Evidence for the existence of $ud\bar{b}\bar{b}$ and the nonexistence of $ss\bar{b}\bar{b}$ and $cc\bar{b}\bar{b}$ tetraquarks from lattice QCD

Pedro Bicudo,[1] Krzysztof Cichy,[2,3] Antje Peters,[2] Björn Wagenbach,[2] and Marc Wagner[2]

[1]*CFTP, Departamento de Física, Instituto Superior Técnico, Universidade de Lisboa,*
Avenida Rovisco Pais, 1049-001 Lisboa, Portugal
[2]*Johann Wolfgang Goethe-Universität Frankfurt am Main, Institut für Theoretische Physik,*
Max-von-Laue-Straße 1, D-60438 Frankfurt am Main, Germany
[3]*Adam Mickiewicz University, Faculty of Physics, Umultowska 85, 61-614 Poznan, Poland*

We combine lattice QCD results for the potential of two static antiquarks in the presence of two quarks qq of finite mass and quark model techniques to study possible existing $qq\bar{b}\bar{b}$ tetraquarks within the Born-Oppenheimer approximation. While there is strong indication for a bound four-quark state for $qq = (ud - du)/\sqrt{2}$, i.e., isospin $I = 0$, which is stable with respect to QCD, i.e., can only decay weakly, we find clear evidence against the existence of corresponding tetraquarks with $qq \in \{uu, (ud + du)/\sqrt{2}, dd\}$, i.e., isospin $I = 1$, $qq = ss$, and $qq = cc$.

I. INTRODUCTION

Exotic hadrons have been proposed many years ago. As soon as quarks were found in the sixties, it became clear that systems more complex than standard mesons ($q\bar{q}$ states) and baryons (qqq states) could possibly exist. However, exotic hadrons are very elusive systems. Confirming their existence or nonexistence still remains one of the main challenges of particle physics.

Frequently discussed exotic hadrons are tetraquarks [1,2], which are four-quark bound states composed of two quarks and two antiquarks. There are several hadronic resonances which are tetraquark candidates. Among them are the light scalar mesons σ, κ, $f_0(980)$, and $a_0(980)$ as well as the heavier mesons D_{s0}^* and D_{s1}. However, these systems have quantum numbers also consistent with a standard $q\bar{q}$ structure and their masses are not too different from what is expected in a $q\bar{q}$ picture. Thus, it is hard to rigorously argue that they are indeed predominantly tetraquarks. On the other hand, there are also candidates which have quantum numbers or masses typical for tetraquarks, but not for standard $q\bar{q}$ mesons. For example π_1^{-+} has exotic quantum numbers $J^{PC} = 1^{-+}$ or $Z_c^\pm$ and $Z_b^\pm$ masses and decay products strongly suggest hidden $c\bar{c}$ or $b\bar{b}$ pairs, while their electrical charge ± 1 indicates isospin $I = 1$. While the evidence for π_1^{-+} is not conclusive and the $Z_b^\pm$ claimed by the Belle collaboration [3] remains to be confirmed by different experimental collaborations, the $Z_c^\pm$ has received a series of experimental observations by the Belle collaboration [4,5], the Cleo-C collaboration [6], the BESIII collaboration [7–11], and the LHCb collaboration [12]. Nevertheless, the $Z_c^\pm$ would profit by more comprehensive measurements of its decay channels. We expect the existing and future experimental collaborations

to continue the study of present tetraquark candidates and to possibly also discover further ones.

The theoretical study of tetraquarks is crucial to confirm and correctly interpret corresponding experimental observations and could as well provide information in which channels tetraquarks may be found. However, tetraquark studies face a number of difficulties, e.g., (i) tetraquarks are usually open to meson-meson decay, (ii) tetraquarks are complex relativistic four-body systems, (iii) quark models still fail to reproduce sectors of standard hadronic spectra and, thus, are not yet sufficiently well calibrated to reliably predict tetraquarks.

In this work, we study the existence/nonexistence of tetraquarks with two heavy bottom antiquarks $\bar{b}\bar{b}$. To this end, we use potentials of two static antiquarks in the presence of two quarks qq of finite mass, which we compute using lattice QCD. We extend recent studies of $qq\bar{b}\bar{b}$ tetraquarks [13,14], where $qq \in \{(ud-du)/\sqrt{2}, uu, (ud+du)/\sqrt{2}, dd\}$, to similar systems with heavier quarks, $qq = ss$ and $qq = cc$. In the future, we also plan to extend our investigations to the $b\bar{b}$ tetraquarks claimed by the Belle Collaboration [3]. Such tetraquarks with a $b\bar{b}$ pair are, however, rather difficult to study with lattice QCD, since they couple to several decay channels.

We avoid some of the technical difficulties of studying tetraquarks following a strategy already identified in the eighties [15]. We search for bound states rather than for resonances, to avoid open decay channels. Moreover, by using $\bar{b}\bar{b}$ potentials obtained by lattice QCD computations, we largely avoid the calibration problem of quark models. Very heavy antiquarks such as $\bar{b}$ allow for the Born-Oppenheimer approximation [16]. For the two lighter quarks qq, the heavy antiquarks $\bar{b}\bar{b}$ can be approximated as static color charges, which allows us to determine the

light quark energy using lattice QCD. On the other hand, once the energy of the light quarks qq is determined, it can be utilized as an effective potential for the heavy antiquarks $\bar{b}\bar{b}$.

Our lattice QCD computation goes beyond computations with four static quarks, which show a clear evidence for four-body tetraquark potentials [17,18] and tetraquark flux tubes [19,20]. On the other hand, lattice QCD computations with four quarks of finite mass are extremely difficult and have found neither evidence for charmed tetraquark bound states with $\bar{u}\,\bar{d}\,cc$ flavor [21] nor for resonances in the Z_c family [22].

This paper is organized as follows. In Sec. II, we briefly review the quark model and discuss qualitative expectations regarding $qq\bar{b}\bar{b}$ four-quark systems. In Sec. III, we discuss the lattice QCD computation of $\bar{b}\bar{b}$ potentials in the presence of two lighter quarks qq and provide parametrizations of these results by continuous functions. In Sec. IV, we use these parametrizations in model calculations and check for the existence of bound states, which would indicate the existence of tetraquarks. We conclude in Sec. V.

II. MODELING THE $\bar{b}\bar{b}$ INTERACTION IN THE PRESENCE OF TWO LIGHT QUARKS qq

In the following, we discuss quark model expectations regarding the qualitative behavior of a $qq\bar{b}\bar{b}$ four-quark systems, where q denotes either a light u, d, s, or c quark [23]. In particular, we are interested in the $\bar{b}\bar{b}$ interaction in the presence of two light quarks qq. The qualitative expectations are confirmed by corresponding lattice QCD results, which are discussed in Sec. III. The main purpose of these model considerations is to motivate a suitable fit function for the lattice QCD $\bar{b}\bar{b}$ potential results, which is used in Sec. IV in the Schrödinger equation to check whether and in which channels bound four-quark states, i.e., tetraquarks, exist.

