Relativistic corrections to J/ψ polarization in photo- and hadroproduction

Zhi-Guo He and Bernd A. Kniehl

II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany

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We systematically calculate the relativistic corrections to the polarization variables of prompt J/ψ photoproduction and hadroproduction using the factorization formalism of nonrelativistic QCD. Specifically, we include the ${}^3S_1^{[1]}$ and ${}^3P_J^{[1]}$ color-singlet and the ${}^3S_1^{[8]}$, ${}^1S_0^{[8]}$, and ${}^3P_J^{[8]}$ color-octet channels as well as the effects due to the mixing between the ${}^3S_1^{[8]}$ and ${}^3D_1^{[8]}$ channels. We provide all the squared hard-scattering amplitudes in analytic form. Assuming the nonrelativistic-QCD long-distance matrix elements to satisfy the velocity scaling rules, we find the relativistic corrections to be appreciable, especially at small transverse momentum p_T and large inelasticity z. The results obtained here and in our previous work on the unpolarized yield [Phys. Rev. D 90, 014045 (2014)] will help to render global analyses of prompt J/ψ production data more complete and hopefully to shed light on the J/ψ polarization puzzle.

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I. INTRODUCTION

Through concerted efforts from both the experimental and theoretical sides, we have deepened our understanding of the mechanism of J/ψ production at high energies in recent years (for a review, see Ref. [1] and references cited therein). Yet, there are still major challenges to the theoretical models. One of them is the long-standing J/ψ polarization puzzle. The polarization of promptly produced J/ψ mesons, which are produced either directly or via the feed down from higher excited states such as χ_{cJ} $(J = 0, 1, 2)$ and ψ' mesons, was measured by several experiments in different environments, including CDF at the Fermilab Tevtron [2,3]; HERA-B [4], ZEUS [5], and H1 [6] at DESY HERA; PHENIX at BNL RHIC [7]; and ALICE [8], CMS [9], and LHCb [10] at CERN LHC. Unfortunately, these measurements could not yet be explained by theoretical analyses in a way consistent with the world data on the unpolarized J/ψ yield. The production of heavy-quarkonium states, like the J/ψ meson, involves the creation of the heavy-quark pair $(Q\bar{Q})$ and its subsequent transition to the hadronic bound state. Different ways of treating these two parts result in different models. Polarization variables are more sensitive to the fine details of the hadronization of the $Q\bar{Q}$ pair than production rates and, therefore, play a key role in testing the theoretical models.

Due to the fact that the heavy-quark mass m_Q is much larger than the asymptotic scale parameter of QCD Λ_{QCD}, the heavy-quarkonium state can be approximately treated as a nonrelativistic system. The factorization formalism based on the effective field theory of nonrelativistic QCD (NRQCD) [11–13] is nowadays considered to be the most favorable theoretical approach to study heavy-quarkonium production and decay. In this framework, the heavy-quarkonium production cross sections are factorized into process-dependent short-distance coefficients (SDCs) and supposedly universal long-distance matrix elements (LDMEs) [13]. The SDCs, which describe the production of the $Q\bar{Q}$ pair at energy scales $2m_Q$ or larger, can be calculated perturbatively through expansions in the strong-coupling constant α_s. The LDMEs, which measure the probability of the hadronization of the $Q\bar{Q}$ pair, are weighted by definite powers of the relative velocity v of the heavy quarks in the heavy-meson rest frame [12]. In this way, theoretical calculations are organized as double expansions in α_s and v. Quantum corrections come as terms of higher orders in α_s and relativistic corrections as terms of higher orders in $v^2 \approx \alpha_s$. Thus, next-to-leading-order (NLO) results include corrections of orders $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(v^2)$ relative to the leading-order (LO) result. Next-to-next-to-leading-order results also include terms of relative orders $\mathcal{O}(\alpha_s^2)$, $\mathcal{O}(\alpha_s v^2)$, $\mathcal{O}(v^4)$, and so on. A crucial difference between the NRQCD factorization formalism and the conventional color-singlet (CS) model is that the former allows for the $Q\bar{Q}$ pair to be in any possible Fock state $n = {}^{2S+1}L_J^{[a]}$ with total spin S, orbital angular momentum L, total angular momentum J, and color multiplicity $a = 1, 8$, while the latter is restricted to the CS state, with $a = 1$, sharing the quantum numbers S, L, and J with the heavy meson. The color-octet (CO) mechanism, i.e., the appearance of CO states, with $a = 8$, is a distinctive feature of NRQCD factorization.

Two decades after the introduction of NRQCD factorization [13], all the relevant observables of prompt J/ψ production are available at NLO in α_s. In particular, these include yield [14] and polarization [15] in photoproduction, yield [16–18] and polarization [19–23] in hadroproduction, and observables in other production modes [24–26]. Since the LDMEs of the η_c meson are related to those of the J/ψ meson via heavy-quark spin symmetry, the η_c yield [27–29] must be included in this list, too. The J/ψ LDMEs determined at NLO in NRQCD via a global fit to measurements of the unpolarized J/ψ yields in photoproduction, hadroproduction, and other production modes [26] lead to predictions of J/ψ polarization in hadroproduction [19] that are incompatible with measurements at the Tevatron [2,3] and the LHC [9,10]. On the other hand, J/ψ LDMEs determined only from hadroproduction data [20,21] are incompatible with data from photoproduction and other production modes [30]. In other words, the long-standing J/ψ polarization crisis of NRQCD, which had already been observed at LO [31], is substantiated at NLO in α_s. This crisis has recently been aggravated by the observation [27] that all the up-to-date J/ψ LDME sets [20,21,26,32] lead to NLO NRQCD predictions for the η_c yield in hadroproduction that significantly overshoot the recent LHCb measurement [33]. Unfortunately, attempts [28] to reconcile the measured η_c and J/ψ yields of hadroproduction by resorting to pre-LHC LDMEs [17] fail to describe the J/ψ polarization measured at the Tevatron [23] and the LHC [28] in the regime of large p_T values and central rapidities y. The analysis of Ref. [29] also misses its goal of achieving a coherent description of the J/ψ and η_c yields and the J/ψ polarization in prompt hadroproduction. Obviously, NRQCD factorization presently faces severe challenges with regard to the predicted universality of the LDMEs at NLO in α_s.

To resolve the J/ψ polarization puzzle and to clarify the universality problem of the NRQCD LDMEs, we systematically calculated, in our previous work [34], the relativistic corrections, of relative order $\mathcal{O}(v^2)$, to the unpolarized J/ψ yields in photoproduction and hadroproduction by including both the direct and feed-down contributions. We found that the $\mathcal{O}(v^2)$ corrections are appreciable, except for the CS ${}^3S_1^{[1]}$ channel of hadroproduction, and that their line shapes greatly differ between photoproduction and hadroproduction, which may offer a chance to improve the goodness of the state-of-the-art determinations of the LDMEs. In this paper, we take the next logical step by calculating the $\mathcal{O}(v^2)$ corrections to the J/ψ polarization observables in photoproduction and hadroproduction. All these $\mathcal{O}(v^2)$ corrections must be included on top of the respective $\mathcal{O}(\alpha_s)$ corrections [14–23] to render the NLO predictions complete.

We organize the remainder of this paper as follows. In Sec. II, we shall briefly describe how to calculate the SDCs for the polarization parameters in the direct and feed-down processes of photoproduction and hadroproduction. In Sec. III, we shall present and discuss our numerical results. A brief summary will be given in Sec. IV. All the relevant analytic expressions will be collected in the Appendix.

II. NRQCD FACTORIZATION FORMULA

The polarization of the J/ψ meson is measured through the angular distribution of its leptonic decay, $J/\psi \to l^+ + l^-$, which may be parametrized as

$$W(\theta,\phi) = 1 + \lambda_\theta \cos^2\theta + \lambda_\phi \sin^2\theta \cos(2\phi) + \lambda_{\theta\phi}\sin(2\theta)\cos\phi, \tag{1}$$

where θ and ϕ are the polar and azimuthal angles of lepton l^+ in the J/ψ rest frame, respectively, which depend on the choice of coordinate frame. For example, in the recoil or s-channel helicity frame, the z axis is chosen to be the J/ψ flight direction. The parameters λ_θ, λ_ϕ, and $\lambda_{\theta\phi}$ in Eq. (1) may be related to the helicity density matrix $d\sigma_{\lambda\lambda'}$, which describes the interferences between the states of helicity λ and λ' in J/ψ production [35]

$$\lambda_\theta = \frac{d\sigma_{1,1} - d\sigma_{0,0}}{d\sigma_{1,1} + d\sigma_{0,0}},\quad \lambda_\phi = \frac{d\sigma_{1,-1}}{d\sigma_{1,1} + d\sigma_{0,0}},\quad \lambda_{\theta\phi} = \frac{\sqrt{2}\mathrm{Re}\,d\sigma_{1,0}}{d\sigma_{1,1} + d\sigma_{0,0}}. \tag{2}$$

Invoking the Weizsäcker–Williams approximation and the factorization theorems of the QCD parton model and NRQCD [13], we may write the hadronic helicity density matrices of both the direct and feed-down processes in the general form

$$d\sigma_{\lambda\lambda'}(A+B \to J/\psi + X) = \sum_{i,j,H}\int dx_1 dy_1 dx_2 f_{i/A}(x_1) f_{k/i}(y_1) f_{j/B}(x_2) \times d\hat{\sigma}_{\lambda\lambda'}(k+j \to H+X)\mathrm{Br}(H \to J/\psi + X), \tag{3}$$

where $f_{i/A}(x)$ is the parton density function (PDF) of the parton i in the hadron $A = p, \bar{p}$ or the flux function of the photon $i=\gamma$ in the charged lepton $A = e^-, e^+$, $f_{j/i}(y_1)$ is $\delta_{ij}\delta(1-y_1)$ or the PDF of the parton j in the resolved photon $i=\gamma$, $\mathrm{Br}(H \to J/\psi + X)$ is the branching ratio for $H = J/\psi, \chi_{cJ}, \psi'$ with the understanding that $\mathrm{Br}(J/\psi \to J/\psi + X) = 1$, and $d\hat{\sigma}_{\lambda\lambda'}(i+j \to H+X)$ is the partonic helicity density matrix. Through relative order $\mathcal{O}(v^2)$, the latter may be factorized as[1]

$$d\hat{\sigma}_{\lambda\lambda'}(i+j \to H+X) = \sum_n \left(\frac{dF^{ij}_{\lambda\lambda'}(n)}{m_c^{d_{\mathcal{O}(n)}-4}}\langle \mathcal{O}^H(n)\rangle + \frac{dG^{ij}_{\lambda\lambda'}(n)}{m_c^{d_{\mathcal{P}(n)}-4}}\langle \mathcal{P}^H(n)\rangle\right), \tag{4}$$

where $\mathcal{O}^H(n)$ is the four-quark operator pertaining to the transition $n \to H$ at LO, with dimension $d_{\mathcal{O}(n)}$, $\mathcal{P}^H(n)$ is related to its $\mathcal{O}(v^2)$ correction and carries dimension $d_{\mathcal{P}(n)} = d_{\mathcal{O}(n)} + 2$, and $F^{ij}_{\lambda\lambda'}(n)$ and $G^{ij}_{\lambda\lambda'}(n)$ are the appropriate SDCs of the partonic subprocess $i + j \to c\bar{c}(n) + X$. The definitions of the $\mathcal{O}$ and $\mathcal{P}$ operators and some of their properties may be found in Refs. [13,34]. Working in the

[1]The spin-flip interactions are $\mathcal{O}(v^3)$ suppressed [36], so that NRQCD factorization still holds for the helicity density matrix through relative order $\mathcal{O}(v^2)$.

fixed-flavor-number scheme, the parton i runs over the gluon g and the light quarks $q = u, d, s$ and antiquarks $\bar{q}$.

As in the unpolarized case [34], the relevant partonic subprocesses include

$$\begin{aligned} g+\gamma &\to c\bar{c}({}^3S_1^{[1,8]}, {}^1S_0^{[8]}, {}^3P_J^{[8]}) + g, \\ q(\bar{q})+\gamma &\to c\bar{c}({}^3S_1^{[8]}, {}^1S_0^{[8]}, {}^3P_J^{[8]}) + q(\bar{q}), \\ g+g &\to c\bar{c}({}^3S_1^{[1,8]}, {}^1S_0^{[8]}, {}^3P_J^{[1,8]}) + g, \\ q(\bar{q})+g &\to c\bar{c}({}^3S_1^{[8]}, {}^1S_0^{[8]}, {}^3P_J^{[1,8]}) + q(\bar{q}), \\ \bar{q}+q &\to c\bar{c}({}^3S_1^{[8]}, {}^1S_0^{[8]}, {}^3P_J^{[1,8]}) + g. \end{aligned} \quad (5)$$

The SDCs $F^{ij}_{\lambda\lambda'}(n)$ and $G^{ij}_{\lambda\lambda'}(n)$ may still be calculated with the help of the spinor projection method as explained in Ref. [34] when the polarization four-vector $\epsilon^\mu(\lambda)$ of the $c\bar{c}$ pair is kept generic. In the following, we only explain the differences in computing the helicity density matrix elements with respect to Ref. [34]. We only need to discuss the nontrivial cases of $S = 1$. In the case of direct J/ψ production, the total spin S of the $c\bar{c}$ pair can safely be identified with the spin S of the J/ψ meson through $\mathcal{O}(v^2)$ because spin-flip effects occur only at $\mathcal{O}(v^3)$ [36], and the index L_z related to the orbital angular momentum of the $c\bar{c}$ pair may be summed over. Starting from the partonic scattering amplitude $M_\lambda(n) = M(i + j \to c\bar{c}(n,\lambda) + X)$, where λ is the helicity related to the total spin S of the $c\bar{c}$ pair, the helcity density matrix element is thus evaluated as $\rho^{ij}_{\lambda\lambda'}(n) = \overline{\sum} M_\lambda(n) M^*_{\lambda'}(n)$ by summing over the orbital angular momentum L_z of the $c\bar{c}$ pair and the spins and colors of the other outgoing partons, contained in system X, and averaging over the spins and colors of the incoming partons i and j. Through $\mathcal{O}(v^2)$, the general Lorentz structure of $\rho^{ij}_{\lambda\lambda'}(n)$ implies the decomposition

$$\rho^{ij}_{\lambda\lambda'}(n) = \sum_k [A^{ij}_k(n) + \mathbf{q}^2 B^{ij}_k(n)] s_k^{\mu\nu} \epsilon^*_\mu(\lambda)\epsilon_\nu(\lambda'), \quad (6)$$

where k labels the process-independent four-tensors

$$\begin{aligned} s_1^{\mu\nu} &= g^{\mu\nu}, \qquad s_2^{\mu\nu} = k_1^\mu k_1^\nu, \\ s_3^{\mu\nu} &= k_2^\mu k_2^\nu, \qquad s_4^{\mu\nu} = k_1^\mu k_2^\nu + k_2^\mu k_1^\nu, \end{aligned} \quad (7)$$

with k_1 and k_2 being the four-momenta of the incoming partons; $A^{ij}_k(n)$ and $B^{ij}_k(n)$ are the SDCs, which are functions of the partonic Mandelstam variables s, t, and u and depend on the considered partonic subprocess, but not on the choice of coordinate frame; and $\mathbf{q}$ is the relative three-momentum within the $c\bar{c}$ pair. Because $P\cdot\epsilon(\lambda) = 0$, where P is the total four-momentum of the $c\bar{c}$ pair, and $P^2 = 4E_q^2$ [34], $\epsilon^\mu(\lambda)$ also implicitly depends on $\mathbf{q}^2$. To obtain the complete results at $\mathcal{O}(v^2)$, the contractions between $s_k^{\mu\nu}$ and $\epsilon^*_\mu(\lambda)\epsilon_\nu(\lambda')$ also need to be expanded as series in $\mathbf{q}^2$ after choosing a coordinate frame. Writing

$$s_k^{\mu\nu}\epsilon^*_\mu(\lambda)\epsilon_\nu(\lambda') = c^k_{\lambda\lambda'} + \mathbf{q}^2 d^k_{\lambda\lambda'} + \mathcal{O}(\mathbf{q}^4), \quad (8)$$

it is straightforward to obtain

$$\begin{aligned} \frac{F^{ij}_{\lambda\lambda'}(n)}{m_c^{d_{\mathcal{O}(n)}-4}} &= \frac{1}{2s}\int d\text{LIPS} \sum_k A^{ij}_k(n) c^k_{\lambda\lambda'}, \\ \frac{G^{ij}_{\lambda\lambda'}(n)}{m_c^{d_{\mathcal{P}(n)}-4}} &= \frac{1}{2s}\int d\text{LIPS} \sum_k \Big[A^{ij}_k(n)(K c^k_{\lambda\lambda'} + d^k_{\lambda\lambda'}) \\ &\quad + B^{ij}_k(n) c^k_{\lambda\lambda'} \Big], \end{aligned} \quad (9)$$

where the definition of the factor K is the same as in Ref. [34]. Note that $c^k_{\lambda\lambda'}$ and $d^k_{\lambda\lambda'}$ are frame dependent.

