Testing the molecular nature of $D_{s0}^*(2317)$ and $D_0^*(2400)$ in semileptonic B_s and B decays

Fernando S. Navarra,[1,*] Marina Nielsen,[1,†] Eulogio Oset,[2,‡] and Takayasu Sekihara[3,§]

[1]*Instituto de Física, Universidade de São Paulo, C.P. 66318, 05389-970 São Paulo, SP, Brazil*
[2]*Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC,*
Institutos de Investigación de Paterna, Apartado 22085, 46071 Valencia, Spain
[3]*Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka, 567-0047, Japan*

We study the semileptonic B_s and B decays into the $D_{s0}^*(2317)$ and $D_0^*(2400)$ resonances, respectively. With the help of a chiral unitary model in coupled channels we compute the ratio of the decay widths of both processes. Using current values of the width for the $\bar{B}^0 \to D_0^*(2400)^+ \bar{\nu}_l l^-$ we make predictions for the rate of the $\bar{B}_s^0 \to D_{s0}^*(2317)^+ \bar{\nu}_l l^-$ decay and for the DK invariant mass distribution in the $\bar{B}_s^0 \to DK\bar{\nu}_l l^-$ decay.

I. INTRODUCTION

The recent discovery of many mesons with charm contributed to the revival of hadron spectroscopy (for recent reviews, see Ref. [1]). Two interesting examples of these mesons are the $D_{s0}^*(2317)$ [2] and $D_0^*(2400)$ [3] scalar resonances. As it happened to other states, their measured masses and widths do not match the predictions from potential-based quark models. This disagreement motivated several nonconventional (exotic) interpretations of these states. Among them the most popular are multi-quark configurations in the form of tetraquarks or meson molecules [1]. Since their masses are located below the DK and $D_s K$ thresholds it is quite natural to think that they are bound states of DK and $D_s K$ meson pairs. In the case of the $D_{s0}^*(2317)$, additional support to the molecular interpretation came recently [4] from lattice QCD simulations. In all previous lattice studies of the $D_{s0}^*(2317)$, it was treated as a conventional quark-antiquark state, the mass was obtained directly from the two point meson correlation function, and no states with the correct mass (below the DK threshold) were found. A review of early lattice work is done in [5], reporting on calculations with $q\bar{q}$ interpolators which obtain a state with the quantum numbers of the $D_{s0}^*(2317)$. Already at that time one of these papers, Ref. [6], concluded that these lattice results gave support to a non-$\bar{c}s$ interpretation of the $D_{s0}^*(2317)$. On the other hand, as noted in [5], the large value of the leptonic decay constant $f_{D_{s0}^+}$ obtained in lattice evaluations favored the $q\bar{q}$ interpretation of the state [7]. A recent review of further work is done in [4] where for the first time the effect of threshold of meson pairs is considered. In Ref. [4], with the

introduction of DK meson interpolators in addition to $q\bar{q}$, the right mass was obtained from the scattering phase shifts using the Lüscher formalism [8]. In [9] the scattering length of DK from QCD lattice simulations was extrapolated to physical pion masses and then, using the Weinberg compositeness condition [10,11], the dominance of the DK component in the $D_{s0}^*(2317)$ state was concluded. A reanalysis of the results of [4] has been done in [12] using the information of the three energy levels of [4] and going beyond the effective range formula. The dominance of the DK component of the $D_{s0}^*(2317)$ was firmly established in [12]. On the other hand, the analysis of the $D_{s0}^*(2317) \to D_s^*\gamma$ radiative decay suggests that the $D_{s0}^*(2317)$ is most likely an ordinary $\bar{c}s$ state [13], although this is disputed in [14]. It is therefore important to carry out further studies to clarify this question. One aspect to be investigated is the production of $D_{s0}^*(2317)$ and $D_0^*(2400)$ in B_s and B decays, respectively.

The dominant decay channel of the B_s meson is into the D_s meson plus anything. Therefore various important properties of the $c\bar{s}$ mesons can be studied in the B_s weak decays. In particular, they can shed more light on the controversial $D_{s0}^*(2317)$ meson, whose nature is still under debate, as discussed above. In recent years there has been significant experimental progress in the study of the properties of the B_s mesons. The Belle collaboration considerably increased the number of observed B_s mesons and their decays [15]. Moreover, B_s mesons are copiously produced at the Large Hadron Collider (LHC) and precise data on their properties have been taken by the LHCb collaboration [16]. New data are expected in the near future [17]. The study of weak B_s decays is primarily devoted to the improvement in the determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, but there are several interesting topics in hadron physics to be investigated in these processes.

---

*navarra@if.usp.br
†mnielsen@if.usp.br
‡oset@ific.uv.es
§sekihara@rcnp.osaka-u.ac.jp

The $D_{s0}^*(2317)$ has been measured mostly in B factories and probably because of its narrow width ($\Gamma < 3.8$ MeV) it has not been seen in some channels. One of them is the $\bar{B}_s^0$ semileptonic decay, i.e., $\bar{B}_s^0 \to D_{s0}^{*+}\bar{\nu}_l l^-$ (Fig. 1). There are several theoretical estimates of the branching fraction of this decay channel [18–23] and they predict numbers which differ by up to a factor 2. In all these calculations, a source of uncertainty is in the hadronization. Related work in the nonleptonic B decay into a D or D^* and the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ resonances is done in [24], using effective operators and the factorization approximation. In the present paper, assuming that the $D_{s0}^*(2317)$ and $D_0^*(2400)$ are dynamically generated resonances, we try to improve the hadronization and the treatment of final state meson-meson interactions. Moreover, in order to further reduce uncertainties we compute the ratio of decay widths:

$$R = \frac{\Gamma_{\bar{B}_s^0 \to D_{s0}^{*+}(2317)\bar{\nu}_l l^-}}{\Gamma_{\bar{B}^0 \to D_0^{*+}\bar{\nu}_l l^-}}. \tag{1}$$

We calculate the left side of this equation and then, using the available experimental information about the process $\bar{B}^0 \to D_0^*(2400)^+ \bar{\nu}_l l^-$ (see Fig. 2), we can extract $\Gamma_{\bar{B}^0 \to D_0^*(2317)^+ \bar{\nu}_l l^-}$. We consider also the $B^- \to D_0^*(2400)^0 \bar{\nu}_l l^-$ decay (Fig. 3). The formalism is very similar to the one presented in Refs. [25,26] for nonleptonic B decays.

As it is depicted in Fig. 4, after the W emission the remaining $c - \bar{q}$ pair is allowed to hadronize into a pair of
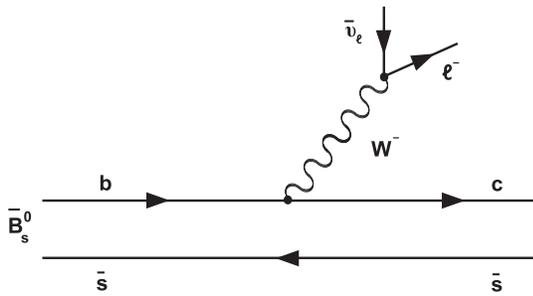
FIG. 1. Semileptonic decay of $\bar{B}_s^0$ into $\bar{\nu}_l l^-$ and a primary $c\bar{s}$ pair.
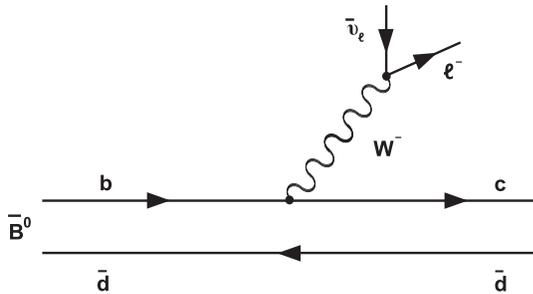
FIG. 2. Semileptonic decay of $\bar{B}^0$ into $\bar{\nu}_l l^-$ and a primary $c\bar{d}$ pair.
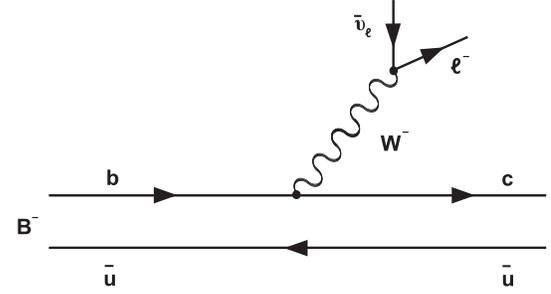
FIG. 3. Semileptonic decay of B^- into $\bar{\nu}_l l^-$ and a primary $c\bar{u}$ pair.
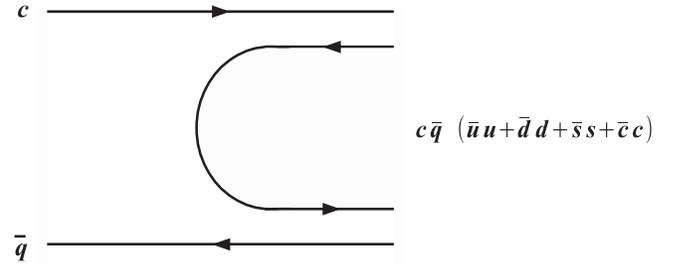
FIG. 4. Schematic representation of the hadronization $c\bar{q} \to c\bar{q}(\bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c)$.

pseudoscalar mesons (the relative weights of the different pairs of mesons is known). Once the meson pairs are produced they are allowed to interact in the way described by the chiral unitary model in coupled channels and automatically the $D_{s0}^*(2317)$ and $D_0^*(2400)$ resonances are produced.

This paper is organized as follows. In Sec. II, we formulate the semileptonic B decay widths into D resonances and give our model of the hadronization. Next in Sec. III we consider D resonance production via meson coalescence after rescattering, and in Sec. IV we calculate the production of two pseudoscalars with prompt production plus rescattering through a D resonance. Then in Sec. V we formulate meson-meson scattering amplitudes to generate the $D_{s0}^*(2317)$ and $D_0^*(2400)$ resonances. In Sec. VI we show our numerical results of the semileptonic B decay widths. Section VII is devoted to drawing the conclusion of this study.

