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Tone-assisted time delay interferometry on GRACE Follow-On

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We have demonstrated the viability of using the Laser Ranging Interferometer on the Gravity Recovery and Climate Experiment Follow-On (GRACE-FO) space mission to test key aspects of the interspacecraft interferometry proposed for detecting gravitational waves. The Laser Ranging Interferometer on GRACE-FO will be the first demonstration of interspacecraft interferometry. GRACE-FO shares many similarities with proposed space-based gravitational wave detectors based on the Laser Interferometer Space Antenna (LISA) concept. Given these similarities, GRACE-FO provides a unique opportunity to test novel interspacecraft interferometry techniques that a LISA-like mission will use. The LISA Experience from GRACE-FO Optical Payload (LEGOP) is a project developing tests of arm locking and time delay interferometry (TDI), two frequency stabilization techniques, that could be performed on GRACE-FO. In the proposed LEGOP TDI demonstration one GRACE-FO spacecraft will have a free-running laser while the laser on the other spacecraft will be locked to a cavity. It is proposed that two one-way interspacecraft phase measurements will be combined with an appropriate delay in order to produce a round-trip, dual one-way ranging (DOWR) measurement independent of the frequency noise of the free-running laser. This paper describes simulated and experimental tests of a tone-assisted TDI ranging (TDIR) technique that uses a least-squares fitting algorithm and fractional-delay interpolation to find and implement the delays needed to form the DOWR TDI combination. The simulation verifies tone-assisted TDIR works under GRACE-FO conditions. Using simulated GRACE-FO signals the tone-assisted TDIR algorithm estimates the time-varying interspacecraft range with a rms error of ± 0.2 m, suppressing the free-running laser frequency noise by 8 orders of magnitude. The experimental results demonstrate the practicability of the technique, measuring the delay at the 6 ns level in the presence of a significant displacement signal.

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I. INTRODUCTION

Space-based gravitational wave detectors have long been proposed as a way to probe the gravitational wave sources that exist outside the frequency band of terrestrial detectors [1,2]. The exciting science cases offered by such proposals have culminated in the selection of a space-based gravitational wave detector for the European Space Agency (ESA)'s L3 mission slot for launch in the early 2030s [3]. One candidate design that has seen over a decade worth of development, the Laser Interferometer Space Antenna (LISA) [2], is comprised of a triangular constellation of three spacecraft, separated by 5×10^6 km, traveling in a heliocentric orbit. LISA will detect gravitational waves by measuring the relative displacement between inertial masses, housed within each spacecraft, using laser interferometry links maintained between spacecraft. The links form six point-to-point displacement measurements and require two key technologies that have not before been demonstrated in space: elimination of spurious forces on inertial masses and interspacecraft laser interferometry.

The first technology is the focus of ESA's upcoming LISA Pathfinder mission. Preparing for launch in July 2015, LISA Pathfinder will test the disturbance reduction systems which will allow drag-free operation of the spacecraft [4]. Laser interferometry will be used over an intraspacecraft laser link of length 38 cm to measure the motion of the spacecraft relative to two inertial sensors. One measurement is used for feedback to the spacecraft, with spacecraft jitter damped using micro-Newton thrusters, while the other will be used as an out-of-loop sensor.

The second technology, interspacecraft laser interferometry, will be demonstrated in the Gravity Recovery and Climate Experiment Follow-On (GRACE-FO) mission. GRACE-FO, due for launch in 2017, is a collaboration between NASA and the German Research Center for Geosciences to perform measurements of Earth's time-varying gravitational field or geoid. Measurement of the geoid is accomplished by monitoring orbital disturbances between two spacecraft in a common low Earth orbit, separated by a nominal 200 km interspacecraft range. GRACE-FO is a continuation of the successful GRACE

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mission [5], which has generated a significant body of work relating to climate change and associated terrestrial mass transport processes using a microwave ranging system [6–8]. GRACE-FO will fly near-identical spacecraft to GRACE, but will supplement the baseline microwave ranging system with a Laser Ranging Interferometer (LRI) technology demonstrator [9]. The LRI will be the first demonstration of interspacecraft interferometry, measuring fluctuations in the interspacecraft range, and will affirm the technology's value for multispacecraft metrology.

The GRACE-FO LRI baseline design benefited directly from LISA technology development activities. Consequently the LRI shares several aspects of the proposed LISA measurement scheme, namely, laser prestabilization, MHz Doppler shifts, ∼100 pW received power, and phase measurement based upon a digital phase-locked loop. A recent proposal [10,11] suggests developing tests of two additional laser ranging techniques for LISA that may be implemented on GRACE-FO. The tests will not modify any existing hardware or impact the science output of the LRI, but instead are viewed as a way to capitalize on the similarities between GRACE-FO and LISA, using the LRI as a "Mission of Opportunity." Specifically, this paper describes a first step to an in-flight demonstration of time delay interferometry (TDI), a method for combining laser measurements to suppress the contribution of laser frequency fluctuations [12]. The other experiment in the LEGOP proposal is laser frequency stabilization using arm locking [13], and will be discussed elsewhere [14].

This paper focuses upon developing and characterizing a TDI experiment suitable for GRACE-FO, with a focus on tone-assisted TDI ranging (TDIR) [15]. We present analysis and simulation verifying the conceptual application of TDI to GRACE-FO's interspacecraft laser link. Results are presented to show that the application of tone-assisted TDIR to GRACE-FO would meet the laser ranging interferometer's displacement sensitivity requirements as well as verifying TDI as a viable method for reducing laser frequency noise in an interspacecraft interferometry measurement. An experiment in an optically sensitive test bed was also performed to validate the operation of TDIR in the presence of large path-length fluctuations and macroscopic optical delays.

The paper is laid out as follows: Sec. II provides some background on TDI; Sec. III discusses GRACE-FO and the LRI in more detail; Sec. IV then outlines the signals involved in a potential GRACE-FO TDI experiment; Sec. V presents a tone-assisted TDIR algorithm; with the results of a simulation testing the algorithm provided in Sec. VI. Section VII then includes a discussion of the experiment and experimental results, with the implications of the simulation and experimental results then discussed in Sec. VIII. The Appendix includes a detailed description of the additional technical delays that had to be measured in order to construct the experimental TDI combination.

II. TIME DELAY INTERFEROMETRY FOR LISA

TDI is a postprocessing technique that allows complex interferometer configurations to be synthesized from a collection of one-way interspacecraft displacement measurements. Of particular interest to LISA are synthetic interferometers which remove the otherwise overwhelming impact of laser frequency noise [16]. TDI is a vital part of realizing the sensitivity required to detect gravitational waves in LISA, where the residual laser frequency noise must be suppressed by over 12 orders of magnitude [17]. TDI has been experimentally demonstrated to meet LISA performance requirements both in an optical test bed with meter-scale delays [18] and in a hardware-based simulation with a multisecond delay [19]. An important outcome of testing TDI on the GRACE-FO optical link would be the demonstration of TDI with a significant, continuously varying delay.

The LISA measurement scheme requires TDI to form synthetic interferometers, such as the Michelson interferometer depicted in Fig. 1, from the six one-way phase measurements made between the three spacecraft. The Michelson configuration shown has two synthetic measurement arms: a dashed clockwise arm and a solid counterclockwise arm. The two arms are matched in path length to cancel laser frequency noise. The counterpropagating arms provide a complementary coupling to the gravitational wave signal. When the interspacecraft arms are of unequal length, the corresponding synthesized interferometers can become quite complex. The TDI combinations require laser frequency fluctuations to be relayed between adjacent interferometer arms. This noise relay is accomplished in postprocessing by aligning data samples taken onboard each spacecraft with samples of the same laser frequency noise at the other end of each interspacecraft laser link. An extensive discussion of this

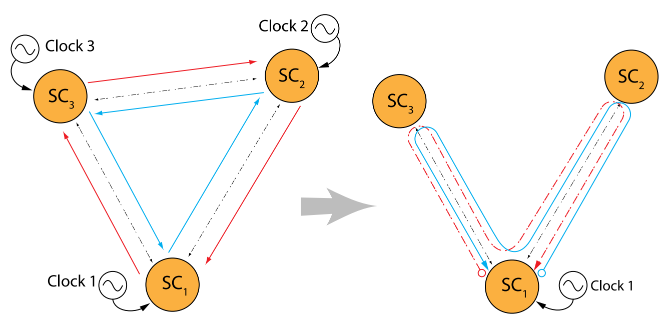

FIG. 1 (color online). LISA will combine the six one-way displacement measurements between spacecraft, shown on the left, to synthesize interferometer configurations with arms of equal length, such as the configuration shown on the right. This combination will cancel laser frequency noise in the gravitational wave measurement. TDI must account for all optical and clock-derived delays in order to correctly "relay" the laser frequency noise of spacecraft 1 (SC_1) around the constellation.

process is available in [20] while experimental demonstration of the technique is presented in [18].

