PHYSICAL REVIEW D **92,** 014018 (2015)

Electroproduction of the $N^*(1535)$ nucleon resonance in QCD

I. V. Anikin,[1,2] V. M. Braun,[1] and N. Offen[1]

[1]*Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany*
[2]*Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Russia*

(Received 26 May 2015; published 15 July 2015)

Following the 12 GeV upgrade, a dedicated experiment is planned with the Hall B CLAS12 detector at Jefferson Laboratory, with the aim to study electroproduction of nucleon resonances at high photon virtualities up to $Q^2 = 12\ \mathrm{GeV}^2$. In this work we present a QCD-based approach to the theoretical interpretation of these upcoming results in the framework of light cone sum rules that combine perturbative calculations with dispersion relations and duality. The form factors are thus expressed in terms of $N^*(1535)$ light-front wave functions at small transverse separations, called distribution amplitudes. The distribution amplitudes can therefore be determined from the comparison with the experimental data on form factors and compared to the results of lattice QCD simulations. The results of the corresponding next-to-leading order calculation are presented and compared with the existing data. We find that the form factors are dominated by the twist-four distribution amplitudes that are related to the p-wave three-quark wave functions of the $N^*(1535)$, i.e. to contributions of orbital angular momentum.

DOI: 10.1103/PhysRevD.92.014018 PACS numbers: 12.38.-t, 12.38.Lg, 13.40.Gp, 14.20.Gk

I. INTRODUCTION

It is generally accepted that studies of baryon form factors at large momentum transfer Q^2 give access to the light-front wave functions at small transverse separations between the constituents, called hadron distribution amplitudes (DAs), although perturbative QCD factorization [1–3] does not seem to be applicable for realistic Q^2 accessible in current or planned experiments. The problem is that the leading contribution involves two hard gluon exchanges and is suppressed by the small factor $(\alpha_s/\pi)^2 \sim 0.01$ compared to the "soft" (end point) contributions which are subleading in the power counting in $1/Q^2$ but do not involve small coefficients. Hence the collinear factorization regime is approached very slowly. Model calculations suggest that "soft" contributions play the dominant role at present energies. Taking into account soft contributions is challenging because they involve a nontrivial overlap of nonperturbative wave functions of the initial and the final state hadrons, and are not factorizable, i.e. cannot be simplified further in terms of simpler quantities.

In this situation the question what exactly do we learn from the studies of form factors is far from trivial. One existing description is to introduce more complicated, transverse-momentum dependent (TMD) quark distributions, taking advantage of Sudakov suppression of large transverse separations, following the technique suggested initially by Li and Sterman [4] for the pion form factor. Another approach that we advocate in this work is to calculate the soft contributions to the form factors as an expansion in terms of nucleon DAs of increasing twist using dispersion relations and duality. This method is known as light cone sum rules (LCSRs) [5] and provides one with the most direct relation of the hadron form factors and DAs that is available at present, with no other nonperturbative parameters.

One attractive feature of the LCSR formalism is that there is no double counting: perturbative QCD contributions [1–3] appear as part of the higher-order perturbative corrections to the LCSRs. This matching of LCSR and perturbative QCD factorization descriptions is shown by explicit calculation and discussed in great detail in Ref. [6] for the case of the pion form factor. For baryon form factors the corresponding terms first occur at the next-to-next-to-leading order level (two gluons) and are beyond the accuracy of the present calculation. Since such terms are suppressed by an additional $\alpha_s/(2\pi)$ factor, it is unlikely that they play a significant role at accessible energies.

The LCSR approach has been used successfully for the calculations of pion electromagnetic and also weak B-decay form factors, see Refs. [6–8] for several recent state-of-the-art calculations. The LCSRs for baryon form factors are more complicated and recent. The first applications for the nucleon electromagnetic form factors were in Refs. [9,10]. Several further studies aimed at finding an optimal nucleon interpolation current [10–13] and extending this technique to other elastic or transition form factors of interest. LCSRs for the axial nucleon form factor were presented in [10,12,14], for the scalar form factor in [14] and tensor form factor in [15]. A generalization to the full baryon octet was considered, e.g. in [16]. Application of the same technique to $N\gamma\Delta$ transitions was suggested in [12,17] and to pion production at threshold in [18]. LCRSs for weak baryon decays $\Lambda_b \to p, \Lambda\ell\nu_\ell$ etc. were studied in [19–22], etc. In the early work only the leading order (LO) contributions to the coefficient functions in the LCSRs have been taken into account. The first complete next-to-leading order (NLO) analysis was done for the electromagnetic nucleon form factors in Ref. [23], and the results appear to be consistent with the constraints on nucleon DAs from lattice calculations [24]. The picture emerging from these studies suggests that the momentum fraction distribution of the valence quarks in the proton is rather broad, with ∼40% of the momentum carried by the u-quark that carries proton helicity, and approximately symmetric to the interchange of the remaining quarks.

Our study is motivated by the dedicated experiment planned with the Hall B CLAS12 detector at Jefferson Laboratory following the 12 GeV upgrade, with the aim to study electroproduction of nucleon resonances at high photon virtualities up to $Q^2 = 12\ \mathrm{GeV}^2$ [25]. The corresponding form factors can be calculated using the LCSR machinery in terms of the DAs of nucleon resonances. Turning this relation around, information on the DAs of resonances can be extracted from the comparison of the LCSR calculations with the experimental data on form factors and compared to the constraints that can come, eventually, from lattice QCD simulations. This program was suggested in Ref. [26], and an exploratory study was made there for the particular case of electroproduction of the lowest negative parity $N^*(1535)$ resonance. In our paper we elaborate on this proposal. Learning about quark distributions in nucleon resonances is an exciting possibility, since existing QCD calculations of resonance properties, e.g. on the lattice, rarely go beyond the mass spectrum.

The case of $N^*(1535)$ is special because the classification and the structure of the light-front wave functions for the states with opposite parity is almost identical. Hence the LCSRs for the corresponding electroproduction form factors are very similar to the LCSRs for electromagnetic nucleon form factors. In particular the NLO expressions derived in Ref. [23] can be overtaken with relatively minor modifications. A detailed analysis of these NLO LCSRs is the main goal of this work.

The presentation is organized as follows. The electroproduction form factors are introduced and the structure of the corresponding LCSRs is explained in Sec. II. Section III contains a detailed numerical analysis and comparison with the existing experimental data, and Sec. IV is reserved for a summary and outlook. A large Appendix A contains a short review of the three-quark light-front wave functions of $J^P = (\frac{1}{2})^-$ nucleon resonances, their relation to DAs, and also explains our conventions. Appendix B contains a simple parametrization of the Q^2 dependence of the coefficient functions in the LCSRs.

II. ELECTROPRODUCTION FORM FACTORS AND LIGHT CONE SUM RULES

The matrix element of the electromagnetic current j_ν^{em} between spin-1/2 states of opposite parity can be parametrized in terms of two independent form factors, which can be chosen as

$$\langle N^*(P')|j_\nu^{\mathrm{em}}|N(P)\rangle = \bar{u}_{N^*}(P')\gamma_5\Gamma_\nu u_N(P),$$
$$\Gamma_\nu = \frac{G_1(q^2)}{m_N^2}(\not{q} q_\nu - q^2\gamma_\nu) - i\frac{G_2(q^2)}{m_N}\sigma_{\nu\rho}q^\rho, \tag{1}$$

where $q = P' - P$ is the momentum transfer. In what follows we use the standard notation $Q^2 = -q^2$. The helicity amplitudes $A_{1/2}(Q^2)$ and $S_{1/2}(Q^2)$ for the electroproduction of $N^*(1535)$ can be expressed in terms of the form factors [27]:

$$A_{1/2} = eB[Q^2G_1(Q^2) + m_N(m_{N^*} - m_N)G_2(Q^2)],$$
$$S_{1/2} = \frac{eBC}{\sqrt{2}}[(m_N - m_{N^*})G_1(Q^2) + m_N G_2(Q^2)]. \tag{2}$$

Here $e = \sqrt{4\pi\alpha}$ is the elementary charge and B, C are kinematic factors defined as

$$B = \sqrt{\frac{Q^2 + (m_{N^*} + m_N)^2}{2m_N^5(m_{N^*}^2 - m_N^2)}},$$
$$C = \sqrt{1 + \frac{(Q^2 - m_{N^*}^2 + m_N^2)^2}{4Q^2 m_{N^*}^2}}. \tag{3}$$

The basic object of the LCSR approach to baryon form factors [9,10] is the correlation function

$$T_\nu(P,q) = i\int dx e^{-iqx}\langle N^*(P')|T\{j_\nu(x)\eta_N(0)\}|0\rangle \tag{4}$$

in which j represents the electromagnetic (or weak) probe and η_N is a suitable local operator with nucleon quantum numbers. The N^* resonance is explicitly represented by its state vector $\langle N^*(P')|$, see a schematic representation in Fig. 1. The LCSR is obtained by comparing (matching) two different representations for the correlation function. On the one hand, when both the momentum transfer $q^2 = -Q^2$ and the momentum $P^2 = (P' - q)^2$ flowing in the η_N vertex are large and negative, the main contribution to the integral comes from the light cone region $x^2 \to 0$ and can be studied using the operator product expansion (OPE)

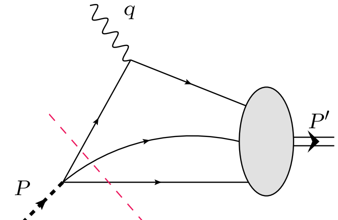

FIG. 1 (color online). Schematic structure of the light cone sum rule for electroproduction form factors.

of the time-ordered product $T\{j(x)\eta_N(0)\}$. The singularity at $x^2 \to 0$ of a particular contribution is governed by the twist of the relevant composite operator whose matrix element $\langle N^*|\ldots|0\rangle$ is related to the N^* DA. On the other hand, one can represent the answer in form of the dispersion integral in P^2 and define the nucleon contribution by the cutoff in the quark-antiquark invariant mass, the so-called interval of duality s_0 (or continuum threshold). The main role of the interval of duality is that it does not allow large momenta $|k^2| > s_0$ to flow through the η_N-vertex; to the lowest order $O(\alpha_s^0)$ one obtains a purely soft contribution to the form factor as a sum of terms ordered by twist of the relevant operators and hence including both the leading- and the higher-twist nucleon DAs. Note that the contribution of higher-twist DAs is suppressed by powers of the continuum threshold (or by powers of the Borel parameter after applying the usual QCD sum rule machinery), but not by powers of Q^2, the reason being that soft contributions are not constrained to small transverse separations.