The radiative decays $\chi_{cJ} \to J/\psi + \gamma$ are not only affected by the E1 transition but also by the higher-multipole M2 (χ_{c1} and χ_{c2}) and E3 (χ_{c2}) ones. A detailed study [37] revealed that only the angular distribution of the photon is very sensitive to the higher-multipole contributions, while the latter are negligible for the angular distribution of the subsequent $J/\psi \to l^+ + l^-$ decay. Therefore, it is sufficient to calculate the helicity density matrix elements of J/ψ production from χ_{cJ} feed down by connecting the J/ψ polarization four-vector to the total spin of the $c\bar{c}$ pair in the helicity density matrix elements of χ_{cJ} production and multiplying the outcome by the branching ratio of $\chi_{cJ} \to J/\psi + \gamma$. At first sight, this is surprising because the Lorentz structure of the helicity density matrix elements of χ_{cJ} production look more complicated [38,39].[2] However, after summing over the polarization J_z of the χ_{cJ} meson, the helicity density matrix elements may indeed be expressed in the form of Eq. (6) as well. To ensure that our simplified method is correct, we compared with previous LO calculations [38,40] to find agreement.

We generate the Feynman diagrams by using the FeynArts package [41] and calculate the amplitude squares with the help of the FeynCalc package [42]. As for direct J/ψ production, we reproduce the LO results for $\rho^{ij}_{\lambda\lambda'}(n)$ in Ref. [35] by setting $\mathbf{q}^2 = 0$, while our results for the $\mathcal{O}(v^2)$ corrections are new. As for J/ψ production via feed down from χ_{cJ} mesons, the LO helicity density matrix elements are found to agree with the results of Ref. [38] after simplifying the latter, but are represented here for the first time in compact form, and the $\mathcal{O}(v^2)$ corrections are again new. We recover our results in Ref. [34] by summing over the helicity indices λ and λ'. In the Appendix, we list all the new results for $A^{ij}_k(n)$ and $B^{ij}_k(n)$ in analytic form.

[2] We caution the reader of the following misprints in Ref. [38]: In Eq. (22), the terms proportional to $\langle\mathcal{O}^{\chi_{c0}}({}^3S_1^{[8]})\rangle B(\chi_{cJ} \to J/\psi + \gamma)$ with $J = 1, 2$ should be multiplied by the respective total-spin multiplicities $(2J + 1)$. Three lines below Eq. (23), $|R'_P(0)|$ should be squared. The last factor in the first line of the expression for c_2 in Eq. (A7) should read $(7\hat{s}^2 + \hat{s}\,\hat{t} + 7\hat{t}^2)$.

III. PHENOMENOLOGICAL RESULTS

We are now in a position to investigate the phenomenological significance of the $\mathcal{O}(v^2)$ corrections to the polarization parameters in prompt J/ψ photoproduction and hadroproduction. In our numerical analysis, we use $m_c = 1.5$ GeV, $\alpha = 1/137.036$, the LO formula for $\alpha_s^{(n_f)}(\mu_r)$ with $n_f = 4$ active quark flavors and asymptotic scale parameter $\Lambda_{\rm QCD}^{(4)} = 215$ MeV [43], the CTEQ6L1 set for proton PDFs [43], the photon flux function given in Eq. (5) of Ref. [44] with $Q^2_{\rm max} = 2.5$ GeV^2 [6], and the choice $\mu_r = \mu_f = \sqrt{p_T^2 + 4m_c^2}$ for the renormalization and factorization scales. In contrast to the case of the unpolarized J/ψ yield in Ref. [34], the size of the $\mathcal{O}(v^2)$ corrections to the J/ψ polarization parameters for a given $c\bar{c}$ Fock state n cannot be directly estimated from the SDC ratios $R_{\lambda\lambda'}(n) = (dF_{\lambda\lambda'}(n)/dx)/(dG_{\lambda\lambda'}(n)/dx)$, where x is some variable, because the $d\sigma_{\lambda\lambda'}$ values enter as ratios in Eq. (2). For instance, if we had $R_{00}(n) = R_{11}(n)$ for some n, then the respective contribution to λ_θ would go unchanged upon the inclusion of the $\mathcal{O}(v^2)$ corrections. Therefore, we shall directly study the shifts induced in the polarization parameters by including the $\mathcal{O}(v^2)$ corrections for each n separately. To standardize the numerical discussion, we quote the $\mathcal{O}(v^2)$ corrections due to the ${}^3S_1^{[8]}$–${}^3D_1^{[8]}$ mixing relative to the corresponding LO results for $n = {}^3S_1^{[8]}$. We exclude from our considerations the trivial $J = 0$ cases $n = {}^1S_0^{[8]}$ and $n = {}^3P_0^{[1]}$. In want of fitted values of the LDMEs $\langle \mathcal{P}^H(n)\rangle$, we estimate them with the help of the velocity scaling rule $\langle \mathcal{P}^H(n)\rangle / \langle \mathcal{O}^H(n)\rangle = m_c^2 v^2$ [12] varying v^2 from 0 to 0.3 to account for uncertainties. Furthermore, we limit ourselves to the direct production of J/ψ mesons and their production via the feed down from χ_{cJ} mesons. In the latter case, we approximately take into account the kinematic effect on the transverse momentum p_T of the J/ψ meson by setting $p_T = p_T^{\chi_{cJ}} M_{J/\psi}/M_{\chi_{cJ}}$, with $M_{J/\psi} = 3.097$ GeV, $M_{\chi_{c1}} = 3.511$ GeV, and $M_{\chi_{c2}} = 3.556$ GeV [45], which

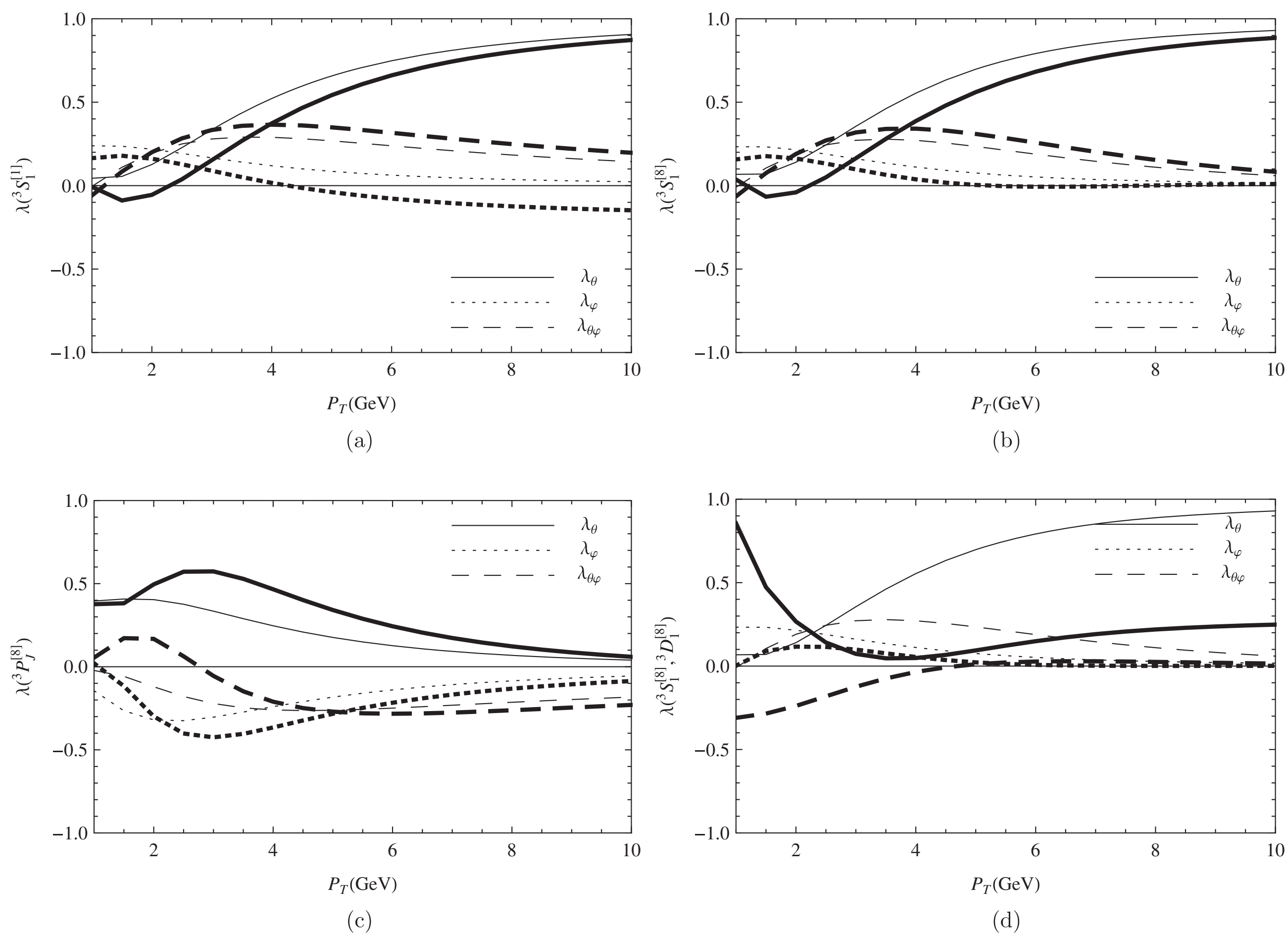

FIG. 1. $\mathcal{O}(v^2)$-corrected polarization parameters λ_θ (solid lines), λ_ϕ (dotted lines), and $\lambda_{\theta\phi}$ (dashed lines) in direct J/ψ photoproduction at HERA Run II [6] through the $c\bar{c}$ Fock states (a) $n = {}^3S_1^{[1]}$, (b) ${}^3S_1^{[8]}$, (c) ${}^3P_J^{[8]}$, and (d) ${}^3S_1^{[8]}$–${}^3D_1^{[8]}$ mixing as functions of p_T. The true results should lie inside the bands encompassed by the evaluations with $v^2 = 0.3$ (thick lines) and the LO results, with $v^2 = 0$ (thin lines).

is justified since $p_T \gg M_{\chi_{cJ}} - M_{J/\psi}$ in typical experimental situations. In the case of γp collisions, we only consider direct photoproduction for illustration because resolved photoproduction shares the partonic subprocesses with hadroproduction, which is studied separately.

The J/ψ polarization was measured in a number of experiments [2–10]. As an illustration, we consider here three representative cases, namely HERA Run II [6], Tevatron Run II [3], and the CMS experimental setup at the LHC [9], limiting ourselves to the helicity frame. In HERA Run II [6], the polarization was measured in prompt J/ψ photoproduction at center-of-mass (c.m.) energy $\sqrt{S} = 319$ GeV differential in p_T and inelasticity $z = p_{J/\psi} \cdot p_p / p_\gamma \cdot p_p$, where p_γ, p_p, and $p_{J/\psi}$ are the four-momenta of the photon, proton, and J/ψ meson, respectively, imposing in turn the acceptance cuts $0.3 < z < 0.9$ and $p_T^2 > 1\ \mathrm{GeV}^2$ always in combination with the acceptance cut 60 GeV $< W <$ 240 GeV on the γp c.m. energy $W = \sqrt{(p_\gamma + p_p)^2}$ [6]. The $\mathcal{O}(v^2)$ corrections to the polarization parameters λ_θ, λ_ϕ, and $\lambda_{\theta\phi}$ in direct photoproduction at HERA Run II [6] through the $c\bar{c}$ Fock states $n = {}^3S_1^{[1,8]}$, ${}^3P_J^{[8]}$, and ${}^3S_1^{[8]}$–${}^3D_1^{[8]}$ mixing are shown in Figs. 1 and 2 as functions of p_T and z, respectively. The true results should lie inside the bands encompassed by the evaluations with $v^2 = 0.3$ and the LO results, corresponding to $v^2 = 0$. We observe from Fig. 1 that the $\mathcal{O}(v^2)$ corrections are generally more significant in the small-p_T range, while they tend to fade out for asymptotically large values of p_T, with the exception of λ_ϕ for $n = {}^3S_1^{[1]}$ and of λ_θ for ${}^3S_1^{[8]}$–${}^3D_1^{[8]}$ mixing. Because of the steep falloff of the p_T distributions, the z distributions in Fig. 2 receive dominant contributions from the p_T region close to the cut at 1 GeV, toward the left ends of the panels in Fig. 1. We observe from Fig. 2 that the $\mathcal{O}(v^2)$ corrections to λ_θ and $\lambda_{\theta\phi}$ tend to be more sizable toward large values of z. The $\mathcal{O}(v^2)$ corrections to λ_ϕ are relatively modest, except for the case of ${}^3S_1^{[8]}$–${}^3D_1^{[8]}$ mixing, where they are appreciable in the vicinity of $z = 0.7$.

The p_T dependence of the polarization in prompt J/ψ hadroproduction was measured in Tevatron Run II at $\sqrt{S} = 1.96$ TeV for $|y| < 0.6$ [3] and under CMS experimental conditions at $\sqrt{S} = 7$ TeV for $|y| < 2.4$ [9]. In both cases, we exclude from our considerations the small-p_T range, $p_T < 3$ GeV, where the application of

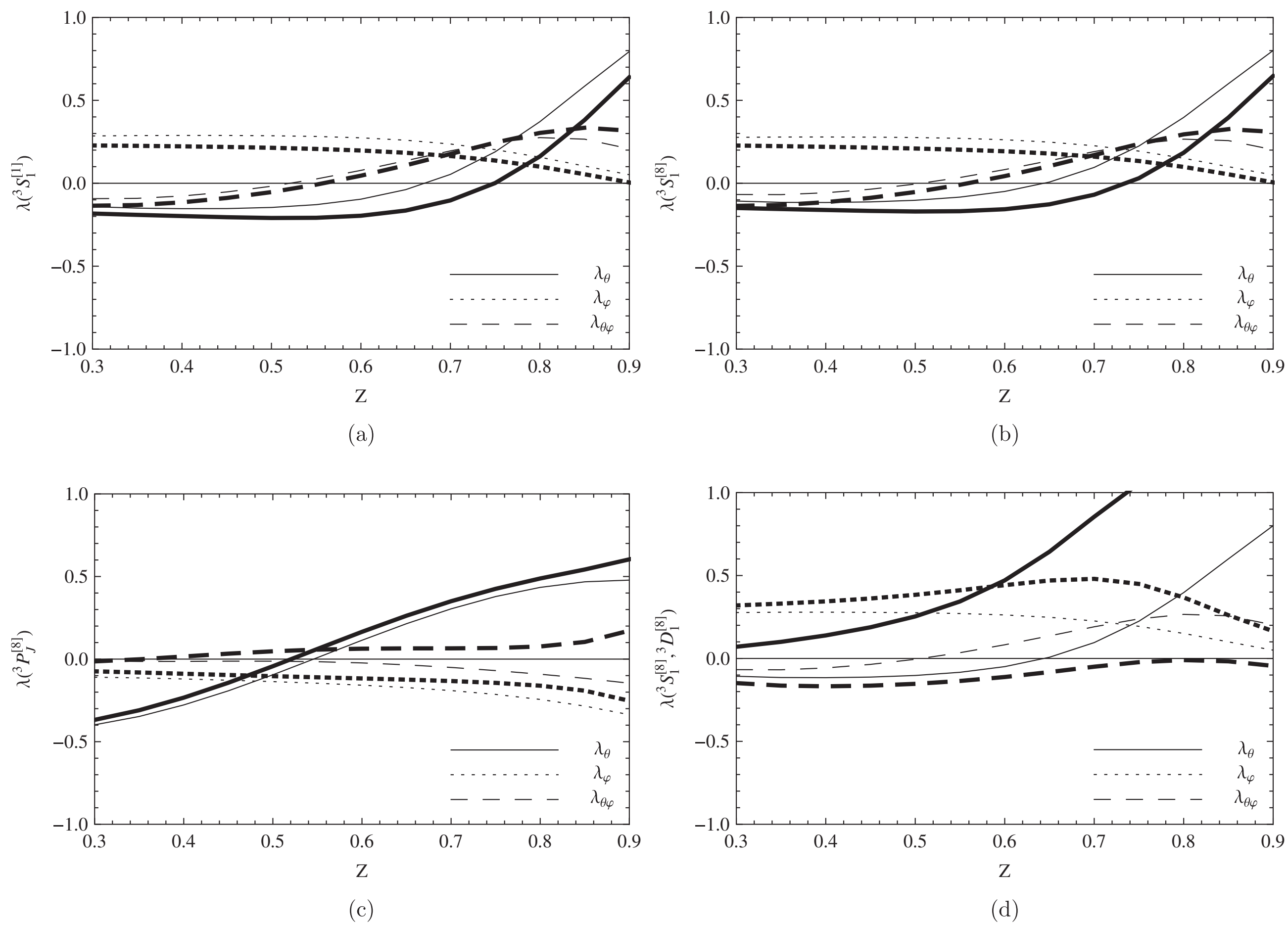

FIG. 2. Same as in Fig. 1, but as functions of z.