A. The quark-antiquark/quark-quark potential at small separations

In the original quark model [24], the quark-antiquark and the quark-quark (or equivalently antiquark-antiquark) potentials at small separations $r = |\mathbf{r}_i - \mathbf{r}_j|$ are dominated by one-gluon exchange similar to the Fermi-Breit interaction,

$$V_{ij}(\mathbf{r}_i, \mathbf{s}_i, \mathbf{r}_j, \mathbf{s}_j)$$
$$= -\frac{C\alpha_s}{4}\left(\frac{1}{r} - \frac{\pi}{2}\delta^3(\mathbf{r})\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{16\mathbf{s}_i\cdot\mathbf{s}_j}{3m_im_j}\right) + \cdots\right) \quad (1)$$

[i, j are the (anti)quark indices, $\mathbf{r}_i$, $\mathbf{s}_i$ and m_i denote their positions, spins, and masses, respectively]. Since we are exclusively interested in ground states, we have specialized Eq. (1) to angular momentum $l = 0$. The quark model has been improved (cf., e.g., [25,26]), but maintains its main ingredients. C depends on the color orientation of the (anti) quarks, which can be specified by a 3×3 matrix Λ. For a quark-antiquark pair $\bar{q}_i\Lambda q_j$

$$C = +\sum_a \text{Tr}(\lambda^a\Lambda\lambda^a\Lambda^\dagger), \quad (2)$$

while for a quark-quark pair $q_i^T\Lambda q_j$

$$C = -\sum_a \text{Tr}(\lambda^a\Lambda\lambda^{aT}\Lambda^\dagger) \quad (3)$$

with the Gell-Mann matrices λ^a, $a = 1, \ldots, 8$. For example, $\Lambda_{AB} = \delta_{AB}/\sqrt{3}$ describes the $q\bar{q}$ color singlet, while $\Lambda_{AB} = \epsilon_{AB3}/\sqrt{2}$ is one of three independent possibilities to realize a qq color triplet. In Table I, the resulting values for C for the $q\bar{q}$ singlet and octet and the qq triplet and sextet color orientations are listed.

Lattice QCD confirms that the static color singlet potential at small separations r can be described reasonably well by one-gluon-exchange (cf., e.g., [27,28], where a matching of lattice QCD and perturbative results is done). At larger separations, it becomes linear with certain $1/r$-corrections due to string vibrations [29]. One can crudely estimate α_s appearing in (1) by considering the color singlet $q\bar{q}$. In that case $C = +16/3$, while string vibrations lead to $V_{ij} \approx -\pi/12r$ at intermediate separations, resulting in $\alpha_s \approx \pi/16$. While this estimate is most appropriate for static quarks, α_s is expected to be somewhat larger for quarks of finite mass [25,26].

The only spin dependent term in (1) is the hyperfine interaction proportional to $\mathbf{s}_i \cdot \mathbf{s}_j$, which is pathological in the original quark model due to the Dirac delta [cf. Eq. (1)]. In the relativistic quark model, however, this interaction is smoother and, hence, well behaved [25,26]. Clearly, the interaction is weaker for a spin triplet than for a spin singlet.

To summarize, whether the potential between a quark and another quark or antiquark is attractive or repulsive depends on their color orientation. For small separations it is approximately Coulomb-like with the color factors C collected in Table I. The hyperfine term enhances the interaction for a spin singlet and decreases it for a spin triplet.

TABLE I. The color factors C for the $q\bar{q}$ singlet and octet and the qq triplet and sextet color orientations.

Color orientation	$q\bar{q}$ singlet 1	$q\bar{q}$ octet 8	qq triplet $\bar{3}$ (and 3)	qq sextet $\bar{6}$ (and 6)
C	+16/3 (attractive)	−2/3 (repulsive)	+8/3 (attractive)	−4/3 (repulsive)

B. Qualitative discussion of the $qq\bar{b}\bar{b}$ system

For the particular case of the $qq\bar{b}\bar{b}$ system, where the $\bar{b}\bar{b}$ pair is significantly heavier than the light qq pair, we utilize the Born-Oppenheimer approximation [16]: for the light quarks, the heavy antiquarks can be regarded as static color charges; once the energy of the light quarks is determined, it can be used as an effective potential for the heavy antiquarks. We assume that at small $\bar{b}\bar{b}$ separations r, the $\bar{b}$ quarks interact according to the quark model discussed in Sec. II A, while at larger separations their interaction is screened by the light quarks, i.e., the four quarks form two rather weakly interacting $B^{(*)}_{(s,c)}$ mesons ($B^{(*)}_{(s,c)}$ denotes either a B, B^*, B_s, B^*_s, B_c, or B^*_c meson).

1. Expectations for the $\bar{b}\bar{b}$ interaction at small separations r

(i) The spin interaction of the $\bar{b}$ quarks is quite small and can possibly be neglected, since it is proportional to $1/m_b^2$ [cf. Eq. (1)].

(ii) In case of a bound $qq\bar{b}\bar{b}$ state, i.e., a tetraquark, the antiquarks $\bar{b}\bar{b}$ are expected to be in a color triplet 3, which is attractive, and not in a color sextet 6, which is repulsive (cf. also Table I). In other words, at small separations r, the antiquarks $\bar{b}\bar{b}$ form an antidiquark as depicted in Fig. 1.

(iii) Because the complete four quark system $qq\bar{b}\bar{b}$ necessarily forms a color singlet, the light quarks qq must be in a color antitriplet $\bar{3}$.

(iv) Since this color antitriplet is antisymmetric, and since the light quarks qq are assumed to be in a spatially symmetric s-wave, the Pauli principle implies a symmetric spin-flavor structure. This can either be a spin singlet with an antisymmetric flavor combination or a spin triplet with a symmetric
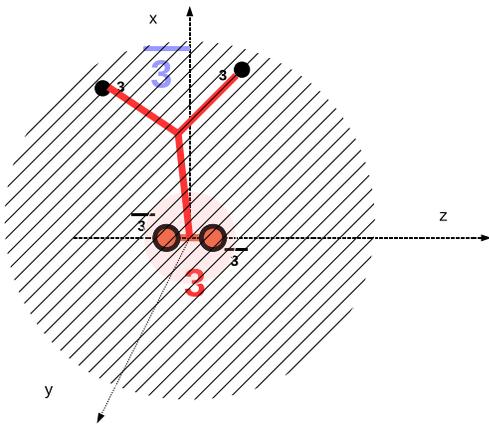

FIG. 1 (color online). At small $\bar{b}\bar{b}$ separations r, the heavy antiquarks $\bar{b}\bar{b}$ form an antidiquark, which corresponds to a color triplet. There is essentially no screening of the $\bar{b}\bar{b}$ interaction due to the much farther separated light quarks qq.

flavor combination. Indeed, when studying light u and d quarks in the presence of two static antiquarks using lattice QCD, two attractive channels have been found [13,30,31]. As expected, these are a (spin) *scalar isosinglet* ($j = 0$, $I = 0$, where j denotes the spin of the light quarks qq) and a (spin) *vector isotriplet* ($j = 1$, $I = 1$). The scalar isosinglet is more attractive, as expected from the hyperfine interaction in Eq. (1), i.e., the lattice QCD results confirm the qualitative quark model expectations.

(v) When studying two identical light quarks $qq = ss$ or $qq = cc$, which are symmetric in flavor, the only attractive channel is a spin triplet. However, it is conceptually interesting to consider two hypothetical degenerate flavors with the mass of strange or charm quarks and then also investigate spin singlets with flavor structure $qq = (s^{(1)}s^{(2)} - s^{(2)}s^{(1)})/\sqrt{2}$ and $qq = (c^{(1)}c^{(2)} - c^{(2)}c^{(1)})/\sqrt{2}$.

2. Expectations for the $\bar{b}\bar{b}$ interaction at large separations r

(i) At large separations r, screening of the $\bar{b}\bar{b}$ interaction is expected due to the light quarks qq, as illustrated in Fig. 2. When the $\bar{b}\bar{b}$ separation is larger than around two times the radius of a $B^{(*)}_{(s,c)}$ meson, there is essentially no overlap between the wave functions of the light quarks and, consequently, the $\bar{b}\bar{b}$ interaction practically vanishes.

(ii) The more massive the light quarks are, the more compact their wave functions in the $B^{(*)}_{(s,c)}$ mesons, as shown in Fig. 2(a), 2(b), and 2(c) and, thus, the stronger the screening. In other words, the corresponding $\bar{b}\bar{b}$ potential becomes more and more narrow and will at some point not anymore be able to host a bound state. Consequently, for a sufficiently heavy pair of light quarks qq the screening should prevent the formation of $qq\bar{b}\bar{b}$ tetraquarks.