The "plus" spinor projection (A8) of the correlation function (4) involving the "plus" component of the electromagnetic current can be parametrized in terms of two invariant functions

$$\Lambda_+ T_+ = p_+\{m_N \mathcal{A}(Q^2, P'^2) + \not{q}_\perp \mathcal{B}(Q^2, P'^2)\} N^+(P), \tag{5}$$

where $Q^2 = -q^2$ and $P'^2 = (P-q)^2$. The correlation functions $\mathcal{A}(Q^2, P'^2)$ and $\mathcal{B}(Q^2, P'^2)$ can be calculated in QCD in terms of N^* DAs for sufficiently large Euclidean momenta $Q^2, -P'^2 \gtrsim 1\ \mathrm{GeV}^2$ using OPE. Schematically,

$$\begin{aligned} \mathcal{A}(Q^2, P'^2) &= \sum_k \int [dx] a_k(Q^2, P'^2, x_i, \mu_F^2) F_k(x_i, \mu_F^2), \\ \mathcal{B}(Q^2, P'^2) &= \sum_k \int [dx] b_k(Q^2, P'^2, x_i, \mu_F^2) F_k(x_i, \mu_F^2), \end{aligned} \tag{6}$$

where the sum goes over all existing DAs, $F_k \in \{V_k, A_k, T_k, S_k, P_k\}$ defined in Eq. (A21), the integration goes over quark momentum fractions and μ_F stands for the factorization scale. The coefficient functions $a_k(Q^2, P'^2, x_i, \mu_F^2)$ and $b_k(Q^2, P'^2, x_i, \mu_F^2)$ are known to the NLO accuracy for twist-three and twist-four DAs [23], and to LO for twist-five and twist-six. In principle this expansion also contains contributions of four-particle DAs with an additional gluon, five-particle with two gluons or a quark-antiquark pair, etc. Such contributions start at twist-four and they are not included in the present calculation because the corresponding DAs are very poorly known (see, however, Ref. [28]). It turns out that the coefficient functions are the same for the states with negative and positive parity, $N^*(1535)$ and the nucleon, if the definitions are chosen as explained in Appendix A. Thus we are able to use the NLO expressions for the electromagnetic nucleon form factors obtained in [23] with trivial modifications, e.g. replacing nucleon mass m_N by m_{N^*}. One difference is that, because of the larger mass, corrections of the type $m_{N^*}^2/Q^2$ become much larger and numerically significant. For this reason in this work we use complete expressions for the LO coefficient functions from Ref. [10] rather than the corresponding expressions from Ref. [23] where the expansion in powers of $m_{N^*}^2/Q^2$ was truncated to match the accuracy of the calculated NLO corrections.

The results of the QCD calculation in Euclidean region can be presented in the form of a dispersion relation

$$\begin{aligned} \mathcal{A}^{\mathrm{QCD}}(Q^2, P'^2) &= \frac{1}{\pi}\int_0^\infty \frac{ds}{s-P'^2} \mathrm{Im}\mathcal{A}^{\mathrm{QCD}}(Q^2, s) + \cdots \\ \mathcal{B}^{\mathrm{QCD}}(Q^2, P'^2) &= \frac{1}{\pi}\int_0^\infty \frac{ds}{s-P'^2} \mathrm{Im}\mathcal{B}^{\mathrm{QCD}}(Q^2, s) + \cdots \end{aligned} \tag{7}$$

where the ellipses indicate possible subtractions. The same correlation functions can be written in terms of physical spectral densities that contain a nucleon (proton) pole at $P'^2 \to m_N^2$, nucleon resonances and the continuum. The nucleon contribution is, obviously, proportional to the electroproduction form factors of interest, whereas for higher mass states one can use quark-hadron duality:

$$\begin{aligned} \mathcal{A}^{\mathrm{phys}}(Q^2, P'^2) &= \frac{2\lambda_1^N Q^2 G_1(Q^2)}{m_N m_{N^*}(m_N^2 - P'^2)} \\ &\quad + \frac{1}{\pi}\int_{s_0}^\infty \frac{ds}{s-P'^2} \mathrm{Im}\mathcal{A}^{\mathrm{QCD}}(Q^2, s) + \cdots \\ \mathcal{B}^{\mathrm{phys}}(Q^2, P'^2) &= \frac{-2\lambda_1^N G_2(Q^2)}{m_N^2 - P'^2} \\ &\quad + \frac{1}{\pi}\int_{s_0}^\infty \frac{ds}{s-P'^2} \mathrm{Im}\mathcal{B}^{\mathrm{QCD}}(Q^2, s) + \cdots \end{aligned} \tag{8}$$

where $s_0 \simeq (1.5\ \mathrm{GeV})^2$ is the interval of duality (also called continuum threshold). Matching the two above representations and making the Borel transformation that eliminates subtractions constants

$$\frac{1}{s - P'^2} \to e^{-s/M^2} \tag{9}$$

one obtains the sum rules

$$\begin{aligned} \frac{2\lambda_1^N Q^2 G_1(Q^2)}{m_N m_{N^*}} &= \frac{1}{\pi}\int_0^{s_0} ds e^{(m_N^2 - s)/M^2} \mathrm{Im}\mathcal{A}^{\mathrm{QCD}}(Q^2, s), \\ -2\lambda_1^N G_2(Q^2) &= \frac{1}{\pi}\int_0^{s_0} ds e^{(m_N^2 - s)/M^2} \mathrm{Im}\mathcal{B}^{\mathrm{QCD}}(Q^2, s). \end{aligned} \tag{10}$$

The dependence on the Borel parameter M^2 is unphysical and has to disappear in the full QCD calculation. It can be used to estimate theoretical uncertainties.

III. NUMERICAL ANALYSIS

Main nonperturbative input to the LCSRs for electroproduction form factors is provided by the DAs of nucleon resonances that can be parameterized by two normalization constants f_{N^*}, $\lambda_1^{N^*}$ and a set of shape parameters φ_{nk}, η_{nk} corresponding to contributions of local operators of increasing dimension, see Eqs. (A27) and (A31). The dependence of the form factors on these parameters is linear so that the results can conveniently be presented as

$$G_1(Q^2) = \frac{\lambda_1^{N^*}}{\lambda_1^N}\Big\{g_1^{00}(Q^2) + g_1^{10}(Q^2)\eta_{10} + g_1^{11}(Q^2)\eta_{11} + \frac{f_{N^*}}{\lambda_1^{N^*}}[f_1^{00}(Q^2) + f_1^{10}(Q^2)\varphi_{10} + f_1^{11}(Q^2)\varphi_{11} + \cdots]\Big\} \quad (11)$$

and similarly

$$G_2(Q^2) = \frac{\lambda_1^{N^*}}{\lambda_1^N}\Big\{g_2^{00}(Q^2) + g_2^{10}(Q^2)\eta_{10} + g_2^{11}(Q^2)\eta_{11} + \frac{f_{N^*}}{\lambda_1^{N^*}}[f_2^{00}(Q^2) + f_2^{10}(Q^2)\varphi_{10} + f_2^{11}(Q^2)\varphi_{11} + \cdots]\Big\} \quad (12)$$

where the ellipses stand for the contributions of second-order polynomials in the leading-twist DAs (A27), terms in φ_{20}, φ_{21}, φ_{22}. The coefficient functions $f_{1,2}^{n,k}(Q^2)$ and $g_{1,2}^{n,k}(Q^2)$ are given by very cumbersome analytic expressions [23] and depend implicitly on the masses of N^* and the nucleon, the continuum threshold s_0, Borel parameter M^2, QCD coupling $\alpha_s(\mu_F)$ and the factorization scale μ_F. Note that, e.g., $f_1^{00}(Q^2)$ includes the sum of contributions of the asymptotic leading-twist DA and the corresponding Wandzura-Wilczek terms in the higher-twist DAs, see Appendix A and Ref. [23] for more details. Note also that the DA Ξ_4 (A25) corresponding to the $\ell_z = 2$ component of the light-front three-quark wave function does not contribute to the LCSRs for our choice of the nucleon interpolating current.

Calculations in this work are done for the "standard" choice of the specific LCSR parameters: continuum threshold $s_0 = (1.5\ \mathrm{GeV})^2$, Borel parameter $M^2 = 2\ \mathrm{GeV}^2$ and factorization (and renormalization) scale $\mu_F^2 = 2\ \mathrm{GeV}^2$. The dependence on these parameters is rather mild; in particular varying the Borel parameter in the range 1.5–2 GeV^2 induces an overall variation of form factor of the order of 10% so that, e.g., the ratio G_2/G_1 is largely unchanged.

The resonance mass corrections enter the LCSRs in a complicated way, as terms in $m_{N^*}^2/Q^2$ and $m_{N^*}^2/s_0$. The latter ones do not decrease at large momentum transfers and in an ideal case have to be resummed to all orders. The corresponding expression exists for the LO LCSRs [10] but not for the NLO corrections. In order to minimize this mismatch we have rescaled the $\mathcal{O}(\alpha_s)$ contributions calculated in [23] by the ratio of the corresponding LO terms calculated with account for $m_{N^*}^2$ corrections and putting $m_{N^*}^2$ to zero. For the numerically important contributions this rescaling corresponds to a reduction of the NLO correction by 10%–20%.

Existing information on the DAs of negative parity resonances is very scarce. The results of the recent lattice calculation [24] are presented in Table I. The most interesting feature of these results is that the corrections to the asymptotic leading twist DAs have alternating signs for the lattice states with increasing mass. In particular the twist-three DA of $N^*(1535)$ has a very small value at the origin and is approximately antisymmetric with respect to the exchange of the two valence quarks forming a scalar "diquark", whereas the DA of $N^*(1535)$ is symmetric and similar in shape to the nucleon DA, see Fig. 9 in Ref. [24]. These results are still exploratory and have to be taken with caution because identification of lattice states with particular physical resonances is not obvious and requires further study. Even with this uncertainty, the lattice values are very helpful as knowing the order of magnitude of the parameters allows one to establish a hierarchy of different contributions to the LCSR.

