Backreaction in growing neutrino quintessence

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We investigate the cosmological effects of neutrino lumps in growing neutrino quintessence. The strongly nonlinear effects are resolved by means of numerical N-body simulations which include relativistic particles, nonlinear scalar field equations, and backreaction effects. For the investigated models with a constant coupling between the scalar field and the neutrinos, the backreaction effects are so strong that a realistic cosmology is hard to realize. This points toward the necessity of a field-dependent coupling in growing neutrino quintessence. In this case realistic models of dynamical dark energy exist which are testable by the observation or nonobservation of large neutrino lumps.

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I. INTRODUCTION

The origin of the observed accelerated expansion of the Universe is still unknown [1,2]. It is usually accounted for by a dark energy (DE) component. The simplest possibility consistent with observations is a cosmological constant Λ, but a lot of alternatives have been proposed [3]. Prime candidates are dynamical dark energy models mediated by a scalar field or modified gravity—the latter being often equivalent to the former [4]. For many alternatives the cosmological constant problem [5,6] of explaining the small value of Λ persists, however. Also the explanation of why DE becomes important in the present cosmological epoch is often not more convincing than for a cosmological constant.

Growing neutrino quintessence (GNQ) [7,8] offers some advantages here. As a quintessence model [9,10], the late time acceleration is driven by a scalar field φ (the cosmon), employing a mechanism similar to inflation. It is possible to unify the late and early time acceleration into a single picture [11–13] so that the same field is responsible for DE and inflation. As an overall description within quantum gravity crossover cosmology [14], GNQ also addresses the cosmological constant problem.

GNQ is able to explain the smallness of the DE component, since the dynamical DE density decays during the cosmic history, just as the other energy densities in the Universe. The DE density being small is then just a matter of time—it is small because the Universe is old. In contrast to simpler quintessence models, GNQ solves the why-now problem. No fine-tuning of the self-interaction potential is needed for this purpose. A coupling between the cosmon and the neutrinos provides a mechanism for stopping the evolution of the cosmon field as soon as the neutrinos become nonrelativistic. The phenomenology of a very slowly evolving scalar field resembles a cosmological constant. The transition from relativistic to nonrelativistic neutrinos acts as a trigger for the DE domination. For neutrino masses allowed by observations, this transition happens in the "recent" past, explaining why DE has become important now.

Despite a background evolution similar to the ΛCDM model for redshift $z \lesssim 5$, GNQ has a phenomenology which is distinct from other models. It predicts a time varying neutrino mass and the formation of neutrino lumps, which might be detectable through their gravitational potentials [15]. The formation of lumps is a consequence of the large coupling between neutrinos and the cosmon, which is required for the stopping mechanism. The resulting additional attraction between neutrinos is about 10^3 times stronger than the gravitational attraction. It can have a natural explanation in a particle physics framework [8].

While the strong coupling on the one hand offers with the lumps a clear and distinct way of testing the model, on the other hand, it renders the model technically difficult to study. In GNQ perturbations in the neutrino density become nonlinear already at $z \approx 1$, this happens on very large scales [15]. This has lead to the development of a comprehensive N-body simulation [16,17] to follow the formation of the neutrino lumps. The simulation is different from the usual cold dark matter (CDM) only simulations: To include backreaction effects, induced by the highly nonlinear nature of the lumps [18], the background is solved simultaneously with the perturbations. Additionally, neutrinos becoming relativistic during the formation of lumps are captured by the simulation. A similar framework for relativistic N-body simulation with focus on the metric perturbations was explored recently in Ref. [19]. With our simulation it was possible to draw a consistent picture of neutrino structures within GNQ. For stable lumps the main characteristic features can be understood within an approximation in terms of a nonrelativistic fluid of neutrino lumps [20].

In this work we investigate if GNQ can provide a realistic expansion history. Therefore, we study the equation of state and the energy density of the cosmon for different model

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parameters. We aim to find model parameters for which the backreaction effect remains compatible with an accelerated expansion with $\Omega_{\mathrm{DE}} \approx 0.7$. At the same time, the accelerated expansion of the Universe must start early enough to be consistent with observations.

A time-dependent neutrino mass related to a scalar dark energy field concerns a wider setting than GNQ. Mass varying neutrino scenarios have been studied earlier in Ref. [21] and share common features with GNQ as the instability of neutrino perturbations [22–24].

This work is organized as follows. We start with a brief review of GNQ in Sec. II. In Sec. III we discuss the formation of lumps and their backreaction on the cosmological expansion. In Sec. IV we describe our simulation, which we use to perform a parameter scan. Results are presented in Sec. V. Finally, we conclude in Sec. VI.

II. GROWING NEUTRINO QUINTESSENCE

A. Basic concepts

In this section we briefly describe GNQ. The ingredients of GNQ are a scalar field φ (the cosmon) and neutrinos. The neutrino mass depends on the value of φ, thereby coupling the cosmon and the neutrinos. The cosmon itself is described by the standard Lagrangian of a scalar field which takes, using the metric signature $(-,+,+,+)$ and setting the reduced Planck mass to unity, $8\pi G = 1$, the form

$$-\mathcal{L}_{\varphi} = \frac{1}{2}\partial_{\mu}\varphi\partial^{\mu}\varphi + V(\varphi). \quad (1)$$

We choose an exponential potential $V(\varphi) \propto e^{-\alpha\varphi}$. As long as the neutrino mass can be neglected, the exponential potential leads to scaling solutions of the cosmon field. The background energy density of the cosmon becomes independent of the initial conditions and mimics matter (radiation) during matter (radiation) domination [9], where the energy density of the cosmon is a constant fraction of the total energy density $\Omega_{\varphi} = 3\frac{1+w}{\alpha^2}$. Here w is the equation of state of the dominating species. Constraints on early dark energy (EDE) require $\alpha \gtrsim 10$ [25–28], where we use a conservative bound in view of possible unexplored parameter degeneracies.