In most scenarios, the limiting factor for TDI's suppression of laser frequency noise is the knowledge of the propagation delay along the link. Given an error in the interspacecraft delay knowledge of $\Delta\tau$, fractional laser frequency noise ($\frac{\delta\tilde{\nu}}{\nu_0}$) will couple into the inertial displacement measurement, $\tilde{x}$, as

$$\delta\tilde{x} = c\Delta\tau\frac{\delta\tilde{\nu}}{\nu_0}(\mathrm{m}/\sqrt{\mathrm{Hz}}), \tag{1}$$

for speed of light c. For LISA to achieve a frequency noise limited measurement sensitivity of 10^{-12} m using a prestabilized laser with fractional stability 10^{-12}, a delay error of $\Delta\tau < 3$ ns is required [17]. The laser interferometry technique utilized by LISA has extreme sensitivity to relative displacement, but the measurement has minimal sensitivity to the propagation range. With this in mind, LISA will operate a dedicated auxiliary laser ranging system [21] to measure the absolute interspacecraft delay. This ranging system requires the laser links to be phase modulated with MHz bandwidth signals. An equivalent dedicated absolute ranging link is not compatible with the GRACE-FO architecture, hence in this paper we propose an alternative macroscopic ranging technique. The proposed ranging technique is better than the LISA method as it is able to recover comparable delay information using a phase modulation much closer to the carrier (sub Hz).

III. GRACE-FO LRI AND TDI

The GRACE-FO mission will place two identical spacecraft in a common low Earth orbit, one behind the other, 200 km apart. Their long baseline separation allows the pair of spacecraft to act as a sensitive gravity gradiometer, with fluctuations in Earth's gravity field modulating the interspacecraft displacement. The LRI measures the total round-trip phase change of an optical signal transmitted between spacecraft, requiring a displacement sensitivity of $80\ \mathrm{nm}/\sqrt{\mathrm{Hz}} \times \mathrm{NSF}(f)$ between fourier frequencies of 2 to 100 mHz. The noise shape function, which accounts for instrumentation noise increasing at low frequency, is $\mathrm{NSF}(f) = \sqrt{(1 + (3\ \mathrm{mHz}/f)^2)(1 + (10\ \mathrm{mHz}/f)^2))}$ [22].

Figure 2 shows the optical layout of the GRACE-FO LRI. On one spacecraft, denoted the "master," the laser's optical frequency is stabilized to a reference cavity before being transmitted to the distant spacecraft [24]. This cavity stabilization reduces laser frequency fluctuations to approximately $30\ \mathrm{Hz}/\sqrt{\mathrm{Hz}} \times \mathrm{NSF}(f)$ over the frequency band of interest. The second, or "slave," spacecraft phase locks its local laser to the incoming signal and transponds the signal back to the master. As such, the remote spacecraft's phase locking imparts the displacement signal onto the outgoing laser, allowing the remote spacecraft to be conceptually treated as a distant, amplifying mirror.

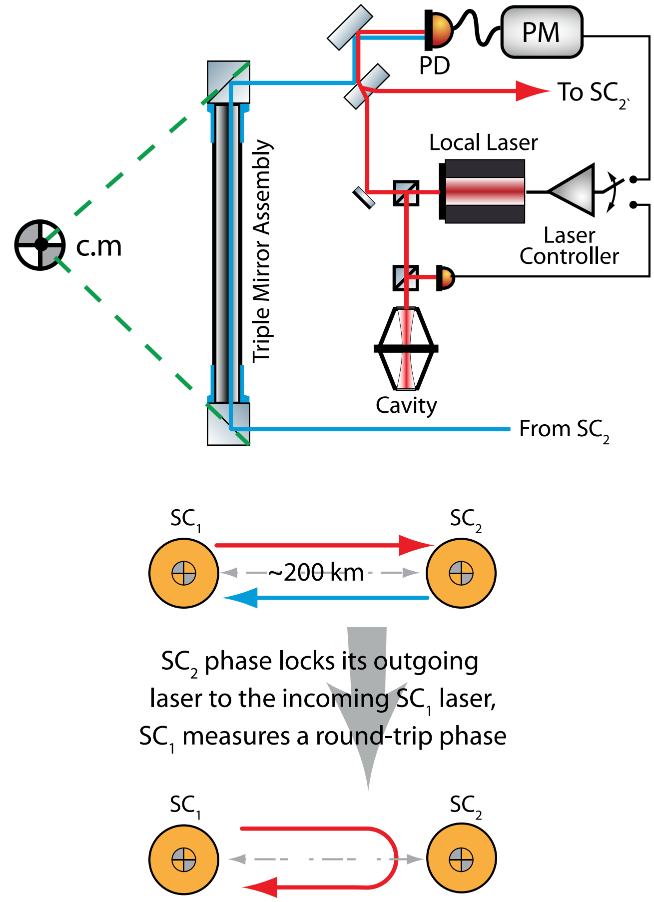

FIG. 2 (color online). GRACE-FO LRI performs a heterodyne laser interferometry displacement measurement between each spacecraft's center of mass (c.m.). Onboard each spacecraft, as shown on the left, the received laser is routed through the triple mirror assembly [23] and is combined with a local laser for measurement upon a photodiode (PD) where the phase of the resulting beat note is measured with a phase meter (PM). The local laser can be either prestabilized to a reference cavity or phase locked with an offset frequency via the phase meter. Phase measurements are made between the two identical spacecraft, allowing the laser onboard one slave spacecraft to be phase locked to the laser on the other master laser, as shown in the lower image.

The round-trip displacement measurement is formed locally on the master, where the local laser phase is compared with the phase of the transponded signal arriving from the distant spacecraft. This measurement is performed by interfering the two fields on a beam splitter and detecting the resulting heterodyne beat note fluctuations with a phase meter [25]. This measurement is sensitive to relative spacecraft displacement as well as the relative drift in the master laser frequency over the round-trip delay between spacecraft. As such, the baseline GRACE-FO science measurement may be formed from the phase measurement at the master alone. For a more detailed discussion of the LRI, the reader is referred to Ref. [9].

Since GRACE-FO only implements one bidirectional interferometer arm, it cannot synthesize the two nonredundant interferometer arms required to cancel laser frequency noise completely. This mandates the use of cavity stabilization for the baseline design. Instead, the goal of applying TDI to GRACE-FO will be the recovery of a round-trip displacement measurement, without requiring high-bandwidth phase locking. This process is equivalent to the laser frequency relay that forms the basis of the LISA TDI schemes to transpond laser frequency noise through selected paths around the interferometer. It is also the measurement strategy employed in both the GRACE and GRACE-FO microwave ranging instruments [26]. Demonstration of this core aspect of TDI in the presence of large dynamic changes in laser link length and propagation delay will retire any residual risk for applying TDI to LISA.