3. Quantum numbers of possibly existing $qq\bar{b}\bar{b}$ tetraquarks

We study exclusively states which correspond for large $\bar{b}\bar{b}$ separations to pairs of $B^{(*)}_{(s,c)}$ mesons in a spatially symmetric s-wave. Therefore, the parity of these states is positive, i.e., $P = +$ (the product of the parity quantum numbers of the two mesons, which are both negative).

As argued above, the two antiquarks $\bar{b}\bar{b}$ are expected to be in an antisymmetric color triplet. Since their flavor is symmetric, their spin j_b must also be symmetric due to the Pauli principle, i.e., $j_b = 1$. Similarly, for an antisymmetric qq flavor combination, i.e., $qq = (ud - du)/\sqrt{2}$, $j = 0$, while for symmetric flavor combinations, i.e., $qq \in \{uu, (ud + du)/\sqrt{2}, dd, ss, cc\}$, $j = 1$. The total spin J of the $qq\bar{b}\bar{b}$ system is the combination of j and j_b.
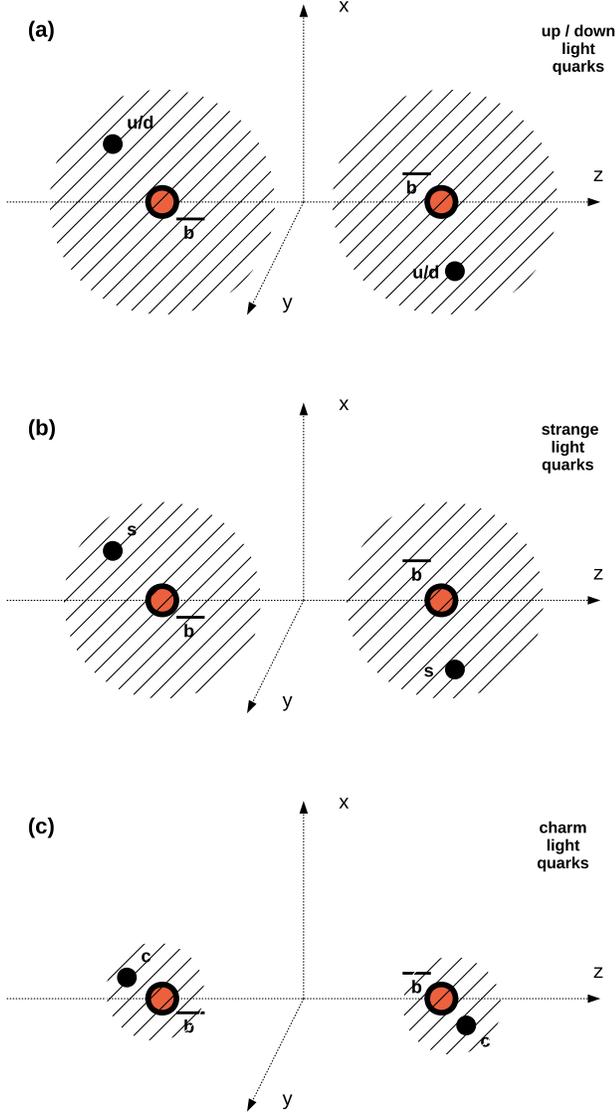
FIG. 2 (color online). At large $\bar{b}\bar{b}$ separations r, the $qq\bar{b}\bar{b}$ system is essentially a system of two $B_{(s,c)}^{(*)}$ mesons. The color charge of each of the antiquarks $\bar{b}$ is almost completely screened by one of the light quarks q. (a) $q \in \{u, d\}$. (b) $q = s$. (c) $q = c$.

Altogether, the possibly existing $qq\bar{b}\bar{b}$ tetraquarks we are going to investigate have the following quantum numbers:

(i) $qq = (ud - du)/\sqrt{2}$:
$\quad I(J^P) = 0(1^+)$.
(ii) $qq \in \{uu, (ud + du)/\sqrt{2}, dd\}$:
$\quad I(J^P) \in \{1(0^+), 1(1^+), 1(2^+)\}$.
(iii) $qq \in \{ss, cc\}$:
$\quad I(J^P) \in \{0(0^+), 0(1^+), 0(2^+)\}$.

C. Fit function for lattice QCD $\bar{b}\bar{b}$ potential results

Using lattice QCD, one can compute $\bar{b}\bar{b}$ potentials in the static limit (i.e., for $m_b \to \infty$) from first principles, i.e.,

from the QCD Lagrangian (cf. [30,31] and Sec. III). Of course, these potentials can be obtained only for a limited number of discrete separations r. Therefore, a suitable fit function is required to interpolate between the lattice QCD results and also to extrapolate beyond them. This fit function is based on the qualitative expectations discussed above and will be used in the Schrödinger equation in Sec. IV, where we determine whether and in which channels bound four-quark states exist.

For two heavy antiquarks $\bar{b}\bar{b}$ inside a cloud of two light quarks qq, i.e., at small $\bar{b}\bar{b}$ separations, we expect a Coulomb-like potential of order $-2\alpha_s/3r \approx -\pi/24r$ corresponding to a color triplet. At larger separations r, the potential will be screened by the light quarks qq. This is due to the decrease of the wave function ψ of each of the light quarks with respect to their separations from the heavy antiquarks. One expects this decrease to follow an exponential of a power of r, i.e., $\psi \propto \exp(-(r/d)^p)$, where d roughly describes the size of each of the $\bar{b}q$ systems, i.e., the size of a $B_{(s,c)}^{(*)}$ meson $\lesssim 0.5$ fm. The parameter p characterizes the radial profile of the light quark wave function inside the $B_{(s,c)}^{(*)}$ meson. Assuming the $q\bar{b}$ interaction inside the $B_{(s,c)}^{(*)}$ meson is dominated by a linear confining potential, one can estimate the parameter p. In the case, where the quark q is rather heavy, e.g., $q = c$, the corresponding nonrelativistic Schrödinger equation is solved by Airy functions, resulting in $p = 3/2$. A similar but relativistic treatment for a lighter quark yields $p = 2$ instead.

These considerations suggest the following fit function for lattice QCD $\bar{b}\bar{b}$ potential results:

$$V(r) = -\frac{\alpha}{r}\exp\left(-\left(\frac{r}{d}\right)^p\right) + V_0, \qquad (4)$$

where it is expected that $\alpha \approx 2\alpha_s/3 \approx \pi/24 \approx 0.13$, $d \lesssim 0.5$ fm and $p \approx 1.5\ldots2.0$. The constant V_0 is necessary to account for twice the mass of the static-light meson. As will be demonstrated in the following section, this fit function is consistent with lattice QCD results and the crude quantitative expectations for α, d, and p are fulfilled.

III. LATTICE QCD COMPUTATION OF THE $\bar{b}\bar{b}$ INTERACTION IN THE PRESENCE OF TWO LIGHT QUARKS qq

To determine the effective $\bar{b}\bar{b}$ potential quantitatively, we use lattice QCD and consider the limit of infinitely heavy $\bar{b}$ quarks, i.e., the static limit. The first lattice computations of such potentials have been performed in the quenched approximation (cf., e.g., [32–36]). Recently, also computations with dynamical sea quarks have been performed [13,14,30,31,37,38]. In this work, we extend our previous computations for light quark combinations

$qq \in \{(ud - du)/\sqrt{2}, uu, (ud + du)/\sqrt{2}, dd\}$ [30,31] by similar computations with strange and charm quarks, i.e., $qq \in \{(s^{(1)}s^{(2)} - s^{(2)}s^{(1)})/\sqrt{2}, ss, (c^{(1)}c^{(2)} - c^{(2)}c^{(1)})/\sqrt{2}, cc\}$.

