Signatures of the very early Universe: Inflation, spatial curvature, and large scale anomalies

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A short inflationary phase may not erase all traces of the primordial Universe. Associated observables include both spatial curvature and "anomalies" in the microwave background or large-scale structure. The present curvature $\Omega_{K,0}$ reflects the initial curvature, $\Omega_{K,\mathrm{start}}$, and the angular size of anomalies depends on k_{start}, the comoving horizon size at the onset of inflation. We estimate posteriors for $\Omega_{K,\mathrm{start}}$ and k_{start} using current data and simulations, and show that if either quantity is measured to have a nonzero value, both are likely to be observable. Mappings from $\Omega_{K,\mathrm{start}}$ and k_{start} to present-day observables depend strongly on the primordial equation of state; $\Omega_{K,0}$ spans 10 orders of magnitude for a given $\Omega_{K,\mathrm{start}}$, while a simple and general relationship connects $\Omega_{K,0}$ and k_{start}. We show that current bounds on $\Omega_{K,0}$ imply that if k_{start} is measurable, the curvature was already small when inflation began. Finally, since the energy density changes slowly during inflation, primordial gravitational wave constraints require that a short inflationary phase be preceded by a nontrivial preinflationary phase with critical implications for the expected value of $\Omega_{K,\mathrm{start}}$.

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I. INTRODUCTION

Standard slow-roll inflation in combination with ΛCDM generically implies that the present-day spatial curvature, $\Omega_{K,0}$, is undetectably small, and current measurements are consistent with a spatially flat Universe [1]. However, there is long-standing interest in "Just Enough" inflation (e.g. Refs. [2–11]). These investigations have many motivations, including possible large-scale anomalies in the cosmic microwave background (CMB) [12,13] as well as a possible future detection of primordial B modes in the CMB.[1] More generally, investigations of Just Enough inflation are necessary for a full understanding of inflationary phenomenology.

Currently, the tightest constraints on $\Omega_{K,0}$ are obtained from the Planck 2015 data set together with baryon acoustic oscillations, type-Ia supernovae, and Hubble constant measurements [17]:

$$100\Omega_{K,0} = 0.08^{+0.40}_{-0.39}. \qquad (1)$$

Bounds on $|\Omega_{K,0}|$ will improve to a level of 10^{-4} with future 21 cm intensity measurements, while $|\Omega_{K,0}| \sim 10^{-5}$ is the effective cosmic variance limit [18].

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[1] The joint analysis of BICEP2, Keck Array, and Planck data has found no evidence for primordial B modes [14]. However, a tensor-to-scalar ratio of $r \sim 0.1$ predicted by simple inflationary models [15] is not excluded, and any significant tensor background would increase the tension between simple primordial power spectra and the observed $\langle TT \rangle$ angular power spectrum at low ℓ [16].

Spatial curvature affects the trajectories of photons propagating from the surface of last scattering toward the observer. Separately, a short inflationary phase may not erase all remnants of the preinflationary Universe, giving rise to CMB anomalies at large angular scales [19] in addition to detectable curvature. If k_{start}, the comoving horizon size at the onset of inflation, is close to the present horizon size, the power spectrum may be modified even if the perturbations are strictly Bunch-Davies.

Accelerated expansion can begin immediately after the big bang, but the energy density changes slowly during inflation. Given bounds on the gravitational wave background inflation must occur at significantly sub-Planckian energies if the number of e-folds is small. Consequently, Just Enough inflation is likely to have a nontrivial preinflationary phase whose remnants may be visible today, providing clues to the nature of the big bang itself. For instance, bounds on $\Omega_{K,0}$ constrain eternal inflation scenarios [18,20] and models in which the big bang is a bubble-nucleation event [21–27].

The cosmic variance of $\Omega_{K,0}$ and the precision with which it can be measured has been studied in Refs. [28–30]. A detectable $\Omega_{K,0}$ may be induced by a local inhomogeneity [31], and some mechanisms which account for possible CMB anomalies suggest that spatial curvature may be measurable in the next generation of experiments [16,19].

Preinflationary remnants obviously reflect the properties of the preinflationary phase. Different scenarios for the preinflationary Universe necessarily have different observational consequences, but k_{start} is likely to be a key parameter in all of them. Likewise, $\Omega_{K,0}$ depends upon the preinflationary curvature $\Omega_{K,\mathrm{start}}$ and the pre- and postinflationary evolution of the Universe. Nucleosynthesis and neutrino freeze-out are described by well-established analyses making well-verified predictions, and inflation is probed via measurements of the perturbation spectrum. Conversely, the behavior of the Universe during the so-called *primordial dark age* is largely unknown, and the equation of state during both the preinflationary and postinflationary phases is unconstrained.

