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Novel parametrization for the leptonic mixing matrix and *CP* violation

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We study leptonic *CP* violation from a new perspective. For Majorana neutrinos, a new parametrization for leptonic mixing of the form $V = O_{23} O_{12} K_a^i \cdot O$ reveals interesting aspects that are less clear in the standard parametrization. We identify several important scenario cases with mixing angles in agreement with experiment and leading to large leptonic *CP* violation. If neutrinos happen to be quasidegenerate, this new parametrization might be very useful, e.g., in reducing the number of relevant parameters of models.

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I. INTRODUCTION

Observations of neutrino oscillations have solidly established the massiveness of the neutrinos and the existence of leptonic mixing. Since neutrinos are strictly massless in the standard model (SM), these observations require necessarily new physics beyond the SM. One is still far from a complete picture of the lepton sector; i.e., many fundamental questions need to be answered. Not only the origin of the leptonic flavor structure remains unknown, but also the leptonic mixing differs tremendously from the observed quark mixing. Moreover, the absolute neutrino mass scale is still missing, one does not know whether neutrinos are Majorana or Dirac particles, and the nature of leptonic *CP* violation is still open (for a recent review, see Ref. [1]).

During the last decades, several attempts were made in order to overcome these fundamental questions. In particular, one may impose family symmetries forbidding certain couplings and at the same time explaining successfully the observed structure of masses and mixings, as well as predicting some other observables [2–15]. Although the structure of leptonic mixing is predicted in such models, the mass spectrum turns out to be unconstrained by such symmetries. The connection of leptonic mixing angles and *CP* phases with neutrino spectra in the context of partially and completely degenerate neutrinos was proposed in [16]. In an alternative approach, the anarchy of the leptonic parameters is assumed so that there is no physical distinction among three generations of lepton doublets [17–20].

From the analysis of neutrino oscillation experiments, one can extract bounds for the light neutrino mass square differences $\Delta m^2_{21} \equiv m^2_2 - m^2_1$ and $\Delta m^2_{31} \equiv m^2_3 - m^2_1$. Recent cosmological observations have constrained the sum of neutrino masses [21], which then imply an upper bound of the lightest neutrino mass. All knowledge on the light neutrino mixing is encoded in the Pontecorvo-Maki-Nakagawa-Sakata matrix (PMNS) [22–24]. In order to further analyze the leptonic flavor structure, it is essential to parametrize all the entries of the full PMNS matrix in terms of six independent parameters. It is clear that the choice of a parametrization does not impose any constraints on the physical observables. However, parametrizations are an important tool—recall the usefulness of the Wolfenstein parametrization [25] in the quark sector—and may play a meaningful role in interpreting underlying symmetries or relations that the data may suggest. In this sense, different parametrizations are certainly equivalent among themselves, although some particular patterns indicated by the data are easer to visualize in some parametrizations than in others. Moreover, special limits suggested by some parametrizations are obfuscated in others.

Among many parametrirmetrization is the most widely used, and the six parameters are three mixing angles, namely, $\theta_{12}, \theta_{13}, \theta_{23} \in [0, \pi/2]$, one Dirac-type phase δ, and two Majorana phases α_1, α_2 in the following form:

$$V^{\text{SP}} = K \cdot O_{23} \cdot K_D \cdot O_{13} \cdot O_{12} \cdot K_M, \qquad (1)$$

where the real-orthogonal matrices O_{12}, O_{13}, and O_{23} are the usual rotational matrices in the (1, 2), (1, 3), and (2, 3) sector, respectively. The diagonal unitary matrices K_D and K_M are given by $K_D \equiv \text{diag}(1, 1, e^{i\alpha_D})$ and $K_M \equiv \text{diag}(1, e^{i\alpha_1^M}, e^{i\alpha_2^M})$. Within the standard parametrization, one may recall that the consistent values for the neutrino mixing angles θ_{12} and θ_{23} together with the smallness of θ_{13} suggest that the neutrino mixing is rather close to the tribimaximal mixing (TBM) [26]. It is important to stress that this parametrization is (modulo irrelevant phases) the same as the one used for the quark sector, despite the fact that leptonic mixing is quite different.

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In this paper, we study leptonic CP violation in the context of a new parametrization for leptonic mixing of the form

$$V = O_{23}O_{12}K_\alpha^i \cdot O, \tag{2}$$

where $K_\alpha^i = \mathrm{diag}(1, i, e^{i\alpha})$, and O is a real-orthogonal matrix parametrized with three mixing angles. We then have a total of six parameters, namely, five mixing angles and one complex phase α, which is the required number of independent parameters for describing the PMNS matrix. We point out again that the nature of leptonic CP violation is still an open question, and it is, thus, not yet clear what, *de facto*, is the most adequate form to express or parametrize this phenomenon. Here, we choose to express leptonic CP violation as combinations of mixing angles and a unique complex phase α and will argue its usefulness in certain cases.

Indeed, due to its specific form, this new parametrization is particularly useful if neutrinos are quasidegenerate Majorana fermions [27]. It may also reflect the specific nature of neutrinos, suggesting that there could be a large contribution to neutrino mixing and CP violation present in the left part of the parametrization, possibly as a result of some symmetry, while the right part, in the form of the orthogonal matrix O, could come from some perturbative effect, reflecting the fact that there are three neutrino families with small mass differences and which result in small mixing, comparable to the mixing in the quark sector and the Cabibbo angle.

The new parametrization permits a new view of large leptonic CP violation. It reveals interesting aspects that are less clear in the standard parametrization. We identify five scenario cases that lead to large Dirac-CP violation, and which have mixing angles in agreement with experimental data. A certain scenario (I-A) is found to be the most appealing, since it only needs two parameters to fit the experimental results on lepton mixing and provides large Dirac-CP violation and large values for the Majorana-CP-violating phases.

The paper is organized as follows. In the next section, we prove the consistency of the new parametrization stated in Eq. (2). In Sec. III, we motivate the use of this new parametrization in the limit of degenerate or quasidegenerate neutrino spectrum. Then in Sec. IV, we present an alternative view of large leptonic CP violation, using the new parametrization for leptonic mixing, discuss its usefulness, and identify several important scenario cases. Results are shown for mixing and CP violation. In Sec. V, we give a numerical analysis of the scenarios described in the previous section, and, for the quasidegenerate Majorana neutrinos, a numerical analysis of their stability. Finally, in Sec. VI, we present our conclusions.

II. A NOVEL PARAMETRIZATION

In this section, we present the new parametrization for the lepton mixing matrix. First, we prove that any unitary matrix can be written with the following structure:

$$V = K_S O_{23}O_{12}K_\alpha^i \cdot O, \tag{3}$$

where $K_S = \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$ is a pure-phase unitary diagonal matrix, O_{23}, O_{12} are two elementary orthogonal rotations in the (23)- and (12)-planes, $K_\alpha^i = \mathrm{diag}(1, i, e^{i\alpha})$ has just one complex phase α (apart from the imaginary unit i), and O is a general orthogonal real matrix described by three angles.