As an illustration, the NLO LCSR result for the form factors at $Q^2 = 2\ \mathrm{GeV}^2$ normalized to the dipole formula

TABLE I. Parameters of the $N^*(1535)$ distribution amplitudes at the scale $\mu^2 = 2\ \mathrm{GeV}^2$. For the lattice results [24] only statistical errors are shown. The set of parameters indicated as LCSR (1) corresponds to the fit to the form factors $G_1(Q^2)$ and $G_2(Q^2)$ extracted from the measurements of helicity amplitudes in Ref. [29] adding the errors in quadrature. The set of parameters indicated as LCSR (2) is obtained from the fit to helicity amplitudes including all available data at $Q^2 \geq 1.7\ \mathrm{GeV}^2$ [29–32].

Method	$\lambda_1^N/\lambda_1^{N^*}$	$f_{N^*}/\lambda_1^{N^*}$	φ_{10}	φ_{11}	φ_{20}	φ_{21}	φ_{22}	η_{10}	η_{11}	Reference
LCSR (1)	0.633	0.027	0.36	−0.95	0	0	0	0.00	0.94	This work
LCSR (2)	0.633	0.027	0.37	−0.96	0	0	0	−0.29	0.23	This work
Lattice	0.633(43)	0.027(2)	0.28(12)	−0.86(10)	1.7(14)	−2.0(18)	1.7(26)			[24]

$$D(Q^2) = \frac{1}{(1+Q^2/a)^2}, \qquad a = 0.71\ \mathrm{GeV}^2 \quad (13)$$

can be written as follows:

$$\frac{G_1^{\rm NLO}(Q^2)}{D(Q^2)} = \frac{\lambda_1^{N^*}}{\lambda_1^N}[0.666 - 2.18\eta_{10} + 0.86\eta_{11} - 0.69\tilde f_{N^*} - 1.76\tilde f_{N^*}\varphi_{10} + 1.05\tilde f_{N^*}\varphi_{11} + 1.3\tilde f_{N^*}\varphi_{20} + 0.66\tilde f_{N^*}\varphi_{21} - 0.06\tilde f_{N^*}\varphi_{22}],$$

$$\frac{G_2^{\rm NLO}(Q^2)}{D(Q^2)} = \frac{\lambda_1^{N^*}}{\lambda_1^N}[-0.466 + 1.84\eta_{10} + 0.06\eta_{11} - 0.82\tilde f_{N^*} - 1.06\tilde f_{N^*}\varphi_{10} - 1.08\tilde f_{N^*}\varphi_{11} + 2.6\tilde f_{N^*}\varphi_{20} + 1.5\tilde f_{N^*}\varphi_{21} + 0.39\tilde f_{N^*}\varphi_{22}]$$

where we use a notation $\tilde f_{N^*}$ for the ratio of twist-three and twist-four couplings

$$\tilde f_{N^*} = \frac{f_{N^*}}{\lambda_1^{N^*}} = 0.027(2) \quad [24]. \qquad (14)$$

For comparison, the similar decomposition of the form factors for the LO LCSRs [26] for the same value $Q^2 = 2\ \mathrm{GeV}^2$ reads

$$\frac{G_1^{\rm LO}(Q^2)}{D(Q^2)} = \frac{\lambda_1^{N^*}}{\lambda_1^N}[0.816 - 2.02\eta_{10} + 0.88\eta_{11} - 0.59\tilde f_{N^*} - 1.60\tilde f_{N^*}\varphi_{10} + 1.19\tilde f_{N^*}\varphi_{11} + 1.26\tilde f_{N^*}\varphi_{20} + 0.70\tilde f_{N^*}\varphi_{21} + 0.12\tilde f_{N^*}\varphi_{22}],$$

$$\frac{G_2^{\rm LO}(Q^2)}{D(Q^2)} = \frac{\lambda_1^{N^*}}{\lambda_1^N}[-0.466 + 1.84\eta_{10} + 0.06\eta_{11} - 1.19\tilde f_{N^*} - 0.78\tilde f_{N^*}\varphi_{10} + 3.82\tilde f_{N^*}\varphi_{11} + 2.9\tilde f_{N^*}\varphi_{20} + 1.6\tilde f_{N^*}\varphi_{21} + 0.28\tilde f_{N^*}\varphi_{22}]$$

so that the NLO corrections are significant.

For convenience we provide a simple parametrization for the coefficient functions $f_{1,2}^{nk}$, $g_{1,2}^{nk}$ (11), (12) as functions of Q^2 in Appendix B. This parametrization was obtained for the region of momentum transfers $2\ \mathrm{GeV}^2 < Q^2 < 12\ \mathrm{GeV}^2$ and should not be used outside this interval. In particular we found that the mass corrections $\sim m_{N^*}^2/Q^2$ become very large for $Q^2 < 2\ \mathrm{GeV}^2$ so that the LCSRs become unstable (and not reliable). In general, different contributions to the LCSRs are distinguished by their Q^2 dependence so that one needs a sufficient lever arm in Q^2 to determine several of them simultaneously.

Since the existing data for $Q^2 \geq 1.5$–$2\ \mathrm{GeV}^2$ are very limited, we put in this work all second-order coefficients in the leading-twist DAs to zero, $\varphi_{20} = \varphi_{21} = \varphi_{22} = 0$, used central lattice values for f_{N^*} and $\lambda_1^{N^*}$, and constrained $\varphi_{10}, \varphi_{11}$ to the lattice values within the given error bars. In this way we are left, essentially, with two free parameters —η_{10} and η_{11}. We expect that much more data will become available after the 12 GeV upgrade at Jefferson Laboratory where a dedicated experiment is planned to study electroproduction of nucleon resonances at high photon virtualities up to $Q^2 = 12\ \mathrm{GeV}^2$ [25].

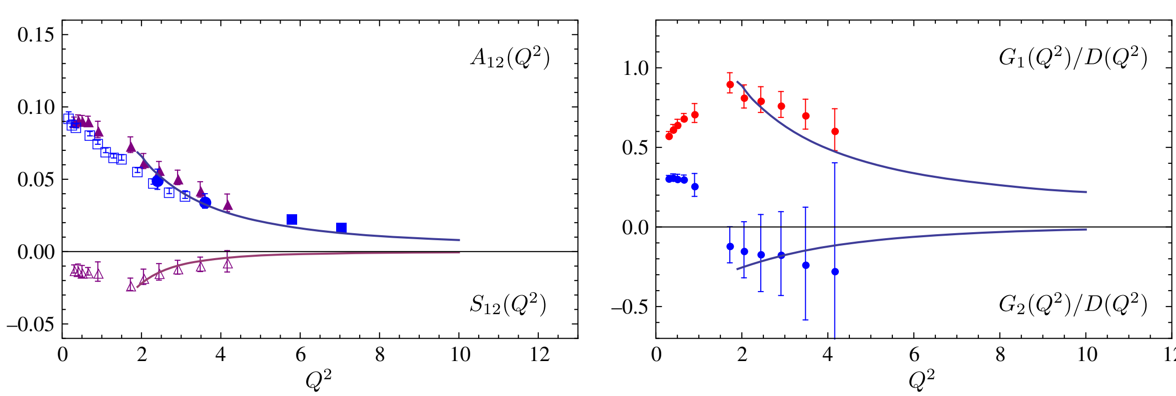

FIG. 2 (color online). Helicity amplitudes A_{12} and S_{12} for electroproduction of $N^*(1535)$ (left panel) and the form factors $G_1(Q^2)$, $G_2(Q^2)$, normalized to the dipole formula (right panel). Experimental data on the left panel are taken from [30] (empty squares) [31] (filled squares) [32] (filled circles) and [29] (triangles). The form factors on the right panel are calculated from the data [29] on helicity amplitudes adding the errors in quadrature. The curves show the results of the NLO LCSR fit to the form factors $G_1(Q^2)$ and $G_2(Q^2)$ for $Q^2 \geq 1.7\ \mathrm{GeV}^2$ with parameters of the $N^*(1535)$ DAs specified in the first line in Table I.

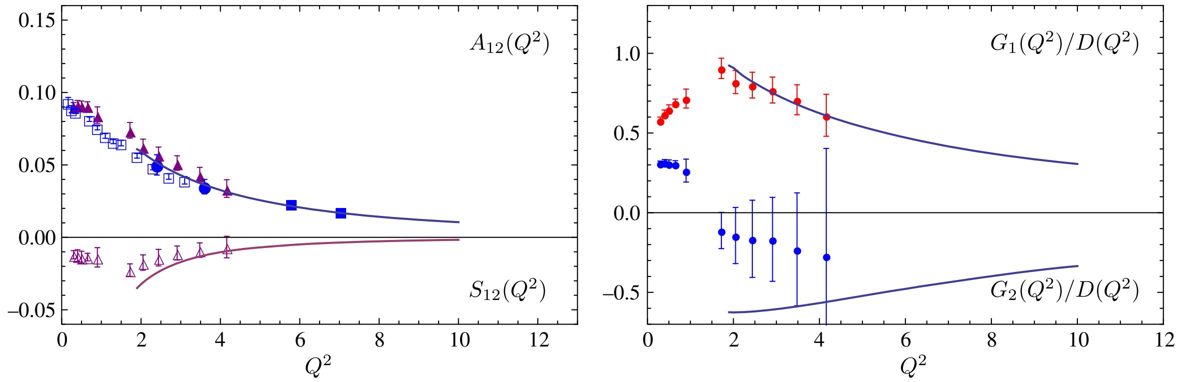

FIG. 3 (color online). The same as in Fig. 2 but for the fit to helicity amplitudes A_{12}, S_{12} including all available data at $Q^2 \geq 1.7\ \mathrm{GeV}^2$. The fitted parameters of the $N^*(1535)$ DAs are specified in the second line in Table I.

