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Partner particles for moving mirror radiation and black hole evaporation

M. Hotta,[1,*] R. Schützhold,[2,†] and W. G. Unruh[3,‡]

[1]*Graduate School of Science, Tohoku University, Aobaku, Aramaki, Aza 6-3, Sendai 980-8678, Japan*

[2]*Fakultät für Physik, Universität Duisburg-Essen, Lotharstrasse 1, 47057 Duisburg, Germany*

[3]*CIAR Cosmology and Gravity Program, Department of Physics, University of British Columbia, Vancouver, British Columbia V6T 1Z1, Canada*

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The partner mode with respect to a vacuum state for a given mode (like that corresponding to one of the thermal particles emitted by a black hole) is defined and calculated. The partner modes are explicitly calculated for a number of cases, in particular for the modes corresponding to a particle detector being excited by turn-on/turn-off transients, or with the thermal particles emitted by the accelerated mirror model for black hole evaporation. One of the key results is that the partner mode in general is just a vacuum fluctuation, and one can have the partner mode be located in a region where the state cannot be distinguished from the vacuum state by any series of local measurements, including the energy density. For example, "information" (the correlations with the thermal emissions) need not be associated with any energy transport. The idea that black holes emit huge amounts of energy in their last stages because of all the information which must be emitted under the assumption of black hole unitarity is found to not necessarily be the case.

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I. INTRODUCTION

Quantum mechanics around a black hole has been one of the most exciting and puzzling aspects of theoretical physics in the past half-century [1]. One of the issues has been that of "information" and how information is carried. Black hole unitarity [2] is the belief that in the space-time outside the black hole, the evolution of a quantum field from a time before the black hole formed to after the black hole completely evaporated, must be unitary—initial states of the quantum field map uniquely to final states of the quantum field. If one believes in "black hole unitarity" (and in this paper we are agnostic about that belief), then there must exist correlations between the early Hawking evaporation emission from the black hole and late time emission. For any particle emitted early on, some correlation between this early mission and the field later on must exist. Given a mode which carries away thermal particles in the early stages, there must be "partner modes" which occur later which are correlated with these early modes in order that "unitarity" be preserved.

The characterization of these partners thus becomes important. In Sec. II we define the partner mode uniquely via suitable conditions. In Sec. III we show that any particle detection measurement of the field also has a partner, even in the case where the detector is stationary but is switched on and off. In Sec. V we look at the partner in the case of the accelerated mirror model of black hole thermal emission.

One of the surprising results is that the partner need not be located near the original mode but can be located in distant regions of the space-time. While recognized in the correlations between the field inside and outside the black hole in the Hawking evaporation process [3], this is a general feature of the partner modes. Our results are somewhat related to recent observations that the long range entanglement in the vacuum can be used to entangle other systems even in spatially separated regions [4] and to energy teleportation studies [5].

First, let us specify how the partner mode can be defined. To this end, we demand the two conditions:

A) The reduced density matrix of the Hawking plus partner modes obtained by integrating out all other degrees of freedom should be a pure state. Since the total state (the initial vacuum) is a pure state, this is equivalent to vanishing entanglement between the Hawking plus partner mode on the one hand and the rest of the system on the other hand.

However, this requirement alone does not define the partner mode uniquely (see below). For example, one could envisage a single-mode squeezing operation and phase rotation acting on the partner mode, which does not change the purity of the combined state (Hawking plus partner). Specifying the partner mode uniquely requires a second condition. There are several reasonable options, and here we list some possibilities.

B1) The quantum state after absorbing (annihilating) one partner particle should be (up to normalization due to possibly different probabilities) the same state as after

*hotta@tuhep.phys.tohoku.ac.jp
†ralf.schuetzhold@uni-due.de
‡unruh@physics.ubc.ca

creating one Hawking particle. This corresponds to the intuitive picture that the Hawking and partner particles always come in pairs.

B2) Alternatively, one could implement the idea that Hawking and partner particles always come in pairs by imposing the requirement the other way around: The quantum state after absorbing one Hawking particle should be (again up to normalization) the same state as after creating one partner particle.

As we shall see below, condition B1 can always be satisfied—unless the Hawking mode contains single-mode squeezing only and thus there would be no need for a partner particle at all—whereas the requirement B2 can only be fulfilled if the single-mode squeezing of the Hawking mode is small enough. As another option (B3), we could demand that the probabilities for detecting Hawking and partner particles should be the same—treating these two modes on a symmetric footing.

As it turns out, in the scenarios we are interested in below (pure two-mode squeezing), all these requirements yield the same answer for the partner particle.

II. DEFINITION OF PARTNER PARTICLE

Now let us show how to satisfy these requirements. As a most general ansatz, we decompose the Hawking mode

$$\hat{a}_H = \int dk(\alpha_k^* \hat{a}_k + \beta_k \hat{a}_k^\dagger), \tag{1}$$

into creation and annihilation operators $\hat{a}_k^\dagger$ and $\hat{a}_k$,

$$[\hat{a}_k, \hat{a}_{k'}] = [\hat{a}_k^\dagger, \hat{a}_{k'}^\dagger] = 0, \qquad [\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta(k, k'), \tag{2}$$

defined with respect to the initial vacuum state

$$\forall_k \ \hat{a}_k |0\rangle = 0, \tag{3}$$

where k denotes some quantum number.

For convenience, let us introduce the usual complex scalar product of two functions or vectors $\boldsymbol{\chi}$ and $\boldsymbol{\zeta}$ via

$$\{\boldsymbol{\chi}|\boldsymbol{\zeta}\} = \int dk \chi_k^* \zeta_k. \tag{4}$$

Accordingly, we define the projection of the initial annihilation operators $\hat{a}_k$ onto one mode $\boldsymbol{\chi}$ via

$$\hat{a}_\chi = \{\boldsymbol{\chi}|\hat{\boldsymbol{a}}\} = \int dk \chi_k^* \hat{a}_k, \tag{5}$$

which gives the commutation relations

$$[\hat{a}_\chi, \hat{a}_\zeta] = [\hat{a}_\chi^\dagger, \hat{a}_\zeta^\dagger] = 0, \qquad [\hat{a}_\chi, \hat{a}_\zeta^\dagger] = \{\boldsymbol{\chi}|\boldsymbol{\zeta}\}. \tag{6}$$

In this notation, the Hawking mode is given by

$$\hat{a}_H = \{\boldsymbol{\alpha}|\hat{\boldsymbol{a}}\} + (\{\boldsymbol{\beta}|\hat{\boldsymbol{a}}\})^\dagger = \{\boldsymbol{\alpha}|\hat{\boldsymbol{a}}\} + \{\hat{\boldsymbol{a}}|\boldsymbol{\beta}\}. \tag{7}$$

Now let us introduce an orthonormal basis $\boldsymbol{n}_\parallel$ and $\boldsymbol{n}_\perp$ in the subspace spanned by the two vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$,

$$\boldsymbol{\alpha} = \alpha \boldsymbol{n}_\parallel, \qquad \boldsymbol{\beta} = \beta_\parallel \boldsymbol{n}_\parallel + \beta_\perp \boldsymbol{n}_\perp, \tag{8}$$

where $|\boldsymbol{n}_\parallel|^2 = \{\boldsymbol{n}_\parallel|\boldsymbol{n}_\parallel\} = 1 = |\boldsymbol{n}_\perp|^2$ and $\boldsymbol{n}_\parallel \perp \boldsymbol{n}_\perp$, i.e., $\{\boldsymbol{n}_\parallel|\boldsymbol{n}_\perp\} = 0$. If $\beta_\perp = 0$, we would have pure single-mode squeezing, and the Hawking mode itself would be in a pure state; i.e., there would be no need for a partner particle. In the general case $\beta_\perp \neq 0$, we can restrict ourselves to the two modes

$$\hat{a}_\parallel = \{\boldsymbol{n}_\parallel|\hat{\boldsymbol{a}}\}, \qquad \hat{a}_\perp = \{\boldsymbol{n}_\perp|\hat{\boldsymbol{a}}\}, \tag{9}$$

which satisfy the usual commutation relations for independent modes due to Eq. (6). These operators annihilate the initial vacuum

$$\hat{a}_\parallel|0\rangle = \hat{a}_\perp|0\rangle = 0, \tag{10}$$

and thus the reduced density matrix of these two modes is a pure state. Now the idea is that everything involving the Hawking mode $\hat{a}_H$ and its partner mode $\hat{a}_P$ will occur in the two-mode space spanned by $\hat{a}_\parallel$ and $\hat{a}_\perp$ and their adjoints $\hat{a}_\parallel^\dagger$ and $\hat{a}_\perp^\dagger$. As we show the Appendix, this is actually the only way to satisfy requirement A. In terms of these operators, the Hawking mode is given by

$$\hat{a}_H = \alpha^* \hat{a}_\parallel + \beta_\parallel \hat{a}_\parallel^\dagger + \beta_\perp \hat{a}_\perp^\dagger. \tag{11}$$

From $[\hat{a}_H, \hat{a}_H^\dagger] = 1$ follows $|\alpha|^2 - |\beta_\parallel|^2 - |\beta_\perp|^2 = 1$. Note that one can make the three Bogoliubov coefficients α, $\beta_\parallel$, and $\beta_\perp$ real by absorbing their phases into $\hat{a}_\parallel$, $\hat{a}_\perp$, and $\hat{a}_H$.

