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Dibaryons in a constituent quark model

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We investigate the properties of dibaryons containing u and d quarks in the constituent quark model. In constructing the ground state wave function, we choose the spatial part to be fully symmetric and the remaining color, isospin, and spin part to be antisymmetric so as to satisfy the Pauli principle. By adapting the isospin $\otimes$ spin (IS) coupling scheme that combines the isospin basis function with the spin basis function, and subsequently coupling this to the color singlet basis function, we construct the color $\otimes$ isospin $\otimes$ spin states compatible with the physical states of the dibaryon. By using the variational method, we then calculate the mass of the dibaryon in a nonrelativistic potential model, involving Coulomb, color confinement, and color-spin hyperfine interaction. In particular, to assess the stability for different types of the confinement potential, we introduce one that is linearly proportional to the interquark distance and another to its square root. For all cases considered, we find that there are no compact bound states against the strong decay.

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I. INTRODUCTION

The recent observation of $d^*(2380)$ with quantum number $I(J^P) = 0(3^+)$ measured by the WASA detector at COSY [1–6] revived interest in the study of multiquark hadrons and led to the renewed investigation of the possible existence of either the $\Delta\Delta$ or six-quark state. Theoretically, starting with the work of Jaffe [7], there were already many studies on the stability of a six-quark system. In particular, in relation to $d^*(2380)$, a study of nonstrange diybaryon was made in Ref. [8], and a work in Ref. [9] was based on the one gluon exchange interaction. Initially, using the bag model with a strange quark, Jaffe [7] predicted that the H dibaryon with $J^P = 0^+$ and $I = 0$ consisting of $uuddss$ could be stable against decay into two Λ baryons when only the color-spin hyperfine interaction was taken into account. Using a similar quark model, Silvestre-Brac and Leandri [10] classified all dibaryon states within the $SU(3)_F$ representation, and they investigated the stability of these states against the decay into the allowed two baryon decay: Through this study, they found that the $\Omega\Omega$ dibaryon is most likely bound, and the H dibaryon could be stable.

Additional models were used to study the stability of the H dibaryon; these include lattice gauge [11], bag model [12], Skyrme model [13], and potential model [14]. In Ref. [15], using the Goldstone boson exchange interaction, the authors predicted that the H dibaryon could not exist. While experimental searches for the H dibaryon seem to suggest that it is not stable against strong decay for realistic quark masses, recent lattice gauge theory calculations suggest that it does become bound when the quark mass increases [16,17].

In addition to the study of the H dibaryon, the dibaryon with strangeness -1 or -3 has been proposed by Maltman [18] and Goldman [19], respectively. Pepin and Stancu [20] investigated the stability of the $uuddsQ$ ($Q = c$ or b) type of quark configuration with a chiral constituent quark model, and they found the dibaryon to be unstable against strong decay. The stability of a multiquark system is known to increase when heavier quarks are included in the tetraquark configuration ($qq\bar{b}\,\bar{b}$) [21–24]. Although the mechanism for stability is different, dibaryons with heavy quarks, such as q^4Q^2 ($Q = c$ or b), have been studied within the simple chromomagnetic model [25].

In this paper, we investigate all dibaryon states containing the u and d quarks, and we calculate their masses in the framework of the nonrelativistic quark model by using the variational method, with a potential that includes the color-spin hyperfine potential introduced in Ref. [26]. In order to examine the stability of the dibaryon for strong decay, we first fit the parameters of the model to reproduce the masses of the baryon multiplet. Then, by comparing the dibaryon masses to the relevant two baryon threshold, we determine whether the dibaryon is bound against strong decay.

The confinement part in our model originates from the effect of the one gluon exchange interaction $\lambda_i^c\lambda_j^c$. But, in principle, the Hamiltonian in the $SU(3)$ symmetric quark model can also have a term proportional to the $SU(3)$ invariant operators in the cubic form which could originate from an intrinsic three-body color confinement interaction. There are two independent three-body color invariant operators: one that can be expressed in terms of two different types of Casimir operators of $SU(3)$, the other that cannot. Since we expect that adding a three-body color invariant operator is very important for the stability of a dibaryon with different flavors, it is necessary to introduce a formula for the operators in terms of the element of the permutation group S_6, based on the established formula by Stancu [27] and Dmitrasinovic [28]. Using this formula, we calculate the matrix element of the operators with respect to the color singlet basis function in a six-quark system and explore the role of the operators in baryon and dibaryon masses.

This paper is organized as following. We introduce the Hamiltonian and fit the baryon spectrum in Sec. II. We construct the spatial function in a simple Gaussian form in Sec. III. We present all of the physical states and construct the color ⊗ flavor ⊗ spin states of a dibaryon in Sec. IV. We show the numerical results obtained from the variational method and deal with the three-body color operators in Sec. V. Finally, we give the summary in Sec. VI.

II. HAMILTONIAN

For the nonrelativistic Hamiltonian, we take the confinement and hyperfine potential for the color and spin degree of freedom given by

$$H = \sum_{i=1}^{6}\left(m_i + \frac{\mathbf{p}_i^2}{2m_i}\right) - \frac{3}{16}\sum_{i<j}^{6}\lambda_i^c\lambda_j^c(V_{ij}^C + V_{ij}^{SS}), \qquad (1)$$

where m_i's are the quark masses, $\lambda_i^c/2$ is the color operator of the ith quark for the color $SU(3)$, and V_{ij}^C and V_{ij}^{SS} are the confinement and the hyperfine potential, respectively. For the confinement potential, we adopt the following two different types.

(a) Type 1:

$$V_{ij}^C = -\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0} - D. \qquad (2)$$

In the following analysis, we take the units for κ, a_0, and D to be MeV fm, $(\mathrm{MeV})^{-1}$ fm, and MeV, respectively.

(b) Type 2:

$$V_{ij}^C = -\frac{\kappa}{r_{ij}} + \frac{(r_{ij})^{1/2}}{a_0} - D. \qquad (3)$$

Here, the unit of a_0 is taken to be $(\mathrm{MeV})^{-1}\,(\mathrm{fm})^{1/2}$. The hyperfine term which effectively splits the multiplets of baryon with respect to spin is given by

$$V_{ij}^{SS} = \frac{\hbar^2 c^2 \kappa'}{m_i m_j c^4}\frac{1}{(r_{0ij})^2 r_{ij}} e^{-(r_{ij})^2/(r_{0ij})^2}\sigma_i\cdot\sigma_j, \qquad (4)$$

where the unit of κ' is taken to be MeV fm. Here, r_{ij} is the distance between the interquarks $|\mathbf{r}_i - \mathbf{r}_j|$ and (r_{0ij}) are chosen to depend on the masses of the interquarks, given by

$$r_{0ij} = 1/\left(\alpha + \beta\frac{m_i m_j}{m_i + m_j}\right), \qquad (5)$$

where the unit of α is $(\mathrm{fm})^{-1}$ and the unit of β is $(\mathrm{MeV\,fm})^{-1}$. We choose to keep the isospin symmetry by requiring that $m_u = m_d$ (MeV). In the Hamiltonian, the parameters have been chosen so that the fitted mass of both the baryon octet and the decuplet are comparable with those of experiments. The fitting parameters are given in Table I, and the masses obtained with these parameters using the variational method are given in Table II. It should be noted that including the pion and sigma exchange potentials are important for a consistent description of three-quark and six-quark states, as discussed in Refs. [29,30]. However, in this work, we are first interested in searching for a possible compact bound dibaryon structure within the purely gluonic exchange potential. Extensions to include pion and sigma exchange potentials will be left for future work.

TABLE I. Parameters fitted to the experimental baryon octet and decuplet masses for the two different types of potentials.

	κ	a_0	D	κ'	α	β	m_u	m_s
Type 1	107.6	0.001062	952.6	107.6	2.36	0.0015	340	610
Type 2	109.6	0.001103	963.6	168.6	2.16	0.0018	348	612

A. Baryon spectrum

In constructing the basis function of a baryon, we restrict the flavor symmetry to isospin part only so that we consider only the u, d quarks as identical quarks, and we find the total wave function of baryon according to the Pauli principle. When we calculate the expectation value of the potential terms for baryon with certain symmetry, it is convenient to introduce the following three Jacobian coordinates so as to reduce our problem to the two-body system in the center of mass frame.

(a) Coordinate I:

$$\boldsymbol{x_1} = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2), \quad \boldsymbol{x_2} = \sqrt{\frac{2}{3}}\left(\mathbf{r}_3 - \frac{1}{2}\mathbf{r}_1 - \frac{1}{2}\mathbf{r}_2\right). \qquad (6)$$

(b) Coordinate II:

$$\boldsymbol{y_1} = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_3), \quad \boldsymbol{y_2} = \sqrt{\frac{2}{3}}\left(\mathbf{r}_2 - \frac{1}{2}\mathbf{r}_1 - \frac{1}{2}\mathbf{r}_3\right). \qquad (7)$$

(c) Coordinate III:

$$\boldsymbol{z_1} = \frac{1}{\sqrt{2}}(\mathbf{r}_2 - \mathbf{r}_3), \quad \boldsymbol{z_2} = \sqrt{\frac{2}{3}}\left(\mathbf{r}_1 - \frac{1}{2}\mathbf{r}_2 - \frac{1}{2}\mathbf{r}_3\right). \qquad (8)$$

By using a simple Gaussian function, we construct the following fully symmetric spatial function for baryons composed of u and d constituents only:

$$R = \exp[-a(\boldsymbol{x_1})^2 - b(\boldsymbol{x_2})^2] + \exp[-a(\boldsymbol{y_1})^2 - b(\boldsymbol{y_2})^2] + \exp[-a(\boldsymbol{z_1})^2 - b(\boldsymbol{z_2})^2], \qquad (9)$$

where a and b are variational parameters. Since the total wave function of the baryon, such as N and P ($I = 1/2$,

$S = 1/2$) or Δ ($I = 3/2$, $S = 3/2$), is fully antisymmetric due to the Pauli principle, the rest of the total wave function must be fully antisymmetric if we choose the spatial function to be fully symmetric.

Concerning the color basis function of the baryon, we consider the color singlet state, as the hadrons are observed to be colorless. The baryon has only one color singlet state, coming from the irreducible representation of $[3]_C \otimes [3]_C \otimes [3]_C$, given by

$$|C\rangle = \begin{array}{|c|}\hline 1\\\hline 2\\\hline 3\\\hline\end{array} = \frac{1}{\sqrt{6}}\epsilon_{ijk}q^i(1)q^j(2)q^k(3), \tag{10}$$

which is fully antisymmetric under the exchange of any two particles among 1, 2, and 3. We note that the Young tableau follows the rule of the standard Young-Yamanouchi representation, which will be shown later in detail.

For the spin basis functions, the baryon can have $S = 1/2$ consisting of two different types, and $S = 3/2$ containing one type, as follows.

(a) $S = 3/2$:

$$|S^{3/2}\rangle = \begin{array}{|c|c|c|}\hline 1&2&3\\\hline\end{array} = \uparrow\uparrow\uparrow.$$

(b) $S = 1/2$:

$$|S_1^{1/2}\rangle = \begin{array}{|c|c|}\hline 1&2\\\hline 3\\\cline{1-1}\end{array} = \tfrac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow),$$

$$|S_2^{1/2}\rangle = \begin{array}{|c|c|}\hline 1&3\\\hline 2\\\cline{1-1}\end{array} = \tfrac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow).$$

As we can see, the spin part of the basis functions of baryon $|S^{3/2}\rangle$ is completely symmetric, while that of $|S_1^{1/2}\rangle$ and $|S_2^{1/2}\rangle$ is partially symmetric; the former being symmetric between particles 1 and 2, and the latter antisymmetric between particles 1 and 2.

Likewise, we construct the isospin basis function of the baryon for $I = 3/2$ and $I = 1/2$.

(a) $I = 3/2$:

$$|I^{3/2}\rangle = \begin{array}{|c|c|c|}\hline 1&2&3\\\hline\end{array} = uuu.$$

(b) $I = 1/2$:

$$|I_1^{1/2}\rangle = \begin{array}{|c|c|}\hline 1&2\\\hline 3\\\cline{1-1}\end{array} = \tfrac{1}{\sqrt{6}}(2uud - udu - duu),$$

$$|I_2^{1/2}\rangle = \begin{array}{|c|c|}\hline 1&3\\\hline 2\\\cline{1-1}\end{array} = \tfrac{1}{\sqrt{2}}(udu - duu).$$

For the baryons with ($I = 3/2$, $S = 3/2$) and ($I = 1/2$, $S = 1/2$), the antisymmetry property of the total wave function can be easily obtained from the direct product of the totally antisymmetric part of the color basis function times the totally symmetric properties of the remain function comprised of the spatial function, the isospin basis functions, and the spin basis functions. The fully symmetric part of the isospin $\otimes$ spin basis function in both cases of the isospin and the spin and can be written as

(a) Δ ($I = 3/2$, $S = 3/2$):
$|I^{3/2}, S^{3/2}\rangle = |I^{3/2}\rangle \otimes |S^{3/2}\rangle$,

(b) N, P ($I = 1/2$, $S = 1/2$):
$|I^{1/2}, S^{1/2}\rangle = \frac{1}{\sqrt{2}}(|I_1^{1/2}\rangle \otimes |S_1^{1/2}\rangle + |I_2^{1/2}\rangle \otimes |S_2^{1/2}\rangle)$.

In the case of ($I = 1/2$, $S = 1/2$), we can see that $|I^{1/2}, S^{1/2}\rangle$ is symmetric between particles 1 and 2. The remaining symmetry for permutation (23) can be deduced from the following formulas, according to the rule to the standard Young-Yamanouchi representation:

$$(23)\begin{array}{|c|c|}\hline 1&2\\\hline 3\\\cline{1-1}\end{array} = -\frac{1}{2}\begin{array}{|c|c|}\hline 1&2\\\hline 3\\\cline{1-1}\end{array} + \frac{\sqrt{3}}{2}\begin{array}{|c|c|}\hline 1&3\\\hline 2\\\cline{1-1}\end{array}$$
$$(23)\begin{array}{|c|c|}\hline 1&3\\\hline 2\\\cline{1-1}\end{array} = \frac{1}{2}\begin{array}{|c|c|}\hline 1&3\\\hline 2\\\cline{1-1}\end{array} + \frac{\sqrt{3}}{2}\begin{array}{|c|c|}\hline 1&2\\\hline 3\\\cline{1-1}\end{array} \tag{11}$$

Now, we can construct the basis function of the color $\otimes$ isospin $\otimes$ spin for ($I = 3/2$, $S = 3/2$) and ($I = 1/2$, $S = 1/2$), which are completely antisymmetric. These are given by

$$|I^{3/2}, S^{3/2}\rangle = |C\rangle \otimes |I^{3/2}, S^{3/2}\rangle,$$
$$|I^{1/2}, S^{1/2}\rangle = |C\rangle \otimes |I^{1/2}, S^{1/2}\rangle. \tag{12}$$

For the hyperon, we treat the strange quark as distinguishable from the u and d quarks and will not require the total wave function to be fully antisymmetric. It is then easy to construct the total wave function for the hyperons with strangeness $s = -1$ and $s = -2$ that satisfy the Pauli principle in the u and d quark sectors only.

TABLE II. This table shows the mass of baryons in octet and decuplet obtained from the variational method. The fourth row indicates the experimental data (unit: MeV).