The dependence of the neutrino mass on the cosmon is given by

$$\beta = -\frac{\mathrm{d}\ln m_{\nu}(\varphi)}{\mathrm{d}\varphi} < 0. \quad (2)$$

In general the coupling β can be φ dependent. We establish in this paper that the size of the backreaction effect depends crucially on the presence or absence of a variation of $\beta(\varphi)$. An investigation of a particle physics motivated variation of β [8] in Ref. [17] has revealed a small backreaction effect and an overall cosmology consistent with present observations. For a constant β, large backreaction effects have been observed [16]. We address here the question if the model remains compatible with observations in this case as well.

A constant coupling implies for the neutrino mass

$$m_{\nu}(\varphi) = m_i e^{-\beta\varphi}, \quad (3)$$

where an additive constant in φ is fixed such that $V(\varphi = 0) = 2.915 \times 10^{-7}$ eV. The φ-dependent neutrino mass allows for energy transfer between neutrinos and the cosmon, which is proportional to the trace of neutrino energy-momentum tensor:

$$\nabla_{\nu}T^{\mu\nu}_{(\varphi)} = +\beta T_{(\nu)}\dot{\varphi}$$
$$\nabla_{\nu}T^{\mu\nu}_{(\nu)} = -\beta T_{(\nu)}\dot{\varphi}. \quad (4)$$

The trace of the energy-momentum tensor $T_{(\nu)} = T^{\mu}_{\mu,(\nu)} = -\rho_{\nu} + 3P_{\nu}$ vanishes for ultrarelativistic neutrinos. The coupling between neutrinos and the cosmon is therefore ineffective for relativistic neutrinos. The neutrino energy-momentum tensor also sources the Klein–Gordon equation which governs the evolution of the cosmon:

$$\nabla_{\mu}\nabla^{\mu}\varphi - V'(\varphi) = \beta T_{(\nu)}. \quad (5)$$

We will describe neutrinos and dark matter by an N-body simulation. The trajectories of classical neutrinos obey a modification of the geodesic equation [16],

$$\frac{\mathrm{d}u^{\mu}}{\mathrm{d}\tau} + \Gamma^{\mu}_{\nu\lambda}u^{\nu}u^{\lambda} = \beta\partial^{\mu}\varphi + \beta u^{\nu}\partial_{\nu}\varphi u^{\mu}, \quad (6)$$

where u^{μ} denotes the 4-velocity and τ the proper time. The left-hand side is the usual gravitational motion, with the Christoffel symbols $\Gamma^{\lambda}_{\mu\nu}$ determined by the metric. Throughout this work we use the Newtonian gauge for the metric:

$$ds^2 = -(1 + 2\Psi)\mathrm{d}t^2 + a^2(1 - 2\Phi)\mathrm{d}\mathbf{x}^2. \quad (7)$$

We will work to first order in the gravitational potentials Φ and Ψ and neglect their time derivatives.

The right-hand side of Eq. (6) describes an additional force due to the coupling to the cosmon. It consists of two parts. First, a velocity-dependent part $\beta u^{\nu}\partial_{\nu}\varphi u^{\mu}$ compensates changes in the mass for neutrinos moving in a varying cosmon field so that momentum is conserved. A neutrino moving into a region with smaller (larger) values of φ will lose (gain) mass. To compensate the loss (gain) of momentum, it will be accelerated (decelerated). Second, the term $\beta\partial^{\mu}\varphi$ is a velocity-independent fifth force. In the nonrelativistic limit, it acts as an attractive force about $2\beta^2$ times stronger than gravity [29].

B. Homogeneous evolution

Let us now turn to the homogeneous limit and discuss how GNQ in its simplest form can lead to an accelerated expansion of the Universe. At early times when the neutrinos are relativistic, the evolution of the cosmon is determined by the potential. Therefore, the cosmon will evolve toward its scaling solution with the DE density decreasing with a^{-3} during matter domination. In view of the growing mass, the neutrinos become nonrelativistic rather late. The interaction becomes important once $\alpha V(\varphi) \approx \beta T_{(\nu)} \approx -\beta\rho_\nu$. It acts as an effective potential barrier stopping the time evolution of the energy density of the cosmon-neutrino fluid. The constant energy density then mimics a cosmological constant. Since the energy density of the neutrinos is small compared to the cosmon energy density, the coupling must be rather large.

Most of the cosmological parameters as $\Omega_{\mathrm{DE}} = \Omega_\varphi + \Omega_\nu$ and m_ν depend to a good approximation only on the ratio $-\frac{\beta}{\alpha}$; see Fig. 1. Demanding a dark energy density of $\Omega_{\mathrm{DE}} \approx 0.7$ enforces $-\frac{\beta}{\alpha} \approx 5$ [7] for a present neutrino mass $m_\nu = O(1\ \mathrm{eV})$, where smaller neutrino masses require large $-\frac{\beta}{\alpha}$. We note that the usual cosmological bounds on the neutrino mass from CMB and large scale structure observations [30,31] do not apply here, since neutrino masses have been substantially smaller in the past. In the homogeneous limit, the neutrino mass is mainly constrained by Earth-based experiments. Also the scale factor at which the neutrinos stop the cosmon evolution has only a moderate dependence on the individual values of α and β. The energy density fraction of the cosmon before stopping is given by $\Omega_\varphi \propto \alpha^{-2}$ and hence becomes smaller for larger α. The time at which the interaction with neutrinos compensates the self-interaction of the cosmon becomes earlier for larger α. The onset of dark energy is therefore earlier for larger values of α and β; see Fig. 1.

As we will discuss later, strong backreaction effects will alter this simple picture. We will see in Sec. III that backreaction effects always counteract the stopping mechanism and the cosmon will evolve again, so that it is not guaranteed that values for α and β which describe a realistic cosmology in the homogeneous limit will also describe a close-to-realistic cosmology including backreaction.