Following our strategy, we can make the following general ansatz for the partner particle:

$$\hat{a}_P = \gamma_\parallel^* \hat{a}_\parallel + \gamma_\perp^* \hat{a}_\perp + \delta_\parallel \hat{a}_\parallel^\dagger + \delta_\perp \hat{a}_\perp^\dagger. \tag{12}$$

In this way, requirement A is automatically satisfied. Since $\hat{a}_P$ should obey the usual commutation relation $[\hat{a}_P, \hat{a}_P^\dagger] = 1$, the above Bogoliubov coefficients should satisfy $|\gamma_\parallel|^2 + |\gamma_\perp|^2 - |\delta_\parallel|^2 - |\delta_\perp|^2 = 1$. Furthermore, since we want the two modes $\hat{a}_H$ and $\hat{a}_P$ to be independent, i.e., $[\hat{a}_P, \hat{a}_H^\dagger] = 0 = [\hat{a}_H, \hat{a}_P^\dagger]$ as well as $[\hat{a}_P, \hat{a}_H] = 0$, we get the conditions $\gamma_\parallel^* \alpha = \beta_\parallel^* \delta_\parallel + \beta_\perp^* \delta_\perp$ and $\gamma_\parallel^* \beta_\parallel + \gamma_\perp^* \beta_\perp = \alpha^* \delta_\parallel$.

As mentioned above, these three equations do not specify the four Bogoliubov coefficients for $\hat{a}_P$ uniquely. We could still apply a single-mode squeezing/phase transformation $\hat{a}_P \to e^{i\varphi} \cosh \zeta \hat{a}_P + e^{i\vartheta} \sinh \zeta \hat{a}_P^\dagger$ within the $\hat{a}_P$-mode for arbitrary (real) values of φ, ζ, and φ without violating any of the conditions above. To fix this remaining degree of freedom, a second requirement is necessary—here, we discuss B1 and B2.

Option B1 corresponds to choosing $\hat{a}_P|0\rangle \propto \hat{a}_H^\dagger|0\rangle$, i.e., $\boldsymbol{\delta}\|\boldsymbol{\alpha}$, which means $\delta_\perp = 0$ and $\delta_\| = \delta$. This then gives $\gamma_\|^* = \beta_\|^*\delta/\alpha$ and $\gamma_\perp^* = (\beta_\perp^{-1} + \beta_\perp^*)\delta/\alpha$ such that the remaining Bogoliubov coefficient δ can be determined by the unitarity condition $|\boldsymbol{\gamma}|^2 - |\boldsymbol{\delta}|^2 = 1$ up to a global phase. Writing this B1 condition $\hat{a}_P|0\rangle \propto \hat{a}_H^\dagger|0\rangle$ in the form $\hat{a}_P|0\rangle = \eta\hat{a}_H^\dagger|0\rangle$ with some constant η, we have

$$(\hat{a}_P - \eta\hat{a}_H^\dagger)|0\rangle = 0. \tag{13}$$

Thus, this linear combination $\hat{a}_P - \eta\hat{a}_H^\dagger$ is composed of initial annihilation operators only.

The other option B2 corresponds to $\hat{a}_H|0\rangle \propto \hat{a}_P^\dagger|0\rangle$, i.e., $\boldsymbol{\gamma}\|\boldsymbol{\beta}$, which allows us to determine the Bogoliubov coefficients in a completely analogous manner. Note, however, that there is an important difference: As shown above, condition B1 can always be fulfilled unless $\beta_\perp = 0$, in which case we would have pure single-mode squeezing within the $\hat{a}_H$-mode, and there would be no need for a partner mode. In contrast, it can be shown that requirement B2 cannot be satisfied if the amount of single-mode squeezing becomes too large.

In case of vanishing single-mode squeezing $\beta_\| = 0$, both requirements give the same partner mode

$$\hat{a}_P = \alpha^*\hat{a}_\perp + \beta\hat{a}_\|^\dagger, \tag{14}$$

where α and β can be made real because their phases can be absorbed into the definition of $\hat{a}_\|$ and $\hat{a}_\perp$. In this case, the initial vacuum state restricted to the two modes $\hat{a}_H$ and $\hat{a}_P$ is a pure two-mode squeezed state

$$|0\rangle = \exp\{\xi\hat{a}_H^\dagger\hat{a}_P^\dagger - \text{H.c.}\}|0\rangle_{HP}, \tag{15}$$

with respect to the zero-particle state $|0\rangle_{HP}$ which is annihilated by $\hat{a}_H$ and $\hat{a}_P$,

$$\hat{a}_H|0\rangle_{HP} = \hat{a}_P|0\rangle_{HP} = 0, \tag{16}$$

where the squeezing parameter ξ satisfies $\alpha = \cosh\xi$ and $\beta = \sinh\xi$. As a result, the initial vacuum $|0\rangle$ can be viewed as a state containing pairs of particles in the modes $\hat{a}_H$ and $\hat{a}_P$. After tracing out (averaging over) the partner mode, this squeezed state (15) yields a thermal-type density matrix for the Hawking mode,

$$\hat{\rho}_H = \frac{1}{Z}\exp\left\{-\frac{\hat{a}_H^\dagger\hat{a}_H}{\mathcal{T}_H}\right\}, \tag{17}$$

with the normalization Z ensuring $\text{Tr}\{\hat{\rho}_H\} = 1$ and

$$\mathcal{T}_H = \frac{1}{2\ln(\cosh\xi)}, \tag{18}$$

which can be regarded as a dimensionless Hawking temperature. Due to the symmetric nature of the squeezed state (15), the same applies to the reduced state of the partner particles (after tracing out the Hawking mode).

Note that the mapping from the Hawking mode $\hat{a}_H$ to its partner mode $\hat{a}_P$ is not linear in general—if $\hat{a}_H$ has the partner mode $\hat{a}_P$ and $\hat{a}'_H$ has the partner mode $\hat{a}'_P$, then the partner mode for $\mu\hat{a}_H + \nu\hat{a}'_H$, for example, is almost never $\mu\hat{a}_P + \nu\hat{a}'_P$, even if we have no single-mode squeezing in both cases.

III. PARTNERS AND DETECTORS

The idea of a partner particle has a broader applicability than just Hawking or acceleration (Unruh) radiation. Consider a model particle detector as suggested by Unruh [6] and developed by De Witt [7]. The detector is taken as occupying a single point in space-time with an internal degree of freedom, often taken to be a spin degree but could equally and more simply be taken to be a harmonic oscillator degree of freedom. The energy difference (in the rest frame of the detector) between the ground state and the first excited state is E. This is coupled to the quantum field of interest. This detector responds to specific degrees of freedom of the field, changing its state from the ground to excited state, which is regarded as a detection. (For example, if the detector is discovered at some time to be in its excited state, it must have absorbed energy and a particle from the field.)

The interaction Lagrangian is given by ($\hbar = c = 1$)

$$L_{\text{int}} = \epsilon(\tau)q(\tau)\partial_\tau\Phi[t(\tau), x(\tau)], \tag{19}$$

where $\epsilon(\tau)$ is the possibly time dependent coupling and $t(\tau), x(\tau)$ is the trajectory of the detector in terms of the proper time along the path τ. After quantization, the internal degree of freedom of the detector corresponds to the operator

$$\hat{q}(\tau) = \hat{c}e^{-iE\tau} + \hat{c}^\dagger e^{iE\tau}, \tag{20}$$

where $\hat{c}$ is the annihilation operator taking the detector from the first excited state of energy E to the ground state of zero energy. Note that if the detector is a harmonic oscillator, then $\hat{c}$ could just be $\sqrt{2E}$ times the usual oscillator annihilation operator. The normalization of $\hat{c}$ is not important because it will cancel out in the following anyway.

One can define a field operator associated with the detector by

$$\hat{a}_D = \mathcal{N}\int d\tau\epsilon(\tau)e^{iE\tau}\partial_\tau\hat{\Phi}[t(\tau), x(\tau)], \tag{21}$$

where $\mathcal{N}$ is chosen so as to make

$$[\hat{a}_D, \hat{a}_D^\dagger] = 1. \tag{22}$$

Furthermore, one can define a mode function associated with this operator by

$$\phi_D(t,x) = [\hat{\Phi}(t,x), \hat{a}_D^\dagger]. \tag{23}$$

Assuming that the field is (initially) in a vacuum state, the excitation probability of the detector will be given by

$$\mathcal{P} = \frac{\langle 0|\hat{a}_D^\dagger \hat{a}_D|0\rangle}{\mathcal{N}^2}. \tag{24}$$

Hence, $\hat{a}_D$ corresponds to the Hawking mode $\hat{a}_H$. This mode, ϕ_D, is the mode that the detector absorbs when it is excited. This mode, by the above argument, has a partner mode, ϕ_P, which is orthogonal to ϕ_D but is perfectly entangled with ϕ_D in the vacuum state $|0\rangle$. If one has a detector to measure various attributes of that partner mode, one will get vacuum values if one ignores the outcome of the measurements of the detector mode ϕ_D. Since $\hat{a}_P$ is a mixture of positive and negative (pseudo)norm vacuum modes, one will find a nonzero probability of finding it in the vacuum. But that probability would be the same as the probability if one looked into the vacuum without measurement of the Hawking/detector mode ϕ_D. There would of course be correlations between the $\hat{a}_P$ and $\hat{a}_D$ modes. For example, if the Hawking mode were detected (the detector was found in its excited state), the detector measuring the partner would also be excited.