Large-scale anomalies, spatial curvature and Just Enough inflation all receive significant attention. However, these phenomena are not widely discussed in combination. In this paper, we examine the full range of phenomenology associated with Just Enough inflation, and possible correlations between different observables. There is a simple relationship between $\Omega_{K,0}$ and k_{start}, and we show that if k_{start} is measurable from astrophysical observations, the curvature was already small at the beginning of inflation. Moreover, we point out that Just Enough inflation is typically preceded by a significant noninflationary phase. In this case, the preinflationary Universe is thus likely to have either significant curvature or a long phase of "fast roll" inflation, or it is itself fine-tuned. This intuition is incorporated into priors describing models of the preinflationary Universe, which we use to compute posteriors for $\Omega_{K,0}$ and k_{start} from both data and simulations.

This paper is organized as follows: In Sec. II we derive the theoretical dependence of $\Omega_{K,0}$ on the parameters describing the early-Universe physics, in Sec. III we show how the posteriors for $\Omega_{K,0}$ and k_{start} constrain these parameters, and we summarize in Sec. IV.

II. THEORETICAL FRAMEWORK

The overall history of the Universe predicted by slow-roll inflation and ΛCDM is sketched in Fig. 1. Evolution is assumed to begin at near-Planckian energies. Inflation might commence immediately after the big bang, but the energy density at which cosmologically relevant scales leave the horizon is constrained by bounds on the primordial gravitational wave amplitude. Consequently, either Just Enough inflation must be preceded by a nontrivial preinflationary stage, or the evolution of the Universe must begin at a significantly sub-Planckian energy density. The behavior of the Universe in the period following inflation is likewise unconstrained, but it must thermalize in order to permit neutrino production and nucleosynthesis. Finally, as is well known, these phases are followed by matter and eventually dark energy domination.

The current scale factor a_0 is related to its value at the end of inflation a_{end} by the *matching equation*[2] [32]

[2]Throughout this paper, the subscript 0 denotes the current value of a parameter.

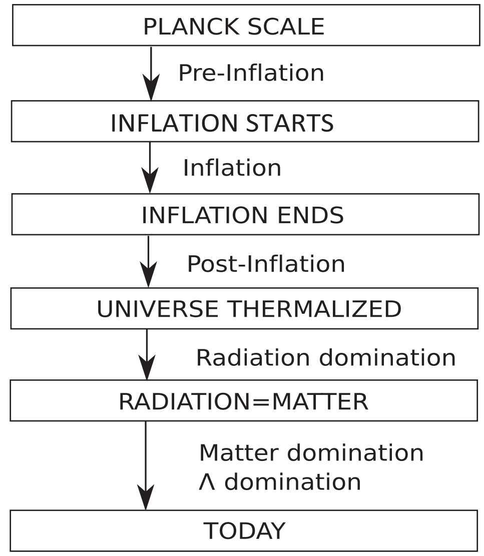

FIG. 1. The evolution history of the Universe.

$$\ln\left(\frac{a_{\mathrm{end}}}{a_0}\right) = \frac{1-3w_{\mathrm{post}}}{12(1+w_{\mathrm{post}})}\ln\left(\frac{\rho_{\mathrm{th}}}{\rho_{\mathrm{end}}}\right) - \frac{1}{4}\ln\left(\frac{\rho_{\mathrm{end}}}{M_{\mathrm{Pl}}^4}\right) - 71.21. \tag{2}$$

Here ρ_{end} is the density at the end of inflation and M_{Pl} is the reduced Planck mass. As in Ref. [32], w_{post} is the effective equation-of-state parameter during reheating, while the density ρ_{th} is the *minimal* acceptable thermalization scale. Nucleosynthesis and the cosmic neutrino background put a lower bound on ρ_{th}, but its specific value forms part of the prior. For a given thermalization history, the value of w_{post} depends on ρ_{th} [32]. The numerical term in equation (2) assumes that the effective number of neutrino species is $N_\nu = 3.04$. The matching equation ignores the impact of the curvature density on the postinflationary Universe. However, $|\Omega_{K,0}| \lesssim 10^{-2}$, so even if Ω_K is not exactly zero, it has no significant effect on the matching between a_{end} and a_0.

The curvature density is

$$\Omega_K \equiv -\frac{KM_{\mathrm{Pl}}^2}{a^2H^2}, \tag{3}$$

where $K = 0, \pm 1$ and H is the Hubble parameter. The current value $\Omega_{K,0}$ is related to the value at the end of inflation $\Omega_{K,\mathrm{end}}$ by

$$\Omega_{K,0} = \Omega_{K,\mathrm{end}}\frac{a_{\mathrm{end}}^2H_{\mathrm{end}}^2}{a_0^2H_0^2}. \tag{4}$$

The curvature is negligible at the end of inflation, so

$$H_{\rm end}^2 = \frac{\rho_{\rm end}}{3M_{\rm Pl}^2}. \tag{5}$$

Putting everything together, we relate $\Omega_{K,0}$ to $\Omega_{K,{\rm end}}$ via

$$\ln\Omega_{K,0} = \ln\Omega_{K,{\rm end}} + \frac{1-3w_{\rm post}}{6(1+w_{\rm post})}\ln\left(\frac{\rho_{\rm th}}{\rho_{\rm end}}\right) - \frac{1}{2}\ln\left(\frac{\rho_{\rm end}}{M_{\rm Pl}^4}\right) + \ln\left(\frac{\rho_{\rm end}}{3M_{\rm Pl}^2H_0^2}\right) - 142.42. \tag{6}$$