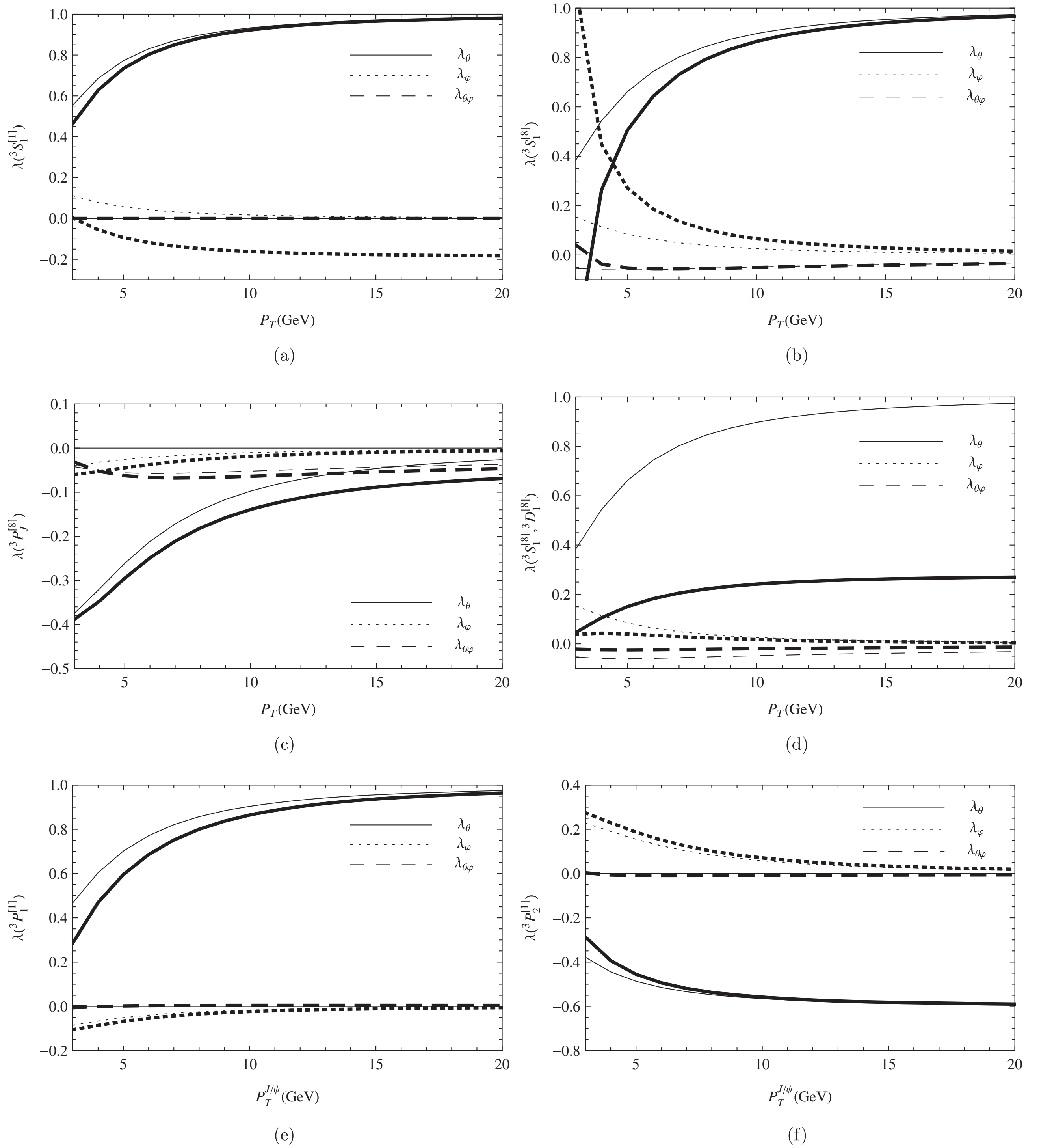

FIG. 3. $\mathcal{O}(v^2)$-corrected polarization parameters λ_θ (solid lines), λ_ϕ (dotted lines), and $\lambda_{\theta\phi}$ (dashed lines) in prompt J/ψ hadroproduction at Tevatron Run II [3] through the $c\bar{c}$ Fock states (a) $n = {}^3S_1^{[1]}$, (b) ${}^3S_1^{[8]}$, (c) ${}^3P_J^{[8]}$, (d) ${}^3S_1^{[8]}$–${}^3D_1^{[8]}$ mixing, (e) ${}^3P_1^{[1]}$, and (f) ${}^3P_2^{[1]}$ as functions of p_T. The true results should lie inside the bands encompassed by the evaluations with $v^2 = 0.3$ (thick lines) and the LO results, with $v^2 = 0$ (thin lines).

fixed-order perturbation theory is problematic. The $\mathcal{O}(v^2)$ corrections to the polarization parameters λ_θ, λ_ϕ, and $\lambda_{\theta\phi}$ in prompt hadroproduction through the $c\bar{c}$ Fock states $n = {}^3S_1^{[1,8]}$, ${}^3P_J^{[1,8]}$, and ${}^3S_1^{[8]}$–${}^3D_1^{[8]}$ mixing are shown as functions of p_T for Tevatron Run II [3] and the CMS setup [9] in Figs. 3 and 4, respectively. We observe from Fig. 3 that the $\mathcal{O}(v^2)$ corrections to λ_θ for $n = {}^3S_1^{[1]}, {}^3S_1^{[8]}, {}^3P_{1,2}^{[1]}$ are significant at small values of p_T but quickly fade out toward large values of p_T, while they increase with p_T for $n = {}^3P_J^{[8]}$ and ${}^3S_1^{[8]}$–${}^3D_1^{[8]}$ mixing. On the other hand, the $\mathcal{O}(v^2)$ corrections to λ_ϕ are generally small and fade out as the value of p_T increases, except for the case of $n = {}^3S_1^{[1]}$, where the $\mathcal{O}(v^2)$ correction tends toward a constant negative value for increasing value of p_T, while the LO result tends to zero. Finally, the $\mathcal{O}(v^2)$ corrections to $\lambda_{\theta\phi}$ are generally very small for all values of p_T. The situation is very similar for the CMS setup [9], as may be seen by comparing Figs. 3 and 4.

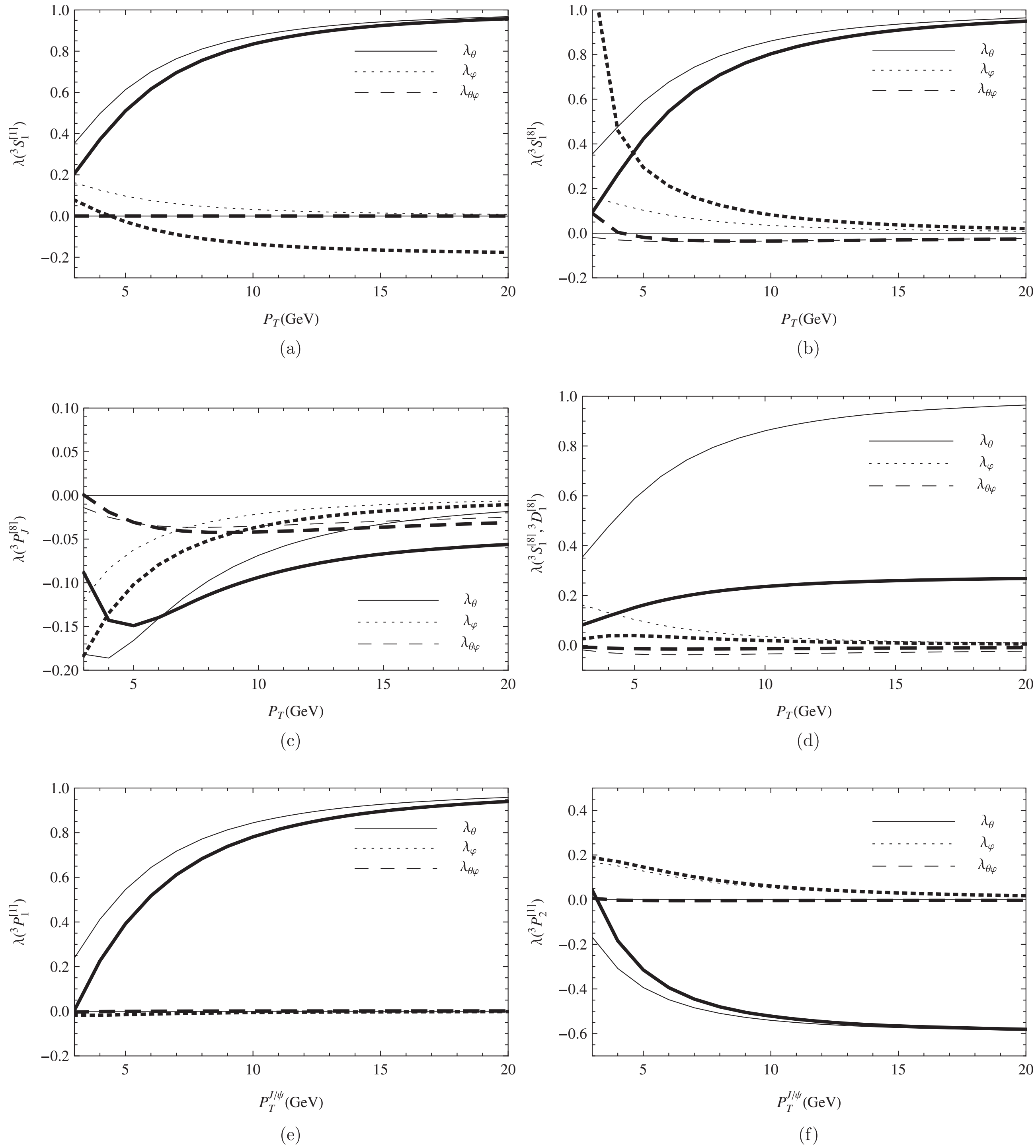

FIG. 4. Same as in Fig. 3, but for the CMS experimental setup at the LHC [9].

IV. SUMMARY

In this work, we systematically calculated the relativistic corrections of relative order $\mathcal{O}(v^2)$ to the polarization observables λ_θ, λ_ϕ, and $\lambda_{\theta\phi}$ in prompt J/ψ photoproduction and hadroproduction, including both the direct mode and the feed down from χ_{cJ} and ψ' mesons, and provided in analytic form the SDCs of all the relevant partonic subprocesses, which are listed in Eq. (5). This study is a natural extension of the one presented in Ref. [34], where the $\mathcal{O}(v^2)$ corrections to the unpolarized J/ψ yields in photoproduction and hadroproduction were considered. All these $\mathcal{O}(v^2)$ corrections are to be included along with the respective $\mathcal{O}(\alpha_s)$ corrections [14–23] to complete the NLO treatments. We obtained the χ_{cJ} feed-down contributions in a compact form with the structure that is familiar from direct production. We found agreement with the LO results for direct production [35] and χ_{cJ} feed down [38,40]. Our results for the $\mathcal{O}(v^2)$ corrections are new. Upon summation over the helicity labels, we recover from them our $\mathcal{O}(v^2)$ results for the unpolarized yield of prompt J/ψ production [34]. We numerically estimated the $\mathcal{O}(v^2)$ corrections to the J/ψ polarization parameters λ_θ, λ_ϕ, and $\lambda_{\theta\phi}$ in direct photoproduction at HERA [6] and prompt hadroproduction at the Tevatron [3] and the LHC [9] due to the relevant $c\bar{c}$ Fock states assuming that their LMDEs obey the velocity scaling rules [12]. We found that the shifts in λ_θ, λ_ϕ, and $\lambda_{\theta\phi}$ due to the $\mathcal{O}(v^2)$ corrections are significant in most cases, typically reaching ±0.2. Due to their nontrivial dependencies on p_T and z and the characteristic differences between photoproduction and hadroproduction, their inclusion in global data analyses of J/ψ yield and polarization, in addition to the available $\mathcal{O}(\alpha_s)$ corrections [14–23], may help to improve the quality of the determinations of the CO LDMEs and hopefully to remedy the J/ψ polarization crisis.

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APPENDIX: ANALYTIC RESULTS

In this Appendix, we present in compact analytic form the coefficients $A_k^{ij}(n)$ and $B_k^{ij}(n)$ defined in Eq. (6) for the partonic subprocesses in Eq. (5). We refrain from presenting $A_k^{ij}(n)$ for $n = {}^3S_1^{[1,8]}$, ${}^1S_0^{[8]}$, ${}^3P_J^{[8]}$ because these expressions may be found in Appendix B of Ref. [35]. We list $A^{ij}(n)$ for $n = {}^3P_{1,2}^{[1]}$ and $B^{ij}(n)$ for $n = {}^3S_1^{[1,8]}$, ${}^3P_J^{[8]}$, ${}^3P_{1,2}^{[1]}$, and ${}^3S_1^{[8]}$–${}^3D_1^{[8]}$ mixing. It is convenient to pull out common factors by writing $A_k^{ij}(n) = a^{ij}(n)g_k$ and $B_k^{ij}(n) = b^{ij}(n)h_k$. The Mandelstam variables s, t, and u appearing below are defined as $s = (k_1 + k_2)^2$, $t = (k_1 - P)^2|_{\mathbf{q}=0}$, and $u = (k_2 - P)^2|_{\mathbf{q}=0}$, respectively, and satisfy $s + t + u = 4m_c^2$.

1. Photoproduction

$g + \gamma \to c\bar{c}({}^3S_1^{[1]}) + g$:

$$b^{g\gamma}({}^3S_1^{[1]}) = \frac{4096\pi^3\alpha\alpha_s^2}{729m_c(4m_c^2 - s)^3(4m_c^2 - t)^3(s + t)^3} \tag{A1a}$$

$$\begin{aligned} h_1 = {} & 2048m_c^{10}(3s^2 + 2st + 3t^2) - 256m_c^8(5s^3 - 12s^2t + 5t^3) - 64m_c^6(15s^4 + 80s^3t \\ & + 90s^2t^2 + 44st^3 + 15t^4) + 16m_c^4(21s^5 + 103s^4t + 182s^3t^2 + 158s^2t^3 + 67st^4 + 21t^5) \\ & - 4m_c^2(7s^6 + 42s^5t + 99s^4t^2 + 124s^3t^3 + 87s^2t^4 + 30st^5 + 7t^6) \\ & + 3st(s + t)(s^2 + st + t^2)^2 \end{aligned} \tag{A1b}$$

$$\begin{aligned} h_2 = {} & 8[4096m_c^{12} - 768m_c^{10}(s - 3t) + 64m_c^8(5s^2 - 50st - 43t^2) + 16m_c^6(8s^3 + 67s^2t \\ & + 98st^2 + 47t^3) + 4m_c^4(2s^4 - 20s^3t - 59s^2t^2 - 58st^3 - 25t^4) + m_c^2t(-6s^4 - 4s^3t \\ & + 15s^2t^2 + 21st^3 + 12t^4) - t^2(s^2 + st + t^2)^2] \end{aligned} \tag{A1c}$$

$$\begin{aligned} h_3 = {} & 8[512m_c^8(s^2 + t^2) + 16m_c^6(13s^3 + 33s^2t + 9st^2 + 13t^3) - 4m_c^4(13s^4 + 58s^3t \\ & + 74s^2t^2 + 46st^3 + 25t^4) + m_c^2(s + t)(12s^4 + 29s^3t + 48s^2t^2 + 29st^3 + 12t^4) \\ & - (s + t)^2(s^2 + st + t^2)^2] \end{aligned} \tag{A1d}$$

$$h_4 = 8[128m_c^8(5s^2 - t^2) + 16m_c^6(5s^3 + 19s^2t + 14st^2 + 8t^3) + 4m_c^4(s^4 - 22s^3t - 49s^2t^2 - 36st^3 - 14t^4) + m_c^2t(7s^4 + 29s^3t + 48s^2t^2 + 34st^3 + 12t^4) - t(s+t)(s^2 + st + t^2)^2]. \tag{A1e}$$

$g + \gamma \to c\bar{c}({}^3S_1^{[8]}) + g$:

$$b^{g\gamma}({}^3S_1^{[8]}) = \frac{15}{8} b^{g\gamma}({}^3S_1^{[1]}). \tag{A2}$$

The coefficients h_k are the same as those in Eq. (A1).

