# Kinetically modified nonminimal chaotic inflation

Constantinos Pallis
*Departament de Física Teòrica and IFIC, Universitat de València-CSIC, E-46100 Burjassot, SPAIN*
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We consider *supersymmetric* (SUSY) and non-SUSY models of chaotic inflation based on the $\phi^n$ potential with $2 \leq n \leq 6$. We show that the coexistence of a nonminimal coupling to gravity $f_{\mathcal{R}} = 1 + c_{\mathcal{R}}\phi^{n/2}$ with a kinetic mixing of the form $f_{\rm K} = c_{\rm K} f_{\mathcal{R}}^m$ can accommodate inflationary observables favored by the BICEP2/*Keck Array* and *Planck* results for $0 \leq m \leq 4$ and $2.5 \times 10^{-4} \leq r_{\mathcal{R}\rm K} = c_{\mathcal{R}}/c_{\rm K}^{n/4} \leq 1$, where the upper limit is not imposed for $n = 2$. Inflation can be attained for sub-Planckian inflaton values with the corresponding effective theories retaining the perturbative unitarity up to the Planck scale.

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## I. INTRODUCTION

It is well known [1–3] that the presence of a nonminimal coupling function

$$f_{\mathcal{R}}(\phi) = 1 + c_{\mathcal{R}}\phi^{n/2}, \tag{1}$$

between the inflaton $\phi$ and the Ricci scalar $\mathcal{R}$, considered in conjunction with a monomial potential of the type

$$V_{\rm CI}(\phi) = \lambda^2\phi^n/2^{n/2}, \tag{2}$$

provides, at the strong $c_{\mathcal{R}}$ limit with $\phi < 1$—in the reduced Planck units with $m_{\rm P} = M_{\rm P}/\sqrt{8\pi} = 1$, an attractor [3] towards the spectral index, $n_{\rm s}$, and the tensor-to-scalar ratio, $r$, respectively

$$n_{\rm s} \simeq 1 - 2/\hat{N}_\star = 0.965 \quad \text{and} \quad r \simeq 12/\hat{N}_\star^2 = 0.0036, \tag{3}$$

for $\hat{N}_\star = 55$ $e$-foldings with negligible $n_{\rm s}$ running, $a_{\rm s}$. Although perfectly consistent with the present combined BICEP2/*Keck Array* and *Planck* results [4,5],

$$n_{\rm s} = 0.968 \pm 0.0045 \quad \text{and} \quad r = 0.048^{+0.035}_{-0.032}, \tag{4}$$

$r$ in Eq. (3) lies well below its central value in Eq. (4) and the sensitivity of the present experiments searching for primordial gravity waves—for an updated survey see [6]. Nonetheless, this model—called henceforth "nonminimal chaotic inflation" (non-MCI)—exhibits also a weak $c_{\mathcal{R}}$ regime, with $\phi > 1$ and $c_{\mathcal{R}}$-dependent observables [3,7] approaching for decreasing $c_{\mathcal{R}}$'s their values within MCI [8]. Focusing on this regime, we would like to emphasize that solutions covering nicely the 1-$\sigma$ domain of the present data in Eq. (4) can be achieved, even for $\phi < 1$, by introducing a suitable noncanonical kinetic mixing $f_{\rm K}(\phi)$. For this reason we call this type of non-MCI "*kinetically modified*." Although a new parameter $c_{\rm K}$, included in $f_{\rm K}$, may take relatively high values within this scheme, no problem with the perturbative unitarity arises.

## II. NON-SUSY FRAMEWORK

Non-MCI is formulated in the *Jordan frame* (JF) where the action of $\phi$ is given by

$$\mathsf{S} = \int d^4x\sqrt{-\mathfrak{g}}\left(-\frac{f_{\mathcal{R}}}{2}\mathcal{R} + \frac{f_{\rm K}}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V_{\rm CI}(\phi)\right). \tag{5}$$

Here $\mathfrak{g}$ is the determinant of the background Friedmann-Robertson-Walker metric, $g^{\mu\nu}$ with signature $(+,-,-,-)$ and we allow for a kinetic mixing through the function $f_{\rm K}(\phi)$. By performing a conformal transformation [2] according to which we define the *Einstein frame* (EF) metric $\hat{g}_{\mu\nu} = f_{\mathcal{R}}g_{\mu\nu}$ we can write $\mathsf{S}$ in the EF as follows,

$$\mathsf{S} = \int d^4x\sqrt{-\hat{\mathfrak{g}}}\left(-\frac{1}{2}\hat{\mathcal{R}} + \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi} - \hat{V}_{\rm CI}(\hat{\phi})\right), \tag{6a}$$

where hat is used to denote quantities defined in the EF. We also introduce the EF canonically normalized field, $\hat{\phi}$, and potential, $\hat{V}_{\rm CI}$, defined as follows,

$$\frac{d\hat{\phi}}{d\phi} = J = \sqrt{\frac{f_{\rm K}}{f_{\mathcal{R}}} + \frac{3}{2}\left(\frac{f_{\mathcal{R},\phi}}{f_{\mathcal{R}}}\right)^2} \quad \text{and} \quad \hat{V}_{\rm CI} = \frac{V_{\rm CI}}{f_{\mathcal{R}}^2}, \tag{6b}$$

where the symbol $\phi$ as subscript denotes derivation *with respect to* (w.r.t) the field $\phi$. In the pure non-MCI [1–3] we take $f_{\rm K} = 1$ and so, as shown from Eq. (6b), the role of $f_{\mathcal{R}}$ in Eq. (1) is twofold: (i) it determines the canonical normalization of $\hat{\phi}$; and (ii) it controls the shape of $\hat{V}_{\rm CI}$ affecting thereby the observational predictions.