Using the standard notation

$$H_0 = 100h\frac{\rm km/s}{\rm Mpc},$$

we get

$$\ln\Omega_{K,0} = \ln\Omega_{K,{\rm end}} + \frac{1-3w_{\rm post}}{6(1+w_{\rm post})}\ln\left(\frac{\rho_{\rm th}}{\rho_{\rm end}}\right) + \frac{1}{2}\ln\left(\frac{\rho_{\rm end}}{M_{\rm Pl}^4}\right) - 2\ln h + 133.06. \tag{7}$$

Now, we focus on the evolution from the Planck scale until the onset of inflation. Primordial gravitational wave constraints imply that the energy scale of inflation is 2×10^{16} GeV or less [15]. We model the preinflationary Universe as a combination of the inflaton field and a fluid with equation-of-state parameter $w_{\rm pre}$ and energy density

$$\rho = \rho_{\rm pre}a^{-3(1+w_{\rm pre})},$$

where $\rho_{\rm pre}$ is a constant. The inflaton energy density $\rho_{\rm infl}$ is approximately but not exactly constant, as discussed in Sec. II C. This formalism describes scenarios with a fast roll phase with $w_{\rm pre}\approx -1/3$, as well as conventional, noninflationary expansion.

Adding all the sources of energy, the preinflationary Friedmann equation has the following form:

$$H^2 = -\frac{KM_{\rm Pl}^2}{a^2} + \frac{\rho_{\rm infl}}{3M_{\rm Pl}^2} + \frac{\rho_{\rm pre}}{3M_{\rm Pl}^2a^{3(1+w_{\rm pre})}}. \tag{8}$$

Slow-roll inflation (as opposed to the generic inflationary criterion of accelerated expansion) starts when the second term on the right-hand side of (8) becomes dominant. For concreteness, we define this to occur when $\rho_{\rm infl}$ contributes 90% of the energy budget, i.e.

$$\frac{\rho_{\rm infl}}{3M_{\rm Pl}^2} = 0.9H^2. \tag{9}$$

The number of e-folds following this instant is $N_{\rm total}$,

$$\ln a_{\rm end} = \ln a_{\rm start} + N_{\rm total}. \tag{10}$$

In this context, $N_{\rm total}$ refers only to the slow-roll phase.

When inflation ends, the curvature density $\Omega_{K,{\rm end}}$ is related to its initial value $\Omega_{K,{\rm start}}$ by

$$\Omega_{K,{\rm end}} = \Omega_{K,{\rm start}}\frac{a_{\rm start}^2H_{\rm start}^2}{a_{\rm end}^2H_{\rm end}^2}, \tag{11}$$

and

$$\ln\Omega_{K,{\rm end}} = \ln\Omega_{K,{\rm start}} - 2N_{\rm total} + 2\ln\left(\frac{H_{\rm start}}{H_{\rm end}}\right). \tag{12}$$

We compute $\Omega_{K,{\rm start}}$ by solving the preinflationary Friedmann equation (8) for the scale factor and the Hubble parameter at the onset of inflation; $\Omega_{K,{\rm start}}$ follows via Eq. (3).

The second key quantity is $k_{\rm start}$, the largest scale that crosses the horizon during inflation, or the horizon size at the moment inflation begins. CMB anomalies generated by preinflationary relics depend on the detailed configuration of the preinflationary Universe, but $k_{\rm start}$ will determine the current angular size of these anomalies [19]. Likewise, if the apparent lack of large-scale power in the temperature maps is attributed to a short period of slow-roll inflation [21], $k_{\rm start}$ determines the scale beyond which the power is suppressed. In comoving coordinates,

$$k_{\rm start,com} = a_{\rm start}H_{\rm start}, \tag{13}$$

and its physical value today is

$$k_{\rm start} = \frac{a_{\rm start}H_{\rm start}}{a_0} = \frac{a_{\rm start}}{a_0}\frac{H_{\rm start}}{M_{\rm Pl}}\cdot 3.81\times 10^{53}\ {\rm Mpc}^{-1}. \tag{14}$$

We express $k_{\rm start}$ in units of ${\rm Mpc}^{-1}$, and with our definitions,

$$\ln k_{\rm start} = -N_{\rm total} + \ln\left(\frac{a_{\rm end}}{a_0}\right) + \ln\left(\frac{H_{\rm start}}{M_{\rm Pl}}\right) + 123.37. \tag{15}$$

If the physical scale corresponding to the horizon size at the onset of inflation is not much larger than the radius of the last scattering surface, imprints of preinflationary perturbations may be detectable in the CMB [19,33]. This possibility can be probed via careful analyses of CMB anomalies. Separately, a deficit of large-scale power in temperature maps may be statistically significant if the CMB B modes prove to have a large primordial component [16,34]. In this case a new physical scale will become apparent in CMB data and can be mapped to $k_{\rm start}$ for specific models of the preinflationary epoch.

