Effective field theory analysis of Higgs naturalness

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Assuming the presence of physics beyond the Standard Model (SM) with a characteristic scale $M \sim \mathcal{O}(10)$ TeV, we investigate the naturalness of the Higgs sector at scales below M using an effective field theory (EFT) approach. We obtain the complete set of higher-dimensional effective operators (at any dimension $n \geq 5$) that give the leading one-loop EFT contributions to the Higgs mass with a Wilsonian-like hard cutoff and discuss the (fine-) tuning between these terms and the SM one-loop contribution, which is required in order to alleviate the *little hierarchy problem*. We then show that this tuning can be translated into a condition for naturalness in the underlying new physics, a condition we denote by "EFT naturalness" and which we express as constraints on the corresponding higher-dimensional operator coefficients up to the scale of the effective action $\Lambda < M$. We also determine the types of physics that can lead to EFT naturalness and discuss the current experimental constraints on the relevant operator coefficients. We show that these types of new physics are best probed in vector-boson and multiple-Higgs production.

I. INTRODUCTION

The recent LHC discovery of a light 126 GeV scalar particle [1] brought us one step closer to understanding the mechanism of electroweak symmetry breaking. Indeed, the measurements of its production and decays to the SM gauge bosons [2] are consistent (within large errors) with the SM. Moreover, in view of the fact that no evidence for new physics has been observed yet up to energies of $\sim$1–2 TeV and that the SM with a 126 GeV Higgs seems to be a consistent theory up to the Planck scale (favoring a metastable EW vacuum [2–4]), this discovery exacerbates the long-standing fundamental difficulty of the SM known as the hierarchy problem. Simply put, the presence of a fundamental Higgs with an EW-scale mass appears unnatural, since if the SM is the only physics present up to some high scale Λ, it is then hard to see why the Higgs boson mass m_h does not receive large corrections of $O(\Lambda)$. This technical difficulty is also known as the naturalness or fine-tuning problem of the SM. It becomes evident when one calculates the SM's leading $O(\Lambda^2)$ one-loop corrections to the Higgs mass squared with a hard cutoff:

$$\delta m_h^2(\text{SM}) = \frac{\Lambda^2}{16\pi^2}[24x_t^2 - 6(2x_W^2 + x_Z^2 + x_h^2)] \sim 8.2\frac{\Lambda^2}{16\pi^2},$$
$$x_i \equiv \frac{m_i}{v}(v \simeq 246 \text{ GeV}), \qquad (1)$$

where the dominant contribution is generated by the top-quark loop. This gives $\delta m_h^2(\text{SM}) \approx m_h^2$ already for $\Lambda \sim 550$ GeV when $m_h \sim 125$ GeV, and the Higgs mass is then said to be unnatural above this scale.

In Wilson's approach [5], the hard cutoff Λ in Eq. (1) corresponds to the scale of the effective action—in the following we will use this picture to investigate the behavior of δm_h^2 in the presence of new physics (NP) with a mass scale $M > \Lambda$. Indeed, this approach [Eq. (1)] has been the underlying rationale for intensive theoretical studies of physics beyond the SM over the past four decades [6–8]. In particular, although the cutof-dependent $\delta m_h^2(\text{SM})$ is an unobservable quantity, the question of naturalness in the Higgs sector becomes meaningful if one suspects that there is an actual physical cutoff at high energy. In such a case, the insight provided by Eq. (1) is regularization-scheme independent, since even within dimensional regularization a new heavy mass threshold M will yield an additive (one-loop) correction to the Higgs mass of $\mathcal{O}(M^2/(16\pi^2))$ [7,9], which in Eq. (1) is represented by $\Lambda^2/(16\pi^2)$.

The first to address the naturality consequences of the quadratic cutoff behavior in $\delta m_h^2(\text{SM})$ was Veltman [6], who suggested that a conspiracy or symmetry (possibly supersymmetry) leads to the cancellation $24x_t^2 - 6(2x_W^2 + x_Z^2 + x_h^2) = 0$ in Eq. (1). Our approach in this work bares some similarity to Veltman's concept.

In particular, let us imagine the presence of NP with a characteristic scale $M > \Lambda > v$. As mentioned above, in this case Eq. (1) will be modified as the heavy excitations will generate new contributions to δm_h^2. These contributions can be derived within specific models such as

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little-Higgs and supersymmetric theories or within phenomenological extensions of the SM which contain additional heavy scalars and/or fermions [9–12]. In such a case, one assumes full knowledge of the physics up to a yet higher scale (i.e., larger than M), above which the selected model breaks down (or is subsumed by a more fundamental theory); accordingly, in model-dependent studies the scale of the effective action, Λ, can be extended beyond M, i.e., beyond the typical mass scale of the new particles of a specific theory.

Alternatively, one can adopt a model-independent approach using an effective field theory (EFT). The EFT approach refrains from selecting a specific model but allows a reliable calculation of δm_h^2 *only when the cutoff is below* M; this approach can be used to impose general restrictions on the parameters of the unknown underlying theory by imposing the condition that the EFT remains natural for all scales $\Lambda < M$.

In this paper we wish to investigate naturalness using this EFT approach, following Wilson's prescription with a hard cutoff and assuming also that the NP is weakly coupled and renormalizable (or, alternatively, that all nonrenormalizable terms in the theory are suppressed by inverse powers of a much higher scale $\gg M$). Thus, at scales below M the NP is not directly observable,[1] but it can have important virtual effects that generate both renormalization of the SM parameters and an infinite tower of effective operators with dimension ≥ 5.

We find the complete set of higher-dimensional effective operators (at any dimension $n \geq 5$) that can yield $\mathcal{O}(\Lambda^2)$ contributions to δm_h^2 and classify the underlying heavy theories that can generate these operators at tree level. We then evaluate their effect on δm_h^2 and discuss the tuning between these EFT terms and the SM contribution that is required in order to obtain "EFT-naturalness" conditions. Namely, the conditions and relations among the EFT parameters for which naturalness in the SM Higgs sector can be ameliorated, i.e., addressing the little hierarchy problem of the SM Higgs sector up to the scale of the effective action Λ^2; if the tuning needed is of $\mathcal{O}(10^{-4})$ or worse, then we implicitly assume that there is an underlying symmetry responsible for our EFT-naturalness conditions.