A. Lattice QCD setup

We have performed computations using two ensembles of gauge link configurations generated by the European Twisted Mass Collaboration (ETMC) with 2 dynamical quark flavors. The quark action is Wilson twisted mass tuned to maximal twist, while the gluon action is tree-level Symanzik improved. Most importantly, this guarantees automatic $\mathcal{O}(a)$ improvement of spectral quantities, i.e., discretization errors in the resulting $b\bar{b}$ potentials appear only quadratically in the lattice spacing a. Information about these ensembles is collected in Table II. Further details, in particular regarding their generation, can be found in [39,40].

For $b\bar{b}$ potentials in the presence of two light quarks qq with $q \in \{u, d\}$, we reuse our lattice QCD results from [30,31], which were obtained using the ensemble with the coarser lattice spacing $a \approx 0.079$ fm. For $q \in \{s, c\}$, the $b\bar{b}$ interaction is screened at significantly smaller $\bar{b}\bar{b}$ separations (cf. the discussion in Sec. II B and Fig. 2). To be able to resolve the corresponding potentials properly, we decided to use for flavor combinations $qq \in \{(s^{(1)}s^{(2)} - s^{(2)}s^{(1)})/\sqrt{2}, ss, (c^{(1)}c^{(2)} - c^{(2)}c^{(1)})/\sqrt{2}, cc\}$ another ensemble with a finer lattice spacing $a \approx 0.042$ fm. Although the physical extent of the lattice for this ensemble is much smaller than for the other one, this should not introduce significant finite volume effects at the rather small separations we are interested in.

Note that for both ensembles, the u/d quarks are unphysically heavy, corresponding to a pion mass $m_\pi \approx 340$ MeV. Moreover, there are no s and c sea quarks, i.e., our lattice QCD results are obtained in a partially quenched approximation. For the computation of $b\bar{b}$ potentials in the presence of light s and c quarks, we also use a much smaller number of gauge link configurations. The reason is that the propagators of the heavier s and c quarks introduce less statistical noise than those for lighter u/d quarks.

B. Lattice QCD computation of $b\bar{b}$ potentials

We determine $b\bar{b}$ potentials in the presence of two light quarks qq from the exponential decay of temporal correlation functions,

TABLE II. Ensembles of gauge link configurations used for the computation of $b\bar{b}$ potentials (β: inverse gauge coupling; size: number of lattice sites; μ_l: bare u/d quark mass in lattice units; a: lattice spacing; m_π: pion mass; configurations: number of gauge link configurations used).

β	Size	μ_l	a in fm	m_π in MeV	Configurations
3.90	$24^3 \times 48$	0.0040	0.079	340	480
4.35	$32^3 \times 64$	0.00175	0.042	352	100

$$C(t, |\mathbf{r}_1 - \mathbf{r}_2|) = \langle\Omega|\mathcal{O}^\dagger(t)\mathcal{O}(0)|\Omega\rangle \qquad (5)$$

of four-quark creation operators

$$\mathcal{O}(t) = (\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}(\bar{Q}_C(\mathbf{r}_1)q_A^{(1)}(\mathbf{r}_1))(\bar{Q}_D(\mathbf{r}_2)q_B^{(2)}(\mathbf{r}_2)) \qquad (6)$$

at sufficiently large $t_{\min} \le t \le t_{\max}$. Here $\bar{Q}$ denotes a static antiquark operator approximating a $\bar{b}$ quark, q is a light quark operator, A, B, C, D are spin indices, (1), (2) are flavor indices and $\mathcal{C} = \gamma_0\gamma_2$ is the charge conjugation matrix. For the static antiquarks, the only relevant variable is their separation. Their spin components can be combined with $\tilde{\Gamma} \in \{(1 - \gamma_0)\gamma_5, (1 - \gamma_0)\gamma_j\}$, $j = 1, 2, 3$, where the resulting $b\bar{b}$ potential does not depend on which $\tilde{\Gamma}$ matrix is chosen. The spin components of the two light quarks can be coupled in 16 independent ways via Γ, which should be an appropriately chosen combination of γ matrices to realize definite quantum numbers $|j_z|$ (angular momentum with respect to the axis of separation), P (parity) and P_x (behavior under reflections across an axis perpendicular to the axis of separation). For a more detailed discussion of symmetries and quantum numbers, cf. [30].

Note that the creation operators (6), when applied to the vacuum $|\Omega\rangle$, do not only generate definite quantum numbers $(|j_z|, P, P_x)$, but also a structure resembling two $B_{(s,c)}^{(*)}$ mesons separated by $r = |\mathbf{r}_1 - \mathbf{r}_2|$. Such operators should be well suited to excite the ground state of the corresponding $(|j_z|, P, P_x)$ sector, in particular for large $\bar{Q}\bar{Q}$ separations r, where one expects two weakly interacting $B_{(s,c)}^{(*)}$ mesons (cf. the discussion in Sec. II B). Note, however, that the arrangement of the four quarks $qq\bar{Q}\bar{Q}$ in the ground state is decided by QCD dynamics, i.e., automatically realized in the lattice result according to QCD and not by the structure of the employed creation operators. For example, in recent lattice QCD work on tetraquark candidates, it has been demonstrated that operators similar to (6) generate significant overlap to a variety of different four-quark structures, including mesonic molecules, diquark-antidiquark pairs and two essentially noninteracting mesons [41,42].

In previous computations [13,30,31], we have considered light quarks $q \in \{u, d\}$ (due to technical reasons, the quark mass $m_{u,d}$ was chosen unphysically heavy corresponding to a pion mass $m_\pi \approx 340$ MeV; cf. also the first line in Table II). We studied the scalar isosinglet with antisymmetric spin $j = 0$ and flavor $qq = (ud - du)/\sqrt{2}$ (in the following denoted as the scalar u/d channel), as well as the vector isotriplet with symmetric spin $j = 1$ and flavor $qq \in \{uu, (ud + du)/\sqrt{2}, dd\}$ (in the following denoted as the vector u/d channel), which are the two attractive channels between ground state mesons (B and B^*). Note that the scalar u/d channel was found to be more attractive

than the vector u/d channel, as expected from quark model considerations [cf. Eq. (1) and the discussion in Sec. II B].

In this work, we extend these computations to heavier pairs of light quarks $qq = ss$ and $qq = cc$. For these symmetric flavor combinations, the only attractive channel for two ground state mesons $(B_{s,c}^{(*)})$ is the vector channel, i.e., with light quark spin $j = 1$. It corresponds to $\Gamma = (1 + \gamma_0)\gamma_j$, $j = 1, 2, 3$, in the creation operator (6).

To be able to study also the scalar channel, i.e., $j = 0$, with strange and charm quarks, we consider two hypothetical degenerate flavors with the mass of the strange or the charm quark, which allow us to form antisymmetric flavor combinations $qq = (s^{(1)}s^{(2)} - s^{(2)}s^{(1)})/\sqrt{2}$ and $qq = (c^{(1)}c^{(2)} - c^{(2)}c^{(1)})/\sqrt{2}$. It corresponds to $\Gamma = (1 + \gamma_0)\gamma_5$ in the creation operator (6).

For further details regarding the lattice QCD computation of $\bar{b}\bar{b}$ potentials, we refer to [30,31]. Examples for $qq = (ud - du)/\sqrt{2}$ (scalar u/d channel) and for $qq \in \{uu, (ud + du)/\sqrt{2}, dd\}$ (vector u/d channel) are shown in [13], Fig. 1.