IV. TDI ON GRACE-FO

We propose to use TDI techniques to suppress the laser frequency contributions of one laser on GRACE-FO. In this scenario, the master spacecraft will still lock its own laser to its onboard reference cavity, however the slave spacecraft will now operate with weakened or disabled laser phase locking, aiming to only maintain optical beat notes within the MHz bandwidth of the phase meter [27]. The interspacecraft displacement measurements formed at both ends of this link are now corrupted by the significant laser frequency noise of the free-running laser onboard the slave spacecraft. Fortunately, this noise is highly correlated between the two measurements, allowing suppression using TDI. The resulting TDI combination will suppress the free-running noise of the remote laser down to the level of the "reference" stabilized laser, recovering a measurement sensitivity equal to the phase-locked case.

The GRACE-FO TDI combination, described in terms of interspacecraft optical phase measurements, is given as follows. We let $\phi_1(t)$ be the phase measured on the master spacecraft (SC_1) and $\phi_2(t)$ be the phase measured on the slave spacecraft (SC_2) at time t. With the inclusion of the interspacecraft separation delay τ, the phase measurements are given by

$$\phi_1(t) = \phi_{11}(t) - \phi_{12}(t-\tau) - \phi_{\mathrm{path}}(t), \tag{2}$$

$$\phi_2(t) = \phi_{11}(t-\tau) - \phi_{12}(t) + \phi_{\mathrm{path}}(t), \tag{3}$$

where $\phi_{11}(t), \phi_{12}(t)$ are the phase noise of the SC_1 and SC_2 lasers and $\phi_{\mathrm{path}}(t)$ is the path-length phase due to the changing spacecraft separation. To simplify the analysis we assume light from both spacecraft measure the same delay $\tau \approx 0.6$ ms as relative spacecraft velocities are sufficiently small (~m/s). The GRACE-FO dual one-way ranging (DOWR) [28] TDI combination, denoted $\phi_{\mathrm{DOWR}}(t)$, is then given by

$$\phi_{\mathrm{DOWR}}(t) = \phi_1(t) - D_{12}\phi_2(t). \tag{4}$$

We use the operator D_{12} to indicate a time delayed signal [29]. The delay operator is defined by $D_{12}a(t) = a(t - L_{12}/c)$, where L_{12} is the optical path delay between the two spacecraft. To verify that this combination will suppress the free-running laser phase noise we substitute Eqs. (2) and (3) into Eq. (4) to give

$$\begin{aligned}\phi_{\mathrm{DOWR}}(t) = &\underbrace{[\phi_{11}(t) - \phi_{11}(t-\tau-\hat{\tau})]}_{\text{stabilized}} \\ &- \underbrace{[\phi_{12}(t-\tau) - \phi_{12}(t-\hat{\tau})]}_{\text{free running}} \\ &- \underbrace{[\phi_{\mathrm{path}}(t) + \phi_{\mathrm{path}}(t-\hat{\tau})]}_{\text{displacement}},\end{aligned} \tag{5}$$

where we have used $\hat{\tau}$ to represent the estimated spacecraft delay. If the estimated delay is correct ($\hat{\tau} = \tau$) then the contribution from the free-running SC_2 laser will cancel. The dual one-way ranging TDI combination then includes a round-trip, differential measurement of the stabilized laser phase and a round-trip displacement signal with two passes along the interferometer arm.

Importantly, any error in the estimate delay $\hat{\tau}$ will introduce residual free-running laser noise into the round-trip displacement measurement. GRACE-FO has a minimum displacement requirement for residual frequency noise of $x_{\text{laser residual}}(f) = 30\ \mathrm{nm}/\sqrt{\mathrm{Hz}}$ [22]. From Eq. (1) this corresponds to a maximum delay uncertainty for GRACE-FO of 6 ns given a frequency stability of $\nu(f)/\nu = 10^{-10}/f \times 1$ Hz. This 6 ns uncertainty is equivalent to a range uncertainty of approximately 2 m.

As a consequence, to construct the DOWR measurement requires an accurate estimate of the interspacecraft delay. However, unlike LISA's dedicated ranging system, GRACE-FO will not directly measure this delay as it is not required in the baseline phase-locked configuration. Although GRACE-FO possesses an onboard GPS system, for the purposes of this work we choose to neglect the onboard GPS knowledge because (1) it is not representative of the LISA instrument and (2) it does not measure the relevant delays in the phase measurement chain. Instead, the delay dependence of the laser frequency noise contribution may itself be used as a ranging mechanism through a technique termed TDIR [15].

TDIR is a postprocessing technique that computes the delays needed for a TDI combination using nonlinear optimization, avoiding the need for a direct measurement of the interspacecraft range. This noise minimization is achieved by exploiting the correlated laser frequency noise that couples into each one-way displacement measurement.

TDI combinations are formed using an estimate of the interspacecraft delay, with the "in-band" rms power of the resulting signal used as a minimization cost metric for optimization of the delay estimate. This procedure has been shown to produce a delay estimate for LISA that canceled the laser frequency noise to a level below the other sources of noise [15].

Unfortunately, the rms minimization technique described in [15] cannot be directly applied to GRACE-FO as the interspacecraft displacement signal (the primary science signal) masks the laser frequency noise below 10 mHz. Figure 3 shows a breakdown of several displacement spectra over the GRACE-FO LRI measurement band. In particular, Fig. 3(b) shows the GRACE displacement spectra for four days (January 1–4, 2012) of microwave measurement data from the 2002 mission [30]. Features of the range spectra are the roll-off of the displacement signal above 10 mHz and the microwave thermal noise floor of $\sim\mu\mathrm{m}/\sqrt{\mathrm{Hz}}$. GRACE-FO is expected to show similar behavior. Also shown is the single link free-running laser phase noise, for a laser with noise spectra $\delta\nu_1 = 30\ \mathrm{kHz}/f\,\mathrm{Hz}/\sqrt{\mathrm{Hz}}$ [31], and stabilized laser phase noise with residual fluctuations at $\delta\nu_2 = 30\ \mathrm{Hz}/\sqrt{\mathrm{Hz}}$ [32]. The trace labeled *residual* laser phase noise is the displacement equivalent noise caused by the suppression of the stabilized laser frequency noise in a round-trip measurement. For comparison, 5 orders below this level is the single link *shot noise* for 100 pW of received power, which represents the limiting displacement noise source for LISA [9].

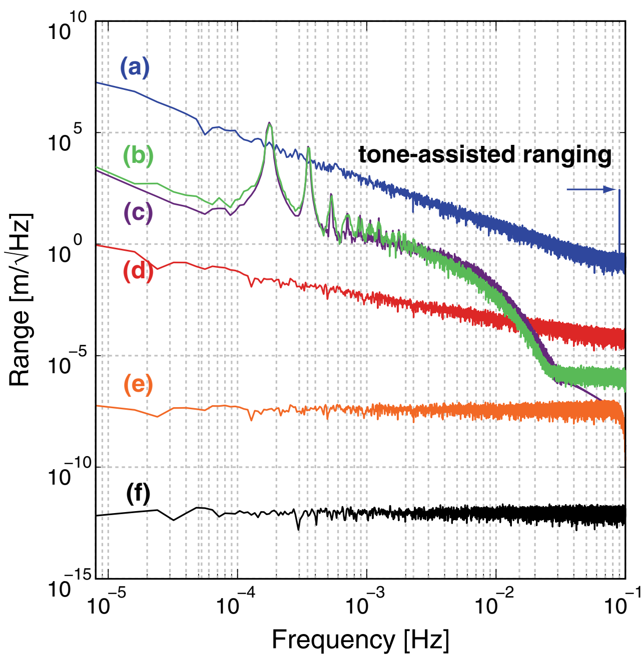

FIG. 3 (color online). Simulated GRACE-FO displacement noise spectrum. The signals are (a) free-running laser phase noise; (b) GRACE range; (c) simulated GRACE range; (d) stabilized laser phase noise; (e) residual laser phase noise; and (f) single link shot noise. With one laser free running and the other stabilized the TDI combination, assuming zero error in delay, would suppress the laser noise down to the residual laser phase noise curve. The single link shot noise is quantum noise in both measurements due to the random arrival time of photons on the detector. We have assumed $(3\times10^4)\times(1\ \mathrm{Hz}/f)\ \mathrm{Hz}/\sqrt{\mathrm{Hz}}$ free-running laser frequency noise and a stabilized laser frequency noise of $30\ \mathrm{Hz}/\sqrt{\mathrm{Hz}}$. The tone used in the tone-assisted TDIR is shown on the free-running laser phase noise curve. The shot noise is assumed to be $1\ \mathrm{pm}/\sqrt{\mathrm{Hz}}$. The GRACE range trace shows four days of GRACE microwave displacement data from early 2012, demonstrating the expected signal spectra for the LRI.