Let us denote the higher-dimensional operators by $\mathcal{O}_i^{(n)}$ (n denotes the dimension and i all other distinguishing labels), which are local, gauge- and Lorentz-invariant

combinations of SM fields and their derivatives. They result from integrating out the heavy degrees of freedom of the heavy NP theory that underlies the SM and expanding in inverse powers of M after appropriate renormalization of the SM parameters.[3]

The effective Lagrangian then takes the form [16–19]

$$\mathcal{L}_{\text{eff}} = \sum_{n=5}^{\infty} \frac{1}{M^{(n-4)}} \sum_i f_i^{(n)} \mathcal{O}_i^{(n)}. \qquad (2)$$

Different types of NP can generate the *same* operators, but, in general, with *different* coefficients, so that the SM renormalization constants and the operator coefficients parametrize all possible types of NP. Moreover, some of the $\mathcal{O}$'s are *necessarily* generated by loops involving only heavy particles [17,19], and we label such operators "loop-generated" (LG); this is a useful distinction because graphs involving LG operators and ℓ SM loops are considered to be at least $\ell + 1$ loop diagrams.

II. EFT AND THE ONE-LOOP HIGGS MASS CORRECTIONS

In general, *all* (SM and NP) one-loop corrections to m_h^2 are generated by the graphs in Fig. 1. In the scenarios we are interested in here, these corrections can be separated into three categories:

$\delta m_h^2(\text{SM})$: When all internal lines are the light SM fields. The contributions from this category are given in Eq. (1).

$\delta m_h^2(\text{Hvy})$: When all internal lines are heavy fields of the underlying NP. The contributions from this category are contained in the renormalization of the parameters of the SM that follows upon integration of the heavy particles. This is included in what we denote here as "tree-level" parameters, i.e., $m_h^2(\text{tree}) = m_h^2(\text{bare}) + \delta m_h^2(\text{Hvy})$.

$\delta m_h^2(\text{eff})$: When one line is heavy and the other is light (in graphs (b) and (c) in Fig. 1). The contributions in this category are generated by the effective Lagrangian in Eq. (2) and are the ones we are interested in here.

As noted earlier, the "little hierarchy problem" of the SM refers to the fact that $\delta m_h^2(\text{SM}) > m_h^2(\text{tree})$ when $\Lambda \gtrsim 500$ GeV, assuming $m_h(\text{tree})$ is close to the observed value $m_h(\text{tree}) \simeq m_h \simeq 125$ GeV. Our aim here is only to address this problem at scales below Λ, viewed as the scale of the Wilsonian effective action; we will not be concerned with the issues related to the UV completion of the SM or EW-Planck hierarchy or with any details of the underlying theory giving rise to Eq. (2). Specifically, we will study the role that the effective interactions in Eq. (2) may play in

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[1]At scales above M the NP becomes manifest, and naturalness issues related to quadratic divergences may arise in connection with new heavy scalar particles that might be present, but such complications will not affect the conditions under which heavy new physics may tame the little hierarchy problem at scales *below* Λ, which is our only concern in this paper. Note that fermionic solution(s) do not suffer from this difficulty.

[2]We note in passing that higher-dimensional NP operators in the SM Higgs sector may also have a significant effect on the stability of the EW vacuum [13].

[3]We adopt the minimal coupling scheme in constructing the higher dimensional effective operators below, which is consistent with the assumption of a weakly coupled and renormalizable underlying NP (see e.g., [14]); the compatibility of minimal coupling with EFT was recently discussed in the literature (see [14,15]).
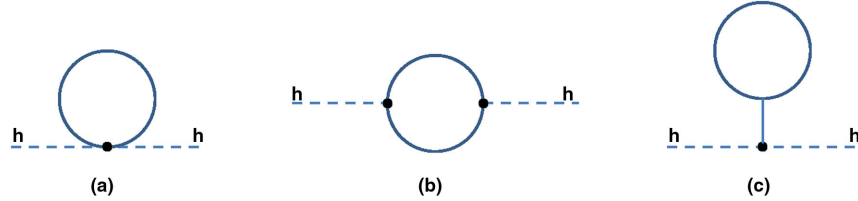
FIG. 1 (color online).   The one-loop graphs generating δm_h^2. The internal lines represent bosons or fermions from either the SM or the heavy NP.
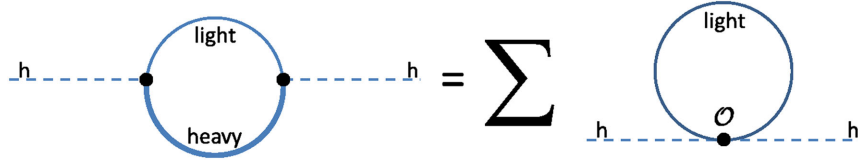

FIG. 2 (color online).   Description of the manner in which the effective Lagrangian in Eq. (2) generates graphs in category $\delta m_h^2(\text{eff})$ defined in the text.

restoring naturalness to the Higgs sector at any given intermediate scale $v < \Lambda < M$[4] and determine the conditions under which $\delta m_h^2(\text{SM}) + \delta m_h^2(\text{eff}) \lesssim m_h^2$ when $m_h \ll \Lambda \leq M$. We now proceed to the calculation of $\delta m_h^2(\text{eff})$.

To illustrate the manner in which the effective operators contribute to δm_h^2, we consider the contributions to the Higgs mass generated by the diagrams in Fig. 2. Expanding the heavy propagator in powers of its (large) mass M, one generates an infinite series of vertices suppressed by inverse powers of M (see Fig. 2 for a schematic depiction). As mentioned above, we will evaluate loop graphs using a cutoff prescription (with Λ being the cutoff) so that this expansion remains valid for the graphs in Fig. 2 (recall that $M \geq \Lambda$) and the effective vertices correspond to those generated by the effective operators in Eq. (2). We, therefore, need the set of operators $\mathcal{O}$ which are not LG, and contribute to graph Fig. 1(a), where the vertex is generated by the effective operator.

The non-LG operators that give $O(\Lambda^2)$ contributions to δm_h^2 can be characterized using the following arguments: the internal lines in the graphs on the right-hand side of Fig. 2 (with the $\mathcal{O}$-generated vertices) can be either the SM scalar, fermions or vectors. For the first case, $\mathcal{O}$ must contain at least four SM scalar doublets; but if it contains more than four such scalar fields, the corresponding contributions to δm_h^2 are suppressed by powers of (v/M) and are, therefore, subdominant. Thus, leading contributions with a scalar internal line are generated by effective operators with precisely four scalar doublets. Similarly, if

the internal lines are fermions or vectors, the operators must contain two SM scalar doublets. Lastly, it is straightforward to show [17,19] that operators with two SM scalar doublets, no fermions and any number of vectors are LG and are also subdominant.