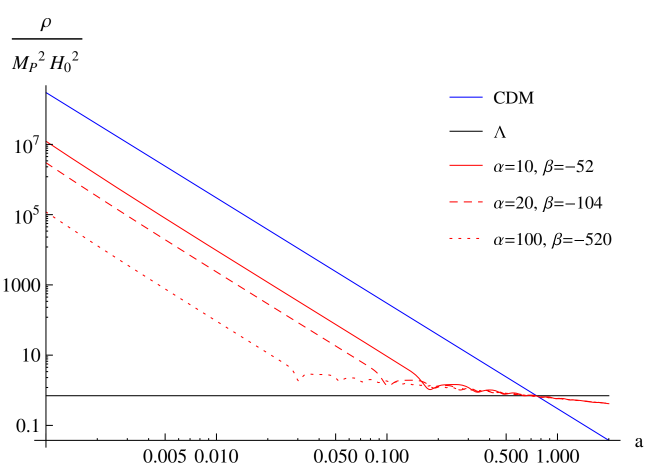

FIG. 1 (color online). Energy density of the cosmon-neutrino fluid, for different parameters α and β. We compare to the CDM density and the density of a cosmological constant Ω_Λ. The parameters were chosen to match Ω_Λ today. The stopping occurs earlier for larger α, with a smaller amount of EDE.

Since backreaction effects can only be important after the neutrinos became nonrelativistic, the homogeneous description remains valid at early times. Large values for α are preferred by bounds on EDE. For large α the stopping mechanism acts earlier, and hence also the backreaction becomes important earlier. From these qualitative considerations, we already find some tension between reducing the backreaction effects, which spoil the stopping of the cosmon evolution, and satisfying bounds on EDE.

III. BACKREACTION AND EFFECTIVE EQUATION OF STATE

A. Neutrino lumps

In GNQ it is important to understand structure formation, not only in view of using large scale structure observation as a probe for our cosmological models, especially to test DE models or "measure" the neutrino mass. It is crucial to understand the formation and evolution of neutrino lumps before being able to judge about the viability of GNQ as a DE model. In this section we shortly review the progress toward an understanding of the neutrino lumps, for details we refer to previous work [15,16,18,20,24,29,32–34]. Our main focus lies on the strong backreaction effects from nonlinear perturbations in the neutrino-cosmon fluid.

The large nonlinearities have their origin in the large coupling $\beta = O(10^2)$. Therefore, the additional force between neutrinos will be about $10^3 - 10^4$ times larger than the gravitational interaction between neutrinos and between neutrinos and CDM. In turn the neutrino perturbations grow very quickly as soon as neutrinos become nonrelativistic. This implies that the fluctuations in the neutrino energy density become nonlinear even at large scales. The scale factor a_{NL} at which this happens for a neutrino perturbation of a given wavelength k^{-1} can be estimated by the value of a at which the linear dimensionless neutrino power spectrum $\Delta_\nu(k) = k^3 P_\nu(k)/(2\pi^2)$ becomes order unity. Looking at Fig. 2, we see that for the particular choice of parameters $\alpha = 10$ and $\beta = -52$ already at $a \sim 0.4$ scales around $k_{\mathrm{NL},\nu} \sim 0.01\ \mathrm{h\,Mpc}^{-1}$ become nonlinear, while today scales around $k_{\mathrm{NL},\nu} \sim 0.002\ \mathrm{h\,Mpc}^{-1}$ are nonlinear. The exact value of the nonlinear scale of neutrino-cosmon perturbations depends on the chosen parameters, but it is a generic finding that $k_{\mathrm{NL},\nu}$ is smaller than the corresponding wave vector for CDM perturbations, $k_{\mathrm{NL,C,0}} \sim 0.1\ \mathrm{h\,Mpc}^{-1}$. These can be traced back to instabilities in the neutrino perturbations already present at linear order. These instabilities are stabilized nonperturbatively by the formation of neutrino lumps.

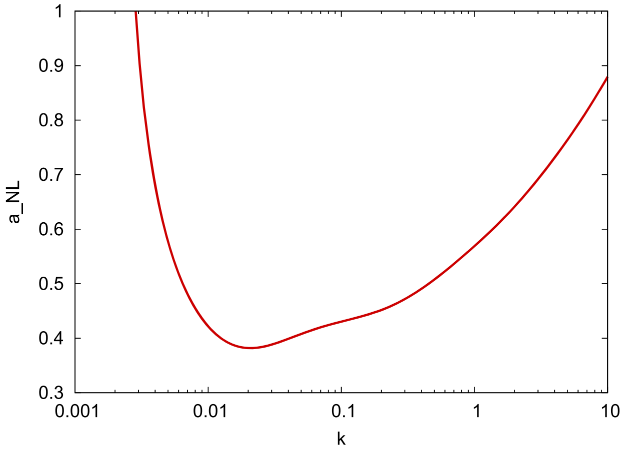

FIG. 2 (color online). The scale factor $a_{\rm NL}$ at which the dimensionless linear neutrino power spectrum becomes unity, $\Delta(k, a_{\rm NL}) = 1$, as a function of scale, for the parameters $\alpha = 10$ and $\beta = -52$. Already at $a \sim 0.40$ scales around $k \sim 0.02$ are nonlinear, demonstrating the failure of standard perturbative methods compared to the same figure in Ref. [17].

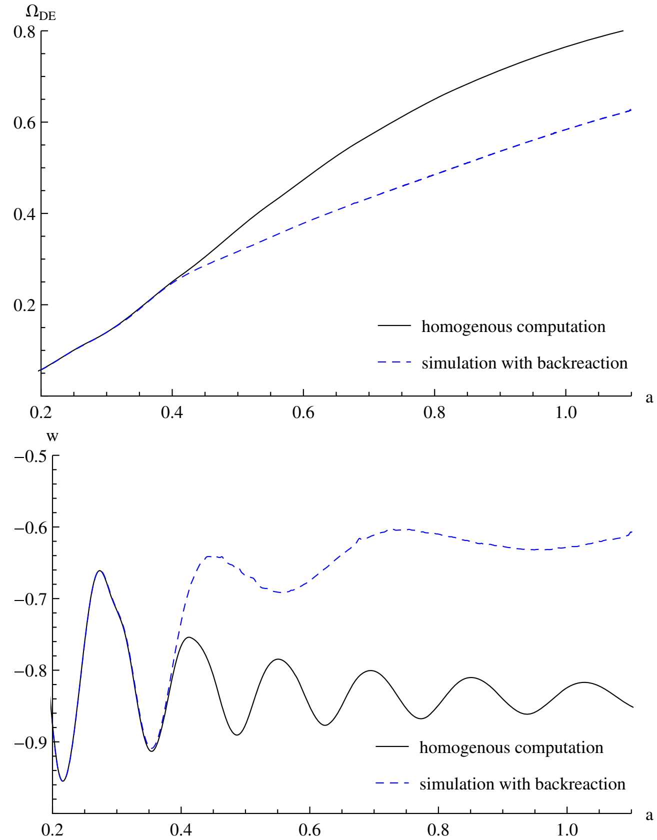

FIG. 3 (color online). Dark energy density fraction $\Omega_{\rm DE}$ (top) and equation of state w (bottom) as a function of the scale factor, for $\alpha = 10$ and $\beta = -52$, with and without backreaction.

