Yang-Mills theory in the maximal Abelian gauge in presence of scalar matter fields

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We address the issue of the all order multiplicative renormalizability of $SU(2)$ Yang-Mills theories quantized in the maximal Abelian gauge in presence of scalar matter fields. The nonlinear character of the maximal Abelian gauge requires the introduction of quartic interaction terms in the Faddeev-Popov ghosts, a well-known feature of this gauge. We show that, when scalar matter fields are introduced, a second quartic interaction term between scalar fields and Faddeev-Popov ghosts naturally arises. A Becchi-Rouet-Stora-Tyutin invariant action accounting for those quartic interaction terms is identified and proven to be multiplicative renormalizable to all orders by means of the algebraic renormalization procedure.

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I. INTRODUCTION

Nowadays, the maximal Abelian gauge [1–3] is widely employed in order to investigate nonperturbative aspects of Yang-Mills theories. This gauge turns out to be suitable for the study of the dual superconductivity mechanism for color confinement [4], according to which Yang-Mills theories in the low energy region should be described by an effective Abelian theory [5–9] in the presence of monopoles. The condensation of these magnetic charges leads to a dual Meissner effect resulting in quark confinement. In the maximal Abelian gauge, the Abelian configuration is identified with the diagonal components A^3_μ of the gauge field corresponding to the diagonal generator of the Cartan subgroup of $SU(2)$. The remaining off-diagonal components A^a_μ, $a = 1, 2$, corresponding to the off-diagonal generators of $SU(2)$, are expected to acquire a mass through a dynamical mechanism, thus decoupling at low energies. This phenomenon is known as Abelian dominance and is the object of intensive investigation, both from analytic and from numerical lattice simulations.

From the analytic side, evidence for the dynamical mass generation for the off-diagonal components of the gauge field can be found in [10–12], while [13–15] are devoted to numerical studies.

Besides being a renormalizable gauge [16–18], the maximal Abelian gauge enjoys the important property of exhibiting a lattice formulation [13–15,19,20], a property which allows us to compare analytic and numerical results. In particular, this important feature of the maximal Abelian gauge has made possible the study, from the numerical lattice point of view, of the behavior of the two-point gluon correlation function in the nonperturbative infrared region, providing evidence for the Abelian dominance as well as for the confining character of the propagator of the Abelian gluon component [13–15,19,20]. This issue has also been addressed through analytical methods by taking into account the existence of the Gribov copies [21] which, as in any covariant and renormalizable gauge, affect the maximal Abelian gauge [22–24]. Here, proceeding in a way similar to the Landau gauge [25,26], a few properties of the so-called Gribov region have been derived together with the restriction of the domain of integration in the functional integral to the Gribov horizon; see for instance Refs. [27–31] for the details of the Gribov issue on the maximal Abelian gauge. Remarkably, the agreement between the lattice numerical results and the analytic calculations based on the restriction to the Gribov region looks quite good [19,29], confirming the expectation that the study of the Gribov problem is of great relevance for gluon confinement.

Nevertheless, so far, the study of the correlation function in the maximal Abelian gauge has been done only for the gluon sector, without including matter fields, i.e. spinor and scalar fields. To our knowledge, unlike the Landau gauge, no available nonperturbative studies of the two-point matter correlation functions are available in the maximal Abelian gauge, and this from analytical results and numerical data simulations.

This work aims at starting an analytic study of the nonperturbative behavior of the correlation functions for matter fields in the maximal Abelian gauge, along the lines recently outlined in the case of the Landau gauge [32–34], where it has been possible to recover the behavior of the propagators for scalar and spinor fields observed in lattice simulations [35–38] from an analytic point of view [34]. This study might be of relevance for several reasons, such as, for instance, to investigate to what extent the Abelian dominance affects the matter sector, to make a prediction for the propagator of scalars and quark fields which might be compared with lattice numerical simulations, or to study the confining character of the correlation functions. As a first

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step in this endeavor, we need to establish the all orders multiplicative renormalizability of the maximal Abelian gauge in presence of matter fields, a topic which, till now, has not yet been addressed. This is the goal of the present paper. Although the renormalizability of the maximal Abelian gauge in presence of the matter fields is an expected feature, we shall see that it is not a straightforward matter, requiring in fact a nontrivial analysis. This is due to the nonlinear character of the maximal Abelian gauge which gives rise to a rather complex Faddeev-Popov operator. It was already pointed out that the structure of this operator requires the introduction of a quartic interaction between ghosts [16–18]. Only at the very end of the whole renormalization process can the gauge parameter entering the quartic interaction be set to zero [16–18], thus recovering the genuine maximal Abelian gauge condition. In this work, we shall see that this feature generalizes to the case of scalar matter fields; i.e. a quartic interaction between scalar fields and Faddeev-Popov ghosts naturally arises due to the nonlinearity of the gauge condition. As a consequence, a second gauge parameter associated to this new term has to be introduced. As in the case of the quartic ghost term, this second gauge parameter can be set to zero only at the very end of the renormalization process.

The present work is organized as follows. In Sec. II we briefly discuss the maximal Abelian gauge and the corresponding gauge fixing. In Sec. III we elaborate on the quartic interactions required to renormalize the theory. Section IV is devoted to establishing the set of Ward identities needed for the all orders proof of the renormalizability. In Sec. V we present the algebraic characterization of the most general invariant local counterterm, establishing the all orders multiplicative renormalizability of the theory. Section VI collects our conclusions.