Summarizing: the operators that generate one-loop $O(\Lambda^2)$ contributions to δm_h^2 can be of *only* two types:

(i) Type I: $\mathcal{O}$ contains four scalar fields, any number of derivatives and is not LG.

(ii) Type II: $\mathcal{O}$ contains two fermions and two scalar fields, any number of derivatives and is not LG.

The simplest way to determine the form of the operators of types I and II is by recalling that these operators are generated at tree level in the underlying heavy theory by the graphs in Fig. 3, where the relevant $\mathcal{O}$ is obtained by expanding the propagators in inverse power of the internal heavy mass and imposing gauge invariance. We can also eliminate operators with derivatives that act on scalar fields connected to the external legs, since δm_h^2 is evaluated at zero momentum.[5]

A further simplification follows from a more careful study of the diagram shown in Fig. 3(c), which contributes to δm_h^2 only through graphs of the type depicted in Fig. 1(c), for which the heavy boson must be a scalar. This heavy scalar must also be an $SU(2)$ triplet or singlet (since it couples to two SM isodoublets), which implies that the fermions must have the same chirality (since a pair of fermions with different chirality cannot form a singlet or a triplet). It then follows that the loop in Fig. 1(c) must involve a chirality flip, so that its contribution to δm_h^2 will

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[4]It is important to note that Eq. (2) can be used to calculate such NP effects provided all energies (including those that appear within loop calculations) are kept below M so the cutoff must obey $\Lambda < M$.

[5]If δm_h^2 is evaluated at some other low scale, e.g., at $\mu = m_h$, then vertices where a derivative acts on an external leg are also subdominant since their contribution will be suppressed by a factor of m_h/Λ.
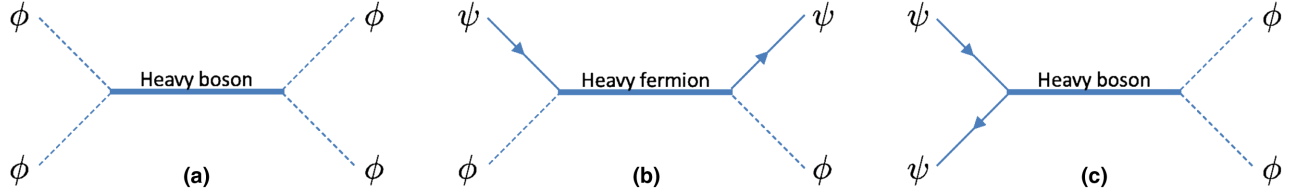
FIG. 3 (color online). Tree-level graphs that generate the effective operators of type I (diagram a) and II (diagrams b and c) that can produce leading corrections to δm_h^2. ϕ and ψ denote the SM scalar doublet and fermions, respectively, and all vertices are understood to be invariant under SM gauge transformations.

be suppressed by a factor of m_ψ/Λ and is, therefore, also subdominant. We, thus, conclude that we can neglect the effective operators associated with Fig. 3(c).

With the above comments, it is a straightforward exercise to obtain the relevant set of effective operators of interest. Those generated by heavy scalar exchanges in Fig. 3(a) are

$$\mathcal{O}_S^{(2k+4)} = \frac{1}{2}|\phi|^2\Box^k|\phi|^2,$$
$$\mathcal{O}_\chi^{(2k+4)} = \frac{1}{2}(\phi^\dagger\tau_I\phi)D^{2k}(\phi^\dagger\tau_I\phi),$$
$$\mathcal{O}_{\tilde\chi}^{(2k+4)} = \frac{1}{4}(\phi^\dagger\tau_I\tilde\phi)D^{2k}(\tilde\phi^\dagger\tau_I\phi), \quad (3)$$

which correspond to the cases where the heavy scalar is a SM gauge singlet (labeled S) or an isotriplet of hypercharge 0 or 1 (labeled χ and $\tilde\chi$, respectively). There are no other operators of this type since S,χ, and $\tilde\chi$ are the only possible three states that can be formed with two SM scalar isodoublets. In the following, we denote these heavy scalars collectively by Φ.

Similarly the operators generated by heavy vector exchanges in Fig. 3(a) are

$$\mathcal{O}_v^{(2k+6)} = \frac{1}{2}j_\mu\Box^k j^\mu, \qquad \mathcal{O}_{\tilde v}^{(2k+6)} = \tilde j_\mu^\dagger\Box^k\tilde j^\mu,$$
$$\mathcal{O}_\mathbf{V}^{(2k+6)} = \frac{1}{6}J_{I\mu}D^{2k}J_I^\mu, \quad (4)$$

where the currents are

$$j^\mu = i\phi^\dagger D^\mu\phi + \text{H.c.}, \qquad \tilde j^\mu = i\tilde\phi^\dagger D^\mu\phi,$$
$$J_I^\mu = i\phi^\dagger\tau^I D^\mu\phi + \text{H.c.}, \quad (5)$$

and the labels in Eq. (4) refer to heavy vector isosinglets $(v, \tilde v)$ of hypercharge 0 or 1, respectively, and a heavy vector isotriplet ($\mathbf{V}$) of hypercharge 0. In the following we will collectively denote these heavy vectors by X.[6]

Finally, the graph in Fig. 3(b) involves an exchange of a heavy fermion Ψ which may or may not be colored and has the same quantum numbers as $\phi\psi$ or $\tilde\phi\psi$. That is, Ψ can be an isosinglet, doublet or triplet heavy lepton or quark of hypercharge $y_\Psi = y_\psi \pm 1/2$ (y_r denotes the hypercharge of r). These Ψ-generated operators are

$$\mathcal{O}_{\Psi-\psi}^{(2k+4)} = |\phi|^2\bar\psi(iD)^{2k-1}\psi, \quad (k \geq 1), \quad (6)$$

where ψ is any SM fermion.[7] Another type of operator that may be generated by the heavy-fermion exchange is $(\phi^\dagger\tau_I\phi)(\bar\psi\tau_I D^{2k-1}\psi)$, where ψ is an isodoublet. However, this operator will yield a contribution to δm_h^2 which is suppressed by a factor of m_ψ^2/Λ^2 and is, therefore, also subdominant.