Inspired by Ref. [9,10], where noncanonical kinetic terms assist in obtaining inflationary solutions for $\phi < 1$, we liberate $f_{\mathcal{R}}$ from its first role above implementing it by a kinetic function of the form

$$f_{\rm K}(\phi) = c_{\rm K} f_{\mathcal{R}}^m \quad \text{where } c_{\rm K} = (c_{\mathcal{R}}/r_{\mathcal{R}\rm K})^{4/n}, \tag{7}$$

with $r_{\mathcal{R}\rm K}$ being introduced for later convenience. The form of $f_{\rm K}$ in Eq. (7) is chosen so that the perturbative unitarity is preserved up to Planck scale. Its most general form could be $f_{\rm K} = c_{\rm K}\tilde{f}$ with $\tilde{f}$ being an arbitrary function such that $\tilde{f}(\langle\phi\rangle = 0) = 1$—see below. However, the variation of $f_{\rm K}$ generated by $\tilde{f}$ can be covered by the parametrization of Eq. (7) selecting conveniently $m = \ln\tilde{f}/\ln f_{\mathcal{R}}$.

Plugging, finally, Eqs. (7) and (2) into Eq. (6b), we obtain

$$J^2 = \frac{c_{\rm K}}{f_{\mathcal{R}}^{1-m}} + \frac{3n^2 c_{\mathcal{R}}^2 \phi^{n-2}}{8 f_{\mathcal{R}}^2} \simeq \frac{c_{\rm K}}{f_{\mathcal{R}}^{1-m}} \quad \text{and} \quad \hat{V}_{\rm CI} = \frac{\lambda^2\phi^n}{2^{n/2} f_{\mathcal{R}}^2}, \tag{8}$$

assuming $c_{\rm K} \gg c_{\mathcal{R}}$. In contrast to Ref. [10] the presence of both $f_{\rm K}$ and $f_{\mathcal{R}}$ plays a crucial role within our proposal.

## III. SUPERGRAVITY EMBEDDINGS

The supersymmetrization of the above models requires the use of two gauge singlet chiral superfields, i.e., $z^\alpha = \Phi, S$, with $\Phi$ ($\alpha = 1$) and $S$ ($\alpha = 2$) being the inflaton and a "stabilized" field, respectively. The EF action for $z^\alpha$'s within *supergravity* (SUGRA) [11] can be written as

$$\mathsf{S} = \int d^4x\sqrt{-\hat{\mathfrak{g}}}\left(-\frac{1}{2}\hat{\mathcal{R}} + K_{\alpha\bar\beta}\hat{g}^{\mu\nu}\partial_\mu z^\alpha \partial_\nu z^{*\bar\beta} - \hat{V}\right), \tag{9a}$$

where summation is taken over the scalar fields $z^\alpha$, star (*) denotes complex conjugation, $K$ is the Kähler potential with $K_{\alpha\bar\beta} = K_{,z^\alpha z^{*\bar\beta}}$ and $K^{\alpha\bar\beta}K_{\bar\beta\gamma} = \delta^\alpha_\gamma$. Also $\hat{V}$ is the EF F–term SUGRA potential given by

$$\hat{V} = e^K\left(K^{\alpha\bar\beta}(D_\alpha W)(D^*_{\bar\beta}W^*) - 3|W|^2\right), \tag{9b}$$

where $D_\alpha W = W_{,z^\alpha} + K_{,z^\alpha}W$ with $W$ being the superpotential. Along the inflationary track determined by the constraints

$$S = \Phi - \Phi^* = 0, \quad \text{or} \quad s = \bar{s} = \theta = 0, \tag{10}$$

if we express $\Phi$ and $S$ according to the parametrization

$$\Phi = \phi e^{i\theta}/\sqrt{2} \quad \text{and} \quad S = (s + i\bar{s})/\sqrt{2}, \tag{11}$$

$V_{\rm CI}$ in Eq. (2) can be produced, in the flat limit, by

$$W = \lambda S\Phi^{n/2}. \tag{12}$$

The form of $W$ can be uniquely determined if we impose two symmetries: (i) an $R$ symmetry under which $S$ and $\Phi$ have charges 1 and 0 and (ii) a global $U(1)$ symmetry with assigned charges $-1$ and $2/n$ for $S$ and $\Phi$.