Proof: Let us start from a general unitary matrix V and compute the following symmetric unitary matrix S,

$$S = V^* V^\dagger. \tag{4}$$

Assuming that S is not trivial, i.e., it is not a diagonal unitary matrix, one can rewrite the matrix S as

$$S = K_S^* S_0 K_S^*, \tag{5}$$

with a pure-phase diagonal unitary matrix $K_S = \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$ so that the first row and the first column of S_0 become real. In fact, the diagonal matrix K_S has no physical meaning, since it only rephases the PMNS matrix V on the left. This can be clearly seen in the weak basis where the charged lepton mass matrix is diagonal, and through a weak-basis transformation the phases in K_S can be absorbed by the redefinition of the right-handed charged lepton fields. One can now perform a (23)-rotation on S_0 as

$$S_0' = O_{23}^\intercal S_0 O_{23}, \tag{6}$$

with a orthogonal matrix O_{23} given by

$$O_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix}, \tag{7}$$

so that the (13)- and (31)-elements of the resulting matrix vanish. Making use of unitarity conditions, one concludes that automatically the (13)- and (31)-elements become also zero, and, therefore, the (12)-sector of S_0' decouples, and one obtains that

$$S_0 = O_{23}O_{12} \cdot \mathrm{diag}(1, -1, e^{-2i\alpha}) \cdot O_{12}^\intercal O_{23}^\intercal, \tag{8}$$

where O_{12} is given by

$$O_{12} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{9}$$

The matrix S_0 is then written as

$$S_0 = U_0^* U_0^\dagger, \tag{10}$$

where the unitary matrix U_0 is given by

$$U_0 = O_{23} O_{12} K_\alpha^i, \tag{11}$$

with $K_\alpha^i = \mathrm{diag}(1, i, e^{i\alpha})$. Thus, given V, we can compute explicitly the matrices K_S, O_{23}, O_{12}, and K_α^i. In order to obtain the form given in Eq. (3), we factorize the leptonic mixing V as

$$V = (K_S U_0) W_0^*, \tag{12}$$

and we demonstrate that W_0 is real and orthogonal. By definition, the matrix W_0,

$$W_0 \equiv (U_0^\intercal K_S) \cdot V^*, \tag{13}$$

is obviously unitary since it is the product of unitary matrices. Let us then verify that W_0 is indeed orthogonal by computing the product

$$W_0 \cdot W_0^\intercal = (U_0^\intercal K_S) \cdot V^* V^\dagger \cdot (K_S U_0) = U_0^\intercal K_S S K_S U_0, \tag{14}$$

where we have used Eq. (4). Inserting into this expression the other expression for S given in Eq. (5) and making use of Eq. (10), we find

$$W_0 \cdot W_0^\intercal = \mathbb{1}, \tag{15}$$

which means that W_0 is real and orthogonal. We, thus, write W_0 explicitly as

$$W_0 \equiv (U_0^\intercal K_S) \cdot V^* \equiv O, \tag{16}$$

where the O stands for the fact that it is a real-orthogonal matrix. Finally, rewriting this equation, we find for the general unitary matrix V,

$$V = K_S \cdot U_0 \cdot O, \tag{17}$$

or with Eq. (11),

$$V = K_S \cdot O_{23} O_{12} K_\alpha^i \cdot O. \tag{18}$$

We have, thus, derived a new parametrization for the lepton mixing matrix, i.e.,

$$V = O_{23} O_{12} K_\alpha^i \cdot O, \tag{19}$$

where we have discarded the unphysical pure-phase matrix K_S. It is clear that, as with the standard parametrization in Eq. (1), this parametrization has also six physical parameters, but some are now of a different nature: two angles in O_{23} and O_{12}, three other angles in O, but just one complex phase α in K_α^i. From now on, we use explicitly the following full notation

$$V = O_{23}^L O_{12}^L \cdot K_\alpha^i \cdot O_{23}^R O_{13}^R O_{12}^R, \tag{20}$$

where we have identified each of the elementary orthogonal rotations, either on the left or on the right of the CP-violating pure-phase matrix K_α^i, with a notation superscript L, R.

The angles θ_{23}^L and θ_{12}^L can be easily calculated from the PMNS matrix V as

$$|\tan\theta_{23}^L| = \frac{r_3}{r_2}, \qquad |\tan 2\theta_{12}^L| = \frac{(r_2^2 + r_3^2)|\cos\theta_{23}^L|}{r_1 r_2}, \tag{21}$$

where the real numbers r_1, r_2, and r_3 are given by

$$r_1 = |V_{11}^2 + V_{12}^2 + V_{13}^2|, \tag{22}$$

$$r_2 = |V_{11}V_{21} + V_{12}V_{22} + V_{13}V_{23}|, \tag{23}$$

$$r_3 = |V_{11}V_{31} + V_{12}V_{32} + V_{13}V_{33}|. \tag{24}$$

The phase α in K_i^α is given by $\arg[(O_{23}^\intercal S_0 O_{23})_{33}]$.

A. *CP* violation

In the standard parametrization (SP), we may distinguish two types of CP-violating phases: Dirac- and Majorana-CP-violating phases. The Dirac-type phases are determined by the four independent arguments of the quartets $\arg(V_{1i}V_{kj}V_{1j}^*V_{ki}^*)$, with $i \neq j \neq k$, and the Majorana-type phases are given by the six independent arguments of the bilinears $\arg(V_{ij}V_{ik}^*)$, with $j \neq k$. In the SP, these phases are the minimal CP-violating quantities when neutrinos are Majorana particles [28–34].

However, we remind again that the nature of leptonic CP violation is still open, and it is, thus, not yet clear what could be the most adequate form to express or parametrize this phenomenon. Here, as an alternative, we choose to express CP violation in a different way, namely, as combinations of mixing angles and the unique complex phase α. It is worth to note that, in our new parametrization, even when $\alpha = 0$ or π, we still have CP violation due to the presence of an imaginary unit in the diagonal matrix K_i^α. In particular, setting $\alpha = 0$ the Dirac-CP-violating invariant $I_{CP} \equiv \mathrm{Im}(V_{12}V_{23}V_{22}^*V_{13}^*)$ yields

$$I_{CP} = \frac{1}{32}(\sin 2\theta^L_{23}\cos 2\theta^R_{23}(\sin^2\theta^L_{12}\cos\theta^L_{12}\sin 2\theta^R_{12}(3\sin 3\theta^R_{13} - 5\sin\theta^R_{13}) + 8\sin^2\theta^L_{12}\cos\theta^L_{12}\cos 2\theta^R_{12}\cos 2\theta^R_{13}\sin 2\theta^R_{23} + (7\cos\theta^L_{12} + \cos 3\theta^L_{12})\sin 2\theta^R_{12}\sin\theta^R_{13}\cos^2\theta^R_{13}) + 2\sin 2\theta^L_{12}\cos 2\theta^R_{23}\cos\theta^R_{23}(\sin 2\theta^R_{12}\cos\theta^R_{13}(\cos 2\theta^R_{13} - 3)\cos 2\theta^R_{23} + 2\cos^2\theta^R_{13}) - 2\cos 2\theta^R_{12}\sin 2\theta^R_{13}\sin 2\theta^R_{23})), \quad (25)$$

which vanishes when $\theta^L_{12} = \theta^L_{23} = 0$ [i.e., omitting the left orthogonal matrices in Eq. (20)] and when $\theta^L_{12} = \theta^R_{23} = 0$.

B. Other parametrizations

It should also be mentioned that lepton mixing matrices expressed as a product of two orthogonal and pure-phase diagonal matrices OKO' were first proposed in Ref. [35], however, with limited usefulness, and later in Ref. [36] in the context of a type-I seesaw. In regard to this, we point out, following a similar reductive procedure outlined here, that one can also obtain other forms [from the one in Eqs. (19) and (20)] for the parametrization of the lepton mixing matrix. E.g., one can have a parametrization where $V = O_{23}O_{13}K^\alpha_i \cdot O$, with $K^\alpha_i = \mathrm{diag}(1, e^{i\alpha}, i)$, or even other variations such as $V = O_{12}O_{23}\tilde{K}^\alpha_i \cdot O$, with $\tilde{K}^\alpha_i = \mathrm{diag}(e^{i\alpha}, i, 1)$. On the contrary, and due to its specific form, it will be shown that our new parametrization of Eqs. (19) and (20) is particularly useful.

C. Usefulness

Why a new parametrization? Does it add anything useful to the standard parametrization? We give several motivations.