Note that the graphs in Fig. 3 represent the possible types of NP that can generate the effective operators in Eqs. (3), (4) and (6) at tree level. There are other types of NP that can also generate these operators, but only via loop diagrams. It then follows that the coefficients of the operators associated with the same heavy particle are correlated; we return to this point below.

Calculating the one-loop quadratic corrections to m_h^2 which are generated by the operators in Eqs. (3), (4) and (6), we obtain

$$\delta m_h^2(\text{eff}) = -\frac{\Lambda^2}{16\pi^2}F^{(\text{eff})}, \quad (7)$$

where ($\Phi = S,\chi,\tilde\chi$ and $X = v,\tilde v,\mathbf{V}$)[8]

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[6]There is a fourth current that can be constructed using two scalar fields, namely, $\tilde{\mathbf{J}}^\mu = i\tilde\phi^\dagger\tau D^\mu\phi$; however, since $\tilde{\mathbf{J}}^\mu = D^\mu\mathbf{P}$ with $\mathbf{P} \equiv (i/2)\tilde\phi^\dagger\tau\phi$ holds identically, and since vector bosons do not have tree-level couplings to total derivatives, there are no tree-level operators involving $\tilde{\mathbf{J}}$. None of the other currents in Eq. (5) can be written as derivatives of scalar operators.

[7]It is, in principle, possible to eliminate the operator in Eq. (6) using the "equivalence theorem" of [19]. However in a cutoff scenario like the one we consider, this procedure involves a nontrivial Jacobian that will generate terms of the form $|\phi|^2\Lambda^2$, which will reproduce the contributions to δm_h^2 generated by $\mathcal{O}_{\Psi-\psi}$.

[8]When evaluating the contributions associated with the heavy vectors (sum over X) it is convenient to use a renormalizable gauge. The calculation in the unitary gauge is more involved since, in this case, the longitudinal component of the vector propagators are proportional to $1/v^2$, and this complicates isolating the leading contributions.

$$F^{(\text{eff})} = \sum_{k=0}^{\infty} \frac{(\Lambda/M)^{2k}}{k+1} \sum_{\Phi} f_{\Phi}^{(2k+4)}$$
$$- \sum_{k=0}^{\infty} \frac{(\Lambda/M)^{2k+2}}{k+2} \sum_{X} f_X^{(2k+6)}$$
$$- \sum_{k=1}^{\infty} \frac{(-1)^k (\Lambda/M)^{2k}}{k+1} \sum_{\Psi,\psi} f_{\Psi-\psi}^{(2k+4)}. \qquad (8)$$

Defining the measure for fine-tuning to be $\Delta_h \equiv |\delta m_h^2|/m_h^2$, where $\delta m_h^2 = \delta m_h^2(\text{SM}) + \delta m_h^2(\text{eff})$ and m_h^2 is the physical mass, $m_h^2 = m_h^2(\text{tree}) + \delta m_h^2$, we have[9]

$$\Delta_h = \frac{\Lambda^2}{16\pi^2 m_h^2} |F^{(\text{eff})} - 8.2|. \qquad (9)$$

It is evident from Eq. (9) that in order to restore naturalness at the effective action scale $\Lambda < M$, there must be a cancellation[10] between the $O(\Lambda^2)$ one-loop expressions generated in the SM [Eq. (1)] and those produced by the effective operators [Eq. (7)]; this cancellation can be partial or exact (e.g., due to a model or symmetry), leading to $\delta m_h^2 = O(v^2)$—in this case the tree-level contribution $m_h^2(\text{tree})$ will be of this order as well. It should be emphasized again that $F^{(\text{eff})}$ is the one-loop contribution to the Higgs mass calculated only from effective operators which are generated at tree level in the underlying NP [the ones given in Eqs. (3), (4) and (6)]; it is, thus, a dimensionless function of the NP parameters and the ratio Λ/M, so that the cancellation conditions will depend on Λ as well. Our requirement that Λ be the scale below which the SM little-hierarchy problem is solved is consistent within our EFT-naturalness scenario because of the requirement $M > \Lambda$.

Rewriting the above defined fine-tuning condition as $|m_h^2(\text{tree})/\delta m_h^2 + 1| = 1/\Delta_h$, it is evident that this cancellation must occur to a precision of $1/\Delta_h$, so that a larger Δ_h corresponds to a less natural theory. Therefore, a theory (i.e., $F^{(\text{eff})}$) for which $\Delta_h = 1$ is natural, while one with $\Delta_h = 10(100)$ suffers from fine-tuning of 10%(1%).

In Fig. 4 we plot regions in the $F^{(\text{eff})} - \Lambda$ plane that correspond to an effective action which is natural (i.e., enclosed within the $\Delta_h = 1$ region) and those that suffer from fine-tuning of no worse than 10% and 1%, corresponding to $\Delta_h = 10$ and $\Delta_h = 100$, respectively. This figure illustrates the (fine-) tuning between the EFT and SM one-loop terms which is required to alleviate the little

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[9]Our measure for naturalness corresponds to what is known as the Barbieri-Giudice criteria [20]: $\Delta = |\partial \ln O|/|\partial \ln f|$, for $O = m_h^2$ and $f = F^{(\text{eff})} - 8.2$.
[10]If no cancellation occurs between the radiative corrections then $\delta m_h^2 = O(\Lambda^2)$, which must be balanced by an $O(\Lambda^2)$ tree-level contribution to the Higgs mass in order to explain the experimentally observed value of the physical mass $m_h^2 = O(v^2)$.
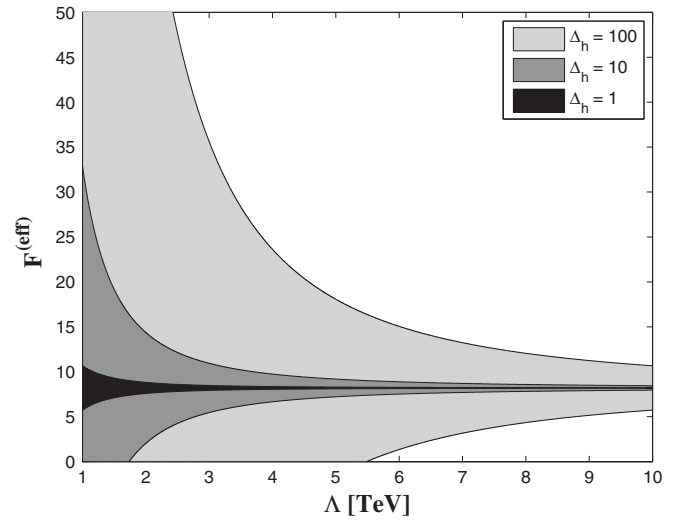