Information on the electrocouplings of nucleon resonances at large momentum transfers is obtained by studying electroproduction of π and η mesons in the respective resonance region [29–32]. The results are usually presented for the helicity amplitudes, and in earlier work only the larger one, $A_{12}(Q^2)$, was studied for large momentum transfers. The latest study [29] also includes the results on $S_{12}(Q^2)$ up to $Q^2 = 4.16\ \mathrm{GeV}^2$ allowing us to extract from these data the Dirac-like and Pauli-like transition form factors $G_1(Q^2)$ and $G_2(Q^2)$ (1) that are more relevant for QCD studies. In this extraction we assumed that the errors for helicity amplitudes given in Ref. [29] are uncorrelated and added them in quadrature. The results are shown in Figs. 2 and 3 on the right panels; it is seen that the Pauli-like form factor changes sign and becomes negative at large Q^2, although the errors are quite large.

Two different LCSR fits of the experimental data are shown in Figs. 2 and 3. The difference is that in Fig. 2 the fit is done to the form factors extracted from the data on helicity amplitudes reported in Ref. [29], and in Fig. 3 we make a fit to the data on helicity amplitudes $A_{12}(Q^2)$ and $S_{12}(Q^2)$ themselves including all existing data for $Q^2 \geq 1.7\ \mathrm{GeV}^2$. In the second case the fit is driven by the data [30–32] on $A_{12}(Q^2)$ that have smaller errors and not entirely consistent with [29], so that a worse description of the form factors in this fit is not a surprise. The corresponding parameters are listed in Table I.

Because of the small value of the leading twist normalization constant suggested by lattice calculations (14), the results for $A_{12}(Q^2)$ and $G_1(Q^2)$ prove to be almost insensitive to the leading twist DA of the $N^*(1535)$ resonance and are dominated by the twist-four contributions corresponding to the p-wave parts of the three-quark light-front wave functions (see Appendix A). Moreover, sensitivity of the results to the shape parameters of the twist-four DAs, η_{10} and η_{11}, is rather mild, cf. two last columns in the first and the second line in Table I. Thus $A_{12}(Q^2)$ and $G_1(Q^2)$ are both sensitive mostly to the ratio of the normalization constants $\lambda_1^N/\lambda_1^{N^*}$ which we fix to the lattice value 0.633 [24]. The LCSR predictions for these observables are very stable, and the agreement of the existing data with the normalization suggested by lattice calculations is encouraging.

The $S_{12}(Q^2)$ amplitude and especially the Pauli-like form factor $G_2(Q^2)$ are much more sensitive to the nonperturbative input and in particular to the shape parameters of the twist-four DAs, compare Figs. 2 and 3. Also the leading-twist contributions play some role in this case because of strong cancellations. More precise data and a larger interval in Q^2 are needed to make this comparison quantitative.

IV. CONCLUSIONS AND OUTLOOK

In this work we argue that the LCSR approach can provide one with quantitative information on the wave functions of nucleon resonances at short distances. The basic idea behind this technique is that soft Feynman contributions to the form factors are calculated in terms of small transverse distance quantities using dispersion relations and duality. The form factors are thus expressed in terms of light-front wave functions at small transverse separations, called DAs, without additional parameters. Alternatively, the distribution amplitudes can be extracted from the comparison with the experimental data on form factors and compared to the results of lattice QCD simulations or other nonperturbative approaches based on, e.g., QCD sum rules or Dyson-Schwinger equations. The results of the corresponding NLO calculation for the particular case of the $N^*(1535)$ resonance are presented and compared with the existing data. We find that the form factors are dominated by twist-four DAs that are related to the p-wave three-quark wave functions, i.e., to the distribution of orbital angular momentum.

Interestingly enough the LCSRs have the same form for spin-1/2 resonances of both parities so that apart from the (calculable) effects of resonance mass corrections the difference in observed form factors of, say, $N^*(1535)$ and $N^*(1650)$ can be attributed to the difference in the wave functions, which is of major interest. The differences between nucleon elastic form factors and electroexcitation of the Roper resonance can be studied in a similar manner; however, it is likely that in the latter case interpretation of the results may require a better understanding and more sophisticated models of twist-five DAs than are available at present.

ACKNOWLEDGMENTS

V. B. is grateful to A. Belitsky for the correspondence concerning the relation of light-front wave functions and DAs. The work by I. A. was partially supported by the Heisenberg-Landau Program of the German Research Foundation (DFG).

APPENDIX A: LIGHT-FRONT WAVE FUNCTIONS AND DAS OF $J^P = (\frac{1}{2})^-$ NUCLEON RESONANCES

In the light-front description [3] a hadron is represented by the superposition of Fock states with different number of partons. Restricting ourselves to the three-quark (valence) components we view, e.g., the proton with positive helicity as a superposition of states with different values of the quark orbital angular momentum projection on the direction of motion, $\ell_z = -1, 0, 1, 2$,

$$|N\uparrow\rangle = \sum_{\ell_z} |N\uparrow\rangle^{\ell_z}_{uud}. \tag{A1}$$

A nonzero value of ℓ_z accounts for the mismatch between the proton helicity and the sum of helicities of the valence quarks λ_i so that $1/2 = \lambda_1 + \lambda_2 + \lambda_3 + \ell_z$. The four different contributions can be written in terms of six independent scalar light-front wave functions as [33–35]

$$\begin{aligned}
|N\uparrow\rangle^{\ell_z=0}_{uud} &= \frac{\epsilon^{ijk}}{\sqrt{6}} \int \frac{[dx][dk_\perp]}{\sqrt{x_1x_2x_3}} [\psi^{(0)}_{N;1}(1,2,3) + i\epsilon^{\alpha\beta} k^\perp_{1\alpha} k^\perp_{2\beta} \psi^{(0)}_{N;2}(1,2,3)] b^{\dagger i}_{u\uparrow}(1)(b^{\dagger j}_{u\downarrow}(2) b^{\dagger k}_{d\uparrow}(3) - b^{\dagger j}_{d\downarrow}(2) b^{\dagger k}_{u\uparrow}(3))|0\rangle, \\
|N\uparrow\rangle^{\ell_z=1}_{uud} &= \frac{\epsilon^{ijk}}{\sqrt{6}} \int \frac{[dx][dk_\perp]}{\sqrt{x_1x_2x_3}} [k^\perp_1 \psi^{(1)}_{N;1}(1,2,3) + k^\perp_2 \psi^{(1)}_{N;2}(1,2,3)](b^{\dagger i}_{u\uparrow}(1) b^{\dagger j}_{u\downarrow}(2) b^{\dagger k}_{d\downarrow}(3) - b^{\dagger i}_{d\uparrow}(1) b^{\dagger j}_{u\downarrow}(2) b^{\dagger k}_{u\downarrow}(3))|0\rangle, \\
|N\uparrow\rangle^{\ell_z=-1}_{uud} &= \frac{\epsilon^{ijk}}{\sqrt{6}} \int \frac{[dx][dk_\perp]}{\sqrt{x_1x_2x_3}} [\bar{k}^\perp_1 \psi^{(-1)}_{N}(1,2,3)] b^{\dagger i}_{u\uparrow}(1)(b^{\dagger j}_{u\uparrow}(2) b^{\dagger k}_{d\uparrow}(3) - b^{\dagger j}_{d\uparrow}(2) b^{\dagger k}_{u\uparrow}(3))|0\rangle, \\
|N\uparrow\rangle^{\ell_z=2}_{uud} &= \frac{\epsilon^{ijk}}{\sqrt{6}} \int \frac{[dx][dk_\perp]}{\sqrt{x_1x_2x_3}} [k^\perp_1 k^\perp_3 \psi^{(2)}_{N}(1,2,3)](b^{\dagger i}_{u\downarrow}(1) b^{\dagger j}_{u\downarrow}(2) b^{\dagger k}_{d\downarrow}(3) - b^{\dagger i}_{u\downarrow}(1) b^{\dagger j}_{d\downarrow}(2) b^{\dagger k}_{u\downarrow}(3))|0\rangle.
\end{aligned} \tag{A2}$$

Here $b^{\dagger i}_{u\uparrow}(1)$ etc. are creation operators for the quarks of specific flavor with positive $\uparrow$ or negative $\downarrow$ helicity; the argument (1) stands for the dependence on longitudinal momentum fractions and transverse momenta of the given quark, i.e. $u_{\uparrow i}(1) = u_{\uparrow i}(x_1, k^\perp_1)$, and so on. We use the notation for transverse momenta

$$\begin{aligned}
k^\perp &= k^\perp_x + ik^\perp_y, \\
\bar{k}^\perp &= k^\perp_x - ik^\perp_y.
\end{aligned} \tag{A3}$$

The light-front wave functions $\psi^{(\ell_z)}_{N;i}(1,2,3)$ depend on momentum fractions x_i and transverse momenta squared $|k_{\perp,i}|^2 = k_i \bar{k}_i$ of all partons. The integration measure is chosen as [28]

$$[dx] = \prod_{k=1}^{3} dx_k \delta\left(1 - \sum x_k\right), \tag{A4}$$

and

$$[dk_\perp] = \frac{1}{4(2\pi)^6} \prod_{k=1}^{3} d^2 k^\perp_k \delta^{(2)}\left(\sum k^\perp_i\right). \tag{A5}$$