II. SEMILEPTONIC B DECAYS

Let us first formulate the semileptonic B decays into D resonances in the following decay modes:

$$\bar{B}_s^0 \to D_{s0}^*(2317)^+ \bar{\nu}_l l^-,$$
$$\bar{B}^0 \to D_0^*(2400)^+ \bar{\nu}_l l^-,$$
$$B^- \to D_0^*(2400)^0 \bar{\nu}_l l^-, \tag{2}$$

where the lepton flavor l can be e and μ. For this purpose we express the decay amplitudes and widths in a general

form in Sec. II A and give our model of the hadronization in Sec. II B.

A. Semileptonic decay widths

In general, by using the propagation of the W boson and its couplings to leptons and quarks, we can express the decay amplitude of $B \to \bar{\nu} l^- \text{hadron(s)}$, T_B, in the following manner:

$$-iT_B = \bar{u}_l i \frac{g_W}{\sqrt{2}} \gamma^\alpha \frac{1 - \gamma_5}{2} v_\nu \times \frac{-ig_{\alpha\beta}}{p^2 - M_W^2}$$
$$\times \bar{u}_c i \frac{g_W V_{bc}}{\sqrt{2}} \gamma^\beta \frac{1 - \gamma_5}{2} u_b \times (-iV_{\text{had}}), \quad (3)$$

where u_l, v_ν, u_c, and u_b are Dirac spinors corresponding to the lepton l^-, neutrino, charm quark, and bottom quark, respectively, g_W is the coupling constant of the weak interaction, V_{bc} is the Cabibbo-Kobayashi-Maskawa matrix element, and M_W is the W boson mass. The factor V_{had} consists of the wave function of quarks inside the B meson and the hadronization contribution in the final state, and it will be evaluated in the sections below. In the following we neglect the squared momentum of the W boson (p^2) which should be much smaller than M_W^2 in the B decay process, and therefore the decay amplitude becomes

$$T_B = -i\frac{G_F V_{bc}}{\sqrt{2}} L^\alpha Q_\alpha \times V_{\text{had}}, \quad (4)$$

where we have introduced the Fermi coupling constant $G_F \equiv g_W^2/(4\sqrt{2}M_W^2)$ and defined the lepton and quark parts of the W boson couplings as

$$L^\alpha \equiv \bar{u}_l \gamma^\alpha (1 - \gamma_5) v_\nu, \qquad Q_\alpha \equiv \bar{u}_c \gamma_\alpha (1 - \gamma_5) u_b, \quad (5)$$

respectively.

Now let us calculate the decay widths of the semileptonic B mesons into D resonances. In the calculation of the decay widths, we take the absolute value of the decay amplitude T_B and average (sum) the polarizations of the initial-state quarks (final-state leptons and quarks). Therefore, in terms of the amplitude in Eq. (4), we can obtain the squared decay amplitude as

$$\frac{1}{2}\sum_{\text{pol}}|T_B|^2 = \frac{|G_F V_{bc} V_{\text{had}}|^2}{4}\sum_{\text{pol}}|L^\alpha Q_\alpha|^2, \quad (6)$$

where the factor $1/2$ comes from the average of the bottom quark polarization. Then, by using the conventions of the Dirac spinors and traces of Dirac matrices summarized in the Appendix, we can calculate the lepton part of the amplitude (5), which reads

$$\sum_{\text{pol}} L^\alpha L^{\dagger\beta} = \text{tr}\left[\gamma^\alpha(1 - \gamma_5)\frac{\not{p}_\nu - m_\nu}{2m_\nu}(1 + \gamma_5)\gamma^\beta\frac{\not{p}_l + m_l}{2m_l}\right]$$
$$= 2\frac{p_\nu^\alpha p_l^\beta + p_l^\alpha p_\nu^\beta - p_\nu \cdot p_l g^{\alpha\beta} - i\epsilon^{\rho\alpha\sigma\beta}p_{\nu\rho}p_{l\sigma}}{m_\nu m_l}, \quad (7)$$

where p_ν and p_l (m_ν and m_l) are momenta (masses) of the neutrino and lepton l^-, respectively. In a similar manner, we can calculate the quark part of the amplitude, which is given by Eq. (5),

$$\sum_{\text{pol}} Q_\alpha Q_\beta^\dagger = \text{tr}\left[\gamma_\alpha(1 - \gamma_5)\frac{\not{p}_b + m_b}{2m_b}(1 + \gamma_5)\gamma_\beta\frac{\not{p}_c + m_c}{2m_c}\right]$$
$$= 2\frac{p_{b\alpha}p_{c\beta} + p_{c\alpha}p_{b\beta} - p_b \cdot p_c g_{\alpha\beta} - i\epsilon_{\rho\alpha\sigma\beta}p_b^\rho p_c^\sigma}{m_b m_c}, \quad (8)$$

with the momenta (masses) of the bottom and charm quarks, p_b and p_c (m_b and m_c), respectively. Contracting Eqs. (7) and (8) we obtain

$$\sum_{\text{pol}}|L^\alpha Q_\alpha|^2 = \frac{16(p_b \cdot p_\nu)(p_c \cdot p_l)}{m_\nu m_l m_b m_c}. \quad (9)$$

The dominant terms come from the energy of the heavy quarks but we can also take into account the momentum of the quarks by using

$$\frac{p_b^\mu}{m_b} = \frac{p_B^\mu}{m_B}, \qquad \frac{p_c^\mu}{m_c} = \frac{p_R^\mu}{m_R}, \quad (10)$$

where we have neglected the relative internal three momenta of the quarks, which are small relative to the masses of the heavy quarks. Here m_B and m_R (p_B^μ and p_R^μ) are the masses (momenta) of the B and D^* mesons, respectively. As a consequence, the square of $L^\alpha Q_\alpha$ with polarization summation is expressed as

$$\sum_{\text{pol}}|L^\alpha Q_\alpha|^2 = \frac{16(p_B \cdot p_\nu)(p_R \cdot p_l)}{m_\nu m_l m_B m_R}. \quad (11)$$

Finally we obtain the squared decay amplitude:

$$\frac{1}{2}\sum_{\text{pol}}|T_B|^2 = \frac{4|G_F V_{bc} V_{\text{had}}|^2}{m_\nu m_l m_B m_R}(p_B \cdot p_\nu)(p_R \cdot p_l). \quad (12)$$

With the above squared amplitude we can compute the decay width. We will be interested in two types of decays: three-body decays, such as $\bar{B}_s^0 \to D_{s0}^+ \bar{\nu}_l l^-$, and four-body decays, such as $\bar{B}_s^0 \to D^+ K^0 \bar{\nu}_l l^-$ and also for the similar $\bar{B}^0$ and B^- initiated processes. As it will be seen, both decay

types can be described by the amplitude T_B with different assumptions for V_{had}. The final formulas for three- and four-body decays are then given by

$$\Gamma_3 = \frac{4m_\nu m_l}{2m_B} \int d\Phi_3 \overline{\sum_{\text{pol}}} \sum_{\text{pol}} |T_3|^2, \tag{13}$$

$$\Gamma_4 = \frac{4m_\nu m_l}{2m_B} \int d\Phi_4 \overline{\sum_{\text{pol}}} \sum_{\text{pol}} |T_4|^2, \tag{14}$$

respectively. In the equations, m_ν and m_l are respectively the masses of the neutrino ν and lepton l, $T_{3(4)}$ is the three-(four-)body decay amplitude, and the summation symbols represent the average of the polarizations in the initial state and the sum over the polarizations in the final state. Moreover, the n-body phase space $d\Phi_n$ has been introduced as

$$d\Phi_n \equiv \prod_{i=1}^{n} \left[\frac{d^3 p_i}{(2\pi)^3} \frac{1}{2E_i} \right] (2\pi)^4 \delta^4(p_B - p_{\text{tot}}), \tag{15}$$

where $E_i \equiv \sqrt{\mathbf{p}_i^2 + m_i^2}$ is the on-shell energy of the ith particle with its mass m_i, p_B^μ is the four-momentum of the initial B meson, and p_{tot}^μ is the sum of the final-state momentum:

$$p_{\text{tot}}^\mu \equiv \sum_{i=1}^{n} p_i^\mu, \qquad p_i^\mu = (E_i, \mathbf{p}_i). \tag{16}$$

In order to proceed with the calculation we need a prescription of hadronization, i.e., after the W emission in Figs. 1–3 we must specify a way to convert the outgoing quarks into hadrons and compute V_{had}. This will be done in the next subsection.

B. Hadronization

The conversion of quarks into hadrons in the final stage of hadron reactions is a long-standing problem which up to now has no definitive solution. Since the energies involved are of the order of a few GeV or less, this is a non-perturbative process. For particles produced in very high energy collisions and with high transverse momentum, we can use fragmentation functions, which are extracted from data phenomenologically and then refined with a perturbative QCD treatment. In some cases, one can develop an approach based on effective Lagrangians [27]. In the process considered here, in contrast to the high energy case where many particles are produced along with the formed hadron, only one quark-antiquark pair is produced and hadronization is mostly a recombination process which binds together the existing quarks. Here we follow [25] and describe hadronization as depicted in Fig. 4. An extra $q\bar{q}$ pair with the quantum numbers of the vacuum, $\bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c$, is added to the already existing quark pair. The probability of producing the pair is assumed to be given by a number which is the same for all light flavors and which will cancel out when taking ratios of decay widths. We can write this $c\bar{q}(\bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c)$ combination in terms of pairs of mesons. For this purpose we follow the work of [26] and define the $q\bar{q}$ matrix M:

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} & u\bar{c} \\ d\bar{u} & d\bar{d} & d\bar{s} & d\bar{c} \\ s\bar{u} & s\bar{d} & s\bar{s} & s\bar{c} \\ c\bar{u} & c\bar{d} & c\bar{s} & c\bar{c} \end{pmatrix} \tag{17}$$

which has the property

$$M \cdot M = M \times (\bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c). \tag{18}$$

Now, in terms of mesons the matrix M corresponds to [28]

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 & D^- \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' & D_s^- \\ D^0 & D^+ & D_s^+ & \eta_c \end{pmatrix}. \tag{19}$$

