Neutrino propagation in media: Flavor, helicity, and pair correlations

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(Received 10 April 2015; published 16 June 2015)

Neutrinos propagating in media (matter and electromagnetic fields) undergo flavor and helicity oscillations, where helicity transitions are instigated both by electromagnetic fields and matter currents. In addition, it has been shown that correlations between neutrinos and antineutrinos of opposite momentum can build up in anisotropic media. We rederive the neutrino equations of motion in the mean-field approximation for homogeneous yet anisotropic media, confirming previous results except for a small correction in the Majorana case. Furthermore, we derive the mean-field Hamiltonian induced by neutrino electromagnetic interactions. We also provide a phenomenological discussion of pair correlations in comparison with helicity correlations.

DOI: 10.1103/PhysRevD.91.125020 PACS numbers: 14.60.Pq, 13.15.+g, 97.60.Bw

I. INTRODUCTION

Neutrino flavor conversion in vacuum [1,2], in matter [3–8], or self-induced flavor conversion in a gas of interacting neutrinos [9–27] provide a rich phenomenology of very practical experimental and astrophysical importance. The data leave no room for doubt that neutrinos have small but nonvanishing masses. One consequence is that neutrinos have small electromagnetic dipole and transition moments [28]. These lead to spin and spin-flavor oscillations in strong electromagnetic fields [29–32]. Actually, polarized matter or matter currents alone instigate spin and spin-flavor transitions of massive neutrinos, having effects similar to electromagnetic fields [33–37].

For neutrinos streaming from a supernova core, the background medium may contain currents. Moreover, the neutrino stream itself provides an unavoidable non-isotropic background. In addition, self-induced flavor conversion in an interacting neutrino gas requires unstable modes in flavor space (run-away solutions). If such solutions exist, even small perturbations or otherwise small effects can grow exponentially. In this sense, it is never obvious if a seemingly small effect can get amplified by an instability to play an important role after all. Therefore, it is interesting to study if an interacting neutrino gas can amplify helicity conversion effects [38] which otherwise are very small.

Flavor oscillations lead to correlations building up between neutrinos of different flavor. If $a_\alpha^\dagger$ is the creation operator of a neutrino in flavor state α with a certain momentum $\mathbf{p}$, the initially prepared system can be described by the occupation number $\langle a_\alpha^\dagger a_\alpha\rangle$. One way of looking at flavor oscillations is that "flavor off-diagonal" occupation numbers of the type $\langle a_\alpha^\dagger a_\beta\rangle$ develop and oscillate [39–41]. One unifies these expressions in a density matrix ρ with components $\rho_{\alpha\beta} = \langle a_\beta^\dagger a_\alpha\rangle$. It evolves according to the commutator equation $i\dot\rho = [\mathsf{H},\rho]$, where H is the Hamiltonian matrix, consisting of oscillation frequencies. For vacuum oscillations we have $\mathsf{H} = \mathsf{M}^2/2E$, where M^2 is a matrix of squared neutrino masses. Similar descriptions pertain to spin and spin-flavor oscillation, where the indices now indicate various states of spin and/or flavor.

It was recently stressed that yet another form of correlations, hitherto neglected in the context of neutrino propagation, can build up in nonisotropic media [42–44]. If $a_\mathbf{p}^\dagger$ is the creation operator of a massless neutrino in mode $\mathbf{p}$ and $b_{-\mathbf{p}}^\dagger$ is the one for an antineutrino with opposite momentum, correlators of the form $\kappa_\mathbf{p} = \langle b_{-\mathbf{p}} a_\mathbf{p}\rangle$ and $\kappa_\mathbf{p}^\dagger = \langle a_\mathbf{p}^\dagger b_{-\mathbf{p}}^\dagger\rangle$ will build up, the latter corresponding to the creation of a particle-antiparticle pair with vanishing total momentum. Because massless neutrinos and antineutrinos have opposite helicity, this pair has total spin 1 so that its creation requires a medium current transverse to $\mathbf{p}$ to satisfy angular-momentum conservation. This requirement is analogous to the case of helicity transitions where we also need a transverse current or magnetic field for the same reason. Including flavor and spin degrees of freedom expands the "pair correlations" κ and $\kappa^\dagger$ to become matrices similar to ρ.

To develop more intuition about the meaning of the pair correlations, we consider a single mode $\mathbf{p}$ of neutrinos and $-\mathbf{p}$ of antineutrinos. We define $\rho_\mathbf{p} = \langle a_\mathbf{p}^\dagger a_\mathbf{p}\rangle$ and for antineutrinos $\bar\rho_\mathbf{p} = \langle b_{-\mathbf{p}}^\dagger b_{-\mathbf{p}}\rangle$ involving the opposite momentum. Following the earlier literature [42–44], we unify these expressions in an extended density matrix

$$\mathsf{R} = \begin{pmatrix} \rho & \kappa \\ \kappa^\dagger & 1-\bar\rho \end{pmatrix} = \begin{pmatrix} \langle a_\mathbf{p}^\dagger a_\mathbf{p}\rangle & \langle b_{-\mathbf{p}} a_\mathbf{p}\rangle \\ \langle a_\mathbf{p}^\dagger b_{-\mathbf{p}}^\dagger\rangle & \langle b_{-\mathbf{p}} b_{-\mathbf{p}}^\dagger\rangle \end{pmatrix}, \quad (1)$$

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which obeys an equation of motion of the form [42–44]

$$i\dot{\mathsf{R}} = [\mathsf{H}, \mathsf{R}]. \tag{2}$$

If the background is a current moving in the transverse direction with velocity β, the Hamiltonian matrix is found to be

$$\mathsf{H} = E\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + V\begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix}, \tag{3}$$

where $E = |\mathbf{p}|$. For ν_μ or ν_τ neutrinos the usual matter potential is $V = G_{\rm F} n_n/\sqrt{2}$, where n_n is the neutron density.

This commutator equation has the same structure that one encounters for the evolution of any two-level system and in particular for two-flavor or helicity oscillations, of course with a different matrix H for each case. However, what specifically are the two states that are being mixed by the matter current in the pair-correlation case?

The answer becomes evident if one considers the evolution of states rather than correlators. Our simple system is described by the four basis states $|00\rangle$, $|10\rangle$, $|01\rangle$ and $|11\rangle$, where the first entry refers to $\nu(\mathbf{p})$ and the second to $\bar{\nu}(-\mathbf{p})$. A homogeneous background medium cannot mix states of different total momentum, so the single-particle states must evolve independently as $i\partial_t|10\rangle = (E+V)|10\rangle$ and $i\partial_t|01\rangle = (E-V)|01\rangle$, i.e., they simply suffer the usual energy shift by the weak potential of the medium. This leaves us with $|00\rangle$ and $|11\rangle$ which both have zero momentum and therefore can be mixed by a homogeneous medium. The former has spin 0, the latter spin 1, so for the medium to mix them, it must provide a transverse vector in the form of a current or a spin polarization. If A_{00} and A_{11} are the amplitudes of $|00\rangle$ and $|11\rangle$, respectively, we will show later that Eq. (2) corresponds to

$$i\partial_t\begin{pmatrix} A_{00} \\ A_{11} \end{pmatrix} = \begin{pmatrix} 0 & \beta V \\ \beta V & 2E \end{pmatrix}\begin{pmatrix} A_{00} \\ A_{11} \end{pmatrix}. \tag{4}$$

Therefore, it is the empty and the completely filled states that are being mixed and that oscillate. The true ground state of our system is not $|00\rangle$, but a suitable combination of $|00\rangle$ and $|11\rangle$ which follows from diagonalizing the matrix in Eq. (4).

As we have noted, any two-level system is equivalent to an abstract spin-$\frac{1}{2}$ system. In two-flavor oscillations, the "spin" represents the two flavor states. In the pair-correlation case, "spin up" means "empty" and "spin down" means "full with a pair." This interpretation is analogous to Anderson's "pseudo spin" devised to describe Cooper pairs in the context of superconductivity [45]. A coherent superposition of these two spin states, represented in our case by the pair correlations, corresponds to a coherent superposition of $|00\rangle$ and $|11\rangle$.

In analogy to the example of superconductivity, another way to think about these phenomena is in terms of Bogolyubov transformations of the creation and annihilation operators. If we think of a single momentum mode $\mathbf{p}$ of mixed neutrinos in vacuum, the operators a_{ν_e} and a_{ν_μ} in the flavor basis are rotated by a unitary transformation with mixing angle ϑ to form new operators $c_\vartheta a_{\nu_e} + s_\vartheta a_{\nu_\mu}$ and $c_\vartheta a_{\nu_e} - s_\vartheta a_{\nu_\mu}$, and similarly for the creation operators, to form a new set of canonically anticommuting operators, now describing neutrinos in the mass basis. Describing flavor oscillations in terms of time-dependent Bogolyubov transformations can be especially illuminating to understand quantum statistics in mixing phenomena for both bosons and fermions [46]. Pair correlations correspond to the same idea where the mixing is between $a_{\mathbf{p}}$ and $b^\dagger_{-\mathbf{p}}$ with a mixing angle corresponding to the unitary transformation that diagonalizes the matrix in Eq. (4). The state $|00\rangle$ defined in the Bogolyubov-transformed basis is the ground state of the system and no longer oscillates into the new $|11\rangle$ state.

The goal of our paper is two-pronged. On the one hand we reconsider the mean-field equations of motion for massive neutrinos propagating in a background medium that can consist of matter and neutrinos, and that is homogeneous but not isotropic. Besides the usual flavor oscillations in matter, the resulting phenomena include spin and spin-flavor oscillations as well as pair correlations.

As a second goal, we provide a phenomenological discussion of the interpretation of the pair correlations in the context of neutrino oscillation problems in dense media. Ultimately, our community needs to develop an understanding if, from a practical perspective, we need to worry about pair correlations and helicity oscillations in the supernova context.

The supernova environment is characterized by small neutrino energies of at most some 200 MeV (for degenerate ν_e), i.e., small compared to W and Z masses so that it suffices to describe neutrino interactions in terms of an effective current-current Hamiltonian. In the early Universe, where the chemical potentials of background particles are small, one has to worry about corrections from the electroweak gauge-boson propagators even at low temperatures [47]. The supernova environment, in contrast, has large densities of background particles and this concern is moot.

On the mean-field level, the current of background particles is a classical quantity. For example, the neutral-current interaction of a neutrino with neutrons is given by the Hamiltonian density $\mathcal{H} = \sqrt{2}G_{\rm F}[\bar{\nu}\gamma_\mu P_L \nu] I^\mu_n$, where $G_{\rm F}$ is the Fermi constant, ν is the neutrino Dirac field, P_L is the left-handed projector, and I^μ_n is the neutron current. If the current is homogeneous, $H = \int d^3\mathbf{x}\mathcal{H}$ is effectively a "forward" Hamiltonian: it couples, e.g., $a^\dagger$ and a of equal momenta. Following the previous literature [40,41,44], the evolution of, e.g., the annihilator for a neutrino of mass eigenstate i in mode $\mathbf{p}$ is given by the Heisenberg equation of motion $i\partial_t a_i(t,\mathbf{p}) = [a_i(t,\mathbf{p}), H]$. It is then straightforward to find the equations of motion of bilinears of the form $a_i^\dagger(t,\mathbf{p})a_j(t,\mathbf{p})$, of their expectation value $\langle a_i^\dagger(t,\mathbf{p})a_j(t,\mathbf{p})\rangle$, of the entire matrix ρ, and then of the extended matrix R which also includes pair correlations.

It is largely a cumbersome bookkeeping exercise to obtain, for neutrinos with mass, all the components of the Hamiltonian matrix H appearing in Eq. (2) when R involves all components of spin and flavor. We perform this task separately for Dirac neutrinos in Sec. II, for Majorana neutrinos in Sec. III, and for Weyl neutrinos (massless two-component case) in Sec. IV. These derivations closely parallel the recent paper by Serreau and Volpe [44] and we will largely follow their notation to avoid confusion. In the Majorana case, we find a small correction, but otherwise our results agree.

The density matrix formalism allows one to treat helicity oscillations induced by magnetic fields and by matter currents on equal footing for both Dirac and Majorana fermions. We derive the mean-field Hamiltonian induced by electromagnetic fields in Sec. V. Concerning helicity oscillations induced by matter currents, which we analyze in Sec. VI, our results coincide with those of Volpe and Serreau, and parallel those of Vlasenko, Fuller, and Cirigliano [36–38] as far as the mean-field limit is concerned. These authors have derived the neutrino kinetic equations starting from first field-theoretic principles and have carried the results beyond the mean-field limit to include (nonforward) collision terms, generalizing previous derivations [39–41]. We note in passing that one of their findings—helicity oscillations in a nonisotropic matter background—had been anticipated in several papers by Studenikin and collaborators who have worked out the one-to-one correspondence to the effect of electromagnetic fields [33,35]. Of course, Vlasenko, Fuller, and Cirigliano also included neutrino-neutrino interactions as an agent of helicity conversion and carried their results beyond the mean-field limit.

Pair correlations have been studied in detail in condensed matter and nuclear physics, as well as in the context of pair creation in quantum field theory. On the other hand, in neutrino physics these concepts are less familiar. They have been addressed only in a handful of papers in the context of leptogenesis, where pair correlations have been studied from first principles in a series of papers by Fidler, Herranen, Kainulainen and Rahkila [48–52]. In the context of neutrino propagation in supernovae, the only discussions so far appear in a series of papers by Volpe and collaborators [42–44]. We address phenomenological aspects of pair correlations in Sec. VII and compare them to helicity correlations.

Finally, in Sec. VIII we summarize the results and present our conclusions.

II. DIRAC NEUTRINO

Our first goal is to derive the components of the Hamiltonian matrix H which governs the evolution equation (2) for the extended density matrix R including flavor, helicity, and pair correlations. In this rather technical section, we begin with the conceptually simplest case of three neutrino flavors which are assumed to have Dirac masses. Therefore, helicity correlations involve the sterile components of the neutrino field, which otherwise are completely decoupled.

A. Two-point correlators and kinetic equations

In the simplest approximation, one can describe the state of a neutrino gas in terms of one-particle distribution functions. They are extended to include flavor and helicity coherence effects by promoting the one-particle distribution functions to density matrices [36,39–41,44]. In terms of the usual creation and annihilation operators, their components are

$$(2\pi)^3\delta(\mathbf{p}-\mathbf{k})\rho_{ij,sh}(t,\mathbf{p}) = \langle a_{j,h}^\dagger(t,+\mathbf{k})a_{i,s}(t,+\mathbf{p})\rangle, \quad (5a)$$

$$(2\pi)^3\delta(\mathbf{p}-\mathbf{k})\bar\rho_{ij,sh}(t,\mathbf{p}) = \langle b_{i,s}^\dagger(t,-\mathbf{p})b_{j,h}(t,-\mathbf{k})\rangle, \quad (5b)$$

where i and j are flavor indices in the mass basis, and s and $h \in \{+,-\}$ denote helicities. In this convention, the density matrix for antineutrinos $\bar\rho_{ij,sh}(t,\mathbf{p})$ for momentum $\mathbf{p}$ actually corresponds to the occupation numbers of antineutrinos with physical momentum $-\mathbf{p}$.

This convention is necessary to combine ρ and $\bar\rho$ with the pair correlations which are defined as [43,44]

$$(2\pi)^3\delta(\mathbf{p}-\mathbf{k})\kappa_{ij,sh}(t,\mathbf{p}) = \langle b_{j,h}(t,-\mathbf{k})a_{i,s}(t,+\mathbf{p})\rangle, \quad (6a)$$

$$(2\pi)^3\delta(\mathbf{p}-\mathbf{k})\kappa_{ij,sh}^\dagger(t,\mathbf{p}) = \langle a_{j,h}^\dagger(t,+\mathbf{p})b_{i,s}^\dagger(t,-\mathbf{k})\rangle, \quad (6b)$$

and which involve opposite-momentum modes.

The kinetic equations for Eqs. (5) and (6) are obtained with the Heisenberg equation of motion. As we will show below, in the mean-field approximation, and assuming spatial homogeneity, the Hamiltonian of charged- and neutral-current neutrino interactions can be written in the compact form

$$H_{\mathrm{mf}} = \int d^3\mathbf{x}\,\bar\nu_i(t,\mathbf{x})\Gamma_{ij}\nu_j(t,\mathbf{x}), \quad (7)$$

where summation over repeated indices is implied. The kernel takes account of the background medium and is

$$\Gamma_{ij} = \gamma_\mu P_L V_{ij}^\mu, \quad (8)$$

where $P_L = (1-\gamma_5)/2$ is the usual left-chiral projector. The current of background matter V_{ij}^μ will be defined in Eq. (24).

The momentum-mode decomposition of a Dirac neutrino field reads

$$\nu_i(t,\mathbf{x}) = \int_{\mathbf{p},s} e^{i\mathbf{p}\cdot\mathbf{x}}\nu_{i,s}(t,\mathbf{p}), \tag{9}$$

where $\int_{\mathbf{p},s}$ denotes the phase-space integration $\int d^3\mathbf{p}/(2\pi)^3$ and the summation over helicities. In the mass basis, the individual momentum modes are

$$\nu_{i,s}(t,\mathbf{p}) = a_{i,s}(t,\mathbf{p})u_{i,s}(\mathbf{p}) + b^\dagger_{i,s}(t,-\mathbf{p})v_{i,s}(-\mathbf{p}). \tag{10}$$

The chiral spinors u and v are given in Appendix A, and the creation and annihilation operators satisfy the usual equal-time anticommutation relation,

$$\{a_{i,s}(t,\mathbf{p}), a^\dagger_{j,h}(t,\mathbf{k})\} = (2\pi)^3\delta(\mathbf{p}-\mathbf{k})\delta_{ij}\delta_{sh}. \tag{11}$$

Similar relations hold for the antiparticle operators b and $b^\dagger$.