B. Backreaction

Usually backreaction in cosmology is assumed to be negligible. In the last years, several quantitative estimates [35–37] came to the conclusion that backreaction is indeed small in the ΛCDM model. In contrast, backreaction effects are crucial in GNQ. We demonstrate this in Fig. 3, where we compare the numerical results for the clumping neutrinos with the pure background evolution for which the effects of nonlinear neutrino perturbations are neglected. We choose the parameters $\alpha = 10$ and $\beta = -52$ that have often been employed in the literature.

We find two types of backreaction effects. First, the Friedmann equation involves the volume averaged energy density, which we will define below. Second, the average value of the cosmon $\bar{\varphi}$ cannot be obtained by solving the homogeneous equation of motion. The Klein–Gordon equation needs to be modified to include backreaction effects from the neutrino lumps. The reason is that the typical velocities and masses of the neutrinos do not coincide with their counterparts of the homogeneous calculation. While the first effect mainly affects the expansion history of the Universe, the second effect is also important for an understanding of the lump dynamics.

Let us first discuss the second effect. Due to the strong interaction, most of neutrinos are bound in the lumps. Inside gravitational bound objects, the gravitational potential has a well. Similarly, inside neutrino lumps the local field value is smaller than its average by an amount of $\delta\varphi$. The mass of a neutrino inside a lump is therefore smaller than the mass of a free-streaming neutrino $m(\bar{\varphi} + \delta\varphi) < m(\bar{\varphi})$. As a consequence most of the neutrinos have a mass substantially smaller than the mass estimated from the homogenous calculation. Due to the velocity-dependent force, the loss of mass during the formation of lumps is accompanied by an acceleration to relativistic velocities. These two effects lead to a mismatch between the energy-momentum tensor of neutrinos from the homogeneous calculation and its average value, as soon as the formation of lumps has started.

We account for the backreaction effects by using the volume averaged energy-momentum tensor. The Klein–Gordon equation for the average field is given approximately by

$$\ddot{\bar{\varphi}} + 3H\dot{\bar{\varphi}} + \alpha V(\bar{\varphi}) = -\beta \bar{T}_{(\nu)}, \tag{8}$$

where the volume average is defined as

$$\bar{T}_{(\nu)} = \frac{1}{V}\int d^3x \sqrt{g^{(3)}}\, T_{(\nu)} \approx \frac{a^3}{V}\int d^3x (1 - 3\Phi) T_{(\nu)}. \tag{9}$$

The determinant of the spatial 3-metric up to first order in metric perturbations is given by $\sqrt{g^{(3)}} \approx a^3(1 - 3\Phi)$. The integration is to be understood over the whole simulation box. The volume is given by $V \approx a^3 \int d^3x (1 - 3\Phi)$. Taking backreaction effects consistently into account and evolving the volume averaged field $\bar{\varphi}$, additional modifications arise in the equation. However, we will neglect these terms for the qualitative discussion of backreaction in this section and postpone a more detailed discussion to Sec. IV.

The right-hand side of Eq. (8) can be written as

$$\beta\bar{T}_{(\nu)} = \beta(-\bar{\rho}_\nu + 3\bar{P}_\nu) = -\beta\bar{\rho}_\nu(1-3w_\nu) < -\beta\bar{\rho}_\nu, \qquad (10)$$

where the energy density and pressure are understood as volume averages. We use them to define the equation of state w_ν. The neutrino pressure is positive ($w_\nu \geq 0$) such that pressure effects lower the effective potential barrier which stops the cosmon evolution. As a consequence, the time at which the cosmon evolution stops is postponed toward the future. If the evolution has already stopped, the effective reduction of the barrier can have the effect that the cosmon will evolve again. The weaker interaction between the neutrinos and the cosmon after the formation of lumps can also be interpreted as a lower effective coupling β_l, which gets renormalized by integrating out short wavelength modes [20]. In a qualitative sense, β_l can be interpreted as the effective coupling between a fluid of neutrino lumps and the homogenous cosmon field. The smaller value of β_l as compared to β is the dominant backreaction effect in our model.

We next turn to the backreaction effect for the evolution of the background metric. One needs to replace the background density of neutrinos and the cosmon by their volume average, such that the Friedmann equation becomes

$$H^2 = \bar{\rho}_{\mathrm{CDM}} + \bar{\rho}_\nu + \bar{\rho}_\varphi. \qquad (11)$$

In the presence of lumps, ρ_ν has contributions from the neutrino velocities, and ρ_φ involves additional gradient contributions. The observable DE component is the combined neutrino-cosmon fluid ρ_{DE}. The neutrinos are typically subdominant but still contribute a significant fraction $\frac{\bar{\rho}_\nu}{\rho_{\mathrm{DE}}} \sim 0.1$. With an equation of state $w_\nu \sim 0.1$, the neutrinos lift the dark energy equation of state away from $w \approx -1$ to some higher value.