II. QUANTIZING GAUGE THEORIES IN THE MAXIMAL ABELIAN GAUGE

In order to introduce the maximal Abelian gauge, we start by considering a Lie algebra valued gauge field $\mathcal{A}_\mu$ for the gauge group $SU(2)$, whose generators $T^A (A = 1, .., 3)$

$$[T^A, T^B] = \varepsilon^{ABC} T^C \tag{1}$$

are chosen to be anti-Hermitian and to obey the orthonormality condition $\mathrm{Tr}(T^A T^B) = \delta^{AB}$. Following [1–3] we decompose $\mathcal{A}_\mu$ into off-diagonal and diagonal components

$$\mathcal{A}_\mu = \mathcal{A}^A_\mu T^A = A^a_\mu T^a + A_\mu T^3, \tag{2}$$

where $a = 1, 2$ and T^3 is the diagonal generator of the Cartan subgroup sf $SU(2)$. Analogously, decomposing the field strength, we obtain

$$\mathcal{F}_{\mu\nu} = \mathcal{F}^A_{\mu\nu} T^A = F^a_{\mu\nu} T^a + F_{\mu\nu} T^3, \tag{3}$$

with the off-diagonal and diagonal components given, respectively, by

$$\begin{aligned} F^a_{\mu\nu} &= D^{ab}_\mu A^b_\nu - D^{ab}_\nu A^b_\mu, \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + g\varepsilon^{ab} A^a_\mu A^b_\nu, \end{aligned} \tag{4}$$

where the covariant derivative D^{ab}_μ is defined with respect to the diagonal component A_μ

$$D^{ab}_\mu \equiv \partial_\mu \delta^{ab} - g\varepsilon^{ab} A_\mu, \qquad \varepsilon^{ab} \equiv \varepsilon^{ab3}. \tag{5}$$

For the classical gauge invariant starting action, we have

$$S_{\mathrm{cl}} = S_{\mathrm{YM}} + S_{\mathrm{matter}}, \tag{6}$$

where S_{YM} stands for the Yang-Mills action

$$S_{\mathrm{YM}} = \int d^4x \frac{1}{4} (F^a_{\mu\nu} F^a_{\mu\nu} + F_{\mu\nu} F_{\mu\nu}), \tag{7}$$

while S_{matter} denotes the action of real scalar matter fields in the adjoint representation of the gauge group $SU(2)$, namely

$$\begin{aligned} S_{\mathrm{matter}} &= \int d^4x \left(\frac{1}{2} (D^{AB}_\mu \phi^B)^2 + \frac{m^2_\phi}{2} \phi^A \phi^A + \frac{\lambda}{4!} (\phi^A \phi^A)^2 \right) \\ &= \int d^4x \bigg\{ \frac{1}{2} (\partial_\mu \phi^a)(\partial_\mu \phi^a) + \frac{1}{2} (\partial_\mu \phi)(\partial_\mu \phi) - g^2 \varepsilon^{ab} [(\partial_\mu \phi)\phi^a A^b_\mu - (\partial_\mu \phi^a)\phi A^b_\mu + (\partial_\mu \phi^a)\phi^b A_\mu] \\ &\quad + \frac{g^2}{2} [A^a_\mu A^a_\mu (\phi^b \phi^b + \phi\phi) + A_\mu A_\mu \phi^a \phi^a - A^a_\mu A^b_\mu \phi^a \phi^b - 2A^a_\mu A_\mu \phi^a \phi] \\ &\quad + \frac{m^2_\phi}{2} (\phi^a \phi^a + \phi\phi) + \frac{\lambda}{4!} [(\phi^a \phi^a)^2 + 2\phi^2 \phi^a \phi^a + \phi^4] \bigg\}, \end{aligned} \tag{8}$$

where, as in Eq. (2), the scalar field $\Phi = \phi^A T^A$ is decomposed into off-diagonal and diagonal components, i.e.

$$\phi^A T^A = \phi^a T^a + \phi T^3. \tag{9}$$

The classical action (6) is left invariant by the gauge transformations

$$\delta A^a_\mu = -D^{ab}_\mu \omega^b - g\varepsilon^{ab} A^b_\mu \omega, \quad \delta A_\mu = -\partial_\mu \omega - g\varepsilon^{ab} A^a_\mu \omega^b, \tag{10}$$

and

$$\delta\phi^a = g\varepsilon^{ab}\phi\omega^b - g\varepsilon^{ab}\phi^b\omega, \qquad \delta\phi = -g\varepsilon^{ab}\phi^a\omega^b. \tag{11}$$

The maximal Abelian gauge condition amounts to imposing that the off-diagonal components A^a_μ of the gauge field obey the following nonlinear condition

$$D^{ab}_\mu A^b_\mu = 0, \tag{12}$$

which follows by requiring that the auxiliary functional,

$$\mathcal{R}[A] = \int d^4x A^a_\mu A^a_\mu, \tag{13}$$

be stationary with respect to the gauge transformations (10). Moreover, as it is apparent from the presence of the covariant derivative D^{ab}_μ, Eq. (12) allows for a residual local $U(1)$ invariance corresponding to the diagonal subgroup of $SU(2)$. This additional invariance has to be fixed by means of a further gauge condition on the diagonal component A_μ, which is usually chosen to be of the Landau type, namely

$$\partial_\mu A_\mu = 0. \tag{14}$$

The Faddeev-Popov operator, $\mathcal{M}^{ab}$, corresponding to the gauge condition (12) is easily derived by taking the second variation of the auxiliary functional $\mathcal{R}[A]$, being given by

$$\mathcal{M}^{ab} = -D^{ac}_\mu D^{cb}_\mu - g^2\varepsilon^{ac}\varepsilon^{bd} A^c_\mu A^d_\mu. \tag{15}$$

It enjoys the property of being Hermitian and, as pointed out in [22], is the difference of two positive semidefinite operators given, respectively, by $-D^{ac}_\mu D^{cb}_\mu$ and $g^2\varepsilon^{ac}\varepsilon^{bd} A^c_\mu A^d_\mu$.

It is worth pointing out that the operator $\mathcal{M}^{ab}$ is nonlinear in the gauge fields, a feature which has nontrivial consequences in the renormalization process.