Inserting the usual representation of a massless scalar field in 1 + 1-dimensional flat space-time,

$$\hat{\Phi}(t,x) = \int \frac{dk}{\sqrt{4\pi|k|}}[\hat{a}_k e^{-i(|k|t-kx)} + \hat{a}_k^\dagger e^{i(|k|t-kx)}], \tag{25}$$

we have

$$\hat{a}_D = \frac{\mathcal{N}}{i}\int d\tau\epsilon(\tau)\int dk\sqrt{\frac{|k|}{4\pi}}\left(\frac{dt}{d\tau} - \frac{k}{|k|}\frac{dx}{d\tau}\right) \times (\hat{a}_k e^{-i(|k|[t(\tau)-x(\tau)]+E\tau)} - \hat{a}_k^\dagger e^{i(|k|[t(\tau)-x(\tau)]-E\tau)}). \tag{26}$$

This allows us to read off the Bogoliubov coefficients of Sec. II expressing $\hat{a}_D$, the equivalent of the Hawking mode annihilation operator, in terms of the operators $\hat{a}_k$ and $\hat{a}_k^\dagger$,

$$\alpha_k^* = \frac{\mathcal{N}}{i}\int d\tau\epsilon(\tau)\sqrt{\frac{|k|}{4\pi}}\left(\frac{dt}{d\tau} - \frac{k}{|k|}\frac{dx}{d\tau}\right) \times e^{-i(|k|[t(\tau)-x(\tau)]+E\tau)}, \tag{27}$$

$$\beta_k = i\mathcal{N}\int d\tau\epsilon(\tau)\sqrt{\frac{|k|}{4\pi}}\left(\frac{dt}{d\tau} - \frac{k}{|k|}\frac{dx}{d\tau}\right) \times e^{i(|k|[t(\tau)-x(\tau)]-E\tau)}. \tag{28}$$

In general, both the α coefficients and β coefficients will be nonzero, and α_k will not be proportional to β_k. Thus, there is a partner mode.

Let us now restrict attention to the case where the detector is at rest at $x = 0$ but $\epsilon(t)$ is nontrivial. We choose $\epsilon(t)$ such that

$$\{\boldsymbol{\alpha}|\boldsymbol{\beta}\} = \int dk\alpha_k^*\beta_k = 0, \tag{29}$$

so that the detector mode corresponds to pure two-mode squeezing. Defining

$$\cosh^2 r = \{\boldsymbol{\alpha}|\boldsymbol{\alpha}\} = \int dk|\alpha_k|^2,$$
$$\sinh^2 r = \{\boldsymbol{\beta}|\boldsymbol{\beta}\} = \int dk|\beta_k|^2, \tag{30}$$

the partner mode is

$$\phi_P = \int \frac{dk}{\sqrt{4\pi|k|}}(\beta_k e^{-i(|k|t-kx)}\coth r + \alpha_k^* e^{i(|k|t-kx)}\tanh r). \tag{31}$$

Now, the term in the first line is just the positive frequency part of $-\phi_D^*$, which we can write as

$$\int \frac{dk}{\sqrt{4\pi|k|}}\beta_k e^{-i|k|t} = -\int dt' \frac{\phi_D^*(t', x=0)}{2\pi i(t-t'-i0^+)}. \tag{32}$$

Hence, we can also write the partner mode as

$$\phi_P(t, x=0) = -\coth r\int dt' \frac{\phi_D^*(t', x=0)}{2\pi i(t-t'-i0^+)} + \tanh r\int dt' \frac{\phi_D^*(t', x=0)}{2\pi i(t-t'+i0^+)}. \tag{33}$$

Thus, in general $\phi_P(t,0)$ will have a long tail falling off as $1/t$ for large $|t|$. Only if the moments $\int dtt^n\phi_D(t)$ are zero for all n will the partner fall off faster than any power. (If those are zero for all $n < N$ but nonzero thereafter, then ϕ_D will fall off as $1/|t|^{N+1}$.)

Let us now give an example. Let us assume that

$$\epsilon(t) = \epsilon_0(t) + \epsilon_0(t-T)\lambda\cos[3E(t-T)], \tag{34}$$

where λ is very small. Again E is the energy difference between the two states of the detector. The remaining function $\epsilon_0(t)$ is supposed to be a smooth switching function. Furthermore let us assume that the Fourier transform of $\epsilon_0(t)$, namely $\tilde{\epsilon}_0(\omega)$ is real and nonzero only in a compact region $-E/4 < \omega < E/4$. The Fourier transform of $\epsilon(t)e^{iEt}$, which occurs in the expression for $\hat{a}_D$, will have three peaks, one small one centered at $-2E$, one large one at E, and another small one at $4E$. Thus, β_k will have two small peaks with amplitude proportional to λ at $k \approx \pm 2E$, while α_k will have two large peaks of amplitude $\mathcal{O}(1)$ at $k \approx \pm E$ and two of amplitude λ at $k \approx \pm 3E$. Because of the limited width of each of these peaks, none overlap, and $\boldsymbol{\beta}$ will be orthogonal to $\boldsymbol{\alpha}$. Thus, the "detector mode" will be a pure two-mode squeezed state. The $\boldsymbol{\beta}$-coefficient and thus $\tanh r$ will be of order λ, while $\coth r$ will be of order $1/\lambda$. The partner mode will have a temporal Fourier transform with a single peak centered at $2E$ of amplitude $\mathcal{O}(1)$, two smaller peaks of amplitude λ at $-E$, and one of amplitude $\mathcal{O}(\lambda^2)$ at $-3E$. Thus, the partner mode will be approximately given by

$$\begin{aligned}\phi_P(t, x = 0) = \epsilon_0(t - T)e^{2iE(t-T)} \times \mathcal{O}(1) \\ + \epsilon_0(t)e^{-iEt} \times \mathcal{O}(\lambda) \\ + \epsilon_0(t - T)e^{-3iE(t-T)} \times \mathcal{O}(\lambda^2). \qquad (35)\end{aligned}$$

The envelope of the partner mode will thus be dominated by $\epsilon_0(t - T)$, i.e., displaced from the detector mode by a time T. As a result, the partner mode will be centered around a time arbitrarily displaced from the maximum of the detector mode. Of course, this is somewhat misleading since the part $\propto \epsilon_0(t - T)$ of detector mode which leads to detection (the β^* part of the detector mode) and the α^* part of the partner do overlap.

However, if one chooses some other mode, ϕ_X, which has an overlap with the partner but, let us assume, none with the detector mode, there will be correlations between measurements made on this mode and the outcomes of the detector measurements.

We note that this example shows that partner modes are not a unique feature of black holes, or accelerated detectors. All detectors, which have a finite probability of detecting something in the state of interest, even if due to "switch on/off" transients, will have both a detector mode and a partner mode associated with them. If the partner is well separated from the Hawking mode (which it is if we are interested in the detection of radiation from say a black hole, where the partner is behind the horizon and the other is far from the black hole), then any measurements made on the partner mode will give results indistinguishable from the results in that vacuum state. There will, however, be correlations between the results for measurements on the partner and on the "Hawking mode" and not with any other modes orthogonal to these two. If one were able to communicate between the detectors, one could for example measure the partner whenever the detector detected a particle. This would absorb a particle from the vacuum, leaving the vacuum in a lower energy state, thus extracting energy from the vacuum—a form of energy teleportation. In our case, the ability to communicate the result to somewhere where the partner could be detected would be difficult (due to causality), but in some cases [5] one can actually carry out such a procedure and extract energy from the vacuum state, leaving the system with locally less energy than the vacuum (but leaving the system as a whole of course with higher energy).