First, we still do not know whether neutrinos are hierarchical or quasidegenerate. However, if neutrinos happen to be quasidegenerate, then the new parametrization is very useful.

Second, in this case, the new parametrization may reflect some specific nature of neutrinos. Heuristically, it may suggest that there is some large contribution to neutrino mixing and CP violation present in the left part $O^L_{23}O^L_{12}K^i_\alpha$ of Eq. (20), possibly as a result of some symmetry, while the right part in the form of the real-orthogonal matrix $O = O^R_{23}O^R_{13}O^R_{12}$ with the three angles could come from some perturbative effect, reflecting that there are three neutrino families with small mass differences and results in small mixing. Indeed, we consider that our parametrization incorporates well-diverse fixed structures for the lepton mixing [26,37–39] in the limit $V_{13} = 0$, and, in particular, the case of TBM which, e.g., in [40,41] occurs as the result of a family symmetry. If such a family symmetry exists, once it is broken at the electroweak scale, the reactor angle gets a small contribution of the order of the Cabibbo angle, possibly related to the small neutrino mass differences.

The third motivation is that this parametrization permits a different view of large leptonic CP violation from a new perspective. It reveals interesting aspects that were less clear in the standard parametrization. The Dirac- and Majorana-CP-violating quantities are here simply related to just one complex phase α present in $K^i_\alpha = \mathrm{diag}(1, i, e^{i\alpha})$. We discuss these issues in the next subsection, first in the limit of degenerate and quasidegenerate Majorana neutrinos.

III. DEGENERATE AND QUASIDEGENERATE MAJORANA NEUTRINOS

A. Degenerate neutrino masses

In the weak basis where the charged lepton mass matrix is diagonal and real positive, the matrix S_0 has a special meaning in the limit of exact neutrino mass degeneracy [27,42]. In this limit, the neutrino mass matrix M_0 assumes the following form:

$$M_0 = \mu S_0 = \mu U^*_0 U^\dagger_0, \quad (26)$$

where μ is the common neutrino mass. The matrix U_0 accounts for the leptonic mixing. Thus, within the parametrization given in Eq. (19), degeneracy of Majorana neutrino masses corresponds to setting the orthogonal matrix O to the identity matrix. In the limit of exact degenerate neutrinos, the orthogonal matrix O on the right of the new parametrization in Eq. (19) has no physical meaning. It can be absorbed in the degenerate neutrino fields. This has motivated our proposal for the use of the new parametrization.

As stated in Ref. [27], in the limit of exact degeneracy for Majorana neutrinos, leptonic mixing and CP violation can exist irrespective of the nature of neutrinos. Leptonic mixing can only be rotated away if and only if there is CP invariance and all neutrinos have the same CP parity [43,44]. This is clearly the case when S_0 is trivial. It is also clear that even in the limit of exact degeneracy with CP conservation, but with different CP parities ($\alpha = 0$ or $\alpha = \frac{\pi}{2}$), one cannot rotate U_0 away through a redefinition of the neutrino fields. Thus, even in this limit (within the degeneracy limit), leptonic mixing may occur.

B. Quasidegenerate neutrino masses

The usefulness of the new parametrization is particularly interesting if neutrinos are quasidegenerate. When the degeneracy is lifted, i.e., for quasidegenerate neutrinos, the full neutrino mass matrix becomes slightly different from the exact limit in Eq. (26):

$$M = \mu(S_0 + Q^{\epsilon}), \tag{27}$$

where Q^{ϵ} is some small perturbation. In general, this perturbation may significantly modify the mixing result for the exact case in Eq. (26). In view of our new parametrization, now the full lepton mixing matrix diagonalizing M is described by

$$V = U'_o \cdot O, \tag{28}$$

where U'_o is of the same form as U_o. It is not guaranteed that this U'_o is the exactly same as U_o. It may differ from U_o because of the perturbation, just as the matrix O, which can be either a small or possibly large general orthogonal matrix. In Sec. V, we shall quantify this more explicitly, using numerical simulations.

C. *CP* violation of quasidegenerate neutrinos

It was pointed out in Ref. [27] that if neutrinos are quasidegenerate (or even exact degenerate), CP violation continues to be relevant. This can be understood if one defines weak-basis invariant quantities sensitive to CP violation. An important invariant quantity in this case is

$$G_m \equiv |\mathrm{Tr}([M_\nu H_l M_\nu^*, H_l^*]^3)|, \tag{29}$$

where $H_l = M_l M_l^\dagger$ is the squared charged lepton mass matrix. Contrary to the usual quantity $I = \mathrm{Tr}([M_\nu^\dagger M_\nu, H_l]^3)$, which is proportional to the Dirac-CP-violating quantity I_{CP}, we find that the quantity G_m signals CP violation even if neutrinos are exact degenerate. In fact, we obtain in this limit

$$G \equiv \frac{G_m}{\Delta_m} = \frac{3}{4}|\sin 2\theta_{12}^L \sin 4\theta_{12}^L \sin^2 2\theta_{23}^L \sin 2\alpha|, \tag{30}$$

where

$$\Delta_m \equiv \mu^6 (m_\tau^2 - m_\mu^2)^2 (m_\tau^2 - m_e^2)^2 (m_\mu^2 - m_e^2)^2, \tag{31}$$

with μ the common neutrino mass. θ_{12}^L and θ_{23}^L are, respectively, the angles of O_{12}^L and O_{23}^L in Eq. (20), and α is the complex phase of $K_\alpha^i = \mathrm{diag}(1, i, e^{i\alpha})$. Obviously, with the new parametrization for the lepton mixing in Eq. (20), this invariant takes on a new and relevant meaning. It is a curious fact that G is so specifically (and in such a clean way) dependent on only, what we have called, the left part of Eq. (20) and on $\sin 2\alpha$.

D. Quasidegenerate neutrinos and double-beta decay

Another result which we obtain in the case of quasidegenerate neutrinos is the fact that the parameter M_{ee} measuring double-beta decay depends in our new parametrization mainly on the matrix U_o. From Eq. (27), it is clear that

$$|M_{ee}| = |\mu (S_o)_{11}| = |\mu \cos 2\theta_{12}^L|, \tag{32}$$

in zeroth order in ε. This is an interesting result for M_{ee} when confronting it with the one calculated directly from the standard parametrization in Eq. (1). In the case of quasidegenerate neutrinos, we have the approximation

$$|M_{ee}| = |\mu(\cos^2\theta_{\mathrm{sol}} + e^{2i\alpha_1^M} \sin^2\theta_{\mathrm{sol}})|, \tag{33}$$

neglecting the terms with V_{13}^2.

The point here is that, with possible separate future results for μ and M_{ee}, we may deduce if there is any significant Majorana-type phase α_1^M. Subsequently, by comparing Eq. (33) with Eq. (32), we may know if θ_{12}^L can be identified with the solar mixing angle θ_{sol}. If, however, this is not the case, then we also know that the perturbation in Eq. (26) produces large effects. E.g., suppose that inserting the (future) experimental results in Eq. (33) yields $\alpha_1^M = 0$, which from Eq. (32) results in $\theta_{12}^L = 0$. Then a large solar angle must come mainly from the O in Eq. (19).

IV. LEPTONIC *CP* VIOLATION FROM A NEW PERSPECTIVE

Maximum Dirac-CP violation in lepton mixing can be obtained in the standard parametrization of Eq. (1) when choosing the Dirac phase $\alpha_D = \pi/2$ in the diagonal unitary matrix $K_D = \mathrm{diag}(1, 1, e^{i\alpha_D})$. If neutrinos are Dirac, then there is no other form of leptonic CP violation. If neutrinos are Majorana, then there are two more CP-violating phases in $K_M = \mathrm{diag}(1, e^{i\alpha_1^M}, e^{i\alpha_2^M})$. These Majorana phases may be large or small, and one finds that leptonic CP violation is apparently limited to these two considerations if one chooses the standard parametrization. On the contrary, if one switches to the new parametrization of Eq. (20), one gets a much richer structure for leptonic CP violation, particularly, if neutrinos are quasidegenerate.

