Unified dark matter with intermediate symmetry breaking scales

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Asymmetric symmetry breaking models dynamically break the $G \times G$ gauge symmetries of mirror models to distinct subgroups in the two sectors. The coincidental abundances of visible and dark matter, $\Omega_{\rm DM} \simeq 5\Omega_{\rm VM}$, motivates asymmetric dark matter theories where similar number densities of baryons in each sector are explained by their connected origins. However, the question of why the baryons of two sectors should have similar mass remains. In this work we develop an alternative class of asymmetric symmetry breaking models which unify the dark and visible sectors while generating a small difference in the mass scale of the baryons of each sector. By examining the different paths that the SO(10) GUT group can take in breaking to gauge symmetries containing SU(3), we can adapt the mechanism of asymmetric symmetry breaking to demonstrate models in which originally unified visible and dark sectors have isomorphic color gauge groups at low energy yet pass through different intermediate gauge groups at high energy. Through this, slight differences in the running coupling evolutions and, thus, the confinement scales of the two sectors are generated.

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I. INTRODUCTION

The present understanding of our Universe places all of the matter that we understand as just a small fraction of the total amount of matter and energy that make up the cosmos. This visible matter (VM), made up of the particles of the standard model (SM) interacting under the gauge forces described by the group $SU(3) \times SU(2) \times U(1)$ accounts for only 4.9% of the total mass energy while the remainder is made up of the currently unknown dark matter (DM), with 26.8%, and dark energy which drives the acceleration of the Universe accounting for 68.3%. The similarity in the abundances of visible and dark matter suggests a common origin, since if the two forms of matter were completely independent their relative abundances would likely be dissimilar. Asymmetric dark matter (ADM) models seek to explain this observed ratio of visible and dark matter of $\Omega_{\rm DM} \simeq 5\Omega_{\rm VM}$ by the conservation of a single global quantum number. By establishing a symmetry in a linear combination of baryon and dark baryon numbers, the matter-antimatter annihilations and chemical reprocessing taking place result in a relation between the number densities of visible and dark baryons [1–3]. Though armed with an explanation for similar baryon number densities, ADM models are still in need of an explanation as to why visible and dark baryons should have similar mass. In [4], the mechanism of *asymmetric symmetry breaking (ASB)* was introduced to develop a way of connecting these masses by generating an SU(3) confinement scale in each sector which is similar but slightly different due to a grand unification of the two sectors for which the originally unified $G_V \times G_D$ gauge group is broken differently for each sector. This was used in the context of an $SU(5) \times SU(5)$ model where the hidden sector was broken to a dark $SU(3)$. Such a model then allows for variations in the masses of fermions in each sector and alters the running couplings of the surviving SU(3) gauge symmetries that confine the visible and dark baryons. This model worked by using the fact that since the two sectors unify into a single $G \times G$ group at high energy with a $\mathbb{Z}_2$ mirror symmetry, the values of the coupling constants at the GUT scale are the same and different quark masses of the two sectors differentiate the evolution of the couplings slightly to produce confinement scales of similar but distinct value.

In this work we explore the ability of spontaneous symmetry breaking to generate similar results from different GUT breaking chains in the two sectors in an $SO(10) \times SO(10)$ theory. These different gauge symmetry breaking chains can result from a simple extension of the mechanism of ASB and allows one to create regions where the coupling evolution differs in the two sectors without considering fermion mass generation. The models that we explore here are larger extensions to mirror symmetric models which have been explored in many contexts [5–22], where in this work the mirror symmetry serves only at high energy and the low-energy features of the two sectors can be vastly different. We use this to develop a way of explaining the similarity of DM mass, the focus of this paper. In further work it would be interesting to see more complete theories that explore the baryogenesis of the two sectors such as in the recent work of [23] where an $SO(10) \times SO(10)$ model explored baryogenesis via leptogenesis with visible and dark QCD scales set at similar values. Our work is also related to other investigations into the possibility of a dark QCD such as [24–30].

The next section will review the motivation for such models by examining how the running of coupling

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constants in gauge theories with unification can be used to link the color confinement scales of the two sectors. From there Sec. III will discuss SO(10) models and their appeal as the choice of GUT group to be implemented with this method. Following this Sec. IV and Sec. V will discuss the paths of symmetry breaking that we can take within SO(10) models and how these can be used in ASB models to create the standard model in one sector with an SU(3) group in the dark sector (DS). We will then move on to Sec. VI where we will explore similar models within the supersymmetric framework, while Sec. VII will examine the results from a broad range of these possible scenarios and their effect on the dark QCD scale. Finally, in Sec. VIII we will discuss the constraints on some of these models and the outlook for such theories.

II. DIMENSIONAL TRANSMUTATION

Our objective is to develop $SO(10) \times SO(10)$ models that can account for the similarity in mass of visible and dark matter. The overwhelming majority of the mass of VM comes from dimensional transmutation where a dimensionful parameter is created at the scale at which a coupling begins to diverge and the theory becomes nonperturbative. The masses of the protons and neutrons which dominate the visible sector (VS) in the present Universe come from the confinement scale of QCD where the coupling constant of the color force becomes large at low energy. This feature of asymptotically free theories presents an elegant way to introduce mass scales into a theory. The capacity to yield such scales at low energy comes from the negative sign of the beta function of a non-Abelian gauge theory. The running coupling evolution is described by the logarithmic dependence on energy scale,

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 - (b_0/4\pi)\alpha_s(\mu_0)\ln(\mu^2/\mu_0{}^2)}, \tag{1}$$

such that at low-energy scales the value of α_s grows exponentially. This asymptote sets the energy scale of the proton mass after chiral symmetry breaking when colored particles are confined to bound states. In a general non-Abelian gauge theory for group G, the beta function at one loop is given by

$$\beta(g)_{(1\,\mathrm{Loop})} = \frac{g^3}{16\pi^2}\left(-\frac{11}{3}R_{\mathrm{Gauge}} + \frac{4}{3}R_{\mathrm{Dirac}} + \frac{2}{3}R_{\mathrm{Majorana}} + \frac{2}{3}R_{\mathrm{Weyl}} + \frac{1}{3}R_{\mathrm{C.Scalar}} + \frac{1}{6}R_{\mathrm{Scalar}}\right), \tag{2}$$

where $\beta(g)_{(1\,\mathrm{Loop})} = \frac{g^3}{16\pi^2}b_0$ and the factors of R are the indices for the choice of multiplet(m) defined as

$$\mathrm{Tr}(t^a t^b) = \delta^{ab} \times R(m), \tag{3}$$

and are calculated for each copy of the gauge fields, which are necessarily in the adjoint representation of G, followed by the Dirac, Majorana, and Weyl fermions and finally complex and real scalars. For the familiar QCD group SU(3), the beta function becomes

$$b_0 = -11 + \frac{2}{3}n_f, \tag{4}$$

with n_f the number of flavors. In the standard model the coupling of QCD goes nonperturbative at ≈200 MeV. In this paper we seek to explain the similarity of visible and dark matter masses by assuming that DM similarly gains its mass by dimensional transmutation and that the confinement scales of the two sectors are linked to each other by their different evolution from a common starting point at the GUT scale. These differences can occur spontaneously from a completely mirror symmetric model thanks to ASB where the absolute minima of the potential are such that the vacuum structure of each sector is necessarily different. The goal of this work is to construct a broad outline of the possible models in which a GUT theory with a discrete $\mathbb{Z}_2$ symmetry can naturally explain the similarity of visible and dark matter masses by spontaneously breaking the symmetries of the two sectors through different subgroups while ending with at least one copy of SU(3) in each sector. In this manner the confining scale of the dark QCD is related to that of the standard model through the unified couplings at high scale, but within intermediate symmetry breaking scales the coupling constants run differently due to the contribution from the gauge bosons of their respective groups. It, thus, becomes effectively the first term in Eq. (2) that changes at particular mass scales allowing for the generation of different confinement scales rather than the second term in Eq. (4) at the quark mass thresholds as in [4]. In this work we will not examine any differences resulting from quark mass thresholds though of course the two effects could be utilized in a single theory. We will