The goal of a TDI experiment would be to suppress the free-running phase noise to the $30\ \mathrm{nm}/\sqrt{\mathrm{Hz}}$ level of the residual laser phase noise set by the $30\ \mathrm{Hz}/\sqrt{\mathrm{Hz}}$ cavity stability requirement. However, the science displacement spectra will lie well above this level, preventing the application of TDIR based upon "rms minimization" of the free-running laser noise. Instead, we propose the application of a TDI variant, tone-assisted TDIR [33], where a frequency modulation tone is applied to the free-running laser to provide a signal with a high signal-to-noise ratio for minimization. For all purposes, the frequency modulation appears as laser frequency noise hence it may be suppressed by TDIR.

The frequency and modulation depth of the tone determine the performance of the tone-assisted TDIR. Assuming that the tone may be suppressed to the underlying phase noise floor, the delay error achievable by tone-assisted TDIR is [33]

$$\Delta\tau_{\mathrm{tone}} = \frac{2\phi_{\mathrm{n}}(f_{\mathrm{tone}})}{\nu_{\mathrm{tone}}\sqrt{T f_{\mathrm{s}}}}\,(\mathrm{s}), \qquad (6)$$

where f_{tone} is the frequency of the TDIR tone, $\phi_{\mathrm{n}}(f)$ is the link phase noise amplitude spectral density, $\nu_{\mathrm{tone}}(\mathrm{Hz})$ is the modulation depth of the tone, T is the averaging time and f_{s} is the frequency at which the data was sampled. The primary performance driver is the signal-to-noise ratio of the tone $(\nu_{\mathrm{tone}}/\phi_{\mathrm{n}})$, therefore the tone needs to be added at a frequency with low displacement noise. Fortunately, we are free to place the modulation tone at a frequency above the interesting science data (namely frequencies above the 30 mHz white thermal noise crossover visible in the GRACE microwave ranging spectra of Fig. 3). Additionally, a large modulation depth may be selected to ensure that the tone is clearly visible above the laser's free-running laser noise, allowing the frequency noise to be further suppressed in TDIR processing.

V. TONE-ASSISTED TDI RANGING ALGORITHM

This section presents an example of a TDIR algorithm which meets the LEGOP TDI timing requirements. The algorithm takes spacecraft measurements, $\phi_1(t)$ and $\phi_2(t)$, and forms the TDI combination, $\phi_{\mathrm{DOWR}}(t)$, with an estimated delay, $\hat{\tau}(t)$. The delay estimate is updated using a least-squares fitting algorithm [34] until the amplitude of the TDIR tone in the DOWR combination is minimized.

It is possible to approximate the range delay by forming a simple TDI combination, $\phi_{\mathrm{basic}}(t)$, given by

$$\phi_{\mathrm{basic}}(t) = \phi_1(t) - \phi_2(t). \tag{7}$$

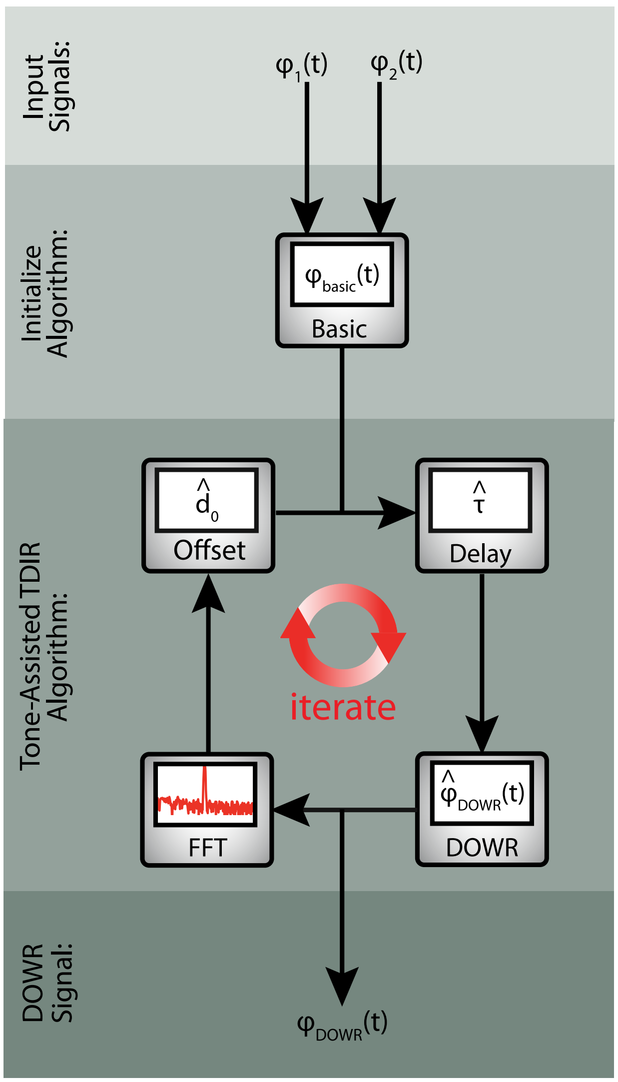

FIG. 4 (color online). Tone-assisted TDIR uses an iterative process to find the best guess of the time-varying interspacecraft delay τ. The delay is initially estimated using $\phi_{\mathrm{basic}}(t)$, a TDI combination proportional to the range $\phi_{\mathrm{path}}(t)$, that is however biased by the unknown initial spacecraft separation. The range ambiguity is compensated for using an offset, d_0, a parameter that is updated using a least-squares fitting algorithm to minimize a tone in the TDI combination, $\phi_{\mathrm{DOWR}}(t)$. The loop iterates until the least-squares fitting algorithm converges producing a best guess of the time-varying spacecraft delay.

Substituting in Eqs. (2) and (3), we can make the approximation

$$\phi_{\mathrm{basic}}(t) \approx -2\phi_{\mathrm{path}}(t). \tag{8}$$

The simple TDI combination, $\phi_{\mathrm{basic}}(t)$, is therefore proportional to the interspacecraft separation. The interspacecraft delay can therefore be estimated as

$$\hat{\tau}(t) \approx -\frac{\lambda\phi_{\mathrm{basic}}(t)}{2c}. \tag{9}$$

Given $\phi_{\mathrm{path}}(t)$ is a measure of the interspacecraft displacement rather than the absolute interspacecraft range, this approximation is, unfortunately, heavily biased by the unknown initial range separation between the spacecraft. It is additionally affected by systematic errors, which enter the phase measurement as delays. These systematic errors are discussed in more detail in the next section. The basic TDI combination, $\phi_{\mathrm{basic}}(t)$, does however provide a good estimate of the higher order range trends.

Figure 4 summarizes the tone-assisted TDIR algorithm. In the algorithm a "best guess" of the time-varying spacecraft delay, over a data set of length L, is formed using an offset, d_0, and a time-varying component, formed using the simple TDI combination, $\phi_{\mathrm{basic}}(t)$. A least-squares fitting algorithm is then applied to resolve the range ambiguity. The algorithm updates the offset, d_0, using a FFT to find the amplitude of the tone in successive DOWR combinations until a minimum is found.

As an alternative, the TDIR algorithm could be combined with a more sophisticated orbital model to find the best guess of the time-varying spacecraft delay. Rather than a one-dimensional search as has been described, a higher dimension search could be used. For the purpose of this implementation though, the orbital fluctuations were found to be sufficiently low in frequency that this simple delay approximation met the ranging requirements.