The experimentally measured mixing angles are given by the paramenters of the new parametrization as

$$|V_{13}|^2 = s^2_{\theta_{12}^L} c^2_{\theta_{13}^R} s^2_{\theta_{23}^R} + c^2_{\theta_{12}^L} s^2_{\theta_{13}^R}, \tag{34a}$$

$$\sin^2\theta_{\mathrm{sol}} = \frac{s^2_{\theta_{12}^L}(c_{\theta_{12}^R} c_{\theta_{23}^R} - s_{\theta_{12}^R} s_{\theta_{13}^R} s_{\theta_{23}^R})^2 + c^2_{\theta_{12}^L} s^2_{\theta_{12}^R} c^2_{\theta_{13}^R}}{1 - |V_{13}|^2}, \tag{34b}$$

$$\sin^2\theta_{\mathrm{atm}} = \frac{c_\alpha^2 c^2_{\theta^R_{13}} c^2_{\theta^R_{23}} s^2_{\theta^L_{23}} - 2c_\alpha c_{\theta^L_{23}} c_{\theta^R_{13}} c_{\theta^R_{23}} s_{\theta^L_{12}} s_{\theta^L_{23}} s_{\theta^R_{13}} + c^2_{\theta^L_{23}} s^2_{\theta^L_{12}} s^2_{\theta^R_{13}} + c^2_{\theta^R_{13}} (s_\alpha c_{\theta^R_{23}} s_{\theta^L_{23}} + c_{\theta^L_{12}} c_{\theta^L_{23}} s_{\theta^R_{23}})^2}{1 - |V_{13}|^2}, \tag{34c}$$

where we have used the identification $c_X = \cos X$ and $s_X = \sin X$. As will be shown, these expressions simplify significantly for several cases near to the experimental data and with large leptonic CP violation.

Next, we identify these important cases leading to large CP violation in lepton mixing using the new parametrization of Eq. (20). We do this by fixing some of the parameters and assume this fixing would arise from a preexisting model and/or symmetry. We choose a starting point for the mixing matrix that has the same mixing angles as the tribimaximal mixing,

$$|V_{13}|^2 = 0, \qquad \sin^2\theta_{\mathrm{atm}} = 1/2, \qquad \sin^2\theta_{\mathrm{sol}} = 1/3. \tag{35}$$

These values are close to the experimental results at the 1σ level [45],

$$\begin{aligned} 0.439 &< \sin^2\theta_{23} < 0.599, \\ 0.0214 &< \sin^2\theta_{13} < 0.0254, \\ 0.307 &< \sin^2\theta_{12l} < 0.339 \end{aligned} \tag{36}$$

given in terms of the standard paramatrization angles. Is easy to observe that $1/3$ is an allowed value for $\sin^2\theta_{12}$, but values slightly lower are better. The central value for $\sin^2\theta_{23}$ is above $1/2$, but values both below and above are preferred.

Given the closeness of tribimaximal mixing with experimental values, we fix some of the parameters such that we can reproduce TBM to zeroth order. The remaining parameters are then small and can be treated as perturbation parameters $\theta_{ij} = \varepsilon t_{ij}$, with ε of the order the Cabibbo angle. We identify five different cases. In Table I, we show the values for the parameters being used in our five different cases. Table II shows the explicit expression for the mixing angles in terms of the perturbation parameters εt_{ij} for each of the cases. All cases can have large Dirac-CP violation.

TABLE I. Values of the parameters for each case.

	O^L_{23}	O^L_{12}	O^R_{23}	O^R_{13}	O^R_{12}
I-A	$-\pi/4$	$\sin^{-1}(1/\sqrt{3})$	εt^R_{23}	εt^R_{13}	εt^R_{12}
I-B	$-\pi/4$	εt^L_{12}	εt^R_{23}	εt^R_{13}	$\sin^{-1}(1/\sqrt{3})$
I-C	$-\pi/4$	$\sin^{-1}(1/2)$	εt^R_{23}	εt^R_{13}	$\sin^{-1}(1/\sqrt{6})$
II-A	εt^L_{23}	εt^L_{12}	$-\pi/4$	εt^R_{13}	$\sin^{-1}(1/\sqrt{3})$
II-B	$\sin^{-1}(1/\sqrt{3})$	εt^L_{12}	$-\pi/4$	εt^R_{13}	$\sin^{-1}(1/\sqrt{3})$

A. Scenario I-A

This scenario yields in leading order a value for the Dirac-type invariant I_{CP}, which may be large:

$$I_{CP} = \frac{\varepsilon}{6\sqrt{3}} |\sqrt{2}t^R_{23}\cos\alpha - 2t^R_{13}\sin\alpha|. \tag{37}$$

All experimental results on mixing, including the central value for the solar angle, can be fit with just the phase α and the small parameter combination εt^R_{23} of the order of the Cabibbo angle. If we take the limit of small t^R_{12} and t^R_{13}, a nonzero value for α is necessary to have a value of $\sin\theta_{\mathrm{atm}} \neq 1/2$. In addition, if the t^R_{12} and t^R_{13} are small, the Majorana-CP-violating phases are large ($\sim\pi/2$). We find for the Majorana phases:

$$\tan\alpha^M_1 = \frac{\sqrt{2}}{\varepsilon t^R_{12}}, \qquad \tan\alpha^M_2 = \frac{t^R_{23}}{\sqrt{2}t^R_{13}}. \tag{38}$$

Clearly, the Majorana phases will decrease if (t^R_{12}, t^R_{13}) assume substantial values, but that will increase the value for the solar angle. We find for the double-beta decay parameter (the leading-order approximation) for the quasidegenerate case,

$$M_{ee} = \frac{\mu}{3}. \tag{39}$$

Another important aspect of this scenario is the form the neutrino mass matrix for the quasidegenerate case. In leading order, we find

$$M = \frac{\mu}{3}\begin{pmatrix} 1 & -2 & -2 \\ -2 & \frac{-1+3e^{-2i\alpha}}{2} & \frac{1+3e^{-2i\alpha}}{2} \\ -2 & \frac{1+3e^{-2i\alpha}}{2} & \frac{-1+3e^{-2i\alpha}}{2} \end{pmatrix}. \tag{40}$$

Furthermore, we obtain for the CP-violating quantity G defined in Eq. (30):

$$G = \frac{4}{9}|\sin(2\alpha)|. \tag{41}$$

B. Scenario I-B

The CP invariant is in this case (in leading order) given by

$$I_{CP} = \frac{\varepsilon}{3\sqrt{2}} |t^R_{13}\cos\alpha|. \tag{42}$$

TABLE II. Mixing angles as a function of the perturbed parameters t_{ij}.

