

Probing the scale dependence of non-Gaussianity with spectral distortions of the cosmic microwave background

Razieh Emami,^{1,2} Emanuela Dimastrogiovanni,³ Jens Chluba,⁴ and Marc Kamionkowski²

¹*School of Physics, Institute for Research in Fundamental Sciences (IPM), P. O. Box 19395-5531, Tehran, Iran*

²*Department of Physics and Astronomy, Johns Hopkins University, 3400 North Charles Street, Baltimore, Maryland 21218, USA*

³*Department of Physics and School of Earth and Space Exploration, Arizona State University, Tempe, Arizona 85827, USA*

⁴*Institute for Astronomy, K30, University of Cambridge, Madingley Road, Cambridge CB3 0HA, United Kingdom*

(Received 5 April 2015; published 22 June 2015)

Many inflation models predict that primordial density perturbations have a nonzero three-point correlation function, or bispectrum in Fourier space. Of the several possibilities for this bispectrum, the most common is the local-model bispectrum, which can be described as a spatial modulation of the small-scale (large-wavenumber) power spectrum by long-wavelength density fluctuations. While the local model predicts this spatial modulation to be scale independent, many variants have some scale dependence. Here we note that this scale dependence can be probed with measurements of frequency-spectrum distortions in the cosmic microwave background (CMB), in particular, highlighting Compton- y distortions. Dissipation of primordial perturbations with wavenumbers $50 \text{ Mpc}^{-1} \lesssim k \lesssim 10^4 \text{ Mpc}^{-1}$ gives rise to chemical-potential (μ) distortions, while dissipation of those with wavenumbers $1 \text{ Mpc}^{-1} \lesssim k \lesssim 50 \text{ Mpc}^{-1}$ gives rise to Compton- y distortions. With local-model non-Gaussianity, the distortions induced by this dissipation can be distinguished from those due to other sources via their cross correlation with the CMB temperature T . We show that the relative strengths of the μT and yT correlations thus probe the scale dependence of non-Gaussianity and estimate the magnitude of possible signals relative to the sensitivities of future experiments. We discuss the complementarity of these measurements with other probes of squeezed-limit non-Gaussianity.

DOI: [10.1103/PhysRevD.91.123531](https://doi.org/10.1103/PhysRevD.91.123531)

PACS numbers: 98.80.Es, 98.65.Dx, 98.70.Vc

The prevailing paradigm for the origin of the primordial density perturbations inferred from fluctuations in the cosmic microwave background (CMB) and from large-scale galaxy surveys is that they arose as quantum fluctuations during a quasi-de Sitter phase (“inflation”) of expansion in the early Universe [1]. Inflation generally predicts a spectrum of primordial perturbations that is nearly, but not precisely, scale invariant, consistent with current measurements. It also generically predicts that these perturbations should be very close to, but not precisely, Gaussian (i.e., have only small, connected correlations beyond the two-point correlation function). The amplitude and detailed form of this non-Gaussianity varies considerably between models and is usually specified in terms of a three-point function, or “bispectrum” in Fourier space [2].

The “local-model” bispectrum, which peaks in the squeezed limit $k_2 \approx k_3 \gg k_1$ (where $k_1 \leq k_2 \leq k_3$ are the three Fourier modes being correlated) appears in a number of simple inflation models [3,4] and has thus become a standard workhorse. Local-model non-Gaussianity has been sought in the CMB [4–7], in large-scale structure [8], and through the scale-dependent biasing [9–12] it induces in galaxy clustering at large scales.

It has also recently been shown [13,14] that local-model non-Gaussianity will induce spatial correlations between the CMB temperature fluctuations and chemical-potential (μ) distortions as a function of position on the sky. These μ distortions arise from the dissipation of primordial perturbations with wavenumbers $50 \text{ Mpc}^{-1} \lesssim k \lesssim 10^4 \text{ Mpc}^{-1}$ at redshifts $z \approx 5 \times 10^4 - 2 \times 10^6$ [15–17]. In the local model, the amplitude of these small-scale perturbations is modulated by the long-wavelength curvature perturbation that also gives rise to large-angle temperature fluctuations, thus inducing a μT correlation [13,14]. These μT correlations therefore probe local-model non-Gaussianity at wavenumbers $50 \text{ Mpc}^{-1} \lesssim k \lesssim 10^4 \text{ Mpc}^{-1}$ far smaller than those accessible with CMB temperature fluctuations, galaxy surveys, or even future 21 cm observations [18].