Hence, in terms of two pseudoscalars we have the correspondence:

$$c\bar{s}(\bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c) \equiv (\phi \cdot \phi)_{43} = D^0 K^+ + D^+ K^0 + D_s^+ \left(-\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' \right) + \eta_c D_s^+, \tag{20}$$

$$c\bar{d}(\bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c) \equiv (\phi \cdot \phi)_{42} = D^0 \pi^+ + D^+ \left(-\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' \right) + D_s^+ \bar{K}^0 + \eta_c D^+, \tag{21}$$

$$c\bar{u}(\bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c) \equiv (\phi \cdot \phi)_{41} = D^0 \left(\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' \right) + D^+ \pi^- + D_s^+ K^- + \eta_c D^0, \tag{22}$$
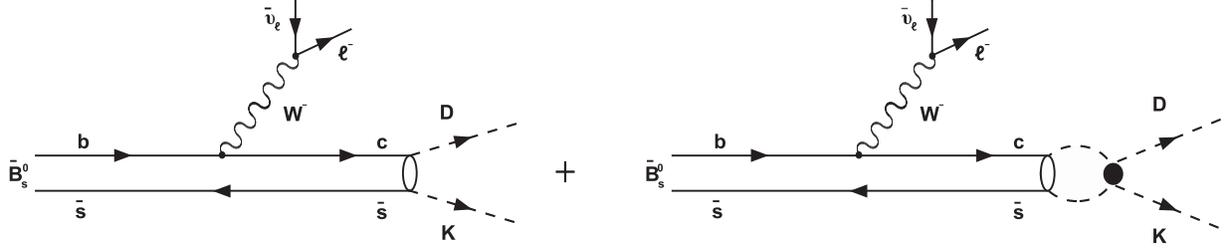
FIG. 5.   Diagrammatic representation of $D_{s0}^{*+}(2317)$ production via meson coalescence after rescattering.

for $D_{s0}^*(2317)^+$, $D_0^*(2400)^+$, and $D_0^*(2400)^0$ production, respectively. Then, for simplicity we concentrate on the relevant channels for the description of the D resonances. In fact, it was pointed out in Ref. [29] that the most important channels for the description of $D_{s0}^*(2317)$ ($D_0^*(2400)$) are DK and $D_s\eta$ ($D\pi$ and $D_s\bar{K}$). Therefore, the weights of the channels to generate the D resonances can be written in terms of the ket vectors as

$$|(\phi\phi)_{43}\rangle = \sqrt{2}|DK(0,0)\rangle - \frac{1}{\sqrt{3}}|D_s\eta(0,0)\rangle, \qquad (23)$$

$$|(\phi\phi)_{42}\rangle = -\sqrt{\frac{3}{2}}|D\pi(1/2,1/2)\rangle + |D_s\bar{K}(1/2,1/2)\rangle, \qquad (24)$$

$$|(\phi\phi)_{41}\rangle = \sqrt{\frac{3}{2}}|D\pi(1/2,-1/2)\rangle - |D_s\bar{K}(1/2,-1/2)\rangle, \qquad (25)$$

where we have used two-body states in the isospin basis, which are specified as (I, I_3) and are summarized in the Appendix. We note that, due to the isospin symmetry, both the charged and neutral $D_0^*(2400)$ are produced with the weight of $|(\phi\phi)_{42}\rangle = -|(\phi\phi)_{41}\rangle$, which means that the ratio of the decay widths into the charged and neutral $D_0^*(2400)$ is almost unity. By using these weights, we can express V_{had} in terms of two pseudoscalars.

Once the quark-antiquark pair hadronizes into two mesons they start to interact and the D resonances can be formed as a result of complex two-body interactions with coupled channels described by the Bethe-Salpeter equation. If the resonance is formed, independent of how it
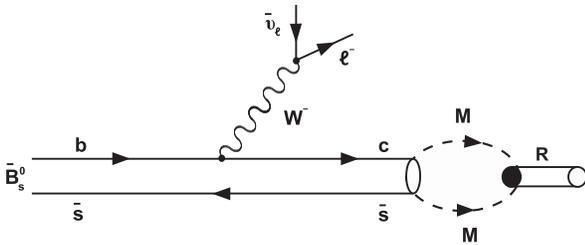
FIG. 6.   Diagrammatic representation of DK production: directly (on the left) and via rescattering (on the right) in $\bar{B}_s^0$ decays.

decays, the process is usually called "coalescence" [30] and it is a reaction with three particles in the final state (see Fig. 5). If we look for a specific two meson final channel we can have it by "prompt" or direct production (first diagram of Fig. 6), and by rescattering, generating the resonance (second diagram of Fig. 6). This process is usually called "rescattering" and it is a reaction with four particles in the final state. Coalescence and rescattering will be discussed in the next sections.

III. COALESCENCE

In this section we consider D resonance production via meson coalescence after rescattering (see Fig. 5). This process has a three-body final state with a lepton, its neutrino and the resonance R. The hadronization factor, V_{had}, can be obtained as

$$V_{\text{had}}(D_{s0}^*(2317)) = C\left(\sqrt{2}G_{DK}g_{DK} - \frac{1}{\sqrt{3}}G_{D_s\eta}g_{D_s\eta}\right), \qquad (26)$$

$$V_{\text{had}}(D_0^*(2400)^+) = -V_{\text{had}}(D_0^*(2400)^0)$$
$$= C\left(-\sqrt{\frac{3}{2}}G_{D\pi}g_{D\pi} + G_{D_s\bar{K}}g_{D_s\bar{K}}\right). \qquad (27)$$

Here g_i is the coupling constant of the D resonance to the ith two meson channel and G_i is the loop function of two meson propagators (see Sec. V)

$$G_i(s) \equiv i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_i^2 + i\epsilon} \frac{1}{(P-q)^2 - m_i'^2 + i\epsilon}, \qquad (28)$$

where P^μ is the total four-momentum of the two-meson system, and thus $P^2 = s$ with s being the invariant mass squared of the two-meson system, and m_i and m_i' are the masses of the two mesons in channel i. An important point to note is that the prefactor C is the same in all decay modes and contains dynamical factors common to all reactions only because we are assuming that in the hadronization the SU(3) flavor symmetry is reasonable, i.e., the quark pairs

$c\bar{s}$ (Fig. 1) and $c\bar{d}$ (Fig. 2) hadronize in the same way. We further assume that C is a constant and therefore is canceled when we take the ratio of decay widths as in Eq. (1). The C factor accounts for the intricate dynamics of the hadronization. However, it does not contain the final state interaction, which we take into account explicitly and is the source of dependence on M_{inv}. This factor is taken constant in the very small range of M_{inv} used in our analysis, and this is a common practice in these reactions [25,31–33]. Empirical support for that is found in related reactions in [34]. In papers like [35] the M_{inv} dependence comes from form factors, which are an equivalent way to ours to take into account the final state interaction of the mesons.

Now we can evaluate the decay widths by using the formula of Eq. (13). Inserting Eq. (26) [or Eq. (27)] into Eq. (12) and the latter into Eq. (13) we can write the decay width of the D resonance production via meson coalescence (Fig. 5) as

$$\Gamma_{\text{coal}} = \frac{m_\nu m_l}{128\pi^5 m_B^2} \int dM_{\text{inv}}^{(\nu l)} p_D^{\text{cm}} \tilde{p}_\nu \int d\Omega_D$$
$$\times \int d\tilde{\Omega}_\nu \frac{4|G_F V_{bc} V_{\text{had}}(D^*)|^2}{m_\nu m_l m_B m_R} (p_B \cdot p_\nu)(p_R \cdot p_l), \tag{29}$$

where p_D^{cm} is the momentum of the D resonance in the B rest frame and $\tilde{p}_\nu$ is the momentum of the neutrino in the νl rest frame, both of which are evaluated as

$$p_D^{\text{cm}} = \frac{\lambda^{1/2}(m_B^2, [M_{\text{inv}}^{(\nu l)}]^2, m_R^2)}{2m_B}, \tag{30}$$

$$\tilde{p}_\nu = \frac{\lambda^{1/2}([M_{\text{inv}}^{(\nu l)}]^2, m_\nu^2, m_l^2)}{2M_{\text{inv}}^{(\nu l)}}, \tag{31}$$

with the Källen function $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$. The tilde on characters indicates that they are evaluated in the νl rest frame unless explicitly mentioned. The solid angles Ω_D and $\tilde{\Omega}_\nu$ are for the D resonance in the B rest frame and for the neutrino in the νl rest frame, respectively, and $M_{\text{inv}}^{(\nu l)}$ is the νl invariant mass, whose integral range is $[m_l + m_\nu, m_B - m_R]$. After performing the angular integral of Ω_D, we obtain the expression of the decay widths for the coalescence of the D resonance:

$$\Gamma_{\text{coal}} = \frac{|G_F V_{bc} V_{\text{had}}(D^*)|^2}{8\pi^4 m_B^3 m_R} \int dM_{\text{inv}}^{(\nu l)} p_D^{\text{cm}} \tilde{p}_\nu$$
$$\times \int d\tilde{\Omega}_\nu (p_B \cdot p_\nu)(p_R \cdot p_l). \tag{32}$$

In the equation, the angular integral of $\tilde{\Omega}_\nu$ is calculated in the following way. Namely, we calculate this integral in the νl rest frame, where we may evaluate the products of the momenta as

$$p_B \cdot p_\nu = \tilde{E}_B \tilde{E}_\nu - \tilde{\boldsymbol{p}}_B \cdot \tilde{\boldsymbol{p}}_\nu, \qquad p_R \cdot p_l = \tilde{E}_R \tilde{E}_l + \tilde{\boldsymbol{p}}_B \cdot \tilde{\boldsymbol{p}}_\nu, \tag{33}$$

where $\tilde{E}_A$ is the energy of particle A in the νl rest frame and we have used $\tilde{\boldsymbol{p}}_\nu = -\tilde{\boldsymbol{p}}_l$ and $\tilde{\boldsymbol{p}}_B = \tilde{\boldsymbol{p}}_R$. For simplicity we neglect lepton masses, so we have

$$\tilde{E}_\nu = \tilde{E}_l = \tilde{p}_\nu = \tilde{p}_l = \frac{M_{\text{inv}}^{(\nu l)}}{2} \quad (\tilde{p}_{\nu,l} \equiv |\tilde{\boldsymbol{p}}_{\nu,l}|). \tag{34}$$

On the other hand, the kinetic condition leads to exact forms of $\tilde{E}_B$ and $\tilde{E}_R$:

$$\tilde{E}_B = \frac{m_B^2 + [M_{\text{inv}}^{(\nu l)}]^2 - m_R^2}{2M_{\text{inv}}^{(\nu l)}}, \tag{35}$$

$$\tilde{E}_R = \frac{m_B^2 - [M_{\text{inv}}^{(\nu l)}]^2 - m_R^2}{2M_{\text{inv}}^{(\nu l)}}. \tag{36}$$