Let us also look at a more complex example. In this case let us assume that the function $\epsilon(t)$ has the form of a trapezoid—it rises linearly from 0 at time $-\tau$ to ϵ_0 at time $-T$, remains constant to time T, and then falls linearly to zero at time τ. The Fourier transform of this $\epsilon(t)$ is

$$\tilde{\epsilon}(\omega) = 2\epsilon_0 \frac{\cos(\omega T) - \cos(\omega\tau)}{\omega^2(\tau - T)}, \qquad (36)$$

which gives

$$\begin{aligned}\alpha_k = \sqrt{\frac{|k|}{4\pi}}\epsilon_0 \frac{\cos[(|k| - E)T] - \cos[(|k| - E)\tau]}{(|k| - E)^2}, \\ \beta_k = \sqrt{\frac{|k|}{4\pi}}\epsilon_0 \frac{\cos[(|k| + E)T] - \cos[(|k| + E]\tau)}{(|k| + E)^2}. \qquad (37)\end{aligned}$$

The requirement that $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ be orthogonal can always be satisfied for suitable values of E. Their overlap,

$$\{\boldsymbol{\alpha}|\boldsymbol{\beta}\}(E) = \int dk\alpha_k^*\beta_k = 2\int_0^\infty dk\alpha_k^*\beta_k, \qquad (38)$$

is plotted in Fig. 1 for $\tau = 1.2T$ as a function of ET, and we see that there are values of ET which make this overlap zero. This is certainly not required, as partners exist even if one does not have a pure two-mode squeezed state, but it makes, as we saw above, the finding of the partner much easier. We will use the zero of $\{\boldsymbol{\alpha}|\boldsymbol{\beta}\}(E)$ for E nearest 40, namely $E = 38.48966$.

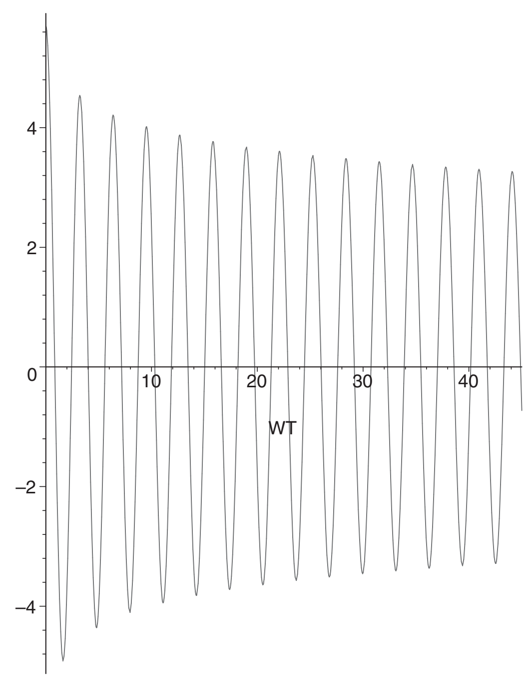

FIG. 1. The evaluation of $\{\boldsymbol{\alpha}|\boldsymbol{\beta}\}(E)$ for the trapezoidal coupling as a function of ET with $\tau = 1.2T$. The zeros correspond to orthogonality and pure two-mode squeezing.

For those values of E where $\{\boldsymbol{\alpha}|\boldsymbol{\beta}\}(E)$ is zero, and thus $\boldsymbol{\alpha}$ is orthogonal to $\boldsymbol{\beta}$, the partner mode will be

$$\phi_P(t) = \int_0^\infty d\omega \left(C_1 \frac{\cos[(\omega+E)\tau] - \cos[(\omega+E)T]}{(\omega+E)^2} e^{i\omega t} + C_2 \frac{\cos[(\omega-E)\tau] - \cos[(\omega-E)T]}{(\omega-E)^2} e^{-i\omega t} \right], \quad (39)$$

where C_1 and C_2 are appropriate normalization factors. For example, if we take

$$c_a = \int d\omega \left(\frac{\cos[(\omega-E)\tau] - \cos[(\omega-E)T]}{(\omega-E)^2} \right)^2 \omega = 5.15932,$$
$$c_b = \int d\omega \left(\frac{\cos[(\omega+E)\tau] - \cos[(\omega+E)T]}{(\omega+E)^2} \right)^2 \omega = 0.0001195, \quad (40)$$

then, defining $\tanh^2 r = c_b/c_a$, we get

$$C_1 = \frac{\cosh r}{\sqrt{c_b}} = 91.47,$$
$$C_2 = \frac{\sinh r}{\sqrt{c_a}} = 1.020 \times 10^{-5}. \quad (41)$$

In this case, the Fourier transform of the partner mode does not vanish at $\omega = 0$ [because $\tilde{\epsilon}(E)$ is not zero] but has a step at $\omega = 0$. This implies that the partner mode ϕ_P will have a slow falloff of order $1/|t|$ for large values of t. In Fig. 2 we have a plot of the magnitude of the partner mode as a function of t for $T = 1, \tau = 1.2$, and $E \approx 40$. In this case, the partner mode is concentrated in the same area as is the original detector mode but with a far longer tail. However, as we saw above, there is no requirement that the partner mode be near the peak in the detector mode.

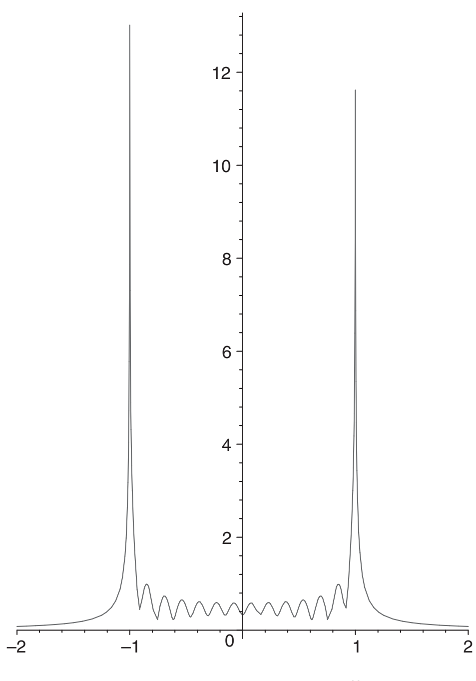

FIG. 2. The amplitude ϕ_P as a function of t.

IV. PARTNERS AND AMPLIFIERS

An example of a system where the detector or Hawking mode is completely separate from the partner more is the case of a phase insensitive amplifier. Let us take the model of such an amplifier as given in Ref. [8], in which the amplifier is represented as the coupling, by a free single degree of freedom q, of two massless one-dimensional fields ϕ and ψ, one (ψ) having a negative action

$$\mathcal{L} = \mathcal{L}_\phi + \mathcal{L}_\psi + \mathcal{L}_q + \mathcal{L}_{\rm int} = \frac{1}{2}[(\partial_t\phi)^2 - (\partial_x\phi)^2] - \frac{1}{2}[(\partial_t\psi)^2 - (\partial_x\psi)^2] + \frac{1}{2}[(\partial_t q)^2 + 2q(\mu\dot{\phi} + \nu\dot{\psi})]\delta(x). \quad (42)$$

These fields are supposed to live on the positive x axis with Neumann boundary conditions at the end point. To avoid that the support of the $\delta(x)$-coupling coincides with this end point, we assume that $x \in [-\varepsilon, \infty)$ and consider the limit $\varepsilon \downarrow 0$. The boundary conditions then read $\partial_x\phi(t,-\varepsilon) = \partial_x\psi(t,-\varepsilon) = 0$.

This model has solutions

$$\phi(t,x) = \phi_0(t,x) - \mu q(t-x), \quad (43)$$

$$\psi(t,x) = \psi_0(t,x) + \nu q(t-x), \quad (44)$$

$$\partial_t^2 q + (\mu^2 - \nu^2)\partial_t q = \partial_t[\mu\phi_0(t,0) + \nu\psi_0(t,0)], \quad (45)$$

where ϕ_0 and ψ_0 are solutions to the free (homogeneous) equation ($\mu = \nu = 0$) with the above boundary conditions. Note that q is damped as long as $\nu^2 < \mu^2$.

The quantum operators $\hat{\Phi}$, $\hat{\Psi}$, and $\hat{Q}$ obey the same equations. Taking the Fourier transform and expressing the solutions in terms of annihilation and creation operators, we have

$$\hat{\Phi}_0(t,x) = \int_0^\infty \frac{d\omega}{\sqrt{\pi\omega}} (\hat{a}_\omega e^{-i\omega t} + \hat{a}_\omega^\dagger e^{i\omega t}) \cos(\omega x), \quad (46)$$

$$\hat{\Psi}_0(t,x) = \int_0^\infty \frac{d\omega}{\sqrt{\pi\omega}} (\hat{b}_\omega^\dagger e^{-i\omega t} + \hat{b}_\omega e^{i\omega t}) \cos(\omega x), \quad (47)$$

$$\hat{Q}(t) = \hat{q}_\omega e^{-i\omega t} + \hat{q}_\omega^\dagger e^{i\omega t}, \quad (48)$$

$$\hat{q}_\omega = \frac{2i}{\sqrt{2\pi}} \frac{\mu\hat{a}_\omega + \nu\hat{b}_\omega^\dagger}{\omega + i(\mu^2 - \nu^2)}. \quad (49)$$

Note that the positions of the creation and annihilation operators of the $\hat{\Psi}$ field is reversed since its conjugate momentum is $\pi_\psi = -\partial_t \psi$. As a result, the inner products of the two fields [see also Eq. (87)],

$$(\phi|\tilde{\phi}) = i\int_0^\infty dx(\phi^* \tilde{\pi}_\phi - \pi_\phi^* \tilde{\phi}),$$

$$(\psi|\tilde{\psi}) = i\int_0^\infty dx(\psi^* \tilde{\pi}_\psi - \pi_\psi^* \tilde{\psi}), \tag{50}$$

are of opposite sign for modes ϕ_ω and ψ_ω with the same ω. The vacuum state for the $\hat{\Psi}_0$ field is a maximum of the energy, rather than a minimum, and is annihilated by the $\hat{b}_\omega$ operators.

