

Target dependence of the annual modulation in direct dark matter searches

Eugenio Del Nobile, Graciela B. Gelmini, and Samuel J. Witte

Department of Physics and Astronomy, UCLA, 475 Portola Plaza, Los Angeles, California 90095, USA

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Due to Earth's revolution around the Sun, the expected scattering rate in direct dark matter searches is annually modulated. This modulation is expected to differ between experiments when given as a function of recoil energy E_R , e.g. due to the gravitational focusing effect of the Sun. A better variable to compare results among experiments employing different targets is the minimum speed v_{\min} a dark matter particle must have to impart a recoil energy E_R to a target nucleus. It is widely believed that the modulation expressed as a function of v_{\min} is common to all experiments, irrespective of the dark matter distribution. We point out that the annual modulation as a function of v_{\min} , and in particular the times at which the rate is maximum and minimum, could be very different depending on the detector material. This would be an indication of a scattering cross section with nonfactorizable velocity and target material dependence. Observing an annual modulation with at least two different target elements would be necessary to identify this type of cross section.

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I. INTRODUCTION

Dark matter (DM) is the most abundant form of matter in the Universe, and its nature still remains a mystery. More than 80% of the mass of our Galaxy resides in a spheroidal DM halo, which extends well beyond the visible disk. Efforts to detect new elementary particles which could constitute the DM are multipronged.

Direct DM detection experiments attempt to detect the energy deposited by DM particles in the dark halo of our Galaxy when they collide with nuclei inside a detector. An unmistakable signature of the expected DM signal is an annual modulation of the rate caused by the rotation of Earth around the Sun [1]. For DM velocity distributions that are smooth and isotropic in the Galactic frame at Earth's location, the expected differential rate for DM scattering onto a target nuclide T in all direct DM detection experiments could be well represented by the first two terms of a harmonic expansion (see e.g. Ref. [2]),

$$\frac{dR_T}{dE_R}(E_R, t) = S_0(E_R) + S_m(E_R) \cos\left(\frac{2\pi}{1 \text{ year}}(t - t_0)\right). \quad (1)$$

Here E_R is the nuclear recoil energy, and t_0 is the time at which the speed of Earth with respect to the Galaxy is maximum, close to 1 June. At high E_R , with S_m positive, t_0 equals the time t_{\max} at which the rate is maximum, while t_{\min} , the time at which the rate is minimum, is six months apart from t_{\max} (except for a shift of about a day due to the eccentricity of Earth's orbit). At low E_R , S_m could become negative, implying t_0 equals t_{\min} instead of t_{\max} (see e.g. Fig. 8.2 of Ref. [3]). Anisotropies in the local DM velocity distribution modify this picture in particular by making t_{\max} and t_{\min} energy dependent. The gravitational focusing (GF) of DM particles due to the Sun inherently makes the local DM halo anisotropic [4]. Reference [5] has shown GF to

have a significant effect on the phase of the modulation at low enough recoil energy.

Since E_R depends on the target nuclide mass, it is not a good variable to compare the annual modulation of the rate among experiments employing different targets. A better variable is v_{\min} , the minimum speed a DM particle must have in Earth's rest frame to impart a recoil energy E_R onto a target nucleus. It is typically assumed that t_{\max} and t_{\min} as functions of v_{\min} do not depend on the target, and consequently they can be used to test the agreement between putative DM signals across multiple detectors.

Here we point out that, in general, the annual modulation of the rate as a function of v_{\min} can vary significantly for different target materials. Specifically, we show that if the velocity and target dependence cannot be factored in the differential scattering cross section, observables associated with the modulation, such as t_{\max} and t_{\min} , may be highly target dependent. Our observation does not rely on any assumption regarding the DM distribution. As an illustration, we show that for DM particles with a magnetic dipole moment, t_{\max} and t_{\min} depend on the target material.

II. DM SIGNAL AND ITS MODULATION

For the spin-independent and spin-dependent contact interactions usually considered, the differential scattering cross section is

$$\frac{d\sigma_T}{dE_R}(E_R, v) = \frac{m_T \sigma_T F_T(E_R)^2}{2\mu_T^2} \frac{1}{v^2}, \quad (2)$$

with m_T the target nuclide mass, μ_T the DM-nucleus reduced mass, σ_T the total cross section for a pointlike nucleus, and $F_T(E_R)$ the appropriate nuclear form factor. The differential scattering rate per unit target mass,

$$\frac{dR_T}{dE_R}(E_R, t) = \frac{C_T \rho}{m_T m} \int_{v \geq v_{\min}(E_R)} v f(\mathbf{v}, t) \frac{d\sigma_T}{dE_R} d^3 v, \quad (3)$$

with Eq. (2) becomes

$$\frac{dR_T}{dE_R}(E_R, t) = C_T \frac{\rho}{m} \frac{\sigma_T F_T(E_R)^2}{2\mu_T^2} \eta(v_{\min}(E_R), t), \quad (4)$$

with ρ and m the local DM particle density and mass, respectively, and C_T the nuclide mass fraction in the detector. Here we define the velocity integral

