3.5$ TeV) *regardless of the messenger scales* that we take. Hence, the wide ranges of UV parameters can yield almost the same values of $m_{h_u}^2$ at low energy. Under such a situation, one can guess that $m_0^2 \approx (4.5\ \text{TeV})^2$ happens to be selected by the Nature, yielding 3–4 TeV stop mass, and so eventually gets responsible for the 126 GeV Higgs mass.

In both cases of Fig. 1, the gluino, wino, and bino (superpartners of the SM gauge bosons) masses at low energy are

$$M_{3,2,1} \approx \{1.7\ \text{TeV}, 660\ \text{GeV}, 360\ \text{GeV}\} \tag{17}$$

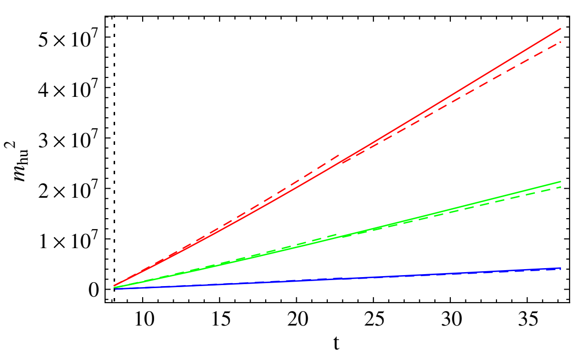

FIG. 1 (color online). RG evolutions of $m_{h_u}^2$ with t [$\equiv \log(Q/\text{GeV})$] for $m_0^2 = (7\ \text{TeV})^2$ [Red], $(4.5\ \text{TeV})^2$ [Green], and $(2\ \text{TeV})^2$ [Blue] when $A_t = -0.2m_0$ and $\tan\beta = 50$. The tilted solid [dotted] lines correspond to the case of $t_M \approx 37$ (or $Q_M \approx 1.3 \times 10^{16}$ GeV, "Case A") [$t_M \approx 23$ (or $Q_M = 1.0 \times 10^{10}$ GeV, "Case B")]. The vertical dotted line at $t = t_T \approx 8.2$ ($Q_T = 3.5$ TeV) indicates the desired stop decoupling scale. The discontinuities of $m_{h_u}^2(t)$ should appear at the messenger scales.

for $m_0^2 = (4.5\ \text{TeV})^2$. They are the prediction of this model. They would be testable at LHC run2. A_t at low energy is about 1 TeV for Case A and B. Consequently, the contributions of $A_t^2/\widetilde{m}_t^2$ to the radiative Higgs mass are smaller than 2.3% of those by the stops.

Table I lists the soft squared masses at $t = t_T$ for the LH and RH stops, and the two MSSM Higgs bosons under the various m_0^2s, when the messenger scale is $Q_M \approx 1.3 \times 10^{16}$ GeV, and $\tan\beta$ is 50 or 25. We can see the changes of $m_{h_u^2}^2$ are quite small [$\ll (550\ \text{GeV})^2$] under the changes of m_0^2 [$(5.5\ \text{TeV})^2$–$(3.5\ \text{TeV})^2$] unlike the other soft squared masses, because $m_{h_u}^2$ is well focused at $t = t_T$. Cases I-IV yield again the same low energy gauginos masses as Eq. (17), because Eq. (16) is valid

TABLE I. Soft squared masses of the stops and Higgs bosons at $t = t_T \approx 8.2$ ($Q_T = 3.5$ TeV) for various trial m_0^2s when the messenger scale is $Q_M \approx 1.3 \times 10^{16}$ GeV. $\Delta_{m_0^2}$ indicates the fine-tuning measure for m_0^2 around $(4.5\ \text{TeV})^2$ for each case. $m_{h_u}^2$ s further decrease to be negative below $t = t_T$. The above mass spectra are generated using SOFTSUSY [12].