In this way $(p_B \cdot p_\nu)(p_R \cdot p_l)$ becomes

$$(p_B \cdot p_\nu)(p_R \cdot p_l) = \frac{\tilde{E}_B \tilde{E}_R [M_{\text{inv}}^{(\nu l)}]^2}{4} + \frac{(\tilde{E}_B - \tilde{E}_R) M_{\text{inv}}^{(\nu l)}}{2}$$
$$\times \tilde{\boldsymbol{p}}_B \cdot \tilde{\boldsymbol{p}}_\nu - (\tilde{\boldsymbol{p}}_B \cdot \tilde{\boldsymbol{p}}_\nu)^2. \tag{37}$$

The second term vanishes when integrated with the neutrino scattering angle in the νl rest frame, and the third term becomes

$$-\int d\tilde{\Omega}_\nu (\tilde{\boldsymbol{p}}_B \cdot \tilde{\boldsymbol{p}}_\nu)^2 = -4\pi \frac{(\tilde{p}_B \tilde{p}_\nu)^2}{3} = -\frac{\pi \tilde{p}_B^2 [M_{\text{inv}}^{(\nu l)}]^2}{3}, \tag{38}$$

where $\tilde{p}_B \equiv \sqrt{\tilde{E}_B^2 - m_B^2}$. As a result, we obtain the angular integral of $\tilde{\Omega}_\nu$ as

$$\int d\tilde{\Omega}_\nu (p_B \cdot p_\nu)(p_R \cdot p_l) = \pi [M_{\text{inv}}^{(\nu l)}]^2 \left(\tilde{E}_B \tilde{E}_R - \frac{\tilde{p}_B^2}{3} \right). \tag{39}$$

Finally we obtain the expression of the decay widths for the coalescence of the D resonance:

$$\Gamma_{\text{coal}} = \frac{|G_F V_{bc} V_{\text{had}}(D^*)|^2}{8\pi^3 m_B^3 m_R}$$
$$\times \int dM_{\text{inv}}^{(\nu l)} p_D^{\text{cm}} \tilde{p}_\nu [M_{\text{inv}}^{(\nu l)}]^2 \left(\tilde{E}_B \tilde{E}_R - \frac{\tilde{p}_B^2}{3} \right). \tag{40}$$

Furthermore, we shall see at the end of Sec. VI that the range of variables in the phase space in the region that we are interested in is rather restricted and this further supports our choice of taking C constant.

IV. RESCATTERING

Next, the production of two pseudoscalars with prompt production plus rescattering through a D resonance is calculated with the diagrams shown in Fig. 6, and its hadronization amplitude V_{had} in the isospin basis is given by

$$
V_{\text{had}}(DK)
= C\left(\sqrt{2} + \sqrt{2}G_{DK}T_{DK \to DK} - \frac{1}{\sqrt{3}}G_{D_s\eta}T_{D_s\eta \to DK}\right),
$$
(41)

$$
V_{\text{had}}(D_s\eta)
= C\left(-\frac{1}{\sqrt{3}} + \sqrt{2}G_{DK}T_{DK \to D_s\eta} - \frac{1}{\sqrt{3}}G_{D_s\eta}T_{D_s\eta \to D_s\eta}\right),
$$
(42)

$$
V_{\text{had}}(D\pi)
= C\left(-\sqrt{\frac{3}{2}} - \sqrt{\frac{3}{2}}G_{D\pi}T_{D\pi \to D\pi} + G_{D_s\bar{K}}T_{D_s\bar{K} \to D\pi}\right),
$$
(43)

$$
V_{\text{had}}(D_s\bar{K})
= C\left(1 - \sqrt{\frac{3}{2}}G_{D\pi}T_{D\pi \to D_s\bar{K}} + G_{D_s\bar{K}}T_{D_s\bar{K} \to D_s\bar{K}}\right).
$$
(44)

Again we see that the prefactor C is the same in all the reactions. In order to calculate decay widths in the particle basis, we have to multiply by the appropriate Clebsch-Gordan coefficients.

Inserting Eq. (41) [or Eqs. (42), (43), and (44)] into Eq. (12) and the latter into Eq. (14) we can derive the differential decay width $d\Gamma_i/dM_{\text{inv}}^{(i)}$, where i represents the two pseudoscalar states and $M_{\text{inv}}^{(i)}$ is the invariant mass of the two pseudoscalars, as

$$
\frac{d\Gamma_i}{dM_{\text{inv}}^{(i)}} = \frac{|G_{\text{F}}V_{bc}V_{\text{had}}(i)|^2}{32\pi^5 m_B^3 M_{\text{inv}}^{(i)}}
$$
$$
\times \int dM_{\text{inv}}^{(\nu l)} P^{\text{cm}} \tilde{p}_\nu \tilde{p}_i [M_{\text{inv}}^{(\nu l)}]^2 \left(\tilde{E}_B \tilde{E}_i - \frac{\tilde{p}_B^2}{3}\right), \quad (45)
$$

where P^{cm} is the momentum of the νl system in the B rest frame, $\tilde{p}_\nu$ is defined in Eq. (31), and $\tilde{p}_i$ is the relative momentum of the two pseudoscalars in their rest frame, both of which are evaluated as

$$
P^{\text{cm}} = \frac{\lambda^{1/2}(m_B^2, [M_{\text{inv}}^{(\nu l)}]^2, [M_{\text{inv}}^{(i)}]^2)}{2m_B},
$$
(46)

$$
\tilde{p}_i = \frac{\lambda^{1/2}([M_{\text{inv}}^{(i)}]^2, m_i^2, m_i'^2)}{2M_{\text{inv}}^{(i)}}.
$$
(47)

Here we note that $\tilde{p}_i$ is a quantity in the rest frame of the two pseudoscalars rather than the νl system. The energies $\tilde{E}_B$ and $\tilde{E}_i$ are evaluated in the νl rest frame as

$$
\tilde{E}_B = \frac{m_B^2 + [M_{\text{inv}}^{(\nu l)}]^2 - [M_{\text{inv}}^{(i)}]^2}{2M_{\text{inv}}^{(\nu l)}},
$$
(48)

$$
\tilde{E}_i = \frac{m_B^2 - [M_{\text{inv}}^{(\nu l)}]^2 - [M_{\text{inv}}^{(i)}]^2}{2M_{\text{inv}}^{(\nu l)}},
$$
(49)

and $\tilde{p}_B = \sqrt{\tilde{E}_B^2 - m_B^2}$.

V. THE DK-$D_s\eta$ AND $D\pi$-$D_s\bar{K}$ SCATTERING AMPLITUDES

In this section we will discuss in more detail the amplitudes which appear in Eqs. (41), (42), (43), and (44). We formulate meson-meson scattering amplitudes for the rescatterings to generate the $D_{s0}^*(2317)$ and $D_0^*(2400)$ resonances in the final state of the B decay. In Ref. [29] it was found that the couplings to DK and $D_s\eta$ are dominant for $D_{s0}^*(2317)$ and the couplings to $D\pi$ and $D_s\bar{K}$ are dominant for $D_0^*(2400)$. Therefore, in the following we concentrate on DK-$D_s\eta$ two-channel scattering in isospin $I = 0$ and $D\pi$-$D_s\bar{K}$ two-channel scattering in $I = 1/2$, extracting essential portions from Ref. [29] and assuming isospin symmetry. Namely, we obtain these amplitudes by solving a coupled-channel scattering equation in an algebraic form

$$
T_{ij}(s) = V_{ij}(s) + \sum_k V_{ik}(s)G_k(s)T_{kj}(s),
$$
(50)

where i, j, and k are channel indices, s is the Mandelstam variable of the scattering, V is the interaction kernel, and G is the two-body loop function.

The interaction kernel V corresponds to the tree-level transition amplitudes obtained from phenomenological Lagrangians developed in Ref. [29]. Here we summarize the tree-level amplitude in the isospin basis (for the two-body states in the isospin basis, see the Appendix). Namely, for the DK-$D_s\eta$ scattering in $I = 0$ we have

$$
V_{DKDK}^{\text{phen}}(s, t, u) = -\frac{1}{3f_\pi f_D}[\gamma(t-u) + s - u + m_D^2 + m_K^2],
$$
(51)

$$
V_{DKD_s\eta}^{\text{phen}}(s, t, u) = V_{\eta D_s KD}^{\text{phen}}(s, t, u)
$$
$$
= -\frac{1}{6\sqrt{3}f_\pi f_D}[\gamma(u-t) - (3+\gamma)(s-u)
$$
$$
- m_D^2 - 3m_K^2 + 2m_\pi^2],
$$
(52)

$$V^{\text{phen}}_{D_s\eta D_s\eta}(s,t,u) = -\frac{1}{9f_\pi f_D}$$
$$\times [\gamma(-s+2t-u)+2m_D^2+6m_K^2-4m_\pi^2], \tag{53}$$

and for the $D\pi$-$D_s\bar{K}$ scattering in $I=1/2$ we have

$$V^{\text{phen}}_{D\pi D\pi}(s,t,u) = -\frac{1}{12f_\pi f_D}[2\gamma(t-u)+(\gamma+4)(s-u)$$
$$+2m_D^2+2m_\pi^2],$$

$$V^{\text{phen}}_{D\pi D_s\bar{K}}(s,t,u) = V^{\text{phen}}_{\bar{K}D_s\pi D}(s,t,u)$$
$$= \frac{1}{2\sqrt{6}f_\pi f_D}[\gamma(t-u)+s-u+m_D^2+m_K^2], \tag{54}$$

$$V^{\text{phen}}_{D_s\bar{K}D_s\bar{K}}(s,t,u) = -\frac{1}{6f_\pi f_D}$$
$$\times [\gamma(t-u)+s-u+m_D^2+2m_K^2-m_\pi^2], \tag{55}$$

where t and u are Mandelstam variables. In these equations, f_π and f_D represent the pion and D meson decay constants, respectively, and m_π, m_K, and m_D are the masses of pion, kaon, and D mesons, respectively. In addition, in order to treat effectively interactions of heavy mesons, we have introduced a parameter $\gamma \equiv (m_L/m_H)^2$ as the squared ratio of the masses of the light to heavy vector mesons (respectively m_L and m_H), which are exchanged between two pseudoscalar mesons. Then we perform the on-shell factorization and the s-wave projection to give the interaction kernel V in Eq. (50):

$$V(s) = \frac{1}{2}\int_{-1}^{1}d\cos\theta V^{\text{phen}}(s,t(s,\cos\theta),u(s,\cos\theta)), \tag{56}$$

where θ is the scattering angle in the center-of-mass frame.