Still, the local-model bispectrum is just one of an infinitude of different types of bispectra. Large classes of bispectra have arisen from different ideas for inflation [19–22], and no shortage of phenomenological parametrizations have been considered.

In this paper we point out that the correlation of CMB Compton- y distortions with the large-angle temperature fluctuation can be used, in tandem with the μT correlation,

to probe different types of non-Gaussianity. Dissipation of primordial perturbations with wavenumbers¹ $1 \text{ Mpc}^{-1} \lesssim k \lesssim 50 \text{ Mpc}^{-1}$ gives rise to Compton- y distortions to the CMB [16]. However, these spectral distortions have been largely disregarded because they are not easily distinguished from larger Compton- y signals from the intergalactic medium at low redshifts. Non-Gaussianity may, however, induce a yT correlation that could allow this dissipation-induced y distortion to be isolated from late-time effects (e.g., [23,24]) by means of its angular dependence. Even without being able to distinguish different contributions, one can still derive conservative upper limits using yT correlations. Since y and μ distortions probe different wavenumbers, the relative strength of the yT and μT distortions can be used to probe the scale dependence of primordial non-Gaussianity, extending previous considerations of just the μT correlation [25].

Below, we first present a simple calculation, based upon the configuration-space description (rather than the Fourier-space description in prior work [13,14]) of local-model non-Gaussianity and the μT correlation. This calculation illustrates clearly that the μT correlation arises from large-scale modulation of small-scale modes. The generalization to yT correlations is thus clear. We then parametrize the type of non-Gaussianity probed by the combination of μT and yT correlations and estimate the sensitivity of future experiments. We close by discussing the complementarity of these measurements with other probes of squeezed-limit non-Gaussianity.

To begin, the average μ and y distortions induced, with Gaussian initial conditions, by dissipation of primordial perturbations are given by

$$\langle \mu \rangle \approx \int d \log k \Delta_{\mathcal{R}}^2(k) W_{\mu}(k), \quad (1a)$$

$$\langle y \rangle \approx \int d \log k \Delta_{\mathcal{R}}^2(k) W_y(k), \quad (1b)$$

where $\Delta_{\mathcal{R}}^2(k)$ is the primordial curvature power spectrum, and $W_{\mu}(k)$ and $W_y(k)$ define k -space window functions to account for the acoustic-damping and thermalization physics. Although more accurate expressions for the k -space window functions have been discussed [17,26,27], here we will use the simple approximations

$$W_{\mu}(k) \approx 2.3[e^{-2k^2/k_{D,\mu}^2} - e^{-2k^2/k_{D,y}^2}], \quad (2a)$$

¹While modes with $k \lesssim 1 \text{ Mpc}^{-1}$ also damp and create a y distortion, after recombination, the effective heating rate for these perturbations is much lower than that of smaller scales [16,17].

$$W_y(k) \approx 0.4[e^{-2k^2/k_{D,\mu}^2} - e^{-2k^2/k_{D,y}^2}], \quad (2b)$$

where $k_D(z) \approx 4.1 \times 10^{-6}(1+z)^{3/2} \text{ Mpc}^{-1}$ is the diffusion damping scale at redshift z . This quantity is evaluated at the initial and final redshifts at which μ and y distortions are produced, assuming that the transition between μ and y happens abruptly at $z \approx 5 \times 10^4$ [28,29]. Thus, $k_{D,\mu} = k_D(2 \times 10^6) \approx 1.1 \times 10^4 \text{ Mpc}^{-1}$, $k_{D,\mu y} = k_D(5 \times 10^4) \approx 46 \text{ Mpc}^{-1}$, and $k_{D,y} = k_D(1090) \approx 0.15 \text{ Mpc}^{-1}$. Evaluating the expressions in Eq. (1) using $\Delta_{\mathcal{R}}^2(k) \approx 2.4 \times 10^{-9}(k/0.002 \text{ Mpc}^{-1})^{n_s-1}$, with $n_s \approx 0.96$ [30], yields $\langle \mu \rangle \approx 1.9 \times 10^{-8}$ and $\langle y \rangle \approx 4.2 \times 10^{-9}$, in good agreement with more detailed computations [16].