C. Fitting Eq. (4) to lattice QCD $\bar{b}\bar{b}$ potential results

To describe the lattice QCD $\bar{b}\bar{b}$ potential results $V^{\text{lat}}(r)$ by continuous functions, we perform uncorrelated χ^2 minimizing fits of Eq. (4), i.e., we minimize

$$\chi^2 = \sum_{r=r_{\min},\ldots,r_{\max}} \left(\frac{V(r) - V^{\text{lat}}(r)}{\Delta V^{\text{lat}}(r)}\right)^2 \qquad (7)$$

with respect to the parameters α, d, and V_0, while keeping $p = 2$ fixed (cf. the discussion in Sec. II C) [43]. ΔV^{lat} denote the corresponding statistical errors.

We perform these fits for the scalar u/d, the vector u/d, the scalar s, the vector s, and the scalar c channel. The lattice QCD $\bar{b}\bar{b}$ potential of the remaining vector c channel is, however, strongly screened and consistent with $V^{\text{lat}}(r) = 0$ for $r > 2a$. Such results are not sufficient to perform a stable fit.

To investigate and quantify systematic errors, we do not only perform a single fit for each of the mentioned five channels, but a large number of fits, where we vary the following parameters:

(i) The range of temporal separations $t_{\min} \leq t \leq t_{\max}$ of the correlation function $C(t, r)$ [Eq. (5)] at which $V^{\text{lat}}(r)$ is read off, according to:
- (a) $t_{\max} - t_{\min} \geq a$;
- (b) for u/d channels:
  $4a \leq t_{\min}, \ t_{\max} \leq 9a$;
- (c) for s and c channels:
  $10a \leq t_{\min} \leq 14a, \ t_{\max} \leq 19a$

(small $t_{\min}$ might lead to a contamination by excited states; large $t_{\min}$ and $t_{\max}$ drastically increase statistical errors).

(ii) The range of spatial $\bar{b}\bar{b}$ separations $r_{\min} \leq r \leq r_{\max}$ considered in the χ^2 minimizing fit (7), according to:
- (a) for the vector u/d channel:
  $r_{\min} = 2a$ [44];
- (b) for all other channels:
  $r_{\min} \in \{2a, 3a\}$;
- (c) for u/d channels:
  $r_{\max} \in \{8a, 9a, 10a\}$;
- (d) for s and c channels:
  $r_{\max} \in \{7a, 8a\}$

[$V^{\text{lat}}(r)$ at small $r < 2a$ are expected to suffer from sizable lattice discretization errors, while $V^{\text{lat}}(r)$ at large r is essentially a constant, i.e., has little effect on the relevant fit parameters α and d].

For each of the fitting parameters α, d, and V_0, we construct a distribution by considering the results of all the above listed fits weighted by $\exp(-\chi^2/\text{dof})$ with χ^2 from Eq. (7). The central values of α, d, and V_0 are then defined as the medians of the corresponding distributions and the lower/upper systematic uncertainties are given by the difference of the 16th/84th percentiles to the medians (in the case of a Gaussian distribution, an uncertainty defined in this way would correspond to its width, i.e., 1σ). Since in general the distributions are asymmetric, the systematic uncertainties are asymmetric as well. For more details regarding this method of estimating systematic errors we refer to [45].
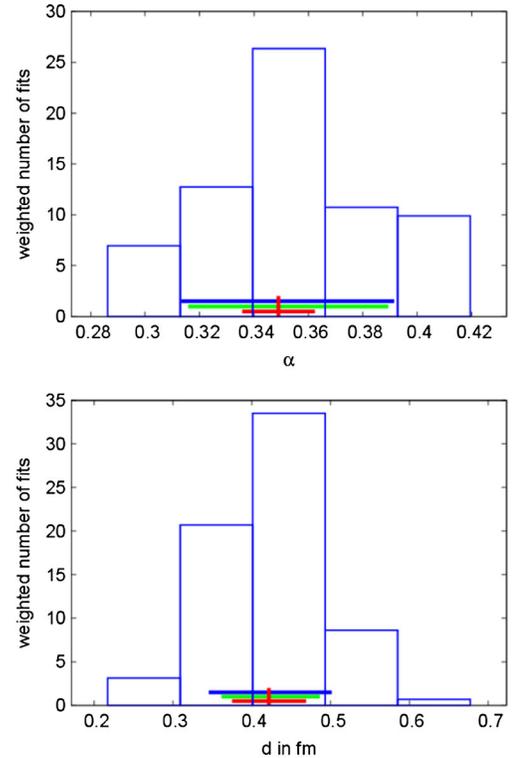

FIG. 3 (color online). Histograms used to estimate systematic errors for α and d for the scalar u/d channel (green, red, and blue bars represent systematic, statistical, and combined errors, respectively).

TABLE III. Parameters α and d obtained from χ^2 minimizing fits of (4) to lattice QCD $\bar{b}\bar{b}$ potential results.

qq	Spin	α	d in fm
$(ud - du)/\sqrt{2}$	scalar	$0.35^{+0.04}_{-0.04}$	$0.42^{+0.08}_{-0.08}$
$uu, (ud + du)/\sqrt{2}, dd$	vector	$0.29^{+0.04}_{-0.06}$	$0.16^{+0.02}_{-0.01}$
$(s^{(1)}s^{(2)} - s^{(2)}s^{(1)})/\sqrt{2}$	scalar	$0.27^{+0.08}_{-0.05}$	$0.20^{+0.10}_{-0.10}$
ss	vector	$0.18^{+0.09}_{-0.05}$	$0.18^{+0.11}_{-0.05}$
$(c^{(1)}c^{(2)} - c^{(2)}c^{(1)})/\sqrt{2}$	scalar	$0.19^{+0.12}_{-0.07}$	$0.12^{+0.03}_{-0.02}$

Finally, to include statistical errors, we compute the jackknife errors of the medians of α, d, and V_0 and add them in quadrature to the corresponding systematic uncertainties.

To illustrate this error estimation procedure, we show in Fig. 3 example histograms representing the distribution of α and d for the scalar u/d channel. The green, red, and blue bars correspond to the systematic, statistical, and combined errors, respectively. In the following, we will always use and quote the combined errors represented by the blue bars.

The final results for α and d are collected in Table III. Note that within errors they agree with the model considerations and crude quantitative expectations discussed in Sec. II. We do not list results for V_0, since it is an irrelevant constant corresponding to twice the mass of a static-light meson. The fit function (4) with the parameter sets from Table III and the corresponding error bands are shown in Fig. 4. Clearly, these results confirm the qualitative expectations discussed in Sec. II B:

(i) The screening of the $\bar{b}\,\bar{b}$ interaction is stronger for heavier light quarks qq.
(ii) The scalar channels are more attractive than the corresponding vector channels.