$$\eta(v_{\min}, t) \equiv \int_{v \geq v_{\min}} \frac{f(\mathbf{v}, t)}{v} d^3 v, \quad (5)$$

where $f(\mathbf{v}, t)$ is the DM velocity distribution in Earth's frame. The time dependence arises due to Earth's revolution around the Sun. The modulation of the rate in Eq. (4) is determined by the time dependence of $\eta(v_{\min}, t)$, which is common to all experiments. Therefore, for the interaction in Eq. (2), t_{\max} and t_{\min} for fixed v_{\min} do not depend on the target material. This remains true for other differential cross sections where the velocity and target dependences can be factored. In general, however, the differential cross section can consist of multiple terms with different velocity dependences and target-dependent coefficients, e.g. with DM particles interacting through a magnetic dipole [6–31] or an anapole moment [6,28–34,34,35]. It also happens with some of the interactions described by the effective operators studied e.g. in Refs. [36–41] (see Refs. [26,42–45] for explicit formulas of scattering amplitudes). In this case the annual modulation of the rate can be strongly target element dependent.

III. AN EXAMPLE: MAGNETIC DIPOLE DM

Here we study in detail the case of a Dirac fermion DM candidate χ that interacts with nuclei through a magnetic dipole moment λ_χ , with interaction Lagrangian $\mathcal{L} = (\lambda_\chi/2)\bar{\chi}\sigma_{\mu\nu}\chi F^{\mu\nu}$. The differential cross section for elastic scattering off a target nucleus T with Z_T protons and spin S_T is

$$\begin{aligned} \frac{d\sigma_T}{dE_R}(v_{\min}, v) = & \alpha \lambda_\chi^2 \left\{ Z_T^2 \frac{m_T}{2\mu_T^2} \left[\frac{1}{v_{\min}^2} - \frac{1}{v^2} \left(1 - \frac{\mu_T^2}{m^2} \right) \right] \right. \\ & \times F_{\text{SI},T}^2(E_R(v_{\min})) \\ & \left. + \frac{\hat{\lambda}_T^2 m_T}{v^2 m_p^2} \left(\frac{S_T + 1}{3S_T} \right) F_{\text{M},T}^2(E_R(v_{\min})) \right\}, \quad (6) \end{aligned}$$

with $\alpha = e^2/4\pi$ the electromagnetic constant, m_p the proton mass, $\hat{\lambda}_T$ the nuclear magnetic moment in units of the nuclear magneton $e/(2m_p) = 0.16 \text{ GeV}^{-1}$, and $E_R(v_{\min}) = 2\mu_T^2 v_{\min}^2/m_T$. The first term is due to DM dipole-nuclear charge interaction, and the corresponding

charge form factor coincides with the usual spin-independent nuclear form factor $F_{\text{SI},T}(E_R)$, while the second term is due to the dipole-dipole interaction and has a nuclear magnetic form factor $F_{\text{M},T}(E_R)$ (both form factors are normalized to 1 at zero momentum transfer). We compute the cross section with the formalism and the form factors provided in Refs. [42,43].

The differential cross section in Eq. (6) contains two terms with different velocity dependence: one with the usual $1/v^2$ factor and another independent of v . The differential rate [see Eq. (3)] is thus also a sum of two terms, one containing $\eta(v_{\min}, t)$ in Eq. (5) and the other containing

$$\tilde{\eta}(v_{\min}, t) \equiv \int_{v \geq v_{\min}} v f(\mathbf{v}, t) d^3 v. \quad (7)$$

For purposes of illustration, we assume the Standard Halo Model (SHM), in which the DM velocity distribution is an isotropic Maxwellian on average at rest with respect to the Galaxy (see e.g. Ref. [46] for details). Under this assumption, the two velocity integrals η and $\tilde{\eta}$ have a very different time dependence. This can be seen in Fig. 1, where their times of maximum τ_{\max} and minimum τ_{\min} are shown. Instead of τ_{\min} , we plot $\tau_{\min} - \hat{\tau}_{\min}$, where $\hat{\tau}_{\min}$ is the time six months apart from τ_{\max} . Figure 1 shows the effect of including (solid lines) and neglecting (dashed lines) GF. Neglecting GF, τ_{\min} is almost indistinguishable from $\hat{\tau}_{\min}$, and thus is not shown. Unless otherwise stated, we include GF and the eccentricity of Earth's orbit in our calculations. Notice that τ_{\max} (τ_{\min}) as a function of v_{\min} coincides with the maximum (minimum) of the differential rate, t_{\max} (t_{\min}), only when the velocity and