For the loop function G, on the other hand, we use the expression in Eq. (28). In this study we employ the dimensional regularization, so we can express the loop function as

$$G_k(s) = \frac{1}{16\pi^2}\left[a_k(\mu_{\text{reg}})+\ln\frac{m_k^2}{\mu_{\text{reg}}^2}+\frac{s+m_k'^2-m_k^2}{2s}\ln\frac{m_k'^2}{m_k^2}\right.$$
$$\left.-\frac{2\lambda^{1/2}(s,m_k^2,m_k'^2)}{s}\operatorname{artanh}\left(\frac{\lambda^{1/2}(s,m_k^2,m_k'^2)}{m_k^2+m_k'^2-s}\right)\right], \tag{57}$$

with the regularization scale μ_{reg} and the subtraction constant a_k, which becomes a model parameter. In this

approach, D resonances can appear as poles of the scattering amplitude $T_{ij}(s)$ with the residue $g_i g_j$:

$$T_{ij}(s) = \frac{g_i g_j}{s-s_{\text{pole}}} + (\text{regular at } s=s_{\text{pole}}). \tag{58}$$

The pole is described by its position s_{pole} and the constant g_i, which can be interpreted as the coupling constant of the D resonance to the i channel. In this study only the subtraction constant in each channel is the model parameter. Actually, the meson masses are fixed as $m_\pi = 138.04$ MeV, $m_K = 495.67$ MeV, $m_\eta = 547.85$ MeV, $m_D = 1867.23$ MeV, and $m_{D_s} = 1968.30$ MeV, and we take

$$f_\pi = 93 \text{ MeV}, \qquad f_D = 165 \text{ MeV}, \tag{59}$$

$$m_L = 800 \text{ MeV}, \qquad m_H = 2050 \text{ MeV}, \tag{60}$$

for the pion and D decay constants and masses of the light and heavy vector mesons, respectively. On the other hand, the subtraction constant, as a model parameter, is determined so as to generate a pole of $D_{s0}^*(2317)$ at the right place, i.e., to reproduce the mass reported by the Particle Data Group from the square root of the pole position, $\sqrt{s_{\text{pole}}}$. In this study we assume that all the subtraction constants take the same value for simplicity, and employ $a_{DK}=a_{D_s\eta}=a_{D\pi}=a_{D_s\bar{K}}=-1.27$ at $\mu_{\text{reg}}=1500$ MeV. Indeed, with these values of the subtraction constant we obtain the pole positions listed in Table I. The values of the coupling constants g_i are also given in Table I.

As one can see from Table I, the $D_{s0}^*(2317)$ state has zero decay width with $\text{Im}\sqrt{s_{\text{pole}}}=0$, since we do not include the $D_s\pi^0$ decay channel, for which the isospin symmetry breaking is necessary. As a result, the coupling constants also become real and have positive values. The DK coupling constant is about 2 times larger than that of the $D_s\eta$ coupling constant, and their values are in agreement with the results obtained in Ref. [29]. On the other hand, the $D_0^*(2400)$ state has a decay width ($2\,\text{Im}\sqrt{s_{\text{pole}}}\approx 320$ MeV) to the $D\pi$ decay channel, and the coupling constant is complex. The magnitude of the $D\pi$ coupling constant is larger than that of the $D_s\bar{K}$ coupling constant, and they are

TABLE I. Pole position $\sqrt{s_{\text{pole}}}$, coupling constant g_i, compositeness X_i, and elementariness Z for the D resonances in the isospin basis.

$D_{s0}^*(2317)$		$D_0^*(2400)$	
$\sqrt{s_{\text{pole}}}$	2317 MeV	$\sqrt{s_{\text{pole}}}$	$2128-160i$ MeV
g_{DK}	10.58 GeV	$g_{D\pi}$	$9.00-6.18i$ GeV
$g_{D_s\eta}$	-6.11 GeV	$g_{D_s\bar{K}}$	$-7.68+4.35i$ GeV
X_{DK}	0.69	$X_{D\pi}$	$0.34+0.41i$
$X_{D_s\eta}$	0.09	$X_{D_s\bar{K}}$	$0.03-0.12i$
Z	0.22	Z	$0.63-0.28i$

very close to the values in Ref. [29]. In the following we will use the coupling constants of the D resonances in Table I for the coalescence of D resonances in the semileptonic B decays [see Eq. (40)] and use the scattering amplitude for the meson-meson invariant mass distributions of the D resonances [see Eq. (45)].

Let us further discuss the structure of the D resonances in this model from the point of view of compositeness, which is defined as the contribution from the two-body part to the normalization of the total wave function and measures the fraction of the two-body state [36–40]. Actually, the coupling constant g_i is found to be the coefficient of the two-body wave function in Refs. [41,42], and the expression of the compositeness in the present model is

$$X_i = -g_i^2 \left[\frac{dG_i}{ds} \right]_{s=s_{\text{pole}}}. \tag{61}$$

On the other hand, the elementariness Z, which measures the fraction of missing channels, is expressed as

$$Z = -\sum_{i,j} g_j g_i \left[G_i \frac{dV_{ij}}{ds} G_j \right]_{s=s_{\text{pole}}}. \tag{62}$$

We note that in general both the compositeness X_i and elementariness Z become complex values for a resonance state and hence one cannot interpret the compositeness (elementariness) as the probability to observe a two-body (missing-channel) component inside the resonance. However, a striking property is that the sum of them coincides with the normalization of the total wave function for the resonance and is exactly unity:

$$\sum_i X_i + Z = 1, \tag{63}$$

which is guaranteed by a generalized Ward identity proved in Ref. [43]. Therefore one can deduce the structure by comparing the value of the compositeness with unity, on the basis of the similarity to the stable bound state case. The values of the compositeness and elementariness of the D resonances in this approach are also listed in Table I. The result indicates that the $D_{s0}^*(2317)$ resonance, which is obtained as a bound state in the present model, is indeed dominated by the DK component. This has been corroborated in the recent analysis of QCD lattice results of [12]. In contrast, we may interpret that the $D_0^*(2400)$ resonance is constructed with missing channels, although the imaginary part for each component is not negligible.

VI. NUMERICAL RESULTS

Now we show our numerical results of the semileptonic B decay widths. As we have seen, we fix the hadronization process of the two mesons in Sec. II B and we employ an

TABLE II. Ratios of decay widths and branching fractions of semileptonic B decays.

R	0.45
$\Gamma_{B^- \to D_0^*(2400)^0 \bar{\nu}_l l^-} / \Gamma_{\bar{B}^0 \to D_0^*(2400)^+ \bar{\nu}_l l^-}$	1.00
$\mathcal{B}[\bar{B}^0 \to D_0^*(2400)^+ \bar{\nu}_l l^-]$	4.5×10^{-3} (input)
$\mathcal{B}[\bar{B}^- \to D_0^*(2400)^0 \bar{\nu}_l l^-]$	4.9×10^{-3}

effective model in Sec. V so as to determine the strength of the couplings of the D resonances to the meson-meson channels. In this way, we can calculate the ratio of the decay widths in the coalescence treatment as well as in the rescattering.

First we consider the coalescence case. The numerical results are summarized in Table II. The most interesting quantity is the ratio $R = \Gamma_{\bar{B}_s^0 \to D_{s0}^*(2317)^+ \bar{\nu}_l l^-} / \Gamma_{\bar{B}^0 \to D_0^*(2400)^+ \bar{\nu}_l l^-}$ in the coalescence treatment, which removes the unknown factor C in the hadronization process. The decay width in the coalescence is expressed in Eq. (40). The coupling constants of the two mesons to the D resonances are determined in Sec. V and listed in Table I. We emphasize that we have no fitting parameters for the ratio R in this scheme. As a result, we obtain the ratio of the decay widths as $R = 0.45$. On the other hand, we find that the ratio $\Gamma_{B^- \to D_0^*(2400)^0 \bar{\nu}_l l^-} / \Gamma_{\bar{B}^0 \to D_0^*(2400)^+ \bar{\nu}_l l^-}$ is 1.00, which can be expected from the same strength of the decay amplitude to the charged and neutral $D_0^*(2400)$ due to the isospin symmetry, as discussed after Eq. (25).

Then, we can fix the absolute value of the common prefactor C by using experimental data of the decay width. Actually, the branching fraction of the semileptonic decay $\bar{B}^0 \to D_0^*(2400)^+ \bar{\nu}_l l^-$ followed by $D_0^*(2400)^+ \to D^0 \pi^+$ to the total decay is reported as $(3.0 \pm 1.2) \times 10^{-3}$ by the Particle Data Group [44]. The branching fraction of $D_0^*(2400)^+$ to $D^0 \pi^+$ can be fixed by the isospin symmetry as $\mathcal{B}[D_0^*(2400)^+ \to D^0 \pi^+] = 2/3$, which indicates $\mathcal{B}[\bar{B}^0 \to D_0^*(2400)^+ \bar{\nu}_l l^-] = (4.5 \pm 1.8) \times 10^{-3}$. By using this mean value we find $C = 7.22$, and the fractions of decays $\bar{B}_s^0 \to D_{s0}^*(2317)^+ \bar{\nu}_l l^-$ and $B^- \to D_0^*(2400)^0 \bar{\nu}_l l^-$ to the total decay widths are obtained as 2.0×10^{-3} and 4.9×10^{-3}, respectively. The values of these fractions are similar to each other. The difference of the fractions of $\bar{B}^0 \to D_0^*(2400)^+ \bar{\nu}_l l^-$ and $B^- \to D_0^*(2400)^0 \bar{\nu}_l l^-$ comes from the fact that the total decay widths of $\bar{B}^0$ and B^- are different.