IV. DEPENDENCE OF THE EXISTENCE OF $qq\bar{b}\bar{b}$ TETRAQUARK STATES ON THE LIGHT QUARK MASS

In [13], we have found evidence for a bound state in the scalar u/d channel, i.e., the existence of a $qq\bar{b}\bar{b} = ud\bar{b}\bar{b}$ tetraquark. For heavier quarks qq, the effective $\bar{b}\bar{b}$ potentials are less attractive. This has qualitatively been anticipated in Sec. II and quantified in Sec. III (in particular cf.
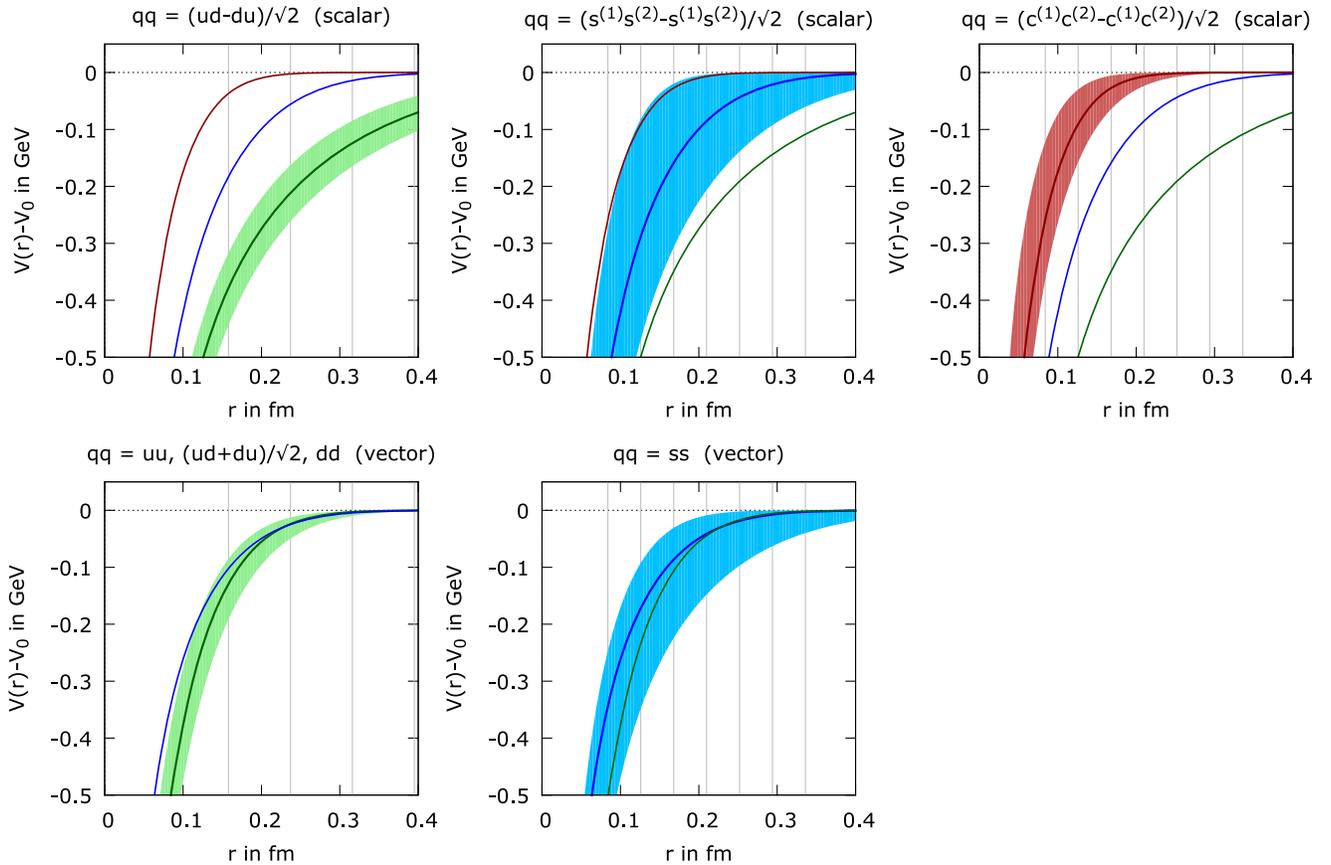

FIG. 4 (color online). $\bar{b}\,\bar{b}$ potentials in the presence of two lighter quarks qq (qq flavor: up/down in green, strange in blue, charm in red; qq spin: $j = 0$, i.e., scalar, in the upper line, $j = 1$, i.e., vector, in the lower line). The plotted curves with the error bands correspond to Eq. (4) with the parameter sets from Table III. Vertical lines indicate lattice separations $r = 2a, 3a, \ldots$ of lattice QCD potential results $V^{\text{lat}}(r)$ used to generate the parameter sets from Table III via χ^2 minimizing fits.

the resulting values for α and d in Table III and the plots in Fig. 4). Thus, for a sufficiently heavy pair of light quarks qq we expect that the $qq\bar{b}\bar{b}$ system will not anymore be able to form a bound state. In the following, we investigate whether this is already the case for strange and/or charm quark masses. We also study the vector channels.

A. The $\bar{b}\bar{b}$ Hamiltonian

We define $U(r) = V(r)|_{V_0=0, p=2}$ with $V(r)$ from of Eq. (4). $U(r)$ with a set of fit parameters α and d from Table III corresponds to the ground state energy of a $qq\bar{b}\bar{b}$ 4-quark system in a specific channel minus the energy of a pair of far separated $B_{(s,c)}^{(*)}$ mesons. Thus, the corresponding Hamiltonian for the relative coordinate of the $\bar{b}\bar{b}$ quarks is

$$H = \frac{\mathbf{p}^2}{2\mu} + 2m_H + U(r), \qquad (8)$$

where $\mu = m_H/2$ is the reduced mass. At large separations, each $\bar{b}$ quark carries the mass of a $B_{(s,c)}^{(*)}$ meson because of screening, and thus $m_H = m_{B_{(s,c)}^{(*)}}$. At small separations, $m_H = m_b$ could be more appropriate. Throughout this section, we always consider two choices, $m_H = m_{B_{(s,c)}}$ and $m_H = m_b$, which yield qualitatively identical results. Note that any dependence on the heavy $\bar{b}$ spins is neglected, because $V(r)$ has been computed in the static limit $m_b \to \infty$. Since the $\bar{b}$ quarks are quite heavy, we expect the static limit to be a reasonable approximation.

In classical mechanics, the $\bar{b}\bar{b}$ separation r would vanish for the ground state, but after quantizing the system, a bound 4-quark state ($E < 2m_H$) may not exist anymore.

B. An analytical estimate for $qq\bar{b}\bar{b}$ binding

In [13], we have derived an approximate analytical rule for the existence/nonexistence of a bound $qq\bar{b}\bar{b}$ state using the Bohr-Sommerfeld quantization condition. If

$$\mu\alpha d \geq \frac{9\pi^2}{128 \times 2^{1/p}(\Gamma(1+1/2p))^2} \qquad (9)$$

is fulfilled, there should be at least one bound state. The right-hand side of this rule has a rather moderate dependence on the exponent p. For example, when p increases from the expected values of 1.5 to 2.0 (cf. Sec. II C), the right-hand side only changes from 0.55 to 0.60. Thus, the existence of a bound state mainly depends on the product of parameters $\mu\alpha d$.

With the medians for the parameters α and d (cf. Table III), we determine the left-hand side of Eq. (9). For the reduced mass, we use both $m_H = m_{B_{(s,c)}}$ ($m_B = 5279$ MeV, $m_{B_s} = 5367$ MeV, $m_{B_c} = 6276$ MeV [46]), which is certainly a good choice for large $\bar{b}\bar{b}$

TABLE IV. Values for $\mu\alpha d$, which represent the left-hand side of Eq. (9). Values > 0.60 [right-hand side of Eq. (9) for $p = 2$] point toward the existence of a bound $qq\bar{b}\bar{b}$ state, while values < 0.60 are an indication against the existence of such a state.

		$\mu\alpha d$	
qq	Spin	$m_H = m_{B_{(s,c)}}$	$m_H = m_b$
$(ud - du)/\sqrt{2}$	scalar	1.97	1.86
uu, $(ud + du)/\sqrt{2}$, dd	vector	0.60	0.57
$(s^{(1)}s^{(2)} - s^{(2)}s^{(1)})/\sqrt{2}$	scalar	0.74	0.69
ss	vector	0.44	0.41
$(c^{(1)}c^{(2)} - c^{(2)}c^{(1)})/\sqrt{2}$	scalar	0.34	0.27

separations, and $\mu = m_b/2$ ($m_b = 4977$ MeV, from quark models [25]), which might be more appropriate for small $\bar{b}\bar{b}$ separations (cf. the discussion in Sec. IV A). The results for $\mu\alpha d$ for the u/d, s and scalar c and vector channels are collected in Table IV. For the scalar u/d channel, there is strong indication for the existence of a tetraquark (i.e., $\mu\alpha d \gg 0.60$), which confirms our findings from [13]. For the vector s channel and for charm quarks, bound $qq\bar{b}\bar{b}$ states are not expected (i.e., $\mu\alpha d \ll 0.60$). For the vector u/d and the scalar s channel the situation is less clear. A more rigorous and quantitative analysis is needed, which is part of the following section.