Since the angle subtended by a causal region at the surface of last scatter is $\sim 1^\circ$, we see in the CMB ~ 40000 causally disconnected Universes. If primordial perturbations are Gaussian, then the amplitude $\Delta_{\mathcal{R}}^2(k_{\text{small}})$ of primordial perturbations on the small scales k_{small} that induce spectral distortions will be the same everywhere. If, however, there is local-model non-Gaussianity, then the power spectrum $\Delta_{\mathcal{R}}^2(k_s, \vec{x})$ for small-scale modes k_s will differ from one causal patch centered at position \vec{x} to another. The fluctuation will, moreover, be correlated with the long-wavelength curvature fluctuation $\mathcal{R}(\vec{x})$.

This can be understood simply from the configuration-space description of local-model non-Gaussianity. The local-model curvature perturbation at position \vec{x} is written as

$$\mathcal{R}(\vec{x}) = r(\vec{x}) + \frac{3}{5} f_{\text{nl}} r^2(\vec{x}), \quad (3)$$

in terms of a Gaussian random variable $r(\vec{x})$. We then write $\mathcal{R}(\vec{x}) = \mathcal{R}_L(\vec{x}) + \mathcal{R}_s(\vec{x})$, where $\mathcal{R}_L(\vec{x})$ is the part of $\mathcal{R}(\vec{x})$ that comes from long-wavelength Fourier modes and $\mathcal{R}_s(\vec{x})$ is the part that comes from short-wavelength Fourier modes, and we similarly write $r(\vec{x}) = r_L(\vec{x}) + r_s(\vec{x})$. By writing

$$\mathcal{R}_L + \mathcal{R}_s = r_L + r_s + \frac{3}{5} f_{\text{nl}} [r_L^2 + 2r_L r_s + r_s^2], \quad (4)$$

we infer that the small-scale curvature fluctuation in the presence of some fixed long-wavelength curvature fluctuation $\mathcal{R}_L(\vec{x})$ is, to linear order in f_{nl} ,

$$\mathcal{R}_s(\vec{x}) = r_s(\vec{x}) \left[1 + \frac{6}{5} f_{\text{nl}} \mathcal{R}_L(\vec{x}) \right]. \quad (5)$$

Thus, the fractional change in small-scale power in a region of a fixed long-wavelength curvature fluctuation is

$$\frac{\delta \langle \mathcal{R}^2 \rangle}{\langle \mathcal{R}^2 \rangle} \approx \frac{12}{5} f_{\text{nl}} \mathcal{R}_L(\vec{x}). \quad (6)$$

We thus infer that, with local-model non-Gaussianity, the (fractional) chemical-potential fluctuation in a given region of the sky is given simply by the long-wavelength curvature perturbation in that region at the surface of last scatter.

The same is true for the large-angle temperature fluctuation—it is determined primarily by the curvature fluctuation at the surface of last scatter and has magnitude $\Delta T/T \approx \mathcal{R}/5$. Therefore, for multipole moments $\ell \lesssim 100$ that probe causally disconnected regions at the surface of last scatter, the cross correlation between the (fractional) chemical-potential fluctuation $\Delta\mu/\mu \approx \delta\langle\mathcal{R}^2\rangle/\langle\mathcal{R}^2\rangle$ and the temperature fluctuation $\Delta T/T$ has a power spectrum

$$C_\ell^{\mu T} \approx 12f_{\text{nl}}C_\ell^{\text{TT}}. \quad (7)$$

This easily obtained result agrees with Ref. [13], noting that their $C_\ell^{\mu T}$ is for the μ fluctuation, rather than the fractional μ fluctuation. The μ autocorrelation caused by non-Gaussianity is $C_\ell^{\mu\mu} \approx 144f_{\text{nl}}^2C_\ell^{\text{TT}}$. Here we assumed that the trispectrum contributions are negligible. As our derivation clarifies, the μT correlation arises from the squeezed limit of the bispectrum, the part of the bispectrum that modulates small-scale power. For completeness, we include the large-angle temperature power spectrum,

$$C_\ell^{\text{TT}} = \frac{2\pi}{25} \frac{\Delta_{\mathcal{R}}^2}{\ell(\ell+1)} \approx \frac{6.0 \times 10^{-10}}{\ell(\ell+1)}. \quad (8)$$

If the non-Gaussianity is scale invariant, the yT and $y\mu$ correlations are the same as the μT and $\mu\mu$ correlations (all in terms of $\Delta y/y$ and $\Delta\mu/\mu$):

$$C_\ell^{yT} \approx 12f_{\text{nl}}C_\ell^{\text{TT}}, \quad C_\ell^{yy} \approx 144f_{\text{nl}}^2C_\ell^{\text{TT}}. \quad (9)$$

If, however, the non-Gaussianity is scale dependent (see, e.g., Refs. [31–37]), then the value of f_{nl} that describes correlations between the long-wavelength ($k_L \lesssim 0.01 \text{ Mpc}^{-1}$) modes responsible for large-angle CMB temperature fluctuations and short-wavelength modes ($50 \text{ Mpc}^{-1} \lesssim k \lesssim 10^4 \text{ Mpc}^{-1}$) responsible for the μ distortion may differ from the value of f_{nl} that describes correlations between long-wavelength modes and $1 \text{ Mpc}^{-1} \lesssim k \lesssim 50 \text{ Mpc}^{-1}$ modes responsible for the dissipation-induced y distortions.