As a next step, we contract the kernels Γ_{ij} with the spinors appearing in the mean-field Hamiltonian (7), leading to the matrices [44]

$$\Gamma^{\nu\nu}_{ij,sh}(\mathbf{p}) \equiv \bar{u}_{i,s}(\mathbf{p})\Gamma_{ij}u_{j,h}(\mathbf{p}), \tag{12a}$$

$$\Gamma^{\nu\bar{\nu}}_{ij,sh}(\mathbf{p}) \equiv \bar{u}_{i,s}(\mathbf{p})\Gamma_{ij}v_{j,h}(-\mathbf{p}), \tag{12b}$$

$$\Gamma^{\bar{\nu}\nu}_{ij,sh}(\mathbf{p}) \equiv \bar{v}_{i,s}(-\mathbf{p})\Gamma_{ij}u_{j,h}(\mathbf{p}), \tag{12c}$$

$$\Gamma^{\bar{\nu}\bar{\nu}}_{ij,sh}(\mathbf{p}) \equiv \bar{v}_{i,s}(-\mathbf{p})\Gamma_{ij}v_{j,h}(-\mathbf{p}), \tag{12d}$$

in component form. We can now bring Eq. (7) to the desired form bilinear in the creation and annihilation operators

$$\begin{aligned} H_{\mathrm{mf}} = \int_{\mathbf{p}} [&a^\dagger_{i,s}(\mathbf{p})\Gamma^{\nu\nu}_{ij,sh}(\mathbf{p})a_{j,h}(\mathbf{p}) + a^\dagger_{i,s}(\mathbf{p})\Gamma^{\nu\bar{\nu}}_{ij,sh}(\mathbf{p})b^\dagger_{j,h}(-\mathbf{p}) \\ &+ b_{i,s}(-\mathbf{p})\Gamma^{\bar{\nu}\nu}_{ij,sh}(\mathbf{p})a_{j,h}(\mathbf{p}) \\ &+ b_{i,s}(-\mathbf{p})\Gamma^{\bar{\nu}\bar{\nu}}_{ij,sh}(\mathbf{p})b^\dagger_{j,h}(-\mathbf{p})], \end{aligned} \tag{13}$$

where we have omitted the time arguments to shorten the notation. Summation over repeated indices is implied.

Using the Heisenberg equation of motion with this Hamiltonian one finds the extended equation of motion $i\dot{\mathsf{R}} = [\mathsf{H},\mathsf{R}]$; see also Eq. (2). The extended density matrix, Eq. (1), and the Hamiltonian, Eq. (3), generalize to [42]

$$\mathsf{R} = \begin{pmatrix} \rho & \kappa \\ \kappa^\dagger & \mathbb{1}-\bar{\rho} \end{pmatrix} \quad \text{and} \quad \mathsf{H} = \begin{pmatrix} \mathsf{H}^{\nu\nu} & \mathsf{H}^{\nu\bar{\nu}} \\ \mathsf{H}^{\bar{\nu}\nu} & \mathsf{H}^{\bar{\nu}\bar{\nu}} \end{pmatrix}, \tag{14}$$

where the submatrices $\mathsf{H}^{\nu\nu} = \Gamma^{\nu\nu}$, $\mathsf{H}^{\nu\bar{\nu}} = \Gamma^{\nu\bar{\nu}}$ etc. and ρ, κ, etc. are 6×6 matrices in helicity and flavor space. The product between such matrices in the commutator is defined in the obvious way $(A\cdot B)_{ij,sh} \equiv A_{in,sr}B_{nj,rh}$ with a summation over repeated indices. In the following we write the matrix structure in the form of 2×2 matrices in helicity space,

$$\begin{pmatrix} \boxed{--}_{ij} & \boxed{-+}_{ij} \\ \boxed{+-}_{ij} & \boxed{++}_{ij} \end{pmatrix}, \tag{15}$$

where each entry is itself a 3×3 matrix in flavor space.

B. Hamiltonian in the mean-field approximation

After having established the overall structure of the kinetic equations we now turn to the interactions contributing to neutrino refraction in the supernova environment. In this subsection we only consider charged- and neutral-current neutrino interactions, whereas the analysis of the electromagnetic interactions is postponed to Sec. V.

1. Charged-current interaction

We begin with charged-current (cc) interactions with background charged leptons. In the low-energy limit and after a Fierz transformation, the usual current-current Hamiltonian density is

$$\mathcal{H}^{\mathrm{cc}} = \sqrt{2}G_{\mathrm{F}}\sum_{\alpha,\beta}[\bar{\nu}_\alpha\gamma^\mu P_L\nu_\beta][\bar{\ell}_\beta\gamma_\mu(1-\gamma^5)\ell_\alpha], \tag{16}$$

where $\alpha,\beta \in \{e,\mu,\tau\}$ are flavor indices.

To obtain the neutrino mean-field Hamiltonian we replace the second bracket by its expectation value. In the supernova environment, the temperature is too low to support a substantial density of muons or tauons, and we use only the electron background. Then we find in the mass basis

$$\mathcal{H}^{\mathrm{cc}}_{\mathrm{mf}} = \sqrt{2}G_{\mathrm{F}}\sum_{i,j}[\bar{\nu}_i\gamma^\mu P_L\nu_j][U^\dagger_{ie}I^\mu_{\mathrm{cc}}U_{ej}], \tag{17}$$

where U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. We have introduced a linear combination of vector and axial-vector charged electron currents,

$$I^\mu_{\mathrm{cc}} \equiv c_V\langle\bar{e}\gamma^\mu e\rangle - c_A\langle\bar{e}\gamma^\mu\gamma^5 e\rangle, \tag{18}$$

where $c_V = c_A = 1$. Because electrons are the only background particles contributing to charged-current interactions and to simplify the notation, an "e" index is implied in I^μ_{cc}. If the electrons are not polarized, the axial current vanishes and $I^\mu_{\mathrm{cc}} = J^\mu_e$, the "convective" electron current.

2. Neutral-current interaction with matter

The neutral-current (nc) interactions with matter are described in the mass basis by the Hamiltonian density

$$\mathcal{H}^{\rm nc} = \sqrt{2}G_{\rm F}\sum_{i,f}[\bar\nu_i\gamma_\mu P_L\nu_i][\bar\psi_f\gamma^\mu(c_V^f - c_A^f\gamma^5)\psi_f], \quad (19)$$

where f denotes electrons, protons, and neutrons. The resulting contribution to the mean-field Hamiltonian is

$$\mathcal{H}_{\rm mf}^{\rm nc} = \sqrt{2}G_{\rm F}\sum_{i}[\bar\nu_i\gamma_\mu P_L\nu_i][I_{\rm nc}^\mu + I_p^\mu + I_n^\mu], \quad (20)$$

where $I_{\rm nc}^\mu$ denotes the electron neutral current (index e implied), whereas the other contributions refer to protons and neutrons as explicitly indicated.

These currents are defined in analogy to Eq. (18) with the appropriate coupling constants. For electrons, they are given by $c_V = -\frac{1}{2} + 2\sin^2\theta_W$ (Weinberg angle θ_W) and $c_A = -\frac{1}{2}$. For protons, $c_V = \frac{1}{2} - 2\sin^2\theta_W$, i.e., the same as for electrons with opposite sign, and for neutrons $c_V = -\frac{1}{2}$. For the nucleon axial vector one often uses $c_A = \pm 1.26/2$ in analogy to their charged current. However, the strange-quark contribution to the nucleon spin as well as modifications in a dense nuclear medium leave the exact values somewhat open [53,54].

In an unpolarized and electrically neutral environment, the axial currents disappear and the electron and proton contributions to the convective neutral current cancel such that in Eq. (20) we have $I_{\rm nc}^\mu + I_p^\mu + I_n^\mu = -\frac{1}{2}J_n^\mu$, where J_n^μ is the neutron convective current. Neutrino refraction in such a medium depends only on the charged electron current and the neutral neutron current.

3. Neutrino-neutrino interaction

The most complicated interaction is the neutral-current neutrino-neutrino one. It is described in the mass basis by the Hamiltonian density

$$\mathcal{H}^{\nu\nu} = \frac{1}{\sqrt{2}}G_{\rm F}\sum_{ij}[\bar\nu_i\gamma_\mu P_L\nu_i][\bar\nu_j\gamma^\mu P_L\nu_j]. \quad (21)$$

To obtain the mean-field Hamiltonian bilinear in the neutrino fields we need to replace products of two of the four neutrino fields in this expression by their expectation value.

The only combinations that do not violate lepton number are of the type $\langle\bar\nu_i\nu_j\rangle$ and $\langle\nu_i\bar\nu_j\rangle$, where i and j can be equal or different. We denote the corresponding mean field as

$$I_{ij}^\mu \equiv \langle\bar\nu_j\gamma^\mu P_L\nu_i\rangle. \quad (22)$$

To simplify notation we avoid an explicit "neutrino" and "nc" index, i.e., expressions of the type I_{ij}^μ always refer to the neutral neutrino current for the mass states i and j. An explicit expression in terms of the density matrices and pair correlators will be given in Eq. (27) below.

For the $i=j$ contractions it is sufficient to take the expectation value of one of the square brackets in Eq. (21), leading to the mean-field Hamiltonian $\sqrt{2}G_{\rm F}\sum_{ij}[\bar\nu_i\gamma_\mu P_L\nu_i]I_{jj}^\mu$. For the $i\neq j$ contractions we use the Fierz identity to rewrite the Hamiltonian as $[\bar\nu_i\gamma_\mu P_L\nu_j][\bar\nu_j\gamma^\mu P_L\nu_i]$ in Eq. (21), leading to the contribution $\sqrt{2}G_{\rm F}\sum_{ij}[\bar\nu_i\gamma_\mu P_L\nu_j]I_{ij}^\mu$. Altogether, we find

$$\mathcal{H}_{\rm mf}^{\nu\nu} = \sqrt{2}G_{\rm F}\sum_{ij}[\bar\nu_i\gamma_\mu P_L\nu_j]\left[I_{ij}^\mu + \delta_{ij}\sum_k I_{kk}^\mu\right] \quad (23)$$

for the neutrino-neutrino mean-field Hamiltonian.

C. Components of the Hamiltonian matrix H

Adding up Eqs. (17), (20), and (23) we find the overall mean-field current

$$V_{ij}^\mu = \sqrt{2}G_{\rm F}\bigg[U_{ie}^\dagger I_{\rm cc}^\mu U_{ej} + \delta_{ij}(I_{\rm nc}^\mu + I_p^\mu + I_n^\mu) + I_{ij}^\mu + \delta_{ij}\sum_k I_{kk}^\mu\bigg]. \quad (24)$$

The spinor contractions defined in Eq. (12) lead to the components of the Hamiltonian matrix H of the form

$$\mathsf{H}_{ij,sh}^{\nu\nu} = (\gamma_\mu P_L)_{ij,sh}^{\nu\nu}V_{ij}^\mu + \delta_{sh}\delta_{ij}E_i, \quad (25a)$$

$$\mathsf{H}_{ij,sh}^{\nu\bar\nu} = (\gamma_\mu P_L)_{ij,sh}^{\nu\bar\nu}V_{ij}^\mu, \quad (25b)$$

$$\mathsf{H}_{ij,sh}^{\bar\nu\nu} = (\gamma_\mu P_L)_{ij,sh}^{\bar\nu\nu}V_{ij}^\mu, \quad (25c)$$

$$\mathsf{H}_{ij,sh}^{\bar\nu\bar\nu} = (\gamma_\mu P_L)_{ij,sh}^{\bar\nu\bar\nu}V_{ij}^\mu - \delta_{sh}\delta_{ij}E_i, \quad (25d)$$

where $E_i = (\mathbf{p}^2 + m_i^2)^{\frac{1}{2}}$ is the neutrino energy, and we have identified $\mathsf{H}^{\nu\nu} = \Gamma^{\nu\nu}$, $\mathsf{H}^{\nu\bar\nu} = \Gamma^{\nu\bar\nu}$, etc. We have used the compact notation

$$(\gamma_\mu P_L)_{ij,sh}^{\nu\nu} \equiv \bar u_{i,s}(+\mathbf{p})\gamma_\mu P_L u_{j,h}(+\mathbf{p}), \quad (26a)$$

$$(\gamma_\mu P_L)_{ij,sh}^{\nu\bar\nu} \equiv \bar u_{i,s}(+\mathbf{p})\gamma_\mu P_L v_{j,h}(-\mathbf{p}), \quad (26b)$$

$$(\gamma_\mu P_L)_{ij,sh}^{\bar\nu\nu} \equiv \bar v_{i,s}(-\mathbf{p})\gamma_\mu P_L u_{j,h}(+\mathbf{p}), \quad (26c)$$

$$(\gamma_\mu P_L)_{ij,sh}^{\bar\nu\bar\nu} \equiv \bar v_{i,s}(-\mathbf{p})\gamma_\mu P_L v_{j,h}(-\mathbf{p}). \quad (26d)$$

Later we will use similar expressions for contractions with other Dirac structures. The neutrino mean-field current itself contains spinor contractions of this type and can be expressed in terms of the density matrices and pair correlations as

$$
\begin{aligned}
I^{\mu}_{ij} = \int_{\mathbf{p},s,h} &[(\gamma^{\mu}P_L)^{\nu\nu}_{ji,hs}\rho_{ij,sh} + (\gamma^{\mu}P_L)^{\nu\bar{\nu}}_{ji,hs}\kappa^{\dagger}_{ij,sh} \\
&+ (\gamma^{\mu}P_L)^{\bar{\nu}\nu}_{ji,hs}\kappa_{ij,sh} + (\gamma^{\mu}P_L)^{\bar{\nu}\bar{\nu}}_{ji,hs}(\delta_{ij}\delta_{sh} - \bar{\rho}_{ij,sh})].
\end{aligned}
\tag{27}
$$

Notice that in this case there is no implied summation over i and j. The fourth term contains a divergent vacuum contribution that must be renormalized.

We finally work out the spinor contractions explicitly to lowest order in neutrino masses. To this end we introduce

$$
n^{\mu} = (1, \hat{\mathbf{p}}), \qquad \bar{n}^{\mu} = (1, -\hat{\mathbf{p}}), \qquad \epsilon^{\mu} = (0, \hat{\boldsymbol{\epsilon}}), \tag{28}
$$

where $\hat{\mathbf{p}}$ is a unit vector in the momentum direction and the complex polarization vector $\hat{\boldsymbol{\epsilon}}$ spans the plane orthogonal to $\mathbf{p}$ (see Appendix A for more details). We also use ϕ to denote the polar angle of $\mathbf{p}$ in spherical coordinates. To lowest order in m_i, the spinor contractions are then found to be

$$
(\gamma_{\mu}P_L)^{\nu\nu}_{ij,sh} \approx \begin{pmatrix} n_{\mu} & -e^{+i\phi}\frac{m_j}{2p}\epsilon^{*}_{\mu} \\ -e^{-i\phi}\frac{m_i}{2p}\epsilon_{\mu} & 0 \end{pmatrix}, \tag{29a}
$$

$$
(\gamma_{\mu}P_L)^{\nu\bar{\nu}}_{ij,sh} \approx \begin{pmatrix} -e^{+i\phi}\frac{m_j}{2p}n_{\mu} & \epsilon^{*}_{\mu} \\ 0 & -e^{-i\phi}\frac{m_i}{2p}\bar{n}_{\mu} \end{pmatrix}, \tag{29b}
$$

$$
(\gamma_{\mu}P_L)^{\bar{\nu}\nu}_{ij,sh} \approx \begin{pmatrix} -e^{-i\phi}\frac{m_i}{2p}n_{\mu} & 0 \\ \epsilon_{\mu} & -e^{+i\phi}\frac{m_j}{2p}\bar{n}_{\mu} \end{pmatrix}, \tag{29c}
$$

$$
(\gamma_{\mu}P_L)^{\bar{\nu}\bar{\nu}}_{ij,sh} \approx \begin{pmatrix} 0 & -e^{-i\phi}\frac{m_i}{2p}\epsilon^{*}_{\mu} \\ -e^{+i\phi}\frac{m_j}{2p}\epsilon_{\mu} & \bar{n}_{\mu} \end{pmatrix}, \tag{29d}
$$

where we use the notation introduced in Eq. (15). As an example, the $\nu\nu$ term, Eq. (29a), reads explicitly

$$
\boxed{--}^{\nu\nu}_{ij} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} n_{\mu}, \tag{30a}
$$

$$
\boxed{-+}^{\nu\nu}_{ij} = -\frac{1}{2p}\begin{pmatrix} m_1 & m_2 & m_3 \\ m_1 & m_2 & m_3 \\ m_1 & m_2 & m_3 \end{pmatrix} e^{+i\phi}\epsilon^{*}_{\mu}, \tag{30b}
$$

$$
\boxed{+-}^{\nu\nu}_{ij} = -\frac{1}{2p}\begin{pmatrix} m_1 & m_1 & m_1 \\ m_2 & m_2 & m_2 \\ m_3 & m_3 & m_3 \end{pmatrix} e^{-i\phi}\epsilon_{\mu}, \tag{30c}
$$

$$
\boxed{++}^{\nu\nu}_{ij} = 0. \tag{30d}
$$

These results agree with those obtained in Ref. [44].

III. MAJORANA NEUTRINO

From a theoretical perspective, it is quite natural for neutrino masses to be of Majorana type. In this case, the two helicity states of a given family coincide with the ν and $\bar{\nu}$ states, the mass term violates lepton number, and there are no sterile degrees of freedom. We work out the modifications of the results of the previous section for the Majorana case, concentrating again on technical issues.