III. BRST SYMMETRY AND EMERGENCE OF QUARTIC INTERACTION TERMS

In order to construct the Faddeev-Popov action corresponding to the gauge conditions (12), (14), we proceed by introducing the nilpotent Becchi-Rouet-Stora-Tyutin (BRST) transformations

$$\begin{aligned}
sA^a_\mu &= -(D^{ab}_\mu c^b + g\varepsilon^{ab} A^b_\mu c),\\
sA_\mu &= -(\partial_\mu c + g\varepsilon^{ab} A^a_\mu c^b)\\
sc^a &= g\varepsilon^{ab} c^b c, \qquad sc = \frac{g}{2}\varepsilon^{ab} c^a c^b,\\
s\bar{c}^a &= b^a, \qquad s\bar{c} = b,\\
sb^a &= sb = 0, \qquad s\phi^a = g\varepsilon^{ab}\phi c^b - g\varepsilon^{ab}\phi^b c,\\
s\phi &= -g\varepsilon^{ab}\phi^a c^b,
\end{aligned} \tag{16}$$

where $(\bar{c}^a, \bar{c}, c^a, c)$ are the Faddeev-Popov ghosts and (b^a, b) are the Nakanishi-Lautrup fields. Further, we introduce the s-exact gauge-fixing term

$$\begin{aligned}
S_{MAG} &= s\int d^4x\{\bar{c}^a D^{ab}_\mu A^b_\mu + \bar{c}\partial_\mu A_\mu\}\\
&= \int d^4x\{b^a D^{ab}_\mu A^b_\mu - \bar{c}^a \mathcal{M}^{ab} c^b\\
&\quad + g\varepsilon^{ab}\bar{c}^a c D^{bc}_\mu A^c_\mu + b\partial_\mu A_\mu \bar{c}\partial_\mu(\partial_\mu c + g\varepsilon^{ab} A^a_\mu c^b)\},
\end{aligned} \tag{17}$$

where $\mathcal{M}^{ab}$ stands for the Faddeev-Popov operator (15). Evidently, the gauge-fixed action

$$S_{\rm cl} + S_{MAG}, \tag{18}$$

with $S_{\rm cl}$ given in Eq. (6), turns out to be BRST invariant. The action (18) is the gauge-fixed action obtained from the BRST construction, usually taken as the starting action in order to evaluate the quantum corrections arising in the renormalization process. However, in the present case, expression (18) has to be supplemented by the introduction of further quartic terms which originate from the nonlinearity of the Faddeev-Popov operator $\mathcal{M}^{ab}$, Eq. (15). In fact, as one can observe from expression (17), the interaction term $g^2\bar{c}^a\varepsilon^{ac}\varepsilon^{bd} A^c_\mu A^d_\mu c^b$ gives rise to divergent Feynman diagrams with four external Faddeev-Popov legs, as one immediately realizes already at one-loop level by considering the divergent one Particle Irreducible (1PI) diagram with four external Faddeev-Popov ghosts and two internal off-diagonal gauge lines. As already pointed out in [16–18], such diagrams give rise to counterterms in the Faddeev-Popov ghosts which are not contained in the action (18). Such additional divergences can be taken into account by introducing the following BRST exact terms [16–18]

$$\begin{aligned}
S_\alpha &= s\int d^4x \frac{\alpha}{2}(\bar{c}^a b^a - 2g\varepsilon^{ab}\bar{c}^a\bar{c}^b c)\\
&= \frac{\alpha}{2}\int d^4x\{b^a b^a - 2g\varepsilon^{ab} b^a \bar{c}^b c + g^2\bar{c}^a\bar{c}^b c^a c^b\},
\end{aligned} \tag{19}$$

where α stands for a suitable gauge parameter. As one can easily figure out, the quartic divergent terms originating from the action (18) can now be reabsorbed in the renormalization of the gauge parameter α.

Nevertheless, the term (19) is not the unique new quartic interaction present in the theory when scalar matter fields are added. In fact, it turns out that, due to the presence of the interaction vertices $(\phi\phi AA)$ and $(\phi(\partial\phi)A)$, a novel quartic term between scalar fields and Faddeev-Popov ghosts, i.e. $(\phi\phi\bar{c}c)$, is generated at the quantum level. For example, the 1PI one-loop diagram with two external ϕ-legs and two external ghost legs connected by two internal gluon lines is logarithmic divergent, giving rise to a quartic divergent term precisely of the kind of $(\phi\phi\bar{c}c)$. Once again, such divergent terms are not contained in the action (18). As such, they would be not reabsorbable. We see therefore that, due to the nonlinearity of the gauge condition, Eq. (12), and of the Faddeev-Popov operator, Eq. (15), a second quartic term is needed for renormalizability. In the present case, this novel term is accounted for by introducing the following exact BRST expression

$$
\begin{aligned}
S_\beta &= s\int d^4x\left(\frac{\beta}{2}\varepsilon^{ab}\phi\phi^a\bar{c}^b\right)\\
&=\frac{\beta}{2}\int d^4x\{g\phi^a\phi^a c^b\bar{c}^b+g\phi^a\phi^b c^a\bar{c}^b\\
&\quad+\phi\phi^a(\varepsilon^{ab}b^b-gc\bar{c}^a)+g\phi\phi c^a\bar{c}^a\},
\end{aligned}
\tag{20}
$$

where β stands for a second gauge parameter. The emergency of divergent terms of the type $(\phi\phi\bar{c}c)$ is now taken into account by an appropriate renormalization of the second gauge parameter β. Relying only on power counting and dimensional analysis, the reader might wonder why in fact expression (20) is the only additional quartic term which is needed as, in principle, other terms can be easily constructed. Here, we have anticipated our main result which shows that expression (20) is in fact the only term which survives the whole set of Ward identities and discrete symmetries which characterize the model. The proof of this statement will be given in detail in Sec. V, where we shall discuss the construction of the most general invariant possible term compatible with all Ward identities and discrete symmetries.