	$\|V_{13}\|^2$	$\sin^2\theta_{\rm atm}$	$\sin^2\theta_{\rm sol}$
I-A	$\frac{\varepsilon^2}{3}(2(t_{13}^R)^2+(t_{23}^R)^2)$	$\frac{1}{2}-\frac{\varepsilon}{\sqrt{3}}(t_{13}^R\cos\alpha-\sqrt{2}t_{23}^R\sin\alpha)$	$\frac{1}{3}+\frac{\varepsilon^2}{9}(3(t_{12}^R)^2+2(t_{13}^R)^2-2(t_{23}^R)^2)$
I-B	$\varepsilon^2(t_{13}^R)^2$	$\frac{1}{2}+\varepsilon t_{23}^R\sin\alpha-\varepsilon^2 t_{12}^L t_{13}^R\cos\alpha$	$\frac{1}{3}+\frac{\varepsilon^2(t_{12}^L)^2}{3}$
I-C	$\frac{\varepsilon^2}{4}(3(t_{13}^R)^2+(t_{23}^R)^2)$	$\frac{1}{2}-\frac{\varepsilon}{2}(t_{13}^R\cos\alpha+\sqrt{3}t_{23}^R\sin\alpha)$	$\frac{1}{3}+\frac{\varepsilon^2}{24}(3(t_{13}^R)^2-2\sqrt{5}t_{13}^R t_{23}^R-3(t_{23}^R)^2)$
II-A	$\frac{\varepsilon^2}{2}((t_{12}^L)^2+2(t_{13}^R)^2)$	$\frac{1}{2}+\varepsilon t_{23}^L\sin\alpha-\frac{\varepsilon^2(t_{12}^L)^2}{4}$	$\frac{1}{3}+\frac{\varepsilon^2(t_{12}^L)^2}{6}$
II-B	$\frac{\varepsilon^2}{2}((t_{12}^L)^2+2(t_{13}^R)^2)$	$\frac{1}{2}+\frac{\sqrt{2}\sin\alpha}{3}-\frac{\varepsilon^2}{12}((t_{12}^L)^2-8t_{12}^L t_{13}^R\cos\alpha)$	$\frac{1}{3}+\frac{\varepsilon^2(t_{12}^L)^2}{6}$

If we want to avoid the central value for the atmospheric mixing angle, then it is clear that we need at least three parameters $\alpha, t_{13}^R, t_{23}^R$ to fit the experimental results on mixing and large Dirac-CP violation. The central value for the solar angle cannot be achieved, not even with the use of all parameters. The Majorana-CP-violating phases are

$$\tan\alpha_1^M=\frac{3}{\sqrt{2}}\varepsilon t_{12}^L,\qquad \tan\alpha_2^M=\frac{\varepsilon t_{12}^L}{\sqrt{2}}\left(1+\frac{\sqrt{2}t_{23}^R}{t_{13}^R}\right). \tag{43}$$

This scenario produces small Majorana-CP-violating phases. If neutrinos are quasidegenerate, we find for the neutrino mass matrix, in leading order

$$M=\mu\begin{pmatrix}1 & 0 & 0\\ 0 & \sin\alpha & \cos\alpha\\ 0 & \cos\alpha & -\sin\alpha\end{pmatrix}. \tag{44}$$

The double-beta decay parameter and the CP-violating quantity G read (in leading order)

$$M_{ee}=\mu\quad\text{and}\quad G=0, \tag{45}$$

respectively.

C. Scenario I-C

An intermediate scenario where both O_{12}^L and O_{12}^R are large. We choose one of the many combinations of these two angles to obtain TBM mixing. Then, three parameters are fixed, which leaves only three free parameters. This scenario yields for I_{CP},

$$I_{CP}=\frac{\varepsilon}{24}|(\sqrt{15}t_{13}^R+\sqrt{3}t_{23}^R)\cos\alpha+(\sqrt{5}t_{23}^R-3t_{13}^R)\sin\alpha|. \tag{46}$$

Again, we may have large Dirac-CP violation. If we want to avoid the central value for the atmospheric mixing angle, it may be seen here that we only need two parameters: the phase α and one of the remaining t_{ij} to fit the experimental results on mixing, but remember that this depends on the choice of the two large angles of O_{12}^L and O_{12}^R. In this context, the Majorana-CP-violating phases can be large:

$$\tan\alpha_1^M=3\sqrt{\frac{3}{5}},\qquad \tan\alpha_2^M=\frac{\sqrt{3}(t_{13}^R+\sqrt{5}t_{23}^R)}{3\sqrt{5}t_{13}^R-t_{23}^R}. \tag{47}$$

We find for the quasidegenerate limit the neutrino mass matrix, the double-beta decay parameter, and the CP-violating quantity G, in leading order:

$$M=\frac{\mu}{2}\begin{pmatrix}1 & -\sqrt{\frac{3}{2}} & \sqrt{\frac{3}{2}}\\ -\sqrt{\frac{3}{2}} & \frac{2e^{-2i\alpha}-1}{2} & \frac{2e^{-2i\alpha}+1}{2}\\ \sqrt{\frac{3}{2}} & \frac{2e^{-2i\alpha}+1}{2} & \frac{2e^{-2i\alpha}-1}{2}\end{pmatrix}, \tag{48}$$

$$M_{ee}=\frac{\mu}{2},\qquad G=\frac{9}{16}|\sin 2\alpha|. \tag{49}$$

D. Limit case II

As in limit case I, we may construct here two opposite and distinctive scenarios: a scenario where O_{23}^L is large, or a scenario where O_{23}^R is large. The scenario where O_{23}^L is large but where O_{23}^R is small, is already contained in the scenario I-A of limit case I (modulo some slight modifications which produce equivalent results). It is, therefore, sufficient to focus on a scenario where O_{23}^L is small and O_{23}^R is large or exceptionally on a scenario between, where both are large.

E. Scenario II-A

A scenario where O_{23}^L is small and O_{23}^R is large. The Dirac-CP invariant is given (in leading order) by

$$I_{CP}=\frac{\varepsilon}{6}|t_{12}^L|. \tag{50}$$

In this case, it is clear that we cannot achieve a central value for the solar angle; we need $t_{12}^L\neq 0$ to have a nonzero value for I_{CP}, but doing so will increase the value of the solar angle above 1σ. The Majorana-CP-violating phases are also obtained in leading order

$$\tan\alpha_1^M=\frac{3}{2}\varepsilon t_{12}^R,\qquad \tan\alpha_2^M=\frac{t_{12}^L}{\sqrt{2}t_{13}^R}, \tag{51}$$

where only the second one can be large. For quasidegenerate neutrinos, we find for the neutrino mass matrix, the double-beta decay parameter and the CP-violating quantity G in leading order:

$$M = \mu \mathbb{1}, \qquad M_{ee} = \mu, \qquad G = 0. \tag{52}$$

F. Scenario II-B

An intermediate scenario where both O_{23}^L and O_{23}^R are large. Also, for this case, only two parameters are needed, e.g., the perturbative parameter t_{13}^R and the phase α, which has to be small of the order of the Cabibbo angle to fit the experimental results on atmospheric mixing and large Dirac-CP violation, but again, here this depends on the choice for the two large angles of O_{23}^L and O_{23}^R. In first order, we have for the Dirac-CP invariant:

$$I_{CP} = \frac{\varepsilon}{18} |t_{12}^L + 4 t_{13}^R \cos\alpha|. \tag{53}$$

As in all of the previous cases, one can have a large value for I_{CP}. For this case, we can make a simple (leading-order) prediction if we take t_{12}^L much smaller than t_{13}^R and α small:

$$I_{CP} = \frac{2}{9} |V_{13}|. \tag{54}$$

The Majorana-CP-violating phases are obtained in leading order

$$\tan\alpha_1^M = \frac{3}{2}\varepsilon t_{12}^R, \qquad \tan\alpha_2^M = \frac{t_{12}^L}{\sqrt{2} t_{13}^R}, \tag{55}$$

where again, the second one can be large. For quasidegenerate neutrinos, we find for the neutrino mass matrix, the double-beta decay parameter and the CP-violating quantity G in leading order:

$$M = \mu \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{e^{-2i\alpha}-2}{3} & \frac{\sqrt{2}(1+e^{-2i\alpha})}{3} \\ 0 & \frac{\sqrt{2}(1+e^{-2i\alpha})}{3} & \frac{2e^{-2i\alpha}-1}{3} \end{pmatrix}, \tag{56}$$

$$M_{ee} = \mu, \qquad G = 0. \tag{57}$$

As already mentioned in this section, it can be seen from Table II that in the cases I-B, II-A, and II-B, the value for $\sin^2\theta_{\rm sol}$ cannot be lower than $1/3$, which is not in agreement with the experimental results given Eq. (36) at the 1σ level. This is, of course, due to our initial choice in Eq. (35), which corresponds to exact tribimaximal mixing. We stress that some of our conclusions with regard to the different scenarios may depend significantly on the initial starting point, while others do not. However, with regard to scenario I-A, very similar results are obtained if one chooses as starting points, e.g., the golden ratio mixing of type I [39] or the hexagonal mixing [37,38], instead of TBM.