A. Two-point correlators and kinetic equations

In the Majorana case, the momentum decomposition of the neutrino field looks the same as for the Dirac case [Eq. (9)]. However, because there are no independent antiparticle degrees of freedom, the field mode $\mathbf{p}$ has the simpler form

$$
\nu_{i,s}(t,\mathbf{p}) = a_{i,s}(t,\mathbf{p})u_{i,s}(\mathbf{p}) + a^{\dagger}_{i,s}(t,-\mathbf{p})v_{i,s}(-\mathbf{p}). \tag{31}
$$

The creation and annihilation operators satisfy the same anticommutation relations [Eq. (11)] and the bispinors are the same as in the Dirac case.

The definitions of the two-point correlation functions are different because of the different particle content,

$$
(2\pi)^3\delta(\mathbf{p}-\mathbf{k})\rho_{ij,sh}(\mathbf{p}) = \langle a^{\dagger}_{j,h}(+\mathbf{k})a_{i,s}(+\mathbf{p})\rangle, \tag{32a}
$$

$$
(2\pi)^3\delta(\mathbf{p}-\mathbf{k})\bar{\rho}_{ij,sh}(\mathbf{p}) = \langle a^{\dagger}_{i,s}(-\mathbf{p})a_{j,h}(-\mathbf{k})\rangle, \tag{32b}
$$

$$
(2\pi)^3\delta(\mathbf{p}-\mathbf{k})\kappa_{ij,sh}(\mathbf{p}) = \langle a_{j,h}(-\mathbf{k})a_{i,s}(+\mathbf{p})\rangle, \tag{32c}
$$

$$
(2\pi)^3\delta(\mathbf{p}-\mathbf{k})\kappa^{\dagger}_{ij,sh}(\mathbf{p}) = \langle a^{\dagger}_{j,h}(+\mathbf{p})a^{\dagger}_{i,s}(-\mathbf{k})\rangle, \tag{32d}
$$

where all operators are taken at the same time t. In the Dirac case, $\kappa^{\dagger}$ has no additional information relative to κ. Here we have additional redundancies

$$
\bar{\rho}_{ij,sh}(t,\mathbf{p}) = \rho_{ji,hs}(t,-\mathbf{p}), \tag{33a}
$$

$$
\kappa_{ij,sh}(t,\mathbf{p}) = -\kappa_{ji,hs}(t,-\mathbf{p}), \tag{33b}
$$

which reflect that Majorana neutrinos have half as many degrees of freedom as Dirac ones. Note that in the Majorana case, the pair correlations violate total lepton number.

The mean-field Hamiltonian, bilinear in the neutrino creation and annihilation operators, has the same form [Eq. (7)] as in the Dirac case. However, as we will demonstrate below, the kernel has a more general structure,

$$
\Gamma_{ij} = \gamma_{\mu}P_L V^{\mu}_{ij} + P_L V^{R}_{ij} + P_R V^{L}_{ij}. \tag{34}
$$

The first piece, V^{μ}_{ij}, is defined as in Eq. (24). In addition, there are two scalar pieces

$$V_{ij}^{L,R} = \sqrt{2} G_{\rm F} I_{ij}^{L,R}, \tag{35}$$

depending, as we will see, on the left-chiral and right-chiral neutrino mean-field scalar background

$$I_{ij}^{L,R} = \langle \bar{\nu}_j P_{L,R} \nu_i \rangle. \tag{36}$$

These scalar pieces are missing in the previous literature.[1] Their explicit form in terms of the density matrices and pair correlators will be given in Eq. (41).

The mean-field Hamiltonian can be written in a form similar to Eq. (13),

$$\begin{aligned} H_{\rm mf} = \int_{\mathbf{p},s,h} [& a_{i,s}^\dagger(\mathbf{p}) \Gamma_{ij,sh}^{\nu\nu}(\mathbf{p}) a_{j,h}(\mathbf{p}) \\ & + a_{i,s}^\dagger(\mathbf{p}) \Gamma_{ij,sh}^{\nu\bar{\nu}}(\mathbf{p}) a_{j,h}^\dagger(-\mathbf{p}) \\ & + a_{i,s}(-\mathbf{p}) \Gamma_{ij,sh}^{\bar{\nu}\nu}(\mathbf{p}) a_{j,h}(\mathbf{p}) \\ & + a_{i,s}(-\mathbf{p}) \Gamma_{ij,sh}^{\bar{\nu}\bar{\nu}}(\mathbf{p}) a_{j,h}^\dagger(-\mathbf{p})], \end{aligned} \tag{37}$$

where the matrices $\Gamma^{\nu\nu}$, $\Gamma^{\nu\bar{\nu}}$, etc. are the spinor contractions defined in Eq. (12). Using the Heisenberg equation of motion with the Hamiltonian (37) one recovers the equation of motion $i\dot{\mathsf{R}} = [\mathsf{H}, \mathsf{R}]$, where R and H have the same structure as in Eq. (14). The components of the effective Hamiltonian now read [44]

$$\mathsf{H}_{ij,sh}^{\nu\nu}(\mathbf{p}) = \Gamma_{ij,sh}^{\nu\nu}(\mathbf{p}) - \Gamma_{ji,hs}^{\bar{\nu}\bar{\nu}}(-\mathbf{p}), \tag{38a}$$

$$\mathsf{H}_{ij,sh}^{\nu\bar{\nu}}(\mathbf{p}) = \Gamma_{ij,sh}^{\nu\bar{\nu}}(\mathbf{p}) - \Gamma_{ji,hs}^{\nu\bar{\nu}}(-\mathbf{p}), \tag{38b}$$

$$\mathsf{H}_{ij,sh}^{\bar{\nu}\nu}(\mathbf{p}) = \Gamma_{ij,sh}^{\bar{\nu}\nu}(\mathbf{p}) - \Gamma_{ji,hs}^{\bar{\nu}\nu}(-\mathbf{p}), \tag{38c}$$

$$\mathsf{H}_{ij,sh}^{\bar{\nu}\bar{\nu}}(\mathbf{p}) = \Gamma_{ij,sh}^{\bar{\nu}\bar{\nu}}(\mathbf{p}) - \Gamma_{ji,hs}^{\nu\nu}(-\mathbf{p}). \tag{38d}$$

Not all of these components are independent. In particular

$$\mathsf{H}_{ij,sh}^{\bar{\nu}\bar{\nu}}(\mathbf{p}) = -\mathsf{H}_{ji,hs}^{\nu\nu}(-\mathbf{p}), \tag{39a}$$

$$\mathsf{H}_{ij,sh}^{\nu\bar{\nu}}(\mathbf{p}) = -\mathsf{H}_{ji,hs}^{\nu\bar{\nu}}(-\mathbf{p}), \tag{39b}$$

so only two of the four submatrices of H are independent.

B. Neutrino-neutrino mean-field Hamiltonian

The Majorana neutrino interaction with matter is described by the same charged- and neutral-current Hamiltonian densities [Eqs. (16) and (19)] which lead to the same mean-field currents of electrons and nucleons—see Eqs. (17) and (20).

The neutrino-neutrino interaction in the Majorana case is also described by Eq. (21). However, Majorana neutrinos violate lepton-number conservation, and in addition to the four lepton-number-conserving combinations considered in Sec. II one should also take into account the lepton-number-violating combinations $\langle \nu_i \nu_j \rangle$ and $\langle \bar{\nu}_i \bar{\nu}_j \rangle$ which were not included in the previous literature.

To calculate these additional contractions, we use the definition of the charge-conjugate field $\nu^c \equiv C\bar{\nu}^T$, where C is the charge-conjugation matrix which has the property $C^T C = 1$. Using this definition $\bar{\nu} = (\nu^c)^T C$ and $\nu = C\gamma^0 (\nu^c)^*$, which further implies $\bar{\nu}\gamma^\mu P_L \nu = -\overline{\nu^c}\gamma^\mu P_R \nu^c$. Therefore, we can rewrite the Hamiltonian as $-[\bar{\nu}_i \gamma^\mu P_L \nu_i][\overline{\nu_j^c}\gamma_\mu P_R \nu_j^c]$ in Eq. (21). The Fierz identity [55] $(\gamma^\mu P_L)[\gamma_\mu P_R] = 2(P_R][P_L)$ further allows us to rewrite it as $2[\bar{\nu}_i P_R \nu_j^c][\overline{\nu_j^c} P_L \nu_i]$, where another sign change was induced by anticommuting the neutrino fields. Taking the expectation value of one of the square brackets we obtain for the new contribution to the mean-field Hamiltonian density

$$\mathcal{H}_{\rm mf}^{\nu\nu} = \sqrt{2} G_{\rm F} \sum_{ij} ([\bar{\nu}_i P_R \nu_j^c] I_{ij}^L + [\overline{\nu_i^c} P_L \nu_j] I_{ij}^R). \tag{40}$$

These new terms supplement the expression for the effective Majorana Hamiltonian obtained in the previous literature [44]. In Appendix B we reproduce this result using two-component notation.

Two comments are in order here. First, for Majorana fermions $\nu^c = \nu$ and therefore the resulting contribution to the kernel reduces to the last two terms in Eq. (34), while the definition of left- and right-chiral neutrino backgrounds reduces to Eq. (35). Second, $\bar{\nu} P_R \nu^c = \bar{\nu}_L \nu_L^c$ and $\bar{\nu}^c P_L \nu = \overline{\nu_L^c} \nu_L$, where $\nu_L \equiv P_L \nu$, which are nothing but components of the Majorana mass term.

C. Components of the Hamiltonian matrix H

The new contributions stemming from neutrino-neutrino interactions can be expressed in terms of the (anti)particle densities and pair correlators,

$$\begin{aligned} I_{ij}^L = \int_{\mathbf{p},s,h} [& (P_L)_{ji,hs}^{\nu\nu} \rho_{ij,sh} + (P_L)_{ji,hs}^{\bar{\nu}\bar{\nu}} (\delta_{ij}\delta_{sh} - \bar{\rho}_{ij,sh}) \\ & + (P_L)_{ji,hs}^{\bar{\nu}\nu} \kappa_{ij,hs} + (P_L)_{ji,hs}^{\nu\bar{\nu}} \kappa_{ij,sh}^\dagger], \end{aligned} \tag{41}$$

where we have again suppressed the common arguments $\mathbf{p}$ and $(t, \mathbf{p})$. The notation for the scalar contractions $(P_L)_{ij,sh}^{\nu\nu}$ etc. is analogous to Eq. (26), except that now there is no γ^μ included.

To lowest order in the small neutrino masses we find, using the explicit form of the chiral spinors of Appendix A,

[1] In a private communication, the authors of Ref. [44] agree that these terms should indeed be present in the Majorana case. Of course, the presence of these terms does not modify the overall structure of the kinetic equations.

$$(P_L)^{\nu\nu}_{ij,sh} \approx \begin{pmatrix} \frac{m_i}{2p} & 0 \\ 0 & \frac{m_j}{2p} \end{pmatrix}, \tag{42a}$$

$$(P_L)^{\nu\bar{\nu}}_{ij,sh} \approx \begin{pmatrix} 0 & 0 \\ 0 & -e^{-i\phi} \end{pmatrix}, \tag{42b}$$

$$(P_L)^{\bar{\nu}\nu}_{ij,sh} \approx \begin{pmatrix} e^{-i\phi} & 0 \\ 0 & 0 \end{pmatrix}, \tag{42c}$$

$$(P_L)^{\bar{\nu}\bar{\nu}}_{ij,sh} \approx \begin{pmatrix} -\frac{m_j}{2p} & 0 \\ 0 & -\frac{m_i}{2p} \end{pmatrix}. \tag{42d}$$

The components of (P_R) can be obtained from these results using the relations $(P_R)^{\nu\nu}_{ij,sh} = [(P_L)^{\nu\nu}_{ji,hs}]^*$ and $(P_R)^{\nu\bar{\nu}}_{ij,sh} = [(P_L)^{\bar{\nu}\nu}_{ji,hs}]^*$, as well as similar relations for the remaining two components.

Using the definitions (38) combined with Eq. (29) and the corresponding definition for the scalar case we obtain for the $\nu\nu$ component of H

$$\begin{aligned}\mathsf{H}^{\nu\nu}_{ij,sh}(\mathbf{p}) = \delta_{sh}\delta_{ij}E_i & \\ &+ (\gamma_\mu P_L)^{\nu\nu}_{ij,sh}(\mathbf{p})V^\mu_{ij} - (\gamma_\mu P_L)^{\bar{\nu}\bar{\nu}}_{ji,hs}(-\mathbf{p})V^\mu_{ji} \\ &+ (P_L)^{\nu\nu}_{ij,sh}(\mathbf{p})V^R_{ij} - (P_L)^{\bar{\nu}\bar{\nu}}_{ji,hs}(-\mathbf{p})V^R_{ji} \\ &+ (P_R)^{\nu\nu}_{ij,sh}(\mathbf{p})V^L_{ij} - (P_R)^{\bar{\nu}\bar{\nu}}_{ji,hs}(-\mathbf{p})V^L_{ji}. \end{aligned} \tag{43}$$

The second line generalizes the Dirac result of Eq. (25a) to the Majorana case and has been obtained in Ref. [44]. The third and fourth lines stem from the contractions (40) and supplement the previous results.

The $\bar{\nu}\bar{\nu}$ term follows from the identity (39). For the $\nu\bar{\nu}$ component we find

$$\begin{aligned}\mathsf{H}^{\nu\bar{\nu}}_{ij,sh}(\mathbf{p}) &= (\gamma_\mu P_L)^{\nu\bar{\nu}}_{ij,sh}(\mathbf{p})V^\mu_{ij} - (\gamma_\mu P_L)^{\nu\bar{\nu}}_{ji,hs}(-\mathbf{p})V^\mu_{ji} \\ &+ (P_L)^{\nu\bar{\nu}}_{ij,sh}(\mathbf{p})V^R_{ij} - (P_L)^{\nu\bar{\nu}}_{ji,hs}(-\mathbf{p})V^R_{ji} \\ &+ (P_R)^{\nu\bar{\nu}}_{ij,sh}(\mathbf{p})V^L_{ij} - (P_R)^{\nu\bar{\nu}}_{ji,hs}(-\mathbf{p})V^L_{ji}. \end{aligned} \tag{44}$$

The $\bar{\nu}\nu$ component follows from replacing $\nu\bar{\nu}$ with $\bar{\nu}\nu$ everywhere in this result.

An inspection of Eqs. (41) and (42) shows that the last two lines of Eq. (43) contain terms proportional to κ and $\kappa^\dagger$ that are linear in the neutrino masses, and additionally terms quadratic in the neutrino masses which we neglect here.

A peculiar feature of Eq. (44) is that its last two lines contain terms proportional to κ and $\kappa^\dagger$ that are not suppressed by the neutrino masses and therefore do not vanish when we set the masses to zero. This is somewhat surprising because we expect that Dirac and Majorana neutrinos are equivalent for $m_\nu \to 0$. Therefore, the components of H must coincide in this limit. We return to this question in Sec. IV, where we study the case of massless two-component neutrinos and demonstrate that in the massless limit these additional terms, which are proportional to the lepton-number-violating correlators, are not produced if they are zero initially.

On the other hand, one important finding of our paper is that for a Majorana neutrino with an arbitrary small mass, lepton-number-violating correlators are automatically produced and, in turn, induce the additional scalar background terms of the mean-field Hamiltonian which then affect the dynamics of the density matrices.

IV. WEYL NEUTRINO

In the previous section we have found that the additional scalar contributions to the mean-field Hamiltonian, that naturally arise for Majorana neutrinos, do not vanish in the massless limit. This is somewhat surprising because we expect no difference between Dirac and Majorana neutrinos in this case. To clarify this paradox we study a single generation of massless neutrinos. The equations presented in this section will also be used later to study particle-antiparticle coherence.

A. Standard two-point correlators and kinetic equations

In the Weyl case, the momentum decomposition of the neutrino field looks the same as for the Dirac case [Eq. (9)]. However, because a Weyl fermion has only two degrees of freedom the field mode $\mathbf{p}$ does not carry a spin index,

$$\nu(t,\mathbf{p}) = a(t,\mathbf{p})u_-(\mathbf{p}) + b^\dagger(t,-\mathbf{p})v_+(-\mathbf{p}). \tag{45}$$

It is automatically left-chiral because the right-chiral components of the chiral spinors $u_-(\mathbf{p})$ and $v_+(-\mathbf{p})$ vanish in the massless limit; see Appendix A.