In conclusion, taking into account the emergency of quartic interaction terms, for the starting gauge-fixed Faddeev-Popov action we have

$$
S=S_{\rm cl}+S_{MAG}+S_\alpha+S_\beta. \tag{21}
$$

Looking at the equations of motion of the field b^a, namely

$$
\frac{\delta S}{\delta b^a}=D^{ab}_\mu A^b_\mu+\alpha(b^a-g\varepsilon^{ab}\bar{c}^b c)+\frac{\beta}{2}g\varepsilon^{ba}\phi\phi^b, \tag{22}
$$

we see that the original maximal Abelian gauge condition (12) is recovered in the limit $\alpha,\beta\to 0$, which has to be taken at the very end of the whole renormalization process. Although being outside of the main aim of the present work, it is worthwhile to spend a few words on what we mean by taking the limit $\alpha,\beta\to 0$. To that end, let us first eliminate the Lagrange multiplier b^a by means of its equation of motion (22), yielding the following action suitable for practical calculations [18]

$$
\begin{aligned}
S=S_{\rm YM}+S_{\rm matter}+\int d^4x\Big\{&-\frac{1}{2\alpha}(D^{ab}_\mu A^b_\mu)^2+b\partial_\mu A_\mu\\
&-\bar{c}^a\mathcal{M}^{ab}c^b+\bar{c}\partial_\mu(\partial_\mu c+g\varepsilon^{ab}A^a_\mu c^b)\\
&+\frac{\alpha}{2}g^2\bar{c}^a\bar{c}^b c^a c^b+\frac{\beta}{2}g(\phi^a\phi^a c^b\bar{c}^b+\phi^a\phi^b c^a\bar{c}^b+\phi^2 c^a\bar{c}^a)\\
&-\frac{\beta}{2\alpha}g\phi\phi^b\varepsilon^{ba}D^{ac}_\mu A^c_\mu-\frac{\beta^2}{8\alpha}g^2\phi^2\phi^a\phi^a\Big\}.
\end{aligned}
\tag{23}
$$

As one immediately sees, the above expression reveals the full complexity of working with a nonlinear gauge such as the maximal Abelian gauge. In fact, already without the second gauge parameter β, one observes the appearance of the term $\frac{1}{2\alpha}(D^{ab}_\mu A^b_\mu)^2$ which, besides the standard quadratic term $\frac{1}{2\alpha}(\partial_\mu A^a_\mu)^2$, gives rise to delicate interaction terms like $\frac{1}{\alpha}(A_\mu A^a_\mu)^2$, for which the existence of the limit $\alpha\to 0$ looks not so obvious. Of course, the same feature remains when the second gauge parameter β is introduced, as one learns from the presence of the terms $\frac{\beta}{2\alpha}(\phi\phi^b\varepsilon^{ba}D^{ac}_\mu A^c_\mu)$ and $\frac{\beta^2}{8\alpha}(\phi^2\phi^a\phi^a)$ in the last line of expression (23). Nevertheless, despite the presence of these terms, all anomalous dimensions of the fields have been evaluated till three-loop in the pure maximal Abelian gauge, i.e. without the inclusion of scalar fields, as reported in Sec. 3 of [18]. From the explicit expressions given in [18][1] one sees that the anomalous dimensions of the fields do admit the limit $\alpha\to 0$. This is precisely what we mean when we speak about the limit $\alpha\to 0$. In other words, quantities like the β-function of the theory as well as the anomalous dimensions of the elementary fields and of suitable gauge invariant composite operators should admit a safe limit $\alpha\to 0$, as shown in the explicit three-loop calculations

[1]See, in particular, Eqs. (3.7)–(3.15) of [18], where the three-loop anomalous dimensions of all fields are explicitly given. From those expressions, it turns out that all field anomalous dimensions admit a safe limit $\alpha\to 0$. Concerning now the anomalous dimension of the gauge parameter itself α, i.e. γ_α, given in Eq. (3.8) of [18], we recall that this quantity enters always in the combination $\alpha\gamma_\alpha$, as one can observe from Eq. (3.6). This precise combination also displays a safe $\alpha\to 0$ limit.

reported in [18]. We expect that such a feature will be maintained also when scalar fields are included; i.e. we expect that the β-functions of the theory as well as the anomalous dimensions of the elementary fields and of gauge invariant composite operators should display a smooth $\alpha, \beta \to 0$ limit.

Having identified a suitable starting action, Eq. (21), we now must prove that it is multiplicative renormalizable to all order, a task which we shall face in the following sections by making use of the algebraic renormalization [39].

IV. WARD IDENTITIES

Having identified a suitable gauge-fixed action, Eq. (21), we proceed to write down the set of Ward identities which we shall employ in the proof of the all orders multiplicative renormalizability of expression (21). To that end, following the algebraic renormalization procedure [39], we need to introduce a set of BRST invariant external sources $(\Omega^a_\mu, \Omega_\mu, L^a, L, F^a, F)$ coupled to the nonlinear BRST variations of the fields $(A^a_\mu, A_\mu, c^a, c, \phi^a, \phi)$, Eqs. (16), namely

$$\begin{aligned} S_{\text{ext}} &= \int d^4x\{\Omega^a_\mu(sA^a_\mu) + \Omega_\mu(sA_\mu) + L^a(sc^a) + L(sc) + F^a(s\phi^a) + F(s\phi)\} \\ &= \int d^4x\Big\{\Omega^a_\mu(-D^{ab}_\mu c^b - g\varepsilon^{ab}A^b_\mu c) + \Omega_\mu(-\partial_\mu c - g\varepsilon^{ab}A^a_\mu c^b) + g\varepsilon^{ab}L^a c^b c \\ &\quad + \frac{g}{2}\varepsilon^{ab}Lc^a c^b + g\varepsilon^{ab}F^a(\phi c^b - \phi^b c) - g\varepsilon^{ab}F\phi^a c^b\Big\}, \end{aligned} \tag{24}$$