A simple relationship between $\Omega_{K,0}$, $\Omega_{K,\mathrm{start}}$, and k_{start} follows from the definitions (3) and (14), so $\Omega_{K,0}$ is simply connected to k_{start} via

$$\ln \Omega_{K,0} = \ln \Omega_{K,\mathrm{start}} + 2 \ln k_{\mathrm{start}} - 2 \ln h + 29.83, \quad (16)$$

where the last term arises from unit conversions (k_{start} is a physical scale expressed in Mpc^{-1}). Critically, this relationship does not depend on the physics of reheating, the inflationary mechanism, its energy scale, or duration. This is a more formal statement of the well-known result that the overall curvature radius of the Universe is given by $(aH)^{-1}/\sqrt{|\Omega_K|}$ (see e.g. Ref. [19]).

A. Priors and preinflationary evolution

As noted previously, the main inflationary phase must occur at substantially sub-Planckian energies, given bounds on the gravitational wave background. Within the Just Enough scenario it is likely that significant expansion occurs before the onset of slow-roll inflation, and Ω_K can evolve during this phase.

Assuming approximate homogeneity and isotropy, we identify three possible scenarios. The first is that $w_{\mathrm{pre}} \approx -1/3$, in which case $\rho \sim 1/a^2$, and Ω_K does not change significantly during the preinflationary phase. Conversely, if $w_{\mathrm{pre}} > -1/3$, the curvature contribution grows until inflation begins. Given our definitions, $\Omega_{K,\mathrm{start}} \leq 0.1$, and if $w_{\mathrm{pre}} \gtrsim 0$ this inequality is easily saturated, even if the initial curvature is small. If both signs of Ω_K are equally likely and $\Omega_{K,\mathrm{start}} > 0$ and $w_{\mathrm{pre}} > 0$, then we need $\Omega_K \ll 1$ initially in order to survive until inflation begins. This provides an avenue for preferentially producing a Universe with negative curvature, independently of the tunneling argument [21]. Finally, it is possible that the initial state of the Universe is effectively set at substantially sub-Planckian scales.

In the light of this analysis, we see three possible priors for $\Omega_{K,\mathrm{start}}$. The first is $\Omega_{K,\mathrm{start}} = 0.1$, which is the natural choice if the initial curvature can take a large range of values with a substantial phase of nonaccelerated expansion before inflation begins; recall that $\Omega_K > 0$ indicates a negatively curved Universe. Conversely, if it is asserted that the Universe undergoes fast-roll inflation with $w_{\mathrm{pre}} \approx -1/3$ or begins evolving at sub-Planckian densities, we can choose uniform or log priors in which the curvature has either sign, provided $|\Omega_{K,\mathrm{start}}| \leq 0.1$.

Given k_{start}, the current spatial curvature can be translated into its value at the beginning of inflation, mapping constraints on $\Omega_{K,0}$ into constraints on $\Omega_{K,\mathrm{start}}$. We plot the relationship between $\Omega_{K,0}$ on k_{start} in Fig. 2, assuming $\Omega_{K,\mathrm{start}} = 0.1$ and $h = 0.7$. We see that the current upper bound $|\Omega_{K,0}| \lesssim 10^{-2}$ implies that $k_{\mathrm{start}} \lesssim 10^{-7}\ \mathrm{Mpc}^{-1}$. Consequently, we can immediately deduce that scenarios in which CMB anomalies are generated by preinflationary relics with $k_{\mathrm{start}} \gtrsim 10^{-7}\ \mathrm{Mpc}^{-1}$ are in tension with bounds on $\Omega_{K,0}$, unless Ω_K is already small when inflation begins.

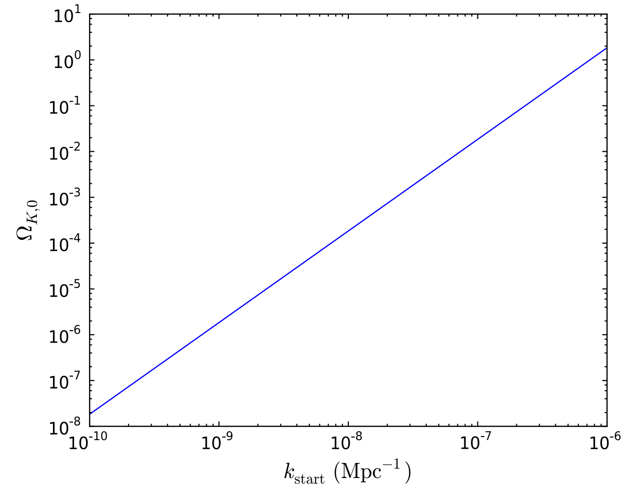

FIG. 2 (color online). The relationship between $\Omega_{K,0}$ and k_{start}, the largest physical scale that crossed the horizon during inflation, plotted for $\Omega_{K,\mathrm{start}} = 0.1$ and $h = 0.7$.