FIG. 4. Regions in the $F^{(\text{eff})} - \Lambda$ plane where naturalness can be restored with no fine-tuning ($\Delta_h = \delta m_h^2/m_h^2 = 1$, in black) and with fine-tuning at the level of 10% (dark gray) and 1% (light gray), corresponding to $\Delta_h = \delta m_h^2/m_h^2 = 10$ and 100, respectively. See also text.

hierarchy problem, following Eq. (9). For example, theories for which $8.17 \lesssim F^{(\text{eff})} \lesssim 8.23$ are natural at $\Lambda \sim 10$ TeV, while theories with $7.95 \lesssim F^{(\text{eff})} \lesssim 8.45$ or $5.73 \lesssim F^{(\text{eff})} \lesssim 10.67$ will suffer from 10% or 1% fine-tuning, respectively, at $\Lambda \sim 10$ TeV. Note also that if the scale of EFT naturalness is $\Lambda \sim 5$ TeV, then a much wider range of theories, those giving $0 \lesssim F^{(\text{eff})} \lesssim 18$, are allowed if one is willing to tolerate 1% fine-tuning. It should also be noted that the EFT-naturalness regions shown in Fig. 4 may, in general, be subject to additional constraints (e.g., from perturbativity), depending on the details of the specific underlying theory.

Given the specific form of the graphs in Fig. 3 which generate the leading operators in Eqs. (3), (4) and (6), it is possible to express the EFT coefficients f in terms of some of the couplings of the heavy particles to the SM. Specifically, defining u_Φ, g_X and $y_{\Psi-\psi}$ to be the couplings of a heavy scalar $\Phi = S, \chi, \tilde{\chi}$ to ϕ^2 (i.e., $u_\Phi \phi^\dagger \Phi \phi$), of a heavy vector boson $X = v, \tilde{v}, \mathbf{V}$ to the currents $J_X = j, \tilde{j}, J_I$ in Eq. (5) (i.e., $g_X X_\mu J_X^\mu$), and of a heavy fermion Ψ to $\psi\phi$ (i.e., $y_{\Psi-\psi} \bar{\psi}\Psi\phi$), respectively, and allowing for the (generic) case of different scales of NP: $M_{\Phi,\Psi,X} \gtrsim \Lambda$ corresponding to the mass scale of the heavy scalars, vectors and fermions, respectively, we find

$$f_{\Phi}^{(2k+4)}(u_\Phi, M_\Phi, M) = \left| \frac{u_\Phi}{M_\Phi} \right|^2 \left(\frac{-M^2}{M_\Phi^2} \right)^k,$$

$$f_{\Psi-\psi}^{(2k+4)}(y_{\Psi-\psi}, M_\Psi, M) = \frac{1}{2} I_\Psi |y_{\Psi-\psi}|^2 \left(\frac{M^2}{M_\Psi^2} \right)^k,$$

$$f_X^{(2k+6)}(g_X, M_X, M) = I_X |g_X|^2 \left(\frac{-M^2}{M_X^2} \right)^{k+1}, \qquad (10)$$

where $I_\zeta = 1, 2, 3$ when the field $\zeta = \Psi$ or X is an isosinglet, doublet or triplet, respectively. Thus, inserting Eq. (10) in Eq. (8) and performing the sum over k, we obtain

$$F^{(\mathrm{eff})}(\Lambda) = \sum_\Phi \frac{|u_\Phi|^2}{M_\Phi^2} A\left(\frac{\Lambda^2}{M_\Phi^2}\right)$$

$$+ \frac{1}{2}\sum_{\Psi,\psi} I_\Psi |y_{\Psi-\psi}|^2 \left[1 - A\left(\frac{\Lambda^2}{M_\Psi^2}\right)\right]$$

$$+ \sum_X I_X |g_X|^2 \left[1 - A\left(\frac{\Lambda^2}{M_X^2}\right)\right], \qquad (11)$$

where $A(x) = \ln(1+x)/x$, so that $1 > A(x) \geq 0$, from which it follows that $F^{(\mathrm{eff})} > 0$.

Assuming that the heavy masses are clustered around a value M, the above expression simplifies to

$$F^{(\mathrm{eff})}(\Lambda) = (\xi - \eta)A\left(\frac{\Lambda^2}{M^2}\right) + \eta; \qquad \xi = \sum_\Phi \frac{|u_\Phi|^2}{M_\Phi^2},$$

$$\eta = \frac{1}{2}\sum_{\Psi,\psi} I_\Psi |y_{\Psi-\psi}|^2 + \sum_X I_X |g_X|^2, \qquad (12)$$

where we expect $\xi, \eta \sim \mathcal{O}(1-10)$, e.g., $u_\Phi \sim 3M_\Phi$ and/or a triplet heavy vectorlike (colored) quark with $y_{\Psi-\psi} \sim 1$ will give $\xi, \eta \sim 10$.