with

$$s\Omega^a_\mu = s\Omega_\mu = sF^a = sF = sL^a = sL = 0. \tag{25}$$

Therefore, for the complete BRST invariant starting action Σ, we get

$$\begin{aligned} \Sigma &= S_{\text{YM}} + S_{\text{matter}} + S_{MAG} + S_\alpha + S_\beta + S_{\text{ext}} \\ &= \int d^4x\Big\{\frac{1}{4}(F^a_{\mu\nu}F^a_{\mu\nu} + F_{\mu\nu}F_{\mu\nu}) + b^a D^{ab}_\mu A^b_\mu - \bar{c}^a\mathcal{M}^{ab}c^b + g\varepsilon^{ab}\bar{c}^a c D^{bc}_\mu A^c_\mu + b\partial_\mu A_\mu \\ &\quad + \bar{c}\partial_\mu(\partial_\mu c + g\varepsilon^{ab}A^a_\mu c^b) + \Omega^a_\mu(-D^{ab}_\mu c^b - g\varepsilon^{ab}A^b_\mu c) + \Omega_\mu(-\partial_\mu c - g\varepsilon^{ab}A^a_\mu c^b) \\ &\quad + g\varepsilon^{ab}L^a c^b c + \frac{g}{2}\varepsilon^{ab}Lc^a c^b + g\varepsilon^{ab}F^a(\phi c^b - \phi^b c) - g\varepsilon^{ab}F\phi^a c^b + \frac{\alpha}{2}[b^a b^a - 2g\varepsilon^{ab}b^a\bar{c}^b c \\ &\quad + g^2\bar{c}^a\bar{c}^b c^a c^b] + \frac{\beta}{2}[g\phi^a\phi^a c^b\bar{c}^b + g\phi^a\phi^b c^a\bar{c}^b + \phi\phi^a(\varepsilon^{ab}b^b - gc\bar{c}^a) + g\phi\phi c^a\bar{c}^a] \\ &\quad + \frac{1}{2}(\partial_\mu\phi^a)(\partial_\mu\phi^a) + \frac{1}{2}(\partial_\mu\phi)(\partial_\mu\phi) - g^2\varepsilon^{ab}[(\partial_\mu\phi)\phi^a A^b_\mu - (\partial_\mu\phi^a)\phi A^b_\mu + (\partial_\mu\phi^a)\phi^b A_\mu] \\ &\quad + \frac{1}{2}g^2[A^a_\mu A^a_\mu(\phi^b\phi^b + \phi\phi) + A_\mu A_\mu\phi^a\phi^a - A^a_\mu A^b_\mu\phi^a\phi^b - 2A^a_\mu A_\mu\phi^a\phi] \\ &\quad + \frac{m^2_\phi}{2}(\phi^a\phi^a + \phi\phi)\frac{\lambda}{4!}[(\phi^a\phi^a)^2 + 2\phi^2\phi^a\phi^a + \phi^4]\Big\}. \end{aligned} \tag{26}$$

Let us display in Table I and Table II the quantum numbers of all elds and sources, respectively (with “B" standing for bosonic and “F” for fermionic nature):The complete action Σ turns out to fulfill a large set of Ward identities, which we enlist below:

(i) The Slavnov-Taylor identity:

$$\mathcal{S}(\Sigma) = 0, \tag{27}$$

with

$$\mathcal{S}(\Sigma) \equiv \int d^4x\Big\{\frac{\delta\Sigma}{\delta\Omega^a_\mu}\frac{\delta\Sigma}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta\Omega_\mu}\frac{\delta\Sigma}{\delta A_\mu} + \frac{\delta\Sigma}{\delta F^a}\frac{\delta\Sigma}{\delta\phi^a} + \frac{\delta\Sigma}{\delta F}\frac{\delta\Sigma}{\delta\phi} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta\Sigma}{\delta c^a} + \frac{\delta\Sigma}{\delta L}\frac{\delta\Sigma}{\delta c} + b^a\frac{\delta\Sigma}{\delta\bar{c}^a} + b\frac{\delta\Sigma}{\delta\bar{c}}\Big\}. \tag{28}$$

Let us also introduce, for further use, the so-called linearized Slavnov-Taylor operator $\mathcal{B}_\Sigma$, defined as [39]

$$\mathcal{B}_\Sigma = \int d^4x \left\{ \frac{\delta\Sigma}{\delta\Omega^a_\mu}\frac{\delta}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta A^a_\mu}\frac{\delta}{\delta\Omega^a_\mu} + \frac{\delta\Sigma}{\delta\Omega_\mu}\frac{\delta}{\delta A_\mu} + \frac{\delta\Sigma}{\delta A_\mu}\frac{\delta}{\delta\Omega_\mu} + \frac{\delta\Sigma}{\delta F^a}\frac{\delta}{\delta\phi^a} + \frac{\delta\Sigma}{\delta\phi^a}\frac{\delta}{\delta F^a} \right.$$
$$\left. + \frac{\delta\Sigma}{\delta F}\frac{\delta}{\delta\phi} + \frac{\delta\Sigma}{\delta\phi}\frac{\delta}{\delta F} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta}{\delta c^a} + \frac{\delta\Sigma}{\delta c^a}\frac{\delta}{\delta L^a} + \frac{\delta\Sigma}{\delta L}\frac{\delta}{\delta c} + \frac{\delta\Sigma}{\delta c}\frac{\delta}{\delta L} + b^a\frac{\delta}{\delta\bar{c}^a} + b\frac{\delta}{\delta\bar{c}} \right\}. \tag{29}$$

The operator $\mathcal{B}_\Sigma$ has the important property of being nilpotent [39], i.e.

$$\mathcal{B}_\Sigma\mathcal{B}_\Sigma = 0. \tag{30}$$

(ii) The diagonal Nakanishi-Lautrup field equation:

$$\frac{\delta\Sigma}{\delta b} = \partial_\mu A_\mu. \tag{31}$$

(iii) The diagonal antighost equation:

$$\frac{\delta\Sigma}{\delta\bar{c}} + \partial_\mu\frac{\delta\Sigma}{\delta\Omega_\mu} = 0. \tag{32}$$

(iv) The local diagonal ghost equation [17]:

$$\frac{\delta\Sigma}{\delta c} + g\varepsilon^{ab}\bar{c}^a\frac{\delta\Sigma}{\delta b^b} = -\partial^2\bar{c} - \partial_\mu\Omega_\mu + g\varepsilon^{ab}(\Omega^a_\mu A^a_\mu - L^a c^b + F^a\phi^b). \tag{33}$$

Notice that the right-hand side of Eq. (33) is linear in the quantum fields. As such, it is a linear breaking, not affected by the quantum correction [39].