The initial (input) fields are those that behave as $\phi_{\rm in}(t+x)$ or $\psi_{\rm in}(t+x)$ while the final (output) fields go as $\phi_{\rm out}(t-x)$ or $\psi_{\rm out}(t-x)$, respectively (remember that $x>0$). Assuming that the input fields are the free fields in Eqs. (46) and (47),

$$\hat{\Phi}_{\rm in} = \hat{\Phi}_0^{\rm in} = \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}}(\hat{a}_\omega e^{-i\omega(t+x)} + \hat{a}^\dagger e^{i\omega(t+x)}), \tag{51}$$

and similarly for $\hat{\Psi}_{\rm in} = \hat{\Psi}_0^{\rm in}$, the output part is, according to Eqs. (43) and (44), given by

$$\hat{\Phi}_{\rm out}(t-x) = \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}} e^{-i\omega(t-x)}\left(\hat{a}_\omega \frac{\omega - i(\mu^2+\nu^2)}{\omega + i(\mu^2-\nu^2)} - \hat{b}_\omega^\dagger \frac{2i\mu\nu}{\omega + i(\mu^2-\nu^2)}\right) + \text{H.c.}, \tag{52}$$

$$\hat{\Psi}_{\rm out}(t-x) = \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}} e^{-i\omega(t-x)}\left(\hat{b}_\omega^\dagger \frac{\omega - i(\mu^2+\nu^2)}{\omega - i(\mu^2-\nu^2)} + \hat{a}_\omega \frac{2i\mu\nu}{\omega - i(\mu^2-\nu^2)}\right) + \text{H.c.}, \tag{53}$$

where h.c. is the Hermitian conjugate. Thus, one can write the output annihilation and creation operators in terms of the input by

$$\hat{A}_\omega = \hat{a}_\omega \frac{\omega - i(\mu^2+\nu^2)}{\omega + i(\mu^2-\nu^2)} - \hat{b}_\omega^\dagger \frac{2i\mu\nu}{\omega + i(\mu^2-\nu^2)}, \tag{54}$$

$$\hat{B}_\omega^\dagger = \hat{b}_\omega^\dagger \frac{\omega - i(\mu^2+\nu^2)}{\omega - i(\mu^2-\nu^2)} + \hat{a}_\omega \frac{2i\mu\nu}{\omega - i(\mu^2-\nu^2)}. \tag{55}$$

Writing the first equation as $\hat{A}_\omega = \alpha_\omega \hat{a}_\omega + \beta_\omega \hat{b}_\omega^\dagger$, we see that the factor $|\alpha_\omega|$ is larger than unity (unless $\nu = 0$) which means that signals in the $\hat{a}_\omega$ channel are amplified. However, as required by unitarity, this goes along with additional noise stemming from the $\hat{b}_\omega^\dagger$ term.

Now let us consider a mode in the ϕ output channel, say, defined by (at late times)

$$f_H(t-x) = \int \frac{d\omega}{\sqrt{4\pi\omega}} \tilde{f}_H(\omega) e^{-i\omega(t-x)}, \tag{56}$$

where we will assume that $\tilde{f}_H(\omega)$ is nonzero only for $\omega > 0$ and is normalized so that $\int d\omega |\tilde{f}_H(\omega)|^2 = 1$. The operator associated with this mode at late times is

$$\hat{a}_H = (f_H|\hat{\Phi}) = \int d\omega \tilde{f}_H^*(\omega)\hat{A}_\omega = \int_0^\infty d\omega \tilde{f}_H^*(\omega) \times \left(\hat{a}_\omega \frac{\omega - i(\mu^2+\nu^2)}{\omega + i(\mu^2-\nu^2)} - \hat{b}_\omega^\dagger \frac{2i\mu\nu}{\omega + i(\mu^2-\nu^2)}\right). \tag{57}$$

As a result, the partner mode will be

$$\hat{a}_P^\dagger = \int_0^\infty d\omega \tilde{f}_H^*(\omega)\left(\hat{a}_\omega \frac{\omega - i(\mu^2+\nu^2)}{\omega + i(\mu^2-\nu^2)} \tanh\vartheta - \hat{b}_\omega^\dagger \frac{2i\mu\nu}{\omega + i(\mu^2-\nu^2)} \coth\vartheta\right), \tag{58}$$

where the mixing angle is given by

$$\sinh^2\vartheta = \int_0^\infty d\omega |\tilde{f}_H^*(\omega)|^2 \frac{4\mu^2\nu^2}{\omega^2 + (\mu^2-\nu^2)^2}. \tag{59}$$

If we define the frequency dependent mixing angles and phases

$$\cosh\theta_\omega = \left|\frac{-i\omega - (\mu^2+\nu^2)}{-i\omega + (\mu^2-\nu^2)}\right|, \tag{60}$$

$$\sinh\theta_\omega = \left|\frac{2\mu\nu}{i\omega + (\mu^2-\nu^2)}\right|, \tag{61}$$

$$e^{i\sigma_\omega} = \frac{-i\omega + (\mu^2-\nu^2)}{|-i\omega + (\mu^2-\nu^2)|}, \tag{62}$$

$$e^{i\lambda} = \frac{(\mu^2+\nu^2) + i\omega}{|i\omega + (\mu^2+\nu^2)|}, \tag{63}$$

then $\sinh^2\vartheta$ is the weighted average

$$\sinh^2\vartheta = \int_0^\infty d\omega |\tilde{f}_H^*(\omega)|^2 \sinh^2\theta_\omega, \tag{64}$$

and we find that we can express the annihilation operator of the partner mode in terms of the outgoing creation and annihilation operators in terms of either the input annihilation operators or of the output,

$$a_P^\dagger = \int_0^\infty d\omega \tilde{f}_H^*(\omega) e^{-i\sigma_\omega}(-\tanh\vartheta \cosh\theta_\omega e^{i\lambda_\omega}\hat{a}_\omega - \coth\vartheta \sinh\theta_\omega \hat{b}_\omega^\dagger) \quad (65)$$

$$= \int_0^\infty d\omega \tilde{f}_H^*(\omega) e^{-i\sigma_\omega}\left(2\frac{\sinh^2\vartheta - \sinh^2\theta_\omega}{\sinh(2\vartheta)}\hat{A}_\omega - \frac{\sinh(2\theta_\omega)}{\sinh(2\vartheta)}\hat{B}_\omega^\dagger\right). \quad (66)$$

If $|\tilde{f}_H^*(\omega)|$ is a highly peaked function about the frequency ω such that $\theta_\omega = \vartheta$, then the first term will be zero, and the partner mode, made up entirely of $\hat{B}_\omega$, will be confined completely to the second output channel—the ψ channel. However, if $\tilde{f}_H(\omega)$ is a broad function (nonzero over a range of order or larger than $\mu^2 - \nu^2$), then the partner mode will have support in both the ψ and the ϕ output channels—mostly in the former, but partially in the latter as well.

This will also be true in the black hole case as well, which behaves exactly like this amplifier, with the output ϕ and ψ channels being the modes travelling to infinity and those falling into the singularity, respectively. For highly peaked functions of frequency, the partner is behind the horizon, while for broadly peaked functions of frequency, the partner has components both inside and outside the horizon. This is another indication of the nonlinear nature of the partner mode. Since any mode is the sum of highly peaked functions, one might expect that the a broadly peaked Hawking mode might still have a partner entirely behind the horizon, but it does not.