G. Scenario I-A and the standard parametrization

We are tempted to find scenario I-A the most appealing. It only needs two extra parameters to fit the experimental results on lepton mixing and provides large Dirac-CP violation and large values for the Majorana-CP-violating phases. The other scenarios need more parameters or need more adjustment. We also point out that scenario I-A would not appear so clearly if one used a different parametrization, e.g., one of the parametrizations mentioned just after Eq. (20).

Given the relevance of scenario I-A, we shall now reproduce this scenario in the standard parametrization given in Eq. (1), where the TBM scheme is obtained with

$$V^{\rm SP} = O_{23}^{\pi/4} \cdot K_D \cdot O_{13} \cdot O_{12}^{\phi_o}, \qquad \sin\phi_o = \frac{1}{\sqrt{3}}, \tag{58}$$

with the angle of O_{13} put to zero. For simplicity, we leave out the Majorana phases. In this parametrization, in order to have a value for $|V_{13}| \neq 0$, we have to switch on the angle O_{13}. However, for the unitary matrix in Eq. (58), one may check that even then, $|V_{23}| = |V_{33}|$, irrespective of the value of the angle of O_{13}. Thus, using this remaining parameter, one cannot adjust the atmospheric mixing angle unless, e.g., from the start, the angle of the O_{23} is chosen to be different from $\pi/4$. One has to correct the atmospheric mixing angle, or from the beginning, or afterwards, with some additional extra contribution which modifies the TBM limit. It is clear, in the standard parametrization, adjusting the TBM limit for the atmospheric mixing angle is not possible using the remaining parameters. This is in clear contrast with our new parametrization and what we obtain for scenario I-A, where the parameters available in the actual parametrization, in this case, via suitable choice for of the parameter εt_{23} in Eq. (58), at the same time adjust the atmospheric mixing angle, generate a small value for $|V_{13}|$, and make possible large values for CP violation. Possibly, this may be useful for some models.

V. NUMERICAL SIMULATION AND STABILITY

For completeness, we give a numerical analysis of some of the scenarios described in the previous section. We choose a fixed scheme, the TBM scheme constructed with the five different scenarios. More precisely, we test

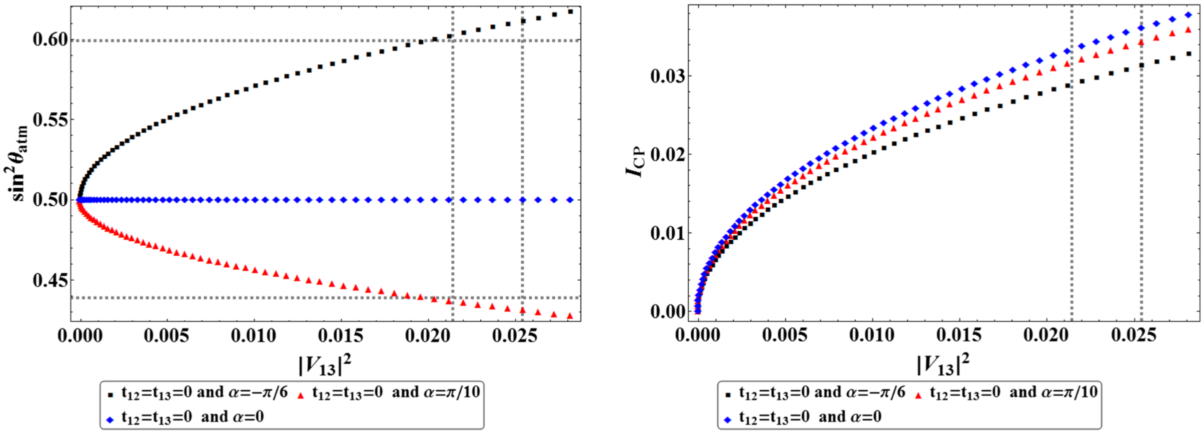

FIG. 1 (color online). Plotting $\sin^2\theta_{\rm atm}$ and the CP invariant I_{CP} as a function of $|V_{13}^2|$ for scenario I-A.

$$
\begin{aligned}
\text{I-A}:\ V_o &= O_{23}^{\pi/4} O_{12}^{\phi_o} K_{\alpha_o}^i,\\
\text{I-B}:\ V_o &= O_{23}^{\pi/4} K_{\alpha_o}^i O_{12}^{\phi_o},\\
\text{I-C}:\ V_o &= O_{23}^{\pi/4} O_{12}^{\phi_1} K_{\alpha_o}^i O_{12}^{\phi_2},\\
\text{II-A}:\ V_o &= O_{23}^{\pi/4} O_{12}^{\phi_o},\\
\text{II-B}:\ V_o &= O_{23}^{\theta_o} K_{\alpha_o}^i O_{23}^{\pi/4} O_{12}^{\phi_o},
\end{aligned}
\tag{59}
$$

where $\sin\phi_o = \sin\theta_o = \frac{1}{\sqrt{3}}$, $\sin\phi_1 = \frac{1}{2}$, $\sin\phi_2 = \frac{1}{\sqrt{6}}$. We define the U_o as the matrix on the left, together with the $K_{\alpha_o}^i$. In the II-A case, this is the identity matrix. We define also the O_o as the matrix on the right of the $K_{\alpha_o}^i$. In the II-A case, this is the whole matrix V_o. For case II-B, $\alpha_o = 0$ as pointed out in the previous section. For the other cases, we assume for α_o, diverse fixed values.

We illustrate in Figs. 1–3 the correlations among the observables for the scenarios I-A, II-A, and II-B. The figures plot for each scenario $\sin^2\theta_{\rm atm}$ and I_{CP} as a function of $|V_{13}|^2$ and I_{CP} as a function of $|V_{13}|^2$, for particular values of the parameters left unconstrained in the definition of each scenario according to Table I. Scenarios I-B and I-C are omitted since they have similar behavior as scenario I-A for these observables. A numerical analysis of scenario I-A was also done in Ref. [42]. We can conclude from Figs. 1–3 that a large CP invariant I_{CP} can be obtained in agreement with the allowed experimental range of the observed parameters.