C. Numerical solution of the Schrödinger equation

To investigate the existence of a bound state more rigorously, we solve the Schrödinger equation with the Hamiltonian (8) numerically. The strongest binding is expected in a s-wave, for which the radial equation is

$$\left(-\frac{1}{2\mu}\frac{d^2}{dr^2} + U(r)\right)R(r) = \underbrace{(E - 2m_H)}_{=E_B}R(r) \qquad (10)$$

with the wave function $\psi = \psi(r) = R(r)/r$. If $E_B = E - 2m_H < 0$, $-E_B$ can be interpreted as the binding energy. We proceed as explained in [13] and solve this equation by imposing Dirichlet boundary conditions $R(r = \infty) = 0$ and using 4th order Runge-Kutta shooting.

For the scalar u/d channel, the lowest eigenvalue $E_B < 0$, which implies the existence of a bound four-quark state. For all other channels, i.e., the vector u/d and the s and c channels, $E_B > 0$, i.e., the corresponding $qq\bar{b}\bar{b}$ tetraquarks will most likely not exist in these channels [47]. These findings confirm the analytical estimates obtained in the previous subsection [Eq. (9) and Table IV].

The central value and the combined systematic and statistical error for the binding energy E_B of the tetraquark state in the scalar u/d channel is obtained by the method discussed in Sec. III C (generating a distribution for E_B from the fits listed in Sec. III C):
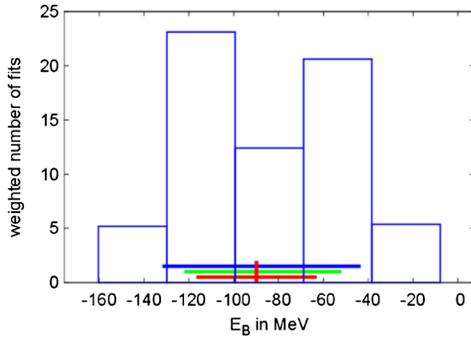
FIG. 5 (color online). Histogram used to estimate the systematic error for the binding energy E_B for the scalar u/d channel and $m_H = m_B$ (green, red, and blue bars represent systematic, statistical, and combined errors, respectively).

$$E_B = -90^{+46}_{-42} \text{ MeV} \quad (\text{for } m_H = m_B), \qquad (11)$$

$$E_B = -93^{+47}_{-43} \text{ MeV} \quad (\text{for } m_H = m_b). \qquad (12)$$

These binding energies are roughly twice as large as their combined systematic and statistical errors. In other words, the confidence level for this $ud\bar{b}\,\bar{b}$ tetraquark state is around 2σ. The corresponding histogram for $m_H = m_B$ is shown in Fig. 5.

To crudely quantify also the nonexistence of bound four-quark states in the remaining channels, we determine numerically by which factors the heavy masses m_H in the Schrödinger equation (10) have to be increased to obtain bound states, i.e., tiny but negative energies E_B [the potentials $U(r)$ are kept unchanged, i.e., we stick to the medians for α and d from Table III]. The resulting factors are collected in Table V. While the scalar s channel is quite close to be able to host a bound state, the scalar c channel and the vector channels are rather far away, since they would require $\bar{b}$ quarks approximately 1.6…3.3 times as heavy as they are in nature. Note that the factors listed in Table V could also be relevant for quark models aiming at studying the binding of tetraquarks quantitatively.

In Fig. 6, we present our results in an alternative graphical way. Binding energy isolines $E_B(\alpha, d) = $ constant are plotted in the $\alpha - d$-plane starting at a tiny energy $E_B = -0.1$ MeV up to rather strong binding,

TABLE V. Factors by which the mass m_H has to be multiplied to obtain a tiny but negative energy E_B. Factors $\ll 1$ indicate strongly bound states, while for values $\gg 1$ bound states are essentially excluded.

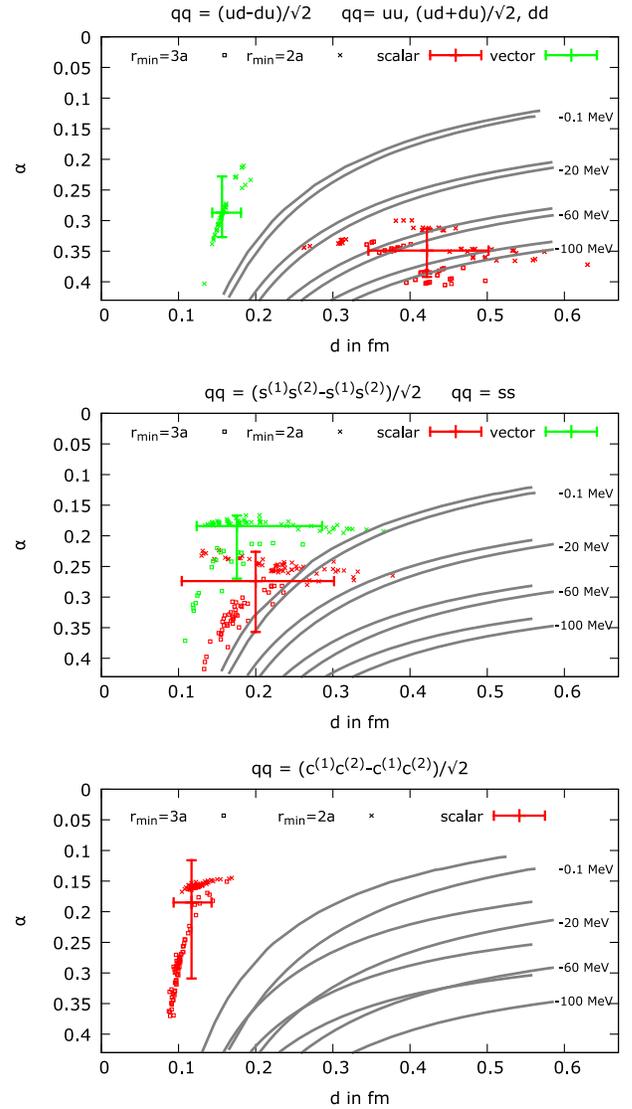| qq	Spin	$m_H = m_{B_{(s,c)}}$	$m_H = m_b$
$(ud - du)/\sqrt{2}$	scalar	0.46	0.49
$uu, (ud + du)/\sqrt{2}, dd$	vector	1.49	1.57
$(s^{(1)}s^{(2)} - s^{(2)}s^{(1)})/\sqrt{2}$	scalar	1.20	1.29
ss	vector	2.01	2.18
$(c^{(1)}c^{(2)} - c^{(2)}c^{(1)})/\sqrt{2}$	scalar	2.57	3.24

FIG. 6 (color online). Binding energy isolines $E_B(\alpha, d) = $ constant in the α-d-plane for u/d, s, and c light quarks qq, respectively (gray dashed lines: $m_H = m_{B_{(s,c)}}$; gray solid lines: $m_H = m_b$). The red and green dots represent the fits of Eq. (4) to lattice QCD $\bar{b}\,\bar{b}$ potential results, while the red and green bars are the corresponding combined systematic and statistical errors.