If we require lepton-number conservation then the only correlators that we can define are

$$(2\pi)^3\delta(\mathbf{p}-\mathbf{k})\rho_{--}(\mathbf{p}) = \langle a^\dagger(\mathbf{k})a(\mathbf{p})\rangle, \tag{46a}$$

$$(2\pi)^3\delta(\mathbf{p}-\mathbf{k})\bar{\rho}_{++}(\mathbf{p}) = \langle b^\dagger(-\mathbf{k})b(-\mathbf{p})\rangle, \tag{46b}$$

$$(2\pi)^3\delta(\mathbf{p}-\mathbf{k})\kappa_{-+}(\mathbf{p}) = \langle b(-\mathbf{k})a(\mathbf{p})\rangle, \tag{46c}$$

$$(2\pi)^3\delta(\mathbf{p}-\mathbf{k})\kappa^\dagger_{+-}(\mathbf{p}) = \langle a^\dagger(\mathbf{p})b^\dagger(-\mathbf{k})\rangle. \tag{46d}$$

Note that we keep helicity indices in these definitions to distinguish the lepton-number-conserving correlators from the lepton-number-violating ones, which we introduce below.We can extract the explicit form of the kinetic equations for these correlators from Eq. (2),

$$i\dot{\rho}_{--} = \mathsf{H}^{\nu\nu}_{--}\rho_{--} - \rho_{--}\mathsf{H}^{\nu\nu}_{--} + \mathsf{H}^{\nu\bar{\nu}}_{-+}\kappa^{\dagger}_{+-} - \kappa_{-+}\mathsf{H}^{\bar{\nu}\nu}_{+-}, \tag{47a}$$

$$i\dot{\bar{\rho}}_{++} = \mathsf{H}^{\bar{\nu}\bar{\nu}}_{++}\bar{\rho}_{++} - \bar{\rho}_{++}\mathsf{H}^{\bar{\nu}\bar{\nu}}_{++} - \mathsf{H}^{\bar{\nu}\nu}_{+-}\kappa_{-+} + \kappa^{\dagger}_{+-}\mathsf{H}^{\nu\bar{\nu}}_{-+}, \tag{47b}$$

$$i\dot{\kappa}_{-+} = \mathsf{H}^{\nu\nu}_{--}\kappa_{-+} - \kappa_{-+}\mathsf{H}^{\bar{\nu}\bar{\nu}}_{++} - \mathsf{H}^{\nu\bar{\nu}}_{-+}\bar{\rho}_{++} - \rho_{--}\mathsf{H}^{\nu\bar{\nu}}_{-+} + \mathsf{H}^{\nu\bar{\nu}}_{-+}, \tag{47c}$$

where we omit the arguments $(t, \mathbf{p})$, which are common to all the functions, to shorten the notation. Note that for a single neutrino generation the first two terms in Eqs. (47a) and (47b) cancel each other and we have retained them only to keep the resemblance with the general form of the kinetic equations.

A peculiar feature of Eq. (47c) is that κ, i.e. the coherence between $|00\rangle$ and $|11\rangle$ states, is automatically induced provided that the mean-field Hamiltonian H has nonzero off-diagonals. The off-diagonals can be induced even if all neutrino two-point functions are zero initially by, for instance, a transverse neutron current.

The explicit form of the mean-field Hamiltonian can be obtained from Eq. (29) by setting the masses to zero,

$$\mathsf{H}^{\nu\nu}_{--}(\mathbf{p}) = E + V^0 - \hat{\mathbf{p}}\mathbf{V}, \tag{48a}$$

$$\mathsf{H}^{\nu\bar{\nu}}_{-+}(\mathbf{p}) = -\hat{\boldsymbol{\epsilon}}^{*}\mathbf{V}, \tag{48b}$$

$$\mathsf{H}^{\bar{\nu}\nu}_{+-}(\mathbf{p}) = -\hat{\boldsymbol{\epsilon}}\mathbf{V}, \tag{48c}$$

$$\mathsf{H}^{\bar{\nu}\bar{\nu}}_{++}(\mathbf{p}) = -E + V^0 + \hat{\mathbf{p}}\mathbf{V}, \tag{48d}$$

where $E = |\mathbf{p}|$. Note that the $\hat{\mathbf{p}}\mathbf{V}$ term in Eq. (48a) accounts for the enhancement (suppression) of the mean-field potential for the matter flowing antiparallel (parallel) to the neutrino momentum. This has been pointed out in Ref. [34].

It remains to express the neutrino current I^{μ} in terms of the density matrices and pair correlations. For its time component we obtain from Eq. (27), $I^0 = \int_{\mathbf{p}} \ell$, where $\ell(t,\mathbf{p}) \equiv \rho(t,\mathbf{p}) - \bar{\rho}(t,-\mathbf{p})$ has the meaning of lepton number in mode $\mathbf{p}$. For the spatial components we find

$$\mathbf{I} = \int_{\mathbf{p}} [\hat{\mathbf{p}}\ell + \hat{\boldsymbol{\epsilon}}\kappa + \hat{\boldsymbol{\epsilon}}^{*}\kappa^{\dagger}], \tag{49}$$

which coincides with the result of Ref. [44].

B. Lepton-number-violating correlators and kinetic equations

If we allow for $\langle\nu\nu\rangle$ and $\langle\bar{\nu}\,\bar{\nu}\rangle$ contractions then, similarly to the Majorana case, the mean-field Hamiltonian receives contributions of the type (40). Because Weyl fields satisfy the condition $P_L\nu = \nu$ we can rewrite Eq. (40) as

$$\mathcal{H}^{\nu\nu}_{\mathrm{mf}} = \sqrt{2}G_{\mathrm{F}} \sum ([\bar{\nu}\nu^c]I^L + [\bar{\nu^c}\nu]I^R) \tag{50}$$

(see Sec. III and Appendix B for more details), where now

$$I^L = \langle\bar{\nu^c}\nu\rangle \quad \text{and} \quad I^R = \langle\bar{\nu}\nu^c\rangle. \tag{51}$$

As has been mentioned above $\bar{\nu^c}\nu$ and $\bar{\nu}\nu^c$ have the structure of the Majorana mass term, which is known to violate lepton number. Therefore, we expect that also for the Weyl neutrino the mean-field Hamiltonian [Eq. (50)] leads to lepton-number violation. However, for Weyl neutrinos the inclusion of these additional terms is somewhat artificial because, as we show below, these correlations are not produced if they are zero initially. They are considered here to better understand the Majorana case, where they are naturally produced by the lepton-number-violating interactions.

The contribution of Eq. (50) to the mean-field Hamiltonian is given by

$$H_{\mathrm{mf}} = \int_{\mathbf{p}} [a^{\dagger}(\mathbf{p})\Gamma^{\nu\bar{\nu}}_{--}(\mathbf{p})a^{\dagger}(-\mathbf{p}) + b^{\dagger}(\mathbf{p})\Gamma^{\nu\bar{\nu}}_{++}(\mathbf{p})b^{\dagger}(-\mathbf{p}) + a(-\mathbf{p})\Gamma^{\bar{\nu}\nu}_{--}(\mathbf{p})a(\mathbf{p}) + b(-\mathbf{p})\Gamma^{\bar{\nu}\nu}_{++}(\mathbf{p})b(\mathbf{p})], \tag{52}$$

and strongly resembles the mean-field Hamiltonian of Majorana neutrinos [Eq. (37)]. From the structure of Eq. (52) it is evident that, as expected, it leads to the violation of lepton number. To take this into account we are forced to introduce the following lepton-number-violating correlators:

$$(2\pi)^3\delta(\mathbf{p}-\mathbf{k})\kappa_{--}(\mathbf{p}) = \langle a(-\mathbf{k})a(\mathbf{p})\rangle, \tag{53a}$$

$$(2\pi)^3\delta(\mathbf{p}-\mathbf{k})\kappa_{++}(\mathbf{p}) = \langle b(-\mathbf{k})b(\mathbf{p})\rangle, \tag{53b}$$

$$(2\pi)^3\delta(\mathbf{p}-\mathbf{k})\kappa^{\dagger}_{--}(\mathbf{p}) = \langle a^{\dagger}(\mathbf{p})a^{\dagger}(-\mathbf{k})\rangle, \tag{53c}$$

$$(2\pi)^3\delta(\mathbf{p}-\mathbf{k})\kappa^{\dagger}_{++}(\mathbf{p}) = \langle b^{\dagger}(\mathbf{p})b^{\dagger}(-\mathbf{k})\rangle, \tag{53d}$$

which also resemble the Majorana definitions (32). These correlators are dictated by the structure of the Hamiltonian (52) and are the only lepton-number-violating correlators we consider in this section. If we wanted to consider all other possible correlators we would be back to the Majorana case with zero neutrino masses.

The lepton-number-violating correlators contribute to the dynamics of the lepton-number-conserving ones,

$$i\dot{\rho}_{--} = \cdots + (\Gamma^{\nu\bar{\nu}}_{--} - [\Gamma^{\nu\bar{\nu}}_{--}]^T)\kappa^{\dagger}_{--} - \kappa_{--}(\Gamma^{\bar{\nu}\nu}_{--} - [\Gamma^{\bar{\nu}\nu}_{--}]^T), \tag{54a}$$

$$i\dot{\bar{\rho}}_{++} = \cdots - (\Gamma^{\bar{\nu}\nu}_{++} - [\Gamma^{\bar{\nu}\nu}_{++}]^T)\kappa_{++} + \kappa^{\dagger}_{++}(\Gamma^{\nu\bar{\nu}}_{++} - [\Gamma^{\nu\bar{\nu}}_{++}]^T), \tag{54b}$$

where ellipses denote terms on the right-hand side of Eq. (47), and the superscript T stands for transposition of the flavor and helicity indices, as well as inversion of the momentum. Comparing Eq. (54) with Eqs. (38b) and (38c) we see that we automatically recover the "Majorana" definitions of the Hamiltonian matrix. Note that to avoid confusion with the definitions of the elements of the mean-field Hamiltonian, which are different for Dirac and Majorana neutrinos, we write the right-hand side of Eq. (54) directly in terms of spinor contractions defined in Eq. (12). The dynamics of κ_{-+} [see Eq. (47c)] does not receive any corrections. The reason is that the components of the mean-field Hamiltonian needed to form the right spin combination with the lepton-number-violating correlators in Eq. (47c) are zero for Weyl neutrinos. The kinetic equations for the lepton-number-violating pair correlations read

$$i\dot{\kappa}_{--} = \Gamma^{\nu\nu}_{--}\kappa_{--} - \kappa_{--}(-[\Gamma^{\nu\nu}_{--}]^T) - \rho_{--}(\Gamma^{\nu\bar{\nu}}_{--} - [\Gamma^{\nu\bar{\nu}}_{--}]^T) - (\Gamma^{\nu\bar{\nu}}_{--} - [\Gamma^{\nu\bar{\nu}}_{--}]^T)[\rho_{--}]^T + (\Gamma^{\nu\bar{\nu}}_{--} - [\Gamma^{\nu\bar{\nu}}_{--}]^T), \tag{55a}$$

$$i\dot{\kappa}_{++} = (-[\Gamma^{\bar{\nu}\bar{\nu}}_{++}]^T)\kappa_{++} - \kappa_{++}\Gamma^{\bar{\nu}\bar{\nu}}_{++} - (\Gamma^{\nu\bar{\nu}}_{++} - [\Gamma^{\nu\bar{\nu}}_{++}]^T)\bar{\rho}_{++} - [\bar{\rho}_{++}]^T(\Gamma^{\nu\bar{\nu}}_{++} - [\Gamma^{\nu\bar{\nu}}_{++}]^T) + (\Gamma^{\nu\bar{\nu}}_{++} - [\Gamma^{\nu\bar{\nu}}_{++}]^T). \tag{55b}$$

Their form can be guessed from Eq. (47c) by replacing components of the mean-field Hamiltonian with their "Majorana" counterparts, taking into account that $\Gamma^{\bar{\nu}\bar{\nu}}_{--} = \Gamma^{\nu\nu}_{++} = 0$, and replacing $\bar{\rho}_{--}$ by $[\rho_{--}]^T$ as well as ρ_{++} by $[\bar{\rho}_{++}]^T$.

Using the explicit form of the chiral spinors (see Appendix A), we obtain

$$\Gamma^{\nu\bar{\nu}}_{--}(\mathbf{p}) = +e^{+i\phi}V^L, \tag{56a}$$

$$\Gamma^{\nu\bar{\nu}}_{++}(\mathbf{p}) = -e^{-i\phi}V^R, \tag{56b}$$

$$\Gamma^{\bar{\nu}\nu}_{--}(\mathbf{p}) = +e^{-i\phi}V^R, \tag{56c}$$

$$\Gamma^{\bar{\nu}\nu}_{++}(\mathbf{p}) = -e^{+i\phi}V^L, \tag{56d}$$

where $V^{L(R)} = \sqrt{2}G_F I^{L(R)}$ are defined analogously to Eq. (35). Let us now recall that I^L and I^R are produced only by neutrino self-interactions and are proportional to the lepton-number-violating pair correlations,

$$I^L = \int_{\mathbf{p}} e^{-i\phi}[\kappa_{--} - \kappa^\dagger_{++}], \tag{57}$$

and a similar expression for I^R. Thus, if the lepton-number-violating correlators are zero initially, then the components in Eq. (56) are zero and κ_{--} and κ_{++} remain zero in the course of the system's evolution. For this reason for Weyl neutrinos the inclusion of lepton-number-violating correlators is rather artificial because they could only exist if they were put in by hand initially.

This observation explains why similar contributions do not vanish for Majorana neutrinos in the limit of zero neutrino masses. While such lepton-number-violating correlators can be introduced by hand as an initial condition, they can dynamically evolve only in the presence of a nonvanishing Majorana mass.

V. ELECTROMAGNETIC BACKGROUND FIELDS

A supernova environment is characterized not only by matter currents, but also by strong magnetic fields. Electromagnetic fields polarize both background media and the vacuum. Although neutrinos do not couple directly to the electromagnetic fields, they feel the induced polarization. The coupling to a polarized background medium has been treated in the previous sections. We now turn to the interaction with the vacuum polarization.

The effect of vacuum polarization is described by electromagnetic form factors. The most prominent examples, the magnetic and electric dipole moments, are inevitable for massive neutrinos and have to be included to obtain consistent evolution equations linear in the neutrino mass. The main effects of electromagnetic fields are spin and spin-flavor oscillations, which can be significant. We treat Dirac and Majorana neutrinos separately.

A. General vertex structure

The coupling of neutrinos to an external vector potential A^μ can be written as an effective vertex $\mathcal{H}^{\rm em} = A_\mu\bar{\nu}\Gamma^\mu\nu$, where Γ^μ contains all irreducible combinations of Lorentz vectors and pseudovectors generated by external momenta and Dirac matrices. Neglecting a hypothetical minicharge, in coordinate space the most general Hamiltonian density can be reduced to

$$\mathcal{H}^{\rm em} = \frac{1}{2}F_{\mu\nu}\bar{\nu}_i(f^{ij}_M\sigma^{\mu\nu} + if^{ij}_E\sigma^{\mu\nu}\gamma_5)\nu_j + \partial^\nu F_{\mu\nu}\bar{\nu}_i(f^{ij}_Q\gamma^\mu + f^{ij}_A\gamma^\mu\gamma_5)\nu_j, \tag{58}$$

where the electromagnetic field-strength tensor is defined as usual, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$. The form factors are f_M (magnetic), f_E (electric), f_Q (reduced charge [28]), and f_A (anapole). The form factors carry generation indices. Diagonal elements describe the usual electromagnetic properties of a neutrino in the mass basis, and reduce to electromagnetic *moments* in the static limit. The off-diagonal elements describe transitions between neutrinos of different masses. Some components of the Hamiltonian matrix have been calculated in [28,56].

Maxwell's equations tell us that $\partial_\nu F^{\mu\nu} = -J^\mu_{\rm em}$, where $J^\mu_{\rm em}$ is some charged matter background that sources electromagnetic fields. In supernovae, the sources are electrons and protons. In the Standard Model with massless neutrinos, the value for the anapole moment has to be $f_A = -f_Q$ to reproduce the left-chiral form of the interaction. For models with neutrino masses, the Hamiltonian matrix might obtain contributions that are not purely left-chiral, but we assume that these are always small so that we can neglect them. The charge and anapole form factors then only yield radiative corrections to the left-chiral tree-level coupling in Eq. (16). We neglect these moments because we are not interested in corrections to leading-order effects. However, for completeness, we give the spinor contractions for right-chiral currents in Appendix C.

B. Dipole moments of Dirac neutrinos

To study the dipole moments, we first turn to the somewhat simpler case of Dirac neutrinos. A Dirac neutrino has diagonal magnetic and electric moments. Because we assume neutrinos to carry no charge, $\mu = f_M(0)$ is defined as the magnetic moment and $\epsilon = f_E(0)$ as the electric dipole moment [28]. In the minimal extension of the Standard Model, the magnetic moments are found to be [57]

$$\mu_{ij} = \frac{3e\sqrt{2}G_{\mathrm{F}}(m_i + m_j)}{2(4\pi)^2}\left(\delta_{ij} - \frac{m_\tau^2}{2m_W^2}\mathcal{F}_{ij}\right), \tag{59a}$$

$$\epsilon_{ij} = i\frac{3e\sqrt{2}G_{\mathrm{F}}}{2(4\pi)^2}(m_i - m_j)\left(\frac{m_\tau^2}{2m_W^2}\right)\mathcal{F}_{ij}, \tag{59b}$$

$$\mathcal{F}_{ij} = \sum_{\alpha=e,\mu,\tau} U^\dagger_{i\alpha}\left(\frac{m_\alpha}{m_\tau}\right)^2 U_{\alpha j}, \tag{59c}$$

where m_τ is the tau mass. Note that the electric dipole moment does not have a diagonal component because it would violate CP [28], and that the transition electric dipole moment carries a phase relative to the transition magnetic dipole moment. Numerically, the above expressions yield for the diagonal magnetic moments

$$\mu_{ii} \simeq 3.2 \times 10^{-19}\left(\frac{m_i}{\mathrm{eV}}\right)\mu_{\mathrm{B}}, \tag{60}$$

where μ_{B} is the Bohr magneton. The transition moments are

$$\mu_{ij} \simeq -3.9 \times 10^{-23}\mathcal{F}_{ij}\left(\frac{m_i + m_j}{\mathrm{eV}}\right)\mu_{\mathrm{B}}, \tag{61a}$$

$$\epsilon_{ij} \simeq 3.9i \times 10^{-23}\mathcal{F}_{ij}\left(\frac{m_i - m_j}{\mathrm{eV}}\right)\mu_{\mathrm{B}}. \tag{61b}$$

Note that the transition moments are much smaller than the diagonal moments due to Glashow-Iliopoulos-Maiani suppression.

C. Hamiltonian matrix for Dirac neutrinos

We treat electromagnetic effects on the same footing as background matter. To this end, we have to evaluate the components of the Hamiltonian matrix, which, for Dirac neutrinos, are equal to the spinor contractions in Eq. (12). For the contractions, we need to evaluate the Lorentz structure of the vertex in Eq. (58).