Case I	$A_t = 0$	$\tan\beta = 50$	$\mathbf{\Delta_{m_0^2} = 1}$
$\mathbf{m_0^2}$	$(\mathbf{5.5}\ \text{TeV})^2$	$(\mathbf{4.5}\ \text{TeV})^2$	$(\mathbf{3.5}\ \text{TeV})^2$
$m_{q_3}^2(t_T)$	$(4363\ \text{GeV})^2$	$(3551\ \text{GeV})^2$	$(2744\ \text{GeV})^2$
$m_{u_3^c}^2(t_T)$	$(3789\ \text{GeV})^2$	$(3098\ \text{GeV})^2$	$(2406\ \text{GeV})^2$
$\mathbf{m_{h_u}^2(t_T)}$	$(\mathbf{431}\ \text{GeV})^2$	$(\mathbf{189}\ \text{GeV})^2$	$-(\mathbf{251}\ \text{GeV})^2$
$m_{h_d}^2(t_T)$	$(2022\ \text{GeV})^2$	$(1512\ \text{GeV})^2$	$(1008\ \text{GeV})^2$
Case II	$A_t = -0.2m_0$	$\tan\beta = 50$	$\mathbf{\Delta_{m_0^2} = 16}$
$\mathbf{m_0^2}$	$(\mathbf{5.5}\ \text{TeV})^2$	$(\mathbf{4.5}\ \text{TeV})^2$	$(\mathbf{3.5}\ \text{TeV})^2$
$m_{q_3}^2(t_T)$	$(4376\ \text{GeV})^2$	$(3563\ \text{GeV})^2$	$(2752\ \text{GeV})^2$
$m_{u_3^c}^2(t_T)$	$(3798\ \text{GeV})^2$	$(3106\ \text{GeV})^2$	$(2413\ \text{GeV})^2$
$\mathbf{m_{h_u}^2(t_T)}$	$(\mathbf{539}\ \text{GeV})^2$	$(\mathbf{361}\ \text{GeV})^2$	$-(\mathbf{44}\ \text{GeV})^2$
$m_{h_d}^2(t_T)$	$(2053\ \text{GeV})^2$	$(1565\ \text{GeV})^2$	$(1046\ \text{GeV})^2$
Case III	$A_t = -0.5m_0$	$\tan\beta = 50$	$\mathbf{\Delta_{m_0^2} = 9}$
$\mathbf{m_0^2}$	$(\mathbf{5.5}\ \text{TeV})^2$	$(\mathbf{4.5}\ \text{TeV})^2$	$(\mathbf{3.5}\ \text{TeV})^2$
$m_{q_3}^2(t_T)$	$(4284\ \text{GeV})^2$	$(3532\ \text{GeV})^2$	$(2630\ \text{GeV})^2$
$m_{u_3^c}^2(t_T)$	$(3755\ \text{GeV})^2$	$(3088\ \text{GeV})^2$	$(2373\ \text{GeV})^2$
$\mathbf{m_{h_u}^2(t_T)}$	$-(\mathbf{363}\ \text{GeV})^2$	$-(\mathbf{41}\ \text{GeV})^2$	$-(\mathbf{546}\ \text{GeV})^2$
$m_{h_d}^2(t_T)$	$(1447\ \text{GeV})^2$	$(1359\ \text{GeV})^2$	$-(950\ \text{GeV})^2$
Case IV	$A_t = 0$	$\tan\beta = 25$	$\mathbf{\Delta_{m_0^2} = 57}$
$\mathbf{m_0^2}$	$(\mathbf{5.5}\ \text{TeV})^2$	$(\mathbf{4.5}\ \text{TeV})^2$	$(\mathbf{3.5}\ \text{TeV})^2$
$m_{q_3}^2(t_T)$	$(4915\ \text{GeV})^2$	$(4025\ \text{GeV})^2$	$(3134\ \text{GeV})^2$
$m_{u_3^c}^2(t_T)$	$(3770\ \text{GeV})^2$	$(3086\ \text{GeV})^2$	$(2400\ \text{GeV})^2$
$\mathbf{m_{h_u}^2(t_T)}$	$(\mathbf{152}\ \text{GeV})^2$	$-(\mathbf{220}\ \text{GeV})^2$	$-(\mathbf{293}\ \text{GeV})^2$
$m_{h_d}^2(t_T)$	$(5057\ \text{GeV})^2$	$(4136\ \text{GeV})^2$	$(3215\ \text{GeV})^2$