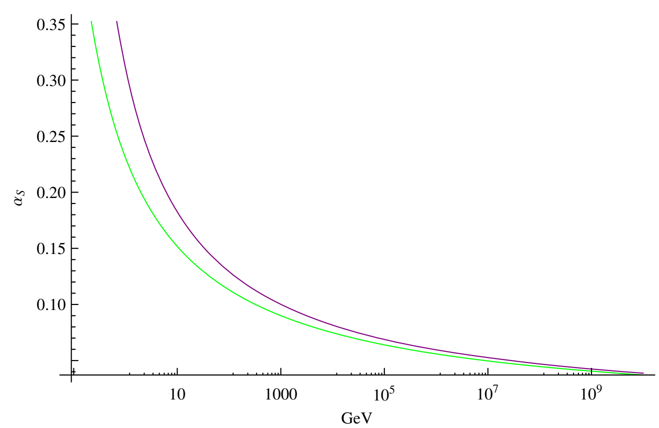

FIG. 1 (color online). The confinement of the dark sector QCD occurs at a higher scale than its visible counterpart after asymmetric symmetry breaking. The top line shows α_D after running as SU(4) for 2 orders of magnitude at a high-energy scale while α_V remains SU(3).

focus on those cases where after the altered running of the two QCDs is established the dark QCD coupling will confine at a higher energy scale as this is more suited to ADM where mass scales of around 1 order of magnitude higher are compatible. Figure 1 shows the divergence of the two SU(3) theories after running at different rates for a segment of the high-energy regime.

A number of other works have explored similar concepts of generating the confinement scale of a dark QCD in order to explain the DM mass coincidence. In particular this work is related to that of [31] where $\mathbb{Z}_2$ symmetric SU(5) GUTs were explored for generating confined states at low scales. The present work, however, seeks to expand the technique of ASB beyond SU(5) theories to the SO(10) gauge group and so we move on to a discussion of its features.

III. SO(10) × SO(10) MODELS

The group SO(10) presents an appealing avenue for GUT extensions to the standard model beyond the minimal cases. It has the benefit of allowing each generation of fermions to fit within a single SO(10) multiplet including the right hand neutrino. Most SO(10) models require at least two Higgs multiplets to break the full symmetry down to the standard model. Typical choices include one set of fields in **45** or **54** representations and another in **10**, **16** or **126** dimensional representations [32]. The choice of 126 for the second is appealing as it allows the generation of fermion masses by Yukawa coupling to the 3 copies of 16_f which contain the fermions of the standard model. Since there are two multiplets required to break SO(10) to the standard model gauge group, the work of [4] can be naturally extended to SO(10) where the visible and dark sectors required two Higgs representations in each sector to carry out ASB. By giving a nonzero vacuum expectation value (VEV) to all four representations in such a manner that representations paired under the $\mathbb{Z}_2$ symmetry gain VEVs of different sizes, the gauge group of each sector will be different for small segments of the range between the GUT scale and the low-energy theory. The parameter space of this particular type of model can be quite small as we shall see in Sec. V and, therefore, leads us to consider nonminimal multistep breaking chains in $SO(10)_V \times SO(10)_D$ models for more than four Higgs multiplets. We are chiefly concerned with paths that can break SO(10) to a gauge sector containing SU(3) in the dark sector while breaking to the SM gauge group in the visible sector. Since our primary goal is to generate dark confinement scales only slightly above that of the visible sector, we will limit ourselves to models where this is the result, that is $\Lambda_D > \Lambda_V$. The case of $\Lambda_D = \Lambda_V$ can also appear, often in the limiting cases where the intermediate scales approach the GUT scale.

To illustrate the concept, consider the case where

$$SO(10)_V \to_{M_X} SU(4) \times SU(2) \times SU(2) \to_{M_I} SU(3) \times SU(2) \times U(1), \quad (5)$$

while in the dark sector

$$SO(10)_D \to_{M_X} SU(5) \to_{M_I} SU(3) \times SU(2) \times U(1). \quad (6)$$

In the visible sector this could be done with with a Higgs multiplet which transforms as a **54** and which gains a VEV at the scale M_X while in the dark sector we have a **45**. Then a pair of $\mathbf{16} + \overline{\mathbf{16}}$ or $\mathbf{126} + \overline{\mathbf{126}}$ representations could gain VEVs in both sectors at the scale M_I where each sector becomes standard model-like. The use of a pair of conjugate representations allows for such fields to be included in the superpotential in supersymmetric theories and also allows us to invoke Michel's conjecture which states that for conjugate pairs such as these, or for real irreducible representations, the symmetry breaking must be to a maximal little group [33,34]. This pair of breaking chains is a particularly simple example where we have only two scales, M_X and M_I; however, in general, it is possible for the intermediate scales of the two sectors to be independent. In such a scenario we have only the distance between the two scales M_X and M_I that determines the size of the difference between the confinement scales between the two sectors. This difference can be approximately determined by calculating the value of the dark sector's Λ after running upward in energy from Λ_V to the lowest breaking scale M_I and then to the second, M_X, before evolving back down in energy until we reach the confinement regime. Using this it can be calculated that at one loop the ratio of the confinement scales is given by

$$\frac{\Lambda_D}{\Lambda_V} = \frac{M_X}{M_I}^{\frac{b_D - b_V}{b_0}}, \quad (7)$$

where the beta functions here are for the intermediate gauge groups in the intermediate range $M_I \leq M \leq M_X$ for the two sectors, and b_0 is the SU(3) beta function given in Eq. (3). This calculation allows us to see that similar but different confinement scales can be generated from a model with different gauge symmetries at high energy, and for this reason we wish to consider the full set of possible symmetry breaking scenarios. In the next section we will examine what breaking chains are possible in each sector.

IV. MULTISTEP BREAKING CHAINS

We wish to systematically explore all the possibilities for the different breaking chains that can occur in each sector for an SO(10) model in order to examine which chains allow for realistic models of both sectors. There are a number of paths through which SO(10) can break down to a gauge theory containing the SM with two of the most notable being through the Pati-Salam $SU(4) \times SU(2) \times SU(2)$ [35] and the Georgi-Glashow SU(5) [36] subgroups. For the visible sector we are mostly concerned with these particular models; however, for the dark sector we are free to choose any breaking which leaves unbroken an SU(3) theory at low energy. This opens up a large number of choices of Higgs multiplet representations in the dark sector. We will limit ourselves to the cases of one and two intermediate scales as additional scales add complexity without necessarily offering more insight into possible outcomes. Below we list all of the possible breaking chains we can consider for the color force in the dark sector. We consider first of all chains with just one intermediate scale, M_I, between the confinement scale, Λ_D, and the GUT scale M_X. These are

$$\begin{aligned}
&SO(10) \to SO(9) \to SU(3) \quad (I)\\
&SO(10) \to SO(8) \to SU(3) \quad (II)\\
&SO(10) \to SO(7) \to SU(3) \quad (III)\\
&SO(10) \to SU(5) \to SU(3) \quad (IV)\\
&SO(10) \to SU(4) \to SU(3) \quad (V)
\end{aligned} \tag{8}$$

and secondly we consider models with two intermediate scales, M_I and M_J, between M_X and the low-energy theory, with $M_J \geq M_I$. These are

$$\begin{aligned}
&SO(10) \to SO(9) \to SO(8) \to SU(3) \quad (VI)\\
&SO(10) \to SO(9) \to SO(7) \to SU(3) \quad (VII)\\
&SO(10) \to SO(9) \to SU(4) \to SU(3) \quad (VIII)\\
&SO(10) \to SO(8) \to SO(7) \to SU(3) \quad (IX)\\
&SO(10) \to SO(8) \to SU(4) \to SU(3) \quad (X)\\
&SO(10) \to SO(7) \to SU(4) \to SU(3) \quad (XI)\\
&SO(10) \to SU(5) \to SU(4) \to SU(3) \quad (XII).
\end{aligned} \tag{9}$$