Considering only magnetic and electric form factors, the Hamiltonian reduces to $\frac{1}{2}F_{\mu\nu}\bar{\nu}_i(f_M^{ij}\sigma^{\mu\nu} + if_E^{ij}\sigma^{\mu\nu}\gamma_5)\nu_j$, which depends on the electric and magnetic fields, $\mathbf{E}$ and $\mathbf{B}$, through $F^{\mu\nu}$. The Lorentz structure can be decomposed into the contractions $(i\gamma^0\boldsymbol{\gamma})_{ij,sh}$ and $(\gamma^0\boldsymbol{\gamma}\gamma_5)_{ij,sh}$, the latter appearing through the identity $\epsilon^{abc}\gamma^0\gamma^c\gamma_5 = \sigma^{ab}$ with spatial indices $a, b, c = 1, 2$ or 3, and the asymmetric tensor ϵ^{abc}. These contractions are three-vectors that are contracted with the electric and magnetic fields. We calculate the contractions in momentum space.

Explicitly, the coupling of the magnetic field through the magnetic form factor (superscript μB) has the structures

$$\mathsf{H}^{\mu\mathrm{B}\nu\nu}_{ij,sh} = -(\gamma^0\boldsymbol{\gamma}\gamma_5)^{\nu\nu}_{ij,sh} f_M^{ij}(q^2)\mathbf{B}, \tag{62a}$$

$$\mathsf{H}^{\mu\mathrm{B}\nu\bar{\nu}}_{ij,sh} = -(\gamma^0\boldsymbol{\gamma}\gamma_5)^{\nu\bar{\nu}}_{ij,sh} f_M^{ij}(l^2)\mathbf{B}, \tag{62b}$$

$$\mathsf{H}^{\mu\mathrm{B}\bar{\nu}\nu}_{ij,sh} = -(\gamma^0\boldsymbol{\gamma}\gamma_5)^{\bar{\nu}\nu}_{ij,sh} f_M^{ij}(l^2)\mathbf{B}, \tag{62c}$$

$$\mathsf{H}^{\mu\mathrm{B}\bar{\nu}\bar{\nu}}_{ij,sh} = -(\gamma^0\boldsymbol{\gamma}\gamma_5)^{\bar{\nu}\bar{\nu}}_{ij,sh} f_M^{ij}(q^2)\mathbf{B}, \tag{62d}$$

where we identify $\mathsf{H}^{\nu\nu} = \Gamma^{\nu\nu}$, $\mathsf{H}^{\nu\bar{\nu}} = \Gamma^{\nu\bar{\nu}}$, etc., and the minus sign in the metric $g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ has already been taken care of. In Eq. (62), the form factors still depend on the momentum transfer. For the $\nu\nu$ and $\bar{\nu}\bar{\nu}$ components, the form factors contain $q^\mu = p^\mu_{\mathrm{out}} - p^\mu_{\mathrm{in}}$, where $q^\mu \to 0$ in the forward-scattering limit. These components are then proportional to the dipole moments. For the neutrino-antineutrino components of the H matrices, the argument of the form factor contains l^2 with $l^\mu = p^\mu_{\mathrm{out}} + p^\mu_{\mathrm{in}}$, the sum of neutrino and antineutrino momenta. In the forward-scattering limit this reduces to $l^2 = (2E)^2$, and the dependence of the form factors on the four-momentum is important.

The coupling of the magnetic field to the electric form factor (superscript ϵB) is

$$\mathsf{H}^{\epsilon\mathrm{B}\nu\nu}_{ij,sh} = -(i\gamma^0\boldsymbol{\gamma})^{\nu\nu}_{ij,sh} f_E^{ij}(q^2)\mathbf{B}, \tag{63a}$$

$$\mathsf{H}^{\epsilon\mathrm{B}\nu\bar{\nu}}_{ij,sh} = -(i\gamma^0\boldsymbol{\gamma})^{\nu\bar{\nu}}_{ij,sh} f_E^{ij}(l^2)\mathbf{B}, \tag{63b}$$

$$\mathsf{H}^{\epsilon\mathrm{B}\bar{\nu}\nu}_{ij,sh} = -(i\gamma^0\boldsymbol{\gamma})^{\bar{\nu}\nu}_{ij,sh} f_E^{ij}(l^2)\mathbf{B}, \tag{63c}$$

$$\mathsf{H}^{\epsilon\mathrm{B}\bar{\nu}\bar{\nu}}_{ij,sh} = -(i\gamma^0\boldsymbol{\gamma})^{\bar{\nu}\bar{\nu}}_{ij,sh} f_E^{ij}(q^2)\mathbf{B}. \tag{63d}$$

The coupling of an electric field to the magnetic form factor is

$$\mathsf{H}^{\mu\mathrm{E}\nu\nu}_{ij,sh} = (i\gamma^0\boldsymbol{\gamma})^{\nu\nu}_{ij,sh} f^{ij}_M(q^2)\mathbf{E}, \tag{64a}$$

$$\mathsf{H}^{\mu\mathrm{E}\nu\bar\nu}_{ij,sh} = (i\gamma^0\boldsymbol{\gamma})^{\nu\bar\nu}_{ij,sh} f^{ij}_M(l^2)\mathbf{E}, \tag{64b}$$

$$\mathsf{H}^{\mu\mathrm{E}\bar\nu\nu}_{ij,sh} = (i\gamma^0\boldsymbol{\gamma})^{\bar\nu\nu}_{ij,sh} f^{ij}_M(l^2)\mathbf{E}, \tag{64c}$$

$$\mathsf{H}^{\mu\mathrm{E}\bar\nu\bar\nu}_{ij,sh} = (i\gamma^0\boldsymbol{\gamma})^{\bar\nu\bar\nu}_{ij,sh} f^{ij}_M(q^2)\mathbf{E}, \tag{64d}$$

which is indicated by μE, and to the electric form factor, ϵE,

$$\mathsf{H}^{\epsilon\mathrm{E}\nu\nu}_{ij,sh} = -(\gamma^0\boldsymbol{\gamma}\gamma_5)^{\nu\nu}_{ij,sh} f^{ij}_E(q^2)\mathbf{E}, \tag{65a}$$

$$\mathsf{H}^{\epsilon\mathrm{E}\nu\bar\nu}_{ij,sh} = -(\gamma^0\boldsymbol{\gamma}\gamma_5)^{\nu\bar\nu}_{ij,sh} f^{ij}_E(l^2)\mathbf{E}, \tag{65b}$$

$$\mathsf{H}^{\epsilon\mathrm{E}\bar\nu\nu}_{ij,sh} = -(\gamma^0\boldsymbol{\gamma}\gamma_5)^{\bar\nu\nu}_{ij,sh} f^{ij}_E(l^2)\mathbf{E}, \tag{65c}$$

$$\mathsf{H}^{\epsilon\mathrm{E}\bar\nu\bar\nu}_{ij,sh} = -(\gamma^0\boldsymbol{\gamma}\gamma_5)^{\bar\nu\bar\nu}_{ij,sh} f^{ij}_E(q^2)\mathbf{E}. \tag{65d}$$

One can see that a magnetic field couples to both, the electric and the magnetic form factor. Also electric fields couple to both form factors. This can be understood as follows. In the neutrino rest frame, the magnetic field only couples to the magnetic dipole moment, and the electric field only couples to the electric dipole moment (if any), as suggested by the nomenclature. Lorentz covariance then demands that both electric and magnetic fields couple to the magnetic form factor in a system where the neutrino moves with nonzero velocity. A moving neutrino also exhibits spin precession in a pure electric field through its magnetic moment [58].

The Lorentz structure of Eqs. (62)–(65) can now be readily calculated. In contrast to the previous sections, we neglect all contributions proportional to the mass since the magnetic and electric form factors are small and, in the models considered here, proportional to the neutrino mass already. The $(\gamma^0\boldsymbol{\gamma}\gamma_5)$ components are

$$(\gamma^0\boldsymbol{\gamma}\gamma_5)^{\nu\nu}_{ij,sh} \approx \begin{pmatrix} 0 & e^{+i\phi}\hat{\boldsymbol{\epsilon}}^* \\ e^{-i\phi}\hat{\boldsymbol{\epsilon}} & 0 \end{pmatrix}, \tag{66a}$$

$$(\gamma^0\boldsymbol{\gamma}\gamma_5)^{\nu\bar\nu}_{ij,sh} \approx \begin{pmatrix} -e^{+i\phi}\hat{\mathbf{p}} & 0 \\ 0 & -e^{-i\phi}\hat{\mathbf{p}} \end{pmatrix}, \tag{66b}$$

$$(\gamma^0\boldsymbol{\gamma}\gamma_5)^{\bar\nu\nu}_{ij,sh} \approx \begin{pmatrix} -e^{-i\phi}\hat{\mathbf{p}} & 0 \\ 0 & -e^{+i\phi}\hat{\mathbf{p}} \end{pmatrix}, \tag{66c}$$

$$(\gamma^0\boldsymbol{\gamma}\gamma_5)^{\bar\nu\bar\nu}_{ij,sh} \approx \begin{pmatrix} 0 & -e^{-i\phi}\hat{\boldsymbol{\epsilon}}^* \\ -e^{+i\phi}\hat{\boldsymbol{\epsilon}} & 0 \end{pmatrix}. \tag{66d}$$

The remaining Lorentz structures are of the form $(i\gamma^0\boldsymbol{\gamma})$. They read

$$(i\gamma^0\boldsymbol{\gamma})^{\nu\nu}_{ij,sh} \approx \begin{pmatrix} 0 & ie^{+i\phi}\hat{\boldsymbol{\epsilon}}^* \\ -ie^{-i\phi}\hat{\boldsymbol{\epsilon}} & 0 \end{pmatrix}, \tag{67a}$$

$$(i\gamma^0\boldsymbol{\gamma})^{\nu\bar\nu}_{ij,sh} \approx \begin{pmatrix} -ie^{+i\phi}\hat{\mathbf{p}} & 0 \\ 0 & ie^{-i\phi}\hat{\mathbf{p}} \end{pmatrix}, \tag{67b}$$

$$(i\gamma^0\boldsymbol{\gamma})^{\bar\nu\nu}_{ij,sh} \approx \begin{pmatrix} ie^{-i\phi}\hat{\mathbf{p}} & 0 \\ 0 & -ie^{+i\phi}\hat{\mathbf{p}} \end{pmatrix}, \tag{67c}$$

$$(i\gamma^0\boldsymbol{\gamma})^{\bar\nu\bar\nu}_{ij,sh} \approx \begin{pmatrix} 0 & ie^{-i\phi}\hat{\boldsymbol{\epsilon}}^* \\ -ie^{+i\phi}\hat{\boldsymbol{\epsilon}} & 0 \end{pmatrix}. \tag{67d}$$

To this level of approximation, the $\nu\bar\nu$ and $\bar\nu\nu$ components are diagonal in helicity space, i.e., electric and magnetic fields mainly couple spin-0 neutrino-antineutrino pairs. Because the diagonal is proportional to $\hat{\mathbf{p}}$, the relevant field components are those parallel to the momentum of the neutrinos. The $\nu\nu$ and $\bar\nu\bar\nu$ components are off-diagonal in helicity space. The dominant effect of magnetic and electric fields on neutrinos and antineutrinos is spin precession. Here the transverse components of the electromagnetic fields contribute. The longitudinal components enter on the diagonals in the next order of the expansion in m/E and are therefore omitted.

D. Dipole moments of Majorana neutrinos

For Majorana neutrinos, electromagnetic transitions always contain two contributions, e.g.,

$$\langle \nu_{\mathbf{p}_{\mathrm{out}}}|\mathcal{H}^{\mathrm{em}}|\nu_{\mathbf{p}_{\mathrm{in}}}\rangle = A_\mu(\bar u_{\mathbf{p}_{\mathrm{out}}}\Gamma^\mu u_{\mathbf{p}_{\mathrm{in}}} - \bar v_{\mathbf{p}_{\mathrm{in}}}\Gamma^\mu v_{\mathbf{p}_{\mathrm{out}}}). \tag{68}$$

This difference of two amplitudes leads to the cancellation of all the diagonal moments except for the anapole moment [28]. This can also be understood by noting that the last two terms (including the minus sign) in Eq. (68) are charge conjugates of each other. Because the Lorentz structure of the magnetic, electric, and charge form factors are C-odd the combination vanishes. The Lorentz structure of the anapole moment is C-even and does not cancel.

Because the magnetic moment of the Majorana neutrino vanishes, it does not couple directly to a magnetic field. However, magnetic fields polarize the background medium, and this effect does lead to helicity oscillations; see Sec. III.

Electromagnetic moments of neutrinos depend on the details of the mechanism that creates the neutrino mass. When neglecting the model-dependent amplitudes, one can compare the moments of Dirac and Majorana neutrinos. The main differences are that the Majorana amplitudes contain Majorana PMNS matrices, which may contain more phases than Dirac PMNS matrices, and that

Eq. (68) has to be taken into account for Majorana neutrinos.

After these adjustments, the off-diagonal form factors of Majorana neutrinos can be obtained from Eq. (59a). One finds that they depend on the relative CP phases of two neutrino species [59]. The relative phase can either be equal or opposite, i.e., the ratio is ± 1. For neutrinos with equal CP phases, the magnetic transition moments vanish [59], while for opposite CP phase the magnetic transition moments are nonzero and can be obtained from Eq. (59a) by substituting $\mathcal{F}_{ij}$ with $2i\mathrm{Im}\mathcal{F}_{ij}$.

For *electric* dipole moments, the role of the CP phases is inverted. Opposite CP phases force the electric transition moments to vanish, while for equal CP phases the electric transition moments are nonzero and are obtained by substituting $\mathcal{F}_{ij}$ with $2\mathrm{Re}\mathcal{F}_{ij}$ [59] in Eq. (59b).

E. Hamiltonian matrix for Majorana neutrinos

The density matrix formalism naturally reproduces the results for the electromagnetic moments discussed in the last section. Similarly to Eq. (68), each component of the Hamiltonian matrix has two contributions from Γ contractions, e.g., $\mathsf{H}^{\nu\nu}_{ij,sh}(\mathbf{p}) = \Gamma^{\nu\nu}_{ij,sh}(\mathbf{p}) - \Gamma^{\bar{\nu}\bar{\nu}}_{ji,hs}(-\mathbf{p})$; see Eq. (38). The spinor contractions $\Gamma^{\nu\nu}$ and $\Gamma^{\bar{\nu}\bar{\nu}}$ have the same structure as for Dirac neutrinos; see Eq. (12). Again neglecting the model dependence, the only difference is that the Dirac PMNS matrix has to be replaced by a Majorana PMNS matrix. For example, a magnetic field coupling to a Majorana neutrino through the magnetic form factor yields

$$\begin{aligned}\mathsf{H}^{\nu\nu}_{ij,-+} &= -[f^{ij}_M(q^2) - \mathrm{c.c.}]e^{+i\phi}\hat{\boldsymbol{\epsilon}}^*\mathbf{B} \\ &= -2i\mathrm{Im}[f^{ij}_M(q^2)]e^{+i\phi}\hat{\boldsymbol{\epsilon}}^*\mathbf{B}, \qquad (69)\end{aligned}$$

where we have used the Hermiticity of the form factors. In the static limit, $2i\mathrm{Im}[f^{ij}_M]$ is the magnetic moment of Majorana neutrinos [57]. It is zero for equal CP phases since $\mathcal{F}_{ij}$ becomes real. It is nonvanishing for opposite CP phases because $\mathcal{F}_{ij}$ becomes imaginary. An analogous argument holds for the electric dipole moment.

VI. HELICITY COHERENCE

In this section we neglect pair correlations and discuss helicity coherence effects. To separate the latter from the usual flavor coherence effects, we consider only one neutrino generation. Furthermore, for definiteness we assume that neutrinos are Dirac particles.

A. Order-of-magnitude estimate

Two different mean-field backgrounds cause spin oscillations and create spin coherence: matter and neutrino currents, and electromagnetic fields. However, it is not clear which of these is dominant in a supernova. In the following we perform a crude estimate.

For Dirac neutrinos without pair correlations, the kinetic equations of neutrinos and antineutrinos decouple, $i\dot{\rho} = [\mathsf{H}^{\nu\nu}, \rho]$ and $i\dot{\bar{\rho}} = [\mathsf{H}^{\bar{\nu}\bar{\nu}}, \bar{\rho}]$, and, for one family, we only have to look at a 2×2 subsystem of the full evolution equation. We start with a matter background with nonrelativistic velocity β, which flows orthogonal to the neutrino's momentum. The Hamiltonian matrix reads

$$\mathsf{H}^{\nu\nu} \approx V\begin{pmatrix} 1 & \frac{m}{2p}\beta \\ \frac{m}{2p}\beta & 0 \end{pmatrix}, \qquad (70)$$

where V is the usual matter potential. For instance for ν_μ or ν_τ it is given by $V = G_\mathrm{F} n_n/\sqrt{2}$, where n_n is the neutron density. We have omitted the neutrino kinetic energy because it is diagonal in helicity space, and for a single generation trivially cancels in the commutator. The dependence of the diagonal terms of the Hamiltonian on the parallel flux and the dependence of the off-diagonal terms on the orthogonal flux were discussed in Refs. [35,60].