(v) The $U(1)$ residual local symmetry:

$$\mathcal{W}^{U(1)}\Sigma = -\partial^2 b, \tag{34}$$

where

$$\mathcal{W}^{U(1)} \equiv \partial_\mu\frac{\delta}{\delta A_\mu} + g\varepsilon^{ab}\left\{ A^a_\mu\frac{\delta}{\delta A^b_\mu} + \phi^a\frac{\delta}{\delta\phi^b} + c^a\frac{\delta}{\delta c^b} + \bar{c}^a\frac{\delta}{\delta\bar{c}^b} + b^a\frac{\delta}{\delta b^b} + \Omega^a_\mu\frac{\delta}{\delta\Omega^b_\mu} + F^a\frac{\delta}{\delta F^b} + L^a\frac{\delta}{\delta L^b} \right\}. \tag{35}$$

As noticed in [17], the $U(1)$ Ward identity (34) can be obtained by anticommuting the diagonal ghost equation, Eq. (33), with the Slavnov-Taylor identity, Eq. (27). This identity shows in a very clear way the fact that the diagonal component A_μ of the gauge field behaves like a $U(1)$ Abelian connection, while all off-diagonal components of the gauge and matter fields play the role of a kind of charged $U(1)$ field, precisely like in a *QED*-like theory. As already mentioned in the Introduction, this identity expresses one of the most important characteristics of the maximal Abelian gauge.

(vi) The discrete symmetry

$$\Psi^1 \to \Psi^1, \qquad \Psi^2 \to -\Psi^2, \qquad \Psi^{\rm diag} \to -\Psi^{\rm diag}, \tag{36}$$

where Ψ^a and $\Psi^{\rm diag}$ stand, respectively, for all off-diagonal and diagonal fields and sources. As pointed out in [17], this discrete symmetry plays the role of the charge conjugation with respect to the $U(1)$ Cartan subgroup of $SU(2)$.

(vii) Finally, looking at the matter sector of the complete action Σ, we have a second discrete symmetry

$$\phi^a \to -\phi^a, \quad \phi \to -\phi, \quad F^a \to -F^a, \quad F \to -F, \tag{37}$$

TABLE I. The quantum numbers of the fields.

Fields	A	ϕ	b	$\bar{c}$	c
Dimension	1	1	2	2	0
Ghost number	0	0	0	1	−1
Nature	B	B	B	F	F

TABLE II. The quantum numbers of the external BRST sources.

Sources	Ω^a_μ	Ω_μ	L^a	L	F^a	F
Dimension	3	3	4	4	2	2
Ghost number	−1	−1	−2	−2	−1	−1
Nature	F	F	B	B	F	F

forbidding the appearance of pure matter terms containing odd powers of the scalar fields (ϕ^a, ϕ).

V. ALGEBRAIC CHARACTERIZATION OF THE INVARIANT COUNTERTERM AND MULTIPLICATIVE RENORMALIZABILITY

In order to prove that the complete action Σ, Eq. (26), is multiplicative renormalizable, we follow the algebraic renormalization setup [39], and characterize, by means of the Ward identities previously derived, the most general invariant local counterterm, Σ_{ct}, which can be freely added to the starting action Σ. According to the power counting, Σ_{ct} is an integrated local polynomial in the fields and external sources of dimension bounded by four and with zero ghost number. Further, we require that the perturbed action, $(\Sigma + \epsilon\Sigma_{\mathrm{ct}})$, satisfies the same Ward identities and constraints of Σ [39], to the first order in the perturbation parameter ϵ, obtaining the following set of constraints:

$$\mathcal{B}_\Sigma \Sigma_{\mathrm{ct}} = 0, \tag{38}$$

and

$$\frac{\delta\Sigma_{\mathrm{ct}}}{\delta\bar{c}} + \partial_\mu \frac{\delta\Sigma_{\mathrm{ct}}}{\delta\Omega_\mu} = 0, \qquad \frac{\delta\Sigma_{\mathrm{ct}}}{\delta c} + g\epsilon^{ab}\bar{c}^a \frac{\delta\Sigma_{\mathrm{ct}}}{\delta b^b} = 0,$$
$$\mathcal{W}^{U(1)}\Sigma_{\mathrm{ct}} = 0, \qquad \frac{\delta\Sigma_{\mathrm{ct}}}{\delta b} = 0. \tag{39}$$

The first constraint, Eq. (38), tells us that Σ_{ct} belongs to the cohomology of the nilpotent linearized operator $\mathcal{B}_\Sigma$ in the space of the integrated local polynomials in the fields and sources bounded by dimension 4. From the general results on the BRST cohomolgy of Yang-Mills theories, it follows that Σ_{ct} can be parametrized as follows:

$$\Sigma_{\mathrm{ct}} = \Sigma_0 + \mathcal{B}_\Sigma \Delta^{-1}, \tag{40}$$

where Σ_0 stands for the nontrivial part of the cohomolgy of the operator $\mathcal{B}_\Sigma$, being given by

$$\Sigma_0 = a_0 S_{\mathrm{YM}} + \int d^4x \left(a_1 \frac{m_\phi^2}{2}\phi^A\phi^A + a_2 \frac{\lambda}{4!}(\phi^A\phi^A)^2 \right), \tag{41}$$

where a_0, a_1, a_2 are free arbitrary coefficients. The second term, Δ^{-1}, in Eq. (40) is a local integrated polynomial in the fields and sources with dimension 4 and ghost number -1. This term represents the trivial part of the cohomolgy, being parametrized as