$g + \gamma \to c\bar{c}({}^3P_J^{[8]}) + g$:

$$b^{g\gamma}({}^3P_J^{[8]}) = \frac{128\pi^3\alpha\alpha_s^2}{15m_c^5(s-4m_c^2)^4(t-4m_c^2)^4(s+t)^4(-4m_c^2+s+t)^2} \tag{A3a}$$

$$h_1 = (-4m_c^2 + s + t)[-524288m_c^{18}(s+t)(3s^2 + 14st + 3t^2) + 65536m_c^{16}(3s - t)(3s^3 - 3s^2t + 25st^2 - 9t^3) + 16384m_c^{14}(39s^5 + 421s^4t + 568s^3t^2 + 908s^2t^3 + 521st^4 + 39t^5) - 4096m_c^{12}(126s^6 + 1064s^5t + 2089s^4t^2 + 3138s^3t^3 + 2909s^2t^4 + 1064st^5 + 126t^6) + 1024m_c^{10}(144s^7 + 1050s^6t + 2473s^5t^2 + 4177s^4t^3 + 4857s^3t^4 + 3273s^2t^5 + 930st^6 + 144t^7) - 256m_c^8(75s^8 + 349s^7t + 816s^6t^2 + 1732s^5t^3 + 2564s^4t^4 + 2432s^3t^5 + 1376s^2t^6 + 249st^7 + 75t^8) + 64m_c^6(s+t)(15s^8 - 66s^7t - 314s^6t^2 - 374s^5t^3 - 230s^4t^4 - 254s^3t^5 + 26s^2t^6 - 106st^7 + 15t^8) + 16m_c^4st(s+t)(39s^7 + 148s^6t + 219s^5t^2 + 193s^4t^3 + 213s^3t^4 + 199s^2t^5 + 88st^6 + 39t^7) + 4m_c^2s^2t^2(s+t)(13s^6 + 90s^5t + 232s^4t^2 + 286s^3t^3 + 212s^2t^4 + 70st^5 + 13t^6) - 11s^3t^3(s+t)^2(s^2 + st + t^2)^2] \tag{A3b}$$

$$h_2 = -16m_c^2[-2097152m_c^{20}(s+t) + 131072m_c^{18}(31s^2 - 70st - 5t^2) - 65536m_c^{16}(18s^3 - 191s^2t - 180st^2 - 61t^3) - 8192m_c^{14}(26s^4 + 1098s^3t + 1287s^2t^2 + 776st^3 + 349t^4) + 4096m_c^{12}(42s^5 + 775s^4t + 1084s^3t^2 + 556s^2t^3 + 238st^4 + 161t^5) - 512m_c^{10}(79s^6 + 972s^5t + 1074s^4t^2 - 584s^3t^3 - 1622s^2t^4 - 1350st^5 - 209t^6) + 256m_c^8(16s^7 + 19s^6t - 602s^5t^2 - 1972s^4t^3 - 2788s^3t^4 - 2582s^2t^5 - 1612st^6 - 365t^7) - 32m_c^6(2s^8 - 246s^7t - 1554s^6t^2 - 4296s^5t^3 - 6520s^4t^4 - 6780s^3t^5 - 5509s^2t^6 - 3008st^7 - 673t^8) - 16m_c^4t(36s^8 + 205s^7t + 664s^6t^2 + 1260s^5t^3 + 1731s^4t^4 + 1755s^3t^5 + 1412s^2t^6 + 702st^7 + 143t^8) + 2m_c^2t^2(-26s^8 - 54s^7t + 5s^6t^2 + 228s^5t^3 + 648s^4t^4 + 790s^3t^5 + 669s^2t^6 + 296st^7 + 48t^8) - st^3(s+t)(6s^6 + 21s^5t + 34s^4t^2 + 42s^3t^3 + 32s^2t^4 + 19st^5 + 6t^6)] \tag{A3c}$$

$$h_3 = -16m_c^2[262144m_c^{16}(s+t)(s^2+t^2) - 8192m_c^{14}(21s^4+218s^3t+114s^2t^2+258st^3 + 21t^4) + 4096m_c^{12}(9s^5+210s^4t+328s^3t^2+648s^2t^3+295st^4+14t^5) + 2048m_c^{10}(34s^6+72s^5t+50s^4t^2-359s^3t^3-490s^2t^4-33st^5+14t^6) - 256m_c^8(s+t)(218s^6+821s^5t+1234s^4t^2+605s^3t^3-31s^2t^4+486st^5+143t^6) + 32m_c^6(s+t)(497s^7+2427s^6t+4644s^5t^2+5088s^4t^3+3648s^3t^4+2494s^2t^5 + 1697st^6+397t^7) - 16m_c^4(s+t)(125s^8+745s^7t+1732s^6t^2+2347s^5t^3+2260s^4t^4 + 1767s^3t^5+1162s^2t^6+575st^7+115t^8) + 2m_c^2(s+t)^2(48s^8+312s^7t+725s^6t^2 + 970s^5t^3+938s^4t^4+770s^3t^5+505s^2t^6+252st^7+48t^8) - st(s+t)^3(6s^6+21s^5t + 38s^4t^2+48s^3t^3+38s^2t^4+21st^5+6t^6)] \tag{A3d}$$

$$h_4 = -16m_c^2[1048576m_c^{18}(s-t)(s+t) - 98304m_c^{16}(5s^3-11st^2-6t^3) + 8192m_c^{14}(s^4-255s^3t-197s^2t^2-93st^3+8t^4) + 2048m_c^{12}(90s^5+897s^4t + 1281s^3t^2+645s^2t^3-87st^4-122t^5) - 512m_c^{10}(226s^6+1467s^5t+2568s^4t^2 + 1908s^3t^3+262s^2t^4-895st^5-372t^6) + 128m_c^8(223s^7+1197s^6t+2055s^5t^2 + 1372s^4t^3-390s^3t^4-1905s^2t^5-2010st^6-614t^7) - 32m_c^6(99s^8+404s^7t+282s^6t^2 - 989s^5t^3-2828s^4t^4-3803s^3t^5-3693s^2t^6-2248st^7-556t^8) + 8m_c^4(18s^9+16s^8t - 385s^7t^2-1503s^6t^3-2951s^5t^4-3933s^4t^5-3790s^3t^6-2794s^2t^7-1296st^8-258t^9) + 2m_c^2t(s+t)(16s^8+145s^7t+409s^6t^2+681s^5t^3+877s^4t^4+836s^3t^5+602s^2t^6 + 274st^7+48t^8) - st^2(s+t)^2(7s^6+23s^5t+39s^4t^2+47s^3t^3+36s^2t^4+20st^5+6t^6)]. \tag{A3e}$$

$g+\gamma \to c\bar{c}({}^3S_1^{[8]}, {}^3D_1^{[8]}) + g$:

$$b^{g\gamma}({}^3S_1^{[8]}, {}^3D_1^{[8]}) = \frac{1024\sqrt{\frac{5}{3}}\pi^3\alpha\alpha_s^2}{81m_c(s-4m_c^2)^4(t-4m_c^2)^4(s+t)^4} \tag{A4a}$$

$$h_1 = -(4m_c^2-s)(4m_c^2-t)(s+t)[2048m_c^{10}(3s^2+5st+3t^2) - 256m_c^8(5s^3+24s^2t + 24st^2+5t^3) - 64m_c^6(15s^4+2s^3t-18s^2t^2+2st^3+15t^4) + 16m_c^4(21s^5+31s^4t + 20s^3t^2+20s^2t^3+31st^4+21t^5) - 4m_c^2(7s^6+12s^5t+9s^4t^2-2s^3t^3+9s^2t^4+12st^5 + 7t^6) - 3st(s+t)(s^2+st+t^2)^2] \tag{A4b}$$

$$h_2 = -4(4m_c^2-t)[81920m_c^{14}(s+t) - 2048m_c^{12}(25s^2+52st+15t^2) + 512m_c^{10}(26s^3+75s^2t+36st^2-13t^3) - 128m_c^8(3s^4+18s^3t-24s^2t^2-92st^3 - 41t^4) - 32m_c^6(12s^5+39s^4t+100s^3t^2+184s^2t^3+166st^4+43t^5) + 8m_c^4(4s^6 + 30s^5t+81s^4t^2+140s^3t^3+161s^2t^4+106st^5+30t^6) - 2m_c^2t(6s^6+12s^5t+29s^4t^2 + 38s^3t^3+41s^2t^4+24st^5+10t^6) - st^2(s+t)(s^2+st+t^2)^2] \tag{A4c}$$

$$h_3 = -4(s+t)[4096m_c^{12}(s^2+t^2) + 512m_c^{10}(29s^3+61s^2t+61st^2+29t^3) - 256m_c^8(s+t)^2(37s^2+43st+37t^2) + 32m_c^6(73s^5+299s^4t+536s^3t^2+536s^2t^3 + 299st^4+73t^5) - 16m_c^4(20s^6+80s^5t+167s^4t^2+212s^3t^3+167s^2t^4+80st^5+20t^6) + 2m_c^2(10s^7+32s^6t+59s^5t^2+77s^4t^3+77s^3t^4+59s^2t^5+32st^6+10t^7) + st(s^3 + 2s^2t+2st^2+t^3)^2] \tag{A4d}$$

$$h_4 = -4[2048m_c^{12}(13s^3 + 7s^2t - 5st^2 - 11t^3) - 512m_c^{10}(14s^4 + 5s^3t - 61s^2t^2 - 95st^3 - 43t^4) - 128m_c^8(s^5 + 43s^4t + 214s^3t^2 + 340s^2t^3 + 265st^4 + 73t^5) + 32m_c^6(2s^6 + 71s^5t + 305s^4t^2 + 536s^3t^3 + 530s^2t^4 + 301st^5 + 71t^6) - 8m_c^4t(40s^6 + 167s^5t + 345s^4t^2 + 413s^3t^3 + 327s^2t^4 + 160st^5 + 40t^6) + 2m_c^2t(10s^7 + 30s^6t + 57s^5t^2 + 77s^4t^3 + 79s^3t^4 + 61s^2t^5 + 32st^6 + 10t^7) + st^2(s^3 + 2s^2t + 2st^2 + t^3)^2]. \tag{A4e}$$

$q(\bar{q}) + \gamma \to c\bar{c}({}^3S_1^{[8]}) + q(\bar{q})$:

$$b^{q(\bar{q})\gamma}({}^3S_1^{[8]}) = \frac{4\pi^3\alpha\alpha_s^2e_q^2}{27m_c^5st(4m_c^2 - s)} \tag{A5a}$$

$$h_1 = -640m_c^6 + 160m_c^4(2s + t) - 4m_c^2(27s^2 + 10st + 5t^2) + 11s(s^2 + t^2) \tag{A5b}$$

$$h_2 = -16m_c^2(44m_c^2 - 5s) \tag{A5c}$$

$$h_3 = -32m_c^2(44m_c^2 - 5s) \tag{A5d}$$

$$h_4 = -16m_c^2(44m_c^2 - 5s). \tag{A5e}$$

$q(\bar{q}) + \gamma \to c\bar{c}({}^3P_J^{[8]}) + q(\bar{q})$:

$$b^{q(\bar{q})\gamma}({}^3P_J^{[8]}) = \frac{256\pi^3\alpha\alpha_s^2}{135m_c^5(4m_c^2 - s)(s + t)^4(-4m_c^2 + s + t)^2} \tag{A6a}$$

$$h_1 = (s + t)(s + t - 4m_c^2)[1280m_c^6(s + 5t) - 128m_c^4t(6s + 5t) + 4m_c^2(21s^3 + 49s^2t + st^2 + 21t^3) - 11s(s^3 + s^2t + st^2 + t^3)] \tag{A6b}$$

$$h_2 = -32m_c^2[1024m_c^8 - 128m_c^6(s - 14t) + 16m_c^4(21s^2 + 14st - 29t^2) - 4m_c^2(4s^3 + 13s^2t + 54st^2 + 3t^3) + s(s^3 + s^2t + 3st^2 + 3t^3)] \tag{A6c}$$

$$h_3 = -64m_c^2(s + t)[80m_c^4(3s + 7t) - 8m_c^2(3s^2 + 11st + 8t^2) + s(s + t)^2] \tag{A6d}$$

$$h_4 = -64m_c^2[64m_c^6(s - t) + 8m_c^4(19s^2 + 72st + 19t^2) - 2m_c^2(s^3 + 27s^2t + 43st^2 + 17t^3) + st(s + t)^2]. \tag{A6e}$$

$q(\bar{q}) + \gamma \to c\bar{c}({}^3S_1^{[8]}, {}^3D_1^{[8]}) + q(\bar{q})$:

$$b^{q(\bar{q})\gamma}({}^3S_1^{[8]}, {}^3D_1^{[8]}) = \frac{8\sqrt{\frac{5}{3}}\pi^3\alpha\alpha_s^2e_q^2}{9m_c^5st} \tag{A7a}$$

$$h_1 = 32m_c^4 - 8m_c^2(s + t) + s^2 + t^2 \tag{A7b}$$

$$h_2 = 16m_c^2 \tag{A7c}$$

$$h_3 = 32m_c^2 \tag{A7d}$$

$$h_4 = 16m_c^2. \tag{A7e}$$

2. Hadroproduction

$g + g \to c\bar{c}({}^3S_1^{[1]}) + g$:

$$b^{gg}({}^3S_1^{[1]}) = \frac{15\alpha_s}{128\alpha} b^{g\gamma}({}^3S_1^{[1]}). \tag{A8}$$

The coefficients h_k are the same as those in Eq. (A1).

$g + g \to c\bar{c}({}^3P_1^{[1]}) + g$:

$$a^{gg}({}^3P_1^{[1]}) = \frac{16\pi^3\alpha_s^3}{9m_c^3(s-4m_c^2)^4(t-4m_c^2)^4(s+t)^4} \tag{A9a}$$

$$\begin{aligned} g_1 = {} & (s-4m_c^2)(4m_c^2-t)(s+t)[16m_c^4(s^2+st+t^2) - 4m_c^2(2s^3+3s^2t+3st^2 \\ & + 2t^3) + (s^2+st+t^2)^2][16m_c^6(s+t) - 4m_c^4(5s^2+6st+5t^2) + m_c^2(8s^3+13s^2t \\ & + 13st^2+8t^3) - (s^2+st+t^2)^2] \end{aligned} \tag{A9b}$$

$$\begin{aligned} g_2 = {} & -4m_c^2(t-4m_c^2)^2[512m_c^{10}(s+t)^2 - 768m_c^8(s+t)^3 + 32m_c^6(13s^4+56s^3t \\ & + 86s^2t^2+58st^3+15t^4) - 16m_c^4(6s^5+40s^4t+82s^3t^2+85s^2t^3+45st^4+10t^5) \\ & + 2m_c^2(4s^6+54s^5t+137s^4t^2+192s^3t^3+153s^2t^4+70st^5+14t^6) \\ & - t(s^2+st+t^2)^2(7s^2+7st+2t^2)] \end{aligned} \tag{A9c}$$

$$\begin{aligned} g_3 = {} & 4m_c^2(s+t)^2[512m_c^{10}(s^2+t^2) - 256m_c^8(3s^3+2s^2t+2st^2+3t^3) \\ & + 32m_c^6(15s^4+18s^3t+10s^2t^2+18st^3+15t^4) - 16m_c^4(10s^5+15s^4t+9s^3t^2+9s^2t^3 \\ & + 15st^4+10t^5) + 2m_c^2(14s^6+22s^5t+17s^4t^2+8s^3t^3+17s^2t^4+22st^5+14t^6) \\ & - (s^2+st+t^2)^2(2s^3-s^2t-st^2+2t^3)] \end{aligned} \tag{A9d}$$

$$\begin{aligned} g_4 = {} & 4m_c^2(s-4m_c^2)[512m_c^{10}(s-t)(s+t)^2 - 128m_c^8(7s^4+14s^3t+2s^2t^2-12st^3 \\ & - 7t^4) + 32m_c^6(18s^5+53s^4t+46s^3t^2-18s^2t^3-44st^4-19t^5) - 8m_c^4(22s^6+86s^5t \\ & + 119s^4t^2+46s^3t^3-60s^2t^4-72st^5-25t^6) + 2m_c^2(14s^7+64s^6t+114s^5t^2+95s^4t^3 \\ & + 3s^3t^4-63s^2t^5-55st^6-16t^7) - (s^2+st+t^2)^2(2s^4+5s^3t+3s^2t^2-4st^3-2t^4)] \end{aligned} \tag{A9e}$$

$$b^{gg}({}^3P_1^{[1]}) = \frac{8\pi^3\alpha_s^3}{45m_c^5(4m_c^2-s)^5(4m_c^2-t)^5(s+t)^5} \tag{A9f}$$