In Fig. 5 we plot the regions in the $\xi - \eta$ plane which correspond to $\Delta_h = 1$ (natural) and $\Delta_h = 10$ (10% fine-tuning), for an EFT-naturalness scale in the range $3 \text{ TeV} < \Lambda < 10 \text{ TeV}$ and NP scale $\Lambda < M < 3\Lambda$. The shaded region in the $\xi - \eta$ scatter plots corresponds to the shaded regions in the $\Lambda - M$ plane (matching colors). In particular, we can find the values of (ξ, η) for which the EFT corrections to Δm_h^2 can restore naturalness in the Higgs sector at a certain Λ for some value M of the NP threshold. For example, extensions of the SM with a typical mass
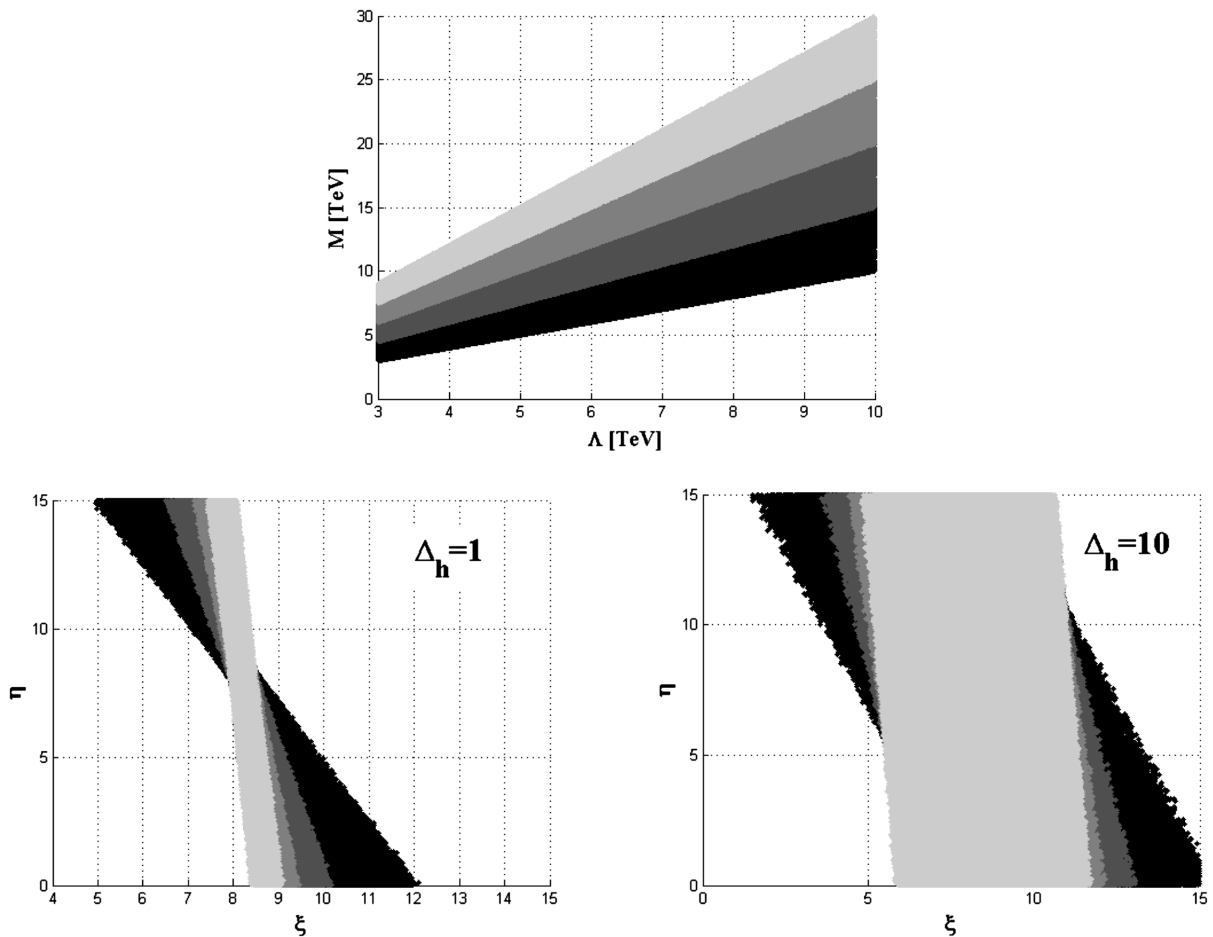

FIG. 5.   Upper plot: regions in the $\Lambda - M$ plane, corresponding to $1 < M/\Lambda < 1.5$ (black), $1.5 < M/\Lambda < 2$ (dark gray), $2 < M/\Lambda < 2.5$ (gray) and $2.5 < M/\Lambda < 3$ (light gray), for a naturalness scale $3 \text{ TeV} < \Lambda < 10 \text{ TeV}$ and M being the typical NP mass scale. Lower plots: scatter plots in the $\xi - \eta$ plane [see Eq. (12)] corresponding to the regions in the $\Lambda - M$ plane (corresponding shading colors), where the NP (with scale M) restores naturalness (left plot) or suffers from 10% fine-tuning (right plot).

scale of $M \sim 7$ TeV that give $\xi \sim 9$ and $\eta \sim 5$ will yield an effective action which is natural up to $\Lambda \sim 5$ TeV (an order of magnitude improvement over the pure SM).

III. SIGNALS OF EFT NATURALNESS

Let us briefly discuss the potential signals of our EFT-naturalness operators or, equivalently, of the tail of the NP that can restore naturalness at energy scales which are accessible to current and future high-energy colliders. In particular, apart from their contribution to δm_h^2, these operators also shift the SM Higgs self-couplings h^3 and h^4, the SM Higgs couplings to the gauge bosons hVV and h^2V^2 ($V = W$ or Z) and the Higgs Yukawa couplings $h\psi\psi$ (ψ being a SM fermion). In addition, they also give rise to new higher-dimensional contact terms such as h^3V^2, h^4V^2 and $h^2\psi^2$.

In Table I we list the expected deviations in the SM couplings and the new contact terms which are generated by each of the operators in Eqs. (3), (4) and (6). Evidently, the tail of the NP generating the EFT-naturalness operators can be searched for in multiboson scattering processes of the form $\psi\bar{\psi}/VV \rightarrow n \cdot h + m \cdot V + X$, where $n, m = 0, 1, 2, \ldots$. In particular, one can search for correlations in the various channels or look for differences between W-boson- versus Z-boson-associated production processes. For example, while $\mathcal{O}_\chi^{(2k+4)}$ will effect Wh production at the LHC, i.e., $pp \rightarrow Wh + X$, the operator $\mathcal{O}_v^{(2k+6)}$ is expected to contribute only to Z-boson-associated production processes such as $pp \rightarrow Zh + X$.

Clearly, though, the search for the tail of these NP effects in Higgs–gauge-boson processes will require a sensitivity to these couplings at the percent level, to be probed at $\Lambda \sim \mathcal{O}(5$–$10$ TeV). This will be challenging even at the high-luminosity LHC and may require future colliders at the high-energy/high-luminosity frontiers, such as a future 30 or 100 TeV hadron collider and/or an $\mathcal{O}(\text{TeV})$ e^+e^- collider. Nonetheless, from our considerations we expect better prospects for detection of NP at the LHC in the low-multiplicity Higgs–gauge-bosons production processes,

e.g., $pp \rightarrow hW, hZ + X$, and in processes involving the new contact terms listed in Table I. This will be studied in a future work.