B. Reheating

Reheating is not well understood, and many scenarios have been discussed [35]. Possibilities include immediate thermalization, a long matter-dominated phase or more exotic scenarios such as cosmic string networks or stiff fluids [32]. The Planck analysis of inflationary models [15,36] lists three representative scenarios, which we adopt here:

1. *Instantaneous entropy generation*, in which the Universe becomes radiation dominated instantly after inflation.
2. *Restrictive entropy generation*, with $\rho_{\mathrm{th}}^{1/4} = 10^9$ GeV, $w_{\mathrm{post}} \in [-1/3, 1/3]$.
3. *Permissive entropy generation*, with $\rho_{\mathrm{th}}^{1/4} = 10^3$ GeV, $w_{\mathrm{post}} \in [-1/3, 1]$.

Note that ρ_{th} is the scale by which the Universe *must* be radiation dominated [32]; actual thermalization can occur at any scale between ρ_{th} and ρ_{end}.

We illustrate the dependence of $\Omega_{K,0}$ on the thermalization energy in Fig. 3. We fix the inflationary energy density at $\rho_{\mathrm{end}}^{1/4} = 10^{16}$ GeV and $\rho_{\mathrm{th}}^{1/4}$ at 10^3 GeV and plot $\Omega_{K,0}/\Omega_*$ for four different values of w_{post}: $-1/3$, 0, 1/3, and 1. For the restrictive entropy generation scenario, $\Omega_{K,0}$ can differ by up to 5 orders of magnitude from the instantaneous entropy generation case, while with the permissive entropy generation scenario the difference can be up to 24 orders of magnitude. For a given inflationary scenario, the required number of e-folds is partially determined by the subsequent expansion history of the Universe [37] while the curvature is exponentially suppressed during slow-roll inflation. Consequently,

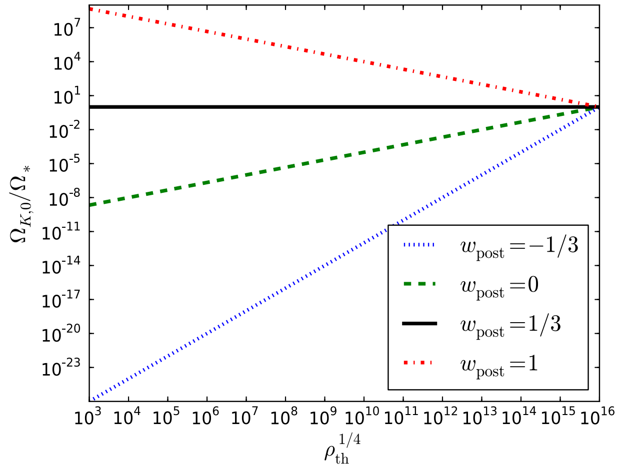

FIG. 3 (color online). The dependence of $\Omega_{K,0}/\Omega_*$ on the thermalization energy scale $\rho_{\rm th}$, where Ω_* denotes the value of $\Omega_{K,0}$ for the instantaneous entropy generation case.

a relatively small change in the number of e-folds leads to a huge variation in $\Omega_{K,0}$ for fixed $\Omega_{K,\rm start}$.

C. Inflation

If the inflationary scale (and thus $\rho_{\rm end}$) is low, fewer e-folds are needed to suppress the curvature, as is evident from equation (7). Thus the answer to the question "how much inflation is just enough?" depends on the energy scale at which inflation occurs.

Using the slow-roll approximation, we relate the variation of the inflationary energy density to the tensor-to-scalar ratio r and the total number of e-folds $N_{\rm total}$, correcting the de Sitter approximation we used above. The slow-roll parameter ϵ is related to the variation of H [38]:

$$\frac{d\ln H}{dN} = -\epsilon. \tag{17}$$

Assuming that ϵ is roughly constant during inflation,[3] we find

$$\ln\frac{H_{\rm start}}{H_{\rm end}} = \epsilon N_{\rm total}. \tag{18}$$

Denoting

$$H_{\rm ratio} \equiv \frac{H_{\rm start}}{H_{\rm end}} \tag{19}$$

and using the slow-roll result [38] $r \approx 16\epsilon$ gives

$$\ln H_{\rm ratio} = \frac{r}{16}N_{\rm total}. \tag{20}$$

[3]This approximation can be improved using the slow-roll hierarchy and is more accurate when ϵ is small [39].

The last equation will allow us to translate the priors on r and $N_{\rm total}$ into a prior for $H_{\rm ratio}$. The energy scale of inflation can also be related to r [38]. Since we are only interested in slow-roll inflation, we will assume that the energy scale at the end of inflation $\rho_{\rm end}$ is similar to the pivot scale, for which r is determined. Therefore, we will use [38]

$$\rho_{\rm end}^{1/4} \approx \left(\frac{r}{0.01}\right)^{1/4} 10^{16}\ \text{GeV}. \tag{21}$$

III. PARAMETER CONSTRAINTS

The current spatial curvature depends on the detailed history of the evolving Universe, making it sensitive to the free parameters that describe the Universe prior to thermalization and radiation domination. Most of these parameters are unknown, so even if $\Omega_{K,0}$ is found to be nonzero, it will not immediately yield constraints on the detailed properties of the preinflationary universe (beyond those that can be learned from the sign of the curvature) unless other parameters contributing to the observed value of $\Omega_{K,0}$ are known independently.