$$\begin{aligned} h_1 = {} & (4m_c^2-s)(4m_c^2-t)(s+t)[65536m_c^{16}(s^3+s^2t+st^2+t^3) - 4096m_c^{14}(7s^4 \\ & - 73s^3t-60s^2t^2-53st^3+7t^4) - 1024m_c^{12}(132s^5+853s^4t+1235s^3t^2+1115s^2t^3 \\ & + 573st^4+92t^5) + 256m_c^{10}(686s^6+3499s^5t+6337s^4t^2+6952s^3t^3+5057s^2t^4 \\ & + 2179st^5+446t^6) - 64m_c^8(1420s^7+7165s^6t+15279s^5t^2+20080s^4t^3+18020s^3t^4 \\ & + 11059s^2t^5+4205st^6+860t^7) + 16m_c^6(1485s^8+7888s^7t+19416s^6t^2+30007s^5t^3 \\ & + 32040s^4t^4+24567s^3t^5+13136s^2t^6+4428st^7+845t^8) - 4m_c^4(780s^9+4522s^8t \\ & + 12768s^7t^2+22889s^6t^3+28781s^5t^4+26401s^4t^5+17649s^3t^6+8348s^2t^7+2482st^8 \\ & + 420t^9) + m_c^2(s^2+st+t^2)^2(164s^6+792s^5t+1609s^4t^2+1762s^3t^3+1289s^2t^4 \\ & + 472st^5+84t^6) - 11st(s+t)(s^2+st+t^2)^4] \end{aligned} \tag{A9g}$$

$$h_2 = 4m_c^2(t-4m_c^2)^2[8192m_c^{14}(s+t)^2(19s+39t) - 2048m_c^{12}(161s^4+696s^3t + 1206s^2t^2+928st^3+257t^4) + 512m_c^{10}(473s^5+2320s^4t+5423s^3t^2+6065s^2t^3 + 3340st^4+723t^5) - 128m_c^8(623s^6+3752s^5t+11313s^4t^2+16704s^3t^3+13939s^2t^4 + 6152st^5+1149t^6) + 32m_c^6(376s^7+3161s^6t+11876s^5t^2+22019s^4t^3+24449s^3t^4 + 16498s^2t^5+6351st^6+1086t^7) - 8m_c^4(84s^8+1328s^7t+6071s^6t^2+13796s^5t^3 + 18908s^4t^4+17172s^3t^5+10043s^2t^6+3536st^7+566t^8) + 2m_c^2t(218s^8+1208s^7t + 3361s^6t^2+5431s^5t^3+6167s^4t^4+4797s^3t^5+2558s^2t^6+832st^7+124t^8) - st^2(s^2+st+t^2)^2(7s^3+14s^2t+9st^2+2t^3)] \tag{A9h}$$

$$h_3 = -4m_c^2(s+t)^2[262144m_c^{16}(s^2+t^2) - 8192m_c^{14}(9s^3-19s^2t+21st^2+9t^3) - 2048m_c^{12}(157s^4+400s^3t+218s^2t^2+120st^3+137t^4) + 512m_c^{10}(683s^5+1914s^4t + 1569s^3t^2+849s^2t^3+994st^4+563t^5) - 128m_c^8(1321s^6+4100s^5t+4181s^4t^2 + 2372s^3t^3+2001s^2t^4+2340st^5+1021t^6) + 32m_c^6(1382s^7+4555s^6t+5490s^5t^2 + 3491s^4t^3+2011s^3t^4+2610s^2t^5+2595st^6+1022t^7) - 8m_c^4(750s^8+2584s^7t + 3563s^6t^2+2580s^5t^3+1242s^4t^4+1100s^3t^5+1703s^2t^6+1424st^7+550t^8) + 2m_c^2(164s^9+600s^8t+926s^7t^2+753s^6t^3+383s^5t^4+183s^4t^5+313s^3t^6+446s^2t^7 + 320st^8+124t^9) - st(2s^2-3st+2t^2)(s^3+2s^2t+2st^2+t^3)^2] \tag{A9i}$$

$$h_4 = 4m_c^2(4m_c^2-s)[8192m_c^{14}(13s^4+76s^3t+42s^2t^2-52st^3-31t^4) - 4096m_c^{12}(92s^5+435s^4t+486s^3t^2+13s^2t^3-322st^4-128t^5) + 512m_c^{10}(729s^6 + 3578s^5t+6026s^4t^2+3382s^3t^3-1629s^2t^4-2648st^5-846t^6) - 256m_c^8(664s^7 + 3613s^6t+7719s^5t^2+7378s^4t^3+1751s^3t^4-2993s^2t^5-2634st^6-722t^7) + 32m_c^6(1324s^8+7870s^7t+19424s^6t^2+24728s^5t^3+14981s^4t^4-1074s^3t^5 - 8562s^2t^6-5580st^7-1351t^8) - 16m_c^4(359s^9+2242s^8t+6074s^7t^2+9229s^6t^3 + 7941s^5t^4+2897s^4t^5-1764s^3t^6-2790s^2t^7-1522st^8-330t^9) + 2m_c^2(164s^{10} + 1060s^9t+3088s^8t^2+5238s^7t^3+5563s^6t^4+3338s^5t^5+368s^4t^6-1476s^3t^7-1411s^2t^8 - 680st^9-132t^{10}) - st(s^2+st+t^2)^2(2s^5+7s^4t+8s^3t^2-s^2t^3-6st^4-2t^5)]. \tag{A9j}$$

$g+g \to c\bar{c}(^3P_2^{[1]})+g$:

$$a^{gg}(^3P_2^{[1]}) = -\frac{16\pi^3\alpha_s^3}{135m_c^3st(s-4m_c^2)^4(t-4m_c^2)^4(s+t)^4(-4m_c^2+s+t)} \tag{A10a}$$

$$
\begin{aligned}
g_1 = {} & -18874368m_c^{20}(s+t)^4 + 9437184m_c^{18}(s+t)^3(3s^2+5st+3t^2) \\
& - 16384m_c^{16}(s+t)^2(1224s^4+3897s^3t+5297s^2t^2+3897st^3+1224t^4) \\
& + 4096m_c^{14}(2160s^7+11913s^6t+29647s^5t^2+44652s^4t^3+44652s^3t^4+29647s^2t^5 \\
& + 11913st^6+2160t^7) - 1024m_c^{12}(2592s^8+14994s^7t+39656s^6t^2+65365s^5t^3 \\
& + 76238s^4t^4+65365s^3t^5+39656s^2t^6+14994st^7+2592t^8) + 256m_c^{10}(2160s^9 \\
& + 13374s^8t+37078s^7t^2+63603s^6t^3+79713s^5t^4+79713s^4t^5+63603s^3t^6+37078s^2t^7 \\
& + 13374st^8+2160t^9) - 64m_c^8(1224s^{10}+8445s^9t+24967s^8t^2+43880s^7t^3+55214s^6t^4 \\
& + 58192s^5t^5+55214s^4t^6+43880s^3t^7+24967s^2t^8+8445st^9+1224t^{10}) + 16m_c^6(432s^{11} \\
& + 3501s^{10}t+11595s^9t^2+21568s^8t^3+26848s^7t^4+27020s^6t^5+27020s^5t^6+26848s^4t^7 \\
& + 21568s^3t^8+11595s^2t^9+3501st^{10}+432t^{11}) - 4m_c^4(72s^{12}+792s^{11}t+3218s^{10}t^2 \\
& + 6815s^9t^3+8979s^8t^4+8574s^7t^5+7840s^6t^6+8574s^5t^7+8979s^4t^8+6815s^3t^9 \\
& + 3218s^2t^{10}+792st^{11}+72t^{12}) + m_c^2st(s^2+st+t^2)^2(60s^7+264s^6t+341s^5t^2+71s^4t^3 \\
& + 71s^3t^4+341s^2t^5+264st^6+60t^7) - s^2t^2(s+t)^2(s^2+st+t^2)^4
\end{aligned}
\tag{A10b}
$$

$$
\begin{aligned}
g_2 = {} & -12m_c^2(4m_c^2-t)[8192m_c^{14}(s+t)^2(12s^2+47st+24t^2) - 2048m_c^{12}(56s^5 \\
& + 345s^4t+722s^3t^2+717s^2t^3+352st^4+68t^5) + 512m_c^{10}(112s^6+741s^5t+1745s^4t^2 \\
& + 2129s^3t^3+1501s^2t^4+552st^5+72t^6) - 128m_c^8(128s^7+919s^6t+2368s^5t^2 \\
& + 3306s^4t^3+2900s^3t^4+1591s^2t^5+480st^6+48t^7) + 32m_c^6(92s^8+704s^7t+1933s^6t^2 \\
& + 2902s^5t^3+2844s^4t^4+1945s^3t^5+905s^2t^6+259st^7+40t^8) - 8m_c^4(40s^9+324s^8t \\
& + 952s^7t^2+1461s^6t^3+1416s^5t^4+983s^4t^5+558s^3t^6+298s^2t^7+132st^8+36t^9) \\
& + 2m_c^2(8s^{10}+80s^9t+274s^8t^2+432s^7t^3+355s^6t^4+133s^5t^5+5s^4t^6+71s^3t^7+132s^2t^8 \\
& + 94st^9+24t^{10}) - t(s^2+st+t^2)^2(4s^6+12s^5t+s^4t^2-18s^3t^3-s^2t^4+10st^5+4t^6)]
\end{aligned}
\tag{A10c}
$$

$$
\begin{aligned}
g_3 = {} & -12m_c^2(s+t)[1572864m_c^{16}(s+t)^3 - 294912m_c^{14}(s+t)^2(7s^2+10st+7t^2) \\
& + 2048m_c^{12}(588s^5+2257s^4t+3917s^3t^2+3917s^2t^3+2257st^4+588t^5) \\
& - 512m_c^{10}(824s^6+3435s^5t+6696s^4t^2+8202s^3t^3+6696s^2t^4+3435st^5+824t^6) \\
& + 128m_c^8(784s^7+3461s^6t+7141s^5t^2+9650s^4t^3+9650s^3t^4+7141s^2t^5+3461st^6 \\
& + 784t^7) - 32m_c^6(524s^8+2459s^7t+5234s^6t^2+7181s^5t^3+7796s^4t^4+7181s^3t^5 \\
& + 5234s^2t^6+2459st^7+524t^8) + 8m_c^4(236s^9+1218s^8t+2722s^7t^2+3705s^6t^3 \\
& + 3905s^5t^4+3905s^4t^5+3705s^3t^6+2722s^2t^7+1218st^8+236t^9) - 2m_c^2(s+t)^2(64s^8 \\
& + 246s^7t+356s^6t^2+337s^5t^3+274s^4t^4+337s^3t^5+356s^2t^6+246st^7+64t^8) \\
& + (s^2+st+t^2)^2(4s^7+18s^6t+23s^5t^2+3s^4t^3+3s^3t^4+23s^2t^5+18st^6+4t^7)]
\end{aligned}
\tag{A10d}
$$

$$g_4 = -12m_c^2[196608m_c^{16}(s+t)^3(3s+5t) - 73728m_c^{14}(s+t)^2(12s^3+33s^2t+35st^2 + 16t^3) + 2048m_c^{12}(280s^6+1369s^5t+3023s^4t^2+3917s^3t^3+3151s^2t^4+1476st^5 + 308t^6) - 512m_c^{10}(416s^7+2129s^6t+5074s^5t^2+7460s^4t^3+7438s^3t^4+5057s^2t^5 + 2130st^6+408t^7) + 128m_c^8(394s^8+2169s^7t+5479s^6t^2+8608s^5t^3+9650s^4t^4 + 8183s^3t^5+5123s^2t^6+2076st^7+390t^8) - 32m_c^6(244s^9+1504s^8t+4073s^7t^2 + 6653s^6t^3+7734s^5t^4+7243s^4t^5+5762s^3t^6+3620s^2t^7+1479st^8+280t^9) + 8m_c^4(92s^{10}+674s^9t+2054s^8t^2+3591s^7t^3+4219s^6t^4+3905s^5t^5+3391s^4t^6 + 2836s^3t^7+1886s^2t^8+780st^9+144t^{10}) - 2m_c^2(16s^{11}+162s^{10}t+606s^9t^2+1224s^8t^3 + 1555s^7t^4+1400s^6t^5+1126s^5t^6+1044s^4t^7+983s^3t^8+680s^2t^9+276st^{10}+48t^{11}) + t(s^2+st+t^2)^2(6s^7+23s^6t+28s^5t^2+3s^4t^3-2s^3t^4+18s^2t^5+16st^6+4t^7)] \tag{A10e}$$

$$b^{gg}({}^3P_2^{[1]}) = \frac{8\pi^3\alpha_s^3}{675m_c^5s^2t^2(4m_c^2-s)^5(4m_c^2-t)^5(s+t)^5(-4m_c^2+s+t)^2} \tag{A10f}$$

$$h_1 = -st(-4m_c^2+s+t)[301989888(s+t)^3(15s^2+38ts+15t^2)m_c^{24} - 8388608(s+t)^2(945s^4+3943ts^3+5985t^2s^2+3763t^3s+945t^4)m_c^{22} + 262144(27624s^7+179241ts^6+503190t^2s^5+788233t^3s^4+755573t^4s^3+450370t^5s^2 + 156201t^6s+24744t^7)m_c^{20} - 65536(65928s^8+438138ts^7+1296615t^2s^6 + 2241746t^3s^5+2524170t^4s^4+1935826t^5s^3+1003615t^6s^2+325218t^7s+50088t^8)m_c^{18} + 16384(108144s^9+742491ts^8+2285700t^2s^7+4194310t^3s^6+5180803t^4s^5 + 4615783t^5s^4+3046670t^6s^3+1457000t^7s^2+456891t^8s+69264t^9)m_c^{16} - 4096(124080s^{10}+900416ts^9+2906497t^2s^8+5577856t^3s^7+7244723t^4s^6 + 6963392t^5s^5+5278403t^6s^4+3209416t^7s^3+1491977t^8s^2+460856t^9s+67920t^{10})m_c^{14} + 1024(99240s^{11}+779395ts^{10}+2690450t^2s^9+5443508t^3s^8+7327604t^4s^7 + 7226103t^5s^6+5774383t^6s^5+4032804t^7s^4+2433368t^8s^3+1137870t^9s^2+343315t^{10}s + 47400t^{11})m_c^{12} - 256(53064s^{12}+463250ts^{11}+1757329t^2s^{10}+3842678t^3s^9 + 5442560t^4s^8+5431020t^5s^7+4269078t^6s^6+3095980t^7s^5+2225920t^8s^4+1409878t^9s^3 + 655689t^{10}s^2+185450t^{11}s+22824t^{12})m_c^{10} + 64(16968s^{13}+173309ts^{12}+751960t^2s^{11} + 1842678t^3s^{10}+2850960t^4s^9+2968913t^5s^8+2256004t^6s^7+1525084t^7s^6 + 1187673t^8s^5+968900t^9s^4+624838t^{10}s^3+270020t^{11}s^2+66749t^{12}s+6888t^{13})m_c^8 - 16(2424s^{14}+34060ts^{13}+183551t^2s^{12}+532924t^3s^{11}+949514t^4s^{10}+1102104t^5s^9 + 872595t^6s^8+540792t^7s^7+397555t^8s^6+391504t^9s^5+330714t^{10}s^4+190964t^{11}s^3 + 68951t^{12}s^2+13300t^{13}s+984t^{14})m_c^6 + 4st(2116s^{13}+19052ts^{12}+74123t^2s^{11} + 165228t^3s^{10}+235243t^4s^9+227226t^5s^8+164836t^6s^7+119456t^7s^6+113946t^8s^5 + 107443t^9s^4+73328t^{10}s^3+32643t^{11}s^2+8372t^{12}s+916t^{13})m_c^4 - s^2t^2(s^3+2ts^2 + 2t^2s+t^3)^2(360s^6+1356ts^5+1421t^2s^4+494t^3s^3+1101t^4s^2+1036t^5s+280t^6)m_c^2 + 7s^3t^3(s+t)^3(s^2+ts+t^2)^4] \tag{A10g}$$

