

Halo Zel'dovich model and perturbation theory: Dark matter power spectrum and correlation function

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Perturbation theory for dark matter clustering has received a lot of attention in recent years, but its convergence properties remain poorly justified and there is no successful model that works both for correlation functions and for power spectra. Here we present the halo Zel'dovich approach combined with perturbation theory, in which we use standard perturbation theory at one-loop order (SPT) at very low k , and connect it to a version of the halo model, for which we adopt the Zel'dovich approximation plus a Padé expansion of a compensated one-halo term. This low- k matching allows us to determine the one-halo term amplitude and redshift evolution, both of which are in an excellent agreement with simulations, and approximately agree with the expected value from the halo model. Our Padé expansion approach of the one-halo term added to the Zel'dovich approximation identifies a typical halo scale averaged over the halo mass function, the halo radius scale of order of 1 Mpc/h, and a much larger halo mass compensation scale, which can be determined from SPT. The model gives better than one-percent-accurate predictions for the correlation function above 5 Mpc/h at all redshifts, without any free parameters. With three fitted Padé expansion coefficients the agreement in the power spectrum is good to a percent up to $k \sim 1$ h/Mpc, which can be improved to arbitrary k by adding higher-order terms in the Padé expansion.

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I. INTRODUCTION

One of the major puzzles in cosmology is understanding the nonlinear formation of structure in the Universe. The current state of the art are the N -body simulations, which have been verified to give reliable answers at the 1% level in the power spectrum up to $k \sim 1$ h/Mpc [1], but are expensive to run, require large allocations, and are often not fully convergent using typical current-generation box sizes and resolutions. The convergence properties become a lot more difficult to achieve for higher-order correlations. For example, recent studies have shown that to reach one percent convergence on the covariance matrix one needs to simulate a volume in excess of 1000 Gpc³ [2], and the covariance matrix depends both on the cosmological model and on the size of the survey one is simulating, making the numerical solution to the full problem an excessively demanding task given the current typical resources. Even more importantly, we want to use clustering statistics to extract information about our Universe, and simulations do not provide much insight into questions such as where is the information content and how to optimally extract it from the data.

An alternative approach is to use perturbation theory (PT), of which the two most prominent examples are standard PT (SPT) and Lagrangian PT (LPT) (see Ref. [3] for a review). In this approach one assumes that the density perturbation is less than unity, and one expands the nonlinear equations perturbatively. Since density perturbations grow with the wave vector k , this approach breaks down for some $k > k_{\text{nl}}$. The current approaches typically underpredict power in LPT and overpredict in one-loop SPT. For example, we know that the Zel'dovich approximation is quite successful in modeling the correlation function on large scales, including the baryonic acoustic oscillation (BAO) wiggles [4]. However, it fails miserably in the power spectrum, underestimating the power at all but the lowest values of k , and giving predictions that are even below the linear theory at $z = 0$ [5]. Various one-loop LPT extensions (one-loop LPT [6], convolved LPT [7], convolved LPTs [5]) somewhat improve this for the power spectrum, but make things worse for the correlation function [5]. Physically, the problem of the Zel'dovich approximation and its extensions is that while it can describe properly the initial streaming of dark matter particles, it fails to account for their capture by the dark matter halos: instead, the particles continue to stream along their trajectories, set by the initial velocity in the Zel'dovich approximation, leading to an excessive smoothing of the power.

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Standard Eulerian PT has the opposite problem. In the power spectrum it quickly overpredicts the amount of power at the one-loop level, specially at low redshifts. The reason for this is that the loop integrals extend over all modes, including those that are in the nonlinear regime $k > k_{\text{nl}}$. These modes are not in the PT regime and typically these contributions are strongly suppressed in the simulations relative to PT, leading to too much power in SPT relative to simulations. Its Fourier transform, using a Gaussian smoothing to obtain a convergent correlation function, results in a worse model than the Zel'dovich approximation around BAO, but is otherwise comparable. Recent work emphasized this point in the context of effective field theory (EFT) [8]. For example, for power-law power spectra these integrals can be divergent, so PT is wrong in such situations [9]. Instead, it is argued that the best that one can do is to introduce EFT parameters that describe the correction from the small-scale physics. For $k < 1/R_h$ these terms can be expressed as a low- k limit of PT, but with free parameters. *A priori* it is unclear what R_h is, and how large these corrections are for our Universe. In particular, realistic cold dark matter (CDM) power spectra have the shape where most of the low- k -limit loop integral comes from scales in the linear regime, where PT is believed to be valid, making the corrections from the nonlinear scales small, although probably not negligible. We will try to quantify this in more detail below. We will argue that the halo model requires R_h to be a typical halo radius.

The philosophy we will advocate in this paper is that any analytic approach must give reliable results both in Fourier space and in configuration space. Failure to do so is a sign of something missing in the model. For example, a Taylor series may give reliable results in Fourier space up to a certain k , but if truncated at a certain order it generally diverges at high k and makes its Fourier transform impossible to calculate. The reason is that the series is not convergent at high k , and one has to adopt a different summation of the terms that has a better convergence. Our goal is to develop a model that is rooted in PT as much as possible, but is also able to reproduce simulations. Since simulations have been verified at 1%, we will strive for this precision in this paper.

II. HALO ZEL'DOVICH MODEL AND PERTURBATION THEORY

The main ingredients of our approach are the following.

- (1) One-loop SPT has loop integrals which, for low k and high z , are entirely in the linear regime and thus reliably computed. There is no guarantee that the entire loop integral is correct at all redshifts, even for low k , but we will make this assumption here and derive the consequences. We will therefore assume that EFT corrections to one-loop SPT are negligible at low k . For higher k and low z SPT predictions

become increasingly unreliable and will not be used. Similarly, two-loop SPT integrals are negligible at high z , while for low z they extend deeply into the nonlinear regime and are grossly overestimated in SPT [5]. Here we will simply ignore two-loop SPT, with one exception, discussed next.