In Table III we compare our predictions for $\mathcal{B}[\bar{B}_s^0 \to D_{s0}^*(2317)^+ \bar{\nu}_l l^-]$ with the results obtained with other approaches. Although not explicitly mentioned by the authors, from the reading of the papers we can see that we should attach an uncertainty of at least 10% to the numbers without theoretical error bars. The discrepancy between the calculated branching fractions can be of a factor 5, showing that there is large room for improvement on the theoretical side. Our approach is the only one where

TABLE III. Branching fraction of the process $\bar{B}_s^0 \to D_{s0}^*(2317)^+ \bar{\nu}_l l^-$ in percentage.

Approach	$\mathcal{B}[\bar{B}_s^0 \to D_{s0}^*(2317)^+ \bar{\nu}_l l^-]$
This paper	0.20
QCDSR + HQET [18]	0.09–0.20
QCDSR (SVZ) [19]	0.10
LCSR [21]	0.23 ± 0.11
CQM [20]	0.49–0.57
CQM [22]	0.44
CQM [23]	0.39

the $D_{s0}^*(2317)^+$ is treated as a mesonic molecule. Looking at Table III we can divide the results in two groups: the first four numbers, which are "small" and the last three, which are "large." In the second group, the constituent quark models (CQM) yield larger branching fractions. Understanding the origin of the discrepancies requires a very careful comparative analysis of all the ingredients of the different approaches and it is beyond the scope of the present paper. However it is tempting, as a first speculation, to attribute these differences to the differences in spatial configurations, which are inherent to each approach. The parent B_s meson is a compact state, with a typical radius of the order of the lowest charmonium radius, i.e. $\langle r \rangle \simeq 0.4$ fm. In constituent quark models all the D_s mesons, being relatively heavy $c\bar{q}$ states, should also be compact and hence the overlap between the initial and final state spatial wave functions is large. In the molecular picture of the $D_{s0}^*(2317)^+$, a bound state of two mesons is expected to have a large radius, of the order of a few fm, and therefore in this case the overlap between initial and final of wave functions is small, reducing the corresponding branching fraction. In the QCD sum rules formalism, there is no explicit mention to the spatial configuration of the interpolating currents and it is difficult to say anything.

In view of the results of Tables I and III, it is interesting to make some observation. In Table I we see that Z is about 20%. This measures the amount in the $D_{s0}^*(2317)$ wave function for components which are not those explicitly taken into account, DK and $D_s\eta$. The rest could be $q\bar{q}$ components or other meson-meson channels not explicitly considered. If we take the extreme limit that all of Z comes from $q\bar{q}$ components, then the rate of B decay into this resonance that we have calculated could be significantly changed. This is because models that take a $q\bar{q}$ component for this resonance listed in Table III give a rate about 2 to 3 times bigger than what we obtain.

This means that the contribution to the rate from this $q\bar{q}$ component could be as big as what we have calculated for the rate. Yet, this would be an extreme, because there can, or most probably will, be contributions to Z from other meson-meson components which would contribute in an amount of about 20% to the rate. In any case there are two extra comments in this respect. First, one must take into account that to a large extend, when one makes the study of the molecular state with certain coupled channels one fits some parameters, in this case the subtraction constant, to get the right binding, and then one is to some extent taking into account in a phenomenological way the missing channels. Also, one must not forget that we have evaluated the rate of the $D_{s0}^*(2317)$ production from the experimental one of the $D_0^*(2400)$ and we obtained a rate of 0.45 as shown in Table II. If the two resonances were of the $q\bar{q}$ type we could expect this rate to be of the order of unity, in which case a 20% contribution of $q\bar{q}$ could only alter that ratio in about 0.2. All this said, we could tentatively accept an uncertainly of about 0.2 in the ratio R, most probably going in the direction of increasing the ratio that we have obtained. This would provide a range of 0.20–0.30 for the rate quoted in Table III. In any case, we are more interested in the shapes of the invariant mass distributions as we discuss below.

Next we consider the rescattering process for the final-state two mesons formulated in Sec. IV. We use the common prefactor $C = 7.22$ fixed from the experimental value of the width of the semileptonic decay $\bar{B}^0 \to D_0^*(2400)^+ \bar{\nu}_l l^-$. The meson-meson scattering amplitude is obtained in Sec. V, and we further introduce the $D_s\pi^0$ channel as the isospin-breaking decay mode of $D_{s0}^*(2317)$. Namely, we calculate the scattering amplitude involving the $D_s\pi^0$ channel as

$$T_{i \to D_s\pi^0} = \frac{g_i g_{D_s\pi^0}}{s - [M_{D_{s0}^*} - i\Gamma_{D_{s0}^*}/2]^2}, \qquad (64)$$

for $i = DK$ and $D_s\eta$. We take the $D_{s0}^*(2317)$ mass as $M_{D_{s0}^*} = 2317$ MeV, while we assume its decay width as $\Gamma_{D_{s0}^*} = 3.8$ MeV, which is the upper limit from experiments [44]. The $D_{s0}^*(2317)$-i coupling constant g_i ($i = DK$, $D_s\eta$) is taken from Table I, and the $D_{s0}^*(2317)$-$D_s\pi^0$ coupling constant $g_{D_s\pi^0}$ is calculated from the $D_{s0}^*(2317)$ decay width as

$$g_{D_s\pi^0} = \sqrt{\frac{8\pi M_{D_{s0}^*}^2 \Gamma_{D_{s0}^*}}{p_\pi}}, \qquad (65)$$

with the pion center-of-mass momentum p_π, and we obtain $g_{D_s\pi^0} = 1.32$ GeV.

The results of the differential decay width $d\Gamma_i/dM_{\text{inv}}^{(i)}$ (45), where i represents the two pseudoscalar states, are shown in Fig. 7. The figure is plotted in the isospin basis. Therefore, when translating into the particle basis we use the relation according to the weight of states given in the Appendix:

$$[D^0K^+] = [D^+K^0] = \frac{1}{2}[DK], \qquad (66)$$

$$[D_s^+\pi^0] = [D_s\pi^0], \qquad (67)$$
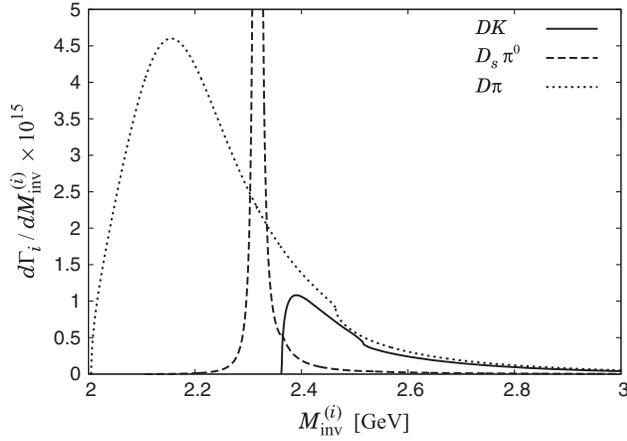
FIG. 7. Differential decay width $d\Gamma_i/dM_{\mathrm{inv}}^{(i)}$ for the two pseudoscalars channel i in the isospin basis. Here we consider the semileptonic decays $\bar{B}_s^0 \to (DK)^+\bar{\nu}_l l^-$, $(D_s\pi^0)^+\bar{\nu}_l l^-$ and $\bar{B}^0 \to (D\pi)^+\bar{\nu}_l l^-$. The DK and $D_s\pi^0$ channels couple to the $D_{s0}^*(2317)^+$ resonance, and $D\pi$ to the $D_0^*(2400)$ resonance. The peak height for the $D_s\pi^0$ channel is $d\Gamma_{D_s\pi^0}/dM_{\mathrm{inv}}^{(D_s\pi^0)} \sim 1.5 \times 10^{-13}$.

$$[D^0\pi^+] = 2[D^+\pi^0] = \frac{2}{3}[D\pi], \tag{68}$$

where $[AB]$ is the partial decay width to the AB channel. An interesting point is that the DK mode shows a rapid increase from its threshold ≈ 2360 MeV due to the existence of the bound state, i.e., the $D_{s0}^*(2317)$ resonance. In experiments, such a rapid increase from the DK threshold would support the interpretation of the $D_{s0}^*(2317)$ resonance as a DK bound state. The strength of the DK contribution in the $M_{\mathrm{inv}}^{(i)} \gtrsim 2.4$ GeV region is similar to that of $D\pi$, which corresponds to the "tail" for the $D_0^*(2400)$ resonance. In fact, the position of the $D_0^*(2400)$ peak in Fig. 7 might be shifted to higher invariant masses, since we have underestimated the mass of the $D_0^*(2400)$ resonance in our model compared to the experimental values 2318 and 2403 MeV for neutral and charged $D_0^*(2400)$, respectively [44] (note that the experimental uncertainties in the position and width of this resonance are large). On the other hand, the $D_s\pi^0$ peak coming from the $D_{s0}^*(2317)$ resonance is very sharp due to its narrow width. The $D_s\pi^0$ peak height is about 30 times larger than the $D\pi$ one coming from $D_0^*(2400)$, but when integrating the bump structure of the differential decay widths we obtain a ratio of semileptonic B decays into $D_{s0}^*(2317)$ to $D_0^*(2400)$ close to 0.45, as obtained in the coalescence treatment above.

We are interested in the shapes of the mass distributions plotted in Fig. 7 with invariant masses around 2.4 GeV. It is then interesting to see from which invariant masses of the lepton neutrino pair one gets most of the contribution. For this, we go back to Eq. (45), fix $M_{\mathrm{inv}}^{(i)}$ at 2.4 GeV, and plot the integrand as a function of $M_{\mathrm{inv}}^{(\nu l)}$. This is plotted in Fig. 8. As we can see, $M_{\mathrm{inv}}^{(\nu l)}$ peaks around the highest mass of the spectrum. This means that the resonance moves with low
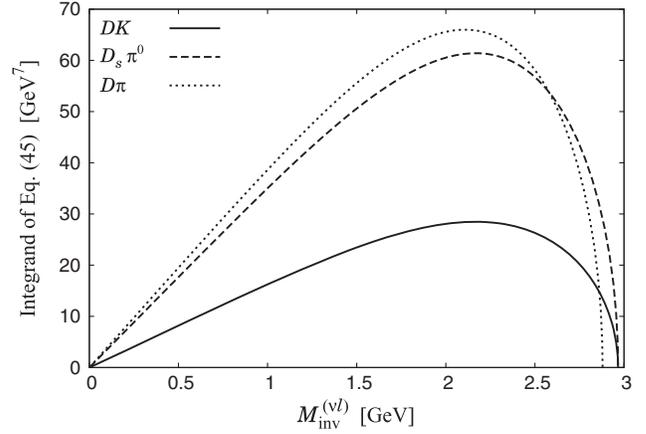
FIG. 8. Integrand of Eq. (45) as a function of the invariant mass of the lepton pair, $M_{\mathrm{inv}}^{(\nu l)}$ (factor outside the integral excluded). Here we consider the semileptonic decays $\bar{B}_s^0 \to (DK)^+\bar{\nu}_l l^-$, $(D_s\pi^0)^+\bar{\nu}_l l^-$ and $\bar{B}^0 \to (D\pi)^+\bar{\nu}_l l^-$. The invariant mass of the two pseudoscalar meson is fixed as $M_{\mathrm{inv}}^{(i)} = 2.4$ GeV in all decay modes.

energy in the B rest frame and we could as well have neglected the three momenta of the heavy quarks. The results of Fig. 8 are interesting because we see that we get the contribution to the process from a limited range of kinematical variables for the particles involved.