The chains we consider in the visible sector are most often IV and V as well as the case where the two intermediate scales are close enough that the symmetry breaking effectively happens at one scale, as per $SO(10) \to SU(3)$. We consider this variety as the limiting case for the magnitude of the difference between the two groups one loop beta functions and is useful for cases where the symmetry breaking chains of the two sectors are in fact the same except for the scales at which breaking occurs. This can be seen as delayed symmetry breaking where at one or more of the scales, M_X, M_I and M_J one sector breaks to a subgroup but the other does not. The analysis is no different than other examples, it is simply that we contrast some intermediate gauge group's running with that of, for instance, the group $SO(10)$ itself. In examining results we choose a breaking chain for each sector from the list, but we will limit ourselves to only those choices for which the dark scale runs faster in the intermediate range of the running for the case of one intermediate scale. These cases demonstrate the key aspect of these theories, that the gauge group of the intermediate energy scale can change the final scale of dimensional transmutation in two SU(3) theories that originate from an originally $\mathbb{Z}_2$ symmetric $G \times G$ theory.

For the sake of proton decay limits the intermediate scale of the visible sector M_I must be above experimental constraints. Additionally it is important for consideration of gauge coupling constant unification in the visible sector which we will return to in Sec. VIII. The scale at which the dark sector becomes SU(3) is not so constrained; however, if it is significantly lower, the confinement scales will distance themselves beyond the desired amount. It may also have consequences for the stability of DM depending on other features of the hidden sector. It is also natural to consider the models mentioned where Higgs multiplets that gain the same VEV in each sector allow for the lower intermediate scale to be the same in the two sectors. Beyond this the next highest intermediate scale M_J is constrained only from above in that $M_J < M_X < M_{\mathrm{Planck}}$. In the next section we present a proof that for non-SUSY models ASB can be realized in potentials that give minima which describe any of the model types we discussed above.

V. MULTISTEP ASYMMETRIC SYMMETRY BREAKING

We will now outline how a Higgs sector can accommodate a large variety of symmetry breaking chains in a GUT model of two sectors. As in [4] ASB can induce nonzero VEVs in Higgs multiplets which have $\mathbb{Z}_2$ partners in the opposing sector that retain a VEV of zero. The simplest example has just two pairs of scalar singlet fields that transform under the $\mathbb{Z}_2$ symmetry as

$$\phi_1 \leftrightarrow \phi_2, \qquad \chi_1 \leftrightarrow \chi_2. \tag{10}$$

We can then write down the general potential without loss of generality as

$$\begin{aligned}
V = {}&\lambda_\phi({\phi_V}^2 + {\phi_D}^2 - v_\phi^2)^2 + \kappa_\phi({\phi_V}^2{\phi_D}^2)\\
&+ \lambda_\chi({\chi_V}^2 + {\chi_D}^2 - v_\chi^2)^2 + \kappa_\chi({\chi_V}^2{\chi_D}^2)\\
&+ \sigma({\phi_V}^2{\chi_V}^2 + {\phi_D}^2{\chi_D}^2)\\
&+ \rho({\phi_V}^2 + {\chi_V}^2 + {\phi_D}^2 + {\chi_D}^2 - v_\phi^2 - v_\chi^2)^2,
\end{aligned} \tag{11}$$

where cubic terms are taken to be absent by additional discrete symmetries. If all of the parameters in Eq. (11) are positive, then each term in the potential is positive definite and, thus, minimized if it is equal to zero. The total potential is then minimized by VEVs that break the $\mathbb{Z}_2$ symmetry in such a way that

$$\langle\phi_1\rangle = v_\phi, \quad \langle\chi_1\rangle = 0, \quad \langle\phi_2\rangle = 0, \quad \langle\chi_2\rangle = v_\chi. \tag{12}$$

This minimum is also degenerate with its $\mathbb{Z}_2$ partner where it is ϕ_2 and χ_1 that gain nonzero VEVs. We can then extend this idea to larger representations of gauge groups by replacing the singlet fields with Higgs multiplets. The set of Higgs multiplets responsible for symmetry breaking in each sector can, thus, be entirely independent for an arbitrary number of representations we add to the theory. Let us first take the case of a set of $2n$ singlet scalar fields, $H_{V1}, H_{D1}, \ldots, H_{Vn}, H_{Dn}$, where under the $\mathbb{Z}_2$ symmetry,

$$H_V \leftrightarrow H_D. \tag{13}$$

We then consider general potentials where again all of the parameters are positive and each individual term is positive definite and cubic terms are taken to be absent by discrete symmetries. For the case of $n = 3$ we have

$$\begin{aligned} V = {}& \lambda_{H_1}(H_{V1}^2 + H_{D1}^2 - v_{H_1}^2)^2 + \kappa_{H_1}(H_{V1}^2 H_{D1}^2) + \lambda_{H_2}(H_{V2}^2 + H_{D2}^2 - v_{H_2}^2)^2 + \kappa_{H_2}(H_{V2}^2 H_{D2}^2) \\ & + \sigma_1(H_{V1}^2 H_{V2}^2 + H_{D1}^2 H_{D2}^2) + \rho_1(H_{V1}^2 + H_{V2}^2 + H_{D1}^2 + H_{D2}^2 - v_{H_1}^2 - v_{H_2}^2)^2 + \lambda_{H_3}(H_{V3}^2 + H_{D3}^2 - v_{H_3}^2)^2 \\ & + \kappa_{H_3}(H_{V3}^2 H_{D3}^2) + \sigma_3(H_{V1}^2 H_{D3}^2 + H_{D1}^2 H_{V3}^2) + \rho_3(H_{V1}^2 + H_{V3}^2 + H_{D1}^2 + H_{D3}^2 - v_{H_1}^2 - v_{H_3}^2)^2 \\ & + \sigma_2(H_{V3}^2 H_{V2}^2 + H_{D3}^2 H_{D2}^2) + \rho_2(H_{V3}^2 + H_{V2}^2 + H_{D3}^2 + H_{D2}^2 - v_{H_3}^2 - v_{H_2}^2)^2. \end{aligned} \tag{14}$$

In this case the minimum is given by

$$\begin{aligned} \langle H_{V1} \rangle &= v_{H_1}, & \langle H_{D1} \rangle &= 0, \\ \langle H_{V2} \rangle &= 0, & \langle H_{D2} \rangle &= v_{H_2}, \\ \langle H_{V3} \rangle &= v_{H_3}, & \langle H_{D3} \rangle &= 0. \end{aligned} \tag{15}$$

The above minima could have been the reverse where the V and D subscripts are interchanged of course. We simply label the sector which develops the features of the SM as the visible sector. This potential demonstrates the general procedure by which we can generate nonsupersymmetric asymmetric symmetry breaking multistep chains. The first two sets of fields form an asymmetric set as in Eq. (11) and for any additional field, such as H_3, we can choose for it to align with either the visible or dark sector based on these choices: For coupling between fields that we want to break similarly we set σ to couple fields in opposing sectors and the ρ term to be that which allows for same sector terms. In this case we choose for H_3 to break the same as H_1 so σ_3 couples fields of different sectors. Then for mixing between H_3 and fields that break differently we set σ to couple the same sector fields, where in Eq. (14) we have σ_2 coupling same sector fields since H_2 is aligned with the opposite sector to H_1. Following this simple prescription allows us to add an arbitrary number of multiplets to each sector with the asymmetry determining which sectors will gain the symmetry breaking aspects of that multiplet. We can then consider representations of SO(10) where now each $H_{Vn} \sim (R_n, 1)$ and its $\mathbb{Z}_2$ partner transforms as $H_{Dn} \sim (1, R_n)$. The general potential will contain additional couplings; however, it will always contain an analogous set of terms to those above for which we can always generate an asymmetric array of VEVs. These will then drive the symmetry breaking of the two sectors to be completely different.