- (2) The Zel'dovich approximation gives an approximately correct physical picture of how the particles are displaced up to the process of halo formation, which stops the particles from displacing. The latter has very little effect on the Zel'dovich displacement: most of the displacement is generated by modes in the linear regime and we will not be correcting the Zel'dovich approximation. In terms of SPT the Zel'dovich approximation receives contributions from loops at all orders, but only from very specific terms related to the linear displacement field correlation function.
- (3) Halo formation has to be an essential part of the complete model. Halos are objects of very high density, leading to a nearly white noise-like contribution to the power spectrum at low k , with the halo profile parameters determining deviations from white noise at higher k . At high k , the halo term contribution dominates the correlations: all of the close pairs are inside the same halos. At a scale $k \sim 1/2R_{\text{vir}}$ the number of close pairs involves all of the pairs inside the virial radius, which must give a contribution of the order of M^2 , where M is the virial mass of the halo. Integrating over all the halos, and weighting by the halo mass function dn/dM , one obtains an estimate of the one-halo amplitude $\bar{\rho}^{-2} \int M^2 (dn/dM) dM$, where $\bar{\rho}$ is the mean density of the Universe. At redshift 0 the integral is dominated by cluster mass halos with virial radius of 1–2 h^{-1} Mpc. We expect the one-halo term to be approximately of this amplitude at $k \sim 0.2\text{--}0.4$ h/Mpc . However, the mass has to be conserved so the halos have to be compensated, which forces the one-halo term to vanish at very low k . There is no unique way to do this, since it depends on what we compensate against. The compensation is by definition a two-halo term, since the mass is being compensated by the particles outside the virial radius. Here we compensate against Zel'dovich and demand that the total agrees with SPT, which then automatically enforces mass and momentum conservation.
- (4) More specifically, we will assume that 1) can be connected to 2) and 3) at some low k , which is low enough that SPT can be assumed to be valid, yet large enough to still be close to the scale which dominates the compensation. We will match the two on this scale where both descriptions are valid, and use 2) + 3) at higher k . Since we believe that the Zel'dovich approximation is a good starting point for any modeling we will include it as one ingredient

of the theory, and decompose the power spectrum and correlation function into two parts,

$$P(k) = P_{\text{Zel}} + P_{\text{BB}}, \quad \xi(r) = \xi_{\text{Zel}}(r) + \xi_{\text{BB}}(r). \quad (1)$$

Here the subscript Zel stands for Zel'dovich and *BB* stands for broadband beyond Zel'dovich [5], which is our one-halo term. While there may be a residual BAO wiggle signature that is not captured by Zel'dovich, it is essentially negligible in the power spectrum and at most a few percent in the correlation function around BAO (100 Mpc/h), probably too small to be observed by existing or future redshift surveys due to large sampling variance errors. Here we will thus focus on the modeling of the broadband one-halo component and ignore the wiggle part. Note that we do not assume that the Zel'dovich part is uncorrelated with the one-halo term. In this sense our one halo term is not the so-called stochastic term uncorrelated with Zel'dovich.

As we argued the key physics ingredient missing in PT is the halo formation, which leads to a large contribution from the near-zero lag correlations, also called the one-halo term in the halo model [10–13]. The halo model postulates that the nonlinear evolution leads to halo formation, and that all the dark matter particles belong to collapsed halos, with the halo mass distribution given by the halo mass function $dn(M)$. The correlations between the dark matter particles can be simply split into correlations within the same halo, the one-halo term, and between halos, the two-halo term. On large scales the latter reduces to linear theory $P_L(k)$. The one-halo term in the halo model has a simple physical interpretation on small scales, which is that all dark matter particles are inside the dark matter halos distributed with a radial halo density profile, and this leads to a power spectrum that is simply an integral over the halo mass function times the Fourier transform of the halo profile squared. The halo profile has a compact support, extending out to roughly the virial radius, and its correlation function is a convolution of the profile with itself, extending to roughly twice that. In the power spectrum the convolution becomes a square of the Fourier transform of the profile, which can be expanded as a series of even powers of k [14],

$$P_{\text{BB}}(k) = F(k)A_0(1 - R_{1h,2}^2 k^2 + R_{1h,4}^4 k^4 + \dots). \quad (2)$$

Here the parameters $A_0 R_{1h,n}$ have a specific interpretation in terms of the integrals over the halo mass function $n(M)$ times the halo mass M squared, and times $2n$ moments of the halo radius averaged over the halo density profile [14]. Specifically, $A_0 = \bar{\rho}^{-2} \int M^2 dn(M)$ is just a weighted halo mass squared divided by the mean density $\bar{\rho}$ and does not depend on the halo density profile. These arguments however do not yet account for the halo compensation.

Above we introduced $F(k)$, which is the compensation function, which is required to vanish in the $k \rightarrow 0$ limit as long as the two-halo term converges to linear theory in the same limit: mass conservation requires that the leading nonlinear one-halo term cannot be a constant A_0 [14]. Thus the one-halo term has to be generalized to include mass compensation effects: nonlinear effects cause the dark matter to collapse into dark matter halos, bringing in mass from larger scales, so it has to be compensated by a mass deficit at large scales to satisfy the mass (and momentum) conservation. Because of this one can show that the one-halo term has to scale as k^4 at low k [10,15,16].

The two-halo term can also be expressed as a convolution of the linear theory over the halo profiles, and the resulting Taylor expansion is given by a similar series

$$P_{2h}(k) = P_L(k)(1 - k^2 R_{2h,2}^2 + k^4 R_{2h,4}^4 \dots). \quad (3)$$

The leading-order correction scales as $k^2 P_L(k) R_{2h,2}^2$, where $R_{2h,2}$ is also related to an average second moment of the halo density profile, although with a different mass and halo bias weighting [10]. Note that this gives at the leading-order correction the usual EFT term [8]. It is clear that both the one-halo and two-halo Taylor expansions break down for $k > R_{h,2}^{-1}$. The breakdown of the two-halo term does not matter: at the relevant k the correlations are dominated by the one-halo term. For the latter however, a different expansion is advantageous, as we discuss below.