We now apply the formalism developed here to the problem of extracting early-Universe parameters from data. We work with both Planck data and simulations for which some or all of $\Omega_{K,0}$, $k_{\rm start}$, and r are detectable.

In this initial treatment the simulations are implemented via a synthetic likelihood function for $\Omega_{K,0}$ and $k_{\rm start}$. In practice, a "detection" of $k_{\rm start}$ would be made with respect to a specific scenario for the preinflationary Universe; we are simply assuming that such a detection has been made, and are not performing analyses based on detailed simulations of the sky.

The priors used in our analysis are summarized in Table I. We allow a wide range of values for $\Omega_{K,\rm start}$ with a uniform logarithmic prior. We use the restrictive entropy generation prior for the reheating parameters. The $N_{\rm total}$ prior is consistent with the Planck Collaboration analysis of inflationary models [15,36], and the h prior is derived from recent observational bounds [17]. For convenience we consider only positive $\Omega_{K,\rm start}$ (or negative curvature), but the generalization to a two-sided bound is straightforward. The upper bound on $\Omega_{K,\rm start}$ follows from our definition.

We perform parameter estimations for four scenarios with different choices for $\Omega_{K,0}$, $k_{\rm start}$, and r. The first

TABLE I. Parameter priors used in our simulations.

Param.	Prior	Distribution
$\Omega_{K,\rm start}$	$[10^{-10}, 0.1]$	Uniform log
$w_{\rm post}$	$[-1/3, 1/3]$	Uniform
$\rho_{\rm th}^{1/4}$	10^9 GeV	Fixed
$N_{\rm total}$	[20, 90]	Uniform
h	0.677 ± 0.005	Gaussian

TABLE II. Simulation parameters. Gaussian priors are used for $\Omega_{K,0}$. A uniform prior is used for $k_{\rm start}$ for simulations 1 and 3, and a Gaussian prior is used for simulations 2 and 4. A uniform logarithmic prior is used for r for simulations 1–3, and a Gaussian prior is used for simulation 4.

Param.	Sim. 1	Sim. 2	Sim. 3	Sim. 4
$100\Omega_{K,0}$	0.08 ± 0.40	0.08 ± 0.40	0.1 ± 0.02	0.1 ± 0.02
$10^6 k_{\rm start}$	<10	1.0 ± 0.2	<10	1.0 ± 0.2
r	$[10^{-26}, 0.1]$	$[10^{-26}, 0.1]$	$[10^{-26}, 0.1]$	0.1 ± 0.02

simulation uses the most recent experimental constraints [17], while the other simulations assume detections of some of these parameters. Parameter ranges for the different simulations are given in Table II. We use the Metropolis-Hastings sampler in the numerical library Cosmo + + [40] for our analysis.

Simulation 1 is consistent with current experimental bounds, and the likelihood incorporates constraints on $\Omega_{K,0}$ estimated from Planck data. Constraints on $k_{\rm start}$ depend on a specific model of the preinflationary Universe, and $k_{\rm start}$ is only bounded above by present data. For the purposes of this investigation we take $k_{\rm start} < 10^{-5}\ {\rm Mpc}^{-1}$, which roughly corresponds to the largest presently observable scales. We assume a uniform logarithmic prior for r with the upper end consistent with current bounds [14,17] and the lower end matching an inflationary energy of $\sim 10^{10}$ GeV [see Eq. (21)], consistent with our assumptions about reheating. Simulation 2 is identical to simulation 1, but assumes a measured $k_{\rm start}$ at the 5σ level, with $k_{\rm start} = 10^{-6} \pm 2\times 10^{-7}\ {\rm Mpc}^{-1}$. Simulation 3, on the other hand, assumes a 5σ measurement of $\Omega_{K,0}$ without a measurement of $k_{\rm start}$. Finally, in simulation 4 we assume that all three parameters $\Omega_{K,0}$, $k_{\rm start}$, and r have been detected at the 5σ level.

The posteriors for $\Omega_{K,\rm start}$, $N_{\rm total}$, and $w_{\rm post}$ are shown in Figs. 4, 5, and 6, respectively. Current observational bounds (simulation 1, black solid lines) do not restrict the early-Universe parameters, yielding a flat posterior for $\Omega_{K,\rm start}$ and a lower bound for $N_{\rm total}$. The lower bound for $N_{\rm total}$ is consistent with expectations, given that inflation needs to solve the flatness problem which is reflected in $\Omega_{K,0}$ and the horizon problem, reflected in $k_{\rm start}$. Unsurprisingly, the posteriors are more restrictive with simulation 2 (green dashed lines), which assumes a detection of $k_{\rm start}$. In this case we get an upper bound on $\Omega_{K,\rm start}$, and a constraint interval for $N_{\rm total}$. The upper bound on $\Omega_{K,\rm start}$ is easy to understand for this case, since we are assuming $\Omega_{K,0}$ has not been detected. On the other hand, if we assume a detection of $\Omega_{K,0}$ instead (simulation 3, blue dotted lines), we get a lower bound on $\Omega_{K,\rm start}$, but the posterior of $N_{\rm total}$ is similar to the previous case (simulation 2). Finally, if both $\Omega_{K,0}$ and $k_{\rm start}$ are measured (simulation 4, red dash-dotted line), then the posterior distribution of