We therefore parametrize the scale-dependent non-Gaussianity in terms of a scale-dependent non-Gaussianity parameter $f_{\text{nl}}(k_s)$ defined by the squeezed-limit ($k_s \equiv k_2 = k_3 \gg k_1 \equiv k_L$) curvature bispectrum,

$$B_{\mathcal{R}}(k_1, k_2, k_3) \approx \frac{12}{5} f_{\text{nl}}(k_s) P_{\mathcal{R}}(k_s) P_{\mathcal{R}}(k_L), \quad (10)$$

defined from

$$\langle \mathcal{R}_{\vec{k}_1} \mathcal{R}_{\vec{k}_2} \mathcal{R}_{\vec{k}_3} \rangle \equiv (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_{\mathcal{R}}(k_1, k_2, k_3) \quad (11)$$

for $k_L \lesssim 0.01 \text{ Mpc}^{-1}$. We then define two parameters, $f_{\text{nl}}^y \equiv f_{\text{nl}}(k_s \approx 7 \text{ Mpc}^{-1})$ and $f_{\text{nl}}^\mu \equiv f_{\text{nl}}(k_s \approx 740 \text{ Mpc}^{-1})$, to parametrize the non-Gaussianity on y and μ scales, respectively. Here, the f_{nl} parameter is evaluated roughly at the log midpoints of the y - and μ -distortion k intervals, which define the y - and μ -distortion pivot wavenumbers $k_y \approx 7 \text{ Mpc}^{-1}$ and $k_\mu \approx 740 \text{ Mpc}^{-1}$, respectively. Although the scale dependence of f_{nl} is often modeled in the literature as a power law in wavenumber k , this parametrization does not accommodate the possibility, which arises in curvaton and multifield models [34,38–40], that f_{nl} may change sign with scale. The above parametrization is therefore more general. We note that the $k_1 \equiv k_L \ll k_2 = k_3 \equiv k_s$ bispectrum in Eq. (10) is the squeezed limit of the common bispectrum parametrization (e.g., [41]), $B_{\mathcal{R}}(k_1, k_2, k_3) = (6/5)[f_{\text{nl}}(k_1)P(k_2)P(k_3) + \text{perms}]$, of scale-dependent non-Gaussianity. Our results are thus easily compared with many other results on scale-dependent non-Gaussianity.

We estimate the detectability of the μT signal following Ref. [13], assuming a noise power spectrum $C_\ell^{\mu n} \approx w_\mu^{-1} e^{\ell^2/\ell_{\text{max}}^2}$ with $w_\mu^{-1/2} \approx \sqrt{4\pi}(\mu_{\text{min}}/\langle\mu\rangle)$ and $\ell_{\text{max}} \approx 100$. Note again that ours is a power spectrum for $\Delta\mu/\mu$, as opposed to the power spectrum for μ in Ref. [13], where μ_{min} is the smallest detectable μ monopole, which is estimated to be $\mu_{\text{min}} \approx 10^{-8}$ for PIXIE [42] and $\mu_{\text{min}} \approx 10^{-9}$ for PRISM [43]. We then obtain the smallest detectable (at $\sim 1\sigma$) μT correlation to arise for

$$f_{\text{nl}}^\mu \approx 220 \left(\frac{\mu_{\text{min}}}{10^{-9}} \right) \left(\frac{\langle\mu\rangle}{2 \times 10^{-8}} \right)^{-1}. \quad (12)$$