In this paper we argue that the natural way to connect SPT to small-scale nonlinear effects is in the context of the Zel'dovich approximation plus a compensated one-halo term. In this picture we can think of P_{Zel} as the leading-order two-halo term, and P_{BB} as the one-halo term. The motivation for this is that the Zel'dovich correlation function is almost exact for $r > 5$ Mpc/h, and that the correction relative to it is negative, suggesting a compensation of a nonlinear term $\xi_{\text{BB}}(r)$. All the corrections to the Zel'dovich model thus go into the compensation term $F(k)$. In the halo model these corrections would arise from the compensation of the halo term and from two-halo term correlations of particles inside the halos. We do not try to separate these into the latter that one expects to scale as $k^2 P_L(k)$ at low k , and the former that scales as k^4 at very low k . In general it is difficult to do this separation, as both of these arise from the two-halo term correlations. It is also not clear that in the regime where it matters ($k > 0.1$ h/Mpc) these low- k expansions still apply. We will however use them at very low $k \sim 0.02$ h/Mpc, where we expect SPT to be valid. However, it should be clear that the term P_{BB} is not just the one-halo term in the traditional sense, because of these compensation corrections at low k .

In Ref. [14] this function $F(k)$ was simply fitted using a polynomial form. We will begin by keeping this function

completely general, and then choosing a very simple form for it. Our one-halo term, and thus the compensation form of $F(k)$, is defined relative to the Zel'dovich term. If we adopted a different form for the two-halo term we would obtain a different form of $F(k)$. In principle, the Zel'dovich approximation itself could contain some halos with the correct mass and so a part of what we usually call the one-halo term could already be contained there. However, we will see that this must be a minor effect and the one-halo term we derive agrees with the expected value from the halo model.

While the one-halo expansion in even powers of k above works in Fourier space up to the virial radius scale of order $k \sim R_{1h,2}^{-1} \sim 1$ h/Mpc, it breaks down above that. Moreover, powers of k diverge at high k and do not have a well-defined Fourier transform, making this form unsuitable for correlation function predictions without resumming it first. Instead we will use in this paper the Padé series ansatz

$$P_{BB} = A_0 F(k) \frac{1 + \sum_{m=1}^{n_{\max}-1} (kR_m)^{2m}}{1 + \sum_{n=1}^{n_{\max}} (kR_{nh})^{2n}}. \quad (4)$$

By requiring the series in the denominator to run to a higher power than in the numerator we guarantee that the series does not diverge at high k and has a finite Fourier transform. Here we will explore the truncation of the series at $n_{\max} = 0, 1, 2$. In terms of the halo model A_0 has the same interpretation as before; it is the halo mass squared averaged over the halo mass function. It is the only quantity that has units of power spectrum; all the other parameters have units of length. For the R_m and R_{nh} parameters we expect that they will be related to a typical scale of the halos. There are several halo scales one can define. One is the scale radius R_s , defined as the scale where the slope of the density profile is -2 . Another scale we can define is the virial radius $R_{\text{vir}} = cR_s$, where c is the concentration parameter with values around 3–4 for the most massive halos and increasing towards less massive halos [17]. The mean overdensity at R_{vir} is 200 by definition, while the typical mean overdensity at R_s can be of the order of 1000 or more. In the halo model approach we integrate over these with the halo mass function, and the number of pairs for each is proportional to M^2 , which gives most of the weight to the very massive halos. As a result, if we use $n_{\max} = 1$ and only have one scale parameter, which we denote R_{h1}^0 , then we find that the typical scale, when averaged over all halos, is of the order of 1 Mpc/h at $z = 0$, a typical scale of a cluster.

While the halo model has been successful as a phenomenological model, its connection and consistency with PT has not been explored. In this paper we propose a halo Zel'dovich model applied to perturbation theory (HZPT) approach, in which we connect the halo model to PT in the regime where both can be expected to be approximately valid. We take the approach that one-loop SPT has a regime

of validity on very large scales and gives us the correct description of the onset of nonlinearity. This is not guaranteed by SPT: the one-loop SPT may receive contributions from small scales which are nonlinear and thus not reliably computed. For CDM-type power spectra, the integrals are convergent and for sufficiently high redshift all of the one-loop integral contributions come from linear scales, the prediction is reliable, and no EFT correction is needed. For now we will simply assume there are no corrections to SPT at low k , and return to this discussion later.

Let us therefore assume that one-loop SPT is correct at very low k , and that it can be matched to the halo model ansatz in the equation in the regime of its validity. The low- k limit of the Zel'dovich approximation is [5] $P_{\text{Zel}} = (1 - k^2 \sigma_L^2 + k^4 \sigma_L^4/2)P_L + Q_3/2$, where P_L is the linear power spectrum, $\sigma_L^2 = 1/(6\pi^2) \int dq P(q)$ is the square of the linear displacement field dispersion and $Q_3 = 1/(10\pi^2)k^4 \int dq P^2(q)/q^2 = C_3 k^4$ is a mode coupling integral, as defined in Ref. [6]. It has been shown in Ref. [5] that this expansion is valid for $k < 0.1$ h/Mpc. Note that we kept terms beyond one loop in the Zel'dovich approximation. One-loop SPT can be written as $P_{\text{SPT}} = P_L + P_{13} + P_{22}$, which at low k is $P_{13} = -61/105 k^2 \sigma_L^2 P_L$ and $P_{22} = 45 Q_3/98$.