In a derivation similar to the one that leads to Eq. (70), we obtain the 2×2 subsystem of the Hamiltonian matrix for a neutrino in a transverse magnetic field

$$\mathsf{H}^{\nu\nu} \approx -\mu B\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad (71)$$

(see Sec. V for more details). Spin coherence is instigated by the off-diagonals of Eqs. (70) and (71), and to find the relative importance of the matter and magnetic contributions it is sufficient to estimate their relative size. Typical magnetic fields in a supernova are of order 10^{12} G and much larger in magnetars. Using the standard value for the magnetic moment given in Eq. (60), and assuming a neutrino mass of 0.1 eV, we find for the contribution of the magnetic field $\mu B \sim 10^{-16}$ eV. For a typical neutron mass density 10^{12} g/cm^3, which corresponds to a number density $n_n \sim 10^4$ MeV^3, the matter potential is of the order of the neutrino mass, $V \sim 0.1$ eV. Thus, for a typical momentum $p \sim 30$ MeV we obtain $V\beta m/(2p) \sim 10^{-10}\beta$ eV. For maximal background velocities of 3000 km/s, $\beta \sim 0.01$, the matter contribution dominates. Surprisingly, the magnetic field is only important if the background moves very slowly, if the matter density has decreased sufficiently, or if the magnetic moment is enhanced.

Turning now to the density matrix, the size of the off-diagonal elements depends on the initial conditions and history of the evolution. To obtain a rough estimate, we can assume that the system has reached equilibrium and, hence, its previous evolution is irrelevant. In equilibrium, the system is in an eigenstate of the Hamiltonian, i.e., $\mathsf{H}^{\nu\nu}$ and ρ commute. This condition alone allows us to express the off-diagonals of the density matrix in terms of the diagonals and components of the Hamiltonian matrix,

$$\rho_{-+} = \frac{\mathsf{H}^{\nu\nu}_{-+}}{\mathsf{H}^{\nu\nu}_{--} - \mathsf{H}^{\nu\nu}_{++}} (\rho_{--} - \rho_{++}). \tag{72}$$

Keeping only the (dominant) matter contribution, Eq. (70), we find $\rho_{-+} = (\rho_{--} - \rho_{++}) m\beta/2p \sim m\beta/2p$. For $m \sim 0.1$ eV and a typical momentum $p \sim 30$ MeV this results in $\rho_{-+} \sim 10^{-11}$, where we have used $\beta \sim 0.01$.

The same result can be obtained by noting that if ρ and $\mathsf{H}^{\nu\nu}$ commute, they can be simultaneously diagonalized by a rotation that mixes positive- and negative-helicity states. The rotation angle is $\tan 2\vartheta = m\beta/p$. Considering e.g. the $\rho_{--} = 1$ eigenstate of the diagonalized Hamiltonian and rotating back to the basis where the Hamiltonian has the form (70) we find to leading order

$$\rho \sim \begin{pmatrix} 1 & \frac{m}{2p}\beta \\ \frac{m}{2p}\beta & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 10^{-11} \\ 10^{-11} & 0 \end{pmatrix}. \tag{73}$$

The corrections to the diagonals are not included in Eq. (73) because they are of the order of $\delta\rho_{--} \sim \delta\bar{\rho}_{++} \sim \rho^2_{-+} \sim 10^{-22}$ and are therefore negligibly small.

B. Resonant enhancement

For a magnetic field, the diagonal elements of the Hamiltonian matrix, Eq. (71), are zero for very relativistic neutrinos. This allows for the magnetic fields to completely flip the spin of a population of neutrinos. On the other hand, the diagonals of Eq. (70) are in general nonzero and suppress a complete conversion. In general, the matter contribution is given by

$$\mathsf{H}^{\nu\nu} \approx \begin{pmatrix} V^0 - V_\parallel & \frac{m}{2p} V_\perp \\ \frac{m}{2p} V_\perp & 0 \end{pmatrix} \tag{74}$$

[see Eq. (43)], where $V_\parallel \equiv \hat{\mathbf{p}}\mathbf{V}$ and $V_\perp \equiv \hat{\boldsymbol{\epsilon}}\mathbf{V}$ are components of the matter flux parallel and orthogonal to the neutrino momentum. Thus, if there are relativistic currents parallel to the momentum of the neutrino such that the diagonals vanish, Eq. (72) implies that a resonant enhancement of the spin conversion is possible. The possibility to generate the spin conversion by an orthogonal flux of matter, and the cancellation of the matter effect for relativistic matter moving along the direction of the neutrino momentum were first discussed in Refs. [35,60] on the basis of the Lorentz-covariant Bergmann-Michel-Telegdi equation. In Refs. [36,38] these effects have also been studied using the formalism of nonequilibrium quantum field theory. In the context of resonant leptogenesis the formation of flavor and helicity correlations in medium and the derivation of flavor-covariant transport equations able to account for helicity correlations has been discussed in Ref. [61].

For vanishing diagonals, Eq. (74) can be rotated into its diagonal form with a rotation angle $\vartheta = \pi/4$. In other words, mixing of the helicity states becomes maximal, similarly to the Mikheyev–Smirnov–Wolfenstein resonance mixing, and hence in equilibrium

$$\rho \sim \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}, \tag{75}$$

where we have again assumed that the system is in an eigenstate of the diagonalized Hamiltonian. Outside of the core, a supernova is far from equilibrium, but nonlinear feedback can enhance the spin-flipping processes [38].

Making use of Eq. (24), we can rewrite the resonance condition, $\mathsf{H}^{\nu\nu}_{--} - \mathsf{H}^{\nu\nu}_{++} = V^0 - V_\parallel = 0$, in the form [38]

$$Y_e + \frac{4}{3}\left(Y_\nu - \frac{V_\parallel}{2n_b}\right) = \frac{1}{3}, \tag{76}$$

where $Y_e \equiv n_e/n_B$ and $Y_\nu = (n_\nu - n_{\bar{\nu}})/n_B$ are the electron and neutrino asymmetry fractions respectively and n_B is the baryon number density, The resonance condition can potentially be fulfilled in or near the protoneutron star in a core-collapse supernova, or near the central region of a compact object merger; see Ref. [38] and references therein.

C. Lorentz covariance

Helicity coherence builds up only if the off-diagonal elements of the Hamiltonian matrix differ from zero. On the other hand, because the off-diagonals are proportional to the component of the matter flow orthogonal to the neutrino momentum, one can always find a frame where the off-diagonals vanish and no helicity coherence builds up. In other words, at first sight physical results seem to depend on the frame. This raises the question of Lorentz covariance of the kinetic equations.

To be specific, let us consider the following simple example. We have two identical observers moving with velocity β with respect to each other. In the frame of the first observer, the neutrino has momentum $\mathbf{p}$ along the z axis and the matter is at rest, i.e. $V_\parallel = V_\perp = 0$,

$$\mathsf{H}^{\nu\nu} \approx V \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{77}$$

Thus no helicity coherence builds up. In the frame of the second observer which moves with velocity β along the x axis the Hamiltonian is no longer diagonal,

$$\mathsf{H}^{\nu\nu} \approx \frac{V}{\gamma} \begin{pmatrix} 1 & \frac{m}{2p}\beta \\ \frac{m}{2p}\beta & 0 \end{pmatrix}, \tag{78}$$

and we expect helicity coherence to build up. Here γ is the usual Lorentz factor and V and p denote the potential and neutrino momentum in the frame of the first observer.

Do the Hamiltonian matrices (77) and (78) lead to different physical results? The answer is no, but to demonstrate this point we need to take into account that a helicity state is also not Lorentz invariant. Let the neutrino be in a state of definite helicity in the frame of the first observer, e.g. $|p\hat{\mathbf{z}},-\rangle$, where p is the absolute value of the neutrino momentum and $\hat{\mathbf{z}}$ is the unit vector along the z axis. The corresponding density matrix reads

$$\rho = |p\hat{\mathbf{z}},-\rangle\langle p\hat{\mathbf{z}},-| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{79}$$

The Hamiltonian matrix (77) and the density matrix commute and therefore the latter is constant in time. The boost to the frame of the second observer transforms $|p\hat{\mathbf{z}},-\rangle$ into a mixed helicity state with momentum $\mathbf{q}$,

$$|\psi\rangle = c_{\theta/2}|\mathbf{q},-\rangle - s_{\theta/2}|\mathbf{q},+\rangle, \tag{80}$$

where θ is the angle of Wigner rotation around $\hat{\mathbf{y}}$ with $\tan\theta = -m\beta/p$. Note that the rotation angle vanishes in the limit of zero neutrino mass which reflects chirality conservation. The density matrix develops off-diagonal elements,

$$\rho = |\psi\rangle\langle\psi| = \frac{1}{2}\begin{pmatrix} 1+c_\theta & -s_\theta \\ -s_\theta & 1-c_\theta \end{pmatrix}. \tag{81}$$

The Hamiltonian matrix and the density matrix again commute. In other words, the second observer sees a mixed helicity state which, as expected, is also time independent. This result reflects Lorentz covariance of the kinetic equations, the lesson being that one has to transform the initial conditions consistently to obtain covariant results.

Let us now consider this result from a slightly different viewpoint. In each frame, we can diagonalize the effective Hamiltonian by performing a Bogolyubov transformation that mixes annihilation (creation) operators of the positive- and negative-helicity states, $a_s \to c_\vartheta a_s + s_\vartheta a_{-s}$. In particular Eq. (78) is diagonalized by a Bogolyubov transformation with the angle $\tan 2\vartheta = m\beta/p$. This transformation brings the density matrix (81) back to the form (79). In other words, there is a connection between the Lorentz and Bogolyubov transformations. In particular, if in every frame we diagonalize the Hamiltonian then the transformed density matrix remains invariant under the boosts.

To summarize, as far as helicity coherence is concerned, both the Hamiltonian and the density matrix transform under Lorentz boosts in such a way that the kinetic equation is Lorentz covariant. We will rely on this result in the discussion of particle-antiparticle coherence whose Lorentz transformation properties are not as evident as for the helicity coherence.

VII. PARTICLE-ANTIPARTICLE COHERENCE

In this section we discuss particle-antiparticle coherence. In contrast to helicity coherence, which requires nonzero neutrino masses, and flavor coherence, which in addition to nonzero masses requires the existence of several neutrino generations, particle-antiparticle coherence arises already for a single massless neutrino generation. As has been discussed in the previous section, for a massless neutrino the only "natural" correlators are ρ_{--}, ρ_{++} and κ_{-+}. To shorten the notation in this section we suppress the spin indices.

A. Quantum-mechanical example

To clarify the meaning of the particle-antiparticle coherence, let us first study in more detail the simple quantum-mechanical example briefly discussed in the Introduction. We consider a system that can be in a linear combination of one of four pure states. These are i) the empty state $|00\rangle$ without particles; ii) the paired state $|11\rangle = a^\dagger(\mathbf{p})b^\dagger(-\mathbf{p})|00\rangle$, which contains a neutrino with momentum $\mathbf{p}$ and an antineutrino with momentum $-\mathbf{p}$; iii) the one neutrino state $|10\rangle$; and iv) the one antineutrino state $|01\rangle$. Note that in all these states the antineutrinos stream in the direction opposite to that of neutrinos. A general state can be expressed in terms of these four states, $|\psi\rangle = A_{00}|00\rangle + A_{11}|11\rangle + A_{10}|10\rangle + A_{01}|01\rangle$, where the coefficients A_{ij} are time dependent and normalized to unity, $|A_{00}|^2 + |A_{11}|^2 + |A_{10}|^2 + |A_{01}|^2 = 1$.

In analogy to Eq. (13) we write the Hamiltonian in the form

$$\begin{aligned} H = {} & a^\dagger(\mathbf{p})\mathsf{H}^{\nu\nu}a(\mathbf{p}) + a^\dagger(\mathbf{p})\mathsf{H}^{\nu\bar\nu}b^\dagger(-\mathbf{p}) \\ & + b(-\mathbf{p})\mathsf{H}^{\bar\nu\nu}a(\mathbf{p}) - b^\dagger(-\mathbf{p})\mathsf{H}^{\bar\nu\bar\nu}b(-\mathbf{p}). \end{aligned} \tag{82}$$

The Schrödinger equation for the coefficients A_{ij} then splits into three independent equations,

$$i\partial_t\begin{pmatrix} A_{00} \\ A_{11} \end{pmatrix} = \begin{pmatrix} 0 & \mathsf{H}^{\bar\nu\nu} \\ \mathsf{H}^{\nu\bar\nu} & \mathsf{H}^{\nu\nu} - \mathsf{H}^{\bar\nu\bar\nu} \end{pmatrix}\begin{pmatrix} A_{00} \\ A_{11} \end{pmatrix}, \tag{83a}$$

$$i\partial_t A_{10} = \mathsf{H}^{\nu\nu}A_{10}, \tag{83b}$$

$$i\partial_t A_{01} = -\mathsf{H}^{\bar\nu\bar\nu}A_{01}. \tag{83c}$$

Thus the evolution of the single-particle states completely decouples because a homogeneous background medium cannot mix states of different total momentum. On the other hand, the $|00\rangle$ and $|11\rangle$ states have zero momenta and therefore can be mixed by a homogeneous medium through the $\mathsf{H}^{\bar\nu\nu}$ term of the Hamiltonian. However, the $|00\rangle$ and

$|11\rangle$ states have different angular momentum. Hence, an anisotropic background medium, e.g. a transverse matter flux, is needed to absorb the angular momentum and to mix the two states.

To make the connection to the density matrix equations, we note that the number of neutrinos and antineutrinos is given by $\rho = |A_{11}|^2 + |A_{10}|^2$ and $\bar{\rho} = |A_{11}|^2 + |A_{01}|^2$ respectively. Their time evolution can be derived from Eq. (83) and takes the form expected from Eq. (47),

$$\dot{\rho} = -2\mathrm{Im}(\mathsf{H}^{\bar{\nu}\nu}\kappa), \tag{84a}$$

$$\dot{\bar{\rho}} = -2\mathrm{Im}(\mathsf{H}^{\bar{\nu}\nu}\kappa), \tag{84b}$$

if we identify $\kappa = A_{00}^* A_{11}$. Equation (83) also leads to an evolution equation for κ

$$i\dot{\kappa} = (\mathsf{H}^{\nu\nu} - \mathsf{H}^{\bar{\nu}\bar{\nu}})\kappa + \mathsf{H}^{\nu\bar{\nu}}(1 - \rho - \bar{\rho}), \tag{85}$$

which can be obtained by using the normalization of the state $|\psi\rangle$. Equation (85) is again consistent with Eq. (47) and coincides with the result of Ref. [44] in the one-flavor limit.

From these kinetic equations we can infer that while ρ and $\bar{\rho}$ are not separately conserved in the presence of nonzero κ, their difference is conserved [44]. Because $\rho(t, \mathbf{p})$ describes neutrinos with momentum $\mathbf{p}$ whereas $\bar{\rho}(t, \mathbf{p})$ describes antineutrinos with momentum $-\mathbf{p}$, the conservation of $\rho - \bar{\rho}$ implies that κ induces the production of neutrino-antineutrino pairs with opposite momentum.

The kinetic equation for κ describes a driven harmonic oscillator with frequency $\mathsf{H}^{\nu\nu} - \mathsf{H}^{\bar{\nu}\bar{\nu}} \sim 2E$. Hence κ oscillates with twice the neutrino energy as expected.

From the definition $\kappa = A_{00}^* A_{11}$ we see that nonzero particle-antiparticle coherence means that the system is not in an eigenstate of the unperturbed Hamiltonian, but instead in a mixture of the $|00\rangle$ and $|11\rangle$ states, i.e., in a squeezed state. Such states do not have a definite particle number. This observation clarifies the physical meaning of particle-antiparticle coherence.

B. Order-of-magnitude estimate

As a next step we perform an order-of-magnitude estimate of κ. For a single neutrino generation the extended density matrix reduces to a 2×2 matrix of the form [see Eq. (1)],

$$\mathsf{R} = \begin{pmatrix} \rho & \kappa \\ \kappa^\dagger & 1 - \bar{\rho} \end{pmatrix}, \tag{86}$$

where now ρ and $\bar{\rho}$ are real numbers and κ is a complex number. We again start with an example of a matter background with nonrelativistic velocity β, which flows orthogonal to the neutrino's momentum. Then, as follows from Eq. (48), the Hamiltonian matrix reads [see also Eq. (3)],

$$\mathsf{H} = E\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + V\begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix}. \tag{87}$$

Unlike for helicity coherence, the neutrino kinetic energy E no longer cancels out in the commutator.

Similarly to the case of spin coherence we can get a crude estimate of the κ magnitude by assuming that the system has reached equilibrium and hence $\dot{\kappa} = 0$. Equation (85) then gives

$$\kappa = -\frac{\mathsf{H}^{\nu\bar{\nu}}}{\mathsf{H}^{\nu\nu} - \mathsf{H}^{\bar{\nu}\bar{\nu}}}(1 - \rho - \bar{\rho}). \tag{88}$$

If we insert this result into Eq. (84) and use the Hermiticity of the Hamiltonian matrix, we see that indeed $\dot{\rho} = \dot{\bar{\rho}} = 0$.

Let us assume for a moment that the neutrino-neutrino interactions are small compared to the neutrino interaction with matter. For typical supernova parameters $V \sim 0.1$ eV and $E \sim 30$ MeV and we then find $V/E \sim 10^{-9}$. Thus to a good approximation $\mathsf{H}^{\nu\bar{\nu}}/(\mathsf{H}^{\nu\nu} - \mathsf{H}^{\bar{\nu}\bar{\nu}}) \sim V\beta/2E \sim 10^{-11}$, where we have used $\beta \sim 0.01$. Because typically $|1 - \rho - \bar{\rho}| \sim 1$ we conclude that the "natural" size of the particle-antiparticle coherence is $\kappa \sim 10^{-11}$.