$E_B = -100$ MeV (gray dashed lines have been computed with $m_H = m_{B_{(s,c)}}$, gray solid lines with $m_H = m_b$). The three plots correspond to u/d, s and c light quarks qq, respectively. Each fit of Eq. (4) to lattice QCD $\bar{b}\,\bar{b}$ potential results (cf. the detailed discussion about systematic error estimation for α and d in Sec. III C) is represented by a dot (red: scalar channels; green: vector channels; crosses: $r_{\min} = 2a$; boxes: $r_{\min} = 3a$). The extensions of these point clouds represent the systematic uncertainties with respect to α and d. If a point cloud is localized above or left of the isoline with $E_B = -0.1$ MeV (approximately the binding threshold), the corresponding four quarks $qq\bar{b}\,\bar{b}$ will not form a bound state. A localization below or right of

that isoline is a strong indication for the existence of a tetraquark. In case the point cloud is intersected by that isoline, the estimated systematic error is too large to make a definite statement regarding the existence or nonexistence of a bound four-quark state. The big red and green bars in horizontal and vertical direction represent the combined systematic and statistical errors of α and d, as quoted in Table III. One can observe and conclude the following from Fig. 6:

  (i) There is clear evidence for a tetraquark state in the scalar u/d channel.

 (ii) The scalar s channel is close to binding/unbinding. A definite statement with our currently available lattice QCD data is not possible.

(iii) The scalar c and all vector channels do not host a bound four-quark state.

These findings are consistent with the results presented above in Table IV and Table V.

V. CONCLUSIONS AND OUTLOOK

In a previous publication [13], we have found indication for the existence of a $qq\bar{b}\bar{b}$ tetraquark with $qq = (ud - du)/\sqrt{2}$ (i.e., in the scalar u/d channel). In this work, we extend these studies by considering for qq not only u/d, but also heavier s and c quarks. To compute the $\bar{b}\bar{b}$ potential, the $\bar{b}$ quarks are treated in the static limit, while the $u/d/s/c$ quarks are treated as quarks of finite mass (the u/d quarks are unphysically heavy corresponding to a pion mass of approximately 340 MeV). We use the Born-Oppenheimer approximation, which means that we consider the $\bar{b}$ quarks in the potential of the light $u/d/s/c$ quarks. In order to determine whether a bound state exists, we insert the resulting potential to the radial Schrödinger equation of two $\bar{b}$ quarks, or B mesons, in an s-wave. In contrast to [13], we investigate and quantify systematic uncertainties in detail, associated with the extraction of $\bar{b}\bar{b}$ potentials from lattice QCD correlation functions.

Our main results are the following:

  (i) We confirm the $ud\bar{b}\bar{b}$ tetraquark state in the scalar u/d channel predicted in our previous paper [13] with confidence level $\approx 2\sigma$. The overall quantum numbers of this state are $I(J^P) = 0(1^+)$. Note that this state is stable with respect to the strong interactions, but can, of course, decay weakly.

 (ii) There is no bound four-quark state in the vector u/d channel $(I(J^P) = \in \{1(0^+), 1(1^+), 1(2^+)\})$. Note, however, that we have been using unphysically heavy u/d quarks ($m_\pi \approx 340$ MeV). Since decreasing the light quark mass should enhance binding, it will be interesting to explore in the future whether a bound four-quark state exists at physically light u/d quark mass.

(iii) $ss\bar{b}\bar{b}$ and $cc\bar{b}\bar{b}$ tetraquarks, which correspond to the vector s and c channels ($J^P \in \{0^+, 1^+, 2^+\}$), do not exist.

(iv) It is of conceptual interest to introduce a hypothetical second s or c quark flavor. Then it is possible to also study the scalar s and c channels, i.e., $((s^{(1)}s^{(2)} - s^{(2)}s^{(1)})/\sqrt{2})\bar{b}\bar{b}$ and $((c^{(1)}c^{(2)} - c^{(2)}c^{(1)})/\sqrt{2})\bar{b}\bar{b}$ systems ($J^P = 1^+$). While in the scalar c channel there is no bound four-quark state, the situation is less clear for s quarks. Improved lattice QCD results (less statistical errors, finer resolution of $\bar{b}\bar{b}$ separations) are needed before a definite statement can be made. Binding in the hypothetical scalar s channel would indicate *a fortiori* binding for four-quark systems $((us - su)/\sqrt{2})\bar{b}\bar{b}$ and $((ds - sd)/\sqrt{2})\bar{b}\bar{b}$. Such light-strange channels would then be highly relevant for experimental tetraquark searches.

We consider these results to be important because they indicate both to experimental collaborations and to quark model phenomenologists which $qq\bar{b}\bar{b}$ tetraquarks are expected to exist and which are not.

To supply data for future quark model studies of tetraquarks, we also provide parametrizations of the potential of two static antiquarks $\bar{b}\bar{b}$ in the presence of two lighter quarks qq, where $qq \in \{(ud - du)/\sqrt{2}, uu, (ud + du)/\sqrt{2}, dd, (s^{(1)}s^{(2)} - s^{(2)}s^{(1)})/\sqrt{2}, ss, (c^{(1)}c^{(2)} - c^{(2)}c^{(1)})/\sqrt{2}\}$. Moreover, we have determined quantitatively for these channels by which factor the heavy quark or meson mass m_H has to be increased to obtain a tetraquark state.

It is also interesting to compare our findings to other groups studying the same or similar systems using, however, different theoretical approaches. For instance in [48], in the framework of QCD sum rules, binding for flavors equivalent to $ud\bar{b}\bar{b}$, $us\bar{b}\bar{b}$, and $ss\bar{b}\bar{b}$ has been found, and no binding for doubly charmed tetraquarks. However, these bound systems have $J^P = 0^-$ and $J^P = 1^-$ different from our results. Another example using the Dyson-Schwinger framework is [49], where a tetraquark composed of four charm quarks, i.e., $cc\bar{c}\bar{c}$, has recently been predicted with a mass significantly lighter than $2m_{\eta_c}$. In principle our static antiquarks can also be considered as a crude approximation of $\bar{c}\bar{c}$. Since we do not find a bound state for $qq = cc$, there seems to be a qualitative discrepancy to our results, which would be interesting to understand and to resolve.

As an outlook, it would be interesting to decrease the light u/d quark mass to their physical value, since this should increase the radius of a B meson, reduce screening and, therefore, lead to a larger binding energy. As mentioned above, a tetraquark could then also exist in the vector u/d channel. Additionally, lighter u/d quark masses may also allow the study of light meson exchange interactions between the two B mesons. Because simulations and computations at lighter u/d quark masses are computationally very expensive, we leave them for a future publication.

Since there is a bound state for $qq = (ud - du)/\sqrt{2}$, and possibly even for $qq = (s^{(1)}s^{(2)} - s^{(2)}s^{(1)})/\sqrt{2}$, it will be very interesting to investigate $us\bar{b}\bar{b}$ (or equivalently $ds\bar{b}\bar{b}$)

systems. This will, however, require additional computations and also the implementation of certain modifications in our analysis procedure. We plan to study such flavor combinations in the near future.

Another interesting, but very challenging task, is to include corrections due to the heavy $\bar{b}\bar{b}$ spins. While in principle it is possible to compute such corrections using lattice QCD (cf. [50,51], where this has been pioneered for the standard static quark-antiquark potential), in practice we expect this to be extremely hard for $qq\bar{b}\bar{b}$ systems. Therefore, a more promising and realistic approach seems to include such spin-dependent interactions in the Schrödinger equation, which will result in a coupled channel differential equation. We are currently in the process of exploring this approach, where first promising qualitative results have recently been presented at a conference [52].

Once these techniques are fully developed for $qq\bar{b}\bar{b}$ systems, it will be most interesting to extend them to $q\bar{q}b\bar{b}$

systems and to study the crypto-exotic $b\bar{b}$ tetraquark candidates observed by the Belle collaboration [3].

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