Although the dissipation-induced average y distortion is expected to be ≈ 5 times smaller than the μ distortion (see the estimate above), the experimental sensitivity to y is $\approx 5 - 10$ times better [42,44]. Assuming a PRISM detection limit of $y_{\text{min}} \approx 2 \times 10^{-10}$, we thus infer a comparable smallest non-Gaussianity parameter,

$$f_{\text{nl}}^y \approx 220 \left(\frac{y_{\text{min}}}{2 \times 10^{-10}} \right) \left(\frac{\langle y \rangle}{4 \times 10^{-9}} \right)^{-1}, \quad (13)$$

that will engender a detectable yT correlation. These estimates are obtained assuming a roughly scale-invariant spectrum of primordial perturbations. If for some reason the small-scale power-spectrum amplitude is increased in the y range ($1 \text{ Mpc}^{-1} \lesssim k \lesssim 50 \text{ Mpc}^{-1}$) or μ range ($50 \text{ Mpc}^{-1} \lesssim k \lesssim 10^4 \text{ Mpc}^{-1}$), the smallest detectable f_{nl}^y and f_{nl}^μ , respectively, will be decreased by a similar factor. In this case, the homogeneous values of y and μ will also be increased [17]. The detection limits obtained from the μ

and y autocorrelations are typically a few times weaker, since, in contrast to the temperature cross correlations, noise dominates the μ distortion measurements.

The μ and y spectral distortions in any given causal patch at the surface of last scatter come from the dissipation of small-scale modes within that patch. There is a finite number $N \sim (k\lambda)^3$ of such modes in this patch, where k is the largest relevant wavenumber (10^4 Mpc^{-1} and 50 Mpc^{-1} for μ and y distortions, respectively), and $\lambda \sim 100 \text{ Mpc}$ is the size of the causal patch at the surface of last scatter. There will therefore be Poisson fluctuations of amplitude $\sim N^{-1/2}$ in the value of the μ and y distortions in any such patch. The mean-squared fractional μ fluctuation in a patch of angular size θ will thus be

$$\left(\frac{\Delta\mu}{\mu}\right)_\theta^2 = \int \frac{\ell d\ell}{2\pi} C_\ell |W_\ell|^2 \sim \ell^2 C_\ell |_{\ell \sim \theta^{-1}} \sim \left(\frac{H_0}{k_{\text{max}}\theta}\right)^3,$$

where W_ℓ is the window function for a circle on the sky of radius θ , and the last (approximate) equality is the Poisson-fluctuation amplitude. We therefore infer fluctuations $C_\ell^{yy} \sim 6 \times 10^{-15} (\ell/100)$ and $C_\ell^{\mu\mu} \sim 6 \times 10^{-25} (\ell/100)$ well below those from measurement noise.

Before the yT correlations considered here can be used to probe or constrain primordial non-Gaussianity, it will be necessary to consider the cross correlation of the Compton- y distortion from intergalactic gas in the late Universe with the contribution to the large-angle temperature fluctuation from the integrated Sachs-Wolfe effect. We anticipate that lensing reconstruction can be used to separate out this late-time yT correlation. We also anticipate that this contribution can be distinguished by a different ℓ dependence.

An additional source of yT correlations could arise from the damping of primordial magnetic fields [45]. In spite of the large uncertainty in the amplitude of primordial magnetic fields, the scale dependence is again generally expected to differ from the one caused by non-Gaussianity. However, a more detailed study is required.

As Fig. 1 shows, the constraints on the squeezed-limit bispectrum from μT and yT correlations considered here will be complemented on the longer- and shorter-wavelength ends by other measurements. Searches for non-Gaussianity in CMB fluctuations and in galaxy surveys probe wavenumbers $k \lesssim 0.1 \text{ Mpc}^{-1}$. There are other searches for the scale-dependent bias that arises from non-Gaussianity [9–11]. These probe squeezed-limit non-Gaussianity primarily on wavenumbers $k \lesssim 1 \text{ Mpc}^{-1}$ [12] (those that are most important for determining the abundance of the galaxies being correlated). Dissipation of acoustic modes with wavelengths $10^4 \text{ Mpc}^{-1} \lesssim k \lesssim 10^5 \text{ Mpc}^{-1}$ produce entropy in the primordial plasma after big-bang nucleosynthesis [46,47]. The long-wavelength modulation induced by squeezed-limit non-Gaussianity on these scales will then give