at low energy, independent of A_t and $\tan\beta$. A_t at low energy turns out to be around 1 TeV or smaller for $m_0^2 = (4.5\ \text{TeV})^2$, and so its contribution to the Higgs boson mass is still suppressed. By Eq. (4) $m_{h_u}^2$s further decrease to be negative below $t = t_T$. With Eq. (2) $|\mu|$ are determined as $\{485\ \text{GeV}, 392\ \text{GeV}, 516\ \text{GeV}, 586\ \text{GeV}\}$ for Case I, II, III, and IV, respectively. Actually the RG running of μ is completely separated from other soft parameters. Moreover, the generation scale of μ is quite model-dependent. So we do not discuss them here. To avoid a potential fine-tuning issue associated with μ, however, we confine our discussion to cases of $|\mu| < 600$ GeV.

From Table I, we can read the A_t dependence of the fine-tuning measure $\Delta_{m_0^2}$ ($\equiv|\frac{\partial \log m_Z^2}{\partial \log m_0^2}| = |\frac{m_0^2}{m_Z^2}\frac{\partial m_Z^2}{\partial m_0^2}|$ [13]) around $m_0^2 = (4.5\ \text{TeV})^2$. Case I gives almost the minimum of $\Delta_{m_0^2}$ ($= 1$) when $\tan\beta = 50$. On the other hand, Δ_{A_t} ($= |\frac{A_t}{m_Z^2}\frac{\partial m_Z^2}{\partial A_t}|$) are $\{0, 10, 118, 0\}$ for Case I, II, III, and IV, respectively. When $A_t/m_0 = +0.1$, $\{\Delta_{m_0^2}, \Delta_{A_t}, |\mu|\}$ turn out to be about $\{22, 33, 569\ \text{GeV}\}$. Therefore, we can conclude the parameter range

$$-0.5 < A_t/m_0 \lesssim +0.1 \quad \text{and} \quad \tan\beta \gtrsim 25 \qquad (18)$$

allows $\{\Delta_{m_0^2}, \Delta_{A_t}\}$ and $|\mu|$ to be smaller than 100 and 600 GeV, respectively. We see that a larger $\tan\beta$ would be preferred for a smaller $\Delta_{m_0^2}$. It is basically because $m_{h_d}^2$ is not focused unlike $m_{h_u}^2$, even though it also contributes to m_Z^2 as seen in Eq. (2). Actually $\tan\beta = 50$ is easily obtained, e.g., from the minimal SO(10) GUT [10].

In the above cases, the sleptons and sbottom (superpartners of the leptons and b-quark) turn out to be quite heavier than 3 TeV. The first two generations of SUSY particles would be much heavier than them. Hence, the bino is the lightest superparticle (LSP). To avoid overclose of the bino dark matter in the Universe, some entropy production [14] or other lighter dark matter such as the axino and axion is needed [15]. Further numerical analyses on the parameter space will be found in other literatures [9].

In conclusion, we have noticed that a FP of $m_{h_u}^2$ appears at 3–4 TeV, when the mGrM and mGgM effects are combined at the GUT scale for a common SUSY breaking source parametrized with m_0, and the gaugino masses are dominantly generated by the mGgM effects. Even for a 3–4 TeV stop mass explaining the 126 GeV Higgs mass, thus, the fine-tuning measures significantly decrease well below 100 for $-0.5 < A_t/m_0 \lesssim +0.1$ and $\tan\beta \gtrsim 25$ in the minimal mixed mediation. In this range, $|\mu|$ is smaller than 600 GeV. The expected gluino mass is about 1.7 TeV, which could readily be tested at LHC run2.

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