IV. CONSTRAINTS FROM EW PRECISION DATA AND HIGGS SIGNALS

Let us now examine the limits that the current data impose on the coefficients of our effective operators. Since the most important effects are generated by the lowest-dimensional operators, we will only investigate the limits on the dimension-six coefficients $f_{\Phi,\Psi-\psi,X}^{(6)}$, which are mainly of two types [to simplify the expressions we define $\epsilon = (v/M)^2$].

1. *A shift to the ρ parameter:*
   The scalar-triplet operators $\mathcal{O}_{\chi\tilde{\chi}}^{(6)}$ and the vector operators $\mathcal{O}_X^{(6)}$ modify the SM gauge-boson masses according to

$$\frac{\delta M_Z^2}{M_Z^2} = \epsilon\left(f_v^{(6)} + \frac{1}{3}f_{\mathbf{V}}^{(6)} - f_{\tilde{\chi}}^{(6)}\right),$$

$$\frac{\delta M_W^2}{M_W^2} = \epsilon\left(\frac{1}{2}f_{\tilde{v}}^{(6)} + \frac{1}{3}f_{\mathbf{V}}^{(6)} - f_\chi^{(6)} - \frac{1}{2}f_{\tilde{\chi}}^{(6)}\right), \quad (13)$$

which shift the ρ parameter accordingly:

$$\delta\rho = \epsilon\left(\frac{1}{2}f_{\tilde{v}}^{(6)} - f_v^{(6)} + \frac{1}{2}f_{\tilde{\chi}}^{(6)} - f_\chi^{(6)}\right). \quad (14)$$

This is the strongest constraint from precision EW observables; it is suppressed by a factor of ϵ because the heavy physics being considered here (the heavy Φ, X and Ψ states) does not break the SM gauge invariance (this happens only at the EW scale v). If we assume that the operator coefficients f are $O(1)$ and that there are no cancellations, then the constraint $|\delta\rho| < 0.0007$ [21] implies $M \gtrsim 9.3$ TeV; but this can be considerably reduced if some cancellations do occur.

TABLE I. Vertices involving the Higgs, gauge-bosons and fermions which are generated by the operators in Eqs. (3), (4) and (6). A check mark is used to indicate that the vertex is affected by the specific operator.

Operator	h^3	h^4	hWW	h^2W^2	h^3W^2	h^4W^2	hZZ	h^2Z^2	h^3Z^2	h^4Z^2	$h\psi\psi$	$h^2\psi^2$
$\mathcal{O}_S^{(2k+4)}$	✓	✓										
$\mathcal{O}_\chi^{(2k+4)}$	✓	✓	✓	✓	✓	✓						
$\mathcal{O}_{\tilde{\chi}}^{(2k+4)}$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		
$\mathcal{O}_v^{(2k+6)}$							✓	✓	✓	✓		
$\mathcal{O}_{\tilde{v}}^{(2k+6)}$			✓	✓	✓	✓						
$\mathcal{O}_{\mathbf{V}}^{(2k+6)}$			✓	✓	✓	✓	✓	✓	✓	✓		
$\mathcal{O}_{\Psi-\psi}^{(2k+4)}$											✓	✓

2. *A shift of the Higgs boson couplings to fermions and SM gauge bosons:* This effect can be divided into three parts:

(i) The scalar operators $\mathcal{O}_\Phi^{(6)}$ modify the Higgs kinetic term. Thus, in order to recover a canonically normalized Higgs field we need to rescale $h \to [1 + (\epsilon/2)\sum f_\Phi^{(6)}]h$. This modifies the Higgs couplings to all other SM fields (denoted by x),

$$\delta_\Phi \equiv \frac{\delta g_{hxx}}{g_{hxx}^{\rm SM}} = \frac{\epsilon}{2}\sum_\Phi f_\Phi^{(6)}, \qquad (15)$$

and changes all Higgs decay widths into any final state x by the same factor (so that branching ratios remain the same): $\delta\Gamma(h \to xx) \approx \epsilon \sum f_\Phi^{(6)}\Gamma_{\rm SM}(h \to xx)$ (to lowest order in Λ).

(ii) The fermion operators $\mathcal{O}_{\Psi-\psi}^{(6)}$ modify the Higgs-Yukawa coupling to the SM fermions (ψ):

$$\delta_{\Psi-\psi} \equiv \frac{\delta g_{h\psi\psi}}{g_{h\psi\psi}^{\rm SM}} = -\epsilon \sum_\Psi f_{\Psi-\psi}^{(6)}. \qquad (16)$$

Note that this shift in the htt coupling also modifies the top-quark loop contribution in the gluon-fusion Higgs production cross section as well as in the one-loop decays $h \to \gamma\gamma$ and $h \to Z\gamma$.

(iii) The scalar-triplet operators $\mathcal{O}_{\chi\tilde{\chi}}^{(6)}$ and the vector operators $\mathcal{O}_X^{(6)}$ modify the couplings of the Higgs to the vector bosons,

$$\delta_W \equiv \frac{\delta g_{hWW}}{g_{hWW}^{\rm SM}} = \epsilon\left(f_{\tilde{v}}^{(6)} + \frac{2}{3}f_{\mathbf{V}}^{(6)} - \frac{1}{2}f_\chi^{(6)} - \frac{1}{4}f_{\tilde{\chi}}^{(6)}\right), \qquad (17)$$

$$\delta_Z \equiv \frac{\delta g_{hZZ}}{g_{hZZ}^{\rm SM}} = \epsilon\left(2f_{\tilde{v}}^{(6)} + \frac{2}{3}f_{\mathbf{V}}^{(6)} - \frac{1}{2}f_{\tilde{\chi}}^{(6)}\right), \qquad (18)$$

and therefore changes the Higgs decay width to a pair of SM gauge bosons.