The volume average of the cosmon energy density is given by

$$\bar{\rho}_\varphi = \frac{1}{2}\overline{\dot{\varphi}^2} + \frac{1}{2a^2}\overline{(1+2\Phi)(\partial_i\varphi)(\partial_j\varphi)\delta^{ij}} + \overline{V(\varphi)}, \qquad (12)$$

where we only keep metric perturbations up to first order, neglect their time derivatives, and use that the volume average of the gravitational potentials vanishes $\bar{\Phi} = \bar{\Psi} = 0$. Also assuming that time derivatives of the cosmon perturbation $\delta\varphi$ are small allows us to approximate $\overline{\dot{\varphi}^2} \approx \dot{\bar{\varphi}}^2$. Using the quasistatic approximation is justified although the individual neutrino velocities are large. For the quasistatic approximation to hold, it is sufficient that the energy-momentum tensor for all neutrinos does not evolve quickly, so that there are no quickly varying sources for the cosmon. A nonzero $\delta\dot{\varphi}$ results in a positive contribution to the pressure, making it even harder to achieve an almost constant energy density for the cosmon-neutrino fluid.

Without the gradient term, one has the usual competition between potential and kinetic energy. The potential energy should be dominant in order to have an accelerated expansion. The averaged potential energy $\overline{V(\varphi)}$ differs from the potential energy $V(\bar{\varphi})$ of the averaged field $\bar{\varphi}$ only by a few percent, such that no major backreaction effect arises from this source. In contrast, the gradient term can be almost as large as the potential energy. From the expression for the pressure

$$\bar{P}_\varphi \approx \frac{1}{2}\dot{\bar{\varphi}}^2 - \frac{1}{6a^2}\overline{(1+2\Phi)(\partial_i\varphi)(\partial_j\varphi)\delta^{ij}} - \overline{V(\varphi)}, \qquad (13)$$

we see that a gradient term dominated equation of state would be $w_\nu = -\frac{1}{3}$. We emphasize that all backreaction effects individually lead to an evolving energy density of neutrino-cosmon fluid and typically push w away from -1.

For models with constant β, the lumps have the tendency to stabilize and to remain present once formed. The neutrino-cosmon fluid can be understood as an effective fluid of nearly virialized neutrino lumps with parameters differing from the microscopic ones [20]. The observable DE is then the sum of a neutrino lump fluid and a homogenous background field. For virialized lumps the pressure between relativistic neutrinos and cosmon gradients is expected to cancel [20]. Therefore, the equation of state of the lump fluid is close to zero, similar to the fluid of nonrelativistic neutrinos. The backreaction effect that remains even in this limit is the reduced effective coupling β_l between neutrino lumps and the cosmon background field. Due to the not completely virialized lumps, the pressure contribution from the neutrinos and the cosmon gradients do not cancel exactly, adding a small but relevant additional backreaction effect. This is different to gravitationally bound objects, for which a nonrenormalization theorem states that small virialized objects decouple completely from the background evolution and there is no backreaction effect from small virialized objects at all [36].

IV. N-BODY SIMULATION

The highly nonlinear nature of the neutrino lumps makes their description nonamenable to standard perturbative techniques. Instead we use a N-body simulation specially designed for GNQ. The N-body simulation solves the background and the inhomogeneities simultaneously and therefore allows us to study the backreaction effect of lumps on the homogeneous background evolution. The concept and many details of the simulation were already described in Refs. [16,17], and we focus here on the equation of motion for the average cosmon field $\bar{\varphi}$ and its perturbation $\delta\varphi$.

In our simulation we follow the usual motion of nonrelativistic CDM particles and their clustering due to gravity. In contrast to the standard picture of structure formation, the two gravitational potentials differ, $\Phi \neq \Psi$, because of the anisotropic stress from the neutrinos. This is accounted for by solving the Poisson equation for $\Phi - \Psi$, which yields Φ once the Newtonian potential Ψ is known. The Poisson equation for Ψ is sourced by the energy density of CDM, neutrinos, and to a small part by the one of the cosmon perturbations.

The neutrinos are evolved using Eq. (6). The cosmon evolution is governed by the Klein–Gordon equation (5). We split the cosmon into the volume average $\bar{\varphi} = \frac{1}{V}\int d^3x\sqrt{g^{(3)}}\varphi$ and a perturbation $\delta\varphi = \varphi - \bar{\varphi}$. Neglecting time derivatives of the gravitational potentials, time derivatives commute with the process of averaging $\bar{\dot{\varphi}} \approx \dot{\bar{\varphi}}$. The averaged equation (5) is

$$\ddot{\bar{\varphi}} + 3H\dot{\bar{\varphi}} + \alpha\overline{(1+2\Psi)V(\varphi)} = -\beta\overline{(1+2\Psi)T_{(\nu)}} + a^{-2}\overline{\delta^{ij}(\partial_j\Psi)(\partial_i\delta\varphi)}, \quad (14)$$

where we expanded up to first order in metric perturbations. Equation (14) is the full version of Eq. (8). As already discussed in Sec. III, the most important difference as compared to a naive homogeneous calculation is the use of the actual average of the neutrino momentum tensor. Including the gravitational potential in the average gives only a minor correction. Also the averaged potential term agrees up to a few percent with the homogeneous estimate. The gradient terms are roughly 1 order of magnitude smaller than the potential term and therefore only subdominant. Nevertheless, our numerical code includes all these effects.

By subtracting Eq. (14) from the Klein–Gordon equation (5), we find the equation for the perturbation:

$$\begin{aligned}\delta\ddot{\varphi} &+ 3H\delta\dot{\varphi} - a^{-2}\delta^{ij}\partial_i\partial_j\delta\varphi(1+2\Phi)\\ &- a^{-2}\delta^{ij}(\partial_j(\Psi-\Phi))(\partial_i\varphi) + a^{-2}\overline{\delta^{ij}(\partial_j\Psi)(\partial_i\varphi)}\\ &+ \alpha((1+2\Psi)V(\varphi) - \overline{(1+2\Psi)V(\varphi)})\\ &= -\beta((1+2\Psi)T_{(\nu)} - \overline{(1+2\Psi)T_{(\nu)}}). \quad (15)\end{aligned}$$

This equation is a nonlinear wave equation, which is, due to the averaging, nonlocal in position space. To be able to solve this equation, we need to make some approximations. We employ a quasistatic approximation for the cosmon perturbation for which we neglect the second-order time derivative $\delta\ddot{\varphi}$. Simply neglecting all time derivatives is not a consistent approximation. Doing so the resulting equation does not ensure that the perturbation has a vanishing mean $\overline{\delta\varphi} = 0$. This can be seen by averaging Eq. (15). Taking into account the Φ dependence in the volume element and only keeping terms to first order in the metric perturbations, all terms except the time derivatives cancel:

$$\ddot{\overline{\delta\varphi}} + 3H\dot{\overline{\delta\varphi}} = 0. \quad (16)$$

This relation ensures that the if the average vanishes initially it will vanish at all times. This is still true if we neglect the second time derivative while keeping the first one. This approximation is consistent with the approximation for the kinetic term of the average energy density and pressure

$$\overline{\dot{\varphi}^2} = \dot{\bar{\varphi}}^2 + \overline{\delta\dot{\varphi}^2}, \quad (17)$$

where we neglected the $\overline{\delta\dot{\varphi}^2}$ term, which is also second order in time derivatives of the cosmon perturbation.