As we mentioned earlier the simplest variety of SO(10) model is one where the asymmetry in the VEVs of the potential is not limited to distinguishing between zero and nonzero, but rather creates an asymmetry in the size of the VEVs which are all nonzero. Consider a potential of just two pairs as in Eq. (11) but with each of $\kappa_\phi, \kappa_\chi < 0$. In this scenario we can create asymmetries of the form

$$\langle \phi_1 \rangle = \langle \chi_2 \rangle, \qquad \langle \chi_1 \rangle = \langle \phi_2 \rangle. \tag{16}$$

We found it possible to generate a ratio of $\langle \phi_1 \rangle / \langle \chi_1 \rangle \approx 10^3$ for a very constrained region of parameter space. Such a potential can minimally accommodate exactly the number of Higgs multiplets necessary to break two copies of SO(10) to the same final gauge group but with different gauge groups in the intermediate range depending on the choice of Higgs multiplet. While simple in the number of multiplets, this minimal theory suffers from a much smaller allowed parameter space than the previously discussed ASB mechanisms. In particular the size of parameters must be fine tuned slightly such that we very nearly have $\kappa_\chi \simeq \kappa_\phi$ and $-\kappa_\phi - \kappa_\chi \simeq \sigma$. If we remove the condition of having just two breaking scales and allow each of the four fields to attain different VEVs then a much broader range of the parameter space is compatible.

We can also develop models in which additional pairs of multiplets that transform under the $\mathbb{Z}_2$ symmetry are added as in Eq. (14) but break in such a way that they both gain nonzero VEVs and, thus, both contribute to the symmetry breaking in each sector. This is in fact the simplest method in some kinds of cases where the same dimensional representation is useful for the symmetry breaking needed in each sector. This can be always be accomplished by, for example, having these added fields couple only weakly to the previously added fields. We illustrate this asymmetric breaking with a particular $SO(10) \times SO(10)$ potential which breaks the mirror symmetric GUT group to $[SU(4) \times SU(2) \times SU(2)]_V \times [SU(5) \times U(1)]_D$. Within the context of the standard model, such a theory would need at least one more Higgs multiplet in order to break the Pati-Salam group to $SU(3) \times SU(2) \times U(1)$. Within our variety of models we would require at least one additional mirror symmetric pair of representations to break the symmetry in each sector to one containing SU(3). Since the important results from this work are the generation of different symmetries in the intermediate range we focus on constructing a potential that asymmetrically generates the first step of the breaking chain. We consider a set of fields transforming as

$$\phi_V \sim (45,1), \qquad \chi_V \sim (54,1), \qquad \phi_D \sim (1,45), \qquad \chi_D \sim (1,54). \tag{17}$$

With these we can follow the procedure detailed in the toy model and construct an asymmetric potential. Each of the terms in the toy model has a direct analogue and in addition to these there will be new terms from unique contractions of the Higgs multiplets. The general renormalizable fourth-order potential is

$$\begin{aligned}
&-\frac{\mu_\phi^2}{2}(\phi_{Vij}\phi_{Vji}+\phi_{Dij}\phi_{Dji})+\frac{\lambda_\phi}{4}((\phi_{Vij}\phi_{Vji})^2+(\phi_{Dij}\phi_{Dji})^2)+\frac{\alpha_\phi}{4}(\phi_{Vij}\phi_{Vjk}\phi_{Vkl}\phi_{Vli}+\phi_{Dij}\phi_{Djk}\phi_{Dkl}\phi_{Dli})\\
&+\kappa_\phi(\phi_{Dij}\phi_{Dji}\phi_{Vkl}\phi_{Vlk})-\frac{\mu_\chi^2}{2}(\chi_{Vij}\chi_{Vji}+\chi_{Dij}\chi_{Dji})+\frac{\lambda_\chi}{4}((\chi_{Vij}\chi_{Vji})^2+(\chi_{Dij}\chi_{Dji})^2)\\
&+\frac{\alpha_\chi}{4}(\chi_{Vij}\chi_{Vjk}\chi_{Vkl}\chi_{Vli}+\chi_{Dij}\chi_{Djk}\chi_{Dkl}\chi_{Dli})+\kappa_\chi(\chi_{Dij}\chi_{Dji}\chi_{Vkl}\chi_{Vlk})\\
&+\frac{\beta\mu_\chi}{3}(\chi_{Vij}\chi_{Vjk}\chi_{Vki}+\chi_{Dij}\chi_{Djk}\chi_{Dki})+c_1(\phi_{Dij}\phi_{Dji}\chi_{Vkl}\chi_{Vlk}+\phi_{Vij}\phi_{Vji}\chi_{Dkl}\chi_{Dlk})\\
&+c_2(\phi_{Dij}\phi_{Dji}\chi_{Dkl}\chi_{Dlk}+\phi_{Vij}\phi_{Vji}\chi_{Vkl}\chi_{Vlk})+c_3(\phi_{Dij}\phi_{Djk}\chi_{Dkl}\chi_{Dli}+\phi_{Vij}\phi_{Vjk}\chi_{Vkl}\chi_{Vli})\\
&+c_4(\phi_{Dij}\phi_{Djk}\chi_{Dki})+\phi_{Vij}\phi_{Vjk}\chi_{Vki})+c_5(\mathrm{Tr}[(\phi_{Vik}\chi_{Vkm}-\chi_{Vil}\phi_{Vlm})^2]+\mathrm{Tr}[(\phi_{Dik}\chi_{Dkm}-\chi_{Dil}\phi_{Dlm})^2]).
\end{aligned}\tag{18}$$

The addition of the cubic term is necessary for the pattern of symmetry breaking we have chosen. This differs from the toy model cases where an additional $\mathbb{Z}_2$ symmetry protected the potentials from such cubic terms. Relaxing this condition still allows for asymmetric solutions for the VEVs of the two sectors, however, as discussed in the Appendix. For the sake of simplicity we also set the parameters c_3, c_4, c_5 to be zero as large values will remove the asymmetric VEV structure. The analysis can be simplified by transforming the fields into a simplified VEV form. For the adjoint representation this becomes a block diagonal matrix with each block being a 2×2 antisymmetric matrix. For the 54 we have a traceless diagonal matrix. For the region of parameter space discussed in the Appendix, the potential is minimized with VEVs

$$\langle\phi_V\rangle = M_i\begin{pmatrix}
0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
-a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -a & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -a & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -a & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -a & 0
\end{pmatrix}$$

$$\langle\phi_D\rangle = 0$$

$$\langle\chi_V\rangle = 0$$

$$\langle\chi_D\rangle = M_j\begin{pmatrix}
b & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & b & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & b & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & b & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & b & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & b & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & c & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & c & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c
\end{pmatrix}. \tag{19}$$

In the above $SO(10)_V$ breaks by the VEV of the **54** to $SO(6)\times SO(4)\sim SU(4)\times SU(2)\times SU(2)$ and the **45** serves to break $SO(10)_D$ to $SU(5)\times U(1)$[37,38]. Following this symmetry breaking we would then need additional Higgs multiplets to break each of the gauge groups to SU(3) color theories after which the running couplings will be parallel. Due to the complexity of analyzing potentials with increasing numbers of large dimensional Higgs multiplets we leave such detailed models to more specific theories. We have, however, completed our stated objective of constructing an SO(10) asymmetric potential, built according to the principles of ASB, and showing that by choosing the breaking scales in the two sectors and the breaking chains listed in the previous section, asymmetric potentials can be constructed such that exactly that scenario is the minimum of the potential. In the next section we attempt to generalize such possibilities for supersymmetric models. We specifically look at the general case of real representations which we can examine in an illustrative model.