	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	$(0, \frac{1}{2})$	$(1, \frac{1}{2})$	$(\frac{1}{2}, \frac{3}{2})$	$(1, \frac{3}{2})$	$(\frac{3}{2}, \frac{3}{2})$
(I, S)	N, P	Ξ	Λ	Σ	Ξ^*	Σ^*	Δ
Type 1	977.1	1315.3	1115.6	1206.0	1530.2	1403.4	1267.5
Type 2	976.3	1380.3	1115.6	1238.2	1593.0	1419.3	1237.2
Exp	938.2–939.5	1314.8–1321.7	1115.6	1189.3–1197.4	1530–1531.8	1382.8–1387.2	1230–1234

III. SPATIAL FUNCTION

In order to construct a fully antisymmetric wave function of dibaryon containing only identical u and d particles, we choose the spatial function to be fully symmetric such that the rest of the wave function represented by color ⊗ flavor ⊗ spin should be antisymmetric. Since we restrict the $SU(3)$ flavor of the dibaryon to the isospin symmetry only, the flavor state can be identified with the isospin quantum number. In describing the system consisting of six quarks, it is convenient to deal with the system in the center of mass frame, reducing the number of suitable Jacobian coordinates of the system to five. The five Jacobian coordinates are given by

$$\begin{aligned} \boldsymbol{x}_1^1 &= \frac{1}{\sqrt{2}}(\mathbf{r}_1-\mathbf{r}_2), \qquad \boldsymbol{x}_2^1 = \frac{1}{2}(\mathbf{r}_3-\mathbf{r}_4+\mathbf{r}_5-\mathbf{r}_6), \\ \boldsymbol{x}_3^1 &= \frac{1}{2}(\mathbf{r}_3-\mathbf{r}_4-\mathbf{r}_5+\mathbf{r}_6), \qquad \boldsymbol{x}_4^1 = \frac{1}{2}(\mathbf{r}_3+\mathbf{r}_4-\mathbf{r}_5-\mathbf{r}_6), \\ \boldsymbol{x}_5^1 &= \frac{1}{\sqrt{12}}(\mathbf{r}_3+\mathbf{r}_4+\mathbf{r}_5+\mathbf{r}_6-2\mathbf{r}_1-2\mathbf{r}_2). \end{aligned} \tag{13}$$

The variational method for calculating the mass of the dibaryon turns out to be easy when the Gaussian form with respect to the Jacobian coordinates is used for the spatial wave function. Using the Jacobian coordinates given in Eq. (13) in the Gaussian wave function, we find the form to be symmetric under the exchange of any two particles among 3, 4, 5, 6 and at the same time symmetric under the exchange of two particles between 1 and 2; these symmetry properties are denoted as [3456][12]. Introducing the variational parameters a, b, c, the spatial function is then given by

$$R^{s_1} = \exp[-(a(\boldsymbol{x}_1^1)^2 + b(\boldsymbol{x}_2^1)^2 + b(\boldsymbol{x}_3^1)^2 + b(\boldsymbol{x}_4^1)^2 + c(\boldsymbol{x}_5^1)^2)]. \tag{14}$$

In addition to this Gaussian function, the full symmetry of the spatial function requires the linear sum of 14 additional Gaussian functions, each of which has a specific symmetry under particle exchange. The next set of five Jacobian coordinates with the symmetry of [2456][13] is given by

$$\begin{aligned} \boldsymbol{x}_1^2 &= \frac{1}{\sqrt{2}}(\mathbf{r}_1-\mathbf{r}_3), \qquad \boldsymbol{x}_2^2 = \frac{1}{2}(\mathbf{r}_2-\mathbf{r}_4+\mathbf{r}_5-\mathbf{r}_6), \\ \boldsymbol{x}_3^2 &= \frac{1}{2}(\mathbf{r}_2-\mathbf{r}_4-\mathbf{r}_5+\mathbf{r}_6), \qquad \boldsymbol{x}_4^2 = \frac{1}{2}(\mathbf{r}_2+\mathbf{r}_4-\mathbf{r}_5-\mathbf{r}_6), \\ \boldsymbol{x}_5^2 &= \frac{1}{\sqrt{12}}(\mathbf{r}_2+\mathbf{r}_4+\mathbf{r}_5+\mathbf{r}_6-2\mathbf{r}_1-2\mathbf{r}_3). \end{aligned} \tag{15}$$

The corresponding Gaussian function specifying the symmetry of [2456][13] is given by

$$R^{s_2} = \exp[-(a(\boldsymbol{x}_1^2)^2 + b(\boldsymbol{x}_2^2)^2 + b(\boldsymbol{x}_3^2)^2 + b(\boldsymbol{x}_4^2)^2 + c(\boldsymbol{x}_5^2)^2)]. \tag{16}$$

We find that the set of Jacobian coordinates necessary to obtain the fully symmetric wave function under the exchange of any two particles among 1, 2, 3, 4, 5, and 6—and consequently the corresponding Gaussian functions—is the one with the following symmetry: [3456][12], [2456][13], [2356][14], [2346][15], [2345][16], [1456][23], [1356][24], [1346][25], [1345][26], [1256][34], [1246][35], [1245][36], [1236][45], [1235][46], [1234][56].

Combining these Gaussian functions with the symmetry into a linear form, we obtain the spatial function with three variational parameters a, b, c, which is fully symmetric as follows:

$$\begin{aligned} R^s = {} & R^{s_1} + R^{s_2} + R^{s_3} + R^{s_4} + R^{s_5} \\ & + R^{s_6} + R^{s_7} + R^{s_8} + R^{s_9} + R^{s_{10}} \\ & + R^{s_{11}} + R^{s_{12}} + R^{s_{13}} + R^{s_{14}} + R^{s_{15}}. \end{aligned} \tag{17}$$

It is easy to check the symmetry of the spatial function with respect to all of the permutations of S_6 by considering only the five permutations (12), (23), (34), (45), and (56), as these permutations generate all of the permutations of S_6.

IV. CLASSIFICATION OF DIBARYON WITH ISOSPIN SYMMETRY

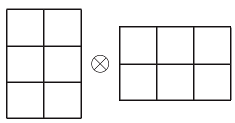
A. Isospin and spin state of the dibaryon

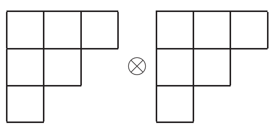
In this section, we investigate the state of the dibaryon consisting of identical u, d quarks, whose flavor part is characterized by isospin symmetry. Since the color ⊗ isospin ⊗ spin state of each quark can be represented by $[3]_C \otimes [2]_I \otimes [2]_S$, the direct product of six quarks enables us to classify all of the states of the dibaryon with respect to the state of isospin and spin, denoted by $|I, S\rangle$. In our notation, the $[3]_C$ indicates the fundamental representation of $SU(3)_C$, $[2]_I$ the fundamental representation of $SU(2)_I$, and $[2]_S$ the fundamental representation of $SU(2)_S$. In our case, where we choose the spatial function of dibaryon to be fully symmetric, the color ⊗ isospin ⊗ spin state of the dibaryon will be chosen to be fully antisymmetric. The fully antisymmetric state of the color ⊗ isospin ⊗ spin state can be easily obtained from the classifying of the multiplets of the direct six product of $[12]_{CIS}$, which is the fundamental representation of $SU(12)_{CIS}$, and gives the multiplet with dimension 924 represented by Young tableau $[1^6]$. Using the original representation of $[3]_C \otimes [4]_{IS}$, which we will equivalently represent as

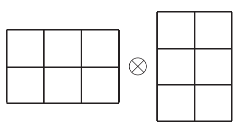
$([3]_C, [4]_{IS})$, the totally antisymmetric multiplet of $[1^6]_{CIS}$ can be decomposed as

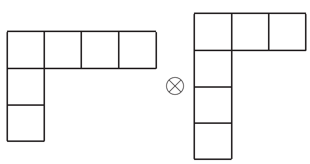
$$[1^6]_{CIS} = ([1]_C, [50]_{IS}) \oplus ([8]_C, [64]_{IS}) \oplus ([\bar{10}]_C, [\bar{10}]_{IS}) \oplus ([10]_C, [10]_{IS}) \oplus ([27]_C, [6]_{IS}). \quad (18)$$

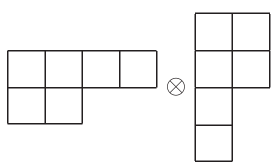
By using the Young tableau, we can easily find that the multiplets on the right-hand side of Eq. (18) are fully antisymmetric. According to the permutation group theory, for a given Young tableau, the fully antisymmetric function can be constructed by multiplying the Young tableau by its conjugate of the Young tableau, where the conjugate representation of a given Young tableau can be obtained by exchanging the row and column in the Young tableau:

$([1]_C, [50]_{IS}) =$  ,

$([8]_C, [64]_{IS}) =$  ,

$([\bar{10}]_C, [\bar{10}]_{IS}) =$  ,

$([10]_C, [10]_{IS}) =$  ,

$([27]_C, [6]_{IS}) =$  .

Because the dibaryon is supposed to be a physically observable color singlet state, the dibaryon belongs to the five independent color singlet states represented by the Young tableau of [2,2,2] from the classification of the multiplets of the direct six times $[3]_C$. Hence, only the $([1]_C, [50]_{IS})$ states in Eq. (18) are allowed as the physical states of the dibaryon. We can also find the decomposition of $[50]_{IS}$ with respect to the multiplets of $[2]_I \otimes [2]_S$, given by

$$[50]_{IS} = ([1]_I, [3]_S) \oplus ([3]_I, [1]_S) \oplus ([3]_I, [5]_S) \oplus ([5]_I, [3]_S) \oplus ([7]_I, [1]_S) \oplus ([1]_I, [7]_S). \quad (19)$$

TABLE III. The decay mode for the dibaryon into two baryons with respect to the (I, S) states.

	$\Delta\Delta$	ΔN	ΔN	NN	$\Delta\Delta$	NN
(I, S)	(3,0)	(2,1)	(1,2)	(1,0)	(0,3)	(0,1)

These $|I, S\rangle$ states are all the possible states of dibaryon with isospin symmetry which satisfy the antisymmetry property and the color singlet requirement as an observable. As we see in the Eq. (19), the isospin and the spin of the dibaryon with isospin symmetry have $I = 0(S = 0)$, $I = 1(S = 1)$, $I = 2(S = 2)$, and $I = 3(S = 3)$, coming from the classification of $SU(2)$ of the dibaryon, given by

$$[2]_I \otimes [2]_I \otimes [2]_I \otimes [2]_I \otimes [2]_I \otimes [2]_I = [1]_{I=0} \otimes F_{[3,3]} \oplus [3]_{I=1} \otimes F_{[4,2]} \oplus [5]_{I=2} \otimes F_{[5,1]} \oplus [7]_{I=3} \otimes F_{[6]}, \quad (20)$$

where F indicates the number of times the corresponding state (I) appears in the product. Moreover, the subscript of F represents the Young tableau for the state (I), $F_{[3,3]} = 5$, $F_{[4,2]} = 9$, $F_{[5,1]} = 5$, and $F_{[6]} = 1$. Consequently, there are six different kinds of $|I, S\rangle$ states of the dibaryon which dictate the possible two baryon decay mode in the strong force as shown in Table III. In order to investigate the stability of the dibaryon against a strong decay, we will examine energies in relation to the threshold for the decay mode.

It is very important to understand the property of the basis functions of the dibaryon in calculating the expectation values of both the confinement and the hyperfine potential, proportional to $\lambda_i^c \lambda_j^c$ and $\lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j$, respectively. So we will now establish the basis functions of the color, isospin, and spin based on the Young tableau, which are very useful for constructing completely antisymmetric states. Then, the expectation values can be easily calculated by using the complete antisymmetry properties.

B. Color basis functions

The dibaryon of our interest with isospin symmetry has a color singlet function represented by the Young tableau of [2,2,2], as we mentioned earlier. Since the dimension of the Young tableau of [2,2,2] is 5, there are five color singlet functions corresponding to the Young tableau of [2,2,2]. We define the color singlet functions as follows:

$$
\begin{aligned}
|C_1\rangle = \begin{array}{|c|c|}\hline 1&2\\\hline 3&4\\\hline 5&6\\\hline\end{array} &= \{[(12)_6 3]_8 [4(56)_6]_8\}_1 \\
&= \frac{1}{3\sqrt{2}}(\epsilon_{ijk}q^i(1)q^j(3)q^k(5)\epsilon_{lmn}q^l(2)q^m(4)q^n(6) \\
&\quad - \frac{1}{2}\epsilon_{ijk}q^i(1)q^j(2)q^k(5)\epsilon_{lmn}q^l(3)q^m(4)q^n(6) \\
&\quad - \frac{1}{2}\epsilon_{ijk}q^i(1)q^j(3)q^k(4)\epsilon_{lmn}q^l(2)q^m(5)q^n(6) \\
&\quad + \frac{1}{4}\epsilon_{ijk}q^i(1)q^j(2)q^k(4)\epsilon_{lmn}q^l(3)q^m(5)q^n(6) \\
&\quad - \frac{3}{4}\epsilon_{ijk}q^i(1)q^j(2)q^k(3)\epsilon_{lmn}q^l(4)q^m(5)q^n(6)),
\end{aligned}
$$

$$
\begin{aligned}
|C_2\rangle = \begin{array}{|c|c|}\hline 1&3\\\hline 2&4\\\hline 5&6\\\hline\end{array} &= \{[(12)_{\bar{3}} 3]_8 [4(56)_6]_8\}_1 \\
&= \frac{1}{2\sqrt{6}}(\epsilon_{ijk}q^i(1)q^j(2)q^k(5)\epsilon_{lmn}q^l(3)q^m(4)q^n(6) \\
&\quad - \frac{1}{2}\epsilon_{ijk}q^i(1)q^j(2)q^k(4)\epsilon_{lmn}q^l(3)q^m(5)q^n(6) \\
&\quad + \frac{1}{2}\epsilon_{ijk}q^i(1)q^j(2)q^k(3)\epsilon_{lmn}q^l(4)q^m(5)q^n(6)),
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
|C_3\rangle = \begin{array}{|c|c|}\hline 1&2\\\hline 3&5\\\hline 4&6\\\hline\end{array} &= \{[(12)_6 3]_8 [4(56)_{\bar{3}}]_8\}_1 \\
&= \frac{1}{2\sqrt{6}}(\epsilon_{ijk}q^i(1)q^j(3)q^k(4)\epsilon_{lmn}q^l(2)q^m(5)q^n(6) \\
&\quad - \frac{1}{2}\epsilon_{ijk}q^i(1)q^j(2)q^k(4)\epsilon_{lmn}q^l(3)q^m(5)q^n(6) \\
&\quad + \frac{1}{2}\epsilon_{ijk}q^i(1)q^j(2)q^k(3)\epsilon_{lmn}q^l(4)q^m(5)q^n(6)),
\end{aligned}
$$

$$
\begin{aligned}
|C_4\rangle = \begin{array}{|c|c|}\hline 1&3\\\hline 2&5\\\hline 4&6\\\hline\end{array} &= \{[(12)_{\bar{3}} 3]_8 [4(56)_{\bar{3}}]_8\}_1 \\
&= \frac{1}{4\sqrt{2}}(\epsilon_{ijk}q^i(1)q^j(2)q^k(4)\epsilon_{lmn}q^l(3)q^m(5)q^n(6) \\
&\quad - \frac{1}{3}\epsilon_{ijk}q^i(1)q^j(2)q^k(3)\epsilon_{lmn}q^l(4)q^m(5)q^n(6)),
\end{aligned}
$$

$$
\begin{aligned}
|C_5\rangle = \begin{array}{|c|c|}\hline 1&4\\\hline 2&5\\\hline 3&6\\\hline\end{array} &= \{[(12)_{\bar{3}} 3]_1 [4(56)_{\bar{3}}]_1\}_1 \\
&= \frac{1}{6}\epsilon_{ijk}q^i(1)q^j(2)q^k(3)\epsilon_{lmn}q^l(4)q^m(5)q^n(6).
\end{aligned}
$$

We note that the color singlet functions followed by the standard Young-Yamanouchi representation are symmetric with respect to any adjacent particles that lie in the same row, and the functions are antisymmetric with respect to any particles that lie in the same column. The definition next to the Young tableau expresses the convenient intermediate states for constructing the color singlet, and it originates from the $[8] \otimes [8]$ and the $[1] \otimes [1]$. The orthogonality of the color singlet functions, $\langle C_i | C_j \rangle = \delta_{ij}$, is easily obtained by using the tensor form and, in fact, results from the orthogonality of the standard Young-Yamanouchi bases.

There are several ways for calculating the expectation value of $\lambda_i^c \lambda_j^c$ with respect to the color singlet functions. Among those, it is very useful to consider one that is based on the irreducible matrix representation of the permutation group with respect to the standard Young-Yamanouchi bases whose irreducible matrix for the transposition is symmetric. Moreover, when considering the operator for describing three gluon exchange, through either $if_{abc}\lambda_i^a \lambda_j^b \lambda_j^c$ or $d_{abc}\lambda_i^a \lambda_j^b \lambda_j^c$, as we shall show in the detailed calculation in Appendix B, this method gives us a simple form if we know the irreducible matrix representation of the permutation group. However, in our case with the fully antisymmetric color $\otimes$ isospin $\otimes$ spin state, labeled by $|C_i I_j S_k\rangle$, we only need the expectation value of $\lambda_1^c \lambda_2^c$ because the expectation value of $\lambda_i^c \lambda_j^c$ can be obtained from the former using the antisymmetric property of the basis functions. In fact, this calculation is performed using the formula, $\sum_{i<j}^6 \lambda_i^c \lambda_j^c = -8/3N$, where N is the number of the participant particle in this dibaryon, $N = 6$, resulting in $15\langle \lambda_1^c \lambda_2^c \rangle = -16$. Here, 15 is the number of ways of pairing between particle i and particle j ($i < j$, $i, j = 1, 2, 3$, 4, 5, and 6).