Turning now to the overall effect of the above modifications on the Higgs couplings to the SM fermions and gauge bosons, let us analyze the constraints that can be imposed from the recently measured Higgs signals (see also [22]). In particular, defining

$$w_{xx} \equiv \frac{\Gamma(h \to xx)}{\Gamma(h_{\rm SM} \to xx)}, \qquad R^{\rm Total} \equiv \frac{\Gamma_h^{\rm Total}}{\Gamma_{h_{\rm SM}}^{\rm Total}}, \qquad (19)$$

the normalized branching ratios for each channel are given by

$$R_{xx}^{\rm BR} \equiv \frac{{\rm BR}(h \to xx)}{{\rm BR}(h_{\rm SM} \to xx)} = \frac{w_{xx}}{R^{\rm Total}}. \qquad (20)$$

The "signal strength" for each Higgs production and decay mode is then given by

$$\mu_{xx}^{ii} \equiv \frac{\sigma(ii \to h \to xx)}{\sigma(ii \to h_{\rm SM} \to xx)} = \frac{w_{ii}w_{xx}}{R^{\rm Total}}, \qquad (21)$$

so that

Higgs signal	$gg \to \gamma\gamma$	$gg \to ZZ^\star$	$gg \to WW^\star$	$gg \to Z\gamma$	$gg \to \tau^+\tau^-$	$VV \to \gamma\gamma$	$q\bar{q} \to Vh \to Vb\bar{b}$	$q\bar{q} \to V_1h \to V_1V_2V_2^\star$
$\mu_{xx}^{ii} \cdot R^{Total}$	$w_{gg}w_{\gamma\gamma}$	$w_{gg}w_{ZZ}$	$w_{gg}w_{WW}$	$w_{gg}w_{Z\gamma}$	$w_{gg}w_{\tau\tau}$	$w_{VV}w_{\gamma\gamma}$	$w_{VV}w_{bb}$	$w_{V_1V_1}w_{V_2V_2}$

$$(22)$$

Note that, since the one-loop hgg coupling is controlled primarily by the top-quark Yukawa coupling, we have $w_{gg} = w_{tt}$. To lowest order in ϵ, we then find

$$w_{\psi\psi} \approx 1 + 2\tilde{\delta}_\psi, \quad w_{gg} \approx w_{tt}, \quad w_{WW} \approx 1 + 2\tilde{\delta}_W, \quad w_{ZZ} \approx 1 + 2\tilde{\delta}_Z, \quad w_{\gamma\gamma} \approx 1 + 2.56\tilde{\delta}_W - 0.56\tilde{\delta}_t, \quad w_{Z\gamma} \approx 1 + 2.1\tilde{\delta}_Z - 0.1\tilde{\delta}_t,$$

$$(23)$$

and

$$R^{\rm Total} \approx 1 + 2\left({\rm BR}_{\rm SM}^{WW}\tilde{\delta}_W + {\rm BR}_{\rm SM}^{ZZ}\tilde{\delta}_Z + {\rm BR}_{\rm SM}^{gg}\tilde{\delta}_t + \sum_\psi {\rm BR}_{\rm SM}^{\psi\psi}\tilde{\delta}_\psi\right), \qquad (24)$$

where ${\rm BR}_{\rm SM}^{xx} \equiv {\rm BR}(h_{\rm SM} \to xx)$ are the SM branching ratios, $\tilde{\delta}_\psi \equiv \delta_\Phi + \delta_{\Psi-\psi}$, $\tilde{\delta}_W \equiv \delta_\Phi + \delta_W$, $\tilde{\delta}_Z \equiv \delta_\Phi + \delta_Z$ and $\delta_\Phi, \delta_{\Psi-\psi}, \delta_W, \delta_Z$ are given in Eqs. (15)–(18).

We see that if the coefficients of the higher-dimensional operators $f_\Psi^{(6)}, f_\Phi^{(6)}$ and $f_X^{(6)}$ are of $O(1)$, then the typical correction to the Higgs signal strengths is of $O(\epsilon)$. Thus, given that the LHC is expected to probe the Higgs couplings to at most 10% accuracy (this includes the high luminosity run of the LHC [23]), a rather weak bound of $M \gtrsim O(1\,{\rm TeV})$ can be imposed on the scale of the new

heavy physics that can lead to EFT naturalness in the Higgs sector. A future e^+e^- collider may be able to improve the accuracy to $O(0.01)$ [23], in which case EFT naturalness can be probed up to $\Lambda \sim 2$–3 TeV. We conclude that precision Higgs measurements are not expected to impose significant constraints on our EFT-naturalness scenario.

Finally, we note that none of the operators in Eqs. (3), (4) and (6) breaks any of the global symmetries of the SM, so no strong limits can be obtained from CP, flavor or lepton and baryon number conservation experiments.

V. SUMMARY

We have used EFT techniques to study the little hierarchy problem of the SM Higgs sector assuming the presence of weakly coupled and decoupling heavy physics with scale M. Following Wilson's approach, we calculate the one-loop contribution to the Higgs mass in the EFT as a function of the scale of the effective action Λ ($\Lambda \gg m_W$) and discuss the tuning between these EFT terms and the SM one-loop contribution, which is required in order to alleviate the *little hierarchy problem*. In particular, we determine the conditions under which these quadratic new heavy-physics contributions can balance those generated by the SM at the scale Λ, where we implicitly assume that there is an underlying symmetry responsible for that, especially if the needed tuning is of $\mathcal{O}(10^{-4})$ or worse.

We analyze the complete set of higher-dimensional effective operators (at any dimension $n \geq 5$) that can yield $\mathcal{O}(\Lambda^2)$ contributions to δm_h^2 in the EFT and classify the underlying heavy theories that can generate these operators at tree level. In particular, we find that heavy new physics theories that can lead to EFT naturalness (i.e., that can restore naturalness in the effective action at, e.g., $\Lambda \sim 5$–10 TeV) must contain one or more singlet or triplet heavy bosons or else a singlet, doublet or triplet fermions, all heaving typical masses larger than Λ. We then estimate the coefficients of the EFT-naturalness higher-dimensional operators using the relevant phenomenological interactions of these heavy particles.

We have also studied the constraints that precision electroweak data and the recently measured Higgs signals impose on our EFT-naturalness setup and find that heavy scalar singlets and/or heavy fermions (singlets, doublets or triplets) are more likely to play a role in softening the fine-tuning in the SM Higgs sector if the scale of the new heavy physics is below $\sim$10 TeV.

Finally, we have discussed the expected signatures that the tail of the NP (responsible for EFT naturalness) can have at the LHC and at future colliders. In particular, we find that signals of EFT naturalness are likely to be manifest as deviations in processes involving Higgs + gauge-boson production, e.g., pp or $e^+e^- \to hh$, hW, hZ, WW, $ZZ + X$ and/or processes with higher Higgs/ gauge-boson multiplicities in the final state.

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