VI. SUPERSYMMETRIC THEORIES

As in [4] this analysis is predicated on the unification of coupling constants and for this, among other reasons such as the gauge hierarchy problem, we will explore supersymmetric varieties of these models in this section. As discussed in [4], Supersymmetric ASB requires more fields than the non-SUSY case, specifically gauge singlets. Here we will outline a general scheme to create asymmetric symmetry breaking chains from the superpotential. In general additional fields are required to allow for the scalar potential to have the necessary terms that drive ASB since only including nonsinglet Higgs multiplets does not allow us to couple fields from the different sectors at all in the scalar potential. The method that we outline below is not necessarily the simplest way to generate such breaking for any specific choice of representations or breaking chains; indeed for many simple models as few as one additional singlet is required [4]. The purpose of this discussion is to provide an existence proof that for any symmetry breaking chain we may consider in Sec. VII, a scalar potential can be created which allows for such a vacuum solution.

We wish to consider a supersymmetric extension to the argument of Sec. V wherein pairs of Higgs multiplets can be added one at a time to a model in a $\mathbb{Z}_2$ symmetric manner while allowing us to choose which sector its VEVs will favor by appropriate choice of couplings. Take the case of the fields $H_{1V}, H_{1D}, H_{2V}, H_{2D}, H_{3V}, H_{3D}, X_1, X_2, Y_1, Y_2, Z_1, Z_2, \phi, \theta$, where under the $\mathbb{Z}_2$ symmetry

$$\begin{aligned} &X_1 \leftrightarrow X_2, \qquad Y_1 \leftrightarrow Y_2, \\ &Z_1 \leftrightarrow Z_2, \qquad \phi \leftrightarrow \phi, \qquad \theta \leftrightarrow \theta. \end{aligned} \tag{20}$$

We then consider the general, renormalizable, gauge invariant superpotential that respects the $\mathbb{Z}_2$ symmetry between the sectors. In this case we are assuming that the Higgs multiplets form real representations though a similar argument likely exists for complex representations as well. We do not write down all of the terms in such a superpotential, only those which directly contribute to the ASB terms as in Eq. (14):

$$\begin{aligned} W = {} & \rho_1(H_{2V}^2\phi + H_{2D}^2\phi) + \rho_2(H_{2V}^2 Y_1 + H_{2D}^2 Y_2) + \rho_3(H_{1V}^2 Z_2 + H_{1D}^2 Z_1) + \rho_4(H_{2V}^2 Z_2 + H_{2D}^2 Z_1) \\ & + \rho_5(H_{1V}^2\theta + H_{1D}^2\theta) + \rho_6(H_{1V}^2 X_1 + H_{1D}^2 X_2) + \rho_7({Z_1}^3 + {Z_2}^3) + \rho_8\theta^3 + \rho_9\phi^3 \\ & + \rho_{10}({X_1}^3 + {X_2}^3) + \rho_{11}({Y_1}^3 + {Y_2}^3) + \rho_{12}(H_{3V}^2 X_2 + H_{3D}^2 X_1) + \rho_{13}(H_{3V}^2 Y_1 + H_{3D}^2 Y_2) \\ & + \rho_{14}(H_{3V}^2\theta + H_{3D}^2\theta) + \rho_{15}(H_{3V}^2\phi + H_{3D}^2\phi) + \dots \end{aligned} \tag{21}$$

The scalar potential then comes from the sum of soft terms and $W^{i*}W_i$ where we ignore the D-terms for this analysis, though in general such terms will add positive definite quartic interactions among those fields which are nonsinglets which will not negatively affect the results discussed here. We examine the extreme case of the parameter space where the terms shown dominate and all other parameters in the superpotential are at or very close to zero. In this case the scalar potential minimally contains only those terms that would exist without the purely singlet fields, as in Eq. (14) and which are necessary for ASB, in addition to a number of other terms which contain the purely singlet fields. If the sum of the soft mass terms and mass terms from the superpotential F-terms for the singlet fields X, Y, Z, θ, ϕ is positive then these fields can maintain a VEV of zero at the minimum. In this case the dependencies among the remaining fields is entirely that of N pairs of fields under the $\mathbb{Z}_2$ symmetry exactly like that of Sec. V where the symmetry breaking of added fields can be chosen by the couplings to the previously added fields and we only have quartic and quadratic terms to deal with. Again we have that the symmetry breaking of an added multiplet such as H_3 can be chosen by its coupling strength to previously added fields, in this case the X and Y fields. In Eq. (21) we have chosen to include the couplings for which, upon taking the derivatives with respect to X_1, Y_1, X_2, Y_2 create the terms that follow the prescription discussed in Sec. V. If one wished to have the field H_3 break similarly to H_2 instead of H_1 we simply reduce the magnitude of the parameters $\rho_{12,13}$ and replace them with larger couplings for the terms $(H_{3V}^2 X_1 + H_{3D}^2 X_2)$ and $(H_{3V}^2 Y_2 + H_{3D}^2 Y_1)$ which were previously among the omitted terms. Z_1 and Z_2 set the initial asymmetry between the first two pairs of fields $H_{1V,D}$ and $H_{2V,D}$ while the F-terms from θ and ϕ create the remaining couplings in the ρ terms from Eq. (14). The magnitude of the VEVs of the Higgs multiplets will, however, depend on the size of the soft mass terms that we add and so it may be difficult to construct models with very different mass scales. This may, however, work to our benefit as large differences in the values of Λ_{QCD} can be generated in short ranges if the difference in the beta functions is large. One can take this example as a proof of concept that asymmetric models of any number of Higgs multiplets can be built in SUSY with the addition of singlet fields. Now that we have demonstrated such possible models in both supersymmetric and nonsupersymmetric cases we will move on to displaying the numerical results for the dark confinement scale for different choices of representations of the Higgs multiplets.

VII. DARK QCD SCALE FROM ASYMMETRIC SYMMETRY BREAKING

We first consider the set of models with just one intermediate scale which allows just one energy range over which the beta functions of the two SU(3) groups differ. In this case we are, thus, only considering models where the group in the dark sector has a larger beta function. We consider both SUSY and non-SUSY models here since for this part of the analysis the only discerning feature is the size of the beta functions which for the SUSY case contains supersymmetric partners to consider as per Eq. (2). We take the unification point to be where both sectors become $SO(10)$, the GUT scale M_X in our context. We can, however, have cases where the dark sector remains as an SO(10) for the range between M_X and the intermediate scale M_I while the visible sector changes group. The analysis is the same with the intermediate gauge group of the dark sector being simply SO(10).

There are three possibilities for the visible sector's QCD parent group. It can remain SU(3) up until M_X while the dark sector changes at M_I or it can become $SU(4)$ or $SU(5)$ at the M_I and continue to the unification point. For the dark sector group we examined the cases of the chains from Sec. IV. For the case of just two scales M_X and M_I we plot the ratio of confinement scales by using Eq. (7). We look at the scale M_I and the difference between the two scales $\delta M = M_X - M_I$. We display in Figs. 2 and 3 the minimal and maximal cases in terms of group choice, that is the largest and smallest difference in beta functions for each of the possible breaking chains in the VS. The color scale of each graph gives the ratio $\frac{\Lambda_D}{\Lambda_V}$. We see in these figures that quite a large range in the distance between the breaking