Let us begin by first dropping the two-loop term $k^4 \sigma_L^4 P_L/2$ from the Zel'dovich approximation. Then all of the one-loop terms scale as the square of the power spectrum. Matching at low k gives

$$P_{\text{SPT}}(k) - P_{\text{Zel}}(k) = \frac{44}{105} k^2 \sigma_L^2 P_L - \frac{2}{49} Q_3 = A_0 F(k), \quad (5)$$

where the linear order cancels in the difference and we have dropped all higher-order terms since they are negligible at low k . What does this imply for the amplitude dependence of A_0 and $F(k)$? We will adopt the standard σ_8 normalization for the amplitude of fluctuations, where $\sigma_8(z)$ is the rms fluctuation of spheres of radius of 8 Mpc/h, and which is redshift dependent. Often this is phrased in terms of the redshift dependence of the growth factor $D(z)$, and in linear theory we would write $\sigma_8(z) = D(z)\sigma_8(z=0)$. But the amplitude dependence is more general than the redshift dependence, since it encompasses the idea that changing the redshift or changing the amplitude should give the same result in the context of PT. Since both SPT and Zel'dovich at low k scale as a square of the power spectrum, which itself scales as σ_8^2 , requiring Eq. (5) to be valid over a broad range of k where $F(k)$ is rapidly changing, there can only be one solution to the amplitude dependence, $A_0 \propto \sigma_8^4$ and $F(k) = \text{const}$. This simple result is in very close agreement with simulations spanning a wide range of redshifts and models [14], where the slope of 3.9 was derived. We will see below that including the full Zel'dovich instead of its lowest order

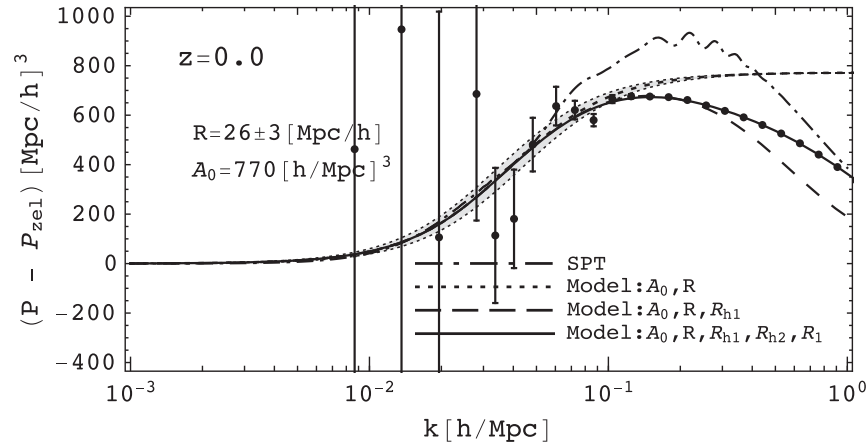


FIG. 1. The HZPT model for $n_{\max} = 0, 1, 2$ (dotted, dashed and solid lines respectively), together with SPT (dot-dashed line) are shown as a function of k . We match SPT to HZPT around $k \sim 0.02$ h/Mpc to derive A_0 and R , while the remaining parameters of our model (R_1, R_{1h}, R_{2h}) are fitted to simulations (points) at scales $k > 0.1$ h/Mpc. The gray band shows the change in the HZPT, $n_{\max} = 0$ model in the case where the R value varies for ± 3 Mpc/h. This plot is for $z = 0$, and higher z versions are similar. Simulation points are taken from Ref. [5], and are for the $\sigma_8 = 0.807$ Lambda-CDM model.

further improves the agreement on the slope. Note that this is valid for a general compensation function $F(k)$.

To proceed we need to assume a specific functional form for the compensation term. A simple way to achieve general compensation is to use $F(k) = 1 - 1/(1 + k^2 R^2)$, which has an analytic Fourier transform and vanishes as $k \rightarrow 0$, so that the overall model for the one-halo term is

$$P_{BB} = A_0 \left(1 - \frac{1}{1 + k^2 R^2} \right) \frac{1 + \sum_{m=1}^{n_{\max}-1} (kR_m)^{2m}}{1 + \sum_{n=1}^{n_{\max}} (kR_{nh})^{2n}}. \quad (6)$$

The parameter R governs the transition of the one-halo term to 0 at low k : this is the compensation scale parameter, and we expect it to be very large compared to the typical halo size. Here we simply chose the simplest form for this function $F(k) = (1 - 1/(1 + k^2 R^2))$, but we do not expect this to be the correct form at all k , since, as seen from Eq. (5), the residual between SPT and Zel'dovich at low k is not k^2 , but $k^2 P_L$. We evaluate SPT and Zel'dovich in the low- k limit and fit for the parameters A_0 and R . We find that there is a range of k where the fit is good (Fig. 1) and we use $0.02 < k < 0.05$ h/Mpc for the fits. The fit is not perfect, a consequence of the simplified form of $F(k)$ term, but it is a good fit for $0.02 < k < 0.05$ h/Mpc. Even for the more general forms of $F(k)$ one can still define a typical compensation scale on which $F(k)$ goes from unity to zero. The current form of compensation is sufficient for our purposes and we will not explore more general forms. We find

$$\begin{aligned} A_0 &= 750 \left(\frac{\sigma_8(z)}{0.8} \right)^{3.75} (\text{h/Mpc})^3 \\ R &= 26.0 \left(\frac{\sigma_8(z)}{0.8} \right)^{0.15} (\text{Mpc/h}). \end{aligned} \quad (7)$$

This is a remarkably simple result. Moreover, it agrees well with the recent numerical determination of the A_0 amplitude from a suite of 38 emulator simulations at different redshifts in Ref. [14], where a scaling $A_0 = 670(\sigma_8/0.8)^{3.9} (\text{Mpc/h})^3$ has been derived over the redshift range 0–1. The amplitude is a bit different because the form of compensation used in this paper is a bit different than in Ref. [14], but they both provide an equally good fit to the simulations, as shown in Fig. 1. The two parameters are correlated, and the parameter R is less well determined than A_0 : R comes with a 5 times larger relative error than the error on A_0 . Figure 1 shows the band over which R is varied by 12%: a reasonable estimate of its error is 3–5% (and the error on A_0 is 1% or less). We find the slopes of A_0 and R to be uncertain at the 0.1 level. The value of the amplitude also agrees very well with the expected amplitude of the one-halo term in the halo model, which is $\bar{\rho}^{-2} \int M^2 dn$, where M is the halo mass, dn/dM is the halo mass function and $\bar{\rho}$ is the mean density of the Universe. This suggests that the Zel'dovich approximation by itself does not contribute much to the one-halo term.