The proton light cone DAs, in turn, are defined as matrix elements of gauge-invariant nonlocal operators with the three quark fields separated by a lightlike distance. Standard decomposition [36] involves 24 invariant functions:

$$
\begin{aligned}
&4\langle 0|\epsilon^{ijk}u_\alpha^i(a_1n)u_\beta^j(a_2n)d_\gamma^k(a_3n)|N(P,\lambda)\rangle\\
&\quad= S_1^N m_N C_{\alpha\beta}(\gamma_5 u_N^+)_\gamma + S_2^N m_N C_{\alpha\beta}(\gamma_5 u_N^-)_\gamma + P_1^N m_N(\gamma_5 C)_{\alpha\beta}(u_N^+)_\gamma + P_2^N m_N(\gamma_5 C)_{\alpha\beta}(u_N^-)_\gamma\\
&\qquad + V_1^N(\not{p}C)_{\alpha\beta}(\gamma_5 u_N^+)_\gamma + V_2^N(\not{p}C)_{\alpha\beta}(\gamma_5 u_N^-)_\gamma + \frac{1}{2}V_3^N m_N(\gamma_\perp C)_{\alpha\beta}(\gamma^\perp\gamma_5 u_N^+)_\gamma\\
&\qquad + \frac{1}{2}V_4^N m_N(\gamma_\perp C)_{\alpha\beta}(\gamma^\perp\gamma_5 u_N^-)_\gamma + V_5^N\frac{m_N^2}{2pn}(\not{n}C)_{\alpha\beta}(\gamma_5 u_N^+)_\gamma + \frac{m_N^2}{2pn}V_6^N(\not{n}C)_{\alpha\beta}(\gamma_5 u_N^-)_\gamma\\
&\qquad + A_1^N(\not{p}\gamma_5 C)_{\alpha\beta}(u_N^+)_\gamma + A_2^N(\not{p}\gamma_5 C)_{\alpha\beta}(u_N^-)_\gamma + \frac{1}{2}A_3^N m_N(\gamma_\perp\gamma_5 C)_{\alpha\beta}(\gamma^\perp u_N^+)_\gamma\\
&\qquad + \frac{1}{2}A_4^N m_N(\gamma_\perp\gamma_5 C)_{\alpha\beta}(\gamma^\perp u_N^-)_\gamma + A_5^N\frac{m_N^2}{2pn}(\not{n}\gamma_5 C)_{\alpha\beta}(u_N^+)_\gamma + \frac{m_N^2}{2pn}A_6^N(\not{n}\gamma_5 C)_{\alpha\beta}(u_N^-)_\gamma\\
&\qquad + T_1^N(i\sigma_{\perp p}C)_{\alpha\beta}(\gamma^\perp\gamma_5 u_N^+)_\gamma + T_2^N(i\sigma_{\perp p}C)_{\alpha\beta}(\gamma^\perp\gamma_5 u_N^-)_\gamma + T_3^N\frac{m_N}{pn}(i\sigma_{pn}C)_{\alpha\beta}(\gamma_5 u_N^+)_\gamma\\
&\qquad + T_4^N\frac{m_N}{pn}(i\sigma_{np}C)_{\alpha\beta}(\gamma_5 u_N^-)_\gamma + T_5^N\frac{m_N^2}{2pn}(i\sigma_{\perp n}C)_{\alpha\beta}(\gamma^\perp\gamma_5 u_N^+)_\gamma + \frac{m_N^2}{2pn}T_6^N(i\sigma_{\perp n}C)_{\alpha\beta}(\gamma^\perp\gamma_5 u_N^-)_\gamma\\
&\qquad + \frac{1}{2}m_N T_7^N(\sigma_{\perp\perp'}C)_{\alpha\beta}(\sigma^{\perp\perp'}\gamma_5 u_N^+)_\gamma + \frac{1}{2}m_N T_8^N(\sigma_{\perp\perp'}C)_{\alpha\beta}(\sigma^{\perp\perp'}\gamma_5 u_N^-)_\gamma, \qquad \text{(A6)}
\end{aligned}
$$

In this expression α,β,γ are spinor indices, n_μ is an auxiliary lightlike vector, $n^2=0$,

$$p_\mu = P_\mu - \frac{1}{2}\frac{m_N^2}{Pn}, \qquad p^2 = 0, \qquad \text{(A7)}$$

where P_μ is the proton momentum, $P^2 = m_N^2$. Further, $u_N^\pm = \Lambda_\pm u_N(P,\lambda)$ where $u_N(P,\lambda)$ is the usual Dirac spinor in relativistic normalization, the projectors are defined as

$$
\begin{aligned}
&\Lambda_+ = \frac{\not{p}\not{n}}{2pn}, \qquad \Lambda_- = \frac{\not{n}\not{p}}{2pn},\\
&g_{\mu\nu}^\perp = g_{\mu\nu} - \frac{p_\mu n_\nu + p_\nu n_\mu}{pn} \qquad \text{(A8)}
\end{aligned}
$$

and C is the charge-conjugation matrix. We use a shorthand notation $\sigma_{\perp n}\otimes\gamma^\perp = \sigma_{\mu\nu}n^\nu g_\perp^{\mu\alpha}\otimes\gamma_\alpha$, etc. The invariant functions $F = V_i, A_i, T_i$ correspond to contributions of a given *collinear twist* and can be written as Fourier integrals

$$F(a_j, pn) = \int [dx] e^{-i(pn)\sum_i x_i a_i} F(x_i) \qquad \text{(A9)}$$

where $F(x_i)$ depend on the three valence quark momentum fractions x_i.

Using various symmetries these functions can be combined in eight independent light cone DAs [36]. There exists a single DA for the leading twist-three [3]

$$
\begin{aligned}
&\langle 0|\epsilon^{ijk}(u_i^\uparrow(a_1n)C\not{n}u_j^\downarrow(a_2n))\not{n}d_k^\uparrow(a_3n)|N(P,\lambda)\rangle\\
&\quad= -\frac{1}{2}f_N(pn)\not{n}u_N^\uparrow(P)\int[dx]e^{-i(pn)\sum x_i a_i}\varphi_N(x_i), \qquad \text{(A10)}
\end{aligned}
$$

such that [37]

$$
\begin{aligned}
V_1(1,2,3) &= \frac{1}{2}f_N[\varphi_N(1,2,3)+\varphi_N(2,1,3)],\\
A_1(1,2,3) &= \frac{1}{2}f_N[\varphi_N(2,1,3)-\varphi_N(1,2,3)],\\
T_1(1,2,3) &= \frac{1}{2}f_N[\varphi_N(1,3,2)+\varphi_N(2,3,1)], \qquad \text{(A11)}
\end{aligned}
$$

and for twist-four there are three independent DAs [36]

$$
\begin{aligned}
&\langle 0|\epsilon^{ijk}(u_i^\uparrow(a_1n)C\not{n}u_j^\downarrow(a_2n))\not{p}d_k^\uparrow(a_3n)|N(P,\lambda)\rangle\\
&\quad= -\frac{1}{4}(pn)\not{p}u_N^\uparrow(P)\int[dx]e^{-i(pn)\sum x_i a_i}\\
&\qquad\times[f_N\Phi_4^{N,WW}(x_i)+\lambda_1^N\Phi_4^N(x_i)], \qquad \text{(A12)}\\
&\langle 0|\epsilon^{ijk}(u_i^\uparrow(a_1n)C\not{n}\gamma_\perp\not{p}u_j^\downarrow(a_2n))\gamma^\perp\not{n}d_k^\uparrow(a_3n)|N(P,\lambda)\rangle\\
&\quad= -\frac{1}{2}(pn)\not{n}m_N u_N^\uparrow(P)\int[dx]e^{-i(pn)\sum x_i a_i}\\
&\qquad\times[f_N\Psi_4^{N,WW}(x_i)-\lambda_1^N\Psi_4^N(x_i)], \qquad \text{(A13)}\\
&\langle 0|\epsilon^{ijk}(u_i^\uparrow(a_1n)C\not{p}\not{n}u_j^\uparrow(a_2n))\not{n}d_k^\uparrow(a_3n)|N(P,\lambda)\rangle\\
&\quad= \frac{\lambda_2^N}{12}(pn)\not{n}m_N u_N^\uparrow(P)\int[dx]e^{-i(pn)\sum x_i a_i}\Xi_4^N(x_i), \qquad \text{(A14)}
\end{aligned}
$$

where $\Phi_4^{N,WW}(x_i)$ and $\Psi_4^{N,WW}(x_i)$ are the so-called Wandzura-Wilczek contributions that can be expressed in terms of the leading-twist DA $\varphi_N(x_i)$ [38,39]. The constants f_N, λ_1^N and λ_2^N are defined in such a way that the integrals of the DAs φ_N, Φ_4, Ψ_4, Ξ_4 are normalized to unity:

$$\int [dx] F(x_i) = 1, \qquad F \in \{\varphi_N, \Phi_4, \Psi_4, \Xi_4\}. \tag{A15}$$

Using the canonical expansion of the quark fields in (A10), (A12) in terms of creation and annihilation operators and Dirac equation to eliminate "bad" quark field components it is easy to calculate the required matrix elements from the set of light-front wave functions in Eqs. (A2). In this way one obtains for our normalization (cf. [35,40])

$$\begin{aligned}
f_N \varphi_N(x_1,x_2,x_3) &= -4\sqrt{6}\int [dk_\perp] \psi^{(0)}_{N;1}(1,2,3),\\
[\lambda_1^N \Phi_4^N + f_N \Phi_4^{N,WW}](x_2,x_1,x_3) &= -8\sqrt{6}\int \frac{[dk_\perp]}{x_3 m_N} k_3^\perp \cdot [\bar{k}_1^\perp \psi^{(1)}_{N;1} + \bar{k}_2^\perp \psi^{(1)}_{N;2}](1,2,3),\\
[\lambda_1^N \Psi_4^N - f_N \Psi_4^{N,WW}](x_1,x_2,x_3) &= -8\sqrt{6}\int \frac{[dk_\perp]}{x_2 m_N} \bar{k}_2^\perp \cdot [k_1^\perp \psi^{(1)}_{N;1} + k_2^\perp \psi^{(1)}_{N;2}](1,2,3),\\
\lambda_2^N \Xi_4^N(x_1,x_2,x_3) &= -24\sqrt{6}\int \frac{[dk_\perp]}{x_1 m_N} k_1^\perp \cdot [\bar{k}_1^\perp(\psi^{(-1)}_N(1,3,2) - \psi^{(-1)}_N(1,2,3))\\
&\quad + \bar{k}_2^\perp(\psi^{(-1)}_N(2,3,1) - \psi^{(-1)}_N(2,1,3))]
\end{aligned} \tag{A16}$$

so that DAs correspond to integrals over the light-front wave functions over transverse momenta, with some prefactors. One has to have in mind that these relations are somewhat schematic since transverse momentum integrals on the right-hand side (rhs) are divergent and have to be regulated e.g. introducing a cutoff. In turn, the DAs are usually defined using dimensional regularization and the minimal subtraction so that a matching coefficient can be necessary. Also the wave function renormalization factors have to be added for the quark fields. The twist-four DAs include additional contributions from the four-particle Fock states with an extra gluon [28,34]. If these contributions are taken into account, the four-particle quark-gluon nucleon DAs have to be added as well [28,38].

The complete set of nucleon DAs carries the full information on the nucleon structure, in the same manner as the complete basis of light-front wave functions. In practice, however, both expansions have to be truncated and the usefulness of a truncated version, taking into account either the first few Fock states or a few lowest twist contributions, may depend on the concrete physics application.