On the other hand, the derivation of $\hat{V}_{\rm CI}$ in Eq. (8) via Eq. (9b) requires a judiciously chosen $K$. Namely, along the track in Eq. (10), the only surviving term in Eq. (9b) is

$$\hat{V}_{\rm CI} = \hat{V}(\theta = s = \bar{s} = 0) = e^K K^{SS^*}|W_{,S}|^2. \tag{13}$$

The incorporation $f_{\mathcal{R}}$ in Eq. (1) and $f_{\rm K}$ in Eq. (7) dictates the adoption of a logarithmic $K$ [11], including the functions

$$F_{\mathcal{R}}(\Phi) = 1 + 2^{\frac{n}{4}}\Phi^{\frac{n}{2}}c_{\mathcal{R}} \quad \text{and} \quad F_{\rm K} = (\Phi - \Phi^*)^2. \tag{14a}$$

Here $F_{\mathcal{R}}$ is an holomorphic function reducing to $f_{\mathcal{R}}$, along the path in Eq. (10), and $F_{\rm K}$ is a real function which assists us to incorporate the noncanonical kinetic mixing generating by $f_{\rm K}$ in Eq. (7). Indeed, $F_{\rm K}$ leaves intact $\hat{V}_{\rm CI}$, since it vanishes along the trajectory in Eq. (10), but it contributes to the normalization of $\Phi$—contrary to the naive kinetic term $|\Phi|^2/3$ [11] which influences both $J$ and $\hat{V}_{\rm CI}$ in Eq. (6b). Although $F_{\rm K}$ is employed in Ref. [3] too, its importance in implementing nonminimal kinetic terms within non-MCI has not been emphasized so far. We also include in $K$ the typical kinetic term for $S$, considering the next-to-minimal term for stability reasons [11]—see below, i.e.

$$F_S = |S|^2/3 - k_S|S|^4/3. \tag{14b}$$

Taking for consistency all the possible terms up to fourth order, $K$ is written as

$$K = -3\ln\left(\frac{c_{\rm K}}{2^m 6}(F_{\mathcal{R}} + F^*_{\mathcal{R}})^m F_{\rm K} + \frac{1}{2}(F_{\mathcal{R}} + F^*_{\mathcal{R}}) - F_S + \frac{k_\Phi}{6}F_{\rm K}^2 - \frac{k_{S\Phi}}{3}F_{\rm K}|S|^2\right). \tag{15a}$$

Alternatively, if we do not insist on a pure logarithmic $K$, we could also adopt the form

$$K = -3\ln\left(\frac{1}{2}(F_{\mathcal{R}} + F^*_{\mathcal{R}}) - F_S\right) - \frac{c_{\rm K}}{2^m}\frac{F_{\rm K}}{(F_{\mathcal{R}} + F^*_{\mathcal{R}})^{1-m}}. \tag{15b}$$

Note that for $m = 0$ [$m = 1$] $F_K$ and $F_{\mathcal{R}}$ in $K$ given by Eq. (15a) [Eq. (15b)] are totally decoupled, i.e. no higher order term is needed. Our models, for $c_{\rm K} \gg c_{\mathcal{R}}$, are completely natural in the 't Hooft sense because, in the limits $c_{\mathcal{R}} \to 0$ and $\lambda \to 0$, the theory enjoys the following enhanced symmetries (cf. Ref. [12]),

$$\Phi \to \Phi^*, \qquad \Phi \to \Phi + c \quad \text{and} \quad S \to e^{i\alpha}S, \tag{16}$$

where $c$ is a real number. Therefore, the terms proportional to $c_{\mathcal{R}}$ can be regarded as a gravity-induced violation of the symmetries above.

To verify the appropriateness of $K$ in Eqs. (15a) and (15b), we can first remark that, along the trough in Eq. (10), it is diagonal with nonvanishing elements $K_{\Phi\Phi^*} = J^2$, where $J$ is given by Eq. (8), and $K_{SS^*} = 1/f_{\mathcal{R}}$. Upon substitution of $K^{SS^*} = f_{\mathcal{R}}$ and $\exp K = f_{\mathcal{R}}^{-3}$ into Eq. (13) we easily deduce that $\hat{V}_{\rm CI}$ in Eq. (8) is recovered. If we perform the inverse of the conformal transformation described in Eqs. (6a) and (5) with frame function $\Omega/3 = -\exp(-K/3)$ we end up with the JF potential $V_{\rm CI} = \Omega^2\hat{V}_{\rm CI}/9$ in Eq. (2). Moreover, the conventional Einstein gravity at the SUSY vacuum, $\langle S\rangle = \langle\Phi\rangle = 0$, is recovered since $-\langle\Omega\rangle/3 = 1$.