C. Flavor basis functions

Since the flavor of the dibaryon is in the irreducible representation of $SU(2)$, we consider the flavor basis functions in terms of the isospin representation that is allowed for the dibaryon. As in the case of color basis functions, the isospin part can be obtained using similar techniques based on the Young tableau. Moreover, it is convenient to establish the orthogonal basis functions with a certain symmetry, making use of the standard Young-Yamanouchi bases. For the dibaryon, as mentioned above, the representation of the isospin has $I = 0$, whose Young tableau is [3,3] with a dimension of 5, $I = 1$, whose Young tableau is [4,2] with a dimension of 9, $I = 2$, whose Young tableau is [5,1] with a dimension of 5, and $I = 3$, whose Young tableau is [6] with a dimension of 1.

(a) $I = 0$: five basis functions with Young tableau [3,3],

$$
|I_1^0\rangle = \begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5&6\\\hline\end{array}\quad
|I_2^0\rangle = \begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3&5&6\\\hline\end{array}\quad
|I_3^0\rangle = \begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2&5&6\\\hline\end{array},
$$

$$
|I_4^0\rangle = \begin{array}{|c|c|c|}\hline 1&2&5\\\hline 3&4&6\\\hline\end{array}\quad
|I_5^0\rangle = \begin{array}{|c|c|c|}\hline 1&3&5\\\hline 2&4&6\\\hline\end{array}
$$

(b) $I=1$: nine basis functions with Young [4,2],

$$|I_1^1\rangle=\begin{array}{cccc}1&2&3&4\\5&6&&\end{array}\quad |I_2^1\rangle=\begin{array}{cccc}1&2&3&5\\4&6&&\end{array}\quad |I_3^1\rangle=\begin{array}{cccc}1&2&4&5\\3&6&&\end{array}$$
$$|I_4^1\rangle=\begin{array}{cccc}1&3&4&5\\2&6&&\end{array}\quad |I_5^1\rangle=\begin{array}{cccc}1&2&3&6\\4&5&&\end{array}\quad |I_6^1\rangle=\begin{array}{cccc}1&2&4&6\\3&5&&\end{array},$$
$$|I_7^1\rangle=\begin{array}{cccc}1&3&4&6\\2&5&&\end{array}\quad |I_8^1\rangle=\begin{array}{cccc}1&2&5&6\\3&4&&\end{array}\quad |I_9^1\rangle=\begin{array}{cccc}1&3&5&6\\2&4&&\end{array}$$

(c) $I=2$: five basis functions with Young tableau [5,1],

$$|I_1^2\rangle=\begin{array}{ccccc}1&2&3&4&5\\6&&&&\end{array}\quad |I_2^2\rangle=\begin{array}{ccccc}1&2&3&4&6\\5&&&&\end{array}$$
$$|I_3^2\rangle=\begin{array}{ccccc}1&2&3&5&6\\4&&&&\end{array}\quad |I_4^2\rangle=\begin{array}{ccccc}1&2&4&5&6\\3&&&&\end{array},$$
$$|I_5^2\rangle=\begin{array}{ccccc}1&3&4&5&6\\2&&&&\end{array}$$

(d) $I=3$: one basis function with Young tableau [6],

$$|I^3\rangle=\begin{array}{cccccc}1&2&3&4&5&6\end{array}.$$

We can also see the orthogonality of the isospin states $\langle I_i^l|I_j^l\rangle=\delta_{ij}$ in a given irreducible representation of isospin from the orthogonality of the standard Young-Yamanouchi bases as well as the orthogonality between any two different irreducible representations, according to the group theory.

D. Spin basis functions

For the spin states of the dibaryon, the representation of spin states of the dibaryon is the same as that of the isosopin states because the irreducible representation of $SU(2)$ should also be applied in this case:

(a) $S=0$: five basis functions with Young tableau [3,3],

$$|S_1^0\rangle=\begin{array}{ccc}1&2&3\\4&5&6\end{array}\quad |S_2^0\rangle=\begin{array}{ccc}1&2&4\\3&5&6\end{array}\quad |S_3^0\rangle=\begin{array}{ccc}1&3&4\\2&5&6\end{array},$$
$$|S_4^0\rangle=\begin{array}{ccc}1&2&5\\3&4&6\end{array}\quad |S_5^0\rangle=\begin{array}{ccc}1&3&5\\2&4&6\end{array}$$

(b) $S=1$: nine basis functions with Young tableau [4,2],

$$|S_1^1\rangle=\begin{array}{cccc}1&2&3&4\\5&6&&\end{array}\quad |S_2^1\rangle=\begin{array}{cccc}1&2&3&5\\4&6&&\end{array}\quad |S_3^1\rangle=\begin{array}{cccc}1&2&4&5\\3&6&&\end{array}$$
$$|S_4^1\rangle=\begin{array}{cccc}1&3&4&5\\2&6&&\end{array}\quad |S_5^1\rangle=\begin{array}{cccc}1&2&3&6\\4&5&&\end{array}\quad |S_6^1\rangle=\begin{array}{cccc}1&2&4&6\\3&5&&\end{array},$$
$$|S_7^1\rangle=\begin{array}{cccc}1&3&4&6\\2&5&&\end{array}\quad |S_8^1\rangle=\begin{array}{cccc}1&2&5&6\\3&4&&\end{array}\quad |S_9^1\rangle=\begin{array}{cccc}1&3&5&6\\2&4&&\end{array}$$

(c) $S=2$: five basis functions with Young tableau [5,1],

$$|S_1^2\rangle=\begin{array}{ccccc}1&2&3&4&5\\6&&&&\end{array}\quad |S_2^2\rangle=\begin{array}{ccccc}1&2&3&4&6\\5&&&&\end{array}$$
$$|S_3^2\rangle=\begin{array}{ccccc}1&2&3&5&6\\4&&&&\end{array}\quad |S_4^2\rangle=\begin{array}{ccccc}1&2&4&5&6\\3&&&&\end{array},$$
$$|S_5^2\rangle=\begin{array}{ccccc}1&3&4&5&6\\2&&&&\end{array}$$

(d) $S=3$: one basis function with Young tableau [6],

$$|S^3\rangle=\begin{array}{cccccc}1&2&3&4&5&6\end{array}.$$

We are now in a position to construct the completely antisymmetric function of the color ⊗ isospin ⊗ spin state of dibaryon, which we will denote by $|C_i,I_j,S_k\rangle$. In particular, we choose the $|I,S\rangle$ basis function to be in the [3,3] representation, and in the conjugate of the color singlet [2,2,2], so that the color singlet ⊗ isospin ⊗ spins becomes fully antisymmetric. With this IS coupling scheme, we can find the fully antisymmetric function of $|C,I,S\rangle$ for all (I,S)'s, by using the Clebsch-Gordan (CG) coefficient for making the representation of [3,3] of isospin ⊗ spin function. In calculating the Clebsch-Gordan coefficients of the element of the permutation group, S_6, it is convenient to use the factorization property that factorizes the CG coefficients of S_n into an isoscalar factor, which is called a K matrix, and the CG coefficients of S_{n-1}. For example, the isoscalar factor can be defined by [31]

$$\begin{aligned}S([f']p'q'y'[f'']p''q''y''|[f]pqy)\\=K([f']p'[f'']p''|[f]p)S([f'_{p'}]q'y'[f''_{p''}]q''y''|[f_p]qy),\end{aligned}\tag{22}$$

where S in the left-hand (right-hand) side is a CG coefficient of S_n (S_{n-1}). In this notation, $[f_p]$ is the Young tableau associated with S_{n-1} which can be obtained from $[f]$, the Young tableau of S_n, by removing the nth particle characterized by pqy; p is the position of the nth particle in the row, q the position of the $(n-1)$th particle in the row, and y the position of the $(n-2)$th particle in the row, respectively. In our case, by repeating the process of factorizing the CG coefficients of S_6 further, we find the CG coefficients from the following formula,

$$\begin{aligned}S([f']p'q'y'r'[f'']p''q''y''r''|[f]pqyr)\\=K([f']p'[f'']p''|[f]p)K([f'_{p'}]q'[f''_{p''}]q''|[f_p]q)\\\times K([f'_{p'q'}]y'[f''_{p''q''}]y''|[f_{pq}]y)\\\times S([f'_{p'q'y'}]r'[f''_{p''q''y''}]r''|[f_{pqy}]r),\end{aligned}\tag{23}$$

where S in the third row is the CG coefficient of S_3. When we calculate the CG coefficients, we use the relevant isoscalar factors for S_4, S_5, and S_6 in Eq. (23) which were obtained by Stancu and Pepin [32]. Then, we find the five-dimensional basis function of $|I^i, S^j\rangle$ for $(I = i, S = j)$ corresponding to the Young tableau of [3.3]:

$$|[I^i, S^j]_1\rangle = \begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5&6\\\hline\end{array}, \quad |[I^i, S^j]_2\rangle = \begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3&5&6\\\hline\end{array},$$
$$|[I^i, S^j]_3\rangle = \begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2&5&6\\\hline\end{array}, \quad |[I^i, S^j]_4\rangle = \begin{array}{|c|c|c|}\hline 1&2&5\\\hline 3&4&6\\\hline\end{array},$$
$$|[I^i, S^j]_5\rangle = \begin{array}{|c|c|c|}\hline 1&3&5\\\hline 2&4&6\\\hline\end{array}.$$

Here, the detailed representations of all five of the basis functions in terms of the IS coupling scheme are given in Appendix A. As discussed above, we find the color $\otimes$ isospin $\otimes$ spin state satisfying the fully antisymmetric property for $(I, S) = (i, j)$, to be given by

$$|C, I^i, S^j\rangle = \frac{1}{\sqrt{5}}(|C_1\rangle \otimes |[I^i, S^j]_5\rangle - |C_2\rangle \otimes |[I^i, S^j]_4\rangle - |C_3\rangle \otimes |[I^i, S^j]_3\rangle + |C_4\rangle \otimes |[I^i, S^j]_2\rangle - |C_5\rangle \otimes |[I^i, S^j]_1\rangle). \quad (24)$$

In dealing with the expectation value of $-\lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j$ with respect to the $|C, I, S\rangle$ state for all (I, S)'s, the symmetry property of the state makes this calculation simple in that $\langle -\sum_{i<j}^6 \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j\rangle = 15\langle -\lambda_1^c \lambda_2^c \sigma_1 \cdot \sigma_2\rangle$, as argued previously. Moreover, using the symmetry properties of the wave function, one can derive the effective formula which is expressed in terms of the Casimir operators of the isospin, spin, and color, given by [33]

$$-\sum_{i<j}^{N} \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j = \left[\frac{4}{3}N(N-6) + 4I(I+1) + \frac{4}{3}S(S+1) + 2C_c\right], \quad (25)$$

where N is the total number of quarks in this system, and $C_c = \frac{1}{4}\lambda^c \lambda^c$, that is, the first kind of Casimir operator of $SU(3)$ in the system of the N quarks. Since we must consider only the color singlet as a physical observable, the term C_c vanishes. For practical purposes, our calculation of $\langle -\lambda_1^c \lambda_2^c \sigma_1 \cdot \sigma_2\rangle$ can be easily performed with the symmetry between particles 1 and 2 in the $|C, I, S\rangle$ state, whose property of the symmetry is definitely derived from the Young tableau. Then we find the following formula:

$$\lambda_1^c \lambda_2^c \left|\begin{array}{|c|}\hline 1\\\hline 2\\\hline\end{array}\right\rangle = -\frac{8}{3}\left|\begin{array}{|c|}\hline 1\\\hline 2\\\hline\end{array}\right\rangle, \quad \lambda_1^c \lambda_2^c \left|\begin{array}{|c|c|}\hline 1&2\\\hline\end{array}\right\rangle = \frac{4}{3}\left|\begin{array}{|c|c|}\hline 1&2\\\hline\end{array}\right\rangle,$$
$$\sigma_1 \cdot \sigma_2 \left|\begin{array}{|c|}\hline 1\\\hline 2\\\hline\end{array}\right\rangle = -\left|\begin{array}{|c|}\hline 1\\\hline 2\\\hline\end{array}\right\rangle, \quad \sigma_1 \cdot \sigma_2 \left|\begin{array}{|c|c|}\hline 1&2\\\hline\end{array}\right\rangle = \left|\begin{array}{|c|c|}\hline 1&2\\\hline\end{array}\right\rangle. \quad (26)$$

V. NUMERICAL RESULTS

In this section, we analyze the numerical results obtained from the variational method by using the completely symmetric spatial function as the trial function. Table V shows the result of the analysis with the trial spacial wave function given in Eq. (13) after adding 14 additional forms, as discussed before. Among all the dibaryons with (I, S), it is the dibaryon with $(I = 0, S = 3)$ that is most likely to be stable against the strong decay. However, as we see in Table V, we find that even for this state the two baryon threshold lies below the dibaryon mass.

Although a simple comparison of $\langle -\sum_{i<j}^6 \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j\rangle$ in the hyperfine interaction between the dibaryon and the two baryons, as given in Table IV, shows the splitting is minimal in the $(I = 0, S = 3)$ channel, the lowest mass of the dibaryon for both types of the potential considered is far above the threshold of two Δ baryons.

Table V shows that the dibaryon mass is larger for the type 1 potential than for the type 2 one. This is because the size of the dibaryon in the former is smaller than that in the latter, as determined by the inverse of the variational parameters a, b, c. To better understand this point, we show the values of each energy term of the dibaryon with $(I = 0, S = 3)$ in Table VI.

Since the size of the dibaryon in both types of potential are similar to that of a single baryon as shown in Table VII, the value of the kinetic part of the dibaryon is comparatively larger than the sum of that of the two Δ baryons due to additional kinetic terms. Moreover, the value of kinetic part of dibaryon in type 2 is much smaller than that in type 1 due to its relatively small increase in size.

In addition to the kinetic term, the effect of the half-power confinement part of the type 2 potential causes the mass of the dibaryon to decrease compared to the case of type 1. Nevertheless, the lowest mass in type 2 is still about 155 MeV above the threshold of two Δ baryons, and no other choice for the confinement potential, such as one-third power, is expected to change the stability of the dibaryon.

In investigating the stability in the present work, we can consider the three-body color confinement operators which are mentioned in the Introduction, as this approach may change the stability of multiquark configurations, such as that of the dibaryon. The two types of operators can be expressed in terms of the permutation operators given by

TABLE IV. The expectation value of $-\sum_{i<j}^6 \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j$ with respect to $|C, I, S\rangle$ for all possible (I,S) configurations in the dibaryon, which will be denoted by V_d. ΔV is $V_d - (V_{b1} + V_{b2})$, in which V_{b1} and V_{b2} are those baryons to which the dibaryon can decay.

(I, S)	(3,0)	(2,1)	(1,2)	(1,0)	(0,3)	(0,1)
V_d	48	$\frac{80}{3}$	16	8	16	$\frac{8}{3}$
ΔV	32	$\frac{80}{3}$	16	24	0	$\frac{56}{3}$

TABLE V. The mass of the dibaryon in (I, S) state with the two types of potentials given in Eqs. (2) and (3). The binding energy E_B is taken to be the difference between the mass of the dibaryon and the two baryon threshold. The dimension of the variational parameters are given in fm^{-2}.