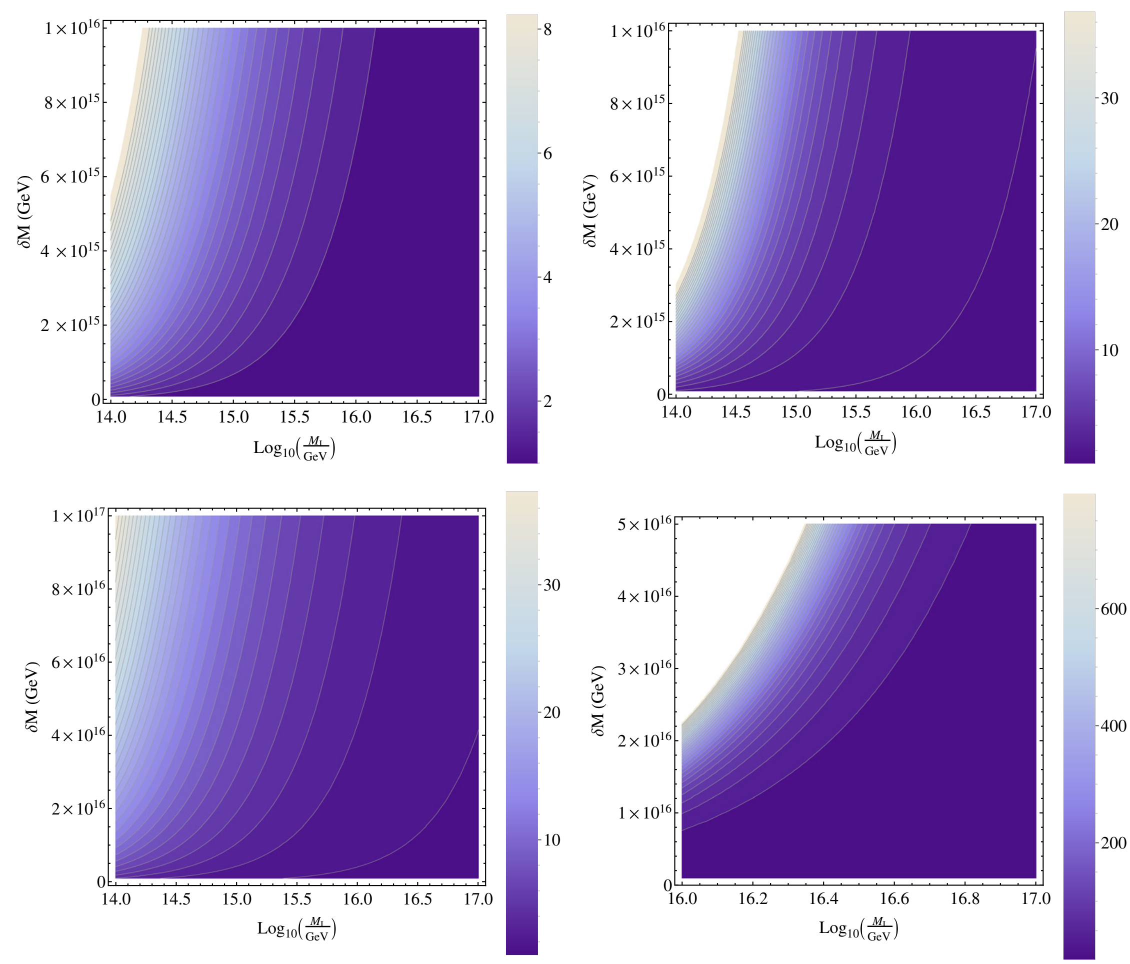

FIG. 2 (color online). The ratio of confinement scales Λ_D/Λ_V in the two sectors for a sample of nonsupersymmetric breaking chains. The top left figure is generated from $SU(3)_V$ and $SU(4)_D$ as the groups above the scale M_I. The top right features $SU(3)_V$ and $SU(5)_D$ above M_I while the bottom left has $SU(4)_V$ and $SU(5)_D$ followed by the bottom right with $SU(3)_V$ and $SO(10)_D$. In each graph the vertical scale is δM while the horizontal scale is M_I below which both sectors contain SU(3) subgroups.

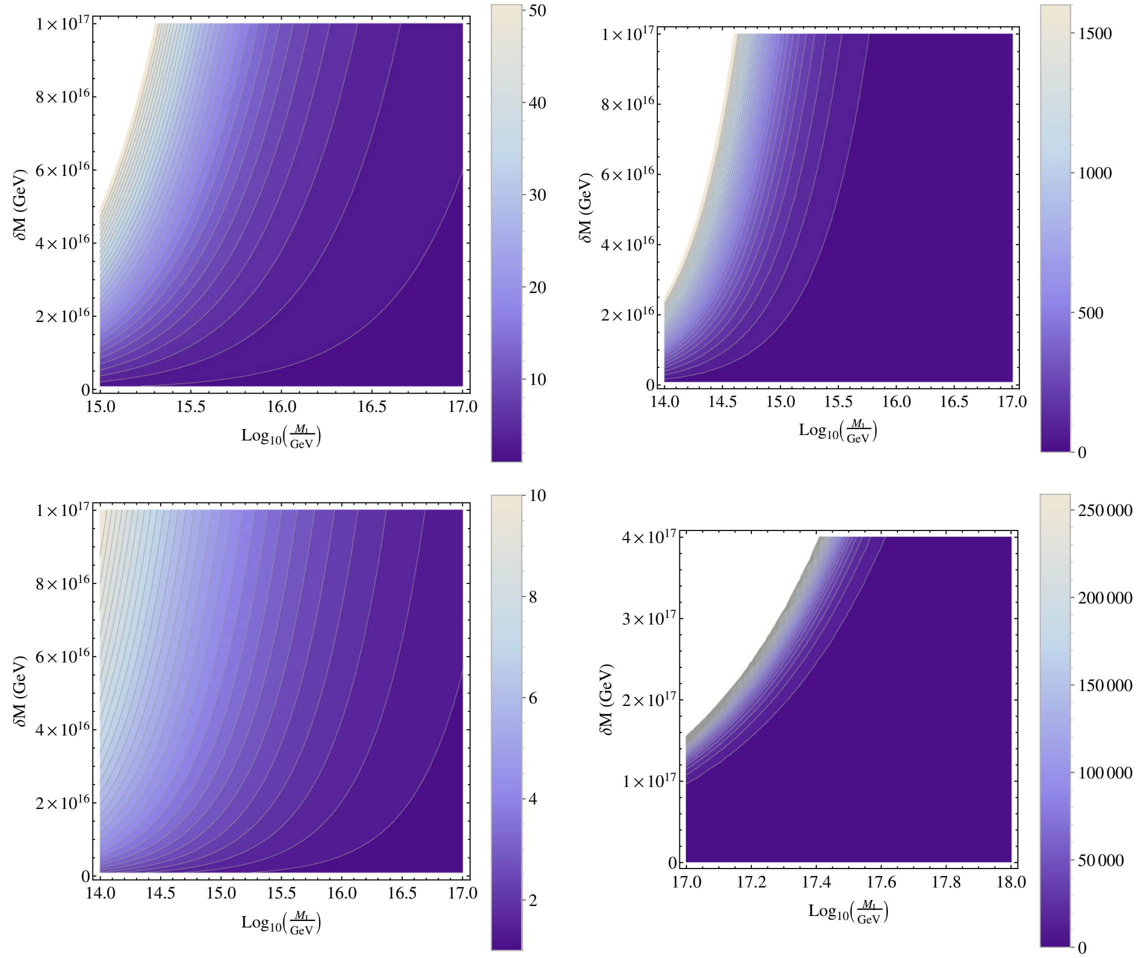

FIG. 3 (color online). The ratio of confinement scales Λ_D/Λ_V in the two sectors for a sample of supersymmetric breaking chains. The top left figure is generated from $SU(3)_V$ and $SU(4)_D$ as the groups above the scale M_I. The top right features $SU(3)_V$ and $SU(5)_D$ above M_I while the bottom left has $SU(4)_V$ and $SU(5)_D$ followed by the bottom right with $SU(5)_V$ and $SO(10)_D$.

scales is acceptable if the beta functions are not very different in size, for example in the case of SU(3) and SU(4). The magnitude of this difference may be smaller depending on the particle content of a specific theory though the $\mathbb{Z}_2$ symmetry between the sectors prevents these matter terms in Eq. (2) from generating large differences. For the limiting case of SU(3) and SO(10), on the other hand, we have a much more constrained parameter space for the choice of breaking scales.