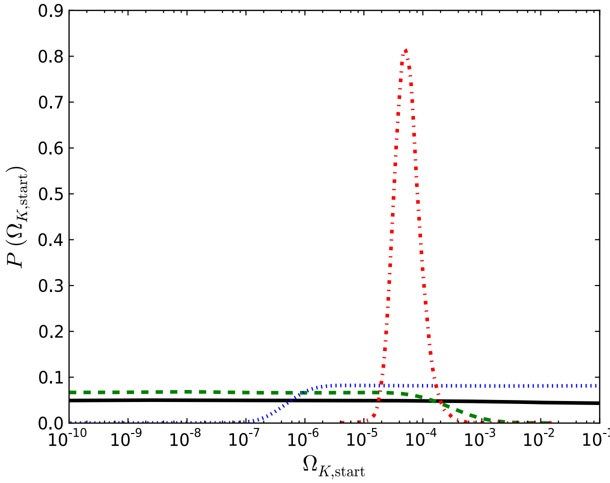

FIG. 4 (color online). Posteriors of $\Omega_{K,\rm start}$. The black solid line corresponds to simulation 1, the green dashed line to simulation 2, the blue dotted line to simulation 3, and the red dash-dotted line to simulation 4.

$\Omega_{K,\rm start}$ becomes very sharp, as expected from Eq. (16). The posterior distribution of $N_{\rm total}$ does not improve significantly, but it shifts slightly as simulation 4 assumes a measurement of the tensor-scalar ratio, r.

Posterior distributions of $w_{\rm post}$ are shown in Fig. 6. Current observational bounds (simulation 1, black solid line) result in an essentially flat posterior for $w_{\rm post}$. A detection of either $k_{\rm start}$ (simulation 2, green dashed line) or $\Omega_{K,0}$ (simulation 3, blue dotted line) still does not constrain $w_{\rm post}$ well. Even if all three parameters $k_{\rm start}$, $\Omega_{K,0}$, and r have been detected (simulation 4, red dash-dotted line), the posterior distribution of $w_{\rm post}$ is still too wide to impose significant constraints. A hypothetical detection of $\Omega_{K,0}$ gives a higher preference for larger values of $w_{\rm post}$, which is expected, since larger values of $w_{\rm post}$ imply larger values of $\Omega_{K,0}$ (see Fig. 3). However, our results show that reheating

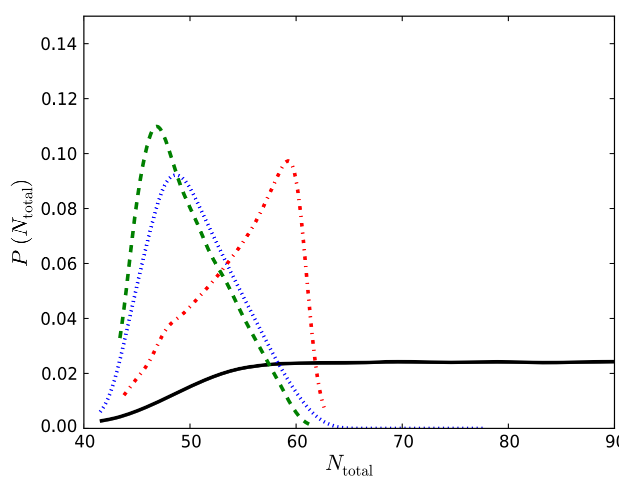

FIG. 5 (color online). Posteriors of $N_{\rm total}$. The labels are the same as in Fig. 4.

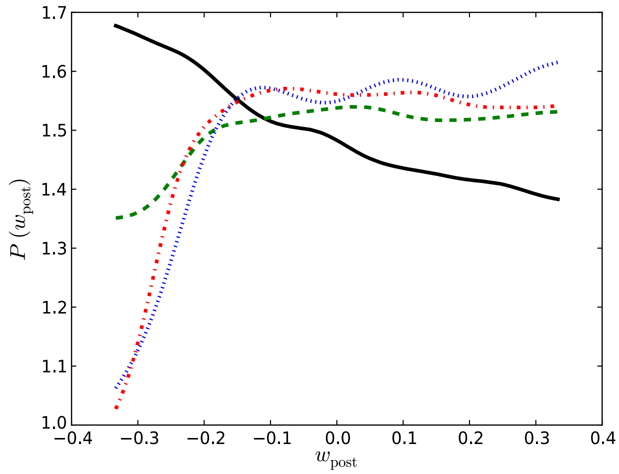

FIG. 6 (color online). Posteriors of $w_{\rm post}$. The labels are the same as in Fig. 4.

physics will not be well constrained even if all of $\Omega_{K,0}$, $k_{\rm start}$, and r have been measured.[4]

The analyses presented here have been performed with a single prior. We have verified that our main conclusions do not change for other reasonable formulations of the prior. The instantaneous entropy generation scenario yields very similar posteriors for $\Omega_{K,{\rm start}}$, so these distributions do not depend on the prior of $w_{\rm post}$. The converse is also true—a very restrictive prior for $\Omega_{K,{\rm start}}$ (for example, assuming that slow-roll inflation starts in a curvature-dominated Universe) does not affect the posterior of $w_{\rm post}$. The posterior of $N_{\rm total}$, on the other hand, becomes narrower if more stringent priors are imposed on either $\Omega_{K,{\rm start}}$ or $w_{\rm post}$.