VI. SIMULATION OF TONE-ASSISTED TDIR

The tone-assisted TDIR algorithm was developed under simulated GRACE-FO conditions. One-way interspacecraft optical phase measurements were simulated with laser phase noise, shot noise and a displacement signal at levels representative of the GRACE-FO LRI. The tone used in the tone-assisted ranging is at 90 mHz with a modulation depth of 2.5 MHz. This was chosen to give good clearance over the other signals in the displacement spectrum and at a frequency where the ranging signal had begun to roll-off.

The interspacecraft range has been simulated by fitting a model to the GRACE microwave ranging data at a sampling rate of 200 mHz. This range model also encodes representative orbital dynamics as harmonics of the orbital period (0.18 mHz), as shown in Fig. 3, and is free of the

TABLE I. Simulation parameters.

Parameter	Value
sampling frequency	200 mHz
free-running frequency noise	30 kHz/f (Hz/$\sqrt{\mathrm{Hz}}$)
stabilized frequency noise	30 Hz/$\sqrt{\mathrm{Hz}}$
laser wavelength (λ)	1064 nm
spacecraft range ambiguity	100 m
tone modulation frequency	90 mHz
tone modulation depth	2.5 MHz
single link shot noise	1 pm/$\sqrt{\mathrm{Hz}}$
data length	5×10^5 s

1 μm/$\sqrt{\mathrm{Hz}}$ microwave thermal noise which would have masked the LRI sensitivity. The initial range between the spacecraft was also varied over (200 ± 100) km to simulate the initial range ambiguity. The one-way spacecraft measurements were formed using Eqs. (2) and (3). The simulated GRACE range model formed the interspacecraft delay, which was applied to the relevant laser frequency noise terms using fractional-delay filtering [35]. Table I summarizes the relevant simulation and algorithm parameters.

Figure 5 shows the displacement spectra of the simulated signals. Traces $x_1(f)$ and $x_2(f)$, which lie on top of each other in the figure, show the interspacecraft one-way displacement measurements, generated from the optical phase measurements $\phi_1(f)$ and $\phi_2(f)$. The displacement signals are dominated by the free-running laser phase noise from SC_1 and the TDIR tone at 90 mHz. The two measurements are combined using Eq. (7) to form $x_{\mathrm{DOWR,basic}}(f)$, which suppresses the laser frequency noise by the 3 orders of magnitude expected for a TDI delay error of $\Delta\tau\approx0.6$ ms (i.e. 200 km interspacecraft range). Below 20 mHz the $x_{\mathrm{DOWR,basic}}(f)$ signal can be seen to follow the simulated GRACE range signal.

Tone-assisted TDIR is applied to optimize the delay estimate and minimize the laser frequency noise contribution. The TDI combination $x_{\mathrm{DOWR}}(f)$ is formed using the best guess of the interspacecraft range. The $x_{\mathrm{DOWR}}(f)$ signal is dominated by the range signal. Subtracting the known range signal reveals the residual noise due to the stabilized laser [Eq. (5)], labeled $x_{\mathrm{DOWR,laser\,residual}}(f)$ in Fig. 5. This signal contains the error introduced by laser frequency and phase noise sources. The residual noise in $x_{\mathrm{DOWR,laser\,residual}}(f)$ is ~8 orders below the free-running noise and reaches the laser residual requirement of 30 nm/$\sqrt{\mathrm{Hz}}$, indicating that TDIR has successfully suppressed the free-running laser frequency noise. To confirm that the phase-locking requirement is also met, the DOWR residual when computed with the "true" range is subtracted from $x_{\mathrm{DOWR,laser\,residual}}(f)$ to give $x_{\mathrm{DOWR,phase\,locking\,residual}}(f)$. It can be seen that this line meets the phase-locking requirement of 1 nm/$\sqrt{\mathrm{Hz}}$. The

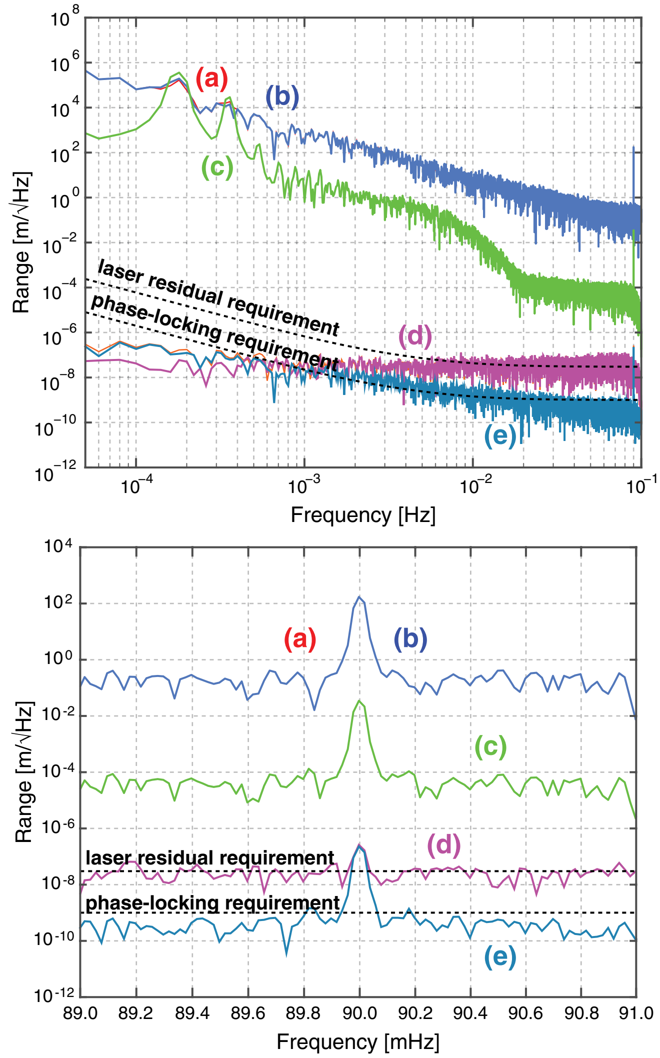

FIG. 5 (color online). Root-power spectral density plot of simulated displacement measurements and TDI combinations from a 5×10^5 s simulation sampled at 200 mHz. The traces are (a) SC_1 displacement measurement $x_1(f)$; (b) SC_2 displacement measurement $x_2(f)$; (c) basic DOWR combination $x_{\mathrm{DOWR,basic}}(f)$; (d) residual laser displacement noise $x_{\mathrm{DOWR,laser\,residual}}(f)$; and (e) phase-locking residual displacement noise $x_{\mathrm{DOWR,phase\text{-}locking\,residual}}(f)$. The individual spacecraft displacement measurements are dominated by the free-running laser phase noise from SC_1 with the tone-assisted TDIR tone visible at 90 mHz. The basic DOWR combination suppresses some laser phase noise. Using a best guess of the range delay the tone is suppressed further. The known range signal is subtracted, to reveal the residual laser displacement noise. The trace shows the residual free-running laser noise in the DOWR TDI combination is down to the target level of 30 nm/$\sqrt{\mathrm{Hz}}$. To verify the phase-locking requirement is met, the residual laser noise in a true DOWR is subtracted to give the phase-locking residual displacement noise which itself meets the 1 nm/$\sqrt{\mathrm{Hz}}$ phase-locking requirement. The lower panel shows a zoomed in portion of the root-power spectral density to demonstrate the suppression of the TDIR tone.

lower panel in the figure shows the suppression of the 90 mHz tone through the application of TDI.