$$h_2 = -12m_c^2(4m_c^2-t)[16777216st(s+t)^2(8s^2-9ts+2t^2)m_c^{22}+524288(32s^7-636ts^6-2381t^2s^5-4579t^3s^4 -5103t^4s^3-2877t^5s^2-608t^6s+32t^7)m_c^{20}-131072(224s^8-2256ts^7-13229t^2s^6-34841t^3s^5-50353t^4s^4 -39495t^5s^3-16038t^6s^2-2292t^7s+256t^8)m_c^{18}+32768(672s^9-4044ts^8-34626t^2s^7-111888t^3s^6 -195319t^4s^5-196987t^5s^4-116723t^6s^3-35861t^7s^2-2592t^8s+896t^9)m_c^{16}-8192(1120s^{10}-4264ts^9 -54962t^2s^8-207478t^3s^7-416241t^4s^6-501995t^5s^5-383465t^6s^4-180583t^7s^3-41448t^8s^2+1468t^9s +1792t^{10})m_c^{14}+2048(1120s^{11}-3164ts^{10}-59449t^2s^9-249681t^3s^8-549542t^4s^7-743186t^5s^6-667344t^6s^5 -410272t^7s^4-160085t^8s^3-25309t^9s^2+6312t^{10}s+2240t^{11})m_c^{12}-512(672s^{12}-2176ts^{11}-46457t^2s^{10} -206373t^3s^9-475554t^4s^8-672814t^5s^7-639440t^6s^6-444400t^7s^5-235223t^8s^4-84109t^9s^3-8818t^{10}s^2 +6012t^{11}s+1792t^{12})m_c^{10}+128(224s^{13}-1396ts^{12}-26208t^2s^{11}-119918t^3s^{10}-279940t^4s^9-383196t^5s^8 -319618t^6s^7-174546t^7s^6-88697t^8s^5-63753t^9s^4-36485t^{10}s^3-7707t^{11}s^2+1808t^{12}s+896t^{13})m_c^8-32(32s^{14} -584ts^{13}-9872t^2s^{12}-47988t^3s^{11}-115308t^4s^{10}-146248t^5s^9-68062t^6s^8+62478t^7s^7+108433t^8s^6+47003t^9s^5 -20127t^{10}s^4-29125t^{11}s^3-11012t^{12}s^2-972t^{13}s+256t^{14})m_c^6-8t(104s^{14}+2056ts^{13}+11846t^2s^{12}+32068t^3s^{11} +41516t^4s^{10}+3659t^5s^9-73197t^6s^8-117854t^7s^7-85086t^8s^6-15497t^9s^5+23451t^{10}s^4+20944t^{11}s^3+7342t^{12}s^2 +952t^{13}s-32t^{14})m_c^4+2st^2(144s^{13}+1304ts^{12}+4612t^2s^{11}+7188t^3s^{10}+183t^4s^9-19901t^5s^8-38638t^6s^7 -36862t^7s^6-15497t^8s^5+4991t^9s^4+10688t^{10}s^3+6400t^{11}s^2+1860t^{12}s+232t^{13})m_c^2 -s^2t^3(s^3+2ts^2+2t^2s+t^3)^2(4s^6+12ts^5-83t^2s^4-186t^3s^3-21t^4s^2+74t^5s+20t^6)] \tag{A10h}$$

$$h_3 = 12m_c^2(s+t)[1207959552st(s+t)^2(2s^2+5ts+2t^2)m_c^{22}-2097152st(2003s^5+10179ts^4+20112t^2s^3+19932t^3s^2 +10089t^4s+2093t^5)m_c^{20}-131072(32s^8-26276ts^7-146167t^2s^6-337414t^3s^5-428510t^4s^4-324974t^5s^3 -140887t^6s^2-27476t^7s+32t^8)m_c^{18}+32768(224s^9-54588ts^8-323187t^2s^7-823603t^3s^6-1219834t^4s^5 -1174194t^5s^4-747863t^6s^3-293687t^7s^2-54228t^8s+224t^9)m_c^{16}-8192(672s^{10}-80108ts^9-497784t^2s^8 -1348997t^3s^7-2185606t^4s^6-2420290t^5s^5-1940166t^6s^4-1111197t^7s^3-411864t^8s^2-72268t^9s+672t^{10})m_c^{14} +4096(560s^{11}-43042ts^{10}-281992t^2s^9-800424t^3s^8-1361965t^4s^7-1614273t^5s^6-1466973t^6s^5-1057525t^7s^4 -577964t^8s^3-207612t^9s^2-34462t^{10}s+560t^{11})m_c^{12}-512(1120s^{12}-67652ts^{11}-474251t^2s^{10}-1417688t^3s^9 -2502261t^4s^8-3032698t^5s^7-2861316t^6s^6-2322578t^7s^5-1630861t^8s^4-889688t^9s^3-311491t^{10}s^2-48212t^{11}s +1120t^{12})m_c^{10}+128(672s^{13}-38036ts^{12}-288627t^2s^{11}-922087t^3s^{10}-1708003t^4s^9-2099899t^5s^8-1933612t^6s^7 -1574572t^7s^6-1275799t^8s^5-940063t^9s^4-517827t^{10}s^3-173847t^{11}s^2-24636t^{12}s+672t^{13})m_c^8 -32(224s^{14}-14628ts^{13}-120242t^2s^{12}-415269t^3s^{11}-821945t^4s^{10}-1045419t^5s^9-925223t^6s^8-670140t^7s^7 -542503t^8s^6-510619t^9s^5-406585t^{10}s^4-220149t^{11}s^3-69042t^{12}s^2-8868t^{13}s+224t^{14})m_c^6+8(32s^{15} -3488ts^{14}-30854t^2s^{13}-115762t^3s^{12}-247199t^4s^{11}-330131t^5s^{10}-280925t^6s^9-153705t^7s^8-83045t^8s^7 -107385t^9s^6-142151t^{10}s^5-119459t^{11}s^4-61242t^{12}s^3-17534t^{13}s^2-2048t^{14}s+32t^{15})m_c^4+2st(s+t)^2(392s^{12} +2996ts^{11}+9224t^2s^{10}+14502t^3s^9+10803t^4s^8-1352t^5s^7-10146t^6s^6-7032t^7s^5+2203t^8s^4+6782t^9s^3 +5064t^{10}s^2+1756t^{11}s+232t^{12})m_c^2-s^2t^2(s+t)^3(s^2+ts+t^2)^2(20s^6+46ts^5-91t^2s^4-238t^3s^3-91t^4s^2 +46t^5s+20t^6)] \tag{A10i}$$

$$
\begin{aligned}
h_4 = {} & -12m_c^2[201326592s(s-t)t(s+t)^3m_c^{24} - 4194304st(s+t)^2(291s^3+899ts^2 \\
& + 997t^2s + 285t^3)m_c^{22} + 524288(32s^8 + 3928ts^7 + 22769t^2s^6 + 56744t^3s^5 + 76938t^4s^4 \\
& + 59740t^5s^3 + 25149t^6s^2 + 4444t^7s - 32t^8)m_c^{20} - 131072(224s^9 + 13750ts^8 \\
& + 84023t^2s^7 + 231650t^3s^6 + 369902t^4s^5 + 371442t^5s^4 + 235061t^6s^3 + 86390t^7s^2 \\
& + 13894t^8s - 256t^9)m_c^{18} + 32768(672s^{10} + 30316ts^9 + 191386t^2s^8 + 564744t^3s^7 \\
& + 1001907t^4s^6 + 1182124t^5s^5 + 960430t^6s^4 + 525736t^7s^3 + 174749t^8s^2 + 25168t^9s \\
& - 896t^{10})m_c^{16} - 8192(1120s^{11} + 45670ts^{10} + 298834t^2s^9 + 931048t^3s^8 + 1770223t^4s^7 \\
& + 2303172t^5s^6 + 2179344t^6s^5 + 1526640t^7s^4 + 760913t^8s^3 + 236598t^9s^2 + 30486t^{10}s \\
& - 1792t^{11})m_c^{14} + 2048(1120s^{12} + 48192ts^{11} + 333351t^2s^{10} + 1101380t^3s^9 + 2213799t^4s^8 \\
& + 3049824t^5s^7 + 3122706t^6s^6 + 2517072t^7s^5 + 1614829t^8s^4 + 772192t^9s^3 + 231987t^{10}s^2 \\
& + 27212t^{11}s - 2240t^{12})m_c^{12} - 512(672s^{13} + 35098ts^{12} + 264391t^2s^{11} + 944150t^3s^{10} \\
& + 2026693t^4s^9 + 2927376t^5s^8 + 3093530t^6s^7 + 2615244t^7s^6 + 1916873t^8s^5 + 1212706t^9s^4 \\
& + 586319t^{10}s^3 + 174642t^{11}s^2 + 19394t^{12}s - 1792t^{13})m_c^{10} + 128(224s^{14} + 16740ts^{13} \\
& + 142436t^2s^{12} + 565020t^3s^{11} + 1329468t^4s^{10} + 2056868t^5s^9 + 2236868t^6s^8 + 1858992t^7s^7 \\
& + 1367893t^8s^6 + 1012724t^9s^5 + 691232t^{10}s^4 + 349104t^{11}s^3 + 103807t^{12}s^2 + 11424t^{13}s \\
& - 896t^{14})m_c^8 - 32(32s^{15} + 4676ts^{14} + 47458t^2s^{13} + 216720t^3s^{12} + 577256t^4s^{11} \\
& + 990974t^5s^{10} + 1149214t^6s^9 + 936240t^7s^8 + 606773t^8s^7 + 438198t^9s^6 + 396540t^{10}s^5 \\
& + 310428t^{11}s^4 + 161311t^{12}s^3 + 47012t^{13}s^2 + 5248t^{14}s - 256t^{15})m_c^6 + 8t(576s^{15} \\
& + 8074ts^{14} + 45396t^2s^{13} + 142830t^3s^{12} + 281476t^4s^{11} + 360889t^5s^{10} + 296852t^6s^9 \\
& + 148525t^7s^8 + 60588t^8s^7 + 78887t^9s^6 + 117204t^{10}s^5 + 101911t^{11}s^4 + 51660t^{12}s^3 \\
& + 14268t^{13}s^2 + 1600t^{14}s - 32t^{15})m_c^4 - 2st^2(s+t)^2(396s^{12} + 2764ts^{11} + 8634t^2s^{10} \\
& + 13982t^3s^9 + 10851t^4s^8 - 1158t^5s^7 - 10180t^6s^6 - 7280t^7s^5 + 2283t^8s^4 + 6972t^9s^3 \\
& + 5136t^{10}s^2 + 1712t^{11}s + 232t^{12})m_c^2 + s^2t^3(s+t)^3(s^2+ts+t^2)^2 \\
& (6s^6 + 17ts^5 - 105t^2s^4 - 224t^3s^3 - 62t^4s^2 + 60t^5s + 20t^6)].
\end{aligned} \tag{A10j}
$$

$g + g \to c\bar{c}({}^3S_1^{[8]}) + g$:

$$
b^{gg}({}^3S_1^{[8]}) = -\frac{\pi^3\alpha_s^3}{108m_c^5st(4m_c^2-s)^3(4m_c^2-t)^3(s+t)^3(-4m_c^2+s+t)} \tag{A11a}
$$

$$
\begin{aligned}
h_1 = {} & -st(-4m_c^2+s+t)[622592m_c^{14}(3s^2+2st+3t^2) - 4096m_c^{12}(203s^3 - 12s^2t \\
& + 216st^2 + 203t^3) - 1024m_c^{10}(420s^4 + 2357s^3t + 2358s^2t^2 + 1187st^3 + 258t^4) \\
& + 256m_c^8(2073s^5 + 8653s^4t + 12881s^3t^2 + 10805s^2t^3 + 5215st^4 + 1263t^5) \\
& - 64m_c^6(3265s^6 + 14055s^5t + 26073s^4t^2 + 28978s^3t^3 + 20175s^2t^4 + 8319st^5 + 1807t^6) \\
& + 48m_c^4(774s^7 + 3736s^6t + 8400s^5t^2 + 11507s^4t^3 + 10373s^3t^4 + 6186s^2t^5 + 2170st^6 \\
& + 396t^7) - 108m_c^2(23s^8 + 142s^7t + 392s^6t^2 + 653s^5t^3 + 742s^4t^4 + 569s^3t^5 + 296s^2t^6 \\
& + 88st^7 + 11t^8) + 297st(s+t)(s^2+st+t^2)^3]
\end{aligned} \tag{A11b}
$$

$$h_2 = 8m_c^2 t[4980736m_c^{16}s - 16384m_c^{14}(214s^2 + 13st + 27t^2) + 4096m_c^{12}(125s^3 - 1442s^2t - 799st^2 + 216t^3) + 1024m_c^{10}(1299s^4 + 7096s^3t + 7922s^2t^2 + 1845st^3 - 756t^4) - 256m_c^8(4029s^5 + 17654s^4t + 25019s^3t^2 + 14306s^2t^3 + 1152st^4 - 1512t^5) + 64m_c^6(5497s^6 + 24150s^5t + 40980s^4t^2 + 33774s^3t^3 + 11507s^2t^4 - 1511st^5 - 1863t^6) - 16m_c^4(3672s^7 + 17571s^6t + 35040s^5t^2 + 36622s^4t^3 + 19839s^3t^4 + 2237s^2t^5 - 3125st^6 - 1404t^7) + 4m_c^2(918s^8 + 5400s^7t + 13063s^6t^2 + 16698s^5t^3 + 11639s^4t^4 + 2810s^3t^5 - 2283s^2t^6 - 2057st^7 - 594t^8) + 27t(-10s^8 - 40s^7t - 63s^6t^2 - 49s^5t^3 + 2s^4t^4 + 39s^3t^5 + 39s^2t^6 + 18st^7 + 4t^8)] \tag{A11c}$$

$$h_3 = 8m_c^2(4m_c^2 - s - t)[1024m_c^{10}(27s^4 + 260s^3t + 162s^2t^2 + 260st^3 + 27t^4) - 256m_c^8(162s^5 + 617s^4t + 939s^3t^2 + 1395s^2t^3 + 617st^4 + 162t^5) + 64m_c^6(405s^6 + 1238s^5t + 1841s^4t^2 + 3670s^3t^3 + 2717s^2t^4 + 1172st^5 + 405t^6) - 32m_c^4(270s^7 + 642s^6t + 1190s^5t^2 + 2225s^4t^3 + 2387s^3t^4 + 1757s^2t^5 + 723st^6 + 270t^7) + 4m_c^2(378s^8 + 1193s^7t + 2369s^6t^2 + 3793s^5t^3 + 4964s^4t^4 + 4117s^3t^5 + 2855s^2t^6 + 1193st^7 + 378t^8) - 27(4s^9 + 18s^8t + 39s^7t^2 + 66s^6t^3 + 83s^5t^4 + 83s^4t^5 + 66s^3t^6 + 39s^2t^7 + 18st^8 + 4t^9)] \tag{A11d}$$

$$h_4 = 8m_c^2[221184m_c^{14}(s - t)(s + t)^2 - 2048m_c^{12}(189s^4 - 110s^3t - 162s^2t^2 - 410st^3 - 243t^4) + 512m_c^{10}(567s^5 + 566s^4t - 495s^3t^2 - 2227s^2t^3 - 2374st^4 - 945t^5) - 128m_c^8(945s^6 + 2744s^5t + 2585s^4t^2 - 4606s^3t^3 - 9379s^2t^4 - 6778st^5 - 2079t^6) + 32m_c^6(918s^7 + 4957s^6t + 8354s^5t^2 - 709s^4t^3 - 15709s^3t^4 - 19272s^2t^5 - 10811st^6 - 2808t^7) - 8m_c^4(486s^8 + 3942s^7t + 9139s^6t^2 + 4902s^5t^3 - 11758s^4t^4 - 23880s^3t^5 - 21205s^2t^6 - 9976st^7 - 2322t^8) + 2m_c^2(108s^9 + 918s^8t + 2906s^7t^2 + 2147s^6t^3 - 5453s^5t^4 - 14923s^4t^5 - 17387s^3t^6 - 12136s^2t^7 - 5032st^8 - 1080t^9) + 27t(2s^9 + 4s^8t + 13s^7t^2 + 42s^6t^3 + 83s^5t^4 + 107s^4t^5 + 92s^3t^6 + 53s^2t^7 + 20st^8 + 4t^9)]. \tag{A11e}$$