(I, S)	(3,0)	(2,1)	(1,2)	(1,0)	(0,3)	(0,1)
Type 1	3132.3	2926.5	2808.5	2710.8	2808.5	2639.6
Variational	$a = 1.2$,	$a = 0.9$,	$a = 1.4$,	$a = 3$,	$a = 1.4$,	$a = 3.6$,
parameters	$b = 1.2$,	$b = 1.5$,	$b = 1.8$,	$b = 1.4$,	$b = 1.8$,	$b = 1.4$,
	$c = 1.4$	$c = 1.7$	$c = 0.9$	$c = 1.3$	$c = 0.9$	$c = 1.3$
E_B	597.3	681.9	563.9	756.6	273.5	685.4
Type 2	2845.5	2711.7	2629.9	2558.6	2629.9	2504.1
Variational	$a = 0.6$,	$a = 0.5$,	$a = 0.7$,	$a = 2.2$,	$a = 0.7$,	$a = 2.6$,
parameters	$b = 0.7$,	$b = 0.9$,	$b = 1.1$,	$b = 0.8$,	$b = 1.1$,	$b = 0.8$,
	$c = 0.6$	$c = 0.8$	$c = 0.5$	$c = 0.7$	$c = 0.5$	$c = 0.8$
E_B	371.1	498.2	416.4	606.0	155.5	551.5

TABLE VI. The values of each energy term of the dibaryon with $(I = 0, S = 3)$ and the Δ baryon. ΔE is the difference between the dibaryon and the two Δ baryon in each term.

Type 1	Kinetic	Linear	Coulomb	Hyperfine
Dibaryon	1282.8	2603.6	−446.9	186.9
Δ Baryon	589.2	1214.8	−239.3	111.6
Variational parameters		$a = 1.4$ $b = 2.1$		
ΔE	104.2	173.8	31.6	−36.3
Type 2	Kinetic	1/2 power	Coulomb	Hyperfine
Dibaryon	722.7	2924.1	−346.6	132.5
Δ Baryon	333.2	1410.4	−186.6	81.7
Variational parameters		$a = 1.4$ $b = 0.7$		
ΔE	56.2	103.2	26.7	−30.8

$$d^{abc}F_i^aF_j^bF_k^c = \frac{1}{4}[(ijk)+(ikj)] + \frac{1}{9}I - \frac{1}{6}[(ij)+(ik)+(jk)],$$
$$f^{abc}F_i^aF_j^bF_k^c = -\frac{i}{4}[(ijk)-(ikj)], \tag{27}$$

where I is the identity operator, and (ijk) and (ij) are operators belonging to the permutation group of S_6, which are called 3-cycles, and 2-cycles, respectively, and $F_i^a = 1/2\lambda_i^a$. Also, there is another formula for $d^{abc}F_i^aF_j^bF_k^c$, which can be conveniently shown to be invariant under the $SU(3)$ algebra, given by [27]

$$d^{abc}F_i^aF_j^bF_k^c = \frac{1}{6}\left[C^{(3)}_{i+j+k} - \frac{5}{2}C^{(2)}_{i+j+k} + \frac{20}{3}\right], \tag{28}$$

where $C^{(2)}$ is the first kind of Casimir operator, and $C^{(2)}$ the second kind of Casimir operator of $SU(3)$. Since the baryon consists of three quarks, the $SU(3)$ invariant operators are written by

TABLE VII. The expectation value of the relative distance between any two quarks for the dibaryon with $(I, S) = (0, 3)$ and the Δ baryon. Since the spatial wave function for both the dibaryon and the baryon are fully symmetric, all of the expectation values of $|\mathbf{r}_i - \mathbf{r}_j|$ for i and j ($i < j = 1 \sim 6$) are the same, for both cases. The units are in femtometers.

	Dibaryon	Baryon
Type 1	$\langle|\mathbf{r}_i - \mathbf{r}_j|\rangle = 0.652$	$\langle|\mathbf{r}_i - \mathbf{r}_j|\rangle = 0.608$
Type 2	$\langle|\mathbf{r}_i - \mathbf{r}_j|\rangle = 0.858$	$\langle|\mathbf{r}_i - \mathbf{r}_j|\rangle = 0.799$

$$d^{abc}F_1^aF_2^bF_3^c = \frac{1}{4}[(123)+(132)] + \frac{1}{9}I - \frac{1}{6}[(12)+(13)+(23)],$$
$$f^{abc}F_1^aF_2^bF_3^c = -\frac{i}{4}[(123)-(132)]. \tag{29}$$

For a baryon which has one color singlet represented by the standard Young-Yamanouchi basis of Young tableau [1,1,1], we use the irreducible representation with one dimension given by

$$\sigma \begin{array}{|c|}\hline 1\\\hline 2\\\hline 3\\\hline\end{array} = (-1)^\sigma \begin{array}{|c|}\hline 1\\\hline 2\\\hline 3\\\hline\end{array}, \qquad (\sigma \in S_3) \tag{30}$$

where $(-1)^\sigma$ is 1 if σ is an even permutation, and −1 if σ is an odd permutation. By using this formula, we can easily calculate the action of the operators on the color singlet of the baryon, $|C\rangle = \frac{1}{\sqrt{6}}\epsilon_{ijk}q^i(1)q^j(2)q^k(3)$ in Eq. (10),

$$d^{abc}F_1^aF_2^bF_3^c|C\rangle = \frac{10}{9}|C\rangle, \tag{31}$$

$$f^{abc}F_1^aF_2^bF_3^c|C\rangle = 0. \tag{32}$$

As we can see, while the introduction of $d^{abc}F_1^aF_2^bF_3^c$ can contribute to the baryon mass, $f^{abc}F_1^aF_2^bF_3^c$ will not.

Likewise, for a dibaryon which has five color singlet bases corresponding to the standard Young-Yamanuochi bases of Young tableau [2,2,2], we can calculate the irreducible matrix representation of the permutation operators belonging to S_6 in the form of a 5×5 matrix in terms of the five color singlet bases. Since the irreducible matrix of (ij) and $[(ijk)+(ikj)]$ are symmetric, and the irreducible matrix of (ijk) is antisymmetric, $d^{abc}F_i^aF_j^bF_k^c$ has the form of a symmetric matrix and $f^{abc}F_i^aF_j^bF_k^c$ has the form of a Hermitian matrix with vanishing diagonal elements. As we will show in detail in Appendix B, the matrix of $d^{abc}F_1^aF_2^bF_3^c$ and $f^{abc}F_1^aF_2^bF_3^c$ are given by

$$\begin{aligned}
&\langle C_1|d^{abc}F_1^aF_2^bF_3^c|C_1\rangle = \langle C_2|d^{abc}F_1^aF_2^bF_3^c|C_2\rangle = \\
&\langle C_3|d^{abc}F_1^aF_2^bF_3^c|C_3\rangle = \langle C_4|d^{abc}F_1^aF_2^bF_3^c|C_4\rangle = -\frac{5}{36} \\
&\langle C_5|d^{abc}F_1^aF_2^bF_3^c|C_5\rangle = 10/9, \\
&\langle C_1|f^{abc}F_1^aF_2^bF_3^c|C_1\rangle = \langle C_2|f^{abc}F_1^aF_2^bF_3^c|C_2\rangle = \\
&\langle C_3|f^{abc}F_1^aF_2^bF_3^c|C_3\rangle = \langle C_4|f^{abc}F_1^aF_2^bF_3^c|C_4\rangle = \\
&\langle C_5|f^{abc}F_1^aF_2^bF_3^c|C_5\rangle = 0. \qquad (33)
\end{aligned}$$

Then, we can find the expectation value of the three-body confinement operators $d^{abc}F_1^aF_2^bF_3^c$ and $f^{abc}F_1^aF_2^bF_3^c$ in terms of $|C,I^i,S^j\rangle$ for $(I=i, S=j)$, remembering that when

$$\begin{aligned}
|C,I^i,S^j\rangle = \frac{1}{\sqrt{5}}(&|C_1\rangle\otimes|[I^i,S^j]_5\rangle - |C_2\rangle\otimes|[I^i,S^j]_4\rangle \\
&-|C_3\rangle\otimes|[I^i,S^j]_3\rangle + |C_4\rangle\otimes|[I^i,S^j]_2\rangle \\
&-|C_5\rangle\otimes|[I^i,S^j]_1\rangle),
\end{aligned}$$

one finds the following:

$$\begin{aligned}
\langle C,I^i,S^j|d^{abc}F_1^aF_2^bF_3^c|C,I^i,S^j\rangle &= \frac{1}{5}\left(-4\times\frac{5}{36}+\frac{10}{9}\right) \\
&= \frac{1}{9}, \qquad (34)
\end{aligned}$$

$$\langle C,I^i,S^j|f^{abc}F_1^aF_2^bF_3^c|C,I^i,S^j\rangle = 0. \qquad (35)$$

Because of the complete symmetry of $|C,I^i,S^j\rangle$ under any permutation of S_6, one finds that $\langle C,I^i,S^j|d^{abc}F_l^aF_m^bF_n^c|C,I^i,S^j\rangle = \langle C,I^i,S^j|d^{abc}F_1^aF_2^bF_3^c|C,I^i,S^j\rangle$ for any l,m,n $(l<m<n=1\sim 6)$. A similar relation holds for $\langle C,I^i,S^j|f^{abc}F_l^aF_m^bF_n^c|C,I^i,S^j\rangle$. Consequently, we can make the following conclusion about the effect of the three-body force to the spectrum of both the baryon and the dibaryon.

First, for the f type, as can be seen from Eq. (32), the three-body force does not contribute to the mass of the baryon. For the dibaryon, we have to add all contributions coming from three quarks that can be selected from six quarks inside the dibaryon. However, as can be seen from Eq. (35) and the discussions above, all expectation values vanish and do not contribute to the dibaryon mass.

For the d type of three-body operators, the situation is more involved. As can be seen in Eq. (31), the color factor of the d type of the three-body force is $10/9$ for the baryon. This part has to be multiplied by the expectation value of the spatial part of the three-body force to obtain its contribution to the baryon mass. The color part of the d type of the single three-body force to the dibaryon is $1/9$ for any three quarks, as seen in Eq. (34). However, there are ${}_6C_3=20$ combinations of three quarks within the six-quark state. Therefore, the total color factor is $20/9$ for the dibaryon, which is twice the factor for the baryon. The actual contribution to the mass will depend on the detailed spatial functional form multiplying the $SU(3)$ invariant operators. For the simplest choice, we can choose it to be the sum of a two-body potential such as $(V_{123}=d^{abc}F_1^aF_2^bF_3^c(r_{12}/a_0+r_{13}/a_0+r_{23}/a_0))$. However, for such a simplified form, because the color factor for the dibayon is just twice that of the baryon, as would be the case for the contribution from the two-body confinement part of the potential, the addition will only result in a reparametrization of the confinement parameter and will not change our previous result on the stability of the dibaryon. On the other hand, an intrinsic three-body force will change the situation and, on those grounds, it is of great importance to have some ideas on the understanding of the two-body and three-body confinement.

VI. SUMMARY

In order for the total wave function to be fully antisymmetric in a six-quark system with only u, d quarks, we first consider the spatial function that is fully symmetric and find 15 Jacobian coordinates appropriate for the symmetry. We then construct the spatial function desirable for this scheme with a Gaussian spatial function to perform a variational method in a nonrelativitic Hamiltonian. Second, we classify the physical states with respect to isospin (I) and spin (S), and find the color singlet basis functions, isospin basis functions, and spin basis functions allowed to the six-quark system, and we construct the color $\otimes$ isospin $\otimes$ spin states that should be completely antisymmetric, by means of an IS scheme that couples the color basis function to the IS basis function. We find that there does not exist a compact dibayron system in any system that is stable against the decay into two bayrons with corresponding quantum numbers. Hence, the recently observed peak in the $I=0, S=3$ $B=2$ channel [1–6] should be a molecular configuration composed of two Δ dibaryons [34].

APPENDIX A: COMPLETELY ANTISYMMETRIC COLOR ⊗ ISOSPIN ⊗ SPIN STATE

In this section, we present the isospin ⊗ spin basis function of the dibaryon for $(I,S)=(i,j)$, $|[I^i,S^j]\rangle$, which is obtained from the IS scheme and the corresponding color ⊗ isospin ⊗ spin state, $|C,I^i,S^j\rangle$, which satisfy the fully antisymmetric property. From the CG coefficient in Eq. (23), in the case of $(I,S)=(0,1)$, the $|[I^0,S^1]\rangle$ basis functions belonging to the Young tableau of [3,3] are presented as the following:

$$\begin{aligned}|[I^0,S^1]_1\rangle&=\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5&6\\\hline\end{array}\\&=\frac{\sqrt{6}}{9}|I_1^0\rangle\otimes|S_1^1\rangle+\frac{\sqrt{10}}{9}|I_1^0\rangle\otimes|S_2^1\rangle+\frac{2\sqrt{5}}{9}|I_1^0\rangle\otimes|S_5^1\rangle-\frac{\sqrt{10}}{18}|I_2^0\rangle\otimes|S_3^1\rangle-\frac{\sqrt{5}}{9}|I_2^0\rangle\otimes|S_6^1\rangle\\&\quad+\frac{\sqrt{60}}{36}|I_2^0\rangle\otimes|S_8^1\rangle-\frac{\sqrt{10}}{18}|I_3^0\rangle\otimes|S_4^1\rangle-\frac{\sqrt{5}}{9}|I_3^0\rangle\otimes|S_7^1\rangle+\frac{\sqrt{60}}{36}|I_3^0\rangle\otimes|S_9^1\rangle-\frac{\sqrt{30}}{18}|I_4^0\rangle\otimes|S_3^1\rangle\\&\quad+\frac{\sqrt{60}}{36}|I_4^0\rangle\otimes|S_6^1\rangle-\frac{\sqrt{30}}{18}|I_5^0\rangle\otimes|S_4^1\rangle+\frac{\sqrt{60}}{36}|I_5^0\rangle\otimes|S_7^1\rangle.\end{aligned}\tag{A1}$$

$$\begin{aligned}|[I^0,S^1]_2\rangle&=\begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3&5&6\\\hline\end{array}\\&=-\frac{\sqrt{10}}{18}|I_1^0\rangle\otimes|S_3^1\rangle-\frac{\sqrt{20}}{18}|I_1^0\rangle\otimes|S_6^1\rangle+\frac{\sqrt{15}}{18}|I_1^0\rangle\otimes|S_8^1\rangle+\frac{\sqrt{6}}{9}|I_2^0\rangle\otimes|S_1^1\rangle-\frac{\sqrt{10}}{18}|I_2^0\rangle\otimes|S_2^1\rangle\\&\quad+\frac{\sqrt{5}}{9}|I_2^0\rangle\otimes|S_4^1\rangle-\frac{\sqrt{20}}{18}|I_2^0\rangle\otimes|S_5^1\rangle+\frac{\sqrt{10}}{9}|I_2^0\rangle\otimes|S_6^1\rangle+\frac{\sqrt{30}}{36}|I_2^0\rangle\otimes|S_8^1\rangle+\frac{\sqrt{5}}{9}|I_3^0\rangle\otimes|S_4^1\rangle\\&\quad+\frac{\sqrt{10}}{9}|I_3^0\rangle\otimes|S_7^1\rangle+\frac{\sqrt{30}}{36}|I_3^0\rangle\otimes|S_9^1\rangle-\frac{\sqrt{30}}{18}|I_4^0\rangle\otimes|S_2^1\rangle-\frac{\sqrt{15}}{18}|I_4^0\rangle\otimes|S_3^1\rangle+\frac{\sqrt{15}}{18}|I_4^0\rangle\otimes|S_5^1\rangle\\&\quad+\frac{\sqrt{30}}{36}|I_4^0\rangle\otimes|S_6^1\rangle+\frac{\sqrt{15}}{18}|I_5^0\rangle\otimes|S_4^1\rangle-\frac{\sqrt{30}}{36}|I_5^0\rangle\otimes|S_7^1\rangle.\end{aligned}\tag{A2}$$