Defining the canonically normalized fields via the relations

$$d\hat{\phi}/d\phi = \sqrt{K_{\Phi\Phi^*}} = J, \qquad \hat{\theta} = J\theta\phi, \tag{17}$$

and $(\hat{s}, \hat{\bar{s}}) = \sqrt{K_{SS^*}}(s, \bar{s})$, we can verify that the configuration in Eq. (10) is stable with respect to the excitations of the noninflaton fields. Taking the limit $c_{\rm K} \gg c_{\mathcal{R}}$ we find the expressions of the masses squared $\hat{m}^2_{\chi^\alpha}$ (with $\chi^\alpha = \theta$ and $s$) arranged in Table I, which approach rather well the quite lengthy, exact expressions taken into account in our numerical computation. These expressions assist us to appreciate the role of $k_S > 0$ in retaining positive $\hat{m}^2_s$. Also we confirm that $\hat{m}^2_{\chi^\alpha} \gg \hat{H}^2_{\rm CI} = \hat{V}_{\rm CI0}/3$ for $\phi_{\rm f} \leq \phi \leq \phi_\star$; note that $n_\theta = 4$ or 6 for $K$ taken by Eq. (15a) or Eq. (15b), respectively. In Table I we display the masses $\hat{m}^2_{\psi^\pm}$ of the corresponding fermions too. We define $\hat{\psi}_S = \sqrt{K_{SS^*}}\psi_S$ and $\hat{\psi}_\Phi = \sqrt{K_{\Phi\Phi^*}}\psi_\Phi$ where $\psi_\Phi$ and $\psi_S$ are the Weyl spinors associated with $S$ and $\Phi$, respectively.

Inserting the derived mass spectrum in the well-known Coleman-Weinberg formula, we can find the one-loop radiative corrections, $\Delta\hat{V}_{\rm CI}$ to $\hat{V}_{\rm CI}$. It can be verified that our results are immune from $\Delta\hat{V}_{\rm CI}$, provided that the renormalization group mass scale $\Lambda$, is determined by requiring $\Delta\hat{V}_{\rm CI}(\phi_\star) = 0$ or $\Delta\hat{V}_{\rm CI}(\phi_{\rm f}) = 0$. The possible dependence of our results on the choice of $\Lambda$ can be totally avoided if we confine ourselves to $k_{S\Phi} \sim 1$ and $k_S \sim (0.5\text{–}1.5)$ resulting in $\Lambda \simeq (1-5) \times 10^{14}$ GeV—cf. Ref. [2,13]. Under these circumstances, our results in the SUGRA setup can be exclusively reproduced by using $\hat{V}_{\rm CI}$ in Eq. (8).

TABLE I. Mass spectrum along the path in Eq. (10).

| Fields | Eingestates | Mass Squared |
|---|---|---|
| 1 Real scalar | $\hat{\theta}$ | $\hat{m}^2_\theta \simeq n_\theta\hat{V}_{\rm CI}/3 = n_\theta\hat{H}^2_{\rm CI}$ |
| 2 Real scalars | $\hat{s}, \hat{\bar{s}}$ | $\hat{m}^2_s \simeq 2(6k_S f_{\mathcal{R}} - 1)\hat{H}^2_{\rm CI}$ |
| 2 Weyl spinors | $(\hat{\psi}_S \pm \hat{\psi}_\Phi)/\sqrt{2}$ | $\hat{m}^2_{\psi\pm} \simeq 3n^2\hat{H}^2_{\rm CI}/2c_{\rm K}\phi^2 f_{\mathcal{R}}^{1+m}$ |

## IV. INFLATION ANALYSIS

The period of slow-roll non-MCI is determined in the EF by the condition

$$\max\{\hat{\epsilon}(\phi), |\hat{\eta}(\phi)|\} \leq 1, \tag{18a}$$

where the slow-roll parameters $\hat{\epsilon}$ and $\hat{\eta}$ read

$$\hat{\epsilon} = (\hat{V}_{{\rm CI},\hat{\phi}}/\sqrt{2}\hat{V}_{\rm CI})^2 \quad \text{and} \quad \hat{\eta} = \hat{V}_{{\rm CI},\hat{\phi}\hat{\phi}}/\hat{V}_{\rm CI} \tag{18b}$$

and can be derived employing $J$ in Eq. (6b), without explicitly expressing $\hat{V}_{\rm CI}$ in terms of $\hat{\phi}$. Our results are

$$\begin{aligned}\hat{\epsilon} &= \frac{n^2}{2\phi^2 c_{\rm K} f_{\mathcal{R}}^{1+m}};\\ \frac{\hat{\eta}}{\hat{\epsilon}} &= 2\left(1 - \frac{1}{n}\right) - \frac{4 + n(1+m)}{2n} c_{\mathcal{R}}\phi^{\frac{n}{2}}.\end{aligned} \tag{19}$$

Given that $\phi \ll 1$ and so $f_{\mathcal{R}} \simeq 1$, Eq. (18a) is saturated at the maximal $\phi$ value, $\phi_{\rm f}$, from the following two values,

$$\phi_{1\rm f} \simeq n/\sqrt{2c_{\rm K}} \quad \text{and} \quad \phi_{2\rm f} \simeq \sqrt{(n-1)n/c_{\rm K}}, \tag{20}$$

where $\phi_{1\rm f}$ and $\phi_{2\rm f}$ are such that $\hat{\epsilon}(\phi_{1\rm f}) \simeq 1$ and $\hat{\eta}(\phi_{2\rm f}) \simeq 1$.