$g + g \to c\bar{c}(^3P_J^{[8]}) + g$:

$$b^{gg}(^3P_J^{[8]}) = \frac{\pi^3\alpha_s^3}{m_c^5 s^2 t^2 (s - 4m_c^2)^4 (t - 4m_c^2)^4 (s + t)^4 (-4m_c^2 + s + t)^2} \tag{A12a}$$

$$
\begin{aligned}
h_1 = {} & -st(-4m_c^2+s+t)[1048576m_c^{20}(51s^4+187s^3t+244s^2t^2+127st^3+51t^4) \\
& -262144m_c^{18}(306s^5+1357s^4t+2375s^3t^2+1905s^2t^3+1007st^4+306t^5) \\
& +65536m_c^{16}(911s^6+4526s^5t+9668s^4t^2+10344s^3t^3+7048s^2t^4+3356st^5+831t^6) \\
& -32768m_c^{14}(807s^7+4441s^6t+10914s^5t^2+14369s^4t^3+12229s^3t^4+7369s^2t^5 \\
& +3126st^6+657t^7)+8192m_c^{12}(888s^8+5519s^7t+15179s^6t^2+23051s^5t^3+23276s^4t^4 \\
& +17136s^3t^5+9319s^2t^6+3604st^7+648t^8)-2048m_c^{10}(635s^9+4541s^8t+13692s^7t^2 \\
& +22822s^6t^3+25571s^5t^4+21816s^4t^5+14607s^3t^6+7442s^2t^7+2701st^8+405t^9) \\
& +256m_c^8(627s^{10}+5195s^9t+16915s^8t^2+29976s^7t^3+34868s^6t^4+31408s^5t^5 \\
& +24488s^4t^6+16106s^3t^7+8015s^2t^8+2755st^9+327t^{10})-64m_c^6(210s^{11}+2085s^{10}t \\
& +7585s^9t^2+14784s^8t^3+18290s^7t^4+16790s^6t^5+13650s^5t^6+10810s^4t^7+7164s^3t^8 \\
& +3345s^2t^9+1005st^{10}+90t^{11})+16m_c^4(35s^{12}+506s^{11}t+2338s^{10}t^2+5745s^9t^3 \\
& +9126s^8t^4+10612s^7t^5+9988s^6t^6+8172s^5t^7+5746s^4t^8+3105s^3t^9+1138s^2t^{10} \\
& +246st^{11}+15t^{12})-4m_c^2st(s^2+st+t^2)^2(46s^7+270s^6t+611s^5t^2+761s^4t^3 \\
& +681s^3t^4+451s^2t^5+170st^6+26t^7)+11s^2t^2(s+t)^2(s^2+st+t^2)^4]
\end{aligned}
\tag{A12b}
$$

$$
\begin{aligned}
h_2 = {} & 32m_c^2[4194304s(s-t)^2tm_c^{22}-262144st(11s^3-44ts^2+11t^2s+6t^3)m_c^{20} \\
& -65536(76s^6+237ts^5+436t^2s^4-153t^3s^3-126t^4s^2+118t^5s+36t^6)m_c^{18} \\
& +16384(416s^7+1681ts^6+2772t^2s^5+495t^3s^4-892t^4s^3+346t^5s^2+998t^6s \\
& +288t^7)m_c^{16}-4096(940s^8+4547ts^7+8571t^2s^6+4687t^3s^5-470t^4s^4+878t^5s^3 \\
& +4017t^6s^2+3802t^7s+1008t^8)m_c^{14}+1024(1120s^9+6286ts^8+13654t^2s^7+11090t^3s^6 \\
& +3254t^4s^5+4751t^5s^4+10960t^6s^3+12707t^7s^2+8314t^8s+2016t^9)m_c^{12}-256(740s^{10} \\
& +4700ts^9+11283t^2s^8+11038t^3s^7+5528t^4s^6+9919t^5s^5+21427t^6s^4+26283t^7s^3 \\
& +21360t^8s^2+11286t^9s+2520t^{10})m_c^{10}+64(256s^{11}+1780ts^{10}+4120t^2s^9+3090t^3s^8 \\
& +544t^4s^7+7817t^5s^6+24064t^6s^5+34463t^7s^4+31912t^8s^3+21376t^9s^2+9738t^{10}s \\
& +2016t^{11})m_c^8-16(36s^{12}+244ts^{11}-40t^2s^{10}-2560t^3s^9-5744t^4s^8-1877t^5s^7 \\
& +11279t^6s^6+24269t^7s^5+27618t^8s^4+21672t^9s^3+12769t^{10}s^2+5218t^{11}s+1008t^{12})m_c^6 \\
& +8t(-6s^{12}-192ts^{11}-1024t^2s^{10}-2580t^3s^9-3274t^4s^8-1283t^5s^7+2616t^6s^6 \\
& +5395t^7s^5+5517t^8s^4+3840t^9s^3+2057t^{10}s^2+786t^{11}s+144t^{12})m_c^4+t^2(52s^{12} \\
& +396ts^{11}+1396t^2s^{10}+2700t^3s^9+2943t^4s^8+1412t^5s^7-850t^6s^6-1920t^7s^5-1653t^8s^4 \\
& -916t^9s^3-452t^{10}s^2-188t^{11}s-36t^{12})m_c^2-st^3(s^2+ts+t^2)^2(s^7+4ts^6-4t^2s^5 \\
& -26t^3s^4-22t^4s^3+4t^5s^2+9t^6s+2t^7)]
\end{aligned}
\tag{A12c}
$$

$$h_3 = 32m_c^2[262144m_c^{18}st(28s^4 + 93s^3t + 126s^2t^2 + 73st^3 + 28t^4) - 16384m_c^{16}st(653s^5 + 2802s^4t + 4924s^3t^2 + 4254s^2t^3 + 2407st^4 + 688t^5) - 4096m_c^{14}(116s^8 - 1263s^7t - 8068s^6t^2 - 18556s^5t^3 - 21164s^4t^4 - 15711s^3t^5 - 7768s^2t^6 - 1718st^7 + 36t^8) + 1024m_c^{12}(656s^9 + 369s^8t - 9566s^7t^2 - 32833s^6t^3 - 49800s^5t^4 - 46665s^4t^5 - 30158s^3t^6 - 12591s^2t^7 - 1916st^8 + 216t^9) - 256m_c^{10}(1540s^{10} + 5543s^9t + 1997s^8t^2 - 22185s^7t^3 - 53859s^6t^4 - 66855s^5t^5 - 54509s^4t^6 - 29995s^3t^7 - 9693s^2t^8 - 92st^9 + 540t^{10}) + 64m_c^8(1920s^{11} + 9816s^{10}t + 18914s^9t^2 + 13315s^8t^3 - 10042s^7t^4 - 33052s^6t^5 - 39132s^5t^6 - 27447s^4t^7 - 11120s^3t^8 - 576s^2t^9 + 2236st^{10} + 720t^{11}) - 16m_c^6(1340s^{12} + 8240s^{11}t + 21729s^{10}t^2 + 32953s^9t^3 + 32174s^8t^4 + 21270s^7t^5 + 9410s^6t^6 + 3455s^5t^7 + 3594s^4t^8 + 4948s^3t^9 + 4629s^2t^{10} + 2550st^{11} + 540t^{12}) + 8m_c^4(248s^{13} + 1724s^{12}t + 5352s^{11}t^2 + 10237s^{10}t^3 + 14018s^9t^4 + 15183s^8t^5 + 13805s^7t^6 + 10710s^6t^7 + 7198s^5t^8 + 4463s^4t^9 + 2707s^3t^{10} + 1507s^2t^{11} + 604st^{12} + 108t^{13}) - m_c^2(s+t)^2(76s^{12} + 412s^{11}t + 1004s^{10}t^2 + 1740s^9t^3 + 2541s^8t^4 + 3200s^7t^5 + 3202s^6t^6 + 2320s^5t^7 + 1141s^4t^8 + 440s^3t^9 + 204s^2t^{10} + 132st^{11} + 36t^{12}) - st(s+t)^3(s^2+st+t^2)^2(2s^6 + 5s^5t - 8s^4t^2 - 23s^3t^3 - 8s^2t^4 + 5st^5 + 2t^6)] \tag{A12d}$$

$$h_4 = 32m_c^2[65536st(59s^4 + 150ts^3 + 252t^2s^2 + 182t^3s + 53t^4)m_c^{18} - 16384(96s^7 + 690ts^6 + 1748t^2s^5 + 2626t^3s^4 + 2313t^4s^3 + 1154t^5s^2 + 213t^6s - 36t^7)m_c^{16} + 4096(536s^8 + 3159ts^7 + 8087t^2s^6 + 12811t^3s^5 + 12362t^4s^4 + 7490t^5s^3 + 2561t^6s^2 - 26t^7s - 252t^8)m_c^{14} - 1024(1240s^9 + 7337ts^8 + 19822t^2s^7 + 33719t^3s^6 + 36841t^4s^5 + 25464t^5s^4 + 10399t^6s^3 + 1164t^7s^2 - 1706t^8s - 756t^9)m_c^{12} + 256(1520s^{10} + 9504ts^9 + 27735t^2s^8 + 50970t^3s^7 + 62054t^4s^6 + 49255t^5s^5 + 22995t^6s^4 + 2530t^7s^3 - 4972t^8s^2 - 4387t^9s - 1260t^{10})m_c^{10} - 64(1040s^{11} + 6961ts^{10} + 22156t^2s^9 + 44286t^3s^8 + 58938t^4s^7 + 51617t^5s^6 + 26090t^6s^5 + 904t^7s^4 - 10831t^8s^3 - 10450t^9s^2 - 5487t^{10}s - 1260t^{11})m_c^8 + 16(376s^{12} + 2682ts^{11} + 9390t^2s^{10} + 20757t^3s^9 + 30044t^4s^8 + 27253t^5s^7 + 11565t^6s^6 - 5978t^7s^5 - 15058t^8s^4 - 14380t^9s^3 - 9099t^{10}s^2 - 3812t^{11}s - 756t^{12})m_c^6 - 4(56s^{13} + 408ts^{12} + 1632t^2s^{11} + 4317t^3s^{10} + 7054t^4s^9 + 5921t^5s^8 - 796t^6s^7 - 8944t^7s^6 - 12637t^8s^5 - 11090t^9s^4 - 7241t^{10}s^3 - 3756t^{11}s^2 - 1396t^{12}s - 252t^{13})m_c^4 - t(8s^{13} - 186t^2s^{11} - 472t^3s^{10} - 9t^4s^9 + 1935t^5s^8 + 4462t^6s^7 + 5508t^7s^6 + 4381t^8s^5 + 2457t^9s^4 + 1116t^{10}s^3 + 504t^{11}s^2 + 196t^{12}s + 36t^{13})m_c^2 - st^2(s^3 + 2ts^2 + 2t^2s + t^3)^2 (s^6 + 3ts^5 - 9t^2s^4 - 22t^3s^3 - 6t^4s^2 + 6t^5s + 2t^6)]. \tag{A12e}$$

$g + g \to c\bar{c}({}^3S_1^{[8]}, {}^3D_1^{[8]}) + g$:

$$b^{gg}({}^3S_1^{[8]}, {}^3D_1^{[8]}) = -\frac{\pi^3\alpha_s^3}{54\sqrt{15}m_c^5st(s-4m_c^2)^4(t-4m_c^2)^4(s+t)^4(-4m_c^2+s+t)} \tag{A13a}$$

$$h_1 = st(s-4m_c^2)(4m_c^2-t)(s+t)(4m_c^2-s-t)[49152m_c^{16}(1261s+1459t) - 1024m_c^{14}(34157s^2+40400st+42419t^2) + 512m_c^{12}(5978s^3-29869s^2t-21472st^2 + 11999t^3) + 64m_c^{10}(10841s^4+214882s^3t+450086s^2t^2+196198st^3+5225t^4) + 32m_c^8(19811s^5-64363s^4t-264691s^3t^2-265735s^2t^3-67162st^4+19244t^5) - 4m_c^6(86162s^6+168328s^5t+43407s^4t^2-172806s^3t^3+34263s^2t^4+163270st^5 + 85568t^6) + m_c^4(57024s^7+220986s^6t+406490s^5t^2+459356s^4t^3+459698s^3t^4 + 407102s^2t^5+221256st^6+57024t^7) - 162m_c^2(22s^8+128s^7t+331s^6t^2+535s^5t^3 + 632s^4t^4+535s^3t^5+331s^2t^6+128st^7+22t^8) + 405st(s+t)(s^2+st+t^2)^3] \tag{A13b}$$

$$h_2 = -2m_c^2t(4m_c^2-t)[131072m_c^{18}s(19027s+38911t) - 32768m_c^{16}(34301s^3 + 175284s^2t+117007st^2+1620t^3) - 4096m_c^{14}(81504s^4-265155s^3t-587452s^2t^2 - 154537st^3-25920t^4) + 1024m_c^{12}(340730s^5+674275s^4t+265516s^3t^2+143007s^2t^3 + 77548st^4-90720t^5) - 256m_c^{10}(443414s^6+1706395s^5t+2270361s^4t^2+1548543s^3t^3 + 710607s^2t^4-161108st^5-181440t^6) + 64m_c^8(367548s^7+1961043s^6t+3676724s^5t^2 + 3126628s^4t^3+1228754s^3t^4-268787s^2t^5-686126st^6-223560t^7) - 16m_c^6(235756s^8 + 1490664s^7t+3518089s^6t^2+3849434s^5t^3+1701248s^4t^4-421485s^3t^5-1160629s^2t^6 - 731477st^7-168480t^8) + 4m_c^4(103680s^9+755956s^8t+2167956s^7t^2+3098173s^6t^3 + 2077386s^5t^4+205573s^4t^5-775916s^3t^6-761172s^2t^7-345856st^8-71280t^9) - m_c^2(20736s^{10}+207360s^9t+737224s^8t^2+1361080s^7t^3+1370281s^6t^4+690475s^5t^5 + 23643s^4t^6-195151s^3t^7-151624s^2t^8-60912st^9-12960t^{10}) + 162st(32s^9+160s^8t + 359s^7t^2+476s^6t^3+409s^5t^4+233s^4t^5+96s^3t^6+39s^2t^7+16st^8+4t^9)] \tag{A13c}$$

$$h_3 = -2m_c^2(s+t)(4m_c^2-s-t)[65536m_c^{16}st(6701s+6899t) + 8192m_c^{14}(1620s^4 - 63459s^3t-138920s^2t^2-70317st^3+1620t^4) - 1024m_c^{12}(22680s^5-201895s^4t - 495519s^3t^2-539529s^2t^3-244321st^4+22680t^5) + 256m_c^{10}(68040s^6+28875s^5t - 140260s^4t^2-209950s^3t^3-209740s^2t^4-717st^5+68040t^6) - 64m_c^8(113400s^7 + 353683s^6t+595357s^5t^2+986984s^4t^3+1022588s^3t^4+637657s^2t^5+359587st^6 + 113400t^7) + 16m_c^6(110160s^8+383177s^7t+750828s^6t^2+1345133s^5t^3+1831816s^4t^4 + 1470557s^3t^5+821388s^2t^6+389549st^7+110160t^8) - 4m_c^4(58320s^9+205510s^8t + 354804s^7t^2+556953s^6t^3+851841s^5t^4+878157s^4t^5+598875s^3t^6+370788s^2t^7 + 205888st^8+58320t^9) + m_c^2(s+t)^2(12960s^8+16848s^7t+262s^6t^2-23497s^5t^3 + 8468s^4t^4-26953s^3t^5-8s^2t^6+16848st^7+12960t^8) + 162st(4s^9+24s^8t+75s^7t^2 + 120s^6t^3+149s^5t^4+149s^4t^5+120s^3t^6+75s^2t^7+24st^8+4t^9)] \tag{A13d}$$

$$h_4 = m_c^2[3145728m_c^{20}st(2357s - 3083t) - 131072m_c^{18}(1620s^4 + 105283s^3t + 8642s^2t^2 - 96641st^3 - 1620t^4) + 16384m_c^{16}(25920s^5 + 509621s^4t + 896167s^3t^2 - 85037s^2t^3 - 245935st^4 - 32400t^5) - 8192m_c^{14}(45360s^6 + 320292s^5t + 1056932s^4t^2 + 856843s^3t^3 + 146706s^2t^4 + 583st^5 - 71280t^6) + 1024m_c^{12}(181440s^7 + 599428s^6t + 1988277s^5t^2 + 2840253s^4t^3 + 1083297s^3t^4 + 141039s^2t^5 - 376166st^6 - 362880t^7) - 256m_c^{10}(223560s^8 + 623352s^7t + 977271s^6t^2 + 992272s^5t^3 - 1796934s^4t^4 - 3815074s^3t^5 - 2849597s^2t^6 - 1832690st^7 - 586440t^8) + 64m_c^8(168480s^9 + 592859s^8t + 679396s^7t^2 - 438617s^6t^3 - 4960050s^5t^4 - 10323584s^4t^5 - 9863307s^3t^6 - 6026378s^2t^7 - 2744639st^8 - 615600t^9) - 16m_c^6(71280s^{10} + 312714s^9t + 668005s^8t^2 + 413842s^7t^3 - 2602893s^6t^4 - 8848230s^5t^5 - 12123493s^4t^6 - 9435072s^3t^7 - 5216495s^2t^8 - 2055090st^9 - 408240t^{10}) + 4m_c^4(12960s^{11} + 49248s^{10}t + 294940s^9t^2 + 872129s^8t^3 + 834712s^7t^4 - 1612967s^6t^5 - 5014855s^5t^6 - 5770958s^4t^7 - 4063097s^3t^8 - 2154136s^2t^9 - 805760st^{10} - 155520t^{11}) + m_c^2t(7776s^{11} - 55728s^{10}t - 413458s^9t^2 - 1003935s^8t^3 - 1092466s^7t^4 - 310362s^6t^5 + 540758s^5t^6 + 744113s^4t^7 + 542710s^3t^8 + 309728s^2t^9 + 124416st^{10} + 25920t^{11}) + 324st^2(s+t)^2(8s^8 + 44s^7t + 83s^6t^2 + 85s^5t^3 + 64s^4t^4 + 37s^3t^5 + 31s^2t^6 + 16st^7 + 4t^8)]. \tag{A13e}$$