In these cases the results follow from that of the one intermediate scale case, that is, the final difference in the confinement scales is a function of length of the range over which the couplings run at different rates, and the magnitude of the difference between the beta functions. Because of this it is possible to create a dark sector with an acceptable confinement scale for any breaking chain that is needed to satisfy visible sector GUT constraints. For example, if a specific model requires a large range between the $SO(10)$ scale M_X and the $SU(5)$ scale M_I in a theory like that of breaking chain IV, then we can choose the scale that the dark sector breaks to SU(5) to be similar to M_X and run as SU(5) down to a lower scale than M_I. We have seen that there are a large number of possible cases for the breaking chains in each sector where the confinement scale in the dark sector is just larger than that of the visible sector. We have, however, been treating our GUT scale M_X and intermediate scale M_I as free parameters and so in the next section we will look to constraining the realistic models and look towards possible future work in this area.

VIII. PHENOMENOLOGICAL CONSTRAINTS

The methods detailed here for generating dark sectors with baryons of a mass scale just above that of the proton are generalizable to many breaking chains and GUT models, not all of which will satisfy phenomenological constraints such as current proton decay limits. Here we briefly review some of the recent SO(10) GUT models which can satisfy proton decay constraints in the visible sector. Proton decay bounds typically bounds push the

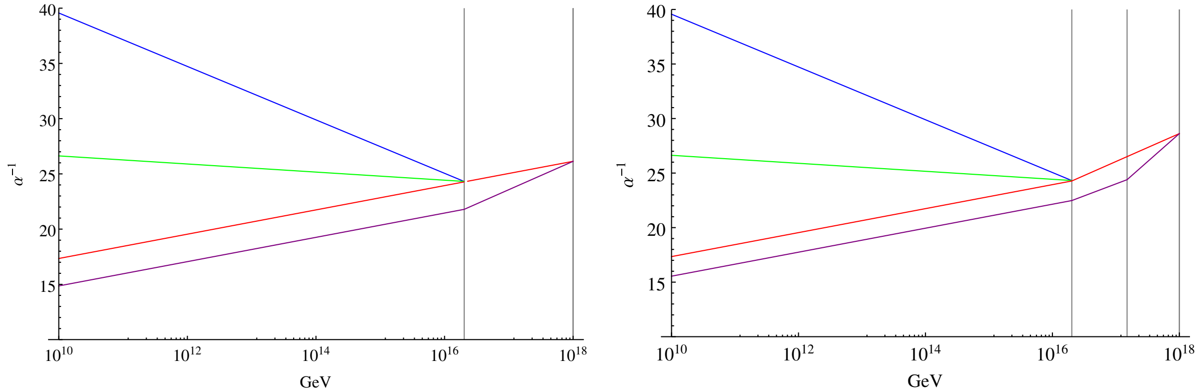

FIG. 4 (color online). Supersymmetric model with one and two intermediate scales. In the left plot we have $SU(5)$ in the visible sector and $SO(8)$ in the dark above the scale $M_X \approx 10^{16}$ GeV while the right plot shows $SU(5)$ in the visible sector and $SO(8)$ breaking to $SU(4)$ at the scale $M_J \approx 10^{17}$ GeV in the dark sector. Each graph displays the running coupling of the SM forces $(\alpha_1, \alpha_2, \alpha_3)$ from top to bottom and that of the color force in the dark sector (the lowest line). The value of the dark confinement scale is 4.1 and 1.9 GeV for the left and right cases, respectively.

scale of unification in SU(5) theories up to energy regimes consistent with the unification of the gauge coupling constants. In some works such as [39] the SU(5) scale is as low as $M_X \approx 4 \times 10^{15}$ after the addition of extra Higgs multiplets. In particular we examine some of the recent work on proton decay constraints in GUT models from [40,41] where while minimal SU(5) theories are ruled out, supersymmetric SU(5) theories may still be viable while both supersymmetric and nonsupersymmetric SO(10) models can generate cases where proton decay is within experimental limits. We have not gone into any depth on any specific choice of representations in this paper so it remains an open question how a particular model of ASB can work in the context of these phenomenological constraints. The construction of realistic models also requires the unification of the coupling constants which places strict constraints on the scale at which the visible sector's QCD parent group starts. We examine such examples for the minimal supersymmetric standard model (MSSM) running and a non-SUSY case. Below we examine the development of a dark QCD in an extension of this model where the SM gauge couplings unify at an intermediate scale and the two sectors unify closer to the Planck scale. Figure 4 shows the case where we have chain IV in the VS and chain X in the dark sector. This could be accomplished with **45** and **16** or **126** Higgs multiplets in the VS, together with a **54** or **210**′ and **16** or **126** in the DS. Figure 5 shows the direct breaking $SO(10) \to SU(3)$ for the color force in the VS and chain XII in the DS which was discussed in Sec. III.

In the MSSM, once we have fixed the scale at which the VS SU(3) is absorbed into SU(5), M_X and any intermediate scale of the dark sector can then be treated as free parameters to generate the dark confinement scale. For the non-SUSY case we examine the work of [42] in which

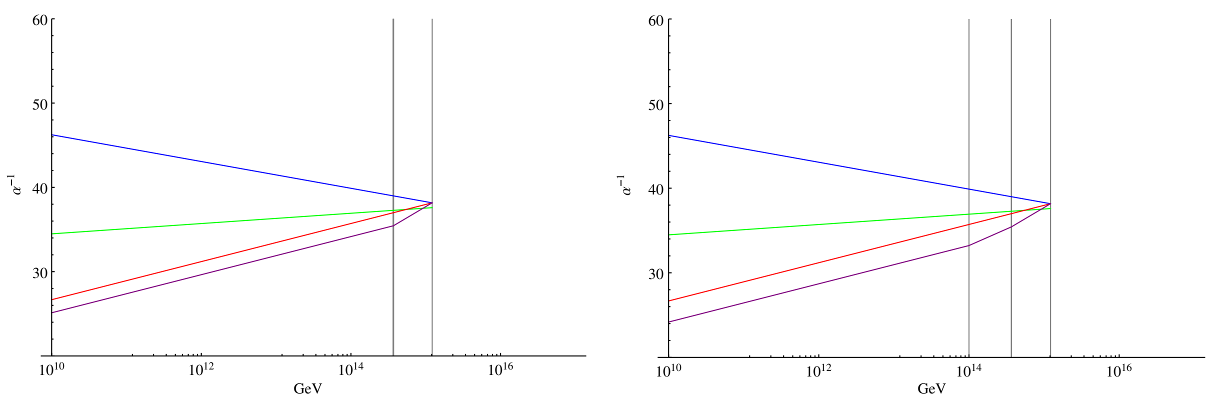

FIG. 5 (color online). Nonsupersymmetric model with one and two intermediate scales. In the left plot we have $SU(3)$ in the visible sector and $SU(5)$ in the dark sector above the scale $M_X \approx 10^{15}$ GeV while the right plot shows $SU(3)$ in the visible sector and $SU(5)$ breaking to $SU(4)$ at a scale $M_J > M_I$ in the dark sector. Each graph displays the running coupling of the SM forces $(\alpha_1, \alpha_2, \alpha_3)$ from top to bottom and that of the color force in the dark sector(the lowest line). The value of the dark confinement scale is 3.2 GeV and 2.5 GeV respectively.

a non-SUSY SO(10) model with a color sextet allows for the unification of the gauge coupling constants. In this case we can also examine a two step process which has one segment working to diverge the couplings after SO(10) breaking, while the next part of the breaking regime brings the couplings closer again to result in a dark QCD scale just 1 order of magnitude greater than the SM for breaking scales which span over 4 orders of magnitude.