$$\begin{aligned}|[I^0,S^1]_3\rangle&=\begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2&5&6\\\hline\end{array}\\&=-\frac{\sqrt{10}}{18}|I_1^0\rangle\otimes|S_4^1\rangle-\frac{\sqrt{20}}{18}|I_1^0\rangle\otimes|S_7^1\rangle+\frac{\sqrt{15}}{18}|I_1^0\rangle\otimes|S_9^1\rangle-\frac{\sqrt{5}}{9}|I_2^0\rangle\otimes|S_4^1\rangle-\frac{\sqrt{10}}{9}|I_2^0\rangle\otimes|S_7^1\rangle\\&\quad-\frac{\sqrt{30}}{36}|I_2^0\rangle\otimes|S_9^1\rangle+\frac{\sqrt{6}}{9}|I_3^0\rangle\otimes|S_1^1\rangle-\frac{\sqrt{10}}{18}|I_3^0\rangle\otimes|S_2^1\rangle-\frac{\sqrt{20}}{18}|I_3^0\rangle\otimes|S_5^1\rangle-\frac{\sqrt{5}}{9}|I_3^0\rangle\otimes|S_3^1\rangle\\&\quad-\frac{\sqrt{10}}{9}|I_3^0\rangle\otimes|S_6^1\rangle-\frac{\sqrt{30}}{36}|I_3^0\rangle\otimes|S_8^1\rangle+\frac{\sqrt{15}}{18}|I_4^0\rangle\otimes|S_4^1\rangle-\frac{\sqrt{30}}{36}|I_4^0\rangle\otimes|S_7^1\rangle-\frac{\sqrt{30}}{18}|I_5^0\rangle\otimes|S_2^1\rangle\\&\quad+\frac{\sqrt{15}}{18}|I_5^0\rangle\otimes|S_4^1\rangle+\frac{\sqrt{15}}{18}|I_5^0\rangle\otimes|S_5^1\rangle+\frac{\sqrt{30}}{36}|I_5^0\rangle\otimes|S_6^1\rangle.\end{aligned}\tag{A3}$$

$$\begin{aligned}|[I^0,S^1]_4\rangle&=\begin{array}{|c|c|c|}\hline 1&2&5\\\hline 3&4&6\\\hline\end{array}\\&=\frac{\sqrt{30}}{18}|I_1^0\rangle\otimes|S_3^1\rangle+\frac{\sqrt{15}}{18}|I_1^0\rangle\otimes|S_6^1\rangle-\frac{\sqrt{30}}{18}|I_2^0\rangle\otimes|S_2^1\rangle-\frac{\sqrt{15}}{18}|I_2^0\rangle\otimes|S_3^1\rangle+\frac{\sqrt{15}}{18}|I_2^0\rangle\otimes|S_5^1\rangle\\&\quad+\frac{\sqrt{30}}{36}|I_2^0\rangle\otimes|S_6^1\rangle-\frac{\sqrt{15}}{18}|I_3^0\rangle\otimes|S_4^1\rangle+\frac{\sqrt{30}}{36}|I_3^0\rangle\otimes|S_7^1\rangle-\frac{\sqrt{6}}{6}|I_4^0\rangle\otimes|S_1^1\rangle+\frac{\sqrt{30}}{12}|I_4^0\rangle\otimes|S_8^1\rangle\\&\quad-\frac{\sqrt{30}}{12}|I_5^0\rangle\otimes|S_9^1\rangle.\end{aligned}\tag{A4}$$

$$
\begin{aligned}
|[I^0,S^1]_5\rangle = &\begin{array}{|c|c|c|}\hline 1&3&5\\\hline 2&4&6\\\hline\end{array}\\
= &-\frac{\sqrt{30}}{18}|I_1^0\rangle\otimes|S_4^1\rangle+\frac{\sqrt{15}}{18}|I_1^0\rangle\otimes|S_7^1\rangle-\frac{\sqrt{15}}{18}|I_2^0\rangle\otimes|S_4^1\rangle+\frac{\sqrt{30}}{36}|I_2^0\rangle\otimes|S_7^1\rangle-\frac{\sqrt{30}}{18}|I_3^0\rangle\otimes|S_2^1\rangle\\
&+\frac{\sqrt{15}}{18}|I_3^0\rangle\otimes|S_3^1\rangle+\frac{\sqrt{15}}{18}|I_3^0\rangle\otimes|S_5^1\rangle-\frac{\sqrt{30}}{36}|I_3^0\rangle\otimes|S_6^1\rangle-\frac{\sqrt{30}}{12}|I_4^0\rangle\otimes|S_9^1\rangle-\frac{\sqrt{6}}{6}|I_5^0\rangle\otimes|S_1^1\rangle\\
&-\frac{\sqrt{30}}{12}|I_5^0\rangle\otimes|S_8^1\rangle.
\end{aligned} \tag{A5}
$$

Coupling the isospin ⊗ spin basis function obtained from the IS scheme to the color singlet basis function, we find the color ⊗ isospin ⊗ spin state satisfying the fully antisymmetry property for $(I,S)=(0,1)$. This is given by

$$
|C,I^0,S^1\rangle=\frac{1}{\sqrt{5}}(|C_1\rangle\otimes|[I^0,S^1]_5\rangle-|C_2\rangle\otimes|[I^0,S^1]_4\rangle-|C_3\rangle\otimes|[I^0,S^1]_3\rangle+|C_4\rangle\otimes|[I^0,S^1]_2\rangle-|C_5\rangle\otimes|[I^0,S^1]_1\rangle). \tag{A6}
$$

In the case of $(I,S)=(1,0)$, the $|[I^1,S^0]\rangle$ basis functions belonging to the Young tableau of [3,3] are presented as the following:

$$
\begin{aligned}
|[I^1,S^0]_1\rangle = &\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5&6\\\hline\end{array}\\
= &\frac{\sqrt{6}}{9}|I_1^1\rangle\otimes|S_1^0\rangle+\frac{\sqrt{10}}{9}|I_2^1\rangle\otimes|S_1^0\rangle+\frac{2\sqrt{5}}{9}|I_5^1\rangle\otimes|S_1^0\rangle-\frac{\sqrt{10}}{18}|I_3^1\rangle\otimes|S_2^0\rangle-\frac{\sqrt{5}}{9}|I_6^1\rangle\otimes|S_2^0\rangle\\
&+\frac{\sqrt{60}}{36}|I_8^1\rangle\otimes|S_2^0\rangle-\frac{\sqrt{10}}{18}|I_4^1\rangle\otimes|S_3^0\rangle-\frac{\sqrt{5}}{9}|I_7^1\rangle\otimes|S_3^0\rangle+\frac{\sqrt{60}}{36}|I_9^1\rangle\otimes|S_3^0\rangle-\frac{\sqrt{30}}{18}|I_3^1\rangle\otimes|S_4^0\rangle\\
&+\frac{\sqrt{60}}{36}|I_6^1\rangle\otimes|S_4^0\rangle-\frac{\sqrt{30}}{18}|I_4^1\rangle\otimes|S_5^0\rangle+\frac{\sqrt{60}}{36}|I_7^1\rangle\otimes|S_5^0\rangle.
\end{aligned} \tag{A7}
$$

$$
\begin{aligned}
|[I^1,S^0]_2\rangle = &\begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3&5&6\\\hline\end{array}\\
= &-\frac{\sqrt{10}}{18}|I_3^1\rangle\otimes|S_1^0\rangle-\frac{\sqrt{20}}{18}|I_6^1\rangle\otimes|S_1^0\rangle+\frac{\sqrt{15}}{18}|I_8^1\rangle\otimes|S_1^0\rangle+\frac{\sqrt{6}}{9}|I_1^1\rangle\otimes|S_2^0\rangle-\frac{\sqrt{10}}{18}|I_2^1\rangle\otimes|S_2^0\rangle\\
&+\frac{\sqrt{5}}{9}|I_4^1\rangle\otimes|S_2^0\rangle-\frac{\sqrt{20}}{18}|I_5^1\rangle\otimes|S_2^0\rangle+\frac{\sqrt{10}}{9}|I_6^1\rangle\otimes|S_2^0\rangle+\frac{\sqrt{30}}{36}|I_8^1\rangle\otimes|S_2^0\rangle+\frac{\sqrt{5}}{9}|I_4^1\rangle\otimes|S_3^0\rangle\\
&+\frac{\sqrt{10}}{9}|I_7^1\rangle\otimes|S_3^0\rangle+\frac{\sqrt{30}}{36}|I_9^1\rangle\otimes|S_3^0\rangle-\frac{\sqrt{30}}{18}|I_2^1\rangle\otimes|S_4^0\rangle-\frac{\sqrt{15}}{18}|I_3^1\rangle\otimes|S_4^0\rangle+\frac{\sqrt{15}}{18}|I_5^1\rangle\otimes|S_4^0\rangle\\
&+\frac{\sqrt{30}}{36}|I_6^1\rangle\otimes|S_4^0\rangle+\frac{\sqrt{15}}{18}|I_4^1\rangle\otimes|S_5^0\rangle-\frac{\sqrt{30}}{36}|I_7^1\rangle\otimes|S_5^0\rangle.
\end{aligned} \tag{A8}
$$

$$
\begin{aligned}
|[I^1,S^0]_3\rangle =& \begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2&5&6\\\hline\end{array}\\
=&-\frac{\sqrt{10}}{18}|I_4^1\rangle\otimes|S_1^0\rangle-\frac{\sqrt{20}}{18}|I_7^1\rangle\otimes|S_1^0\rangle+\frac{\sqrt{15}}{18}|I_9^1\rangle\otimes|S_1^0\rangle-\frac{\sqrt{5}}{9}|I_4^1\rangle\otimes|S_2^0\rangle-\frac{\sqrt{10}}{9}|I_7^1\rangle\otimes|S_2^0\rangle\\
&-\frac{\sqrt{30}}{36}|I_9^1\rangle\otimes|S_2^0\rangle+\frac{\sqrt{6}}{9}|I_1^1\rangle\otimes|S_3^0\rangle-\frac{\sqrt{10}}{18}|I_2^1\rangle\otimes|S_3^0\rangle-\frac{\sqrt{20}}{18}|I_5^1\rangle\otimes|S_3^0\rangle-\frac{\sqrt{5}}{9}|I_3^1\rangle\otimes|S_3^0\rangle\\
&-\frac{\sqrt{10}}{9}|I_6^1\rangle\otimes|S_3^0\rangle-\frac{\sqrt{30}}{36}|I_8^1\rangle\otimes|S_3^0\rangle+\frac{\sqrt{15}}{18}|I_4^1\rangle\otimes|S_4^0\rangle-\frac{\sqrt{30}}{36}|I_7^1\rangle\otimes|S_4^0\rangle-\frac{\sqrt{30}}{18}|I_2^1\rangle\otimes|S_5^0\rangle\\
&+\frac{\sqrt{15}}{18}|I_4^1\rangle\otimes|S_5^0\rangle+\frac{\sqrt{15}}{18}|I_5^1\rangle\otimes|S_5^0\rangle+\frac{\sqrt{30}}{36}|I_6^1\rangle\otimes|S_5^0\rangle.
\end{aligned}
\tag{A9}
$$

$$
\begin{aligned}
|[I^1,S^0]_4\rangle =& \begin{array}{|c|c|c|}\hline 1&2&5\\\hline 3&4&6\\\hline\end{array}\\
=&-\frac{\sqrt{30}}{18}|I_3^1\rangle\otimes|S_1^0\rangle+\frac{\sqrt{15}}{18}|I_6^1\rangle\otimes|S_1^0\rangle-\frac{\sqrt{30}}{18}|I_2^1\rangle\otimes|S_2^0\rangle-\frac{\sqrt{15}}{18}|I_3^1\rangle\otimes|S_2^0\rangle+\frac{\sqrt{15}}{18}|I_5^1\rangle\otimes|S_2^0\rangle\\
&+\frac{\sqrt{30}}{36}|I_6^1\rangle\otimes|S_2^0\rangle-\frac{\sqrt{15}}{18}|I_4^1\rangle\otimes|S_3^0\rangle+\frac{\sqrt{30}}{36}|I_7^1\rangle\otimes|S_3^0\rangle-\frac{\sqrt{6}}{6}|I_1^1\rangle\otimes|S_4^0\rangle+\frac{\sqrt{30}}{12}|I_8^1\rangle\otimes|S_4^0\rangle\\
&-\frac{\sqrt{30}}{12}|I_9^1\rangle\otimes|S_5^0\rangle.
\end{aligned}
\tag{A10}
$$

$$
\begin{aligned}
|[I^1,S^0]_5\rangle =& \begin{array}{|c|c|c|}\hline 1&3&5\\\hline 2&4&6\\\hline\end{array}\\
=&-\frac{\sqrt{30}}{18}|I_4^1\rangle\otimes|S_1^0\rangle+\frac{\sqrt{15}}{18}|I_7^1\rangle\otimes|S_1^0\rangle-\frac{\sqrt{15}}{18}|I_4^1\rangle\otimes|S_2^0\rangle+\frac{\sqrt{30}}{36}|I_7^1\rangle\otimes|S_2^0\rangle-\frac{\sqrt{30}}{18}|I_2^1\rangle\otimes|S_3^0\rangle\\
&+\frac{\sqrt{15}}{18}|I_3^1\rangle\otimes|S_3^0\rangle+\frac{\sqrt{15}}{18}|I_5^1\rangle\otimes|S_3^0\rangle-\frac{\sqrt{30}}{36}|I_6^1\rangle\otimes|S_3^0\rangle-\frac{\sqrt{30}}{12}|I_9^1\rangle\otimes|S_4^0\rangle-\frac{\sqrt{6}}{6}|I_1^1\rangle\otimes|S_5^0\rangle\\
&-\frac{\sqrt{30}}{12}|I_8^1\rangle\otimes|S_5^0\rangle.
\end{aligned}
\tag{A11}
$$

Likewise, we find the color $\otimes$ isospin $\otimes$ spin state satisfying the fully antisymmetry property for $(I,S)=(1,0)$ to be given by

$$
|C,I^1,S^0\rangle=\frac{1}{\sqrt{5}}(|C_1\rangle\otimes|[I^1,S^0]_5\rangle-|C_2\rangle\otimes|[I^1,S^0]_4\rangle-|C_3\rangle\otimes|[I^1,S^0]_3\rangle+|C_4\rangle\otimes|[I^1,S^0]_2\rangle-|C_5\rangle\otimes|[I^1,S^0]_1\rangle).
\tag{A12}
$$

In the case of $(I,S)=(1,2)$, the $|[I^1,S^2]\rangle$ basis functions belonging to the Young tableau of [3,3] are presented as the following:

$$
\begin{aligned}
|[I^1,S^2]_1\rangle =& \begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5&6\\\hline\end{array}\\
=&\frac{\sqrt{15}}{9}|I_1^1\rangle\otimes|S_3^2\rangle+\frac{\sqrt{15}}{9}|I_2^1\rangle\otimes|S_2^2\rangle+\frac{2}{9}|I_2^1\rangle\otimes|S_3^2\rangle-\frac{1}{9}|I_3^1\rangle\otimes|S_4^2\rangle-\frac{1}{9}|I_4^1\rangle\otimes|S_5^2\rangle\\
&+\frac{\sqrt{5}}{5}|I_5^1\rangle\otimes|S_1^2\rangle+\frac{\sqrt{120}}{45}|I_5^1\rangle\otimes|S_2^2\rangle+\frac{2\sqrt{2}}{9}|I_5^1\rangle\otimes|S_3^2\rangle-\frac{\sqrt{2}}{9}|I_6^1\rangle\otimes|S_4^2\rangle-\frac{\sqrt{2}}{9}|I_7^1\rangle\otimes|S_5^2\rangle\\
&-\frac{\sqrt{6}}{9}|I_8^1\rangle\otimes|S_4^2\rangle-\frac{\sqrt{6}}{9}|I_9^1\rangle\otimes|S_5^2\rangle.
\end{aligned}
\tag{A13}
$$

$$
\begin{aligned}
|[I^1,S^2]_2\rangle =& \begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3&5&6\\\hline\end{array} \\
=& \frac{\sqrt{15}}{9}|I^1_1\rangle\otimes|S^2_4\rangle-\frac{1}{9}|I^1_2\rangle\otimes|S^2_4\rangle+\frac{\sqrt{15}}{9}|I^1_3\rangle\otimes|S^2_2\rangle-\frac{1}{9}|I^1_3\rangle\otimes|S^2_3\rangle+\frac{\sqrt{2}}{9}|I^1_3\rangle\otimes|S^2_4\rangle \\
&-\frac{\sqrt{2}}{9}|I^1_4\rangle\otimes|S^2_5\rangle-\frac{\sqrt{2}}{9}|I^1_5\rangle\otimes|S^2_4\rangle+\frac{\sqrt{5}}{5}|I^1_6\rangle\otimes|S^2_1\rangle+\frac{\sqrt{120}}{45}|I^1_6\rangle\otimes|S^2_2\rangle-\frac{\sqrt{2}}{9}|I^1_6\rangle\otimes|S^2_3\rangle \\
&+\frac{2}{9}|I^1_6\rangle\otimes|S^2_4\rangle-\frac{2}{9}|I^1_7\rangle\otimes|S^2_5\rangle-\frac{\sqrt{6}}{9}|I^1_8\rangle\otimes|S^2_3\rangle-\frac{\sqrt{3}}{9}|I^1_8\rangle\otimes|S^2_4\rangle+\frac{\sqrt{3}}{9}|I^1_9\rangle\otimes|S^2_5\rangle.
\end{aligned} \tag{A14}
$$