The spectra shown in Fig. 7 are our predictions and they may be measured at the LHCb. They were obtained in the framework of the chiral unitary approach in coupled channels and their experimental observation would give support to the $D_{s0}^*(2317)$ and $D_0^*(2400)$ as dynamically generated resonances, which is inherent to this approach.

Apart from comparing shapes and relative strength, one can make an analysis of the DK mass distribution as suggested in [45] to determine g_{DK}. With this value and the use of Eq. (61) one can determine the amount of DK component in the $D_{s0}^*(2317)$ wave function. Note that the shape of the DK mass distribution is linked to the potential, with its associated energy dependence, and the mass of the $D_{s0}^*(2317)$ [45]. With the same binding of the resonance, different models that have a different amount of DK component provide different shapes, leading to different values of the g_{DK} coupling, and it is possible to discriminate among models that have a different nature for the $D_{s0}^*(2317)$ resonance.

VII. CONCLUSION

We have extended the formalism developed in [25] and applied it to semileptonic B and B_s decays into resonances, which are interpreted as dynamically generated resonances. As in [25], we start studying the weak process at the quark level and, as a "final state interaction," the outgoing quark-antiquark pair couples to meson pairs, which rescatter and form resonances, which then decay in well-defined channels. This process, discussed in Sec. II B, is a very

economic hadronization mechanism with a single parameter, C. After fixing it with the help of experimental information on the $\bar{B}^0 \to D_0^*(2400)^+\bar{\nu}_l l^-$ decay, we make predictions for the semileptonic decay width of the $D_{s0}^*(2317)$, shown in Table III and also for the invariant mass spectra shown in Fig. 7.

We have added new information related to the nature of the $D_{s0}^*(2317)$ as an object with a dominant DK molecular component, which is the DK mass distribution in the $\bar{B}_s^0 \to (DK)^+\bar{\nu}_l l^-$ decay. The simultaneous measurement of the decay rate into the $D_{s0}^*(2317)$ resonance and the related DK mass distribution are hence strongly encouraged to gain further knowledge on the nature of this resonance. The experimental confirmation of our predictions would give additional support to the $D_{s0}^*(2317)$ and $D_0^*(2400)$ resonances as dynamically generated resonances from the meson-meson interaction.

ACKNOWLEDGMENTS

APPENDIX: CONVENTIONS

In this Appendix we show conventions used in this study. Throughout this paper we employ the metric in four-dimensional Minkowski space defined as $g^{\mu\nu} = g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ and the Einstein summation convention is used unless explicitly mentioned.

We introduce the Dirac spinors $u(\boldsymbol{p},s)$ and $v(\boldsymbol{p},s)$, where $\boldsymbol{p}$ is three-momentum of the field and s represents its spin, as the positive and negative energy solutions of the Dirac equation, respectively:

$$(\not{p} - m)u(\boldsymbol{p},s) = 0, \qquad (\not{p} + m)v(\boldsymbol{p},s) = 0. \quad \text{(A1)}$$

Here m is the mass of the field, $\not{p} \equiv \gamma^\mu p_\mu$ with γ^μ being the Dirac gamma matrices, and $p^\mu \equiv (\sqrt{\boldsymbol{p}^2 + m^2}, \boldsymbol{p})$ is the on-shell four-momentum of the solution. In this study the Dirac spinors are normalized as follows:

$$\bar{u}(\boldsymbol{p},s)u(\boldsymbol{p},s') = \delta_{ss'}, \qquad \bar{v}(\boldsymbol{p},s)v(\boldsymbol{p},s') = -\delta_{ss'}, \quad \text{(A2)}$$

with $\bar{u} \equiv u^\dagger \gamma^0$ and $\bar{v} \equiv v^\dagger \gamma^0$, and hence we have

$$\sum_s u(\boldsymbol{p},s)\bar{u}(\boldsymbol{p},s) = \frac{\not{p} + m}{2m},$$

$$\sum_s v(\boldsymbol{p},s)\bar{v}(\boldsymbol{p},s) = \frac{\not{p} - m}{2m}. \quad \text{(A3)}$$

The trace identities used in this study are summarized as follows:

$$\mathrm{tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma] = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}), \quad \text{(A4)}$$

$$\mathrm{tr}[\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma] = -4i\epsilon^{\mu\nu\rho\sigma}, \quad \text{(A5)}$$

$$\mathrm{tr}[\gamma^\mu \gamma^\nu \gamma^\rho] = \mathrm{tr}[\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho] = 0, \quad \text{(A6)}$$

where $\gamma_5 \equiv i\gamma^0 \gamma^1 \gamma^2 \gamma^3$ and $\epsilon^{\mu\nu\rho\sigma}$ is the Levi-Cività symbol with the normalization $\epsilon^{0123} = 1$. The Levi-Cività symbol satisfies the following identity:

$$\epsilon^{\alpha\beta\mu\nu}\epsilon_{\alpha\beta\rho\sigma} = -2(\delta_\rho^\mu \delta_\sigma^\nu - \delta_\sigma^\mu \delta_\rho^\nu). \quad \text{(A7)}$$

The phase convention for mesons in terms of the isospin states $|I,I_3\rangle$ used in this study is given by

$$|\pi^+\rangle = -|1,1\rangle,$$
$$|K^-\rangle = -|1/2,-1/2\rangle,$$
$$|D^0\rangle = -|1/2,-1/2\rangle, \quad \text{(A8)}$$

while other meson states in this study are represented without phase factors. As a result, we can translate the two-body states used in this study into the isospin basis, which we specify as (I, I_3), as

$$|DK(0,0)\rangle = \frac{1}{\sqrt{2}}|D^0 K^+\rangle + \frac{1}{\sqrt{2}}|D^+ K^0\rangle, \quad \text{(A9)}$$

$$|D_s\eta(0,0)\rangle = |D_s^+ \eta\rangle, \quad \text{(A10)}$$

$$|D\pi(1/2,1/2)\rangle = -\sqrt{\frac{2}{3}}|D^0 \pi^+\rangle + \frac{1}{\sqrt{3}}|D^+ \pi^0\rangle, \quad \text{(A11)}$$

$$|D_s\bar{K}(1/2,1/2)\rangle = |D_s^+ \bar{K}^0\rangle, \quad \text{(A12)}$$

$$|D\pi(1/2,-1/2)\rangle = +\frac{1}{\sqrt{3}}|D^0 \pi^0\rangle + \sqrt{\frac{2}{3}}|D^+ \pi^-\rangle, \quad \text{(A13)}$$

$$|D_s\bar{K}(1/2,-1/2)\rangle = -|D_s^+ K^-\rangle. \quad \text{(A14)}$$

At last we summarize the Feynman rules for the weak interaction used in this study. We express the $W\nu l$ coupling as

$$-iV_{W\nu l}^\mu = i\frac{g_W}{\sqrt{2}}\gamma^\mu \frac{1-\gamma_5}{2}, \quad \text{(A15)}$$

with g_W being the coupling constant of the weak interaction and the Wbc coupling as

$$-iV^\mu_{W\nu l} = i\frac{g_W V_{bc}}{\sqrt{2}}\gamma^\mu\frac{1-\gamma_5}{2}, \qquad (A16)$$

where V_{bc} is the Cabibbo-Kobayashi-Maskawa matrix elements, whose absolute value is $|V_{bc}| \approx 0.041$. The W boson propagator with four-momentum p^μ is written as

$$iP^{\mu\nu}_W(p) = \frac{-ig^{\mu\nu}}{p^2 - M^2_W + i0}, \qquad (A17)$$

with the mass of the W boson M_W. The coupling constant g_W and the mass of the W boson M_W are related to the Fermi coupling constant G_F as

$$G_F = \frac{g^2_W}{4\sqrt{2}M^2_W} \approx 1.166 \times 10^{-5}\ \text{GeV}^{-2}. \qquad (A18)$$

[1] W. Chen, J. Ho, T. G. Steele, R. T. Kleiv, B. Bulthuis, D. Harnett, T. Richards, and S. L. Zhu, Exotic hadron states, arXiv:1411.0585; A. Esposito, A. L. Guerrieri, F. Piccinini, A. Pilloni, and A. D. Polosa, Four-quark hadrons: An updated review, Int. J. Mod. Phys. A **30,** 1530002 (2015); M. Nielsen and F. S. Navarra, Charged exotic charmonium states, Mod. Phys. Lett. A **29,** 1430005 (2014); N. Brambilla, S. Eidelman, B. K. Heltsley, R. Vogt, G. T. Bodwin, E. Eichten, A. D. Frawley, A. B. Meyer *et al.,* Heavy quarkonium: Progress, puzzles, and opportunities, Eur. Phys. J. C **71,** 1534 (2011); M. Nielsen, F. S. Navarra, and S. H. Lee, New charmonium states in QCD sum rules: A concise review, Phys. Rep. **497,** 41 (2010); E. S. Swanson, The new heavy mesons: A status report, Phys. Rep. **429,** 243 (2006); E. E. Kolomeitsev and M. F. M. Lutz, On heavy light meson resonances and chiral symmetry, Phys. Lett. B **582,** 39 (2004); F. K. Guo, P. N. Shen, H. C. Chiang, R. G. Ping, and B. S. Zou, Dynamically generated 0+ heavy mesons in a heavy chiral unitary approach, Phys. Lett. B **641,** 278 (2006); F. K. Guo, C. Hanhart, and U. G. Meissner, Interactions between heavy mesons and Goldstone bosons from chiral dynamics, Eur. Phys. J. A **40,** 171 (2009); M. Cleven, F. K. Guo, C. Hanhart, and U. G. Meissner, Light meson mass dependence of the positive parity heavy-strange mesons, Eur. Phys. J. A **47,** 19 (2011).