The number of $e$-foldings $\hat{N}_\star$ that the scale $k_\star = 0.05/\mathrm{Mpc}$ experiences during this non-MCI and the amplitude $A_{\rm s}$ of the power spectrum of the curvature perturbations generated by $\phi$ can be computed using the standard formulas,

$$\hat{N}_\star = \int_{\hat{\phi}_{\rm f}}^{\hat{\phi}_\star} d\hat{\phi}\, \frac{\hat{V}_{\rm CI}}{\hat{V}_{{\rm CI},\hat{\phi}}} \quad \text{and} \quad A_{\rm s}^{1/2} = \frac{1}{2\sqrt{3}\pi}\frac{\hat{V}_{\rm CI}^{3/2}(\hat{\phi}_\star)}{|\hat{V}_{{\rm CI},\hat{\phi}}(\hat{\phi}_\star)|}, \tag{21}$$

where $\phi_\star[\hat{\phi}_\star]$ are the value of $\phi[\hat{\phi}]$ when $k_\star$ crosses the inflationary horizon. Since $\phi_\star \gg \phi_{\rm f}$, from Eq. (21) we find

$$\hat{N}_\star = \frac{c_{\rm K}\phi_\star^2}{2n}\,{}_2F_1(-m, 4/n; 1 + 4/n; -c_{\mathcal{R}}\phi_\star^{n/2}), \tag{22}$$

where ${}_2F_1$ is the Gauss hypergeometric function [14] which reduces to unity for $m = 0$ (and any $n$) or to the factor $(f_{\mathcal{R}}^{1+m} - 1)/\phi_\star^2 c_{\mathcal{R}}(1+m)$ for $n = 4$ (and any $m$). Concentrating on these cases, we solve Eq. (22) with respect to $\phi_\star$ resulting in

$$\phi_\star \simeq \begin{cases}\sqrt{2n\hat{N}_\star/c_{\rm K}} & \text{for } m = 0,\\ \sqrt{f_{m\star} - 1}/\sqrt{r_{\mathcal{R}\rm K}c_{\rm K}} & \text{for } n = 4,\end{cases} \tag{23}$$

where $f_{m\star}^{1+m} = 1 + 8(m+1)r_{\mathcal{R}K}\hat{N}_\star$. In both cases there is a lower bound on $c_K$, above which $\phi_\star < 1$ and so, our proposal can be stabilized against corrections from higher order terms. From Eq. (21) we can also derive a constraint on $\lambda$ and $c_K$, i.e.

$$\lambda = \sqrt{3A_s}\pi \cdot \begin{cases} (c_K/n\hat{N}_\star)^{\frac{n}{4}}(2nf_{n\star}/\hat{N}_\star)^{\frac{1}{2}} & \text{for } m=0, \\ 16c_K r_{\mathcal{R}K}^{3/2}/(f_{m\star}-1)^{\frac{3}{2}} f_{m\star}^{\frac{1+m}{2}} & \text{for } n=4, \end{cases} \tag{24}$$

where $f_{n\star} = f_\mathcal{R}(\phi_\star) = 1 + r_{\mathcal{R}K}(2n\hat{N}_\star)^{n/4}$.

The inflationary observables are found from the relations

$$n_s = 1 - 6\hat{\epsilon}_\star + 2\hat{\eta}_\star, \qquad r = 16\hat{\epsilon}_\star, \tag{25a}$$

$$a_s = 2(4\hat{\eta}_\star^2 - (n_s-1)^2)/3 - 2\hat{\xi}_\star, \tag{25b}$$

where the variables with subscript $\star$ are evaluated at $\phi = \phi_\star$ and $\hat{\xi} = \hat{V}_{\mathrm{CI},\hat{\phi}}\hat{V}_{\mathrm{CI},\hat{\phi}\hat{\phi}\hat{\phi}}/\hat{V}_{\mathrm{CI}}^2$. For $m = 0$ we find

$$n_s = 1 - (4 + n + n/f_{n\star})/4\hat{N}_\star, \qquad r = 4n/f_{n\star}\hat{N}_\star, \tag{26a}$$

$$a_s = (n^2 - n(n+4)f_{n\star} - 4(n+4)f_{n\star}^2)/16f_{n\star}^2\hat{N}_\star^2. \tag{26b}$$

In the limit $r_{\mathcal{R}K} \to 0$ or $f_{n\star} \to 1$ the results of the simplest power-law MCI, Eq. (2), are recovered—cf. Ref. [8]. The formulas above are also valid for the original non-MCI [3] with $c_K = 1$ and $r_{\mathcal{R}K} = c_\mathcal{R}$ lower than the one needed to reach the attractor's values in Eq. (3). In this limit our results are in agreement with those displayed in Ref. [7] for $n = 4$. Furthermore, for $n = 4$ (and any $m$) we obtain

$$n_s = 1 - 8r_{\mathcal{R}K}\frac{m - 1 - (m+2)f_{m\star}}{(f_{m\star}-1)f_{m\star}^{1+m}}, \tag{27a}$$

$$r = \frac{128 r_{\mathcal{R}K}}{(f_{m\star}-1)f_{m\star}^{1+m}}, \qquad a_s = \frac{64r_{\mathcal{R}K}^2(1+m)(m+2)}{(f_{m\star}-1)^2 f_{m\star}^{4(1+m)}} \cdot f_{m\star}^2\left(f_{m\star}^{2m}\left(\frac{1-m}{m+2} + \frac{2m-1}{m+1}f_{m\star}\right) - f_{m\star}^{2(1+m)}\right). \tag{27b}$$

For $n = 4$ and $m = 1, 2$ and 4 the outputs of Eqs. (26a)–(27b) are specified in Table II after expanding the relevant formulas for $1/\hat{N}_\star \ll 1$. We can clearly infer that increasing $m$ for fixed $r_{\mathcal{R}K}$, both $n_s$ and $r$ increase. Note that this formulas, based on Eq. (23), is valid only for $r_{\mathcal{R}K} > 0$ (and $m \neq 0$).