If one instead neglects the second derivative with respect to conformal time, the Hubble damping changes $3H \to 2H$; we compared both possibilities and found only a small difference. We interpret this as a sign that the quasistatic approximation is justified. To solve the equation for $\delta\varphi$, we use a Newton–Gauss–Seidel (NGS) multigrid relaxation method, already applied to the varying coupling model [17] and originally developed for modified gravity [38]. The quasistatic approximation is crucial for applying the NGS method, which is not applicable to wavelike equations but can be applied to diffusionlike equations [39]. The idea of the NGS solver is to rewrite the equation to be solved into a functional form,

$$\mathcal{L}[\delta\varphi] = D\delta\varphi - F[\delta\varphi] = 0, \quad (18)$$

with some differential operator D and a nonlinear functional F. The root of $\mathcal{L}[\delta\varphi] = 0$ can be obtained by a Newton-like iterative procedure,

$$\delta\varphi^{(n+1)} = \delta\varphi^{(n)} - \mathcal{L}[\delta\varphi^{(n)}]\left(\frac{\partial\mathcal{L}[\delta\varphi^{(n)}]}{\partial\delta\varphi^{(n)}}\right)^{-1}; \quad (19)$$

the derivative is taken at each point individually, and the coupling between different points, induced by the derivatives, is taken into account solely by the iterative procedure. The derivative of the differential operator $\frac{\partial D\delta\varphi}{\partial\delta\varphi}$ is defined by the discretization rule used in the simulation. We define the gradient and the Laplacian by relating a grid point to its neighbors in the j direction by a Taylor expansion, $\delta\varphi(x_i \pm \Delta x\delta_{ij}) = \delta\varphi(x_i) \pm \partial_j\delta\varphi(x_i)\Delta x + \frac{1}{2}\partial_j^2\delta\varphi(x_i)\Delta x^2 + \ldots$, with Δx the spacing between two grid points. The Laplacian is then approximated by a seven-point stencil, and the derivative is $-6/\Delta x^2$. The derivative of the gradient vanishes.

In principle this method can be applied even in the presence of the nonlocal terms present in Eq. (15). In practice this not possible because calculating the nonlocal terms involves an integration over the full simulation box in each iteration step. We account for these terms iteratively. The difference between the values of the average terms of two time steps is small. So we use at a given time step the average terms of the proceeding time step as a first approximation and apply the NGS solver a few times to correct for the difference.

V. RESULTS AND DISCUSSION

Using the N-body simulation described in Sec. IV, we perform a parameter scan and search for parameters describing a realistic universe with accelerated expansion. For the details on the formation of lumps and their characteristics, we refer to previous work [16,20]. We use a simulation box with a comoving volume of $V = (600\ \mathrm{h}^{-1}\,\mathrm{Mpc})^3$, which we divide into $N_c = 128$ cells. The numbers of effective CDM particles N_C and neutrino particles N_ν are chosen to be equal to the number of cells $N_c = N_C = N_\nu$. The initial power spectrum has a spectral index of $n_s = 0.96$ and amplitude of $A_s = 2.3 \times 10^{-9}$ at the pivot scale $k_{\mathrm{pivot}} = 0.05\ \mathrm{Mpc}^{-1}$. We start our simulation with the CDM particles only at $a_{\mathrm{ini,C}} = 0.02$ and add the neutrinos at a later time, after they became nonrelativistic.

In view of the strong backreaction effects, it is no longer clear that the stopping power of neutrinos for the time evolution of the cosmon is sufficient in order to account for a large present fraction of dark energy and an acceleration of the expansion similar to a cosmological constant. If so, the parameter range where this happens may be rather different from the one where the background evolution neglects the effect of neutrino structures.

Our model has three parameters relevant for this investigation, namely α related to the amount of EDE, the cosmon neutrino coupling β and the parameter m_i which is related to the size of the neutrino mass. We have performed a parameter scan in order to search for a parameter range consistent with observations. For this purpose we vary the parameters α and β individually while fixing the mass parameter
to $m_i = 1$ eV. Figure 4 shows that changing the mass parameter by a factor of 10 affects the effective equation of state and the energy density by no more than 10%.

A realistic DE model must certainly assume the benchmark values for the present DE density $\Omega_{\mathrm{DE},0} \approx 0.7$ and the present equation of state $w_0 \approx -1$. In Fig. 5 we show the values of $\Omega_{\mathrm{DE},0}$ and w_0 for a grid in the parameter space for α and β. Sufficient acceleration typically requires rather small values $\alpha \lesssim 5$. A band with an acceptable fraction of present DE is typically found in the range $5 \lesssim \alpha \lesssim 10$, showing some tension already at this stage.

The parameter range yielding an accelerated expansion ($\alpha \lesssim 5$) is problematic also in view of the bounds on EDE, which require $\alpha \gtrsim 10$. In the parameter range where one finds $w_0 < -0.9$, some tension persists if one tries to get both the equation of state and the energy density compatible with observations. For $\alpha = 3$ and $\alpha = 4$, we indeed find $w_0 \lesssim -0.9$, but the energy density exceeds with $\Omega_{\mathrm{DE}} \approx 0.75$ the benchmark value of $\Omega_{\mathrm{DE},0} \approx 0.7$. On the other hand, for $\alpha = 5$ one has $\Omega_{\mathrm{DE}} \approx 0.7$, but the equation of state is $w_0 \approx -0.7$. Although we could not find parameters for which w_0 and $\Omega_{\mathrm{DE},0}$ match the benchmark values precisely, our results are not too far from those values, either. It might be that varying also the mass parameter m_i could bring them into agreement with observations.