V. MOVING MIRROR RADIATION

Before applying the concept of partner particles to what has been taken to be a simple toy model for black hole evaporation—the radiation given off by an exponentially accelerated mirror [9–12]—let us briefly review the basic concepts of this accelerated mirror model. We consider a massless scalar field in 1 + 1-dimensional flat space-time:

$$\Box\phi = 0. \quad (67)$$

At a pointlike mirror with the trajectory $x_\mathrm{m}(t)$, we impose Dirichlet boundary condition

$$\phi(t, x_\mathrm{m}[t]) = 0. \quad (68)$$

In terms of the light-cone coordinates

$$u = t - x, \qquad v = t + x, \quad (69)$$

the general solution of $\Box\phi = 0$ without the boundary condition (68) can be written as a sum of independent left-moving $\phi_\mathrm{left}(v)$ and right-moving $\phi_\mathrm{right}(u)$ contributions $\phi(u,v) = \phi_\mathrm{left}(v) + \phi_\mathrm{right}(u)$. The boundary condition (68) imposes constraints on these two parts, and thus the quantum field can be decomposed as

$$\hat{\phi}(u,v) = \int_0^\infty d\omega \frac{e^{-i\omega v} - e^{-i\omega(2\tau[u]-u)}}{\sqrt{4\pi\omega}}\hat{a}_\omega^\mathrm{in} + \mathrm{H.c.} \quad (70)$$

Here $(\hat{a}_\omega^\mathrm{in})^\dagger$ and $\hat{a}_\omega^\mathrm{in}$ denote the initial creation and annihilation operators, and the function $\tau[u]$ is implicitly determined by the mirror trajectory

$$\tau[u] = u + x_\mathrm{m}(\tau[u]). \quad (71)$$

Hence, the mode functions in Eq. (70) automatically satisfy the boundary condition (68). For a mirror at rest $x_\mathrm{m} = \mathrm{const}$, we find $\tau[u] = u + x_\mathrm{m}$, and thus these mode functions simplify to $e^{-i\omega v} - e^{-i\omega(u+2x_\mathrm{m})}$ which just gives $2ie^{-i\omega t}\sin(\omega[x_\mathrm{m} - x])e^{-i\omega x_\mathrm{m}}$ as one would expect. Thus, assuming that the mirror is a rest initially, the initial vacuum state is determined by

$$\forall_{\omega>0}\hat{a}_\omega^\mathrm{in}|0\rangle = 0. \quad (72)$$

Similarly, for a mirror $x_\mathrm{m} = Vt$ moving with a constant velocity V, we get $\tau[u] = u/(1-V)$. In these cases, no particles are created—but with an accelerated motion of the mirror (resulting in a nontrivial form of $\tau[u]$), one can create particles out of the initial vacuum.

In terms of the light-cone coordinates, the proper acceleration of the mirror $\ddot{x}_\mathrm{m}/(1-\dot{x}_\mathrm{m}^2)^{3/2}$ can be written as $\ddot{v}_\mathrm{m}/(\dot{u}_\mathrm{m}\dot{v}_\mathrm{m})^{3/2}$. Similarly, the redshift factor $\sqrt{(1+\dot{x}_\mathrm{m})/(1-\dot{x}_\mathrm{m})}$ simply reads $\sqrt{\dot{v}_\mathrm{m}/\dot{u}_\mathrm{m}}$. Now, if we choose the mirror trajectory in such a way that the proper acceleration of the mirror is proportional to the redshift factor, an observer at rest sees a stationary thermal spectrum given off by the moving mirror. This situation corresponds to the mirror trajectory

$$t + x_\mathrm{m} = v_\mathrm{m} = -\frac{e^{-\kappa u_\mathrm{m}}}{\kappa} = -\frac{e^{-\kappa(t-x_\mathrm{m})}}{\kappa}, \quad (73)$$

where κ is a proportionality constant which sets the temperature. As a result, the mode functions satisfying the boundary condition (68) are given by

$$\phi_\omega(u,v) = e^{-i\omega v} - \exp\left\{i\frac{\omega}{\kappa}e^{-\kappa u}\right\}. \quad (74)$$

In principle, since the proper acceleration of the trajectory (73) vanishes for very early times $t\downarrow -\infty$, we could consider a mirror moving along the worldline (73) for all times. However, to make the initial behavior as simple as possible, we assume that the mirror is initially at rest and starts accelerating along the trajectory (73) at $u^0 = 0$ which means $v_\mathrm{m}^0 = -1/\kappa$, i.e.,

$$v_{\mathrm{m}}(u) = -\frac{1}{\kappa}\begin{cases} 1-\kappa u & \text{for } u<0 \\ e^{-\kappa u} & \text{for } u>0 \end{cases}. \tag{75}$$

Consequently, incoming light rays with $v < -1/\kappa$ are reflected by the mirror at rest; i.e., initial waves of the form $e^{-i\omega v}$ are simply transformed to final waves of the form $e^{-i\omega u}$. Incoming light rays in the window $-1/\kappa < v < 0$ are reflected by the accelerating mirror. In this region, initial waves of the form $e^{-i\omega v}$ are stretched by the increasing redshift factor and finally behave as $\exp\{i\omega e^{-\kappa u}/\kappa\}$. More generally, an initial wave packet of the form $\phi_{\mathrm{left}}(v)$ in the region $-1/\kappa < v < 0$ is transformed to $\phi_{\mathrm{right}}(-e^{-\kappa u}/\kappa)$, i.e., a final right-moving wavepacket in the region $u > 0$. The remaining light rays with $v > 0$ do not see the mirror at all, and thus their functional form is unchanged. Hence, the null line $v = 0$ is analogous to the black hole horizon.

Starting in the initial vacuum state (72), we can derive the two-point functions in the final state. To avoid artifacts stemming from the infrared divergence of the massless scalar field in two dimensions, we consider the first derivatives of the fields. (This is somewhat similar to considering the electric and magnetic fields instead of the scalar and vector potentials.) As mentioned above, the field can be split up into a left-moving $\hat{\phi}_{\mathrm{left}}(v)$ and a right-moving part $\hat{\phi}_{\mathrm{right}}(u)$. The correlation between the two final left-moving contributions (with $v_{1,2} > 0$) gives

$$\langle 0|\partial_v\hat{\phi}_{\mathrm{left}}(v_1)\partial_v\hat{\phi}_{\mathrm{left}}(v_2)|0\rangle = -\frac{1}{4\pi}\frac{1}{(v_1-v_2)^2}, \tag{76}$$

which just reflects the fact the associated quantum state is locally indistinguishable from vacuum (since it has not "seen" the mirror at all).

Considering the correlation between two right-moving contributions which have been reflected by the accelerated mirror (with $u_{1,2} > 0$), however, gives

$$\langle 0|\partial_u\hat{\phi}_{\mathrm{right}}(u_1)\partial_u\hat{\phi}_{\mathrm{right}}(u_2)|0\rangle = -\frac{\kappa^2}{16\pi}\frac{1}{\sinh^2(\kappa[u_1-u_2]/2)}. \tag{77}$$

As already suggested by the periodicity in imaginary time [Kubo-Martin-Schwinger (KMS) condition], this is locally indistinguishable from a thermal state with the temperature

$$T_H = \frac{\kappa}{2\pi}. \tag{78}$$

The fact that this thermal contribution and the above vacuum part are actually just two regions of the same pure state results in nontrivial cross-correlations between these two contributions,

$$\langle 0|\partial_v\hat{\phi}_{\mathrm{left}}(v_1)\partial_u\hat{\phi}_{\mathrm{right}}(u_2)|0\rangle = -\frac{\kappa^2}{4\pi}\frac{e^{-\kappa u_2}}{(e^{-\kappa u_2}+\kappa v_1)^2}. \tag{79}$$

These results can be generalized to different mirror trajectories $v_{\mathrm{m}}(u)$ in a straightforward manner. In this case, the mode functions read $e^{-i\omega v} - e^{-i\omega v_{\mathrm{m}}(u)}$, and thus the two-point function is given by

$$\langle 0|\hat{\phi}(u_1,v_1)\hat{\phi}(u_2,v_2)|0\rangle = -\frac{1}{4\pi}\ln\left(\frac{[v_1-v_2][v_{\mathrm{m}}(u_1)-v_{\mathrm{m}}(u_2)]}{[v_1-v_{\mathrm{m}}(u_2)][v_{\mathrm{m}}(u_1)-v_2]}\right). \tag{80}$$

VI. PARTNER PARTICLES FOR MIRROR THERMAL RADIATION

Now let us try to determine the partner particles for the thermal radiation created by the mirror as an analog for Hawking radiation. Thus, we define the outgoing Hawking wave function $f_H(u)$ as a linear combination of final positive-frequency right-moving plane waves,

$$f_H(u) = \int_0^\infty d\Omega \tilde{f}_H(\Omega)e^{-i\Omega u}, \tag{81}$$

where $\tilde{f}_H(\Omega)$ is then the Fourier transform of $f_H(u)$. Due to the restriction to positive final frequencies, the support of $f_H(u)$ is unbounded, i.e., extends to negative u as well. However, for simplicity, we assume that $f_H(u)$ lies mostly in the thermal region $u > 0$, i.e., that $f_H(u)$ is exponentially suppressed for $u < 0$. Alternatively, we could consider the case of eternal acceleration of the mirror, where the thermal region extends to negative u.