To qualitatively derive a value for R and A_0 let us look at the low- k limit of SPT, focusing on the leading-order k^2 and k^4 terms at the peak of the power spectrum around $k_{\text{peak}} \sim 0.02$ h/Mpc, where $P_L(k_{\text{peak}}) \sim 27000(\sigma_8/0.8)^2 (\text{h/Mpc})^3$ and $\sigma_L^2 \sim 36(\sigma_8/0.8)^2 (\text{Mpc/h})^2$. Since the difference between Zel'dovich and SPT for low k gives $(44/105) k^2 \sigma_L^2 P_L$, by equating that to $k^2 A_0 R^2$ [the low- k limit of Eq. (5)] we derive $R \sim (0.42 \sigma_L^2 P_L(k_{\text{peak}})/A_0)^{1/2} \sim 24$ Mpc/h for the best-fit value of A_0 , in qualitatively good agreement with the value of 26 Mpc/h derived numerically. Thus the value of R is determined by the linear power spectrum amplitude at the peak, the rms displacement field and the amplitude of the one-halo term A_0 . To determine both A_0 and R we need to expand SPT and Zel'dovich up to k^4

around $P_L(k_{\text{peak}})$, for which we also need to numerically evaluate $C_3 \sim 2 \times 10^9$ Mpc/h. Matching k^4 terms gives $[(1/2 - 45/98)C_3 + (\sigma_L^2)^2 P_L/2] = A_0 R^4$, which, when combined with the k^2 term gives $R \sim 20$ Mpc/h and $A_0 \sim 700$ (Mpc/h)³ at $z = 0$. We note further that there is a considerable contribution beyond k^4 from P_{13} already around $k \sim 0.02$ h/Mpc, and in the numerical fits there is some correlation between A_0 and R .

The Zel'dovich term beyond one loop, $(\sigma_L^2)^2 P_L/2$, cannot be neglected compared to the rest of the k^4 terms even for $k < 0.05$ h/Mpc, and as a consequence the scaling of $A_0 \propto \sigma_8^4$ and $R = \text{const}$ is mildly broken. This term is larger for low redshifts, explaining why the slope of σ_8 scaling is less than 4.0 and closer to 3.8. Remarkably, that is exactly what simulations suggest. Given the uncertainties in the form of $F(k)$ we cannot address in detail the remarkably small scatter of A_0 against the amplitude $\sigma_{11.3}$ when the shape of the power spectrum is varied [14].

The leading order for our compensation term of the one-halo $F(k)$ is k^2 . It is often stated that the mass and momentum conservation effects generate the k^4 tail [18], and so one would naively expect the leading term of one halo to be k^4 . This term is generated by P_{22} in SPT. However, nonlinear evolution also leads to propagator effects contained in P_{13} , which at the leading order give $k^2 P_L$ corrections to the linear theory and we have seen that these effects dominate at low k . We have assumed the two-halo term to be the Zel'dovich approximation, which contains part of the $k^2 P_L$ term in SPT but not all, and so the difference is still given by the $k^2 P_L$ term at the leading order, which has to be the leading-order one-halo term, and happens to coincide with k^2 at the peak of the power spectrum around $k \sim 0.02$ h/Mpc, where we fit to our ansatz. As discussed above, we could use a more general form of $F(k)$ that would make the compensation and two-halo terms exact, but we found that this makes no practical difference to the final results. It is also possible to make a different two-halo ansatz where $k^2 P_L$ is entirely cancelled; for example, our two-halo ansatz could be simply P_{13} of SPT, or its nonlinear propagator version [18]. As shown in Ref. [18], this ansatz leads to a one-halo term that is several times larger than expected in the halo model. It is possible to generate a valid halo model based on a different ansatz for the two-halo term, but we do not pursue this further here.

III. POWER SPECTRUM PREDICTIONS OF HZPT

In this section we fit higher-order parameters of the one-halo term to obtain the best possible agreement against simulations. The best fits for these parameters as powers of the amplitude $\sigma_8(z)$ give

$$\begin{aligned} R_{1h}^0 &= 1.87 \text{ Mpc/h} \left(\frac{\sigma_8(z)}{0.8} \right)^{-0.47}, \\ R_{1h} &= 3.87 \text{ Mpc/h} \left(\frac{\sigma_8(z)}{0.8} \right)^{0.29}, \\ R_1 &= 3.33 \text{ Mpc/h} \frac{\sigma_8(z)^{0.88}}{0.8}, \\ R_{2h} &= 1.69 \text{ Mpc/h} \left(\frac{\sigma_8(z)}{0.8} \right)^{0.43}. \end{aligned} \quad (8)$$

Here R_{1h}^0 refers to the $n_{\text{max}} = 1$ case, while R_{1h} , R_{2h} and R_1 refer to the $n_{\text{max}} = 2$ case. Even though we only fit to one set of simulations, we expect that these parameters are nearly universal and apply well to all cosmological models, just as in the case of the halo plus Zel'dovich model of Ref. [14]. The main difference relative to Ref. [14] is the form of the compensation function $F(k)$, which was fitted to a tenth-order polynomial in Ref. [14], while here we adopt a much simpler form of Eq. (6), and the expansion of the one-halo term, which was in even powers of k in Ref. [14], while we use the Padé expansion here.

In Fig. 2 we show HZPT for $P(k)$ against the N -body simulations for four different redshifts. We find that the model with $n_{\text{max}} = 1$ can fit the simulations at 1% up to $k \sim 0.3$ h/Mpc, while for $n_{\text{max}} = 2$ the fits are good to 1% up to $k \sim 1$ h/Mpc. The parameter R ensures the low- k behavior of the model, while A_0 sets the peak amplitude of P_{BB} which is around $k \sim 0.12$ h/Mpc for all redshifts. In Ref. [19] it was argued that SPT is a relatively good description against N -body simulations for $z > 3$. This is consistent with our results: as shown in Fig. 2, for $z = 2$ SPT differs from simulations by only 2%, and this difference is presumably even smaller for higher z .