From the analytic results above, see Eq. (24) and Eqs. (26a)–(27b), we deduce that the free parameters of our models, for fixed $n$ and $m$, are $r_{\mathcal{R}K}$ and $\lambda/c_K^{n/4}$ and not $c_K$, $c_\mathcal{R}$ and $\lambda$ as naively expected. This fact can be understood by the following observation: If we perform a rescaling $\phi = \tilde{\phi}/\sqrt{c_K}$, Eq. (5) preserves its form replacing $\phi$ with $\tilde{\phi}$ and $f_K$ with $f_\mathcal{R}^m$ where $f_\mathcal{R}$ and $V_{\mathrm{CI}}$ take, respectively, the forms

$$f_\mathcal{R} = 1 + r_{\mathcal{R}K}\tilde{\phi}^{n/2} \quad \text{and} \quad V_{\mathrm{CI}} = \lambda^2\tilde{\phi}^n/2^{n/2}c_K^{n/2}, \tag{28}$$

which, indeed, depend only on $r_{\mathcal{R}K}$ and $\lambda^2/c_K^{n/2}$.

The conclusions above can be verified and extended to others $n$'s and $m$'s numerically. In particular, confronting the quantities in Eq. (21) with the observational requirements [4]

$$\hat{N}_\star \simeq 55 \quad \text{and} \quad A_s^{1/2} \simeq 4.627 \times 10^{-5}, \tag{29}$$

we can restrict $\lambda/c_K^{n/4}$ and $\phi_\star$ and compute the model predictions via Eqs. (25a) and (25b), for any selected $m$, $n$ and $r_{\mathcal{R}K}$. The outputs, encoded as lines in the $n_s - r_{0.002}$ plane, are compared against the observational data [4,5] in Fig. 1 for $m = 0, 1, 2,$ and 4 and $n = 2$ (dashed lines), $n = 4$ (solid lines), and $n = 6$ (dot-dashed lines). The variation of $r_{\mathcal{R}K}$ is shown along each line. To obtain an accurate comparison, we compute $r_{0.002} = 16\hat{\epsilon}(\phi_{0.002})$ where $\phi_{0.002}$ is the value of $\phi$ when the scale $k = 0.002/\mathrm{Mpc}$, which undergoes $\hat{N}_{0.002} = \hat{N}_\star + 3.22$ $e$-foldings during non-MCI, crosses the horizon of non-MCI.

From the plots in Fig. 1 we observe that, for low enough $r_{\mathcal{R}K}$'s—i.e. $r_{\mathcal{R}K} = 10^{-7}, 10^{-4}$, and 0.001 for $n = 6, 4$, and 2—the various lines converge to the $(n_s, r_{0.002})$'s obtained within MCI. At the other end, the lines for $n = 4$ and 6 terminate for $r_{\mathcal{R}K} = 1$, beyond which the theory ceases to be unitarity safe—see below—whereas the $n = 2$ line approaches an attractor value for any $m$. For $m = 0$ we reveal the results of Ref. [3]; i.e., the displayed lines are almost parallel for $r_{0.002} \geq 0.02$ and converge at the values

TABLE II. Inflationary predictions for $n = 4$ and $m = 1, 2,$ and 4.

| | $m=1$ | $m=2$ | $m=4$ |
|---|---|---|---|
| $n_s$ | $1 - 3/2\hat{N}_\star - 3/8(\hat{N}_\star^3 r_{\mathcal{R}K})^{1/2}$ | $1 - 4/3\hat{N}_\star - 1/2(3\hat{N}_\star^4 r_{\mathcal{R}K})^{1/3}$ | $1 - 6/5\hat{N}_\star - 3/5(40\hat{N}_\star^6 r_{\mathcal{R}K})^{1/5} - 3/10(50\hat{N}_\star^7 r_{\mathcal{R}K}^2)^{1/5}$ |
| $r$ | $1/2\hat{N}_\star^2 r_{\mathcal{R}K} + 2/(\hat{N}_\star^3 r_{\mathcal{R}K})^{1/2}$ | $8/3(3\hat{N}_\star^4 r_{\mathcal{R}K})^{1/3} + 4/3(9\hat{N}_\star^5 r_{\mathcal{R}K}^2)^{1/3}$ | $8(4/5\hat{N}_\star^6 r_{\mathcal{R}K})^{1/5}/5 + 4(16/25\hat{N}_\star^7 r_{\mathcal{R}K}^2)^{1/5}/5$ |
| $a_s$ | $-3/2\hat{N}_\star^2 - 9/16(\hat{N}_\star^5 r_{\mathcal{R}K})^{1/2}$ | $-4/3\hat{N}_\star^2 - 2/3(3\hat{N}_\star^7 r_{\mathcal{R}K})^{1/3}$ | $-6/5\hat{N}_\star^2 - 9(4/5\hat{N}_\star^{11} r_{\mathcal{R}K})^{1/5}/25$ |