Next, we test how the lepton mixing matrix changes and the stability of our scenarios, by adding a small random perturbation to a predefined exact degenerate limit. To do this, we construct a neutrino mass matrix M composed of an exact degenerate part in the form of a symmetric unitary matrix S_o related to one of the TBM scenario schemes in Eq. (59) and a part composed of a small random perturbation Q^ϵ. Thus, the full quasidegenerate neutrino mass matrix is as in Eq. (27)

$$M = \mu(S_o + Q^\epsilon), \tag{60}$$

where $S_o = U_o^* U_o^\dagger$, with the U_o's of the different cases, and Q^ϵ is some small complex symmetric random perturbation

$$Q^\epsilon \equiv \epsilon^2 Q, \qquad \epsilon^2 = \frac{(\Delta m_{31}^2)^{\rm exp}}{2\mu^2},$$

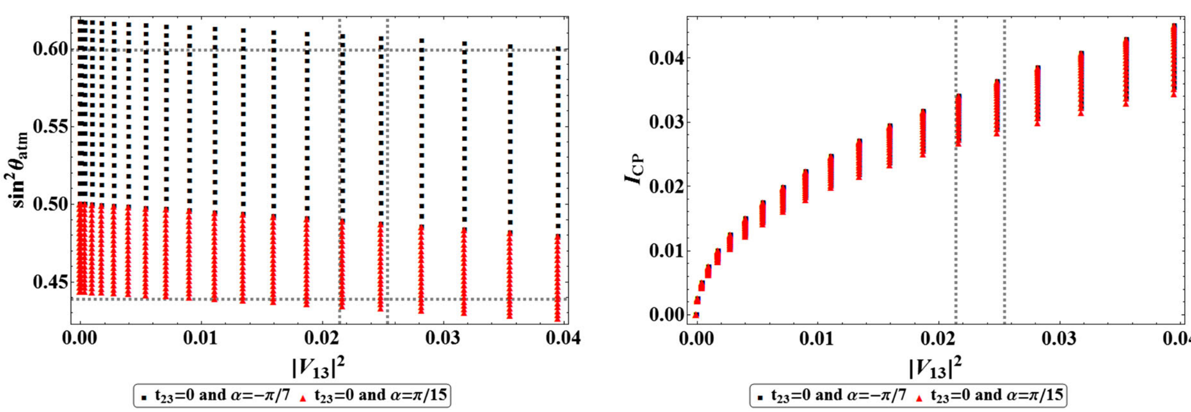

FIG. 2 (color online). Plotting $\sin^2\theta_{\rm atm}$ and the CP invariant I_{CP} as a function of $|V_{13}^2|$ for scenario II-A.

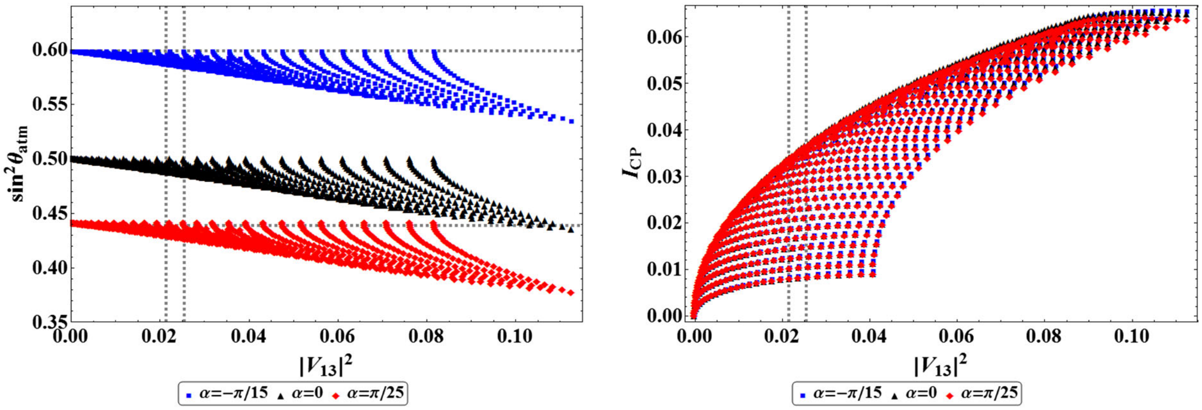

FIG. 3 (color online). Plotting $\sin^2\theta_{\mathrm{atm}}$ and the CP invariant I_{CP} as a function of $|V_{13}^2|$ for scenario II-B.

with Q random perturbations of $O(1)$ generated by our program. We test the stability of lepton mixing of the different scenarios. We do not worry about the exact mass differences, with two (reasonable) exceptions: we take for ε^2 a fixed value. Taking into account the upper bound on the sum of neutrino masses as suggested by the Planck Collaboration [21] obtained in a model-dependent analysis, i.e., $\sum_i m_i < 0.23$ eV, one gets for the common neutrino mass $\mu \lesssim 0.08$ eV. However, if one relaxes this assumption and takes a somewhat larger value for $\mu = 0.14$, together with $(\Delta m_{31}^2)^{\mathrm{exp}} = 2.5 \times 10^{-3}$ eV^2, we obtain $\varepsilon^2 \lesssim 0.064$, which makes ε of the order of the Cabibbo angle. These values make sure that we are in a mass range where the computed output $\Delta m_{31}^2 = O(1) \times 10^{-3}$ eV^2. We discard cases generated by the perturbation where $|\Delta m_{31}^2| < |\Delta m_{21}^2|$. Further, we do not impose any other restrictions on the random perturbation Q other than $\mathrm{Re}(Q_{ij})$ and $\mathrm{Im}(Q_{ij})$ to be real numbers between -1 and 1. However, we have checked that further restrictions on the masses do not change significantly any of the plots.

From the different mixing scenarios and the random Q's in M, we compute the full lepton mixing V, i.e., the corresponding diagonalizing matrix of M, such that $V^\intercal M V = D$ is real and positive. Following the proof in Sec. II, we decompose the full lepton mixing V in the new parametrization obtained as in Eq. (19). We then compare the new $U \equiv O_{23} O_{12} K_\alpha^i$ resulting from the perturbation, with the original U_o (i.e., without the perturbation) of one of the cases in Eq. (59) and evaluate a quantity Δ_U giving a reasonable measure of how much U and U_o differ:

$$\Delta_U = \frac{1}{2}\sum ||U_{ij}| - |(U_o)_{ij}||. \tag{61}$$

Notice that this definition does not "see" the phase factors of the K_α^i of U or of the U_o. For this, we evaluate the changing on the phases α by defining the quantity

$$\Delta_\alpha = ||\sin\alpha| - |\sin\alpha_o|| \tag{62}$$

that compares the phase α of the K_α^i of U, with the phase α_o of the $K_{\alpha_o}^i$ of U_o and discarding differences of π. The II-A case has no α_o phases. We have also estimated how much O in Eq. (19) differs from our original O_o in Eq. (59), with

$$\Delta_O = \frac{1}{2}\sum ||O_{ij}| - |(O_o)_{ij}||, \tag{63}$$

where again we discard any sign difference. The $1/2$ in front of Δ_O (and Δ_U) is a suitable normalization factor chosen such that, e.g., in a case where the original $O_o = \mathbb{1}$ and the new O is such that $O = O_{12}$ (or any other elementary rotation) with an angle $\sin\theta_{12} = 0.2$, then also $\Delta_O \approx 0.2$, of the same order of the Cabibbo angle.

In Figs. 4 and 5, we plot Δ_U as a function of Δ_O and Δ_α as a function of Δ_U, respectively, for the five scenarios. From Figs. 4 and 5, we find that the Δ_U and Δ_α of scenarios I-A and I-C hardly suffer any change with the perturbations. This means also that these quantities hardly depend on the parameter ε and subsequently on the common neutrino mass as given in Eq. (60), which is proportional to the perturbations. In Fig. 6, we show the variation of Δ_O as a function of Δ_U for different values of $\alpha_o = \pi/2, \pi/3, \pi/4,$

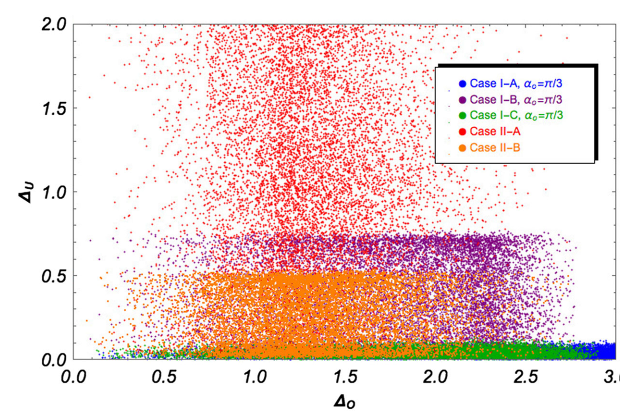

FIG. 4 (color online). Plotting Δ_O versus Δ_U for the five cases identified in Eq. (59) with $\alpha_o = \pi/3$.