We have established that one-loop SPT can determine A_0 and R . Which other coefficients of Eq. (6) can PT determine? Since the coefficients R_{nh} , R_n , $n > 0$, are determined by a typical halo scale averaged over the halo profile and averaged over the halo mass function, they depend on the regime where the overdensity is very large: the halo virial radius R_{vir} is defined at a mean overdensity of 200, and R_s is even smaller (with the correspondingly larger mean overdensity). These cannot be computed from standard PT, which is only supposed to work in the regime where $\delta < 1$. One can show (Fig. 2) that these terms become important at the percent level around $k \sim 0.2$ h/Mpc at $z = 0$. It seems unlikely that PT can make predictions at the percent level for $k > 0.2$ h/Mpc, regardless of which PT formalism we use. It is however remarkable that one-loop SPT can predict the amplitude of the one-halo term A_0 . This is possible because A_0 depends on the total halo mass that has nonlinearly collapsed and not on its profile. At low k the nonperturbative halo profile effects also give rise to the two-halo correction term of order $k^2 R_{2h}^2 P_L$, where $R_{2h} \sim 1$ Mpc/h. This is the EFT term of Ref. [8]. At low k this term is a

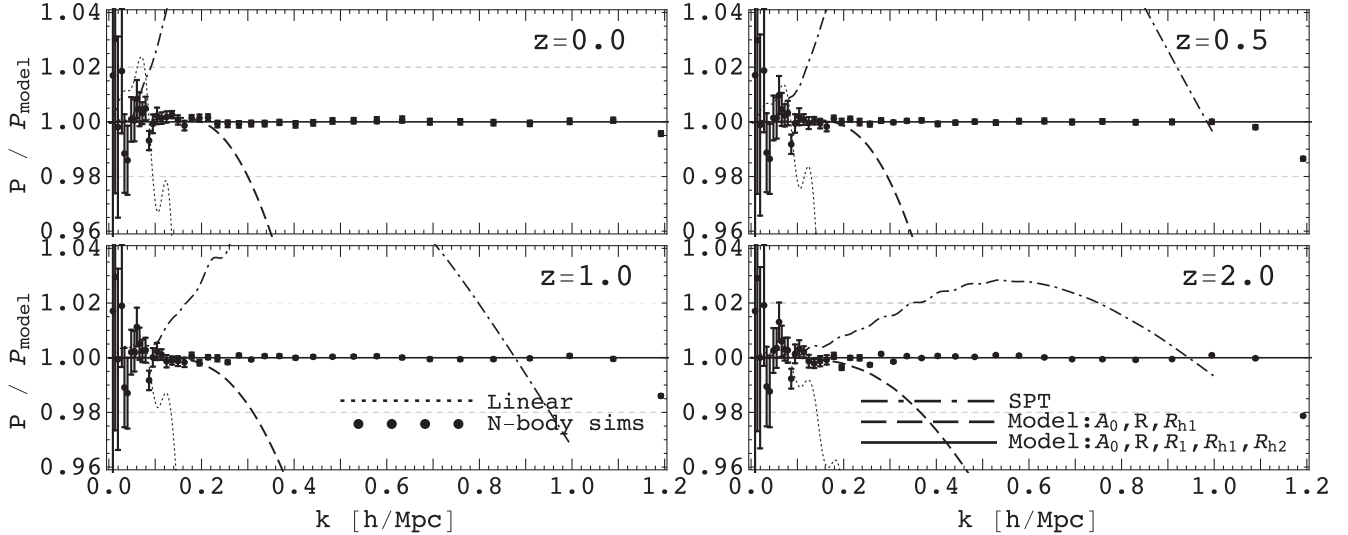


FIG. 2. The simulation power spectrum (points) is shown relative to the HZPT model prediction (P_{model}) with $n_{\text{max}} = 2$ (black solid line) at four different redshifts ($z = 0.0, 0.5, 1.0$ and 2.0). Also shown are SPT (dot-dashed line), HZPT with $n_{\text{max}} = 1$ (dashed line) and linear theory (dotted line). Simulation points are taken from Ref. [5], and are for the $\sigma_8 = 0.807$ Lambda-CDM model.

small, few-percent, correction to the SPT term, which is of course small compared to linear theory. At higher k this term is modified and in our model it is absorbed into the overall compensated one-halo term P_{BB} .

IV. CORRELATION FUNCTION PREDICTIONS OF HZPT

We would like to require from a good PT model that it works both in Fourier space and in configuration space, but so far there has been no successful model achieving this. Typically, the Zel'dovich approximation works quite well in the correlation function but fails in the power spectrum, while SPT does not give very good correlation function predictions, especially around BAO. In the HZPT approach, we expect the Zel'dovich term to dominate the correlation function at large radii. In the absence of compensation the one-halo term would be limited to scales around twice the virial radius and below. With compensation these effects extend to large radii, but as we will show, remain small. There is thus a crucial difference of the effect of the one-halo term between the correlation function and the power spectrum: the one-halo term is mostly a few-percent effect for $r > 5$ Mpc/h in the correlation function, caused by compensation effects. On the other hand, the one-halo term can be very large for $k > 1/R \sim 0.04$ h/Mpc in the power spectrum and dominates $P(k)$ for $k > 0.2$ h/Mpc.