IV. SUMMARY

We have studied how the current curvature density of the Universe $\Omega_{K,0}$ is correlated with both the initial curvature and $k_{\rm start}$, the largest physical scale that left the horizon during inflation, and the evolution of the Universe between inflation and thermalization. The dependence of $\Omega_{K,0}$ on early-Universe physics is neatly summarized in Eqs. (7) and (12), and the relationship between $\Omega_{K,0}$ and $k_{\rm start}$ is given in Eq. (16).

The details of the reheating phase have a strong impact on $\Omega_{K,0}$. Theoretically plausible reheating scenarios permit $\Omega_{K,0}$ to span 10 orders of magnitude for a fixed Just Enough inflation scenario. Consequently, while measuring a nonzero value of $\Omega_{K,0}$ would have enormous implications for our understanding of the early Universe, connecting the measured curvature directly to early-Universe physics requires knowledge of the overall expansion history of the Universe.

In a scenario with detectable curvature, we expect that the horizon size at the onset of inflation $k_{\rm start}$ will be of the same order as the present horizon size. In this case it is plausible that remnants of the preinflationary phase will source "anomalies" in the CMB or large-scale structure [19]. If this scale is not much larger than the radius of the last scattering surface, then the resulting imprints in the CMB maps may be used to determine $k_{\rm start}$. If both $k_{\rm start}$ and $\Omega_{K,0}$ can be measured, then much tighter constraints can be placed on early-Universe physics. In this context, putative anomalies in the CMB temperature data [13] and large-scale power suppression [16,34] would provide a route to detecting $k_{\rm start}$. The Planck polarization data, in particular, will help to pin down this scale [19].

Looking at the relationship between $\Omega_{K,0}$ and $k_{\rm start}$, we can identify four possible scenarios:

1. Both $\Omega_{K,0}$ and $k_{\rm start}$ are too small to be detected, in which case there is no evidence for "Just Enough" inflation, and our scenario reduces to standard slow-roll inflation.
2. The initial horizon size $k_{\rm start}$ is measured via an analysis of large-angle CMB data and large-scale structure information, but $\Omega_{K,0}$ remains consistent with zero. We see from Eq. (16) that this scenario requires $\Omega_{K,{\rm start}} \ll 1$ (see Fig. 4), introducing a preinflationary fine-tuning problem. One solution to this dilemma would be a fast-roll phase, which might also provide a plausible mechanism for large-scale scalar power suppression [21].
3. A nonzero $\Omega_{K,0}$ is detected in future experimental data, but $k_{\rm start}$ is too small to be detected; $\Omega_{K,0} \sim 10^{-4}$ will be detectable using the Planck data in combination with future 21 cm intensity measurements [18]. In this case, if the Universe is curvature dominated before inflation (implying $\Omega_{K,{\rm start}} = 0.1$ with our notation), Eq. (16) implies $k_{\rm start} \gtrsim 10^{-8}\ {\rm Mpc}^{-1}$. This scale is larger than the radius of the last scattering surface by 3 orders of magnitude. However, this scenario requires a narrow range of $N_{\rm total}$ and $w_{\rm post}$ in order to ensure that $\Omega_{K,0}$ is detectable, while $k_{\rm start}$ corresponds to scales significantly larger than our present horizon.
4. Both $\Omega_{K,0}$ and $k_{\rm start}$ are detected in the future experimental data. In this case the curvature density of the Universe at the onset of inflation, $\Omega_{K,{\rm start}}$, can be immediately determined, regardless of the model of reheating [see Eq. (16)]. However, the physics of reheating will remain poorly constrained without additional information about the inflationary phase itself [37].

Our analysis shows that either of $\Omega_{K,0}$ and $k_{\rm start}$ can be detected without the other, but if both parameters are measured together, substantial information can be gleaned

[4]If the inflationary mechanism is known and the running in the spectral index is not vanishingly small, the postinflationary expansion can be constrained via sensitive measurements of the primordial spectrum [37].

about the early Universe. Ideally, these parameters would be estimated simultaneously from the data. We have not investigated the correlations between $\Omega_{K,\mathrm{start}}$ and k_{start} in specific scenarios of the early Universe, but a joint analysis of these two parameters might allow them to be extracted when they are not necessarily detectable with high confidence on their own [19]. Finally, current constraints on $\Omega_{K,0}$ already imply that k_{start} corresponds to scales significantly larger than the radius of the surface of last scattering, if $\Omega_K \sim 1$ when inflation starts. A measurement of k_{start} via large-scale anomalies would imply that Ω_K was already small at the onset of slow-roll inflation.

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