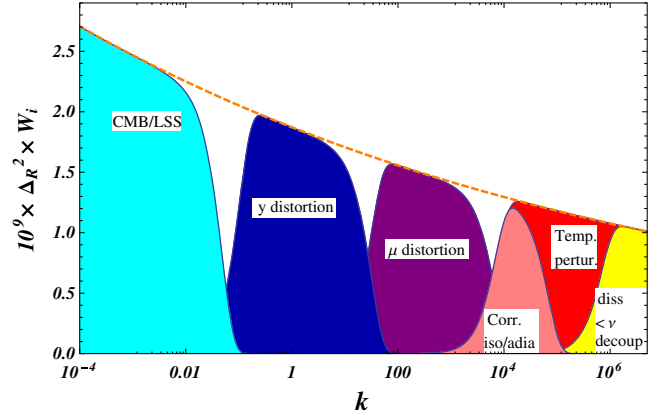


FIG. 1 (color online). The different k -space windows responsible for different observables. The CMB anisotropies are visible at $k \lesssim 0.1 \text{ Mpc}^{-1}$, where the small-scale cut-off is introduced by Silk damping. Compton- y distortions are formed by the damping of modes with $1 \text{ Mpc}^{-1} \lesssim k \lesssim 50 \text{ Mpc}^{-1}$, while μ distortions are created by modes with $50 \text{ Mpc}^{-1} \lesssim k \lesssim 10^4 \text{ Mpc}^{-1}$. Additional large-scale temperature power can be created by modes at $10^4 \text{ Mpc}^{-1} \lesssim k$ [48]. Those with $10^4 \text{ Mpc}^{-1} \lesssim k \lesssim 10^5 \text{ Mpc}^{-1}$ can be constrained using the primordial helium abundance [46,47], while modes with $10^5 \text{ Mpc}^{-1} \lesssim k$ are erased before neutrino decoupling, and thus would only lead to additional large-scale temperature fluctuations.

rise to a small isocurvature fluctuation correlated with the primordial adiabatic perturbation [46]. Finally, modes with $10^4 \text{ Mpc}^{-1} \lesssim k$ can give rise to additional large-scale temperature fluctuations caused by non-Gaussianity [48]. Depending on the sign of the correlation, this could also produce a lack of power on large scales (which could also affect the values of cosmological parameters inferred from these temperature fluctuations).

We find it interesting that there are thus now prospects to probe the amplitude of squeezed-limit non-Gaussianity on a continuum of distance scales from the largest ($\sim \text{Gpc}$) accessible to those on scales nearly 8 orders of magnitude smaller, with CMB, large-scale structure, scale-dependent biasing, yT correlations, μT correlations, and small-scale entropy production. This complement of measurements will thus allow the determination of the functional dependence of $f_{\text{nl}}(k_s)$, without necessarily assuming a specific parametrization (e.g., power law) for its scale dependence.

ACKNOWLEDGMENTS

This work was supported at JHU by NSF Grant No. 0244990, NASA NNX15AB18G, the John Templeton Foundation, and the Simons Foundation, and at ASU by the Department of Energy Grant No. DE-SC0008016. J. C. was supported by the Royal Society, and R. E. acknowledges the support of the New College Oxford–Johns Hopkins Centre for Cosmological Studies.