An advantage of the ansatz in Eq. (6) is the existence of an analytical Fourier transform for low n_{max} . On scales $r \gg R_{\text{halo}}$, and assuming $R \gg R_{nh}, R_n$, one can use $n_{\text{max}} = 0$ and find

$$\xi_{BB}(r) = -\frac{A_0 e^{-\frac{r}{R}}}{4\pi r R^2}. \quad (9)$$

Similarly, if we keep the leading R_{h1} effects in the $n_{\text{max}} = 1$ case we get

$$\xi_{BB}(r) = -\frac{A_0 e^{-\frac{r}{R}}}{4\pi r R^2} \left(1 - \left(\frac{R}{R_{h1}} \right)^2 \exp \left[-\frac{R + R_{h1}}{R R_{h1}} r \right] \right). \quad (10)$$

The first equation effectively reduces the low- k model that we started with in Fourier space to a model in the configuration space with the same two parameters, A_0 and R , that have been determined using SPT. Since the virial radius of the largest halos is about 2 Mpc/h at $z = 0$, we expect the transition between the two regimes to be around 4 Mpc/h. The results for the correlation function are shown in Fig. 3. We see that our model significantly improves upon other PT results, achieving 1–2% agreement down to 5 Mpc/h at all redshifts. A possible exception is the BAO wiggle, $r > 80$ Mpc/h, where there may be an additional wiggle contribution that was discussed in Ref. [5], but it is unclear whether it is real given the large sampling variance fluctuations. It also improves upon the Zel'dovich approximation, which is already very good by itself, with only few-percent deviations from simulations over this range. Our model reduces the correlation function relative to Zel'dovich, as expected by the compensation effects, which take mass from large scales to enhance one-halo terms on small scales. As expected our results agree with SPT on large scales, but only away from BAO, making the range where SPT agrees with simulations and our model at the 1% level only around 50–70 Mpc/h.

Given that the one-halo contribution relative to Zel'dovich is a few percent only, any further corrections have a very small effect on $\xi(r)$ for $r > 5$ Mpc/h. For example, one can improve the agreement of the model with simulations somewhat by increasing R . We can do this at the few-percent level since this is the formal error from the fits to SPT. In the context of our approach, increasing R

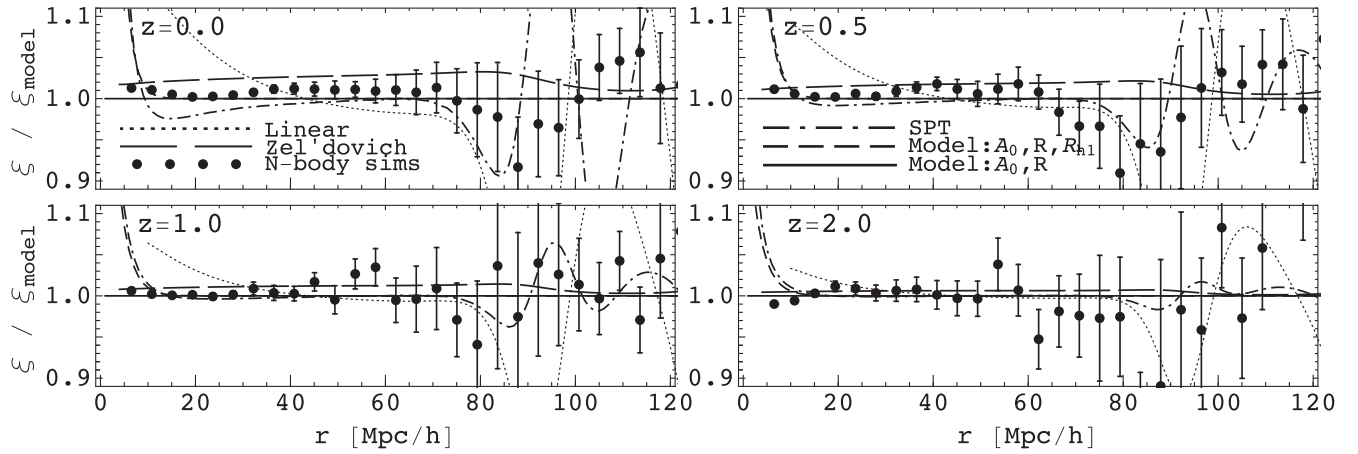


FIG. 3. The simulation correlation function (points) is shown relative to the HZPT model prediction (ξ_{model}) with $n_{\text{max}} = 0$ (black solid line) at four different redshifts ($z = 0.0, 0.5, 1.0$ and 2.0). Note that the HZPT ξ_{model} with $n_{\text{max}} = 0$ is completely determined with the A_0 and R parameters obtained from the low k of the SPT power spectrum. Also shown are HZPT with $n_{\text{max}} = 1$ (dashed line), Zel'dovich (long dashed line), SPT (dot-dashed line), and linear theory (dotted line). Simulation points are taken from Ref. [5], and are for the $\sigma_8 = 0.807$ Lambda-CDM model.

beyond its SPT predicted value at a level more than this would only be possible if there are nonperturbative or two-loop corrections at low k , since an assumption of our model is that one-loop SPT is the correct theory at low k . Note that EFT corrections as advocated by Ref. [8] will reduce $A_0 R^2$, which goes in the opposite direction. Nevertheless, as discussed above, there is no guarantee for SPT to be true even for very low k : the loop integrals can extend into the regime where density perturbations exceed unity. At $z = 0$, for CDM models, this happens around $k_{\text{nl}} \sim 0.1\text{--}0.2$ h/Mpc. The low- k expansion of SPT is dominated by $k^2 \sigma_L^2 P_{\text{lin}}(k)$ terms, which depend only on the convergence of the σ_L^2 integrals. This converges to about 90% of the value for $q \lesssim k_{\text{nl}} \sim 0.2$ h/Mpc at $z = 0$, and converges even more to its full value for $z > 0$ where $k_{\text{nl}} > 0.2$ h/Mpc [5]. So we expect the one-loop SPT to be almost perfectly valid at low k for high redshifts, but there may be low-redshift corrections at the several-percent level, which may allow for additional changes in R or A_0 beyond the predicted value at a comparable level. In addition, our form of the compensation term is just an assumed ansatz, which could be modified for a better agreement.