$$\begin{aligned}\Delta^{-1} = \int d^4x\{&\mathbb{C}_4^{ab}A_\mu^a\Omega_\mu^b + \mathbb{C}_5 A_\mu\Omega_\mu + \mathbb{C}_6^{ab}\phi^a F^b + \mathbb{C}_7\phi F + \mathbb{C}_8^{ab}L^a c^b + \mathbb{C}_9 Lc + \mathbb{C}_{10}^{ab}\bar{c}^a b^b \\ &+ \mathbb{C}_{11}\bar{c}b + \mathbb{C}_{12}^{ab}\bar{c}^a\bar{c}^b c + \mathbb{C}_{13}^{ab}\bar{c}^a\bar{c}c^b + \mathbb{C}_{14}^{ab}\phi^a\phi\bar{c}^b + \mathbb{C}_{15}^{ab}\phi^a\phi^b\bar{c} + \mathbb{C}_{16}^{ab}A_\mu^a A_\mu\bar{c}^b + \mathbb{C}_{17}^{ab}A_\mu^a A_\mu^b\bar{c} \\ &+ \mathbb{C}_{18}^{ab}m_\phi\phi^a\bar{c}^b + \mathbb{C}_{19}m_\phi\phi\bar{c} + \mathbb{C}_{21}^{ab}(\partial_\mu A_\mu^a)\bar{c}^b + \mathbb{C}_{21}(\partial_\mu A_\mu)\bar{c}\},\end{aligned} \tag{42}$$

where $\mathbb{C}_i, i = 4, \ldots, 21$ are free parameters.

After imposition of the conditions (39), of the discrete symmetries (36), (37), and after a rather lengthy algebraic calculation, we get

$$\mathbb{C}_5 = \mathbb{C}_9 = \mathbb{C}_{11} = \mathbb{C}_{13}^{ab} = \mathbb{C}_{15}^{ab} = \mathbb{C}_{17}^{ab} = \mathbb{C}_{18}^a b = \mathbb{C}_{19} = \mathbb{C}_{21} = 0 \tag{43}$$

and

$$\begin{aligned}&\mathbb{C}_4^{ab} = \delta^{ab}\mathbb{C}_4, \qquad \mathbb{C}_6^{ab} = \delta^{ab}\mathbb{C}_6, \qquad \mathbb{C}_7 = -\mathbb{C}_6, \qquad \mathbb{C}_8^{ab} = \delta^{ab}\mathbb{C}_8, \qquad \mathbb{C}_{10}^{ab} = \alpha\delta^{ab}\mathbb{C}_{10}, \\ &\mathbb{C}_{12}^{ab} = -\alpha g\epsilon^{ab}\mathbb{C}_{10}, \qquad \mathbb{C}_{14}^{ab} = \beta\epsilon^{ab}\mathbb{C}_{14}, \qquad \mathbb{C}_{16}^{ab} = \epsilon^{ab}\mathbb{C}_{16}, \qquad \mathbb{C}_{20}^{ab} = \delta^{ab}\mathbb{C}_{20} = -\delta^{ab}\mathbb{C}_{16},\end{aligned} \tag{44}$$

Therefore, for the final expression of the most general counterterm Σ_{ct}, we obtain

$$
\begin{aligned}
\Sigma_{\rm ct} &= \int d^4x\left(\frac{a_0}{4}(F^a_{\mu\nu}F^a_{\mu\nu}+F_{\mu\nu}F_{\mu\nu})+a_1\frac{m_\phi^2}{2}\phi^A\phi^A+a_2\frac{\lambda}{4!}(\phi^A\phi^A)^2\right)\\
&\quad+\mathcal{B}_\Sigma\int d^4x[\mathbb{C}_4A^a_\mu\Omega^a_\mu+\mathbb{C}_6(\phi^aF^a-\phi F)+\mathbb{C}_8L^ac^a+\mathbb{C}_{10}\alpha(\bar c^ab^a-g\varepsilon^{ab}\bar c^a\bar c^bc)]\\
&\quad+\mathcal{B}_\Sigma\int d^4x[\mathbb{C}_{14}\beta\varepsilon^{ab}\phi^a\phi\bar c^b+\mathbb{C}_{16}\bar c^aD^{ab}_\mu A^b_\mu]\\
&=\int d^4x\Bigg\{\frac{a_0}{4}(F^a_{\mu\nu}F^a_{\mu\nu}+F_{\mu\nu}F_{\mu\nu})+a_1\frac{m_\phi^2}{2}\phi^A\phi^A+a_2\frac{\lambda}{4!}(\phi^A\phi^A)^2+\mathbb{C}_4\Bigg[\frac{\delta S_{YM}}{\delta A^a_\mu}A^a_\mu+b^aD^{ab}_\mu A^b_\mu\\
&\quad+g\varepsilon^{ab}(\bar c^acD^{bc}_\mu A^c_\mu-\Omega_\mu A^a_\mu c^b+\bar c\partial_\mu(A^a_\mu c^b))+2g^2(\bar c^ac^a+\phi^a\phi^a+\phi\phi)A^b_\mu A^b_\mu\\
&\quad-2g^2(\bar c^ac^b+\phi^a\phi^b)A^a_\mu A^b_\mu+2g\varepsilon^{ab}A^a_\mu((\partial_\mu\phi)\phi^b-(\partial_\mu\phi^b)\phi)\Bigg]+\mathbb{C}_6[2(\partial_\mu\phi^a)(\partial_\mu\phi^a)\\
&\quad-2(\partial_\mu\phi)(\partial_\mu\phi)-4g^2\varepsilon^{ab}(\partial_\mu\phi^a)\phi A^b_\mu+2g^2A^a_\mu A^a_\mu(\phi^a\phi^a-\phi\phi)+2g^2(A_\mu A_\mu\phi^a\phi^a-A^a_\mu A^b_\mu\phi^a\phi^b)\\
&\quad+m_\phi^2(\phi^a\phi^a-\phi\phi)+\frac{\lambda}{3!}((\phi^a\phi^a)^2-\phi^4)+\beta g(\phi^a\phi^ac^b\bar c^b+\phi^a\phi^bc^a\bar c^b-\phi\phi c^a\bar c^a)]\\
&\quad+\mathbb{C}_8[-\bar c^a\partial^2c^a+2g\varepsilon^{ab}\bar c^aA_\mu\partial_\mu c^b+g^2\bar c^ac^a(A_\mu A_\mu-A^b_\mu A^b_\mu)+g^2\bar c^ac^bA^a_\mu A^b_\mu-g\varepsilon^{ab}Lc^ac^b\\
&\quad+\Omega^a_\mu D^{ab}_\mu c^b+g\varepsilon^{ab}\Omega_\mu c^aA^b_\mu-g\varepsilon^{ab}F^a\phi c^b+g\varepsilon^{ab}F\phi^ac^b-\alpha g^2\bar c^ac^a\bar c^bc^b+\frac{\beta}{2}g\bar c^ac^a(\phi^b\phi^b+\phi^2)\\
&\quad+\frac{\beta}{2}\phi^a\phi^b\bar c^bc^a]+\mathbb{C}_{10}\alpha[b^ab^a-2g\varepsilon^{ab}b^a\bar c^bc+g^2\bar c^ac^a\bar c^bc^b]+\mathbb{C}_{14}\beta[\phi\phi^a(\varepsilon^{ab}b^b+gc\bar c^a)\\
&\quad\times gc^a\bar c^a(\phi^b\phi^b+\phi^2)+g\phi^a\phi^bc^a\bar c^b]+\mathbb{C}_{16}[\bar c^a\partial^2c^a-2g\varepsilon^{ab}\bar c^aA_\mu\partial_\mu c^b-g^2\bar c^ac^bA^a_\mu A^b_\mu\\
&\quad+g^2\bar c^ac^a(A^b_\mu A^b_\mu+A_\mu A_\mu)+2\varepsilon^{ab}\bar c^acD^{bc}_\mu A^c_\mu+b^aD^{ab}_\mu A^b_\mu]\Bigg\}.
\end{aligned}
\tag{45}
$$