The Bogoliubov coefficients can then be defined by the overlap between these modes (81) and the initial positive/negative frequency modes $e^{\pm i\omega v}$. For the trajectory (73), these overlap integrals can be calculated analytically in terms of Γ-functions, etc. However, instead of using these Γ-functions, we do the following trick: Initially, the mode (81) behaved as

$$f_H^{\mathrm{in}}(v<0) = \int_0^\infty d\Omega \tilde{f}_H(\Omega)(-\kappa v)^{i\Omega/\kappa}, \tag{82}$$

and $f_H^{\mathrm{in}}(v > 0) = 0$. Now, let us consider the following linear combinations:

$$f_\Omega^{\pm}(v) = \begin{cases} e^{\pm\pi\Omega/(2\kappa)}|\kappa v|^{i\Omega/\kappa} & \text{for } v<0 \\ e^{\mp\pi\Omega/(2\kappa)}|\kappa v|^{i\Omega/\kappa} & \text{for } v>0 \end{cases}. \tag{83}$$

These linear combinations are chosen such that the function $f_\Omega^+(v)$ is holomorphic in the entire lower half of the complex v plane, i.e., for $\Im(v) < 0$, and has a singularity at $v = 0$ as well as a branch cut from $v = 0$ to $v = \infty$ in the upper half. On the other hand, recalling the structure of the initial mode functions $e^{-i\omega v}$, we find that any solution is exactly decomposed of positive (initial) frequency modes if and only if it is holomorphic in entire lower half of the complex v plane. Thus, the linear combination $f^+_\Omega(v)$ in Eq. (83) contains only positive (initial) frequency modes for all Ω; i.e., it corresponds to an initial annihilation operator $\hat{a}^{\rm in}_\Omega$ with $\hat{a}^{\rm in}_\Omega|0\rangle = 0$. Conversely, the other combination $f^-_\Omega(v)$ is holomorphic in the upper half of the complex v plane and thus contains negative (initial) frequencies only; i.e., it corresponds to an initial creation operator. Using the symmetry $[f^\pm_\Omega(v)]^* = f^\mp_{-\Omega}(v)$, we find that $f^-_\Omega(v)$ corresponds to $(\hat{a}^{\rm in}_{-\Omega})^\dagger$.

Now, we can decompose the function $|\kappa v|^{i\Omega/\kappa}$ as a linear combination of $f^\pm_\Omega(v)$ which gives

$$\frac{e^{+\pi\Omega/(2\kappa)} f^+_\Omega(v) - e^{-\pi\Omega/(2\kappa)} f^-_\Omega(v)}{2\sinh(\Omega/\kappa)} = |\kappa v|^{i\Omega/\kappa} \tag{84}$$

for $v < 0$ and vanishes for $v > 0$. This enables us to directly read off the decomposition of the final Hawking mode (81) into initial creation and annihilation operators

$$\hat{a}_H = \int_0^\infty d\Omega \tilde{f}_H(\Omega)[\alpha_\Omega \hat{a}^{\rm in}_\Omega + \beta_\Omega (\hat{a}^{\rm in}_{-\Omega})^\dagger], \tag{85}$$

with the Bogoliubov coefficients

$$\alpha_\Omega = \frac{e^{+\pi\Omega/(2\kappa)}}{\sqrt{2\sinh(\Omega/\kappa)}}, \qquad \beta_\Omega = \frac{e^{-\pi\Omega/(2\kappa)}}{\sqrt{2\sinh(\Omega/\kappa)}}, \tag{86}$$

where the denominator $\sqrt{2\sinh(\Omega/\kappa)}$ ensures the correct normalization $|\alpha_\Omega|^2 - |\beta_\Omega|^2 = 1$.

This is the starting point for the derivation of the partner mode $\hat{a}_P$. Note that the modes $f^\pm_\Omega(v)$ are orthogonal with respect to the usual inner product for the scalar field,

$$(\phi_1|\phi_2) = i\int d\Sigma^\mu \phi_1^* \overleftrightarrow{\partial}_\mu \phi_2, \tag{87}$$

where $\phi_1^* \overleftrightarrow{\partial}_\mu \phi_2 = \phi_1^*\partial_\mu\phi_2 - \phi_2\partial_\mu\phi_1^*$. For purely right-moving modes, we may align the hypersurface Σ with a null line of constant u (or even $\mathcal{J}^-$) such that the $d\Sigma^\mu$-integral becomes an integration over v and the derivative $\overleftrightarrow{\partial}_\mu$ simplifies to $\overleftrightarrow{\partial}_v$. As explained above, the mode functions $f^+_\Omega(v)$ are decomposed of purely positive frequency initial waves $e^{-i\omega v}$ with $\omega > 0$. Conversely, the mode functions $f^-_\Omega(v)$ are decomposed of purely negative frequency initial waves $e^{+i\omega v}$ with $\omega > 0$. As a result, the contributions $f^+_\Omega(v)$ and $f^-_\Omega(v)$ are orthogonal, and thus the vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are orthogonal; i.e., we have pure two-mode squeezing for all $\tilde{f}_H(\Omega)$.

Using the arguments presented in Sec. II, we find that the partner mode reads

$$\hat{a}_P = \int_0^\infty d\Omega \tilde{f}^*_H(\Omega)[\chi\beta_\Omega \hat{a}^{\rm in}_{-\Omega} + \chi^{-1}\alpha_\Omega(\hat{a}^{\rm in}_\Omega)^\dagger], \tag{88}$$

with the factor $\chi = \alpha/\beta$

$$\chi^2 = \frac{\int_0^\infty d\Omega|\tilde{f}_H(\Omega)|^2\alpha^2_\Omega}{\int_0^\infty d\Omega|\tilde{f}_H(\Omega)|^2\beta^2_\Omega}. \tag{89}$$

If the Hawking mode $\tilde{f}_H(\Omega)$ is well localized and peaked at a given frequency $\Omega = \Omega_0$, then we may approximate this factor by $\chi \approx e^{\pi\Omega_0/\kappa}$. As a result, the wave function of the partner particle $f_P(v)$ contains the following linear combination of the modes $f^\pm_\Omega(v)$, which yields

$$\frac{e^{+\pi\Omega/(2\kappa)} f^+_{-\Omega}(v) - e^{-\pi\Omega/(2\kappa)} f^-_{-\Omega}(v)}{2\sinh(\Omega/\kappa)} = |\kappa v|^{-i\Omega/\kappa} \tag{90}$$

for $v > 0$ and vanishes for $v < 0$. Thus, the wave function of the partner particle is approximately the mirror image of the initial form of the Hawking mode (82) on the other side of the horizon at $v = 0$, i.e.,

$$f_P(v) \approx \int_0^\infty d\Omega \tilde{f}^*_H(\Omega)(\kappa v)^{-i\Omega/\kappa} = f^*_H\left[-\frac{\ln(\kappa v)}{\kappa}\right], \tag{91}$$

for $v > 0$ and zero for $v < 0$. As a result, detecting a Hawking particle with, say, $\Omega = \mathcal{O}(\kappa)$ in the thermal region at late times $\kappa u \gg 1$ yields (up to normalization) approximately the same state as creating a partner particle with exponentially short wavelengths in the left-moving vacuum region at very small but positive values of v. Figure 3 is a plot of a specific outgoing Hawking mode (say the mode detected by some detector) and that mode in the input state, and its partner mode.

If we could signal the measurement result of the Hawking detector at large $u > 0$ on the right-hand side to this vacuum region on the left-hand side, we would (at least in principle) be able to extract energy out of this quantum state, which is locally indistinguishable from vacuum—this is directly related to the concept of "energy teleportation." However, causality prevents us from signaling since these two events are spacelike separated.

Another point is that the cancellation of the contributions in Eq. (90) occurs at one frequency $\Omega = \Omega_0$ only. If we consider a small but finite width $\Delta\Omega \ll \Omega_0$, the partner mode will also have support in the same thermal region as the Hawking particle. For simplicity, let us consider a Gaussian wave packet of the form

$$\tilde{f}_H(\Omega) = \mathcal{N}\exp\left\{-\frac{(\Omega-\Omega_0)^2}{2(\Delta\Omega)^2} + i\Omega u_0\right\}, \tag{92}$$

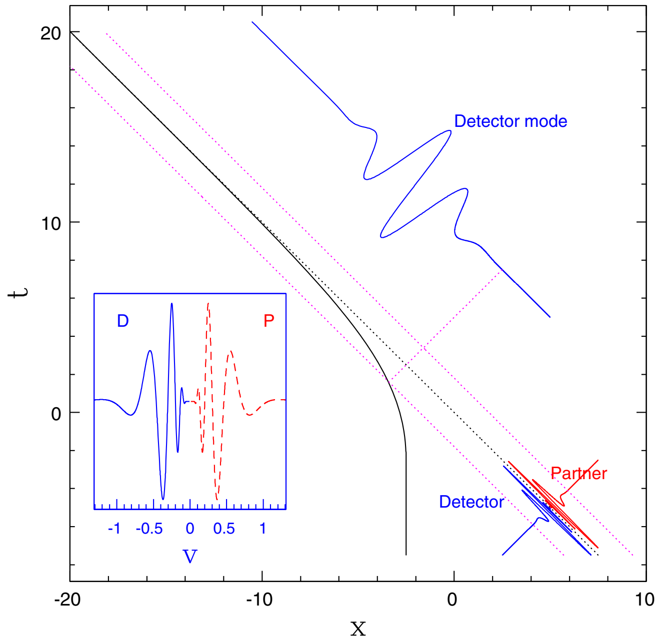

FIG. 3 (color online). Schematic of the outgoing detected Hawking mode (blue wave packet in upper right corner), its shape when traced back to the initial state (lower right corner), and its partner mode (red curve). The insert gives an expanded view of the incoming modes which correspond to the detector mode D (solid blue curve) and its partner mode P (dashed red curve) as a function of $v = t + x$. The solid black curve represents the mirror trajectory, and light rays (including the horizon) are depicted by dotted lines. Note that this schematic is not to scale; e.g., the detector mode is not very narrow in momentum space, and thus the approximation (91) would not be very accurate (i.e., the partner mode would not be the exact mirror image of the detector mode on the other side of the horizon).

which is centered around Ω_0 in frequency space and around u_0 in position space. Even though this $\tilde{f}_H(\Omega)$ is not exactly zero for $\Omega < 0$, this contribution is exponentially small for $\Delta\Omega \ll \Omega_0$ and is thus negligible. Similarly, the tail of this wave packet for $u < 0$ can be made very small by assuming $u_0 \Delta\Omega \gg 1$.