- [1] A. H. Guth and S. Y. Pi, *Phys. Rev. Lett.* **49**, 1110 (1982); A. A. Starobinsky, *Phys. Lett. B* **117**, 175 (1982); J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, *Phys. Rev. D* **28**, 679 (1983); V. F. Mukhanov and G. V. Chibisov, *Pis'ma Zh. Eksp. Teor. Fiz.* **33**, 549 (1981) [*JETP Lett.* **33**, 532 (1981)]; S. W. Hawking, *Phys. Lett. B* **115**, 295 (1982); A. D. Linde, *Phys. Lett. B* **116**, 335 (1982).
- [2] N. Bartolo *et al.*, *Phys. Rep.* **402**, 103 (2004).
- [3] T. Falk, R. Rangarajan, and M. Srednicki, *Astrophys. J.* **403**, L1 (1993); A. Gangui, F. Lucchin, S. Matarrese, and S. Mollerach, *Astrophys. J.* **430**, 447 (1994); A. Gangui, *Phys. Rev. D* **50**, 3684 (1994); L. M. Wang and M. Kamionkowski, *Phys. Rev. D* **61**, 063504 (2000); J. M. Maldacena, *J. High Energy Phys.* **05** (2003) 013; V. Acquaviva, N. Bartolo, S. Matarrese, and A. Riotto, *Nucl. Phys.* **B667**, 119 (2003).
- [4] L. Verde, L. M. Wang, A. Heavens, and M. Kamionkowski, *Mon. Not. R. Astron. Soc.* **313**, L141 (2000).
- [5] E. Komatsu *et al.* (WMAP Collaboration), *Astrophys. J. Suppl. Ser.* **180**, 330 (2009).
- [6] P. A. R. Ade *et al.* (Planck Collaboration), *Astron. Astrophys.* **571**, A24 (2014).
- [7] P. A. R. Ade *et al.* (Planck Collaboration), *arXiv*: 1502.01592.
- [8] L. Verde, A. F. Heavens, W. J. Percival, S. Matarrese, C. M. Baugh, J. Bland-Hawthorn, T. Bridges, R. Cannon *et al.*, *Mon. Not. R. Astron. Soc.* **335**, 432 (2002).
- [9] N. Dalal, O. Dore, D. Huterer, and A. Shirokov, *Phys. Rev. D* **77**, 123514 (2008).
- [10] A. Slosar, C. Hirata, U. Seljak, S. Ho, and N. Padmanabhan, *J. Cosmol. Astropart. Phys.* **08** (2008) 031.
- [11] S. Matarrese and L. Verde, *Astrophys. J.* **677**, L77 (2008); C. Carbone, L. Verde, and S. Matarrese, *Astrophys. J.* **684**, L1 (2008).
- [12] F. Schmidt and M. Kamionkowski, *Phys. Rev. D* **82**, 103002 (2010).
- [13] E. Pajer and M. Zaldarriaga, *Phys. Rev. Lett.* **109**, 021302 (2012).
- [14] J. Ganc and E. Komatsu, *Phys. Rev. D* **86**, 023518 (2012).
- [15] W. Hu, D. Scott, and J. Silk, *Astrophys. J.* **430**, L5 (1994).
- [16] J. Chluba, R. Khatri, and R. A. Sunyaev, *Mon. Not. R. Astron. Soc.* **425**, 1129 (2012).
- [17] J. Chluba, A. L. Erickcek, and I. Ben-Dayan, *Astrophys. J.* **758**, 76 (2012).
- [18] A. Loeb and M. Zaldarriaga, *Phys. Rev. Lett.* **92**, 211301 (2004).
- [19] T. J. Allen, B. Grinstein, and M. B. Wise, *Phys. Lett. B* **197**, 66 (1987); L. A. Kofman and D. Y. Pogosian, *Phys. Lett. B* **214**, 508 (1988); D. S. Salopek, J. R. Bond, and J. M. Bardeen, *Phys. Rev. D* **40**, 1753 (1989); A. D. Linde and V. F. Mukhanov, *Phys. Rev. D* **56**, R535 (1997); P. J. E. Peebles, *Astrophys. J.* **510**, 523 (1999); **510**, 531 (1999).
- [20] S. Mollerach, *Phys. Rev. D* **42**, 313 (1990); D. H. Lyth and D. Wands, *Phys. Lett. B* **524**, 5 (2002); T. Moroi and T. Takahashi, *Phys. Lett. B* **522**, 215 (2001); **539**, 303(E) (2002); D. H. Lyth, C. Ungarelli, and D. Wands, *Phys. Rev. D* **67**, 023503 (2003); K. Ichikawa, T. Suyama, T. Takahashi, and M. Yamaguchi, *Phys. Rev. D* **78**, 023513 (2008); K. Enqvist, S. Nurmi, O. Taanila, and T. Takahashi, *J. Cosmol. Astropart. Phys.* **04** (2010) 009; K. Enqvist and T. Takahashi, *J. Cosmol. Astropart. Phys.* **12** (2009) 001; **09** (2008) 012; K. Enqvist and S. Nurmi, *J. Cosmol. Astropart. Phys.* **10** (2005) 013; A. L. Erickcek, M. Kamionkowski, and S. M. Carroll, *Phys. Rev. D* **78**, 123520 (2008); A. L. Erickcek, C. M. Hirata, and M. Kamionkowski, *Phys. Rev. D* **80**, 083507 (2009); S. Hannestad, T. Haugboelle, P. R. Jarnhus, and M. S. Sloth, *J. Cosmol. Astropart. Phys.* **06** (2010) 001.