$$
\begin{aligned}
|[I^1,S^2]_3\rangle =& \begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2&5&6\\\hline\end{array} \\
=& \frac{\sqrt{15}}{9}|I^1_1\rangle\otimes|S^2_5\rangle-\frac{1}{9}|I^1_2\rangle\otimes|S^2_5\rangle-\frac{\sqrt{2}}{9}|I^1_3\rangle\otimes|S^2_5\rangle+\frac{\sqrt{15}}{9}|I^1_4\rangle\otimes|S^2_2\rangle-\frac{1}{9}|I^1_4\rangle\otimes|S^2_3\rangle \\
&-\frac{\sqrt{2}}{9}|I^1_4\rangle\otimes|S^2_4\rangle-\frac{\sqrt{2}}{9}|I^1_5\rangle\otimes|S^2_5\rangle-\frac{2}{9}|I^1_6\rangle\otimes|S^2_5\rangle+\frac{\sqrt{5}}{5}|I^1_7\rangle\otimes|S^2_1\rangle+\frac{\sqrt{120}}{45}|I^1_7\rangle\otimes|S^2_2\rangle \\
&-\frac{\sqrt{2}}{9}|I^1_7\rangle\otimes|S^2_3\rangle-\frac{2}{9}|I^1_7\rangle\otimes|S^2_4\rangle+\frac{\sqrt{3}}{9}|I^1_8\rangle\otimes|S^2_5\rangle-\frac{\sqrt{6}}{9}|I^1_9\rangle\otimes|S^2_3\rangle+\frac{\sqrt{3}}{9}|I^1_9\rangle\otimes|S^2_4\rangle.
\end{aligned} \tag{A15}
$$

$$
\begin{aligned}
|[I^1,S^2]_4\rangle =& \begin{array}{|c|c|c|}\hline 1&2&5\\\hline 3&4&6\\\hline\end{array} \\
=& \frac{2\sqrt{3}}{9}|I^1_2\rangle\otimes|S^2_4\rangle+\frac{2\sqrt{3}}{9}|I^1_3\rangle\otimes|S^2_3\rangle+\frac{\sqrt{6}}{9}|I^1_3\rangle\otimes|S^2_4\rangle-\frac{\sqrt{6}}{9}|I^1_4\rangle\otimes|S^2_5\rangle-\frac{\sqrt{6}}{9}|I^1_5\rangle\otimes|S^2_4\rangle \\
&-\frac{\sqrt{6}}{9}|I^1_6\rangle\otimes|S^2_3\rangle-\frac{\sqrt{3}}{9}|I^1_6\rangle\otimes|S^2_4\rangle+\frac{\sqrt{3}}{9}|I^1_7\rangle\otimes|S^2_5\rangle+\frac{\sqrt{5}}{5}|I^1_8\rangle\otimes|S^2_1\rangle-\frac{\sqrt{270}}{45}|I^1_8\rangle\otimes|S^2_2\rangle.
\end{aligned} \tag{A16}
$$

$$
\begin{aligned}
|[I^1,S^2]_5\rangle =& \begin{array}{|c|c|c|}\hline 1&3&5\\\hline 2&4&6\\\hline\end{array} \\
=& \frac{2\sqrt{3}}{9}|I^1_2\rangle\otimes|S^2_5\rangle-\frac{\sqrt{6}}{9}|I^1_3\rangle\otimes|S^2_5\rangle+\frac{2\sqrt{3}}{9}|I^1_4\rangle\otimes|S^2_3\rangle-\frac{\sqrt{6}}{9}|I^1_4\rangle\otimes|S^2_4\rangle-\frac{\sqrt{6}}{9}|I^1_5\rangle\otimes|S^2_5\rangle \\
&+\frac{\sqrt{3}}{9}|I^1_6\rangle\otimes|S^2_5\rangle-\frac{\sqrt{6}}{9}|I^1_7\rangle\otimes|S^2_3\rangle+\frac{\sqrt{3}}{9}|I^1_7\rangle\otimes|S^2_4\rangle+\frac{\sqrt{5}}{5}|I^1_9\rangle\otimes|S^2_1\rangle-\frac{\sqrt{270}}{45}|I^1_9\rangle\otimes|S^2_2\rangle.
\end{aligned} \tag{A17}
$$

We find the color $\otimes$ isospin $\otimes$ spin state satisfying the fully antisymmetry property for $(I,S)=(1,2)$ to be given by

$$
|C,I^1,S^2\rangle=\frac{1}{\sqrt{5}}(|C_1\rangle\otimes|[I^1,S^2]_5\rangle-|C_2\rangle\otimes|[I^1,S^2]_4\rangle-|C_3\rangle\otimes|[I^1,S^2]_3\rangle+|C_4\rangle\otimes|[I^1,S^2]_2\rangle-|C_5\rangle\otimes|[I^1,S^2]_1\rangle). \tag{A18}
$$

In the case of $(I,S)=(2,1)$, the $|[I^2,S^1]\rangle$ basis functions belonging to the Young tableau of [3,3] are presented as the following:

$$\begin{aligned}|[I^2,S^1]_1\rangle=&\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5&6\\\hline\end{array}\\ =&\frac{\sqrt{15}}{9}|I_3^2\rangle\otimes|S_1^1\rangle+\frac{\sqrt{15}}{9}|I_2^2\rangle\otimes|S_2^1\rangle+\frac{2}{9}|I_3^2\rangle\otimes|S_2^1\rangle-\frac{1}{9}|I_4^2\rangle\otimes|S_3^1\rangle-\frac{1}{9}|I_5^2\rangle\otimes|S_4^1\rangle\\ &+\frac{\sqrt{5}}{5}|I_1^2\rangle\otimes|S_5^1\rangle+\frac{\sqrt{120}}{45}|I_2^2\rangle\otimes|S_5^1\rangle+\frac{2\sqrt{2}}{9}|I_3^2\rangle\otimes|S_5^1\rangle-\frac{\sqrt{2}}{9}|I_4^2\rangle\otimes|S_6^1\rangle-\frac{\sqrt{2}}{9}|I_5^2\rangle\otimes|S_7^1\rangle\\ &-\frac{\sqrt{6}}{9}|I_4^2\rangle\otimes|S_8^1\rangle-\frac{\sqrt{6}}{9}|I_5^2\rangle\otimes|S_9^1\rangle.\end{aligned}\tag{A19}$$

$$\begin{aligned}|[I^2,S^1]_2\rangle=&\begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3&5&6\\\hline\end{array}\\ =&\frac{\sqrt{15}}{9}|I_4^2\rangle\otimes|S_1^1\rangle-\frac{1}{9}|I_4^2\rangle\otimes|S_2^1\rangle+\frac{\sqrt{15}}{9}|I_2^2\rangle\otimes|S_3^1\rangle-\frac{1}{9}|I_3^2\rangle\otimes|S_3^1\rangle+\frac{\sqrt{2}}{9}|I_4^2\rangle\otimes|S_3^1\rangle\\ &-\frac{\sqrt{2}}{9}|I_5^2\rangle\otimes|S_4^1\rangle-\frac{\sqrt{2}}{9}|I_4^2\rangle\otimes|S_5^1\rangle+\frac{\sqrt{5}}{5}|I_1^2\rangle\otimes|S_6^1\rangle+\frac{\sqrt{120}}{45}|I_2^2\rangle\otimes|S_6^1\rangle-\frac{\sqrt{2}}{9}|I_3^2\rangle\otimes|S_6^1\rangle\\ &+\frac{2}{9}|I_4^2\rangle\otimes|S_6^1\rangle-\frac{2}{9}|I_5^2\rangle\otimes|S_7^1\rangle-\frac{\sqrt{6}}{9}|I_3^2\rangle\otimes|S_8^1\rangle-\frac{\sqrt{3}}{9}|I_4^2\rangle\otimes|S_8^1\rangle+\frac{\sqrt{3}}{9}|I_5^2\rangle\otimes|S_9^1\rangle.\end{aligned}\tag{A20}$$

$$\begin{aligned}|[I^2,S^1]_3\rangle=&\begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2&5&6\\\hline\end{array}\\ =&\frac{\sqrt{15}}{9}|I_5^2\rangle\otimes|S_1^1\rangle-\frac{1}{9}|I_5^2\rangle\otimes|S_2^1\rangle-\frac{\sqrt{2}}{9}|I_5^2\rangle\otimes|S_3^1\rangle+\frac{\sqrt{15}}{9}|I_2^2\rangle\otimes|S_4^1\rangle-\frac{1}{9}|I_3^2\rangle\otimes|S_4^1\rangle\\ &-\frac{\sqrt{2}}{9}|I_4^2\rangle\otimes|S_4^1\rangle-\frac{\sqrt{2}}{9}|I_5^2\rangle\otimes|S_5^1\rangle-\frac{2}{9}|I_5^2\rangle\otimes|S_6^1\rangle+\frac{\sqrt{5}}{5}|I_1^2\rangle\otimes|S_7^1\rangle+\frac{\sqrt{120}}{45}|I_2^2\rangle\otimes|S_7^1\rangle\\ &-\frac{\sqrt{2}}{9}|I_3^2\rangle\otimes|S_7^1\rangle-\frac{2}{9}|I_4^2\rangle\otimes|S_7^1\rangle+\frac{\sqrt{3}}{9}|I_5^2\rangle\otimes|S_8^1\rangle-\frac{\sqrt{6}}{9}|I_3^2\rangle\otimes|S_9^1\rangle+\frac{\sqrt{3}}{9}|I_4^2\rangle\otimes|S_9^1\rangle.\end{aligned}\tag{A21}$$

$$\begin{aligned}|[I^2,S^1]_4\rangle=&\begin{array}{|c|c|c|}\hline 1&2&5\\\hline 3&4&6\\\hline\end{array}\\ =&\frac{2\sqrt{3}}{9}|I_4^2\rangle\otimes|S_2^1\rangle+\frac{2\sqrt{3}}{9}|I_3^2\rangle\otimes|S_3^1\rangle+\frac{\sqrt{6}}{9}|I_4^2\rangle\otimes|S_3^1\rangle-\frac{\sqrt{6}}{9}|I_5^2\rangle\otimes|S_4^1\rangle-\frac{\sqrt{6}}{9}|I_4^2\rangle\otimes|S_5^1\rangle\\ &-\frac{\sqrt{6}}{9}|I_3^2\rangle\otimes|S_6^1\rangle-\frac{\sqrt{3}}{9}|I_4^2\rangle\otimes|S_6^1\rangle+\frac{\sqrt{3}}{9}|I_5^2\rangle\otimes|S_7^1\rangle+\frac{\sqrt{5}}{5}|I_1^2\rangle\otimes|S_8^1\rangle-\frac{\sqrt{270}}{45}|I_2^2\rangle\otimes|S_8^1\rangle.\end{aligned}\tag{A22}$$

$$\begin{aligned}|[I^2,S^1]_5\rangle=&\begin{array}{|c|c|c|}\hline 1&3&5\\\hline 2&4&6\\\hline\end{array}\\ =&\frac{2\sqrt{3}}{9}|I_5^2\rangle\otimes|S_2^1\rangle-\frac{\sqrt{6}}{9}|I_5^2\rangle\otimes|S_3^1\rangle+\frac{2\sqrt{3}}{9}|I_3^2\rangle\otimes|S_4^1\rangle-\frac{\sqrt{6}}{9}|I_4^2\rangle\otimes|S_4^1\rangle-\frac{\sqrt{6}}{9}|I_5^2\rangle\otimes|S_5^1\rangle\\ &+\frac{\sqrt{3}}{9}|I_5^2\rangle\otimes|S_6^1\rangle-\frac{\sqrt{6}}{9}|I_3^2\rangle\otimes|S_7^1\rangle+\frac{\sqrt{3}}{9}|I_4^2\rangle\otimes|S_7^1\rangle+\frac{\sqrt{5}}{5}|I_1^2\rangle\otimes|S_9^1\rangle-\frac{\sqrt{270}}{45}|I_2^2\rangle\otimes|S_9^1\rangle.\end{aligned}\tag{A23}$$

We find the color $\otimes$ isospin $\otimes$ spin state satisfying the fully antisymmetry property for $(I,S)=(2,1)$ to be given by

$$|C, I^2, S^1\rangle = \frac{1}{\sqrt{5}}(|C_1\rangle \otimes |[I^2, S^1]_5\rangle - |C_2\rangle \otimes |[I^2, S^1]_4\rangle - |C_3\rangle \otimes |[I^2, S^1]_3\rangle + |C_4\rangle \otimes |[I^2, S^1]_2\rangle - |C_5\rangle \otimes |[I^2, S^1]_1\rangle). \quad \text{(A24)}$$

For the case of $(I, S) = (3, 0)$ and $(I, S) = (0, 3)$, we find straightforwardly the fully antisymmetric color $\otimes$ isospin $\otimes$ spin state, written by, respectively,

$$|C, I^3, S^0\rangle = \frac{1}{\sqrt{5}}(|C_1\rangle \otimes |I^3\rangle \otimes |S^0_5\rangle - |C_2\rangle \otimes |I^3\rangle \otimes |S^0_4\rangle - |C_3\rangle \otimes |I^3\rangle \otimes |S^0_3\rangle + |C_4\rangle \otimes |I^3\rangle \otimes |S^0_2\rangle - |C_5\rangle \otimes |I^3\rangle \otimes |S^0_1\rangle), \quad \text{(A25)}$$

$$|C, I^0, S^3\rangle = \frac{1}{\sqrt{5}}(|C_1\rangle \otimes |I^0_5\rangle \otimes |S^3\rangle - |C_2\rangle \otimes |I^0_4\rangle \otimes |S^3\rangle - |C_3\rangle \otimes |I^0_3\rangle \otimes |S^3\rangle + |C_4\rangle \otimes |I^0_2\rangle \otimes |S^3\rangle - |C_5\rangle \otimes |I^0_1\rangle \otimes |S^3\rangle). \quad \text{(A26)}$$

APPENDIX B: THREE-BODY COLOR OPERATORS

In this section, we derive the three-body color operators, which is invariant to $SU(3)$, in terms of the relevant permutation operators. This will enable us to represent the three-body color operator with respect to color singlet basis functions of the dibaryon. We can express the algebra of $SU(3)$ as the permutation of two particles, as in the case for the algebra of $SU(2)$:

$$(12) = \frac{1}{2}I + \frac{1}{2}\sigma_1 \cdot \sigma_2,$$
$$(12) = \frac{1}{3}I + \frac{1}{2}\lambda_1^c\lambda_2^c. \quad \text{(B1)}$$

Here, I is the identity operator, and (12) is the 2-cycles permutation. Then, noting that (123) and (132) can be written as $(123) = (23) \cdot (12)$ and $(132) = (23) \cdot (13)$, we can straightforwardly present the 3-cycles permutation as [35]

$$(123) = (23) \cdot (12) = \left(\frac{1}{3}I + \frac{1}{2}\lambda_2^c\lambda_3^c\right)\left(\frac{1}{3}I + \frac{1}{2}\lambda_1^c\lambda_2^c\right) = \frac{1}{9}I + \frac{1}{6}\sum_{i<j}^{3}\lambda_i^c\lambda_j^c + 2d^{abc}F_1^aF_2^bF_3^c + 2if^{abc}F_1^aF_2^bF_3^c. \quad \text{(B2)}$$

$$(132) = (23) \cdot (13) = \left(\frac{1}{3}I + \frac{1}{2}\lambda_2^c\lambda_3^c\right)\left(\frac{1}{3}I + \frac{1}{2}\lambda_1^c\lambda_3^c\right) = \frac{1}{9}I + \frac{1}{6}\sum_{i<j}^{3}\lambda_i^c\lambda_j^c + 2d^{abc}F_1^aF_2^bF_3^c - 2if^{abc}F_1^aF_2^bF_3^c. \quad \text{(B3)}$$