[2] B. Aubert *et al.* (BABAR Collaboration), Observation of a narrow meson decaying to $D^+_s\pi^0$ at a mass of 2.32 GeV/c^2, Phys. Rev. Lett. **90,** 242001 (2003); D. Besson *et al.* (CLEO Collaboration), Observation of a narrow resonance of mass 2.46-**GeV/c^2** decaying to $D^{*+}_s\pi^0$ and confirmation of the $D^*_{sJ}(2317)$ state, Phys. Rev. D **68,** 032002 (2003); **75,** 119908(E) (2007); K. Abe *et al.* (BELLE Collaboration), Upper Bound on the Decay $\tau \to \mu\gamma$ from the Belle Detector, Phys. Rev. Lett. **92,** 171802 (2004).

[3] K. Abe *et al.* (Belle Collaboration), Study of $B- \to D^0\pi -$ $(D^0 \to D(*) + \pi-)$ decays, Phys. Rev. D **69,** 112002 (2004).

[4] D. Mohler, C. B. Lang, L. Leskovec, S. Prelovsek, and R. M. Woloshyn, $D^*_{s0}(2317)$ Meson and D-Meson-Kaon Scattering from Lattice QCD, Phys. Rev. Lett. **111,** 222001 (2013).

[5] C. McNeile, Lattice approach to threshold states, *Conf. Proc.,* C060726 (2006) 903.

[6] G. S. Bali, The D$_{sJ}^+$(2317): What can the lattice say?, Phys. Rev. D **68,** 071501 (2003).

[7] G. Herdoiza, C. McNeile, and C. Michael (UKQCD Collaboration), Decay constants of P-wave heavy-light mesons from unquenched lattice QCD, Phys. Rev. D **74,** 014510 (2006).

[8] M. Luscher, Two particle states on a torus and their relation to the scattering matrix, Nucl. Phys. **B354,** 531 (1991).

[9] L. Liu, K. Orginos, F. K. Guo, C. Hanhart, and U. G. Meissner, Interactions of charmed mesons with light pseudoscalar mesons from lattice QCD and implications on the nature of the $D^*_{s0}(2317)$, Phys. Rev. D **87,** 014508 (2013).

[10] S. Weinberg, Evidence that the deuteron is not an elementary particle, Phys. Rev. **137,** B672 (1965).

[11] V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, and A. E. Kudryavtsev, Evidence that the $a_{0(980)}$ and $f_{0(980)}$ are not elementary particles, Phys. Lett. B **586,** 53 (2004).

[12] A. M. Torres, E. Oset, S. Prelovsek, and A. Ramos, Reanalysis of lattice QCD spectra leading to the $D^*_{s0}(2317)$ and $D^*_{s1}(2460)$, J. High Energy Phys. 05 (2015) 153.

[13] P. Colangelo, F. De Fazio, and A. Ozpineci, Radiative transitions of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$, Phys. Rev. D **72,** 074004 (2005).

[14] M. Cleven, H. W. Griesshammer, F. K. Guo, C. Hanhart, and U. Meissner, Strong and radiative decays of the $D^*_{s0}(2317)$ and $D_{s1}(2460)$, Eur. Phys. J. A **50,** 149 (2014).

[15] R. Louvot *et al.* (Belle Collaboration), Measurement of the Decay $B_s0 \to D_s - \pi^+$ and Evidence for $B_s0 \to D_s \pm K^\pm$ in e^+e_- Annihilation at $\sqrt{s}$ 10.87-GeV, Phys. Rev. Lett. **102,** 021801 (2009).

[16] R. Aaij *et al.* (LHCb Collaboration), Measurement of b-hadron masses, Phys. Lett. B **708,** 241 (2012).

[17] R. Aaij *et al.* (LHCb Collaboration), Implications of LHCb measurements and future prospects, Eur. Phys. J. C **73,** 2373 (2013).

[18] M. Q. Huang, Exclusive semileptonic B_s decays to excited D_s mesons: Search of $D_{sJ}(2317)$ and $D_{sJ}(2460)$, Phys. Rev. D **69,** 114015 (2004).

[19] T. M. Aliev and M. Savci, Analysis of exclusive $B(s) \to D(s0)(2317)$ l anti-nu(l) decay in full QCD, Phys. Rev. D **73,** 114010 (2006).

[20] S. M. Zhao, X. Liu, and S. J. Li, Study on $B(s) \to D(sJ)(2317, 2460)$ l anti-nu semileptonic decays in the CQM model, Eur. Phys. J. C **51,** 601 (2007).

[21] R. H. Li, C. D. Lu, and Y. M. Wang, Exclusive B(s) decays to the charmed mesons $D(s) + (1968, 2317)$ in the standard model, Phys. Rev. D **80,** 014005 (2009).

[22] J. Segovia, C. Albertus, D. R. Entem, F. Fernandez, E. Hernandez, and M. A. Perez-Garcia, Semileptonic B and B_s decays into orbitally excited charmed mesons, Phys. Rev. D **84,** 094029 (2011).

[23] C. Albertus, Weak decays of $\bar{B}_s$ mesons, Phys. Rev. D **89,** 065042 (2014).

[24] A. Datta and P. J. O'donnell, Understanding the nature of $D_{s(2317)}$ and $D_{s(2460)}$ through nonleptonic B decays, Phys. Lett. B **572,** 164 (2003).

[25] W. H. Liang and E. Oset, B^0 and B_s^0 decays into $J/\psi f_0(980)$ and $J/\psi f_0(500)$ and the nature of the scalar resonances, Phys. Lett. B **737,** 70 (2014).

[26] A. Martinez Torres, L. S. Geng, L. R. Dai, B. X. Sun, E. Oset, and B. S. Zou, Study of the $J/\psi \rightarrow \varphi(\omega)f_2(1270)$, $J/\psi \rightarrow \phi(\omega)f_2'(1525)$ and $J/\psi \rightarrow K^{*0}(892\omega)\bar{K}_2^{*0}(1430)$ decays, Phys. Lett. B **680,** 310 (2009).

[27] E. Braaten, Y. Jia, and T. Mehen, The Leading Particle Effect from Heavy Quark Recombination, Phys. Rev. Lett. **89,** 122002 (2002); Phys. Rev. D **66,** 014003 (2002).

[28] D. Gamermann, E. Oset, and B. S. Zou, The radiative decay of $\psi(3770)$ into the predicted scalar state X(3700), Eur. Phys. J. A **41,** 85 (2009).

[29] D. Gamermann, E. Oset, D. Strottman, and M. J. Vicente Vacas, Dynamically generated open and hidden charm meson systems, Phys. Rev. D **76,** 074016 (2007).

[30] E. Braaten, M. Kusunoki, and S. Nussinov, Production of the X(3870) in B Meson Decay by the Coalescence of Charm Mesons, Phys. Rev. Lett. **93,** 162001 (2004); E. Braaten and M. Kusunoki, Production of the X (3870) at the $\Upsilon(4S)$ by the coalescence of charm mesons from B decays, Phys. Rev. D **69,** 114012 (2004).

[31] S. Stone and L. Zhang, Use of $B \rightarrow J/\psi f_0$ Decays to Discern the $q\bar{q}$ or Tetraquark Nature of Scalar Mesons, Phys. Rev. Lett. **111,** 062001 (2013).

[32] W. Ochs, The status of glueballs, J. Phys. G **40,** 043001 (2013).

[33] J. J. Xie, L. R. Dai, and E. Oset, The low lying scalar resonances in the D^0 decays into K_S^0 and $f_0(500)$, $f_0(980)$, $a_0(980)$, Phys. Lett. B **742,** 363 (2015).

[34] J. W. Li, D. S. Du, and C. D. Lu, Determination of $f_0 - \sigma$ mixing angle through $B_s^0 \rightarrow J/\Psi f_0(980)(\sigma)$ decays, Eur. Phys. J. C **72,** 2229 (2012).

[35] J.-P. Dedonder, R. Kaminski, L. Lesniak, and B. Loiseau, Dalitz plot studies of $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays in a factorization approach, Phys. Rev. D **89,** 094018 (2014).

[36] T. Hyodo, D. Jido, and A. Hosaka, Compositeness of dynamically generated states in a chiral unitary approach, Phys. Rev. C **85,** 015201 (2012).

[37] F. Aceti and E. Oset, Wave functions of composite hadron states and relationship to couplings of scattering amplitudes for general partial waves, Phys. Rev. D **86,** 014012 (2012).

[38] C. W. Xiao, F. Aceti, and M. Bayar, The small $K\pi$ component in the K^* wave functions, Eur. Phys. J. A **49,** 22 (2013).

[39] T. Hyodo, Structure and compositeness of hadron resonances, Int. J. Mod. Phys. A **28,** 1330045 (2013).

[40] T. Sekihara, T. Hyodo, and D. Jido, Comprehensive analysis of the wave function of a hadronic resonance and its compositeness, Prog. Theor. Exp. Phys. **2015,** 63D04 (2015).

[41] D. Gamermann, J. Nieves, E. Oset, and E. R. Arriola, Couplings in coupled channels versus wave functions: Application to the X(3872) resonance, Phys. Rev. D **81,** 014029 (2010).

[42] J. Yamagata-Sekihara, J. Nieves, and E. Oset, Couplings in coupled channels versus wave functions in the case of resonances: Application to the two $\Lambda(1405)$ states, Phys. Rev. D **83,** 014003 (2011).

[43] T. Sekihara, T. Hyodo, and D. Jido, Internal structure of resonant $\Lambda(1405)$ state in chiral dynamics, Phys. Rev. C **83,** 055202 (2011).

[44] K. A. Olive *et al.* (Particle Data Group Collaboration), Review of particle physics, Chin. Phys. C **38,** 090001 (2014).

[45] M. Albaladejo, M. Nielsen, and E. Oset, $D_{s0}^{*\pm}(2317)$ and KD scattering from B_s^0 decay, Phys. Lett. B **746,** 305 (2015).