One could examine a limitless number of such models in this context extending the number of breaking scales; however, we can see that for almost any choice in the number of such scales and breaking chains in the VS, a model can be constructed which allows a dark confinement scale through the effect of ASB. In this sense it would be interesting to move on to developing a detailed model which resolves a significant number of other issues associated with GUTs in the VS and then adapt it to an ASB model in the pursuit of explaining DM also. A full theory of baryogenesis in the two sectors can also place strict limits on the size of these intermediate scales particularly in the case of baryogenesis via leptogenesis or GUT baryogenesis where the symmetry breaking scale can affect the amount of baryon number violation in the early Universe. In addition to these constraints we must also consider the current DM constraints on self interaction where the bullet cluster observation sets results on self interaction for nucleon-nucleon–like scattering in [43]. Such nucleonlike scattering has a cross section of $\sigma \sim 10^{-26}$ cm^2 and can be compared to the upper bound of the DM self-interaction cross section $\leq 10^{-23}$ cm^2 [43–45]. In the cases we have considered we were only concerned with maintaining an SU(3) symmetry in the DS and so in many of these models the DM candidate only interacts with itself through short range strong forces and gravity. Such neutral baryon DM particles are, thus, compatible with current detection limits.

IX. CONCLUSIONS

The similarity in the abundances of visible and dark matter leads us to suppose that their origin is not independent but rather that the production of both the number density of DM particles in the early Universe and the DM mass is a result of processes that are deeply connected to the standard model. Mirror symmetric grand unified theories offer a plausible solution to both of these problems by supposing that DM is a result of a hidden sector whose complexity is at least as large as our own yet is derived from an underlying theory that places the two components of the Universe on exactly the same footing in the distant past. By assuming that the mass scale of DM comes from the confinement scale of a dark SU(3), this connection in masses can be realized through grand unified theories. Where mirror symmetric models can give the similarity by having the dark sector be an exact copy of the SM, ASB allows us to establish a much larger set of theories of the Universe which contain completely symmetric sectors whose GUT groups break to subgroups that are necessarily different allowing for the dynamics of visible and dark sectors to appear completely different in the low-energy regime while having the appealing concept of an underlying theory based on symmetries. In this work we have expanded these asymmetric models to allow for SO(10) models to provide the similarity in visible and dark baryon mass by having multistep breaking chains in mirror GUT scenarios give altered coupling constant evolution in the range between intermediate scales and the GUT scale. We have demonstrated a specific $SO(10) \times SO(10)$ potential that breaks the symmetry asymmetrically to allow for such divergences in the coupling constant evolutions. The couplings, once gaining an altered value in the intermediate range, then run parallel all the way down to the low-energy scale of the present day Universe where the divergence of the QCD coupling in the two sectors occurs at separate but similar scales. This, combined with the theory of ADM, has the potential for a natural explanation of the apparently coincidental ratio $\Omega_{\rm DM} \simeq 5\Omega_{\rm VM}$ that characterizes the matter of our Universe.

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APPENDIX: SCALAR POTENTIALS FOR NONSUPERSYMMETRIC SO(10) × SO(10) MODELS

In this appendix we expand on the potential discussed in Sec. V. We first consider the two representations of SO(10) independently. These are the adjoint **45**, denoted by ϕ_{ij} which can be formed from the antisymmetric product of two fundamental representations, and the **54** which we label χ_{ij} which is formed from the completely symmetric product of two fundamentals. The most general quartic potential for a rank two antisymmetric tensor in SO(10) is

$$-\frac{\mu^2}{2}\phi_{ij}\phi_{ji} + \frac{\lambda}{4}(\phi_{ij}\phi_{ji})^2 + \frac{\alpha}{4}\phi_{ij}\phi_{jk}\phi_{kl}\phi_{li}. \quad \text{(A1)}$$

For this potential the symmetry breaking pattern is as follows. For $\lambda > 0$ and $\alpha > 0$, we have

$$SO(10) \to SU(5) \times U(1), \quad \text{(A2)}$$

while for $\lambda > 0$ and $\alpha < 0$, we find

$$SO(10) \to SO(8) \times U(1). \quad \text{(A3)}$$

In the case of the symmetric rank-two representation, we have a similar equation but with the added cubic term $\mathrm{Tr}(\chi^3)$ so that the potential reads

$$-\frac{\mu^2}{2}\chi_{ij}\chi_{ji} + \frac{\lambda}{4}(\chi_{ij}\chi_{ji})^2 + \frac{\alpha}{4}\chi_{ij}\chi_{jk}\chi_{kl}\chi_{li} + \frac{\beta\mu}{3}\chi_{ij}\chi_{jk}\chi_{ki}. \quad \text{(A4)}$$

For this potential the parameter space is such that without the cubic term the possible breaking chains are, when $\lambda < 0$ and $\alpha < 0$,

$$SO(10) \to SO(9), \quad \text{(A5)}$$

while for $\lambda > 0$ and $\alpha > 0$, we have

$$SO(10) \to SO(5) \times SO(5). \quad \text{(A6)}$$

For the parameter space where $\lambda > 0$ and $\alpha > 0$ and the cubic term is nonzero, we have

$$SO(10) \to SO(10-n) \times SO(n), \quad \text{(A7)}$$

where for values of $\beta = 0$ we recover the above result in Eq. (A6) and for $\beta > 0$, n increases as β does until the breaking chain of Eq. (A5) is recovered. The generation of the potential in Eq. (18) then results from the addition of the two potentials given above in Eq. (A1) and Eq. (A4), as well as the analogue terms of the toy potential that mix the fields of the two sectors and the new nontrivial same-sector contractions afforded by the choice of the **45** and **54** representations. Using this potential our numerical results align with the expected minima from the above potentials in the case where the cross terms and the additional cubic terms are sufficiently small. For the choice of parameter space where $\lambda_\phi > 0, \alpha_\phi > 0, \lambda_\chi > 0, \alpha_\chi > 0, \beta > 0, \kappa_\phi > 0, \kappa_\chi > 0, c_2 > c_1 > 0, c_3 \ll c_2, c_4 \ll c_2, c_5 \ll c_2$ we will obtain a potential which breaks asymmetrically with the specific choice of breaking chain for each sector. This agrees with our numerical analysis where for a sample choice of parameters,

$$\begin{aligned} &\lambda_\phi \simeq 1, \quad \kappa_\phi \simeq 0.75, \quad \kappa_\chi \simeq 0.75, \quad \lambda_\chi \simeq 1.6, \\ &\mu_\phi \simeq 1, \quad \mu_\chi \simeq 1, \quad \alpha_\phi \simeq 0.5, \quad \alpha_\chi \simeq 1, \\ &\beta_\chi \simeq 0.35, \quad c_1 \simeq 0.25, \quad c_2 \simeq 0.75, \\ &c_3 \simeq 0, \quad c_4 \simeq 0, \quad c_5 \simeq 0, \end{aligned}$$

we find that minimum preserves the VEVs

$$\langle\phi_V\rangle = 0.3\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\langle\phi_D\rangle = 0$$

$$\langle\chi_V\rangle = 0$$

$$\langle\chi_D\rangle = 0.3\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -6/4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6/4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6/4 \end{pmatrix}, \quad \text{(A8)}$$

which breaks the symmetry according to

$$SO(10)_V \times SO(10)_D \to [SU(4) \times SU(2) \times SU(2)]_V \times [SU(5) \times U(1)]_D. \quad \text{(A9)}$$

The analysis discussed here describes just the first step in asymmetrically breaking an SO(10) mirror symmetric potential to different subgroups for each sector and at different energy scales. While many other possible breaking chains that have been discussed in this paper could be analyzed, we leave such work to future efforts to create a detailed model of an SO(10) GUT model where the choice of representations aligns with choices for fermion mass generation models and considerations of minimality. Due to the complexity in analyzing such Higgs potentials for large gauge groups, we content ourselves at the present juncture with the demonstration of the versatility of such asymmetric symmetry breaking in the context of GUT models. With this specific example and the principles given in the toy model, many of the other breaking chains could be realized in potentials constructed in a like manner.

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