The classification of the three-quark nucleon light-front wave functions in Eq. (A2) can be overtaken for the negative parity isospin-1/2 resonances, e.g. $N^*(1535)$, without modification. The symmetry under parity transformation does not constrain the light-front wave functions but affects the relation between the wave functions of the states with opposite helicity in terms of the helicity-flipped quarks. The corresponding expressions can be worked out using the Jacobi-Wick transformation [41]

$$\hat{\mathbb{Y}}|N,\lambda\rangle = \eta_N (-1)^{1/2-\lambda}|N,-\lambda\rangle \tag{A17}$$

where $\hat{\mathbb{Y}}$ is the parity transformation followed by a 188° rotation along the y-axis, and η_N is internal parity, $\eta_N = 1$ for the nucleon and $\eta_N = -1$ for $N^*(1535)$. Thus $k^\perp \overset{\hat{\mathbb{Y}}}{\mapsto} \bar{k}^\perp$, $|N,\uparrow\rangle \overset{\hat{\mathbb{Y}}}{\mapsto} \eta_N |N,\downarrow\rangle$, whereas for the quark states $q = u, d b^{\dagger i}_{q\uparrow}|0\rangle \overset{\hat{\mathbb{Y}}}{\mapsto} b^{\dagger i}_{q\downarrow}|0\rangle$, but $b^{\dagger i}_{q\downarrow}|0\rangle \overset{\hat{\mathbb{Y}}}{\mapsto} - b^{\dagger i}_{q\uparrow}|0\rangle$. Applying this transformation to the both sides of Eq. (A2) one obtains [33]

$$\begin{aligned}
|N\downarrow\rangle^{\ell_z=0}_{uud} &= -\eta_N \frac{\epsilon^{ijk}}{\sqrt{6}}\int \frac{[dx][dk_\perp]}{\sqrt{x_1x_2x_3}}[\psi^{(0)}_{N;1}(1,2,3) + i\epsilon^{\alpha\beta}\bar{k}^\perp_{1\alpha}\bar{k}^\perp_{2\beta}\psi^{(0)}_{N;2}(1,2,3)] b^{\dagger i}_{u\downarrow}(1)(b^{\dagger j}_{u\uparrow}(2)b^{\dagger k}_{d\downarrow}(3) - b^{\dagger j}_{d\uparrow}(2)b^{\dagger k}_{u\downarrow}(3))|0\rangle,\\
|N\downarrow\rangle^{\ell_z=-1}_{uud} &= \eta_N \frac{\epsilon^{ijk}}{\sqrt{6}}\int \frac{[dx][dk_\perp]}{\sqrt{x_1x_2x_3}}[\bar{k}_1^\perp\psi^{(1)}_{N;1}(1,2,3) + \bar{k}_2^\perp\psi^{(1)}_{N;2}(1,2,3)](b^{\dagger i}_{u\downarrow}(1)b^{\dagger j}_{u\uparrow}(2)b^{\dagger k}_{d\uparrow}(3) - b^{\dagger i}_{d\downarrow}(1)b^{\dagger j}_{u\uparrow}(2)b^{\dagger k}_{u\uparrow}(3))|0\rangle,\\
|N\downarrow\rangle^{\ell_z=1}_{uud} &= \eta_N \frac{\epsilon^{ijk}}{\sqrt{6}}\int \frac{[dx][dk_\perp]}{\sqrt{x_1x_2x_3}}[k_1^\perp\psi^{(-1)}_N(1,2,3)] b^{\dagger i}_{u\downarrow}(1)(b^{\dagger j}_{u\downarrow}(2)b^{\dagger k}_{d\downarrow}(3) - b^{\dagger j}_{d\downarrow}(2)b^{\dagger k}_{u\downarrow}(3))|0\rangle,\\
|N\downarrow\rangle^{\ell_z=-2}_{uud} &= -\eta_N \frac{\epsilon^{ijk}}{\sqrt{6}}\int \frac{[dx][dk_\perp]}{\sqrt{x_1x_2x_3}}[\bar{k}_1^\perp\bar{k}_3^\perp\psi^{(2)}_N(1,2,3)](b^{\dagger i}_{u\uparrow}(1)b^{\dagger j}_{u\uparrow}(2)b^{\dagger k}_{d\uparrow}(3) - b^{\dagger i}_{u\uparrow}(1)b^{\dagger j}_{d\uparrow}(2)b^{\dagger k}_{u\uparrow}(3))|0\rangle.
\end{aligned} \tag{A18}$$

so that, e.g., for the $\ell_z = 0$ states

$$\psi^{(0)}_{N;1}(1,2,3)|_{N\downarrow} = -\eta_N \psi^{(0)}_{N;1}(1,2,3)|_{N\uparrow} \tag{A19}$$

A Lorentz-covariant definition of the DAs of negative parity resonances involves some freedom. It is convenient to choose the definition in such a way that the coefficient functions in the OPE of currents (4) are the same for states of both parities, and also the relations between different DAs imposed by QCD equations of motion remain the same. As noticed in Ref. [26], this can be achieved using invariant decomposition of the $N^*(1535)$ matrix element in terms of the γ_5-rotated quark fields

$$4(\gamma_5)_{\alpha\alpha'}(\gamma_5)_{\beta\beta'}(\gamma_5)_{\gamma\gamma'}\langle 0|\epsilon^{ijk}u^i_{\alpha'}(a_1 n)u^j_{\beta'}(a_2 n)d^k_{\gamma'}(a_3 n)|N^*(P,\lambda)\rangle = S_1^{N^*} m_{N^*} C_{\alpha\beta}(\gamma_5 u^+_{N^*})_\gamma + \cdots \tag{A20}$$

where the expression on the right-hand side is the same as in Eq. (A6) with obvious replacements $m_N \to m_{N^*}$ etc. Projecting out the γ_5 matrices we obtain

$$\begin{aligned}
&4\langle 0|\epsilon^{ijk}u^i_\alpha(a_1 n)u^j_\beta(a_2 n)d^k_\gamma(a_3 n)|N^*(P,\lambda)\rangle \\
&= S_1^{N^*} m_{N^*} C_{\alpha\beta}(u^+_{N^*})_\gamma + S_2^{N^*} m_{N^*} C_{\alpha\beta}(u^-_{N^*})_\gamma + P_1^{N^*} m_{N^*}(\gamma_5 C)_{\alpha\beta}(\gamma_5 u^+_{N^*})_\gamma + P_2^{N^*} m_{N^*}(\gamma_5 C)_{\alpha\beta}(\gamma_5 u^-_{N^*})_\gamma \\
&\quad - V_1^{N^*}(\not{p}C)_{\alpha\beta}(u^+_{N^*})_\gamma - V_2^{N^*}(\not{p}C)_{\alpha\beta}(u^-_{N^*})_\gamma + \frac{1}{2}V_3^{N^*} m_{N^*}(\gamma_\perp C)_{\alpha\beta}(\gamma^\perp u^+_{N^*})_\gamma \\
&\quad + \frac{1}{2}V_4^{N^*} m_{N^*}(\gamma_\perp C)_{\alpha\beta}(\gamma^\perp u^-_{N^*})_\gamma - V_5^{N^*}\frac{m^2_{N^*}}{2pn}(\not{n}C)_{\alpha\beta}(u^+_{N^*})_\gamma - \frac{m^2_{N^*}}{2pn}V_6^{N^*}(\not{n}C)_{\alpha\beta}(u^-_{N^*})_\gamma \\
&\quad - A_1^{N^*}(\not{p}\gamma_5 C)_{\alpha\beta}(\gamma_5 u^+_{N^*})_\gamma - A_2^{N^*}(\not{p}\gamma_5 C)_{\alpha\beta}(\gamma_5 u^-_{N^*})_\gamma + \frac{1}{2}A_3^{N^*} m_{N^*}(\gamma_\perp\gamma_5 C)_{\alpha\beta}(\gamma^\perp\gamma_5 u^+_{N^*})_\gamma \\
&\quad + \frac{1}{2}A_4^{N^*} m_{N^*}(\gamma_\perp\gamma_5 C)_{\alpha\beta}(\gamma^\perp\gamma_5 u^-_{N^*})_\gamma - A_5^{N^*}\frac{m^2_{N^*}}{2pn}(\not{n}\gamma_5 C)_{\alpha\beta}(\gamma_5 u^+_{N^*})_\gamma - \frac{m^2_{N^*}}{2pn}A_6^{N^*}(\not{n}\gamma_5 C)_{\alpha\beta}(\gamma_5 u^-_{N^*})_\gamma \\
&\quad - T_1^{N^*}(i\sigma_{\perp p}C)_{\alpha\beta}(\gamma^\perp u^+_{N^*})_\gamma - T_2^{N^*}(i\sigma_{\perp p}C)_{\alpha\beta}(\gamma^\perp u^-_{N^*})_\gamma + T_3^{N^*}\frac{m_{N^*}}{pn}(i\sigma_{pn}C)_{\alpha\beta}(u^+_{N^*})_\gamma \\
&\quad + T_4^{N^*}\frac{m_{N^*}}{pn}(i\sigma_{np}C)_{\alpha\beta}(u^-_{N^*})_\gamma - T_5^{N^*}\frac{m^2_{N^*}}{2pn}(i\sigma_{\perp n}C)_{\alpha\beta}(\gamma^\perp u^+_{N^*})_\gamma - \frac{m^2_{N^*}}{2pn}T_6^{N^*}(i\sigma_{\perp n}C)_{\alpha\beta}(\gamma^\perp u^-_{N^*})_\gamma \\
&\quad + \frac{1}{2}m_{N^*}T_7^{N^*}(\sigma_{\perp\perp'}C)_{\alpha\beta}(\sigma^{\perp\perp'}u^+_{N^*})_\gamma + \frac{1}{2}m_{N^*}T_8^{N^*}(\sigma_{\perp\perp'}C)_{\alpha\beta}(\sigma^{\perp\perp'}u^-_{N^*})_\gamma,
\end{aligned} \tag{A21}$$

This expression replaces the decomposition (A6) for the nucleon. Note that there are some minus signs and in particular all three leading twist DAs V_1, A_1 and T_1 are defined in our convention with a different sign as compared to the nucleon. As a consequence in the definition of leading-twist DA in terms of the chiral quark fields there is a minus sign as compared to (A10),

$$\langle 0|\epsilon^{ijk}(u_i^\uparrow(a_1 n)C\not{n}u_j^\downarrow(a_2 n))\not{n}d_k^\uparrow(a_3 n)|N^*(P)\rangle = \frac{1}{2}f_{N^*}(pn)\not{n}u^\uparrow_{N^*}(P)\int[dx]e^{-i(pn)\sum x_i a_i}\varphi_{N^*}(x_i), \tag{A22}$$

where, of course, $P^2 = m^2_{N^*}$ and the expressions for the invariant functions $V_1^{N^*}, A_1^{N^*}, T_1^{N^*}$ in terms of φ_{N^*}, are the same as for the nucleon, Eq. (A11).