$q(\bar{q}) + g \to c\bar{c}({}^3P_1^{[1]}) + q(\bar{q})$:

$$a^{q(\bar{q})g}({}^3P_1^{[1]}) = \frac{32\pi^3\alpha_s^3}{81m_c^3(s+t)^4} \tag{A14a}$$

$$g_1 = -(s+t)[-4m_c^2(s+t) + s^2 + t^2] \tag{A14b}$$

$$g_2 = -16m_c^2 s \tag{A14c}$$

$$g_3 = 0 \tag{A14d}$$

$$g_4 = -8m_c^2(s-t) \tag{A14e}$$

$$b^{q(\bar{q})g}({}^3P_1^{[1]}) = \frac{16\pi^3\alpha_s^3}{405m_c^5(4m_c^2 - s)(s+t)^5} \tag{A14f}$$

$$h_1 = -(s+t)[16m_c^4(s^2 + 22st + 21t^2) - 4m_c^2(12s^3 + 43s^2t - 8st^2 + 21t^3) + 11s(s^3 + s^2t + st^2 + t^3)] \tag{A14g}$$

$$h_2 = 16m_c^2 s[4m_c^2(s + 41t) - s(s+t)] \tag{A14h}$$

$$h_3 = 0 \tag{A14i}$$

$$h_4 = 8m_c^2[4m_c^2(s^2 + 50st - 31t^2) - s^3 + st^2]. \tag{A14j}$$

$q(\bar{q}) + g \to c\bar{c}({}^3P_2^{[1]}) + q(\bar{q})$:

$$a^{q(\bar{q})g}({}^3P_2^{[1]}) = -\frac{32\pi^3\alpha_s^3}{1215m_c^3(s+t)^4(-4m_c^2 + s + t)} \tag{A15a}$$

$$g_1 = 16m_c^4(19s^2 + 30st + 19t^2) - 8m_c^2(s^3 + 16s^2t + 16st^2 + t^3) + (s+t)^2(s^2 + t^2) \tag{A15b}$$

$$g_2 = 48m_c^2(32m_c^4 + 4m_c^2(s+2t) - s^2 - st + 2t^2) \tag{A15c}$$

$$g_3 = 96m_c^2(s+t)^2 \tag{A15d}$$

$$g_4 = -24m_c^2(4m_c^2(s-t) + s^2 - 4st - 5t^2) \tag{A15e}$$

$$b^{q(\bar{q})g}({}^3P_2^{[1]}) = \frac{16\pi^3\alpha_s^3}{6075m_c^5(4m_c^2 - s)(s+t)^5(-4m_c^2 + s + t)^2} \tag{A15f}$$

$$\begin{aligned} h_1 = &-(4m_c^2 - s - t)[320m_c^6(19s^3 + 141s^2t + 153st^2 + 95t^3) \\ &+ 16m_c^4(73s^4 + 39s^3t - 923s^2t^2 - 931st^3 - 42t^4) \\ &+ 4m_c^2(s+t)^2(19s^3 + 139s^2t - st^2 + 17t^3) - 7s(s+t)^3(s^2 + t^2)] \end{aligned} \tag{A15g}$$

$$\begin{aligned} h_2 = &\, 48m_c^2[2560m_c^8(s - 7t) - 64m_c^6(49s^2 - 3st + 4t^2) \\ &+ 16m_c^4(37s^3 + 113s^2t + 180st^2 + 8t^3) + 4m_c^2(5s^4 - 21s^3t - 69s^2t^2 + 23st^3 + 66t^4) \\ &- s(s+t)^2(3s^2 - st + 6t^2)] \end{aligned} \tag{A15h}$$

$$h_3 = -96m_c^2(s+t)^2[80m_c^4(3s + 7t) - 4m_c^2(16s^2 + 47st + 31t^2) + s(s+t)^2] \tag{A15i}$$

$$\begin{aligned} h_4 = &\, 24m_c^2[320m_c^6(s^2 + 10st - 7t^2) - 80m_c^4(s^3 + 8s^2t + 31st^2 + 24t^3) \\ &- 4m_c^2(s+t)^2(s^2 + 26st - 155t^2) + s(s - 5t)(s+t)^3]. \end{aligned} \tag{A15j}$$

$q(\bar{q}) + g \to c\bar{c}({}^3S_1^{[8]}) + q(\bar{q})$:

$$b^{q(\bar{q})g}({}^3S_1^{[8]}) = \frac{\pi^3\alpha_s^3}{162m_c^5 st(s - 4m_c^2)(s+t)^3(-4m_c^2 + s + t)} \tag{A16a}$$

$$\begin{aligned} h_1 = &-(4m_c^2 - s - t)[128m_c^6(20s^3 + 69s^2t - 39st^2 + 20t^3) - 32m_c^4(40s^4 + 104s^3t \\ &+ 45s^2t^2 + st^3 + 20t^4) + 4m_c^2(108s^5 + 175s^4t + 77s^3t^2 + 207s^2t^3 + st^4 + 20t^5) \\ &- 11s(4s^5 + 3s^4t + 7s^3t^2 + 7s^2t^3 + 3st^4 + 4t^5)] \end{aligned} \tag{A16b}$$

$$\begin{aligned} h_2 = &-16m_c^2[576m_c^6(s-t)^2 + 16m_c^4(26s^3 + 105s^2t - 3st^2 + 26t^3) - 4m_c^2(55s^4 \\ &+ 155s^3t + 99s^2t^2 + 34st^3 + 35t^4) + s(20s^4 + 44s^3t + 21s^2t^2 + 44st^3 + 11t^4)] \end{aligned} \tag{A16c}$$

$$\begin{aligned} h_3 = &-32m_c^2(4m_c^2 - s - t)[4m_c^2(44s^3 + 87s^2t - 21st^2 + 44t^3) \\ &- 5s(4s^3 + 3s^2t + 3st^2 + 4t^3)] \end{aligned} \tag{A16d}$$

$$\begin{aligned} h_4 = &-8m_c^2[16m_c^4(97s^3 + 147s^2t - 15st^2 + 79t^3) - 4m_c^2(146s^4 + 265s^3t + 189s^2t^2 \\ &+ 77st^3 + 79t^4) + s(49s^4 + 70s^3t + 60s^2t^2 + 70st^3 + 31t^4)]. \end{aligned} \tag{A16e}$$

$q(\bar{q}) + g \to c\bar{c}({}^3P_J^{[8]}) + q(\bar{q})$:

$$b^{q(\bar{q})g}({}^3P_J^{[8]}) = \frac{15\alpha_s}{128\alpha} b^{q(\bar{q})\gamma}({}^3P_J^{[8]}). \tag{A17}$$

The coefficients h_k are the same as those in Eq. (A6).

$q(\bar{q})+g\to c\bar{c}({}^3S_1^{[8]},{}^3D_1^{[8]})+q(\bar{q})$:

$$b^{q(\bar{q})g}({}^3S_1^{[8]},{}^3D_1^{[8]})=-\frac{\pi^3\alpha_s^3}{27\sqrt{15}m_c^5st(s+t)^4(-4m_c^2+s+t)} \tag{A18a}$$

$$h_1=-(s+t)(-4m_c^2+s+t)[160m_c^4(4s^3+3s^2t+3st^2+4t^3)-4m_c^2(40s^4+97s^3t+6s^2t^2+97st^3+40t^4)+5(4s^5+3s^4t+7s^3t^2+7s^2t^3+3st^4+4t^5)] \tag{A18b}$$

$$h_2=8m_c^2[3456m_c^6(s-t)^2-144m_c^4(4s^3-s^2t-16st^2+13t^3)+4m_c^2(22s^4+25s^3t+114s^2t^2+7st^3+112t^4)-40s^5-128s^4t-157s^3t^2-103s^2t^3-83st^4-49t^5] \tag{A18c}$$

$$h_3=8m_c^2(s+t)^2(107s^2-74st+107t^2)(4m_c^2-s-t) \tag{A18d}$$

$$h_4=8m_c^2[216m_c^4(3s^3-5s^2t+5st^2-3t^3)+2m_c^2(17s^4+158s^3t+66s^2t^2+122st^3+197t^4)-(s+t)^2(49s^3+48s^2t-15st^2+58t^3)]. \tag{A18e}$$

$\bar{q}+q\to c\bar{c}({}^3P_1^{[1]})+g$:

$$a^{\bar{q}q}({}^3P_1^{[1]})=\frac{256\pi^3\alpha_s^3}{243m_c^3(s-4m_c^2)^4} \tag{A19a}$$

$$g_1=-(4m_c^2-s)(4m_c^2(s+2t)-s^2-2st-2t^2) \tag{A19b}$$

$$g_2=16m_c^2(4m_c^2-s-t) \tag{A19c}$$

$$g_3=16m_c^2t \tag{A19d}$$

$$g_4=32m_c^4-8m_c^2s \tag{A19e}$$

$$b^{\bar{q}q}({}^3P_1^{[1]})=\frac{128\pi^3\alpha_s^3}{1215m_c^5(4m_c^2-s)^5} \tag{A19f}$$

$$h_1=(4m_c^2-s)[16m_c^4(31s+42t)-8m_c^2(21s^2+32st+21t^2)+11s(s^2+2st+2t^2)] \tag{A19g}$$

$$h_2=-16m_c^2(124m_c^2-s)(4m_c^2-s-t) \tag{A19h}$$

$$h_3=16m_c^2t(s-124m_c^2) \tag{A19i}$$

$$h_4=-8m_c^2(4m_c^2-s)(124m_c^2-s). \tag{A19j}$$

$\bar{q}+q\to c\bar{c}({}^3P_2^{[1]})+g$:

$$a^{\bar{q}q}({}^3P_2^{[1]})=\frac{256\pi^3\alpha_s^3}{3645m_c^3s(s-4m_c^2)^4} \tag{A20a}$$

$$g_1=-4608m_c^8+2304m_c^6(s+t)-16m_c^4(19s^2+66st+36t^2)+8m_c^2s(s^2+16st+15t^2)-s^2(s^2+2st+2t^2) \tag{A20b}$$

$$g_2=-48m_c^2(32m_c^4+4m_c^2(s+2t)-s^2-st+2t^2) \tag{A20c}$$

$$g_3=-48m_c^2(96m_c^4-24m_c^2(s+t)+2s^2+5st+2t^2) \tag{A20d}$$

$$g_4 = -24m_c^2(96m_c^4 - 4m_c^2(s+4t) - s^2 + 4st + 4t^2) \quad \text{(A20e)}$$

$$b^{\bar{q}q}({}^3P_2^{[1]}) = \frac{128\pi^3\alpha_s^3}{18225m_c^5 s^2(4m_c^2 - s)^5} \quad \text{(A20f)}$$

$$\begin{aligned} h_1 = s[&276480m_c^{10} - 512m_c^8(221s + 246t) + 64m_c^6(93s^2 + 950st + 492t^2) \\ &+ 16m_c^4 s(53s^2 - 598st - 458t^2) + 4m_c^2 s^2(33s^2 + 154st + 140t^2) \\ &- 7s^3(s^2 + 2st + 2t^2)] \end{aligned} \quad \text{(A20g)}$$

$$\begin{aligned} h_2 = 48m_c^2[&2048m_c^8 + 128m_c^6(19s - 8t) + 16m_c^4(57s^2 + 54st + 8t^2) \\ &- 4m_c^2 s(20s^2 + st - 58t^2) - s^2(3s^2 + 7st + 10t^2)] \end{aligned} \quad \text{(A20h)}$$

$$\begin{aligned} h_3 = 48m_c^2[&9600m_c^6 s - 32m_c^4(58s^2 + 77st - 4t^2) \\ &+ 4m_c^2 s(52s^2 + 137st + 58t^2) - s^2(6s^2 + 13st + 10t^2)] \end{aligned} \quad \text{(A20i)}$$

$$\begin{aligned} h_4 = 24m_c^2[&128m_c^6(85s - 8t) + 16m_c^4(3s^2 - 100st + 16t^2) \\ &- 8m_c^2 s(3s^2 - 68st - 58t^2) - s^2(7s^2 + 20st + 20t^2)]. \end{aligned} \quad \text{(A20j)}$$

$\bar{q} + q \to c\bar{c}({}^3S_1^{[8]}) + g$:

$$b^{\bar{q}q}({}^3S_1^{[8]}) = \frac{4\pi^3\alpha_s^3}{243stm_c^5(s - 4m_c^2)^3(-4m_c^2 + s + t)} \quad \text{(A21a)}$$

$$\begin{aligned} h_1 = s[&64m_c^4 - 4m_c^2(8s + 9t) + 4s^2 + 9st + 9t^2][704m_c^6 + 16m_c^4(s - 22t) \\ &+ 4m_c^2(23s^2 + 44st + 22t^2) - 11s(s^2 + 2st + 2t^2)] \end{aligned} \quad \text{(A21b)}$$

$$\begin{aligned} h_2 = 16m_c^2[&64m_c^6(77s - 18t) - 96m_c^4(29s^2 + 24st - 12t^2) \\ &+ 36m_c^2(13s^3 + 22s^2t + 9st^2 - 10t^3) - 20s^4 - 36s^3t - 9s^2t^2 + 54st^3 + 36t^4] \end{aligned} \quad \text{(A21c)}$$

$$\begin{aligned} h_3 = 16m_c^2[&4352m_c^6 s - 48m_c^4(49s^2 + 48st - 6t^2) \\ &+ 36m_c^2(10s^3 + 16s^2t + 9st^2 - 6t^3) - 11s^4 + 45s^2t^2 + 90st^3 + 36t^4] \end{aligned} \quad \text{(A21d)}$$

$$h_4 = 72m_c^2(-4m_c^2 + s + 2t)^2(4m_c^2(s - 2t) + s^2 + 2st + 2t^2). \quad \text{(A21e)}$$

$\bar{q} + q \to c\bar{c}({}^3P_J^{[8]}) + g$:

$$b^{\bar{q}q}({}^3P_J^{[8]}) = \frac{16\pi^3\alpha_s^3}{27m_c^5 s^2(s - 4m_c^2)^4} \quad \text{(A22a)}$$

$$\begin{aligned} h_1 = s(4m_c^2 - s)[&3264m_c^6 + 16m_c^4(7s - 30t) \\ &+ 4m_c^2(33s^2 + 52st + 30t^2) - 11s(s^2 + 2st + 2t^2)] \end{aligned} \quad \text{(A22b)}$$

$$h_2 = 32m_c^2[1152m_c^6 + 48m_c^4(5s - 12t) + 8m_c^2 t(5s + 9t) + s(s^2 + 2st + 4t^2)] \quad \text{(A22c)}$$

$$h_3 = 32m_c^2[464m_c^4 s + 8m_c^2(s^2 + 9st + 9t^2) + s(3s^2 + 6st + 4t^2)] \quad \text{(A22d)}$$

$$h_4 = 32m_c^2[16m_c^4(19s - 18t) + 8m_c^2(7s^2 + 7st + 9t^2) + s(s + 2t)^2]. \quad \text{(A22e)}$$

$\bar{q}+q\to c\bar{c}({}^3S_1^{[8]},{}^3D_1^{[8]})+g$:

$$b^{\bar{q}q}({}^3S_1^{[8]},{}^3D_1^{[8]}) = -\frac{8\pi^3\alpha_s^3}{81\sqrt{15}m_c^5 st(s-4m_c^2)^4(-4m_c^2+s+t)} \tag{A23a}$$

$$h_1 = s(s-4m_c^2)[20480m_c^{10} - 1024m_c^8(15s+28t) + 64m_c^6(80s^2+331st + 310t^2) - 16m_c^4(80s^3+411s^2t+705st^2+396t^3) + 4m_c^2(60s^4+277s^3t+570s^2t^2 + 576st^3+198t^4) - 5s(4s^4+17s^3t+35s^2t^2+36st^3+18t^4)] \tag{A23b}$$

$$h_2 = -8m_c^2[512m_c^8(56s-45t) - 64m_c^6(322s^2+117st-360t^2) + 48m_c^4(116s^3 + 120s^2t - 75st^2 - 150t^3) + m_c^2(-712s^4 - 900s^3t + 144s^2t^2 + 936st^3 + 720t^4) + 40s^5 + 72s^4t + 45s^3t^2 + 36st^4] \tag{A23c}$$

$$h_3 = -8m_c^2[10240m_c^8 s - 64m_c^6(115s^2+99st-90t^2) + 48m_c^4(53s^3+48s^2t-39st^2 - 90t^3) - 4m_c^2(133s^4+207s^3t+18s^2t^2-342st^3-180t^4) + s(49s^4+162s^3t+261s^2t^2+144st^3+36t^4)] \tag{A23d}$$

$$h_4 = -72m_c^2(-4m_c^2+s+2t)^2[40m_c^4(s-2t) + 2m_c^2(9s^2+8st+10t^2) + s(-s^2+st+t^2)]. \tag{A23e}$$

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