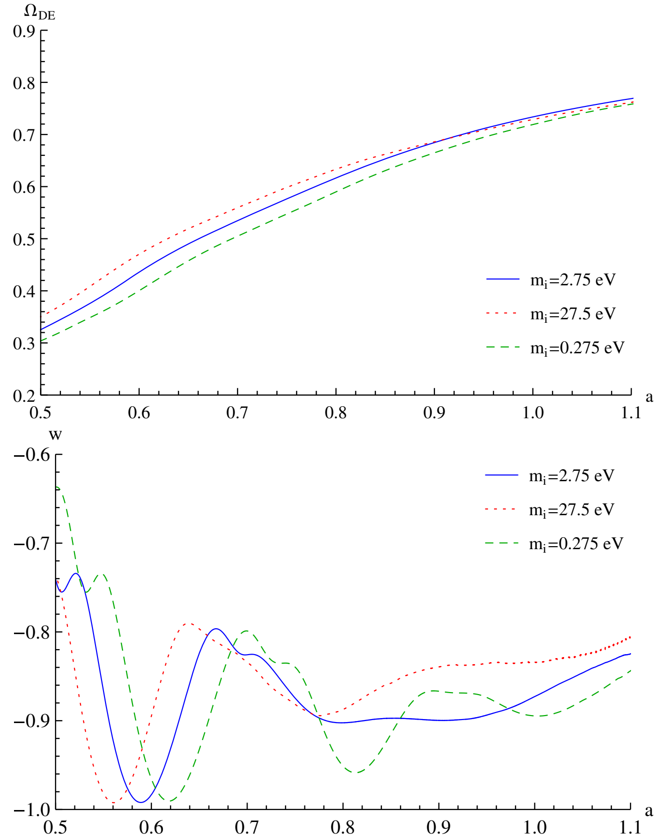

FIG. 4 (color online). Energy density fraction of the cosmon-neutrino fluid Ω_{DE} and effective equation of state w, for different mass parameters m_i, with $\alpha = 5$ and $\beta = -78$. Even for mass parameters different by a factor of 100, the equation of state varies at a maximum of about 10%, indicating that the value of m_i plays only a minor role.

The equation of state is not constant in time, and it can even possess oscillating features; see Fig. 4. It may happen that the present time coincides with a minimum (maximum) of w during an oscillation. In this case the cosmic evolution is actually better described by an average value somewhat larger (smaller) than w_0. The time evolution of the equation of state is shown in Fig. 6 for a range of parameters α and β in the region not too far from the benchmark values. One typically observes a first stop of the scalar field ($w \approx -1$). Due to backreaction this is followed by a slow decrease of the dark energy, typically with $-0.9 \lesssim w \lesssim -0.8$.

Only looking at the energy density and the equation of state today is not sufficient. In the parameter range acceptable for the benchmark, the neutrinos become

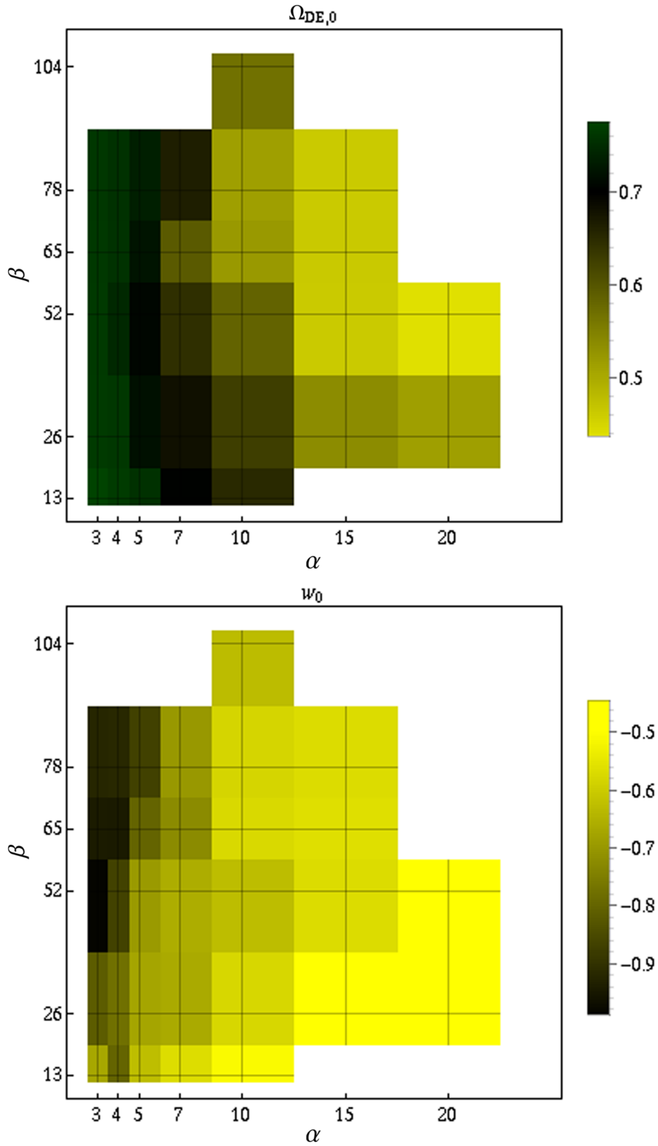

FIG. 5 (color online). Present energy density $\Omega_{\mathrm{DE},0}$ and equation of state w_0 of the cosmon-neutrino fluid. Realistic values ($w_0 \approx -1$, $\Omega_{\mathrm{DE},0} \approx 0.7$) are found for small values of α. It is hard to get both values "correct" simultaneously, for sufficiently large α.