The twist-four DAs also acquire some signs [26]

$$\begin{aligned}
&\langle 0|\epsilon^{ijk}(u_i^\uparrow(a_1 n)C\not{n}u_j^\downarrow(a_2 n))\not{p}d_k^\uparrow(a_3 n)|N^*(P)\rangle \\
&= \frac{1}{4}(pn)\not{p}u^\uparrow_{N^*}(P)\int[dx]e^{-i(pn)\sum x_i a_i} \\
&\quad \times [f_{N^*}\Phi_4^{N^*,WW}(x_i) + \lambda_1^*\Phi_4^{N^*}(x_i)],
\end{aligned} \tag{A23}$$

$$\begin{aligned}
&\langle 0|\epsilon^{ijk}(u_i^\uparrow(a_1 n)C\not{n}\gamma_\perp\not{p}u_j^\downarrow(a_2 n))\gamma^\perp\not{n}d_k^\uparrow(a_3 n)|N^*(P)\rangle \\
&= -\frac{1}{2}(pn)\not{n}m_{N^*}u^\uparrow_{N^*}(P)\int[dx]e^{-i(pn)\sum x_i a_i} \\
&\quad \times [f_{N^*}\Psi_4^{N^*,WW}(x_i) - \lambda_1^*\Psi_4^{N^*}(x_i)],
\end{aligned} \tag{A24}$$

$$\langle 0|\epsilon^{ijk}(u_i^\uparrow(a_1 n) C\not{p}\not{n} u_j^\uparrow(a_2 n))\not{n} d_k^\uparrow(a_3 n)|N^*(p)\rangle = \frac{\lambda_2^*}{12}(pn)\not{n} m_{N^*} u_{N^*}^\uparrow(P)\int [dx] e^{-i(pn)\sum x_i a_i}\Xi_4^{N^*}(x_i), \tag{A25}$$

where $\Phi_4^{N^*,WW}(x_i)$ and $\Psi_4^{N^*,WW}(x_i)$ are given by the same expressions in terms of the expansion of the leading-twist DA $\varphi_{N^*}(x_i)$ as for the nucleon.

The price to pay for universality of correlation functions for positive and negative parities is that the relations between DAs and light-front wave functions in this convention acquire some signs as well,

$$\begin{aligned}
f_{N^*}\varphi_{N^*}(x_1,x_2,x_3) &= +4\sqrt{6}\int [dk_\perp]\psi_{N^*;1}^{(0)}(1,2,3),\\
[\lambda_1^{N^*}\Phi_4^{N^*} + f_N\Phi_4^{N^*,WW}](x_2,x_1,x_3) &= +8\sqrt{6}\int \frac{[dk_\perp]}{x_3 m_{N^*}} k_3^\perp\cdot[\bar{k}_1^\perp\psi_{N^*;1}^{(1)} + \bar{k}_2^\perp\psi_{N^*;2}^{(1)}](1,2,3),\\
[\lambda_1^{N^*}\Psi_4^{N^*} - f_N\Psi_4^{N^*,WW}](x_1,x_2,x_3) &= -8\sqrt{6}\int \frac{[dk_\perp]}{x_2 m_{N^*}} \bar{k}_2^\perp\cdot[k_1^\perp\psi_{N^*;1}^{(1)} + k_2^\perp\psi_{N^*;2}^{(1)}](1,2,3),\\
\lambda_2^{N^*}\Xi_4^{N^*}(x_1,x_2,x_3) &= -24\sqrt{6}\int \frac{[dk_\perp]}{x_1 m_{N^*}} k_1^\perp\cdot[\bar{k}_1^\perp(\psi_{N^*}^{(-1)}(1,3,2) - \psi_{N^*}^{(-1)}(1,2,3))\\
&\quad + \bar{k}_2^\perp(\psi_{N^*}^{(-1)}(2,3,1) - \psi_{N^*}^{(-1)}(2,1,3))],
\end{aligned} \tag{A26}$$

that have to be taken into account for the interpretation of the results.

Parametrization of the DAs of the resonances can be overtaken from that for the nucleon. The leading-twist DA $\varphi_{N^*}(x_i,\mu)$ can be expanded in the set of orthogonal polynomials $\mathcal{P}_{nk}(x_i)$

$$\varphi_N(x_i,\mu) = 120 x_1 x_2 x_3\sum_{n=0}^{\infty}\sum_{k=0}^{n}\varphi_{nk}(\mu)\mathcal{P}_{nk}(x_i),\quad \int [dx] x_1 x_2 x_3 \mathcal{P}_{nk}(x_i)\mathcal{P}_{n'k'}(x_i)\propto \delta_{nn'}\delta_{kk'}, \tag{A27}$$

such that the coefficients are renormalized multiplicatively to one-loop accuracy,

$$f_{N^*}(\mu) = f_{N^*}(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{2/(3\beta_0)},\qquad \varphi_{nk}(\mu) = \varphi_{nk}(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_{nk}/\beta_0}. \tag{A28}$$

Here $\beta_0 = 11 - \frac{2}{3}n_f$ is the first coefficient of the QCD β function and γ_{nk} are the anomalous dimensions. The double sum in Eq. (A27) goes over a complete set of orthogonal polynomials $\mathcal{P}_{nk}(x_i)$, $k = 0,\ldots,n$, of degree n:

$$\begin{aligned}
\mathcal{P}_{00} &= 1,\\
\mathcal{P}_{10} &= 21(x_1 - x_3), \mathcal{P}_{11} = 7(x_1 - 2x_2 + x_3),\\
\mathcal{P}_{20} &= \frac{63}{10}[3(x_1 - x_3)^2 - 3x_2(x_1 + x_3) + 2x_2^2],\\
\mathcal{P}_{21} &= \frac{63}{2}(x_1 - 3x_2 + x_3)(x_1 - x_3),\\
\mathcal{P}_{22} &= \frac{9}{5}[x_1^2 + 9x_2(x_1 + x_3) - 12x_1x_3 - 6x_2^2 + x_3^2]
\end{aligned} \tag{A29}$$

etc., and the corresponding anomalous dimensions are

$$\begin{aligned}
&\gamma_{00} = 0, \qquad \gamma_{10} = \frac{20}{9}, \qquad \gamma_{11} = \frac{8}{3},\\
&\gamma_{20} = \frac{32}{9}, \qquad \gamma_{21} = \frac{40}{9}, \qquad \gamma_{22} = \frac{14}{3}.
\end{aligned} \tag{A30}$$

The normalization condition (A15) implies that $\varphi_{00} = 1$. In the main text we refer to the coefficients $\varphi_{nk}(\mu_0)$ with $n = 1, 2, \ldots$, as shape parameters. The set of these coefficients together with the normalization constant $f_N(\mu_0)$ at a reference scale μ_0 specifies the momentum fraction distribution of valence quarks on the nucleon. They are related to matrix elements of local gauge-invariant three-quark operators and can be calculated, e.g., on the lattice [24,26].

The twist-four DAs can be parameterized as [38]

$$\begin{aligned}\Phi_4^{N^*}(x_i,\mu) &= 24x_1x_2\{1+\eta_{10}(\mu)\mathcal{R}_{10}(x_3,x_1,x_2)\\ &\quad -\eta_{11}(\mu)\mathcal{R}_{11}(x_3,x_1,x_2)\},\\ \Psi_4^{N^*}(x_i,\mu) &= 24x_1x_3\{1+\eta_{10}(\mu)\mathcal{R}_{10}(x_2,x_3,x_1)\\ &\quad +\eta_{11}(\mu)\mathcal{R}_{11}(x_2,x_3,x_1)\},\\ \Xi_4^{N^*}(x_i,\mu) &= 24x_2x_3\left\{1+\frac{9}{4}\xi_{10}(\mu)\mathcal{R}_{10}(x_1,x_3,x_2)\right\},\end{aligned} \tag{A31}$$

where

$$\begin{aligned}\mathcal{R}_{10}(x_1,x_2,x_3) &= 4\left(x_1+x_2-\frac{3}{2}x_3\right),\\ \mathcal{R}_{11}(x_1,x_2,x_3) &= \frac{20}{3}\left(x_1-x_2+\frac{1}{2}x_3\right)\end{aligned} \tag{A32}$$

and $\eta_{10}(\mu)$, $\eta_{11}(\mu)$, $\xi_{10}(\mu)$ are the new shape parameters. The corresponding one-loop anomalous dimensions are [38]

$$\gamma_{10}^{(\eta)} = \frac{20}{9}, \qquad \gamma_{11}^{(\eta)} = 4, \qquad \gamma_{10}^{(\xi)} = \frac{10}{3}. \tag{A33}$$

For the twist-five DAs we take into account contributions of geometric twist-three and twist-four operators as explained in Ref. [23].