By adding Eq. (B3) to Eq. (B2) and subtracting Eq. (B3) from Eq. (B2), we obtain $d^{abc}F_1^aF_2^bF_3^c$ and $f^{abc}F_1^aF_2^bF_3^c$. Also, we can apply this process to any (ijk) $(i < j < k = 1 \sim 6)$ and finally obtain Eq. (27):

$$d^{abc}F_i^aF_j^bF_k^c = \frac{1}{4}[(ijk) + (ikj)] + \frac{1}{9}I - \frac{1}{6}[(ij) + (ik) + (jk)],$$
$$f^{abc}F_i^aF_j^bF_k^c = -\frac{i}{4}[(ijk) - (ikj)]. \quad \text{(B4)}$$

Now, we can construct the matrix representation of the three-body color operators in terms of the standard Young-Yamanouchi bases corresponding to the color singlet bases of dibaryon. As mentioned earlier, the standard Young-Yamanouchi bases which are orthonormal to each other are written by

$$|C_1\rangle = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline 5 & 6 \\ \hline\end{array},\ |C_2\rangle = \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline 5 & 6 \\ \hline\end{array},\ |C_3\rangle = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 5 \\ \hline 4 & 6 \\ \hline\end{array},\ |C_4\rangle = \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 5 \\ \hline 4 & 6 \\ \hline\end{array},\ |C_5\rangle = \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 & 5 \\ \hline 3 & 6 \\ \hline\end{array}. \quad \text{(B5)}$$

The matrix of $d^{abc}F_i^aF_j^bF_k^c$ is given by

$$d_{abc}F_1^aF_2^bF_3^c = \begin{pmatrix} -\frac{5}{36} & 0 & 0 & 0 & 0 \\ 0 & -\frac{5}{36} & 0 & 0 & 0 \\ 0 & 0 & -\frac{5}{36} & 0 & 0 \\ 0 & 0 & 0 & -\frac{5}{36} & 0 \\ 0 & 0 & 0 & 0 & \frac{10}{9} \end{pmatrix},$$

$$d_{abc}F_1^aF_2^bF_4^c = \begin{pmatrix} -\frac{5}{36} & 0 & 0 & 0 & 0 \\ 0 & -\frac{5}{36} & 0 & 0 & 0 \\ 0 & 0 & -\frac{5}{36} & 0 & 0 \\ 0 & 0 & 0 & \frac{35}{36} & \frac{5}{9\sqrt{2}} \\ 0 & 0 & 0 & \frac{5}{9\sqrt{2}} & 0 \end{pmatrix},$$

$$d_{abc}F_1^aF_2^bF_5^c = \begin{pmatrix} -\frac{5}{36} & 0 & 0 & 0 & 0 \\ 0 & \frac{25}{36} & 0 & \frac{5}{6\sqrt{3}} & -\frac{5}{6\sqrt{6}} \\ 0 & 0 & -\frac{5}{36} & 0 & 0 \\ 0 & \frac{5}{6\sqrt{3}} & 0 & \frac{5}{36} & -\frac{5}{18\sqrt{2}} \\ 0 & -\frac{5}{6\sqrt{6}} & 0 & -\frac{5}{18\sqrt{2}} & 0 \end{pmatrix},$$

$$d_{abc}F_1^aF_2^bF_6^c = \begin{pmatrix} -\frac{5}{36} & 0 & 0 & 0 & 0 \\ 0 & \frac{25}{36} & 0 & -\frac{5}{6\sqrt{3}} & \frac{5}{6\sqrt{6}} \\ 0 & 0 & -\frac{5}{36} & 0 & 0 \\ 0 & -\frac{5}{6\sqrt{3}} & 0 & \frac{5}{36} & -\frac{5}{18\sqrt{2}} \\ 0 & \frac{5}{6\sqrt{6}} & 0 & -\frac{5}{18\sqrt{2}} & 0 \end{pmatrix},$$

$$d_{abc}F_1^aF_3^bF_4^c = \begin{pmatrix} -\frac{5}{36} & 0 & 0 & 0 & 0 \\ 0 & -\frac{5}{36} & 0 & 0 & 0 \\ 0 & 0 & \frac{25}{36} & \frac{5}{6\sqrt{3}} & -\frac{5}{6\sqrt{6}} \\ 0 & 0 & \frac{5}{6\sqrt{3}} & \frac{5}{36} & -\frac{5}{18\sqrt{2}} \\ 0 & 0 & -\frac{5}{6\sqrt{6}} & -\frac{5}{18\sqrt{2}} & 0 \end{pmatrix},$$

$$d_{abc}F_1^aF_3^bF_5^c = \begin{pmatrix} \frac{35}{72} & \frac{5}{8\sqrt{3}} & \frac{5}{8\sqrt{3}} & \frac{5}{24} & \frac{5}{12\sqrt{2}} \\ \frac{5}{8\sqrt{3}} & \frac{5}{72} & \frac{5}{24} & \frac{5}{24\sqrt{3}} & \frac{5}{12\sqrt{6}} \\ \frac{5}{8\sqrt{3}} & \frac{5}{24} & \frac{5}{72} & \frac{5}{24\sqrt{3}} & \frac{5}{12\sqrt{6}} \\ \frac{5}{24} & \frac{5}{24\sqrt{3}} & \frac{5}{24\sqrt{3}} & -\frac{5}{72} & \frac{5}{36\sqrt{2}} \\ \frac{5}{12\sqrt{2}} & \frac{5}{12\sqrt{6}} & \frac{5}{12\sqrt{6}} & \frac{5}{36\sqrt{2}} & 0 \end{pmatrix},$$

$$d_{abc}F_1^aF_3^bF_6^c = \begin{pmatrix} \frac{35}{72} & \frac{5}{8\sqrt{3}} & -\frac{5}{8\sqrt{3}} & -\frac{5}{24} & -\frac{5}{12\sqrt{2}} \\ \frac{5}{8\sqrt{3}} & \frac{5}{72} & -\frac{5}{24} & -\frac{5}{24\sqrt{3}} & -\frac{5}{12\sqrt{6}} \\ -\frac{5}{8\sqrt{3}} & -\frac{5}{24} & \frac{5}{72} & \frac{5}{24\sqrt{3}} & \frac{5}{12\sqrt{6}} \\ -\frac{5}{24} & -\frac{5}{24\sqrt{3}} & \frac{5}{24\sqrt{3}} & -\frac{5}{72} & \frac{5}{36\sqrt{2}} \\ -\frac{5}{12\sqrt{2}} & -\frac{5}{12\sqrt{6}} & \frac{5}{12\sqrt{6}} & \frac{5}{36\sqrt{2}} & 0 \end{pmatrix},$$

$$d_{abc}F_1^aF_4^bF_5^c = \begin{pmatrix} \frac{35}{72} & -\frac{5}{8\sqrt{3}} & -\frac{5}{8\sqrt{3}} & \frac{5}{24} & \frac{5}{12\sqrt{2}} \\ -\frac{5}{8\sqrt{3}} & \frac{5}{72} & \frac{5}{24} & -\frac{5}{24\sqrt{3}} & -\frac{5}{12\sqrt{6}} \\ -\frac{5}{8\sqrt{3}} & \frac{5}{24} & \frac{5}{72} & -\frac{5}{24\sqrt{3}} & -\frac{5}{12\sqrt{6}} \\ \frac{5}{24} & -\frac{5}{24\sqrt{3}} & -\frac{5}{24\sqrt{3}} & -\frac{5}{72} & \frac{5}{36\sqrt{2}} \\ \frac{5}{12\sqrt{2}} & -\frac{5}{12\sqrt{6}} & -\frac{5}{12\sqrt{6}} & \frac{5}{36\sqrt{2}} & 0 \end{pmatrix},$$

$$d_{abc}F_1^aF_4^bF_6^c = \begin{pmatrix} \frac{35}{72} & -\frac{5}{8\sqrt{3}} & \frac{5}{8\sqrt{3}} & -\frac{5}{24} & -\frac{5}{12\sqrt{2}} \\ -\frac{5}{8\sqrt{3}} & \frac{5}{72} & -\frac{5}{24} & \frac{5}{24\sqrt{3}} & \frac{5}{12\sqrt{6}} \\ \frac{5}{8\sqrt{3}} & -\frac{5}{24} & \frac{5}{72} & -\frac{5}{24\sqrt{3}} & -\frac{5}{12\sqrt{6}} \\ -\frac{5}{24} & \frac{5}{24\sqrt{3}} & -\frac{5}{24\sqrt{3}} & -\frac{5}{72} & \frac{5}{36\sqrt{2}} \\ -\frac{5}{12\sqrt{2}} & \frac{5}{12\sqrt{6}} & -\frac{5}{12\sqrt{6}} & \frac{5}{36\sqrt{2}} & 0 \end{pmatrix},$$

$$d_{abc}F_1^aF_5^bF_6^c = \begin{pmatrix} -\frac{5}{36} & 0 & 0 & 0 & 0 \\ 0 & -\frac{5}{36} & 0 & 0 & 0 \\ 0 & 0 & \frac{25}{36} & -\frac{5}{6\sqrt{3}} & \frac{5}{6\sqrt{6}} \\ 0 & 0 & -\frac{5}{6\sqrt{3}} & \frac{5}{36} & -\frac{5}{18\sqrt{2}} \\ 0 & 0 & \frac{5}{6\sqrt{6}} & -\frac{5}{18\sqrt{2}} & 0 \end{pmatrix},$$

$$d_{abc}F_2^aF_3^bF_4^c = \begin{pmatrix} -\frac{5}{36} & 0 & 0 & 0 & 0 \\ 0 & -\frac{5}{36} & 0 & 0 & 0 \\ 0 & 0 & \frac{25}{36} & -\frac{5}{6\sqrt{3}} & \frac{5}{6\sqrt{6}} \\ 0 & 0 & -\frac{5}{6\sqrt{3}} & \frac{5}{36} & -\frac{5}{18\sqrt{2}} \\ 0 & 0 & \frac{5}{6\sqrt{6}} & -\frac{5}{18\sqrt{2}} & 0 \end{pmatrix},$$

$$d_{abc}F_2^aF_3^bF_5^c = \begin{pmatrix} \frac{35}{72} & -\frac{5}{8\sqrt{3}} & \frac{5}{8\sqrt{3}} & -\frac{5}{24} & -\frac{5}{12\sqrt{2}} \\ -\frac{5}{8\sqrt{3}} & \frac{5}{72} & -\frac{5}{24} & \frac{5}{24\sqrt{3}} & \frac{5}{12\sqrt{6}} \\ \frac{5}{8\sqrt{3}} & -\frac{5}{24} & \frac{5}{72} & -\frac{5}{24\sqrt{3}} & -\frac{5}{12\sqrt{6}} \\ -\frac{5}{24} & \frac{5}{24\sqrt{3}} & -\frac{5}{24\sqrt{3}} & -\frac{5}{72} & \frac{5}{36\sqrt{2}} \\ -\frac{5}{12\sqrt{2}} & \frac{5}{12\sqrt{6}} & -\frac{5}{12\sqrt{6}} & \frac{5}{36\sqrt{2}} & 0 \end{pmatrix},$$

$$d_{abc}F_2^aF_3^bF_6^c = \begin{pmatrix} \frac{35}{72} & -\frac{5}{8\sqrt{3}} & -\frac{5}{8\sqrt{3}} & \frac{5}{24} & \frac{5}{12\sqrt{2}} \\ -\frac{5}{8\sqrt{3}} & \frac{5}{72} & \frac{5}{24} & -\frac{5}{24\sqrt{3}} & -\frac{5}{12\sqrt{6}} \\ -\frac{5}{8\sqrt{3}} & \frac{5}{24} & \frac{5}{72} & -\frac{5}{24\sqrt{3}} & -\frac{5}{12\sqrt{6}} \\ \frac{5}{24} & -\frac{5}{24\sqrt{3}} & -\frac{5}{24\sqrt{3}} & -\frac{5}{72} & \frac{5}{36\sqrt{2}} \\ \frac{5}{12\sqrt{2}} & -\frac{5}{12\sqrt{6}} & -\frac{5}{12\sqrt{6}} & \frac{5}{36\sqrt{2}} & 0 \end{pmatrix},$$

$$d_{abc}F_2^aF_4^bF_5^c = \begin{pmatrix} \frac{35}{72} & \frac{5}{8\sqrt{3}} & -\frac{5}{8\sqrt{3}} & -\frac{5}{24} & -\frac{5}{12\sqrt{2}} \\ \frac{5}{8\sqrt{3}} & \frac{5}{72} & -\frac{5}{24} & -\frac{5}{24\sqrt{3}} & -\frac{5}{12\sqrt{6}} \\ -\frac{5}{8\sqrt{3}} & -\frac{5}{24} & \frac{5}{72} & \frac{5}{24\sqrt{3}} & \frac{5}{12\sqrt{6}} \\ -\frac{5}{24} & -\frac{5}{24\sqrt{3}} & \frac{5}{24\sqrt{3}} & -\frac{5}{72} & \frac{5}{36\sqrt{2}} \\ -\frac{5}{12\sqrt{2}} & -\frac{5}{12\sqrt{6}} & \frac{5}{12\sqrt{6}} & \frac{5}{36\sqrt{2}} & 0 \end{pmatrix},$$

$$d_{abc}F_2^aF_4^bF_6^c = \begin{pmatrix} \frac{35}{72} & \frac{5}{8\sqrt{3}} & \frac{5}{8\sqrt{3}} & \frac{5}{24} & \frac{5}{12\sqrt{2}} \\ \frac{5}{8\sqrt{3}} & \frac{5}{72} & \frac{5}{24} & \frac{5}{24\sqrt{3}} & \frac{5}{12\sqrt{6}} \\ \frac{5}{8\sqrt{3}} & \frac{5}{24} & \frac{5}{72} & \frac{5}{24\sqrt{3}} & \frac{5}{12\sqrt{6}} \\ \frac{5}{24} & \frac{5}{24\sqrt{3}} & \frac{5}{24\sqrt{3}} & -\frac{5}{72} & \frac{5}{36\sqrt{2}} \\ \frac{5}{12\sqrt{2}} & \frac{5}{12\sqrt{6}} & \frac{5}{12\sqrt{6}} & \frac{5}{36\sqrt{2}} & 0 \end{pmatrix},$$

$$d_{abc}F_2^aF_5^bF_6^c=\begin{pmatrix} -\frac{5}{36} & 0 & 0 & 0 & 0 \\ 0 & -\frac{5}{36} & 0 & 0 & 0 \\ 0 & 0 & \frac{25}{36} & \frac{5}{6\sqrt{3}} & -\frac{5}{6\sqrt{6}} \\ 0 & 0 & \frac{5}{6\sqrt{3}} & \frac{5}{36} & -\frac{5}{18\sqrt{2}} \\ 0 & 0 & -\frac{5}{6\sqrt{6}} & -\frac{5}{18\sqrt{2}} & 0 \end{pmatrix},$$

$$d_{abc}F_3^aF_4^bF_5^c=\begin{pmatrix} -\frac{5}{36} & 0 & 0 & 0 & 0 \\ 0 & \frac{25}{36} & 0 & -\frac{5}{6\sqrt{3}} & \frac{5}{6\sqrt{6}} \\ 0 & 0 & -\frac{5}{36} & 0 & 0 \\ 0 & -\frac{5}{6\sqrt{3}} & 0 & \frac{5}{36} & -\frac{5}{18\sqrt{2}} \\ 0 & \frac{5}{6\sqrt{6}} & 0 & -\frac{5}{18\sqrt{2}} & 0 \end{pmatrix},$$

$$d_{abc}F_3^aF_4^bF_6^c=\begin{pmatrix} -\frac{5}{36} & 0 & 0 & 0 & 0 \\ 0 & \frac{25}{36} & 0 & \frac{5}{6\sqrt{3}} & -\frac{5}{6\sqrt{6}} \\ 0 & 0 & -\frac{5}{36} & 0 & 0 \\ 0 & \frac{5}{6\sqrt{3}} & 0 & \frac{5}{36} & -\frac{5}{18\sqrt{2}} \\ 0 & -\frac{5}{6\sqrt{6}} & 0 & -\frac{5}{18\sqrt{2}} & 0 \end{pmatrix},$$

$$d_{abc}F_3^aF_5^bF_6^c=\begin{pmatrix} -\frac{5}{36} & 0 & 0 & 0 & 0 \\ 0 & -\frac{5}{36} & 0 & 0 & 0 \\ 0 & 0 & -\frac{5}{36} & 0 & 0 \\ 0 & 0 & 0 & \frac{35}{36} & \frac{5}{9\sqrt{2}} \\ 0 & 0 & 0 & \frac{5}{9\sqrt{2}} & 0 \end{pmatrix},$$

$$d_{abc}F_4^aF_5^bF_6^c=\begin{pmatrix} -\frac{5}{36} & 0 & 0 & 0 & 0 \\ 0 & -\frac{5}{36} & 0 & 0 & 0 \\ 0 & 0 & -\frac{5}{36} & 0 & 0 \\ 0 & 0 & 0 & -\frac{5}{36} & 0 \\ 0 & 0 & 0 & 0 & \frac{10}{9} \end{pmatrix}. \tag{B6}$$