A time-domain plot of the range estimate used to form the DOWR TDI combination shown in the figure can be seen in Fig. 6. The range signal is dominated by the orbital period. The range fitted by the tone-assisted TDIR algorithm is shown in the top panel. On the scale of the orbital oscillations it is difficult to discern any difference between the fitted range and the range used in the simulation. For clarity, the difference between these two signals is shown in the lower panel. The TDIR range is clearly within ± 2 m of the true range and therefore meets the timing requirements for the proposed GRACE-FO TDI experiment. The error can be seen to have a periodic structure with a rms of ± 0.015 m, offset from zero. The periodic nature of the error signal is due to the orbital period in the true range, with the regions where the error is largest clearly correlated with where the orbital period has the largest derivative. The offset is consistent with the residual error from the tone-assisted TDIR algorithm that was used to resolve the initial range ambiguity.

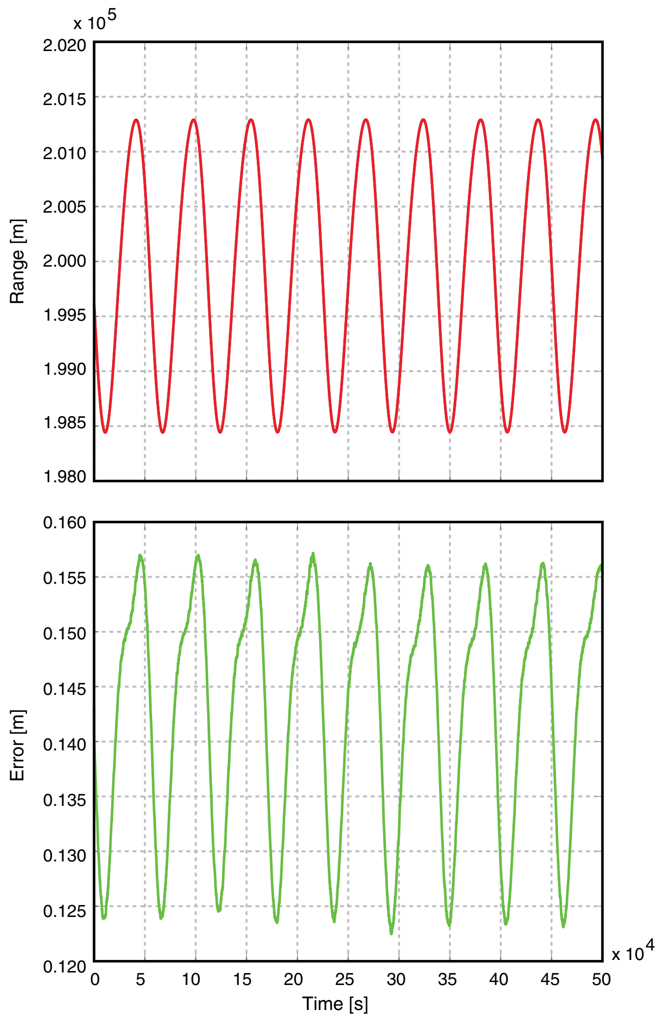

FIG. 6 (color online). The best guess of the interspacecraft range used to form the TDI combinations in Fig. 5. The simulated data used to find the best guess was from a 5×10^5 s simulation sampled at 200 mHz. The range error, found by subtracting the known range from the fitted range, is plotted in the lower panel. The range error is an order of magnitude below the ± 2 m of the known range and therefore meets the timing requirements of the proposed GRACE-FO TDI experiment.

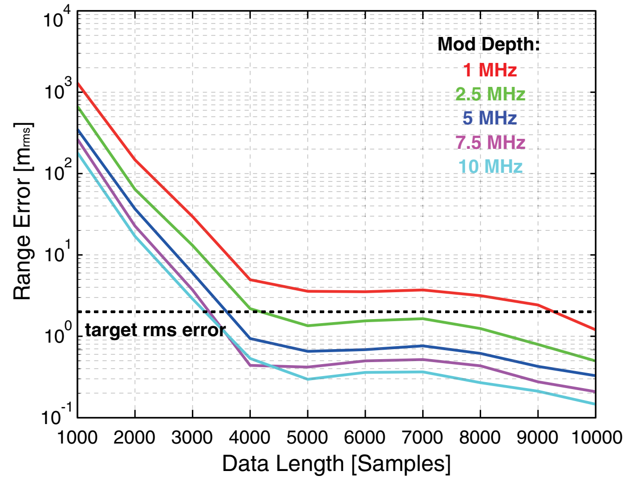

FIG. 7 (color online). The effect of varying data length and modulation depth in the tone-assisted TDIR algorithm. The rms range error can be seen to decrease as the data length increases or as the modulation depth is increased. For reference the 2 m rms ranging requirement needed to meet the timing requirements for a GRACE-FO TDI demonstration is shown. It can be seen that it is possible to meet this requirement for a range of data lengths and modulation depths.

The effect of varying the length of data and the tone modulation depth in the tone-assisted TDIR algorithm was investigated. The rms error in the TDIR range is shown in Fig. 7 as the data length is varied for various modulation depths. Each data point is the average rms error over 25 independent simulation runs. The figure shows that for data sets shorter than 4000 samples (5.6 hrs) the TDIR algorithm fails to meet the ranging requirements using the specified modulation depths. This is because the accuracy of the least-squares fitting algorithm is dependent on the resolution of the tone in the FFT, which scales inversely with length. As can be seen with data lengths greater than 4000, the majority of the modulation depths that were used meet the ranging requirement of 2 m.

The simulation has clearly demonstrated the viability of tone-assisted TDIR under GRACE-FO conditions. Furthermore, the additional tests have shown that the algorithm can meet the timing requirements under a variety of conditions demonstrating the robustness of the algorithm.

VII. EXPERIMENTAL DEMONSTRATION OF TONE-ASSISTED TDIR

An optical experiment was performed to further validate tone-assisted TDIR in the GRACE-FO-like signal environment. The experiment used the JPL LISA interferometry test bed [18,33,36], with modifications to add the key differences between the LISA and GRACE-FO signal environments: chiefly a very large displacement signal. The goal of the experiment was to use the tone-assisted technique defined in Sec. IV to determine the optical delay to less than 6 ns. This accuracy is twice the LISA TDI requirement [17] but ensures that the contribution to the DOWR combination of the residual slave laser phase noise will be less than the LRI slave laser phase-locking requirement.

The schematic of the experiment in Fig. 8 shows a folded version of the LRI measurement with a single laser (a commercial NPRO laser operating at 1064 nm), a beam splitter and a photodetector representing each of the GRACE-FO spacecraft. The laser on the master spacecraft, with phase noise denoted $\phi_{11}(t)$, was stabilized to an optical cavity, with stability of approximately $30\ \mathrm{Hz}/\sqrt{\mathrm{Hz}}$ at 0.1 Hz [37,38]. The laser on the slave spacecraft, with phase noise $\phi_{12}(t)$, was operated without stabilization, unlike the nominal GRACE-FO operation where the slave laser is phase locked to the incoming light. A 0.85 Hz, sinusoidal signal with 550 kHz peak-to-peak amplitude was added to the fast actuator of the laser on the slave spacecraft to provide the TDIR tone, well above the unstabilized laser noise at this frequency. In this experiment, the optical delay between the master and slave spacecraft was provided by an optical fiber of path length 30 m (optical path length ≈45 m). Also in this path, a time-varying phase modulation signal, $s(t)$, was added into one path to mimic the GRACE-FO displacement signal. The signal was added using two additional lasers and the locking scheme shown in Fig. 9, similar to that proposed for the Sagnac interferometer operation of LISA [39]. Here, the displacement signal $s(t)$ was added to the phase locker error point to emulate

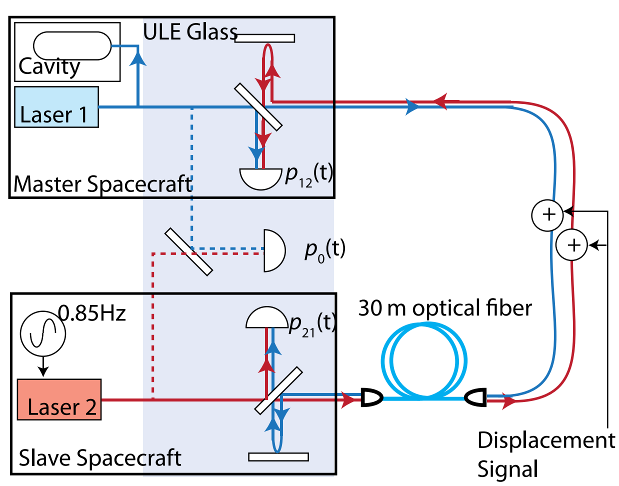

FIG. 8 (color online). Schematic layout of the TDIR experiment. The details of the injected displacement signal can be found in Fig 9.