Note that the asymptotic DAs (at very large scales) for the nucleon and the resonances are the same:

$$\begin{aligned}&\varphi^{\rm as}(x_i) = 120x_1x_2x_3, \qquad \Phi_4^{\rm as}(x_i) = 24x_1x_2,\\ &\Phi_4^{WW,\rm as}(x_i) = 24x_1x_2\left(1+\frac{2}{3}(1-5x_3)\right),\\ &\Psi_4^{WW,\rm as}(x_i) = 24x_1x_3\left(1+\frac{2}{3}(1-5x_2)\right),\\ &\Xi_4(x_i) = 24x_2x_3, \qquad \Psi_4^{\rm as}(x_i) = 24x_1x_3.\end{aligned} \tag{A34}$$

For completeness we also give here the definitions of the normalization constants in terms of matrix elements of local three-quark operators:

$$\begin{aligned}&\langle 0|\epsilon^{ijk}(u_iC\not n u_j)(0)\gamma_5\not n d_k(0)|N^*(P)\rangle\\ &\quad = f_{N^*}(pn)\gamma_5\not n u_{N^*}(P),\\ &\langle 0|\epsilon^{ijk}(u_iC\gamma_\mu u_j)(0)\gamma_5\gamma^\mu d_k(0)|N^*(p)\rangle\\ &\quad = \lambda_1^{N^*}m_{N^*}\gamma_5 u_{N^*}(P),\\ &\langle 0|\epsilon^{ijk}(u_iC\sigma_{\mu\nu} u_j)(0)\gamma_5\sigma^{\mu\nu} d_k(0)|N^*(P)\rangle\\ &\quad = \lambda_2^{N^*}m_{N^*}\gamma_5 u_{N^*}(P).\end{aligned} \tag{A35}$$

APPENDIX B: PARAMETRIZATION OF COEFFICIENT FUNCTIONS

For convenience we provide a simple parametrization for the coefficient functions $f_{1,2}^{nk}$, $g_{1,2}^{nk}$ appearing in (11), (12), for the range $2 < Q^2 < 12$ GeV^2:

$$\begin{aligned}f_{1,2}^{nk}(Q^2) &= D(Q^2)\sum_{p=0}^{4} b_{p;1,2}^{nk}\left(\frac{m_{N^*}^2}{Q^2}\right)^p,\\ g_{1,2}^{nk}(Q^2) &= D(Q^2)\sum_{p=0}^{4} a_{p;1,2}^{nk}\left(\frac{m_{N^*}^2}{Q^2}\right)^p,\end{aligned} \tag{B1}$$

where $D(Q^2)$ is the dipole form factor (13). The coefficients $a_{p;1}^{nk}$, $a_{p;2}^{nk}$, $b_{p;1}^{nk}$ and $b_{p;2}^{nk}$ are collected in Table II.

TABLE II. Coefficient functions in the LCSRs for $N^*(1535)$ production.

	$a_{p;1}^{00}$	$a_{p;1}^{10}$	$a_{p;1}^{11}$	$b_{p;1}^{00}$	$b_{p;1}^{10}$	$b_{p;1}^{11}$	$b_{p;1}^{20}$	$b_{p;1}^{21}$	$b_{p;1}^{22}$
$p=0$	0.0147491	0.251939	0.0256977	0.00716919	0.192078	−0.0271761	−0.340351	−0.653521	0.00864077
$p=1$	0.773867	−6.0864	0.646869	−0.307557	−1.94975	−0.706607	4.61356	3.7677	0.112153
$p=2$	−0.18913	5.62993	0.0535879	−0.242484	0.246661	5.43478	−1.81907	−2.25019	−0.258147
$p=3$	0.	−1.88333	0.	0.	0.	−5.79459	−2.75613	0.	0.0954517
$p=4$	0.	0.	0.	0.	0.	1.99163	1.67516	0.	0.

	$a_{p;2}^{00}$	$a_{p;2}^{10}$	$a_{p;2}^{11}$	$b_{p;2}^{00}$	$b_{p;2}^{10}$	$b_{p;2}^{11}$	$b_{p;2}^{20}$	$b_{p;2}^{21}$	$b_{p;2}^{22}$
$p=0$	0.0469231	−0.365146	−0.0498471	0.13253	0.210365	0.763116	−0.898009	0.978028	−0.408284
$p=1$	−1.35098	10.1647	1.78846	− 1.41541	0.1675	−2.84988	23.7579	19.1668	7.37946
$p=2$	1.30792	−16.9382	−3.92016	0.522203	−3.43005	1.08085	−50.8692	−47.2691	− 14.8858
$p=3$	−0.450538	12.7283	3.3271	0.	2.01318	0.	46.6823	43.7722	12.2449
$p=4$	0.	−3.67258	−1.03533	0.	0.	0.	−16.0002	−14.5341	−3.76771

[1] V. L. Chernyak and A. R. Zhitnitsky, JETP Lett. **25**, 510 (1977); Sov. J. Nucl. Phys. **31**, 544 (1980); V. L. Chernyak, A. R. Zhitnitsky, and V. G. Serbo, JETP Lett. **26**, 594 (1977); Sov. J. Nucl. Phys. **31**, 552 (1980).

[2] A. V. Radyushkin, JINR Report No. R2-10717 (1977) [arXiv: hep-ph/0410276]; A. V. Efremov and A. V. Radyushkin, Theor. Math. Phys. **42**, 97 (1980); Phys. Lett. **94B**, 245 (1980).

[3] G. P. Lepage and S. J. Brodsky, Phys. Lett. **87B**, 359 (1979); Phys. Rev. D **22**, 2157 (1980).

[4] H.-n. Li and G. F. Sterman, Nucl. Phys. **B381**, 129 (1992).

[5] I. I. Balitsky, V. M. Braun, and A. V. Kolesnichenko, Sov. J. Nucl. Phys. **44**, 1028 (1986); Nucl. Phys. **B312**, 509 (1989); V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. **B345**, 137 (1990).

[6] V. M. Braun, A. Khodjamirian, and M. Maul, Phys. Rev. D **61**, 073004 (2000); J. Bijnens and A. Khodjamirian, Eur. Phys. J. C **26**, 67 (2002).

[7] P. Ball and R. Zwicky, Phys. Rev. D **71**, 014015 (2005); Phys. Rev. D **71**, 014029 (2005).

[8] G. Duplancic, A. Khodjamirian, T. Mannel, B. Melic, and N. Offen, J. High Energy Phys. 04 (2008) 014.

[9] V. M. Braun, A. Lenz, N. Mahnke, and E. Stein, Phys. Rev. D **65**, 074011 (2002).

[10] V. M. Braun, A. Lenz, and M. Wittmann, Phys. Rev. D **73**, 094019 (2006).

[11] A. Lenz, M. Wittmann, and E. Stein, Phys. Lett. B **581**, 199 (2004).

[12] T. M. Aliev and M. Savci, Phys. Lett. B **656**, 56 (2007).

[13] T. M. Aliev, K. Azizi, A. Ozpineci, and M. Savci, Phys. Rev. D **77**, 114014 (2008).

[14] Z.-G. Wang, S.-L. Wan, and W.-M. Yang, Eur. Phys. J. C **47**, 375 (2006); Phys. Rev. D **73**, 094011 (2006).

[15] G. Erkol and A. Ozpineci, Phys. Lett. B **704**, 551 (2011); T. M. Aliev, K. Azizi, and M. Savci, Phys. Rev. D **84**, 076005 (2011).

[16] Y.-L. Liu and M.-Q. Huang, Phys. Rev. D **79**, 114031 (2009); G. Erkol and A. Ozpineci, Phys. Rev. D **83**, 114022 (2011).

[17] V. M. Braun, A. Lenz, G. Peters, and A. V. Radyushkin, Phys. Rev. D **73**, 034020 (2006).

[18] V. M. Braun, D. Y. Ivanov, A. Lenz, and A. Peters, Phys. Rev. D **75**, 014021 (2007); V. M. Braun, D. Y. Ivanov, and A. Peters, Phys. Rev. D **77**, 034016 (2008).

[19] M.-q. Huang and D.-W. Wang, Phys. Rev. D **69**, 094003 (2004).

[20] Y.-M. Wang, Y.-L. Shen, and C.-D. Lu, Phys. Rev. D **80**, 074012 (2009).

[21] T. M. Aliev, K. Azizi, and M. Savci, Phys. Rev. D **81**, 056006 (2010).

[22] A. Khodjamirian, C. Klein, T. Mannel, and Y.-M. Wang, J. High Energy Phys. 09 (2011) 106.

[23] I. V. Anikin, V. M. Braun, and N. Offen, Phys. Rev. D **88**, 114021 (2013).

[24] V. M. Braun, S. Collins, B. Gläßle, M. Göckeler, A. Schäfer, R. W. Schiel, W. Söldner, A. Sternbeck, and P. Wein, Phys. Rev. D **89**, 094511 (2014).

[25] I. G. Aznauryan *et al.*, Int. J. Mod. Phys. E **22**, 1330015 (2013); R. W. Gothe *et al.*, Nucleon Resonance Studies with CLAS12, Experiment Report No. E12-09-003.

[26] V. M. Braun *et al.*, Phys. Rev. Lett. **103**, 072001 (2009).

[27] I. G. Aznauryan, V. D. Burkert, and T.-S. H. Lee, arXiv: 0810.0997.

[28] V. M. Braun, T. Lautenschlager, A. N. Manashov, and B. Pirnay, Phys. Rev. D **83**, 094023 (2011).

[29] I. G. Aznauryan *et al.* (CLAS Collaboration), Phys. Rev. C **80**, 055203 (2009).

[30] H. Denizli *et al.* (CLAS Collaboration), Phys. Rev. C **76**, 015204 (2007).

[31] M. M. Dalton, G. S. Adams, A. Ahmidouch, T. Angelescu, J. Arrington, R. Asaturyan, O. K. Baker, N. Benmouna *et al.*, Phys. Rev. C **80**, 015205 (2009).

[32] C. S. Armstrong *et al.* (Jefferson Laboratory E94014 Collaboration), Phys. Rev. D **60**, 052004 (1999).

[33] X. d. Ji, J. P. Ma, and F. Yuan, Nucl. Phys. **B652**, 383 (2003).

[34] X. d. Ji, J. P. Ma, and F. Yuan, Eur. Phys. J. C **33**, 75 (2004).

[35] A. V. Belitsky and A. V. Radyushkin, Phys. Rep. **418**, 1 (2005).

[36] V. Braun, R. J. Fries, N. Mahnke, and E. Stein, Nucl. Phys. **B589**, 381 (2000); **B607**, 433(E) (2001).

[37] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. **112**, 173 (1984).

[38] V. M. Braun, A. N. Manashov, and J. Rohrwild, Nucl. Phys. **B807**, 89 (2009).

[39] I. V. Anikin and A. N. Manashov, Phys. Rev. D **89**, 014011 (2014).

[40] A. V. Belitsky, X. d. Ji, and F. Yuan, Phys. Rev. Lett. **91**, 092003 (2003).

[41] M. Jacob and G. C. Wick, Ann. Phys. **7**, 404 (1959); Ann. Phys. **281**, 774 (2000).