We can prove that $\sum_{i<j<k}^{6} d_{abc}F_i^aF_j^bF_k^c$ is invariant to $SU(3)$, which means that it commutes with the generators of dibaryon $F^a=1/2\ (\lambda_1^a+\lambda_2^a+\lambda_3^a+\lambda_4^a+\lambda_5^a+\lambda_6^a)$ since it is proportional to the identity operator, as can be seen in the following:

$$\sum_{i<j<k}^{6} d_{abc}F_i^aF_j^bF_k^c=\begin{pmatrix} \frac{20}{9} & 0 & 0 & 0 & 0 \\ 0 & \frac{20}{9} & 0 & 0 & 0 \\ 0 & 0 & \frac{20}{9} & 0 & 0 \\ 0 & 0 & 0 & \frac{20}{9} & 0 \\ 0 & 0 & 0 & 0 & \frac{20}{9} \end{pmatrix}. \tag{B7}$$

The matrix of $f_{abc}F_i^aF_j^bF_k^c$ is given by

$$f_{abc}F_1^aF_2^bF_3^c=\begin{pmatrix} 0 & -\frac{i\sqrt{3}}{4} & 0 & 0 & 0 \\ \frac{i\sqrt{3}}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i\sqrt{3}}{4} & 0 \\ 0 & 0 & \frac{i\sqrt{3}}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$f_{abc}F_1^aF_2^bF_4^c=\begin{pmatrix} 0 & \frac{i\sqrt{3}}{4} & 0 & 0 & 0 \\ -\frac{i\sqrt{3}}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{4\sqrt{3}} & \frac{i}{\sqrt{6}} \\ 0 & 0 & \frac{i}{4\sqrt{3}} & 0 & 0 \\ 0 & 0 & -\frac{i}{\sqrt{6}} & 0 & 0 \end{pmatrix},$$

$$f_{abc}F_1^aF_2^bF_5^c=\begin{pmatrix} 0 & 0 & 0 & -\frac{i}{4} & -\frac{i}{2\sqrt{2}} \\ 0 & 0 & \frac{i}{4} & 0 & 0 \\ 0 & -\frac{i}{4} & 0 & \frac{i}{2\sqrt{3}} & -\frac{i}{2\sqrt{6}} \\ \frac{i}{4} & 0 & -\frac{i}{2\sqrt{3}} & 0 & 0 \\ \frac{i}{2\sqrt{2}} & 0 & \frac{i}{2\sqrt{6}} & 0 & 0 \end{pmatrix},$$

$$f_{abc}F_1^aF_2^bF_6^c=\begin{pmatrix} 0 & 0 & 0 & \frac{i}{4} & \frac{i}{2\sqrt{2}} \\ 0 & 0 & -\frac{i}{4} & 0 & 0 \\ 0 & \frac{i}{4} & 0 & \frac{i}{2\sqrt{3}} & -\frac{i}{2\sqrt{6}} \\ -\frac{i}{4} & 0 & -\frac{i}{2\sqrt{3}} & 0 & 0 \\ -\frac{i}{2\sqrt{2}} & 0 & \frac{i}{2\sqrt{6}} & 0 & 0 \end{pmatrix},$$

$$f_{abc}F_1^aF_3^bF_4^c=\begin{pmatrix} 0 & -\frac{i\sqrt{3}}{4} & 0 & 0 & 0 \\ \frac{i\sqrt{3}}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{4\sqrt{3}} & \frac{i}{2\sqrt{6}} \\ 0 & 0 & -\frac{i}{4\sqrt{3}} & 0 & -\frac{i}{2\sqrt{2}} \\ 0 & 0 & -\frac{i}{2\sqrt{6}} & \frac{i}{2\sqrt{2}} & 0 \end{pmatrix},$$

$$f_{abc}F_1^aF_3^bF_5^c=\begin{pmatrix} 0 & 0 & 0 & \frac{i}{4} & -\frac{i}{4\sqrt{2}} \\ 0 & 0 & -\frac{i}{4} & 0 & \frac{i\sqrt{3}}{4\sqrt{2}} \\ 0 & \frac{i}{4} & 0 & -\frac{i}{2\sqrt{3}} & -\frac{i}{4\sqrt{6}} \\ -\frac{i}{4} & 0 & \frac{i}{2\sqrt{3}} & 0 & \frac{i}{4\sqrt{2}} \\ \frac{i}{4\sqrt{2}} & -\frac{i\sqrt{3}}{4\sqrt{2}} & \frac{i}{4\sqrt{6}} & -\frac{i}{4\sqrt{2}} & 0 \end{pmatrix},$$

$$f_{abc}F_1^aF_3^bF_6^c = \begin{pmatrix} 0 & 0 & 0 & -\frac{i}{4} & \frac{i}{4\sqrt{2}} \\ 0 & 0 & \frac{i}{4} & 0 & -\frac{i\sqrt{3}}{4\sqrt{2}} \\ 0 & -\frac{i}{4} & 0 & -\frac{i}{2\sqrt{3}} & -\frac{i}{4\sqrt{6}} \\ \frac{i}{4} & 0 & \frac{i}{2\sqrt{3}} & 0 & \frac{i}{4\sqrt{2}} \\ -\frac{i}{4\sqrt{2}} & \frac{i\sqrt{3}}{4\sqrt{2}} & \frac{i}{4\sqrt{6}} & -\frac{i}{4\sqrt{2}} & 0 \end{pmatrix},$$

$$f_{abc}F_2^aF_3^bF_6^c = \begin{pmatrix} 0 & 0 & 0 & \frac{i}{4} & -\frac{i}{4\sqrt{2}} \\ 0 & 0 & -\frac{i}{4} & 0 & -\frac{i\sqrt{3}}{4\sqrt{2}} \\ 0 & \frac{i}{4} & 0 & \frac{i}{2\sqrt{3}} & \frac{i}{4\sqrt{6}} \\ -\frac{i}{4} & 0 & -\frac{i}{2\sqrt{3}} & 0 & \frac{i}{4\sqrt{2}} \\ \frac{i}{4\sqrt{2}} & \frac{i\sqrt{3}}{4\sqrt{2}} & -\frac{i}{4\sqrt{6}} & -\frac{i}{4\sqrt{2}} & 0 \end{pmatrix},$$

$$f_{abc}F_1^aF_4^bF_5^c = \begin{pmatrix} 0 & 0 & 0 & -\frac{i}{4} & \frac{i}{4\sqrt{2}} \\ 0 & 0 & -\frac{i}{4} & -\frac{i}{2\sqrt{3}} & -\frac{i}{4\sqrt{6}} \\ 0 & \frac{i}{4} & 0 & 0 & \frac{i\sqrt{3}}{4\sqrt{2}} \\ \frac{i}{4} & \frac{i}{2\sqrt{3}} & 0 & 0 & -\frac{i}{4\sqrt{2}} \\ -\frac{i}{4\sqrt{2}} & \frac{i}{4\sqrt{6}} & -\frac{i\sqrt{3}}{4\sqrt{2}} & \frac{i}{4\sqrt{2}} & 0 \end{pmatrix},$$

$$f_{abc}F_2^aF_4^bF_5^c = \begin{pmatrix} 0 & 0 & 0 & \frac{i}{4} & -\frac{i}{4\sqrt{2}} \\ 0 & 0 & \frac{i}{4} & -\frac{i}{2\sqrt{3}} & -\frac{i}{4\sqrt{6}} \\ 0 & -\frac{i}{4} & 0 & 0 & -\frac{i\sqrt{3}}{4\sqrt{2}} \\ -\frac{i}{4} & \frac{i}{2\sqrt{3}} & 0 & 0 & -\frac{i}{4\sqrt{2}} \\ \frac{i}{4\sqrt{2}} & \frac{i}{4\sqrt{6}} & \frac{i\sqrt{3}}{4\sqrt{2}} & \frac{i}{4\sqrt{2}} & 0 \end{pmatrix},$$

$$f_{abc}F_1^aF_4^bF_6^c = \begin{pmatrix} 0 & 0 & 0 & \frac{i}{4} & -\frac{i}{4\sqrt{2}} \\ 0 & 0 & \frac{i}{4} & \frac{i}{2\sqrt{3}} & \frac{i}{4\sqrt{6}} \\ 0 & -\frac{i}{4} & 0 & 0 & \frac{i\sqrt{3}}{4\sqrt{2}} \\ -\frac{i}{4} & -\frac{i}{2\sqrt{3}} & 0 & 0 & -\frac{i}{4\sqrt{2}} \\ \frac{i}{4\sqrt{2}} & -\frac{i}{4\sqrt{6}} & -\frac{i\sqrt{3}}{4\sqrt{2}} & \frac{i}{4\sqrt{2}} & 0 \end{pmatrix},$$

$$f_{abc}F_2^aF_4^bF_6^c = \begin{pmatrix} 0 & 0 & 0 & -\frac{i}{4} & \frac{i}{4\sqrt{2}} \\ 0 & 0 & -\frac{i}{4} & \frac{i}{2\sqrt{3}} & \frac{i}{4\sqrt{6}} \\ 0 & \frac{i}{4} & 0 & 0 & -\frac{i\sqrt{3}}{4\sqrt{2}} \\ \frac{i}{4} & -\frac{i}{2\sqrt{3}} & 0 & 0 & -\frac{i}{4\sqrt{2}} \\ -\frac{i}{4\sqrt{2}} & -\frac{i}{4\sqrt{6}} & \frac{i\sqrt{3}}{4\sqrt{2}} & \frac{i}{4\sqrt{2}} & 0 \end{pmatrix},$$

$$f_{abc}F_1^aF_5^bF_6^c = \begin{pmatrix} 0 & 0 & 0 & -\frac{i}{4} & -\frac{i}{2\sqrt{2}} \\ 0 & 0 & -\frac{i}{4} & -\frac{i}{2\sqrt{3}} & \frac{i}{2\sqrt{6}} \\ 0 & \frac{i}{4} & 0 & 0 & 0 \\ \frac{i}{4} & \frac{i}{2\sqrt{3}} & 0 & 0 & 0 \\ \frac{i}{2\sqrt{2}} & -\frac{i}{2\sqrt{6}} & 0 & 0 & 0 \end{pmatrix},$$

$$f_{abc}F_2^aF_5^bF_6^c = \begin{pmatrix} 0 & 0 & 0 & \frac{i}{4} & \frac{i}{2\sqrt{2}} \\ 0 & 0 & \frac{i}{4} & -\frac{i}{2\sqrt{3}} & \frac{i}{2\sqrt{6}} \\ 0 & -\frac{i}{4} & 0 & 0 & 0 \\ -\frac{i}{4} & \frac{i}{2\sqrt{3}} & 0 & 0 & 0 \\ -\frac{i}{2\sqrt{2}} & -\frac{i}{2\sqrt{6}} & 0 & 0 & 0 \end{pmatrix},$$

$$f_{abc}F_2^aF_3^bF_4^c = \begin{pmatrix} 0 & \frac{i\sqrt{3}}{4} & 0 & 0 & 0 \\ -\frac{i\sqrt{3}}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{4\sqrt{3}} & -\frac{i}{2\sqrt{6}} \\ 0 & 0 & \frac{i}{4\sqrt{3}} & 0 & -\frac{i}{2\sqrt{2}} \\ 0 & 0 & \frac{i}{2\sqrt{6}} & \frac{i}{2\sqrt{2}} & 0 \end{pmatrix},$$

$$f_{abc}F_3^aF_4^bF_5^c = \begin{pmatrix} 0 & 0 & -\frac{i\sqrt{3}}{4} & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{4\sqrt{3}} & \frac{i}{2\sqrt{6}} \\ \frac{i\sqrt{3}}{4} & 0 & 0 & 0 & 0 \\ 0 & -\frac{i}{4\sqrt{3}} & 0 & 0 & \frac{i}{2\sqrt{2}} \\ 0 & -\frac{i}{2\sqrt{6}} & 0 & -\frac{i}{2\sqrt{2}} & 0 \end{pmatrix},$$

$$f_{abc}F_2^aF_3^bF_5^c = \begin{pmatrix} 0 & 0 & 0 & -\frac{i}{4} & \frac{i}{4\sqrt{2}} \\ 0 & 0 & \frac{i}{4} & 0 & \frac{i\sqrt{3}}{4\sqrt{2}} \\ 0 & -\frac{i}{4} & 0 & \frac{i}{2\sqrt{3}} & \frac{i}{4\sqrt{6}} \\ \frac{i}{4} & 0 & -\frac{i}{2\sqrt{3}} & 0 & \frac{i}{4\sqrt{2}} \\ -\frac{i}{4\sqrt{2}} & -\frac{i\sqrt{3}}{4\sqrt{2}} & -\frac{i}{4\sqrt{6}} & -\frac{i}{4\sqrt{2}} & 0 \end{pmatrix},$$

$$f_{abc}F_3^aF_4^bF_6^c = \begin{pmatrix} 0 & 0 & \frac{i\sqrt{3}}{4} & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{4\sqrt{3}} & -\frac{i}{2\sqrt{6}} \\ -\frac{i\sqrt{3}}{4} & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{4\sqrt{3}} & 0 & 0 & \frac{i}{2\sqrt{2}} \\ 0 & \frac{i}{2\sqrt{6}} & 0 & -\frac{i}{2\sqrt{2}} & 0 \end{pmatrix},$$

$$f_{abc}F_3^aF_5^bF_6^c = \begin{pmatrix} 0 & 0 & -\frac{i\sqrt{3}}{4} & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{4\sqrt{3}} & -\frac{i}{\sqrt{6}} \\ \frac{i\sqrt{3}}{4} & 0 & 0 & 0 & 0 \\ 0 & -\frac{i}{4\sqrt{3}} & 0 & 0 & 0 \\ 0 & \frac{i}{\sqrt{6}} & 0 & 0 & 0 \end{pmatrix},$$

$$f_{abc}F_4^aF_5^bF_6^c = \begin{pmatrix} 0 & 0 & \frac{i\sqrt{3}}{4} & 0 & 0 \\ 0 & 0 & 0 & \frac{i\sqrt{3}}{4} & 0 \\ -\frac{i\sqrt{3}}{4} & 0 & 0 & 0 & 0 \\ 0 & -\frac{i\sqrt{3}}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad \text{(B8)}$$

In this case, it turns out that the $SU(3)$ invariant operator is $f_{abc}F_1^aF_2^bF_3^c + f_{abc}F_1^aF_2^bF_4^c + f_{abc}F_1^aF_2^bF_5^c + f_{abc}F_1^aF_2^bF_6^c + f_{abc}F_1^aF_3^bF_4^c + f_{abc}F_1^aF_3^bF_5^c + f_{abc}F_1^aF_3^bF_6^c + f_{abc}F_1^aF_4^bF_5^c + f_{abc}F_1^aF_4^bF_6^c + f_{abc}F_1^aF_5^bF_6^c + f_{abc}F_2^aF_3^bF_4^c + f_{abc}F_2^aF_3^bF_5^c + f_{abc}F_2^aF_3^bF_6^c + f_{abc}F_2^aF_4^bF_5^c + f_{abc}F_2^aF_4^bF_6^c + f_{abc}F_2^aF_5^bF_6^c + f_{abc}F_3^aF_4^bF_5^c + f_{abc}F_3^aF_4^bF_6^c + f_{abc}F_3^aF_5^bF_6^c + f_{abc}F_4^aF_5^bF_6^c$, due to the fact that this operator is

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad \text{(B9)}$$

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