The same result can be obtained by noting that in equilibrium R and H commute and can be simultaneously diagonalized by a Bogolyubov transformation that mixes neutrinos of momentum $\mathbf{p}$ with antineutrinos of momentum $-\mathbf{p}$. Under this transformation the creation and annihilation operators transform as $a(\mathbf{p}) \to e^{-i\phi/2} c_\vartheta a(\mathbf{p}) + e^{i\phi/2} s_\vartheta b^\dagger(-\mathbf{p})$ and $b^\dagger(-\mathbf{p}) \to e^{i\phi/2} c_\vartheta b^\dagger(-\mathbf{p}) - e^{-i\phi/2} s_\vartheta a(\mathbf{p})$ respectively, where the phase $\phi = \arg \mathsf{H}^{\nu\bar{\nu}}$ and the rotation angle is given by $\tan 2\vartheta = 2|\mathsf{H}^{\nu\bar{\nu}}|/(\mathsf{H}^{\nu\nu} - \mathsf{H}^{\bar{\nu}\bar{\nu}}) \sim V\beta/E$. In the basis where the Hamiltonian is diagonal, the system is described by (anti)neutrino densities, which we denote by ϱ and $\bar{\varrho}$ respectively, and a pairing correlator, which we denote by $\varkappa$. From the transformation properties of the creation/annihilation operators, we can infer the following relations:

$$\rho = c_\vartheta^2 \varrho - c_\vartheta s_\vartheta \varkappa - c_\vartheta s_\vartheta \varkappa^\dagger + s_\vartheta^2 (1 - \bar{\varrho}), \tag{89a}$$

$$\bar{\rho} = c_\vartheta^2 \bar{\varrho} - c_\vartheta s_\vartheta \varkappa - c_\vartheta s_\vartheta \varkappa^\dagger + s_\vartheta^2 (1 - \varrho), \tag{89b}$$

$$\kappa = e^{i\phi}[c_\vartheta^2 \varkappa + c_\vartheta s_\vartheta \varrho - c_\vartheta s_\vartheta (1 - \bar{\varrho}) - s_\vartheta^2 \varkappa^\dagger] \tag{89c}$$

(see Ref. [43] for a detailed discussion). Eigenstates of the diagonalized Hamiltonian are characterized by $\varkappa = 0$. Assuming, e.g., that the system is in an eigenstate of the diagonalized Hamiltonian with some ϱ and $\bar{\varrho}$, and rotating back to the basis where the Hamiltonian has the form (87), we find to leading order

$$\mathsf{R} \sim \begin{pmatrix} \varrho & \frac{V\beta}{2E} \\ \frac{V\beta}{2E} & 1-\bar{\varrho} \end{pmatrix} \sim \begin{pmatrix} \varrho & 10^{-11} \\ 10^{-11} & 1-\bar{\varrho} \end{pmatrix}, \quad (90)$$

which again leads to the tiny $\kappa \sim s_\vartheta \sim 10^{-11}$.

Pair correlations themselves are not measurable, and only their effect on the number densities can be observed. A quick inspection of Eq. (89) shows that in equilibrium the difference between e.g. ρ and ϱ is of the order of $s_\vartheta^2 \sim \kappa^2$. In other words, the induced corrections to ρ and $\bar{\rho}$ are quadratic in κ.

This can also be understood from Eq. (85). If the system has not yet reached equilibrium, then κ oscillates around its stationary value (88), provided that the components of the Hamiltonian matrix only vary slowly with time compared to $\mathsf{H}^{\nu\nu} - \mathsf{H}^{\bar{\nu}\bar{\nu}}$. This assumption allows us to approximate the evolution of κ as a driven harmonic oscillator with an amplitude that depends on the initial conditions. Assuming that pairing correlations are not created during neutrino production, the amplitude is of the order of the equilibrium value, Eq. (88). We then find again that the mean number density created by pairing correlations is $\sim\kappa^2$. Therefore, the inclusion of the particle-antiparticle coherence leads to a negligibly small $\delta\rho \sim \delta\bar{\rho} \sim \kappa^2 \sim 10^{-22}$.

C. Including neutrino-neutrino interactions

In the previous subsection we have estimated the "natural" size of κ assuming that the neutrino-neutrino interactions are negligible. However, in a supernova the neutrino density is very large and the neutrino background may play an important role. This complicates the estimate of κ because $\mathsf{H}^{\nu\bar{\nu}}$ in Eq. (88) itself depends on κ once we include neutrino-neutrino interactions,

$$\mathsf{H}^{\nu\bar{\nu}} = -V\beta - 2\sqrt{2}G_{\mathrm{F}}\hat{\boldsymbol{\epsilon}}^* \int_{\mathbf{q}} [\hat{\mathbf{q}}\ell + \hat{\boldsymbol{\epsilon}}\kappa + \hat{\boldsymbol{\epsilon}}^*\kappa^\dagger] \quad (91)$$

[see Eqs. (48) and (49)]. A further complication arises from the fact that the stationary value for κ of one momentum mode $\mathbf{p}$ depends on the pair correlations of all other momentum modes $\mathbf{q}$. Note also that the phase-space integral in Eq. (91) is unbounded. Pairing correlations with a momentum typical for the supernova environment couple to pairing correlations of arbitrary high momentum. This pushes us beyond the limitations of the Fermi approximation, and in principle a fully renormalizable theory has to be studied to make sense of these high-momentum modes. To stay within the realm of applicability of the effective theory, we use a phenomenological cutoff $|\mathbf{q}| = M_W$ in the phase-space integrals.

To estimate the contribution of the κ terms to the integral in Eq. (91), we take into account that pair correlators of different momentum modes oscillate incoherently such that we can replace κ by its approximate mean value, $\kappa \approx -\mathsf{H}^{\nu\bar{\nu}}/2E$, where we use that $V \ll E$ and assume $\rho + \bar{\rho} \ll 1$ in Eq. (88). To proceed we recall that $\mathsf{H}^{\nu\bar{\nu}} = -\hat{\boldsymbol{\epsilon}}^*\mathbf{V}$ [see Eq. (48)], where $\mathbf{V}$ is the total potential that includes matter and neutrino contributions. Note further that $\mathbf{V}$ is momentum independent. With these substitutions, the integrals involving κ in Eq. (91) read

$$\mathrm{Re}\int_{\mathbf{q}} \hat{\boldsymbol{\epsilon}}\kappa \sim \mathrm{Re}\int_{\mathbf{q}} \hat{\boldsymbol{\epsilon}}\frac{\hat{\boldsymbol{\epsilon}}^*\cdot\mathbf{V}}{2E} = \frac{\sqrt{2}}{2}\frac{G_{\mathrm{F}}M_W^2}{3\pi^2}\mathbf{V}, \quad (92)$$

where we have integrated up to the cutoff $|\mathbf{q}| = M_W$. Let us introduce the notation

$$\mathsf{H}_0^{\nu\bar{\nu}} = -V\beta - 2\sqrt{2}G_{\mathrm{F}}\hat{\boldsymbol{\epsilon}}^* \int_{\mathbf{q}} \hat{\mathbf{q}}\ell. \quad (93)$$

Then using Eq. (92) we can write Eq. (91) as

$$\mathsf{H}^{\nu\bar{\nu}} \approx \mathsf{H}_0^{\nu\bar{\nu}}\left(1 - \sqrt{2}\frac{G_{\mathrm{F}}M_W^2}{3\pi^2}\right)^{-1}. \quad (94)$$

In other words the κ terms in Eq. (91) effectively lead to a renormalization of the total potential produced by the matter and neutrino backgrounds. Numerically, the correction is small, $\sqrt{2}G_{\mathrm{F}}M_W^2/(3\pi^2) \approx 3\times10^{-3}$, and can be neglected.

In a supernova the neutrino density is comparable to that of matter. Whereas each individual neutrino is relativistic, the bulk velocity of the neutrino background is also comparable to the matter velocity. Thus, the neutrino density contribution to Eq. (93) is not expected to be larger than the matter contribution. Furthermore, because the direction of the neutrino background flux is more likely to be parallel to the momenta of individual neutrinos, whereas the build up of the particle-antiparticle coherence requires a current component orthogonal to the neutrino momentum, there is an additional suppression as compared to the matter effect. All in all, the estimates of κ presented above remain essentially unaltered by the inclusion of the neutrino-neutrino interactions.

D. Resonance condition

As follows from Eq. (88), particle-antiparticle coherence can be resonantly enhanced if $\mathsf{H}^{\nu\nu} = \mathsf{H}^{\bar{\nu}\bar{\nu}}$. In general for a relativistic matter flow that also includes the neutrino flux, the Hamiltonian matrix reads [see Eq. (48)],

$$\mathsf{H} = E\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} V^0 - V_\parallel & -V_\perp \\ -V_\perp & V^0 + V_\parallel \end{pmatrix}, \quad (95)$$

where, as before, $V_\parallel \equiv \hat{\mathbf{p}}\mathbf{V}$ and $V_\perp \equiv \hat{\boldsymbol{\epsilon}}\mathbf{V}$ are components of the matter flux parallel and orthogonal to the neutrino momentum. The resonance condition then translates into $E = V_\parallel$. Even assuming a relativistic matter flow, for typical supernova parameters $V_\parallel/E \sim 10^{-9}$. In other words, the resonance condition cannot be fulfilled in a supernova and there is no reason to expect κ to be larger than the estimate presented above.

Note also that for $V \sim E$ not only does the Fermi approximation break down, but also the perturbative description is no longer applicable. In other words it is in principle not possible to hit the resonance without rendering the developed formalism meaningless.

E. Initial conditions

All physical processes in which neutrinos are created have time scales much larger than the time scale of κ oscillation. Hence, even during the production process neutrinos would adiabatically adapt to the propagation basis with respect to pair correlations. On the other hand, the time scales of flavor and helicity oscillation are much larger than those associated with production and detection. This separation of time scales is crucial for the idea that neutrinos are produced in an eigenstate of interaction, i.e., in a coherent superposition of propagation eigenstates. For the same physical reason, as neutrinos stream away from the supernova, they have enough time to adiabatically adapt to the external background. Thus, κ does not oscillate but instead closely tracks its equilibrium value. This makes dynamical equations for κ essentially superfluous. As the neutrinos leave the supernova, the mean pair correlations approach zero adiabatically and decouple from the evolution of ρ and $\bar{\rho}$.

F. Lorentz covariance

In the early Universe, the rest frame of the plasma is the only natural reference frame and the question of Lorentz transformation properties of the pair correlators does not arise [52]. In a supernova environment the situation is more complicated. In particular, the comoving frame of the matter can in some cases be more convenient than the rest frame of a distant observer. Similarly to helicity coherence, the particle-antiparticle coherence builds up only if the off-diagonal components of the Hamiltonian matrix are not zero. However, because the off-diagonals are proportional to the component of the matter flow orthogonal to the neutrino momentum, their value depends on the frame. In particular, one can find a frame where the off-diagonals vanish and no particle-antiparticle coherence builds up.

Let us consider the same example as in Sec. VI. We have two identical observers moving with velocity β with respect to each other. In the frame of the first observer, the neutrino has momentum $\mathbf{p}$ along the z axis and the matter is at rest, i.e. $V_\parallel = V_\perp = 0$,

$$\mathsf{H} = E\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + V\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{96}$$

Thus no particle-antiparticle coherence builds up. In the frame of the second observer which moves with velocity β along the x axis the Hamiltonian is no longer diagonal,

$$\mathsf{H} = \gamma E\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + V\begin{pmatrix} 1/\gamma & -\beta \\ -\beta & 1/\gamma \end{pmatrix}, \tag{97}$$

and we expect helicity coherence to build up. In other words, physical results seem to depend on the frame.

As we have learned from the analysis of an analogous problem for helicity coherence, the kinetic equations are covariant only if the initial conditions also transform under the boost. Pair correlations "couple" neutrinos of opposite momenta. The notion of opposite momenta is not Lorentz invariant and is violated by, e.g., a boost orthogonal to the neutrino momentum. This alone implies that the initial conditions, which include specifying κ for all momentum modes, are not Lorentz invariant. At the same time the very fact that the definition of κ involves two momentum modes makes it rather difficult to derive the corresponding Lorentz transformation rules and we will not attempt the derivation here.

We have argued in the previous subsection that neutrinos are produced and propagate in an eigenstate with respect to particle-antiparticle coherence. In Sec. VI we have observed that if in every frame we diagonalize the Hamiltonian then the (transformed) eigenstate of the Hamiltonian remains invariant under the boosts. Here we assume that the same holds true also for particle-antiparticle coherence. As a particularly interesting example let us assume that in the frame of the first observer the system is in the vacuum state of the interacting Hamiltonian, i.e. $\rho = \bar{\rho} = \kappa = 0$,

$$\mathsf{R} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{98}$$

The Hamiltonian matrix (96) and the extended density matrix (98) commute and therefore the latter is constant in time. According to our assumption, after diagonalizing Eq. (97) by a Bogolyubov transformation, the transformed R takes the form (98). Transforming back to the initial basis we obtain,

$$\mathsf{R} = \frac{1}{2}\begin{pmatrix} 1 - c_\vartheta & s_\vartheta \\ s_\vartheta & 1 + c_\vartheta \end{pmatrix}, \tag{99}$$

where ϑ is the angle of the Bogolyubov transformation that diagonalizes Eq. (97), $\tan 2\vartheta = (\beta V/\gamma E)/[1 - \beta^2(V/E)]$. By construction the Hamiltonian matrix (97) and the extended density matrix (99) commute and the latter is time independent as well.

A perplexing feature of Eq. (99) is that it seems to describe a state with a nonzero number of particles and antiparticles. Whereas the first observer would see neither neutrinos nor antineutrinos, the second observer that moves with respect to the first one with a *constant* velocity β seems to observe a nonzero density of neutrinos and antineutrinos. Put in other words, the empty space perceived by the first observer appears to be filled with neutrino-antineutrino pairs in the frame of the second observer. However, it is not entirely clear if the (anti) particle densities in Eq. (99) describe electroweak interaction eigenstates and thus would actually manifest themselves via, e.g., particle production or momentum transfer to nuclei in scattering processes.

G. Interpretation of the Bogolyubov transformation

To better understand the meaning of the Bogolyubov transformation, we solve the equation of motion for a massless neutrino field coupled to a constant classical current V^μ, and demonstrate that this solution reproduces the results obtained using the Bogolyubov transformation.

In the Fermi limit $\mathcal{L} = \nu^\dagger_{\dot\alpha}\bar\sigma^{\mu,\dot\alpha\alpha}(i\partial_\mu - V_\mu)\nu_\alpha$. Varying the Lagrangian with respect to the neutrino field, we obtain the equation of motion, $\bar\sigma^{\mu,\dot\alpha\alpha}(i\partial_\mu - V_\mu)\nu_\alpha = 0$. Its solution can be written in a form similar to Eq. (45),

$$\nu(t,\mathbf{p}) = a(t,\mathbf{p})\chi_-(\hat{\mathbf{p}}_\mathbf{V}) + b^\dagger(t,-\mathbf{p})\chi_+(\hat{\mathbf{p}}_\mathbf{V}), \qquad (100)$$

where $a(t,\mathbf{p}) = a(t,\mathbf{p})e^{-i\omega_+ t}$ and $b^\dagger(t,-\mathbf{p}) = b^\dagger(-\mathbf{p})e^{i\omega_- t}$ satisfy the usual anticommutation relations, $\hat{\mathbf{p}}_\mathbf{V}$ is the unit vector in the direction of $\mathbf{p}-\mathbf{V}$, and the energy spectrum is given by $\omega_\pm \equiv |\mathbf{p}-\mathbf{V}| \pm V^0$.

Using the orthogonal vectors $\hat{\mathbf{p}}$ and $\hat{\boldsymbol{\epsilon}}$ we can write $\omega_\pm$ in the form $\omega_\pm = \sqrt{(E-\hat{\mathbf{p}}\mathbf{V})^2 + |\hat{\boldsymbol{\epsilon}}\mathbf{V}|^2} \pm V^0$, which reproduces the eigenvalues of the Hamiltonian matrix (48). The spinor contractions [see Eq. (12)] now include $\chi_\mp(\hat{\mathbf{p}}_\mathbf{V})$. By construction, $\Gamma^{\nu\bar\nu}$ and $\Gamma^{\bar\nu\nu}$ vanish once we use the solution of the equations of motion. The diagonal elements can be expanded in terms of $\chi_\mp(\hat{\mathbf{p}})$. For example for $\Gamma^{\nu\nu}$ we obtain

$$\chi^\dagger_-(\hat{\mathbf{p}}_\mathbf{V})\bar\sigma^\mu\chi_-(\hat{\mathbf{p}}_\mathbf{V}) = c_1 n^\mu(\hat{\mathbf{p}}) + \mathrm{Re}[c_2\epsilon^\mu(\hat{\mathbf{p}})]. \qquad (101)$$

Multiplied by V_μ, Eq. (101) reproduces the interaction part of the $\mathsf{H}^{\nu\nu}$ element of the diagonalized Hamiltonian matrix. The decomposition coefficients

$$c_1 \equiv \frac{E-\hat{\mathbf{p}}\mathbf{V}}{|\mathbf{p}-\mathbf{V}|} \quad \text{and} \quad c_2 \equiv -\frac{\hat{\boldsymbol{\epsilon}}^*\mathbf{V}}{|\mathbf{p}-\mathbf{V}|}, \qquad (102)$$

are related to the angle of the Bogolyubov transformation by $c_1 = \cos 2\vartheta$ and $|c_2| = \sin 2\vartheta$ respectively. In other words, diagonalizing the Hamiltonian matrix by a Bogolyubov transformation in every frame is equivalent to using the equation of motion. This equivalence suggests interpreting physical particle densities as propagation eigenstates of the full Hamiltonian in line with the discussion in Sec. VII E.