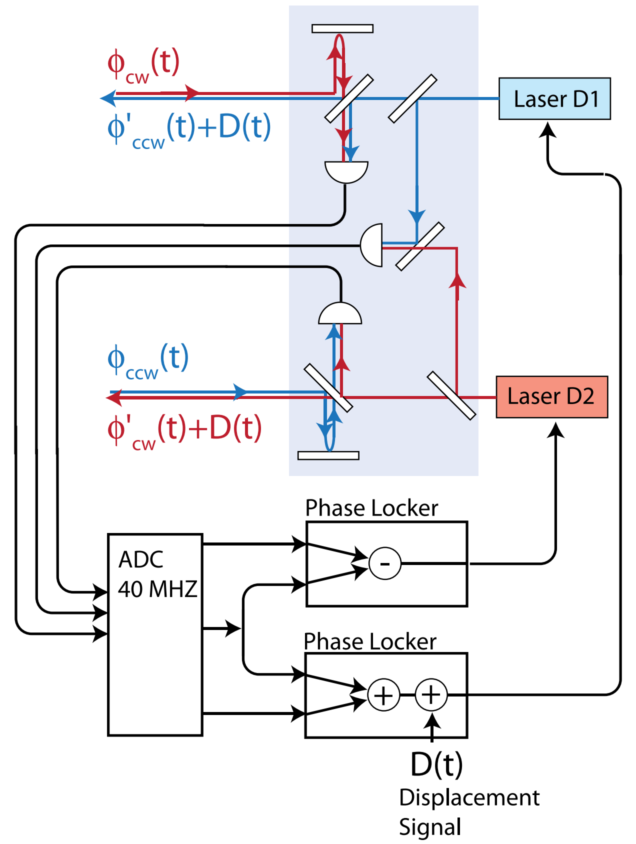

FIG. 9 (color online). Schematic layout showing the injection of the interspacecraft displacement signal. Three optical phase measurements are combined via two phase-locking algorithms to implement the required feedback combinations.

the large displacement signal on GRACE-FO (seen in Fig. 3) which is primarily driven by the slightly different orbits of the two spacecraft. In addition to the intentionally added displacement signal, there was displacement noise of the optical fiber and motion of the optomechanics, $\delta(t)$.

The phase of the interspacecraft measurements shown in Fig. 8 are defined as follows: $\phi_1(t)$ for the master spacecraft and $\phi_2(t)$ for the slave spacecraft. The additional channel, denoted $\phi_0(t)$, provides a truth measurement, the details of which are discussed in Sec. VII B. These signals were detected with low noise commercial detectors and fed into custom built analog-to-digital converters and into commercial field-programmable gate arrays for phase detection. The LISA phase meter [25] was used to track each phase signal. The interspacecraft phase measurements contain the difference of the two laser phases as well as displacement noise in the path length between the spacecraft, $\delta(t)$, which is mostly thermally driven:

$$\phi_1(t) = \phi_{11}(t) - D_{12}\phi_{12}(t) + \delta(t) + s(t), \qquad (10)$$

$$-\phi_2(t) = \phi_{12}(t) - D_{12}\phi_{11}(t) + \delta(t). \qquad (11)$$

In addition to the optical delay, D_{12}, there is a significant delay in the detection electronics that is accounted for by $D_{\mathrm{E}i}a(t) = a(t - \tau_{\mathrm{E}i})$ for the ith detector. The DOWR combination is a sum of the two phase measurements, with the signal on the slave spacecraft delayed by the estimate of the light propagation time $\hat{\tau}$ between the spacecraft plus any differential electronic delays between the measurements.

The delay estimate is $\hat{D}_{12} = (D_{\mathrm{E1}} - D_{\mathrm{E2}})D_{12} + D_{\Delta}$, with the DOWR combination given by

$$\begin{aligned}\phi_{\mathrm{DOWR}}(t) &= \phi_1(t) + \hat{D}_{12}\phi_2(t),\\ &= (1 - D_{12}\hat{D}_{12})\phi_{l1}(t) + (D_{12} - \hat{D}_{12})\phi_{l2}(t)\\ &\quad + (1 + \hat{D}_{12})(s(t) + 2\delta(t))\\ &= (1 - D_{12}^2 D_{\Delta})\phi_{l1}(t) + D_{12}(1 - D_{\Delta})\phi_{l2}(t)\\ &\quad + (1 + \hat{D}_{12})\delta(t) + s(t),\end{aligned} \tag{12}$$

where the error in the time delay estimate $\Delta t = \tau_{12} - \hat{\tau}_{12}$ and the corresponding delay operator is $D_{\Delta}a(t) = a(t - \Delta t)$.

It is clear that in $\phi_{\mathrm{DOWR}}(t)$ the stabilized master laser phase couples via the round-trip propagation (D_{12}^2), while the unstabilized slave laser couples via the error in the one-way time delay estimate ($D_{12} - \hat{D}_{12}$). The recorded signals $\phi_2(t)$ must be delayed for the optical path delay then resampled with a suitable interpolation algorithm [35] so that the DOWR combination cancels the laser frequency noise of the unstabilized slave laser.

A. Experimental results

The experimental results are shown in Fig. 10. These are the root-power spectral densities of the displacement signals: $x_1(f)$ is shown in Fig. 10(a) and is the difference of the master and slave laser phases, which is dominated by the free running noise of the slave laser, plus the TDIR tone at 0.85 Hz. Figure 10(b) shows the DOWR basic combination that is the difference of the detectors $\phi_1(t)$ and $\phi_2(t)$ without any delay added in processing. In Fig. 10(b), laser frequency noise coupling is visible between 0.01 and 0.1 Hz, as well as the 0.85 Hz tone, but the displacement signal $s(t)$ dominates the spectrum. Using the TDIR algorithm to find the delay, the DOWR combination is found and the frequency noise is minimized at all frequencies, this is shown in Fig. 10(c). This improvement is clear at the 0.85 Hz tone frequency has been reduced by more than 8 orders of magnitude from Fig. 10(a). However the large path length noise in the DOWR link has a noise floor much too high to verify this to the 1 millicycle$/\sqrt{\mathrm{Hz}} \times \mathrm{NSF}(f)$ level.

B. Verification of TDIR measured delay via the Sagnac combination

Unlike GRACE-FO, the experiment contains an additional phase measurement (ϕ_0) indicated by the dashed path in Fig. 8: the interference of the two lasers on the same ultralow expansion (ULE) bench. This measurement allows us to estimate the residual noise in $\phi_{\mathrm{DOWR}}(t)$ in the absence of the displacement noise $s(t)$:

$$\phi_0(t) = \phi_{l1}(t) - \phi_{l2}(t). \tag{13}$$

This measurement is the difference of the lasers' frequency noise without any optical delay and with negligible path-length displacement. This additional measurement allows for evaluation of the performance of the range estimate, through forming the Sagnac variable, $\alpha(t)$ [16,18,39]:

$$\alpha(t) = \phi_1(t) + \phi_2(t) - (1 + 2\hat{D}_{21})\phi_0(t) \tag{14}$$

$$= D_{12}(1 - D_{\Delta})(\phi_{l1}(t) + \phi_{l2}(t)) + s(t). \tag{15}$$

Here the common path-length noise of the fiber and other common optics, $\delta(t)$, is subtracted out but the laser frequency noise will couple with the same error as in the DOWR combination $\hat{D}_{21} = D_{21}D_{\Delta}$, giving an upper bound on the error on delay found in the TDIR algorithm for the DOWR link, without displacement noise to obscure the result. However, $\alpha(t)$, as written in Eq. (15) requires two corrections required to make the verification at the millicycle level.