- [21] G. R. Dvali and S. H. H. Tye, *Phys. Lett. B* **450**, 72 (1999); P. Creminelli, *J. Cosmol. Astropart. Phys.* **10** (2003) 003; M. Alishahiha, E. Silverstein, and D. Tong, *Phys. Rev. D* **70**, 123505 (2004).
- [22] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, England 1994); R. Durrer, *New Astron. Rev.* **43**, 111 (1999); R. Durrer, M. Kunz, and A. Melchiorri, *Phys. Rep.* **364**, 1 (2002); A. H. Jaffe, *Phys. Rev. D* **49**, 3893 (1994); A. Silvestri and M. Trodden, *Phys. Rev. Lett.* **103**, 251301 (2009); M. Hindmarsh, C. Ringeval, and T. Suyama, *Phys. Rev. D* **80**, 083501 (2009); **81**, 063505 (2010); D. M. Regan and E. P. S. Shellard, *Phys. Rev. D* **82**, 063527 (2010); N. Turok and D. N. Spergel, *Phys. Rev. Lett.* **66**, 3093 (1991); D. G. Figueroa, R. R. Caldwell, and M. Kamionkowski, *Phys. Rev. D* **81**, 123504 (2010).
- [23] R. Cen and J. P. Ostriker, *Astrophys. J.* **514**, 1 (1999).
- [24] P. J. Zhang, U. L. Pen, and H. Trac, *Mon. Not. R. Astron. Soc.* **355**, 451 (2004).
- [25] M. Biagetti, H. Perrier, A. Riotto, and V. Desjacques, *Phys. Rev. D* **87**, 063521 (2013).
- [26] J. Chluba and D. Grin, *Mon. Not. R. Astron. Soc.* **434**, 1619 (2013).
- [27] J. Chluba, E. Dimastrogiovanni, R. Emami, and M. Kamionkowski (unpublished).
- [28] W. Hu and J. Silk, *Phys. Rev. D* **48**, 485 (1993).
- [29] J. Chluba and R. A. Sunyaev, *Mon. Not. R. Astron. Soc.* **419**, 1294 (2012).
- [30] E. Komatsu *et al.* (WMAP Collaboration), *Astrophys. J. Suppl. Ser.* **192**, 18 (2011).
- [31] M. LoVerde, A. Miller, S. Shandera, and L. Verde, *J. Cosmol. Astropart. Phys.* **04** (2008) 014.
- [32] C. T. Byrnes, M. Gerstenlauer, S. Nurmi, G. Tasinato, and D. Wands, *J. Cosmol. Astropart. Phys.* **10** (2010) 004.
- [33] C. T. Byrnes, S. Nurmi, G. Tasinato, and D. Wands, *J. Cosmol. Astropart. Phys.* **02** (2010) 034.
- [34] J. Bramante and J. Kumar, *J. Cosmol. Astropart. Phys.* **09** (2011) 036.
- [35] A. Riotto and M. S. Sloth, *Phys. Rev. D* **83**, 041301 (2011).
- [36] A. Becker and D. Huterer, *Phys. Rev. Lett.* **109**, 121302 (2012).
- [37] E. Sefusatti, M. Liguori, A. P. S. Yadav, M. G. Jackson, and E. Pajer, *J. Cosmol. Astropart. Phys.* **12** (2009) 022.
- [38] K. Enqvist and S. Nurmi, *J. Cosmol. Astropart. Phys.* **10** (2005) 013.
- [39] C. T. Byrnes, K. Enqvist, S. Nurmi, and T. Takahashi, *J. Cosmol. Astropart. Phys.* **11**(2011) 011.
- [40] J. Meyers and E. R. M. Tarrant, *Phys. Rev. D* **89**, 063535 (2014).
- [41] A. Becker, D. Huterer, and K. Kadota, *J. Cosmol. Astropart. Phys.* **01** (2011) 006.

- [42] A. Kogut, D. J. Fixsen, D. T. Chuss, J. Dotson, E. Dwek, M. Halpern, G. F. Hinshaw, S. M. Meyer *et al.*, *J. Cosmol. Astropart. Phys.* **07** (2011) 025.
- [43] P. André *et al.* (PRISM Collaboration), *J. Cosmol. Astropart. Phys.* **02** (2014) 006.
- [44] J. Chluba and D. Jeong, *Mon. Not. R. Astron. Soc.* **438**, 2065 (2014).
- [45] K. Miyamoto, T. Sekiguchi, H. Tashiro, and S. Yokoyama, *Phys. Rev. D* **89**, 063508 (2014).
- [46] D. Jeong, J. Pradler, J. Chluba, and M. Kamionkowski, *Phys. Rev. Lett.* **113**, 061301 (2014).
- [47] T. Nakama, T. Suyama, and J. Yokoyama, *Phys. Rev. Lett.* **113**, 061302 (2014).
- [48] A. Naruko, A. Ota, and M. Yamaguchi, [arXiv:1503.03722](https://arxiv.org/abs/1503.03722).