VIII. SUMMARY AND CONCLUSIONS

Neutrino flavor conversion is important in supernovae, yet a full understanding remains elusive, largely because of neutrino-neutrino refraction and concomitant self-induced flavor conversion, an effect caused by run-away modes of the interacting neutrino gas. The difficulties in developing a robust phenomenological understanding of even this relatively simple case explains the reluctance to add further complications. Yet other effects could be important as well, caused by inhomogeneities and anisotropies of the medium and by magnetic fields, especially if one broadens the view to include, for example, magnetars or neutron-star mergers. It is often thought that helicity conversion effects will be small, at least if neutrino dipole moments have no additional contributions beyond those provided by their masses, yet one should remain open to such possibilities. Finally, beyond flavor and helicity correlations, it has been stressed recently that pair correlations could also become important.

Motivated by these concerns, we have studied extended kinetic equations that describe flavor, helicity, and pair correlations, limiting ourselves to the mean-field level, i.e., considering only propagation effects for freely streaming neutrinos. Based on the "forward Hamiltonian" of neutrinos interacting with a background medium, we have derived the various terms and have given explicit results up to lowest order in the neutrino mass, similar to previous studies in the literature. For Dirac neutrinos, we confirmed previous results and have extended them to include magnetic-field effects. For Majorana neutrinos, we found a small correction to the mean-field Hamiltonian which arises from lepton-number-violating contractions that appear only in the Majorana case. To analyze the behavior of these additional terms in the limit of vanishing neutrino masses, we have also studied extended kinetic equations for Weyl neutrinos.

The density matrix formalism allows one to treat helicity oscillation induced by matter currents and by magnetic fields on equal footing for both Dirac and Majorana neutrinos. We have derived the mean-field Hamiltonian induced by electromagnetic fields and compared it to that induced by matter currents. Somewhat surprisingly, for typical supernova parameters, matter currents dominate over magnetic fields. In principle, resonant enhancements can be achieved, for example by relativistic flows of matter and background neutrinos.

Flavor and helicity oscillations can be complicated in detail, but they are conceptually straightforward. Their importance arises because charged-current interactions produce neutrinos in flavor eigenstates, and all interactions produce them in almost perfect helicity states. This nonequilibrium distribution which is produced, for example, in the neutrino-sphere region of a supernova, subsequently evolves coherently and leads to the various flavor and helicity oscillation phenomena.

Concerning pair correlations, the mean-field equations produce similar oscillation equations. In the simplest case of massless neutrinos, the pair correlations are between neutrinos and antineutrinos of opposite momenta and the oscillations are between the empty state and the one filled with a neutrino and antineutrino. However, one probably cannot separate production from subsequent propagation. The oscillation frequency is here twice the neutrino energy, so in contrast to flavor and helicity oscillations, there is no separation of scales between the energy of the state and the oscillation frequency. Probably, as far as pair correlations are concerned, one should picture neutrinos as being produced in eigenstates of propagation in the medium and not as eigenstates of the interaction Hamiltonian. Flavor and helicity oscillations become important only because one produces a coherent superposition of different propagation eigenstates. As this crucial characteristic appears to be missing for pair correlations, we are tempted to suspect that pair correlations remain a small correction to neutrino dispersion.

In the simplest case, helicity and pair correlations build up only in anisotropic media because angular-momentum conservation forbids mixing of states with different spin. If the anisotropy is a convective matter current, then there is a seeming paradox. In the frame with the current we expect correlations to build up. On the other hand, we may study these effects in the rest frame of the medium where no correlations build up due to isotropy of the background. As far as helicity correlations are concerned, this paradox is resolved by noting that the handedness of massive neutrinos is not Lorentz invariant. Transforming both the mean-field background and the neutrino states to a different frame, e.g., the rest frame of the medium, leads to consistent physical results. For pair correlations, physical results must also be the same in all frames, yet it is less obvious how to show this point explicitly because the correlated modes of opposite momentum are different ones in every frame. Note, however, that in the supernova context, there is not necessarily a natural coordinate system for the study of neutrino propagation. Explicitly including production and detection processes, i.e., the collision terms in the kinetic equation, may shed more light on this question.

The ultimate ambition of fully understanding neutrino propagation in dense environments and strong magnetic fields requires a more complete development of its theoretical underpinnings. Our paper is meant as a contribution toward this overall goal.

ACKNOWLEDGMENTS

We would like to thank I. Izaguirre, S. Chakraborty, A. Dobrynina, and C. Volpe for fruitful discussions. We acknowledge partial support by the Deutsche Forschungsgemeinschaft (DFG) under Grant No. EXC-153 (Excellence Cluster "Universe"), and by the Research Executive Agency (REA) of the European Union under Grant No. PITN-GA-2011-289442 (FP7 Initial Training Network "Invisibles").

APPENDIX A: CHIRAL SPINORS

Following the conventions of Ref. [62], which differ from the ones used in Ref. [44] by the overall sign of γ^0, the Dirac matrices in the Weyl representation, which is used in this work, are

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \tag{A1}$$

where $\sigma^\mu = (1, \boldsymbol{\sigma})$ and $\bar{\sigma}^\mu = (1, -\boldsymbol{\sigma})$. Here $\boldsymbol{\sigma}$ is a three-vector Pauli matrix and 0 and 1 are 2×2 zero and unity matrices respectively. The chiral projectors are

$$P_L = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad P_R = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{A2}$$

The charge-conjugation matrix is

$$C = -i\gamma^2\gamma^0 = \begin{pmatrix} +\varepsilon & 0 \\ 0 & -\varepsilon \end{pmatrix}, \tag{A3}$$

where

$$\varepsilon = \begin{pmatrix} 0 & -1 \\ +1 & 0 \end{pmatrix}. \tag{A4}$$

Notice that in two-component form, one usually writes $\varepsilon_{\alpha\beta}$ for this antisymmetric 2×2 matrix and $\varepsilon^{\dot{\alpha}\dot{\beta}}$ for $-\varepsilon$ appearing in the lower right position of C.

In the Weyl representation and with these conventions, the Dirac bispinors are

$$u_i(\mathbf{p}, s) = \begin{pmatrix} \mathcal{N}^i_{p,s}\chi_s(\hat{\mathbf{p}}) \\ \mathcal{N}^i_{p,-s}\chi_s(\hat{\mathbf{p}}) \end{pmatrix}, \tag{A5a}$$

$$v_i(\mathbf{p}, s) = s\begin{pmatrix} -\mathcal{N}^i_{p,-s}\chi_{-s}(\hat{\mathbf{p}}) \\ \mathcal{N}^i_{p,s}\chi_{-s}(\hat{\mathbf{p}}) \end{pmatrix}, \tag{A5b}$$

where $\hat{\mathbf{p}}$ is the unit vector in the direction of $\mathbf{p}$, $p \equiv |\mathbf{p}|$, $s = \pm$ is a helicity index, and

$$\mathcal{N}^i_{p,s} = \sqrt{\frac{E_i - sp}{2E_i}} \approx \delta_{s-} + \frac{m_i}{2p}\delta_{s+}, \tag{A6}$$

where $E_i = (p^2 + m_i^2)^{1/2}$ is the energy of a neutrino with mass m_i.

We may describe the modes of the neutrino field in spherical coordinates where the momentum components are $\hat{\mathbf{p}} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. In this case, the standard two-component helicity spinors are explicitly

$$\chi_+(\hat{\mathbf{p}}) = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} \end{pmatrix}, \tag{A7a}$$

$$\chi_-(\hat{\mathbf{p}}) = \begin{pmatrix} -e^{-i\phi}\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{pmatrix}. \tag{A7b}$$

They satisfy the orthogonality condition $\chi_s^\dagger(\hat{\mathbf{p}})\chi_h(\hat{\mathbf{p}}) = \delta_{sh}$.

The matrix elements of $\bar{\sigma}^\mu$ are then found by direct evaluation to be

$$\chi_-^\dagger(\hat{\mathbf{p}})\bar{\sigma}^\mu\chi_-(\hat{\mathbf{p}}) = n^\mu = (1, \hat{\mathbf{p}}), \tag{A8a}$$

$$\chi_+^\dagger(\hat{\mathbf{p}})\bar{\sigma}^\mu\chi_+(\hat{\mathbf{p}}) = \bar{n}^\mu = (1, -\hat{\mathbf{p}}), \tag{A8b}$$

$$\chi_+^\dagger(\hat{\mathbf{p}})\bar{\sigma}^\mu\chi_-(\hat{\mathbf{p}}) = -e^{-i\phi}\epsilon^\mu = -e^{-i\phi}(0, \hat{\boldsymbol{\epsilon}}), \tag{A8c}$$

$$\chi_-^\dagger(\hat{\mathbf{p}})\bar{\sigma}^\mu\chi_+(\hat{\mathbf{p}}) = -e^{i\phi}\epsilon^{\mu*} = -e^{i\phi}(0, \hat{\boldsymbol{\epsilon}}^*), \tag{A8d}$$

where ϵ^μ is a polarization vector orthogonal to n^μ. The explicit components in spherical coordinates are

$$\hat{\boldsymbol{\epsilon}} = \begin{pmatrix} e^{i\phi}\cos^2\frac{\theta}{2} - e^{-i\phi}\sin^2\frac{\theta}{2} \\ -i(e^{i\phi}\cos^2\frac{\theta}{2} + e^{-i\phi}\sin^2\frac{\theta}{2}) \\ -\sin\theta \end{pmatrix}. \tag{A9}$$

Note that the vectors n^μ and ϵ^μ depend on $\hat{\mathbf{p}}$, but we do not show this dependence explicitly to simplify the notation.

APPENDIX B: NEUTRINO-NEUTRINO MEAN-FIELD HAMILTONIAN

Because Majorana and Weyl neutrinos have two degrees of freedom, in many cases it is more convenient to use two-component notation. For Majorana neutrinos,

$$\nu_i = \begin{pmatrix} \nu_{i,\alpha} \\ \nu_i^{\dagger\dot{\alpha}} \end{pmatrix} \quad \text{and} \quad \bar{\nu}_i = (\nu_i^\alpha \nu_{i,\dot{\alpha}}^\dagger), \tag{B1}$$

where $\nu_{i,\alpha}$ and $\nu_{i,\dot{\alpha}}^\dagger$ are two-component fields. They are related by Hermitian conjugation and transform under the $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representations of the Lorentz group respectively. To emphasize the different transformation properties, the conjugated fields, by convention, always carry a dotted spinor index. The spinor indices α and $\dot{\alpha}$ are raised (lowered) using the spinor metric matrices $\epsilon^{\alpha\beta}$ and $\epsilon^{\dot{\alpha}\dot{\beta}}$ ($\epsilon_{\alpha\beta}$ and $\epsilon_{\dot{\alpha}\dot{\beta}}$). Left-handed Weyl fields satisfy the condition $P_L\nu = \nu$. Their explicit form can be obtained from Eq. (B1) by applying the chiral projectors.

Rewritten in terms of the two-component fields, the neutrino-neutrino Hamiltonian density of Eq. (21) is

$$\mathcal{H}^{\nu\nu} = \frac{G_\mathrm{F}}{\sqrt{2}}\sum_{ij}[\nu_{i,\dot{\alpha}}^\dagger \bar{\sigma}^{\mu,\dot{\alpha}\alpha}\nu_{i,\alpha}][\nu_{j,\dot{\beta}}^\dagger \bar{\sigma}_\mu^{\dot{\beta}\beta}\nu_{j,\beta}]. \tag{B2}$$

Taking expectation values of products of two of the four neutrino fields and bearing in mind that fermions anti-commute we obtain for the mean-field Hamiltonian

$$\begin{aligned}\mathcal{H}_\mathrm{mf}^{\nu\nu} = \frac{G_\mathrm{F}}{\sqrt{2}}\sum_{ij}\bar{\sigma}^{\mu,\dot{\alpha}\alpha}\bar{\sigma}_\mu^{\dot{\beta}\beta} &\times \Big[2\nu_{i,\dot{\alpha}}^\dagger\nu_{i,\alpha}\langle\nu_{j,\dot{\beta}}^\dagger\nu_{j,\beta}\rangle - 2\nu_{i,\dot{\alpha}}^\dagger\nu_{j,\beta}\langle\nu_{j,\dot{\beta}}^\dagger\nu_{i,\alpha}\rangle \\ &+ \nu_{i,\dot{\alpha}}^\dagger\nu_{j,\dot{\beta}}^\dagger\langle\nu_{j,\beta}\nu_{i,\alpha}\rangle + \nu_{i,\alpha}\nu_{j,\beta}\langle\nu_{j,\dot{\beta}}^\dagger\nu_{i,\dot{\alpha}}^\dagger\rangle\Big].\end{aligned} \tag{B3}$$

Translating back to the four-component notation we obtain $[\bar{\nu}_i\gamma^\mu P_L\nu_i]\langle\bar{\nu}_j\gamma_\mu P_L\nu_j\rangle$ for the first term in Eq. (B3). Using a Fierz identity [62], $\bar{\sigma}^{\mu,\dot{\alpha}\alpha}\bar{\sigma}_\mu^{\dot{\beta}\beta} = -\bar{\sigma}^{\mu,\dot{\alpha}\beta}\bar{\sigma}_\mu^{\dot{\beta}\alpha}$, and translating back to the four-component notation we can represent the second term in a similar form, $[\bar{\nu}_i\gamma^\mu P_L\nu_j]\langle\bar{\nu}_j\gamma_\mu P_L\nu_i\rangle$. By raising and lowering the spinor indices and reordering the fields the third term can be rewritten as $\sigma_{\dot{\alpha}\alpha}^\mu\bar{\sigma}_\mu^{\dot{\beta}\beta}\nu_{j,\dot{\beta}}^\dagger\nu_i^{\dagger\dot{\alpha}}\langle\nu_i^\alpha\nu_{j,\beta}\rangle$. Using another Fierz identity [62], $\sigma_{\dot{\alpha}\alpha}^\mu\bar{\sigma}_\mu^{\dot{\beta}\beta} = 2\delta_\alpha{}^\beta\delta^{\dot{\beta}}{}_{\dot{\alpha}}$, and translating back to four-component notation we can rewrite the third term in the form $2[\bar{\nu}_j P_R C\bar{\nu}_i^T]\langle\nu_i^T C P_L\nu_j\rangle$. Collecting all terms we obtain in four-component notation

$$\begin{aligned}\mathcal{H}_\mathrm{mf}^{\nu\nu} = \sqrt{2}G_\mathrm{F}\sum_{ij}([\bar{\nu}_i\gamma^\mu P_L\nu_i]\langle\bar{\nu}_j\gamma_\mu P_L\nu_j\rangle &+ [\bar{\nu}_i\gamma^\mu P_L\nu_j]\langle\bar{\nu}_j\gamma_\mu P_L\nu_i\rangle \\ &+ [\bar{\nu}_i P_R C\bar{\nu}_j^T]\langle\nu_j^T C P_L\nu_i\rangle \\ &+ [\nu_i^T C P_L\nu_j]\langle\bar{\nu}_j P_R C\bar{\nu}_i^T\rangle).\end{aligned} \tag{B4}$$

Using the definition of the charge-conjugate field, $\nu^c \equiv C\bar{\nu}^T$, and the resulting $\bar{\nu^c} = \nu^T C$ we can further simplify and write the last two terms in a form which coincides with Eq. (40),

$$\begin{aligned}\mathcal{H}_\mathrm{mf}^{\nu\nu} = \sqrt{2}G_\mathrm{F}\sum_{ij}([\bar{\nu}_i\gamma^\mu P_L\nu_i]\langle\bar{\nu}_j\gamma_\mu P_L\nu_j\rangle &+ [\bar{\nu}_i\gamma^\mu P_L\nu_j]\langle\bar{\nu}_j\gamma_\mu P_L\nu_i\rangle \\ &+ [\bar{\nu}_i P_R\nu_j^c]\langle\bar{\nu_j^c} P_L\nu_i\rangle \\ &+ [\bar{\nu_i^c} P_L\nu_j]\langle\bar{\nu}_j P_R\nu_i^c\rangle).\end{aligned} \tag{B5}$$

Thus, the effective Hamiltonian obtained using the two-component notation is identical to the one obtained using the four-component notation, as expected.

APPENDIX C: RIGHT-CHIRAL CURRENTS

For completeness, we provide the contractions of the Lorentz structure of right-chiral currents ($\gamma^\mu P_R$), which might arise in, e.g., beyond the Standard Model theories with right-handed currents. The contractions are

$$(\gamma_\mu P_R)^{\nu\nu}_{ij,sh} \approx \begin{pmatrix} 0 & e^{+i\phi}\frac{m_i}{2p}\epsilon^*_\mu \\ e^{-i\phi}\frac{m_j}{2p}\epsilon_\mu & n_\mu \end{pmatrix}, \tag{C1a}$$

$$(\gamma_\mu P_R)^{\nu\bar\nu}_{ij,sh} \approx \begin{pmatrix} e^{+i\phi}\frac{m_i}{2p}\bar n_\mu & 0 \\ \epsilon_\mu & e^{-i\phi}\frac{m_j}{2p}n_\mu \end{pmatrix}, \tag{C1b}$$

$$(\gamma_\mu P_R)^{\bar\nu\nu}_{ij,sh} \approx \begin{pmatrix} e^{-i\phi}\frac{m_j}{2p}\bar n_\mu & \epsilon^*_\mu \\ 0 & e^{+i\phi}\frac{m_i}{2p}n_\mu \end{pmatrix}, \tag{C1c}$$

$$(\gamma_\mu P_R)^{\bar\nu\bar\nu}_{ij,sh} \approx \begin{pmatrix} \bar n_\mu & e^{-i\phi}\frac{m_j}{2p}\epsilon^*_\mu \\ e^{+i\phi}\frac{m_i}{2p}\epsilon_\mu & 0 \end{pmatrix}. \tag{C1d}$$

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