A. Renormalization factors

After having identified the most general counterterm, expression (45), we now must check if it can be reabsorbed through a multiplicative redefinition of the fields, sources, coupling constant and parameters of the starting action, according to

$$\Sigma(\Psi_0,\Gamma_0,\xi_0)=\Sigma(\Psi,\Gamma,\xi)+\epsilon\Sigma_{\rm ct}(\Psi,\Gamma,\xi)+O(\epsilon^2), \tag{46}$$

where

$$
\begin{aligned}
\Psi&=\{A^a_\mu,A_\mu,\phi^a,\phi,b^a,b,\bar c^a,c^a\},\\
\Gamma&=\{\Omega^a_\mu,\Omega_\mu,F^a,F,L^a,L,\},\\
\xi&=\{g,m_\phi,\lambda,\alpha,\beta\},
\end{aligned}
\tag{47}
$$

and the so-called bare quantities (Ψ_0,Γ_0,ξ_0) are defined as

$$\Psi_0=Z^{1/2}_\Psi\Psi,\qquad\Gamma_0=Z_\Gamma\Gamma,\qquad\xi_0=Z_\xi\xi. \tag{48}$$

By direct inspection of Eq. (46), for the renormalization factors we obtain

$$Z^{1/2}_A=1+\epsilon(2a_0+\mathbb{C}_4), \tag{49}$$

$$(Z^{\rm diag}_A)^{1/2}=1+2\epsilon a_0, \tag{50}$$

$$Z^{1/2}_b=1+\epsilon(-2a_0+\mathbb{C}_{16}), \tag{51}$$

$$(Z^{\rm diag}_b)^{1/2}=1-2\epsilon a_0, \tag{52}$$

$$Z^{1/2}_c=1-\epsilon\mathbb{C}_8, \tag{53}$$

$$Z^{1/2}_{\bar c}=1+\epsilon\mathbb{C}_{16}, \tag{54}$$

$$Z^{1/2}_\phi=1+\epsilon\mathbb{C}_6, \tag{55}$$

$$(Z^{\rm diag}_\phi)^{1/2}=1-\epsilon\mathbb{C}_6, \tag{56}$$

$$Z_g=1-2\epsilon a_0, \tag{57}$$

$$Z_{m_\phi}=1+\frac{\epsilon}{2}a_1, \tag{58}$$

$$Z_\alpha=1+2\epsilon(2a_0+\mathbb{C}_{10}-\mathbb{C}_{16}), \tag{59}$$

$$Z_\beta = 1 + \epsilon(-2a_0 + 2\mathbb{C}_{14} + \mathbb{C}_{16}), \tag{60}$$

$$Z_\lambda = 1 + \epsilon a_2. \tag{61}$$

It is worth noticing that the diagonal ghosts do not need to be renormalized, a property which follows directly from the diagonal ghost equation (33). This concludes the algebraic proof of the all orders multiplicative renormalizability of the action Σ, Eq. (26). Finally, we note that the nonrenormalization theorem of the maximal Abelian gauge [17]

$$Z_g(Z_A^{\mathrm{diag}})^{1/2} = 1, \tag{62}$$

remains true in the presence of matter fields.

VI. CONCLUSION

In this work we have addressed the issue of the renormalization of Yang-Mills theories in the maximal Abelian gauge in the presence of scalar matter fields. Our main observation is that, due to the nonlinearity of the gauge-fixing condition, Eq. (12), a new quartic interaction term between scalar matter fields and off-diagonal Faddeev-Popov ghosts is required for renormalizabilty. Moreover, this new quartic interaction turns out to be described by an exact BRST invariant term, as expressed by Eq. (20), a feature which ensures that the final gauge-fixed action, Eq. (26), is BRST invariant and multiplicative renormalizable to all orders, as proven in Sec. V.

Although the proof of the renormalizability given here refers to the gauge group $SU(2)$, it can be easily generalized to other gauge groups as well as to other representations of the scalar fields. The inclusion of the usual Dirac action for spinors does not pose any additional problem. Also, unlike the case of scalar matter fields, BRST invariance and power counting do not allow for additional interaction terms between spinors and Faddeev-Popov ghosts.

The analysis of the all orders perturbative renormalizability of the maximal Abelian gauge in presence of matter fields is the first necessary step towards the investigation of the nonperturbative effects of the Gribov copies, which deeply affect the maximal Abelian gauge [27–31]. The study of this issue in presence of matter fields is currently under investigation [40].

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