After inserting this form (92), the formula for the partner mode $f_P(v < 0)$ contains an Ω-integral of which the integrand vanishes (to lowest order) at $\Omega = \Omega_0$. Taylor expanding this integrand around this zero then yields a first-order contribution of the form $(\Omega - \Omega_0)\tilde{f}_H(\Omega)$ which can also be represented by $(\Delta\Omega)^2[iu_0 - \partial_\Omega]\tilde{f}_H(\Omega)$. After the Fourier transformation (81), the ∂_Ω translates into iu, and thus the partner wave function acquires a small contribution with the same support as the Hawking mode

$$f_P(u) \sim \frac{(\Delta\Omega)^2}{\kappa}[u - u_0]f_H(u), \tag{93}$$

in addition to the dominant contribution (91). Due to the term $[u - u_0]$, the two modes are orthogonal as they should be.

VII. CONCLUSIONS

Perhaps the most surprising conclusion of this paper is that the partner particles of the thermal radiation (emitted by a mirror or a black hole) are concentrated in a region which is locally indistinguishable from vacuum. In the black hole evaporation process, the Hawking particles emitted at early or intermediate times can be entangled not with some other energetic emission at late times but with final vacuum fluctuations. This weakens the usual argument in black hole evaporation studies which assume unitarity in the above sense, which states that the large amount of information left in the black hole (entanglement with the emitted thermal emission) must be accompanied by the eventual emission of large amounts of energy.

Thus, if one were to imagine a black hole constantly fed by a pure state designed to just compensate for the energy emitted by the black hole in Hawking thermal emission, for 10^{99} times the natural decay lifetime of the black hole, there must be a huge amount of information inside the black hole, encoded in the entanglement with the outgoing Hawking radiation. When the black hole eventually evaporates that information, which must be emitted at late times, one could expect that it must be accompanied by a large amount of energy as well. However, this paper offers the possibility that eventual emission of information could be in the form of the vacuum, and carrying no energy.

For example, the Bardeen model [13], a response to the Almheiri-Marolf-Polchinski-Sully (AMPS) argument [14] that either unitarity (as defined above) or the regularity of the space-time at horizon must be wrong, has the partner radiation to the Hawking emission trapped within the apparent horizon of the black hole, until eventually that horizon disappears. This would seem to require a massive emission of energy just at the time when that apparent horizon disappears to accompany that massive emission of information (i.e., the entangled partner radiation to the earlier Hawking emission). Our results offer the possibility that those partner modes are, in that final stage, simply a part of the vacuum state with no energy accompanying them.

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APPENDIX: UNIQUENESS PROOF

In the following, we show that the ansatz (12) for the partner particle is the most general ansatz one can make, i.e., that the partner particle is uniquely determined by our two conditions (unless we have pure single-mode squeezing $\boldsymbol{\alpha}\|\boldsymbol{\beta}$). Note that the existence of the partner mode is already demonstrated by the construction in Sec. II which works in all cases (at least using condition B1) apart from those where we have pure single-mode squeezing $\boldsymbol{\alpha}\|\boldsymbol{\beta}$.

For free fields, the initial vacuum state is a Gaussian state, and thus the state restricted to the two modes $\hat{a}_H$ and $\hat{a}_P$ must also be a Gaussian state. In the position representation, i.e., as a function of the position vector $\boldsymbol{x} = (x_H, x_P)$ where $\hat{x}_H = (\hat{a}_H + \hat{a}_H^\dagger)/\sqrt{2}$ and $\hat{x}_P = (\hat{a}_P + \hat{a}_P^\dagger)/\sqrt{2}$, the most general wave function of a pure Gaussian state reads

$$\psi(\boldsymbol{x}) = \mathcal{N} \exp\left\{-\frac{1}{2}\boldsymbol{x}\cdot\boldsymbol{M}\cdot\boldsymbol{x}\right\}, \tag{A1}$$

where $\boldsymbol{M}$ is a symmetric but possibly complex matrix and $\mathcal{N}$ is the corresponding normalization factor. To have a normalizable state, the real part of $\boldsymbol{M}$ must have two positive eigenvalues $\lambda_{1,2}$.

Now, the derivative of ψ yields

$$\frac{\partial}{\partial \boldsymbol{x}}\psi(\boldsymbol{x}) = -\boldsymbol{M}\cdot\boldsymbol{x}\psi(\boldsymbol{x}). \tag{A2}$$

In terms of the momentum operator $\hat{\boldsymbol{p}}$, we get

$$(i\hat{\boldsymbol{p}} + \boldsymbol{M}\cdot\hat{\boldsymbol{x}})|\psi\rangle = 0. \tag{A3}$$

This motivates the introduction of preannihilation operators $\hat{\boldsymbol{A}} = i\hat{\boldsymbol{p}} + \boldsymbol{M}\cdot\hat{\boldsymbol{x}}$ which obey the following commutation relations:

$$[\hat{A}_I, \hat{A}_J] = [\hat{A}_I^\dagger, \hat{A}_J^\dagger] = 0,$$
$$[\hat{A}_I, \hat{A}_J^\dagger] = M_{IJ} + M_{IJ}^* = 2\Re(M_{IJ}). \tag{A4}$$

Since $\Re(\boldsymbol{M})$ is a real symmetric and positive matrix, we may diagonalize it with an orthogonal (rotation) matrix $\boldsymbol{D}$ such that $\boldsymbol{D}\cdot\Re(\boldsymbol{M})\cdot\boldsymbol{D}^\dagger = \mathrm{diag}\{\lambda_I\}$. As a result, the operators $\hat{\boldsymbol{a}} = \boldsymbol{D}\cdot[2\Re(\boldsymbol{M})]^{-1/2}\cdot\hat{\boldsymbol{A}}$, i.e.,

$$\hat{a}_I = \frac{D_{IJ}\hat{A}_J}{\sqrt{2\lambda_I}} \rightsquigarrow \hat{a}_I|\psi\rangle = 0, \tag{A5}$$

satisfy the standard commutation relations and do also annihilate the state $|\psi\rangle$. Since this state $|\psi\rangle$ is just the initial vacuum state reduced to the two modes $\hat{a}_H$ and $\hat{a}_P$, the two operators $\hat{a}_{1,2}$ above must be a linear combination of the initial annihilation operators. From the construction above, we see that the Hawking and partner mode operators $\hat{a}_H$ and $\hat{a}_P$ must be linear combinations of these operators $\hat{a}_1$ and $\hat{a}_2$ as well as their adjoints $\hat{a}_1^\dagger$ and $\hat{a}_2^\dagger$. Thus, the linear subspace spanned by $\hat{a}_1$ and $\hat{a}_2$ can be identified with that of $\hat{a}_\|$ and $\hat{a}_\perp$, and we arrive at the ansatz (12).

Alternatively, one could insert the general ansatz for the modes $\hat{a}_H = \{\boldsymbol{\alpha}|\hat{\boldsymbol{a}}\} + \{\hat{\boldsymbol{a}}|\boldsymbol{\beta}\}$ and $\hat{a}_P = \{\boldsymbol{\gamma}|\hat{\boldsymbol{a}}\} + \{\hat{\boldsymbol{a}}|\boldsymbol{\delta}\}$ into Eq. (A3) which gives the two linear equations

$$\boldsymbol{\beta} - \boldsymbol{\alpha} + M_{11}(\boldsymbol{\beta}+\boldsymbol{\alpha}) + M_{12}(\boldsymbol{\delta}+\boldsymbol{\gamma}) = 0,$$
$$\boldsymbol{\delta} - \boldsymbol{\gamma} + M_{21}(\boldsymbol{\delta}+\boldsymbol{\gamma}) + M_{22}(\boldsymbol{\beta}+\boldsymbol{\alpha}) = 0, \tag{A6}$$

where M_{IJ} are the components of the symmetric matrix $\boldsymbol{M}$ (which also depend on $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$, and $\boldsymbol{\delta}$). Since these two equations are linearly independent for all M_{IJ}, we find that $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$ must lie in the same subspace as $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ (which are assumed to be linearly independent).

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