A related question is whether one should include higher-loop contributions. The next order in PT is the two-loop SPT or, similarly (although not equivalently), one-loop LPT. In both cases the corrections to one-loop SPT can become important at low redshifts, even at low k . For example, at $z = 0$ the two-loop SPT correction to P_{13} is about 15% [5], and since the relative correction of the two-loop to one-loop scales as σ_8^2 , the correction is much smaller at higher redshifts. However, higher-loop contributions are likely to be grossly overestimated in PT. For example, a comparison against simulations suggests that the one-loop LPT contribution to the rms displacement is in reality almost entirely suppressed, such that the total

nonlinear value of σ_{NL}^2 differs by only 1–2% relative to the linear value σ_L^2 at all redshifts [5,20,21]. Physically this can be understood by the process of halo formation, which stops particles from displacing on small scales, and instead traps them inside the dark matter halos: the large-scale displacements, which are one loop in SPT, are correctly predicted, while the small-scale displacements, which are two loop and higher in the SPT sense, are strongly suppressed. It thus seems better to drop two-loop terms entirely, although we have no formal proof of this statement.

In summary, formally one cannot exclude corrections at low k , which will be of the EFT form $k^2 P_L$, but these are likely to be of the order of a few percent only. We see no need for such corrections in our approach: our model is accurate at the current precision of simulations. We thus argue that one-loop SPT is close to the correct theory for $k < R^{-1}$, but this is not a result that can be formally derived.

V. CONCLUSIONS

In this paper we developed a model for the dark matter power spectrum and correlation function that is 1% accurate for both, and that is based on PT as much as possible. We argued that PT approaches to large-scale structure can only have a hope of being valid for very large scales, $k < 0.05$ h/Mpc, a regime that we usually do not focus on when comparing PT to simulations, since deviations from linear theory are very small there. We also argued that the Zel'dovich approximation is a useful starting point for any halo-based model, and that halo formation has to be an essential part of the model. We proposed a model which we called the halo Zel'dovich PT model, in which the Zel'dovich approximation is supplemented with the one-halo term, and the sum of the two is

connected to one-loop standard PT at low k . Within this model we derived the one-halo term amplitude $A_0 = 750 (\text{Mpc}/h)^3 \sigma_8^{3.8}$, which agrees with simulations both in amplitude and in the σ_8 scaling.

The one-halo term needs to be compensated by the other halos for the mass conservation, and there are nonlinear contributions from two-halo correlations, both of which we modeled by using a very simple functional form. This compensation scale of the one-halo term has effects on the power spectrum at the percent level or smaller, but we have argued that it is essential in order to have a self-consistent model that connects the halo model to PT: its introduction gave us one-percent accuracy on both the power spectrum and the correlation function. In particular, the deviations of the correlation function of simulations from Zel'dovich is negative and a few percent only, and this term explains its origin. We have argued that the regime where SPT is valid in the power spectrum, is at best limited to $k \sim 1/R \sim 0.04 \text{ h}/\text{Mpc}$, while for higher k one-halo terms [generalized by the two-halo term corrections encoded in $F(k)$] begin to dominate. It is not possible to formally exclude the presence of nonperturbative or higher-loop correction terms even at very low k , but we see no need to consider them and they are likely at most several percent for our Universe. These terms will also generate a correction to the BAO wiggles [5] that we have ignored in this paper.

We have proposed a Padé-type expansion of the one-halo term as a useful functional form that allows one to go beyond the convergence radius of the Taylor expansion. We have argued that the value of this radius is around 1 Mpc/h, a typical virial radius of halos properly averaged over the halo mass function, and hence the Padé expansion is necessary if one wants a valid description for $k > 1 \text{ h}/\text{Mpc}$. Our approach is similar to the treatment of nonlinear redshift-space distortions (the so-called Fingers of God, FoG), which also require one to have a valid expression for $k > 0.2 \text{ h}/\text{Mpc}$ (the FoG scale is typically 5 Mpc/h). In the context of FoG a Lorentzian distribution is often used, which is just the Padé series at the first order. Padé expansion also has the advantage of being convergent in both the power spectrum and the correlation function, making the Fourier transforms calculable. With this expansion, and keeping terms up to second order, we were able to match the power spectrum to better than 1% against simulations, up to $k = 1 \text{ h}/\text{Mpc}$. The correlation

functions also agree against simulations to this accuracy down to 5 Mpc/h. We expect that a Padé ansatz for the one-halo term will be useful in modeling other correlation functions as well, such as galaxy-dark matter and galaxy-galaxy correlations. In principle a Padé expansion would also be needed for the two-halo terms that scale as P_L , but in practice the two-halo term is irrelevant on scales where this would make any difference ($k \sim 1 \text{ h}/\text{Mpc}$), so a simple Taylor expansion giving rise to $k^2 P_L \dots$ terms suffices.

We have argued that the predictive power of PT beyond Zel'dovich has been reduced to two numbers, A_0 and R . To improve the model further one needs to provide information on the halo profiles in the deeply nonlinear regime, which is unlikely to be predictable by PT. It has been argued in Ref. [14] that these coefficients are also not predictable by N -body simulations, due to the baryonic effects, so PT is not necessarily inferior to N -body simulations. For example, there are baryons inside the dark matter halos in the form of gas and stars and these can redistribute the matter inside halos beyond what the N -body simulations can predict. Baryon gas has pressure and this already changes the total matter profiles significantly, and even more dramatic effects arise from some feedback models where gas is pushed out of the halo center, possibly even dragging dark matter along [22]. These processes will change parameters associated with the halo profile, such as R_{nh} and R_n (the models of Ref. [22] suggest that these coefficients change at a level of 5–10% [14]), but because of mass conservation A_0 changes a lot less [14]. Moreover, most of the cosmological information content is already in the Zel'dovich term and A_0 [14]. So while HZPT, and PT in general, may only be able to determine two numbers beyond the Zel'dovich approximation, this may also be all that can be reliably extracted from N -body simulations at the two-point function level.

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