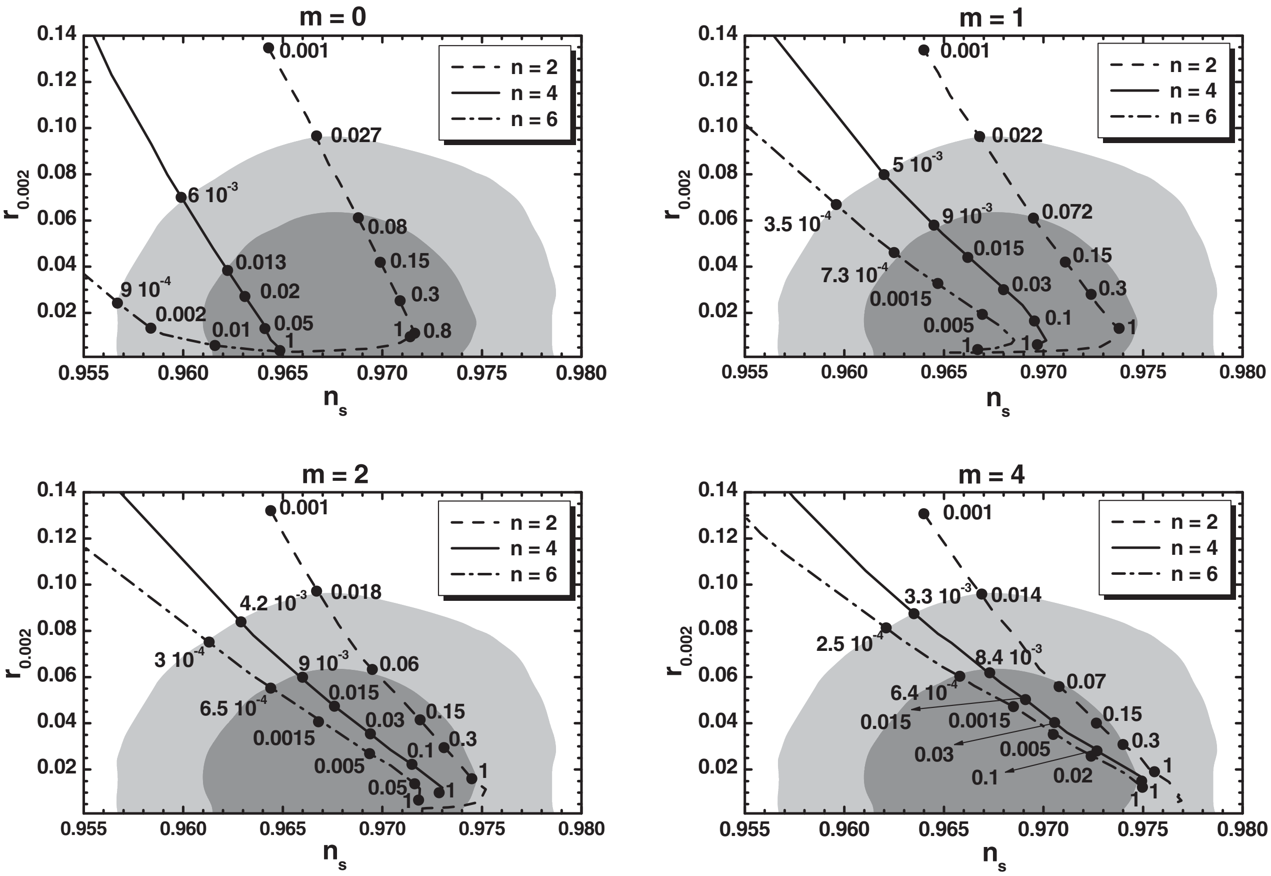


FIG. 1. Allowed curves in the $n_s - r_{0.002}$ plane for $m = 0, 1, 2$ and 4, $n = 2$ (dashed lines), $n = 4$ (solid lines), $n = 6$ (dot-dashed lines) and various $r_{\mathcal{R}K}$'s indicated on the curves. The marginalized joint 68% [95%] regions from *Planck*, BICEP2/*Keck Array* and BAO data are depicted by the dark [light] shaded contours.

in Eq. (3)—for $n = 4$ and 6 this is reached even for $r_{\mathcal{R}K} = 1$. For $m > 0$ the curves move to the right and span more densely the 1-$\sigma$ ranges in Eq. (4) for quite natural $r_{\mathcal{R}K}$'s—e.g. $0.005 \lesssim r_{\mathcal{R}K} \lesssim 0.1$ for $m = 1$ and $n = 4$. It is worth mentioning that the requirement $r_{\mathcal{R}K} \leq 1$ provides a lower bound on $r_{0.002}$, which ranges from 0.0032 (for $m = 0$ and $n = 6$) to 0.015 (for $m = 4$ and $n = 4$). Note, finally, that our estimations in Eqs. (26a)–(26b) are in agreement with the numerical results for $n = 2$ and $r_{\mathcal{R}K} \lesssim 1$, $n = 6[4]$ and $r_{\mathcal{R}K} \lesssim 0.002[0.05]$. For $m > 0$ (and $n = 4$) our findings in Eqs. (27a)–(27b) (and Table II) approximate fairly the numerical outputs for $0.003 \lesssim r_{\mathcal{R}K} \leq 1$.