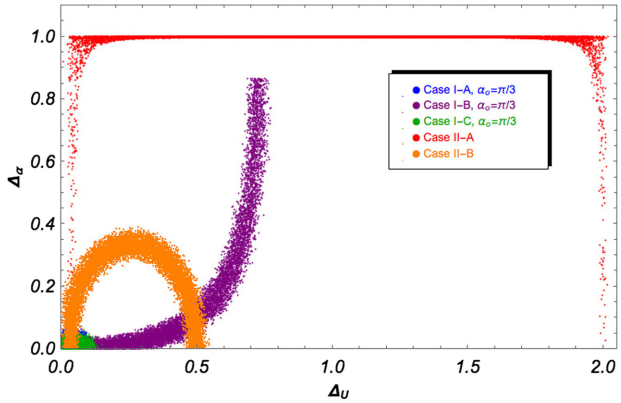

FIG. 5 (color online). Plotting Δ_O versus Δ_U for the five cases identified in Eq. (59) with $\alpha_o = \pi/3$.

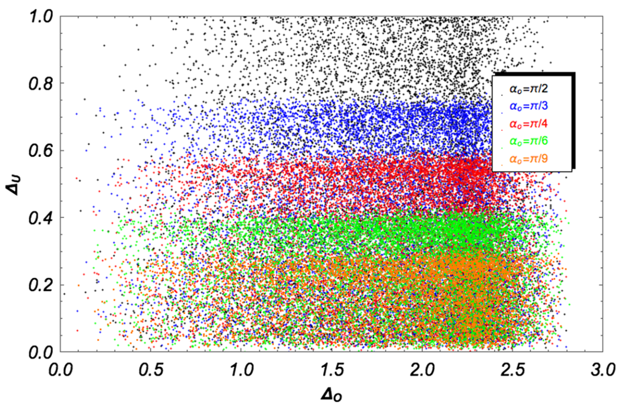

FIG. 6 (color online). Plotting Δ_O versus Δ_U for the case I-B varying $\alpha_o = \pi/2$, $\pi/3$, $\pi/4$, $\pi/6$, and $\pi/9$.

$\pi/6$, and $\pi/9$ for case I-B. Clearly, small α leads to more stability. Case I-A is not shown, since there is no apparent change of these quantities by varying α.

Cases I-A, I-C, and I-B with small α are the most stable with regard to Δ_U and Δ_α. As mentioned previously, Case I-C is somewhat artificial, as it requires a certain conspiracy between two angles ϕ_1 and ϕ_2 to be near the TBM limit. Therefore, we focus on Case I-A. As shown in the previous section, generically, scenario I-A has also the largest Majorana phases.

With regard to the stability and variation of Δ_O, we see that, in general, the perturbations generate large Δ_O contributions for all cases and, in particular, for scenario I-A. This parameter, therefore, depends strongly on the magnitude of the perturbation parameter ε. It seems that this can only be improved if one imposes restrictions on the allowed perturbations forcing smaller Δ_O's. Maybe some kind of symmetry could accomplish this. In Fig. 7 we give an example, where the perturbations Q are restricted: certain elements are taken to be zero, while the imaginary part and the diagonal real part are taken to be 0.1 smaller than the others:

$$Q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{x_{22}}{10} & x_{23} \\ 0 & x_{23} & \frac{x_{33}}{10} \end{pmatrix} + \frac{i}{10}\begin{pmatrix} 0 & 0 & 0 \\ 0 & y_{22} & y_{23} \\ 0 & y_{23} & y_{33} \end{pmatrix}, \qquad (64)$$

where the x's, and y's are random real numbers varying between -1 and 1. For the initial phase α_o, we take $\alpha_o = \pi/9$. We see that most of the deviations Δ_O (from the original $O_o = \mathbb{1}$) are now around 0.2 of the order of the Cabibbo angle, and this does not affect having large values for I_{CP}.

VI. CONCLUSIONS

We have studied some aspects of leptonic CP violation from a new perspective. We have identified several limit scenario cases, with mixing angles in agreement with experiment and leading to large CP violation. We have proposed a new parametrization for leptonic mixing of the form $V = O_{23}O_{12}K^i_\alpha \cdot O$ to accomplish this.

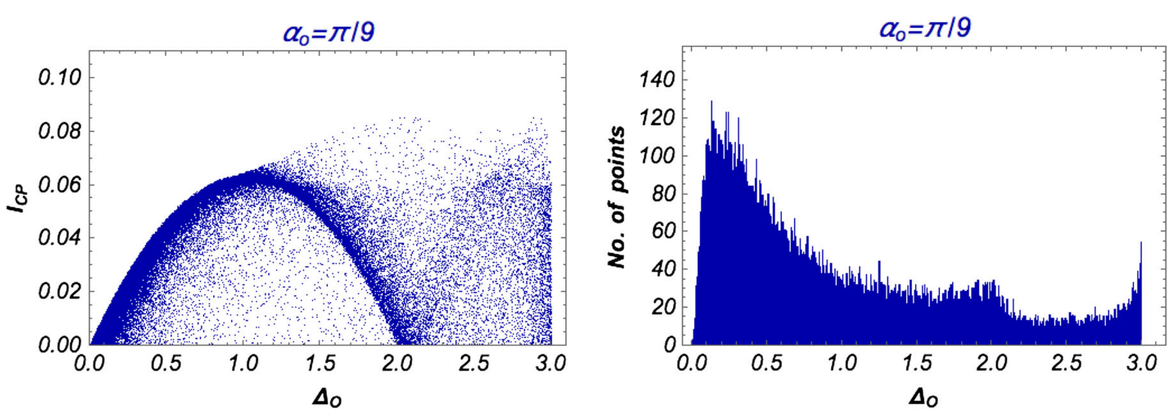

FIG. 7 (color online). Plotting the CP invariant I_{CP} as a function of Δ_O considering restricted perturbations for Q. The right plot is a histogram showing the distribution of the values of Δ_O obtained from 50000 uniformly distributed sets of input values for Q restricted as indicated in Eq. (64).

If neutrinos are quasidegenerate and Majorana, this new parametrization is particularly useful. It may reflect some specific nature of neutrinos, suggesting that there could be a large contribution to neutrino mixing and CP violation present in the left part of the parametrization, possibly as a result of some symmetry, while the right part, in the form of the real-orthogonal matrix O with the three angles could come from some perturbative effect, reflecting the fact that there are three neutrino families with small mass differences and which results in small mixing comparable to the mixing in the quark sector and the Cabibbo angle.

The new parametrization permits a new view of large leptonic CP violation. It shows interesting aspects that were less clear for the standard parametrization. We have identified several limit scenario cases and have shown results for mixing and CP violation. A certain scenario (I-A) was found to be the most appealing. It only needs two extra parameters to fit the experimental results on lepton mixing and provides large Dirac-CP violation and large values for the Majorana-CP-violating phases. We point out that the results for this scenario derive explicitly from the form of the new parametrization.

In addition and for quasidegenerate Majorana neutrinos, the stability of the different scenarios has been tested using random perturbations. We have concluded that the left part of the parametrization behaves quite differently for the diverse scenarios. Scenario I-A was very stable in this respect. With respect to the right part of the parametrization, i.e., the real-orthogonal matrix O, the perturbations generate large contributions for all cases. This unstable part of the mixing (due to the random perturbations) can only be improved if one imposes restrictions on the allowed perturbations. We have shown how to accomplish this.

ACKNOWLEDGMENTS

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