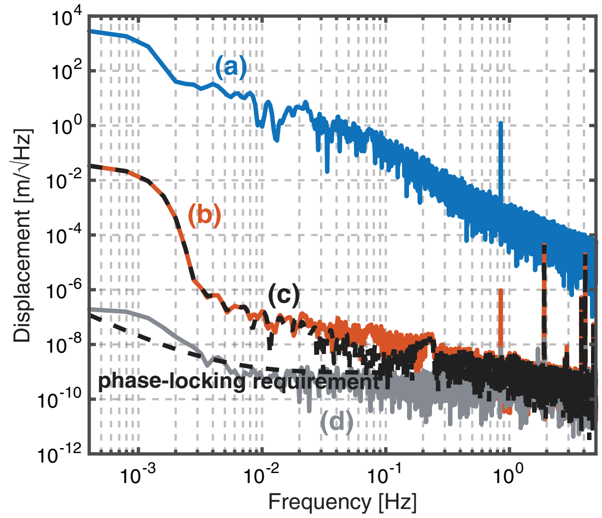

FIG. 10 (color online). The measurement phase noise root-power spectral density for various TDI data combinations. The traces are (a) SC_1 displacement measurement $x_1(f)$; (b) basic DOWR combination $x_{\mathrm{DOWR,basic}}(f)$; (c) DOWR combination $x_{\mathrm{DOWR}}(f)$; and (d) corrected alpha combination $\alpha_{\mathrm{corr}}(f)$. While the 0.85 Hz TDIR tone is suppressed in $x_{\mathrm{DOWR}}(f)$, the TDI combination is dominated by path-length noise and the displacement signal s(t). The corrected alpha combination shows however that the residual laser frequency noise in the DOWR combination is at the level of the 1 nm$/\sqrt{\mathrm{Hz}}$ GRACE-FO phase-locking requirement. The contribution of laser frequency noise is suppressed by 10^9 between the SC_1 displacement measurement and the corrected alpha TDI combination.

First, $\alpha(t)$ contains the injected signal $s(t)$ and the frequency noise suppression is also obscured by this. Figure 10(d) shows a corrected alpha, $\alpha_{\mathrm{corr}}(t)$, which has the injected signal removed:

$$\alpha_{\mathrm{corr}}(t) = \alpha(t) - s(t). \tag{16}$$

Here, the injected signal was measured via a phase measurement of the photodetectors in Fig. 9, $s(t) = \phi_{D1}(t) - \phi_{D0}(t)$.

Second, moving to the three detector combination requires the relative delays of the cables and digital delays through to the phase meter to be measured and taken into account. The details of determining these delays is given in the Appendix, but the methodology was to use the TDIR algorithm for different combinations of detectors.

Thus, $\alpha_{\mathrm{corr}}(f)$ [Fig. 10(d)] can be compared to $x_1(f)$ [Fig. 10(a)] to show the amount of suppression of frequency noise. The results provide experimental verification that the tone-assisted TDIR technique works at the 6 ns level in spite of a large displacement signal.

VIII. CONCLUSION

The LEGOP project proposes to develop tests of arm locking and TDI that may be implemented on GRACE-FO without modifying the existing hardware or interfering with the science measurement. The focus of this paper has been on developing the TDI experiment. A tone-assisted TDIR algorithm has been proposed, with simulation and experimental results demonstrating it can be used to form round-trip measurements of phase that reach the sensitivity of the normal GRACE-FO phase-locking system, requiring no hardware modifications.

The proposed GRACE-FO test of TDI would require the phase locking to be disabled. With one laser frequency stabilized by locking to a stable reference cavity and the other free running, the aim of the TDI experiment would be to recover the displacement sensitivity of the phase-locked system.

Under simulated GRACE-FO conditions, the tone-assisted TDIR algorithm was used to suppress the laser frequency noise by 8 orders of magnitude down to the minimum displacement requirement for residual frequency noise for GRACE-FO of 20 $\mathrm{nm}/\sqrt{\mathrm{Hz}}$. The algorithm was able to estimate the time-varying interspacecraft range to within an rms error of ± 0.2 m from the true range.

The experimental demonstration has shown the capabilities of tone-assisted TDIR in the presence of large path-length fluctuations and macroscopic optical delays.

These results demonstrate that the use of tone-assisted TDIR on GRACE-FO would meet the laser ranging interferometer's displacement sensitivity requirements. This puts us one step closer to realizing an interspacecraft demonstration of TDI as a means of verifying it as a viable method for reducing laser frequency noise for future LISA-like missions.

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APPENDIX: MEASUREMENT OF PHOTODETECTION DELAYS

To verify the delay found in the DOWR combination using the Sagnac combination the delay between each photodetector and the analog-to-digital converter (ADC) input must be taken into account. This appendix determines the correct delay to apply to the individual detectors in the $\alpha(t)$ combination.

1. Free-running noise at each detector

The phase of the slave laser, $\phi_{12}(t)$, is delayed by the electronics behind the nth detector, $D_{\mathrm{E}n}$, and by the one-way light travel time D_{12}, where D's are the delay operators with the usual definition:

$$\phi_0(t) = \phi_{12}(t), \tag{A1}$$

$$\phi_1(t) = D_{\mathrm{E1}} D_{\mathrm{L}} \phi_{12}(t), \tag{A2}$$

$$\phi_2(t) = D_{\mathrm{E2}} \phi_{12}(t). \tag{A3}$$

We have defined the electronic delay of the back link detector to be zero.

2. Laser noise-free combinations

The three detectors can be combined to make slave noise-free combinations using the appropriate delays.

The DOWR combination is

$$\begin{aligned} \mathrm{DOWR}(t) &= \phi_1(t) - D_\Delta \phi_2(t) \\ &= (D_{\mathrm{E1}} D_{12} - D_\Delta D_{\mathrm{E2}}) \phi_{12}(t) \end{aligned} \tag{A4}$$

which cancels the phase noise when the delay

$$D_\Delta = D_{\mathrm{E1}} D_{12} / D_{\mathrm{E2}}. \tag{A5}$$

The $S1$ combination is

$$S1(t) = \phi_0(t) - D_{S1} \phi_1(t) = (1 - D_{S1} D_{\mathrm{E1}} D_{12}) \phi_{12}(t) \tag{A6}$$

which cancels the phase noise when the delay

$$D_{S1} = 1/(D_{\mathrm{E1}}D_{12}). \tag{A7}$$

The $S2$ combination is

$$S2(t) = \phi_0(t) - D_{\mathrm{S2}}\phi_2(t) = (1 - D_{\mathrm{S2}}D_{\mathrm{E2}})\phi_{l2}(t) \tag{A8}$$

which cancels the phase noise when the delay

$$D_{\mathrm{S2}} = 1/(D_{\mathrm{E2}}). \tag{A9}$$

The alpha combination is

$$\begin{aligned}\alpha(t) &= \phi_1(t) + \phi_2(t) - \phi_0(t) - 2D_\alpha\phi_0(t)\\ &= (D_{\mathrm{E1}}D_{12} + D_{\mathrm{E2}} - 1 - 2D_\alpha)\phi_{\mathrm{l2}}(t)\end{aligned} \tag{A10}$$

which cancels the phase noise when the delay

$$2D_{\mathrm{alpha}} = 1/(D_{\mathrm{E1}}D_{12} + D_{\mathrm{E2}}) \tag{A11}$$

$$= 1/(D_\Delta/D_{\mathrm{S2}} + 1/D_{\mathrm{S2}}) \tag{A12}$$

in terms of a delay

$$\tau_\alpha = (\tau_\Delta - 2\tau_{\mathrm{S2}})/2. \tag{A13}$$

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