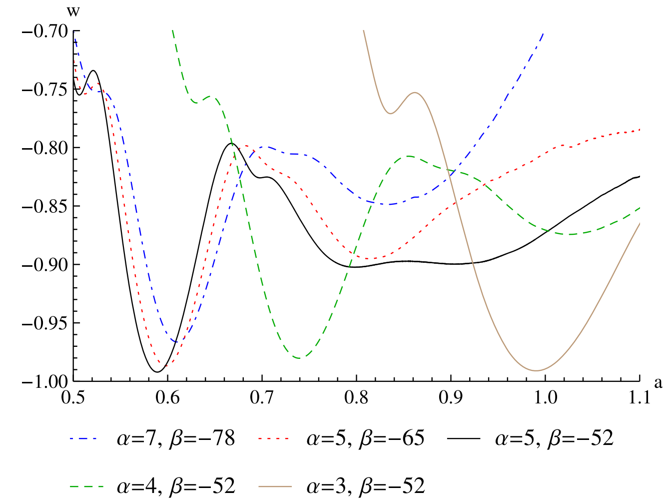

FIG. 6 (color online). Equation of state as a function of the scale factor. The model parameters are chosen such that w and $\Omega_{\mathrm{DE},0}$ are near the benchmark values. Values $w_0 \lesssim -0.9$ are only reached before backreaction effects become important. Thus, $w \approx -0.99$ for $\alpha = 5$ and $\beta = -52$ is not accompanied by large negative w at redshifts relevant for supernova observations.

nonrelativistic late. Consequently the cosmon evolution stops late. This is visible in Fig. 6, where the first pronounced minimum in w precisely corresponds to the time when the increase of φ is first stopped and the oscillations set in. Supernova observations probe the expansion history up to redshifts higher than $z \approx 1$ and prefer an almost constant dark energy [40]. We find that for close-to-realistic models the equation of state reaches values around -1 only for scale factors $a \gtrsim 0.6$, which is difficult to get into agreement with $w \approx -1$ from $a \lesssim 0.5$ until today.

Figure 6 shows the generic evolution of the equation of state: It drops down after the neutrinos become nonrelativistic followed by a few damped oscillations. In the homogeneous evolution, these oscillations are damped away quickly, and the equation of state assumes an almost constant value rather close to $w = -1$. In fact the equation of state grows again due to the backreaction and typically reaches values $w \approx -0.8$. An equation of state of $w_0 \lesssim -0.9$ is only reached before or shortly after backreaction becomes important. This simply means that lumps do not have enough time to grow large enough for being able to induce significant backreaction effects.

From these results we conclude that GNQ with a constant coupling β is probably not a viable DE model. Realistic values for w_0 and $\Omega_{\mathrm{DE},0}$ seem only possible if the cosmon evolution is stopped late, so that backreaction effects have no time to become important. Stopping the cosmon evolution late is in some tension with supernova data and involves a large amount of EDE, probably not consistent with observations.

VI. CONCLUSION

We have performed a numerical analysis of GNQ with a constant cosmon-neutrino coupling β. Due to strong backreaction effects from the formation of large neutrino lumps, these models have difficulties being compatible with the observed properties of dark energy.

A specific choice for the model parameters α, β, and m_i, which appears to be compatible with observations at the homogenous level, is typically no longer viable if backreaction is included. Our parameter scan reveals regions for which the backreaction effects are small enough to allow a slowly evolving cosmon and consequently an almost constant DE density. However, this is only possible if the neutrino lumps form late so that backreaction effects are still small today. In this case an accelerated expansion is only possible for scale factors $a \gtrsim 0.6$, in tension with an almost constant equation of state for scale factors $a \lesssim 0.5$, as preferred by supernova data. Furthermore, the parameter region for which the equation of state is close to -1 and the DE density is not too far from 0.7 requires $\alpha \lesssim 5$. This contradicts constraints on EDE for which $\alpha \gtrsim 10$ is necessary. We conclude that GNQ with a constant coupling β is probably not a viable DE model.

These results for a constant coupling should be contrasted with models where β increases with φ. For this second class of models, the backreaction effect is found to be small since the neutrino lumps form and disrupt periodically [17]. At the present stage this second class of models seems compatible with observations. In certain parameter ranges, it may even be hard to detect a difference from the ΛCDM model and its variants.

These two classes of models may be seen as particular points in a larger class of models where β is allowed to vary with φ. Having established points that are viable with only rather small deviations from ΛCDM, as well as other points where the deviations are so strong that the model is no longer acceptable, we can conclude by continuity that in between there will be models which are still compatible with observations today but also offer highly interesting prospects of finding deviations from ΛCDM. Finding large neutrino lumps, thereby observing the cosmic neutrinos directly, would be a direct hint for GNQ. Even for models with small neutrino perturbations, we expect observable deviations from the ΛCDM model, due to the different evolution of the neutrino sector. First, the transition of relativistic to nonrelativistic standard massive neutrinos is imprinted in the CMB fluctuations as well as in the matter distribution, with a specific scale dependence [30,31]. The signal differs for constant or time-varying neutrino masses. Second, free-streaming standard massive neutrinos attenuate the growth of matter perturbations on small scales and therefore add an additional scale-dependent effect to the matter distribution. Observing these scale-dependent effects as predicted for standard neutrinos with a constant mass would be a strong argument for the ΛCDM model and against GNQ.

The result for models with constant β presented in this note as well the results on the varying β model presented in Ref. [17] suggest that only those models are viable in which the small scale nonlinear neutrino perturbations have only a moderate effect on the large scale dynamics. Nevertheless, the neutrino lumps can have a observable effects on larger scales. One possibility to account for these effects is to construct an effective fluid for the long wavelength perturbations by averaging over small scale nonlinearities as proposed in Ref. [36]. A similar route has already been taken in Ref. [20] to describe the large scale dynamics of virialized neutrino lumps in the constant β model by means of an effective lump fluid. These ideas were already successfully applied to the mildly nonlinear regime of structure formation in the form of the effective field theory of large scale structure [41–46]; see also Ref. [47]. Adopting these ideas to GNQ, we hope that it will become possible to study the dynamics of perturbations in GNQ on large scales qualitatively. It might even become possible to study some effects of lumps on the CMB, without running time-consuming simulations.

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