## V. EFFECTIVE CUTOFF SCALE

The selected $f_K$ in Eq. (7) not only reconciles non-MCI with the 1-$\sigma$ ranges in Eq. (4) but also assures that the corresponding effective theories respect perturbative unitarity up to $m_P = 1$, although $c_K$ may take relatively large values for $\phi < 1$; e.g., for $n = 4, m = 1$ and $r_{\mathcal{R}K} = 0.03$, we obtain $140 \lesssim c_K \lesssim 1.4 \times 10^6$ for $3.3 \times 10^{-4} \lesssim \lambda \lesssim 3.5$. This achievement stems from the fact that $\hat{\phi} = \langle J \rangle \phi$ does not coincide—contrary to the pure non-MCI [15,16] for $n > 2$—with $\phi$ at the vacuum of the theory, given that $\langle J \rangle = \sqrt{c_K}$ or $\langle J \rangle = \sqrt{c_K + 3c_{\mathcal{R}}^2/2}$ for $\langle \phi \rangle = 0$ and $n > 2$ or $n = 2$ [see Eq. (8)]. It is notable that this byproduct of our proposal for $n > 2$ arises without invoking large $\langle \phi \rangle$'s as in Ref. [10,13,17].

To further clarify this point, we analyze the small-field behavior of our models in the EF. We focus on the second term in the right-hand side of Eq. (6a) or (9a) for $\mu = \nu = 0$, and we expand it about $\langle \phi \rangle = 0$ in terms of $\hat{\phi}$; see Eq. (6b). Our result for $m = 0$ and $n = 2, 4,$ and 6 can be written as

$$J^2 \dot{\phi}^2 = \left(1 - r_{\mathcal{R}K} \hat{\phi}^{\frac{n}{2}} + \frac{3n^2}{8} r_{\mathcal{R}K}^2 \hat{\phi}^{n-2} + r_{\mathcal{R}K}^2 \hat{\phi}^n \cdots \right) \dot{\hat{\phi}}^2.$$

Similar expressions can be obtained for the other $m$'s too. Expanding similarly $\hat{V}_{\rm CI}$, see Eq. (8), in terms of $\hat{\phi}$, we have

$$\hat{V}_{\rm CI} = \frac{\lambda^2 \hat{\phi}^n}{2 c_K^{n/2}} \left(1 - 2 r_{\mathcal{R}K} \hat{\phi}^{\frac{n}{2}} + 3 r_{\mathcal{R}K}^2 \hat{\phi}^n - 4 r_{\mathcal{R}K}^3 \hat{\phi}^{\frac{3n}{2}} + \cdots \right),$$

independently of $m$. From the expressions above we conclude that our models do not face any problem with the perturbative unitarity for $r_{\mathcal{R}K} \leq 1$. For $n = 2$ this statement is also valid even for $r_{\mathcal{R}K} > 1$ as shown in Ref. [2,16]. In the latter case, though, the naturalness argument mentioned below Eq. (15b) is invalidated.

## VI. CONCLUSIONS

Prompted by the recent joint analysis of BICEP2/*Keck Array* and *Planck* which, although it does not exclude inflationary models with negligible $r$'s, seems to favor those with $r$'s of order 0.01, we proposed a variant of non-MCI which can safely accommodate $r$'s of this level. The main novelty of our proposal is the consideration of the non-canonical kinetic mixing in Eq. (7)—involving the parameters $m$ and $c_{\rm K}$—apart from the nonminimal coupling to gravity in Eq. (1) which is associated with the potential in Eq. (2). This setting can be elegantly implemented in SUGRA, too, employing the super-and Kähler potentials given in Eqs. (12) and (15a) or (15b). Prominent in this realization is the role of a shift-symmetric quadratic function $F_{\rm K}$ in Eq. (14a) which remains invisible in the SUGRA scalar potential while dominates the canonical normalization of the inflaton. Using $m \geq 0$ and confining $r_{\mathcal{R}K}$ to the range $(2.5 \times 10^{-4} - 1)$, where the upper bound does not apply to the $n = 2$ case, we achieved observational predictions which may be tested in the near future and converge towards the "sweet" spot of the present data—its compatibility with the $m = 1$ case, especially for $n = 4$ and 6, is really impressive (see Fig. 1). These solutions can be attained even with sub-Planckian values of the inflaton requiring large $c_{\rm K}$'s and without causing any problem with the perturbative unitarity. It is gratifying, finally, that a sizable fraction of the allowed parameter space of our models (with $n = 4$) can be studied analytically and rather accurately.


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