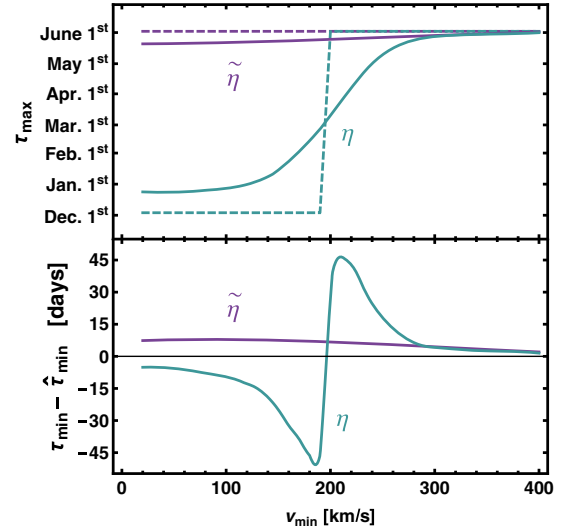


FIG. 1 (color online). Times of maximum τ_{\max} (top) and minimum τ_{\min} (bottom) of η and $\tilde{\eta}$ in the SHM, as functions of v_{\min} , including (solid lines) and neglecting (dashed lines) GF. The bottom panel shows $\tau_{\min} - \hat{\tau}_{\min}$, with $\hat{\tau}_{\min}$ the time six months apart from τ_{\max} . Neglecting GF, τ_{\min} is almost indistinguishable from $\hat{\tau}_{\min}$, and thus is not shown.

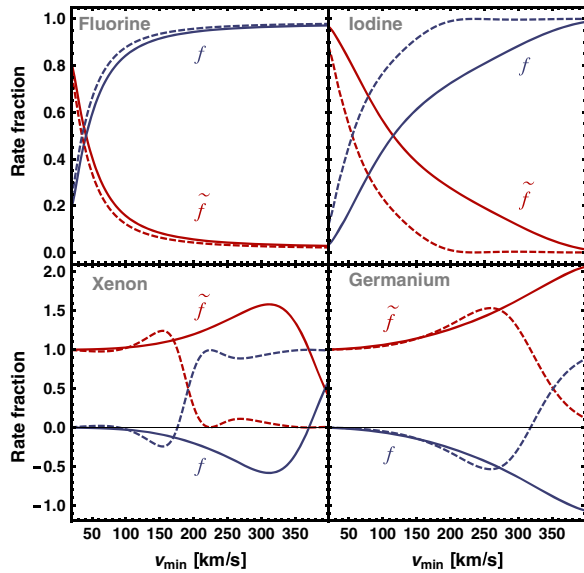


FIG. 2 (color online). Rate fractions $f \equiv r/(r + \tilde{r})$ and $\tilde{f} \equiv \tilde{r}/(r + \tilde{r})$ for fluorine, iodine, xenon, and germanium. Solid (dashed) lines for $m = 100$ GeV (1 TeV).

target dependence can be factored in the differential scattering cross section.

The modulation of the differential rate depends on the interplay of the terms containing η and $\tilde{\eta}$. Since the relative coefficients are in general target dependent, as well as DM particle mass dependent, the modulation also depends on the target and on m . Let us denote with r and \tilde{r} the terms of the expected differential rate containing η and $\tilde{\eta}$, so that $dR_T/dE_R = r + \tilde{r}$. Figure 2 shows the rate fractions $f \equiv r/(r + \tilde{r})$ and $\tilde{f} \equiv \tilde{r}/(r + \tilde{r})$ as functions of v_{\min} for four different target elements (fluorine, iodine, xenon, and germanium) employed by current DM direct detection

experiments. For target elements with more than one isotope (Xe, Ge), we sum Eq. (3) over isotopic composition. Solid (dashed) lines in Fig. 2 correspond to a 100 GeV (1 TeV) DM particle. Notice that because of the negative sign in one of the dipole-charge terms in Eq. (6), r and f are allowed to take negative values. When this happens, $\tilde{f} > 1$, since $f + \tilde{f} = 1$.

Figures 1 and 2 can be used in combination to understand the target-dependent behavior of the times of maximum t_{\max} and minimum t_{\min} of the rate for magnetic DM, shown in Fig. 3 for scattering off fluorine, sodium, iodine, xenon and germanium. Solid (dashed) lines correspond to $m = 100$ GeV (1 TeV). Also shown in Fig. 3 are the E_R thresholds for LUX [47] (3.1 keV, employing Xe), SuperCDMS [48] (1.6 keV, Ge), DAMA [49] (6.7 keV for Na and 22.2 keV for I), and PICO [50] (3.2 keV, F), translated into v_{\min} for m_T averaged over isotopic composition and elastic scattering for $m = 100$ GeV. For larger m , these thresholds move to lower v_{\min} values.

Figure 3 shows that t_{\max} and t_{\min} become essentially target independent above $v_{\min} \approx 300$ km/s. This is due to the fact that the differences between η and $\tilde{\eta}$, which are central to the target dependence of the rate, rapidly vanish at $v_{\min} \gtrsim 300$ km/s (see Fig. 1). The target-independent nature of this region is not specific to magnetic DM and occurs whenever the SHM is assumed, at least with $1/v^2$ and v^n -dependent terms in the differential cross section and $n \geq 0$. This is because all velocity integrals arising from terms going as v^n with $n \geq 0$ in the differential cross section have very similar phases at all v_{\min} values, i.e. they are all comparable to $\tilde{\eta}$ in Fig. 1. The target-dependent effects addressed in this paper thus rely on having both a $1/v^2$ term and a v^n term, $n \geq 0$, in the differential cross section.

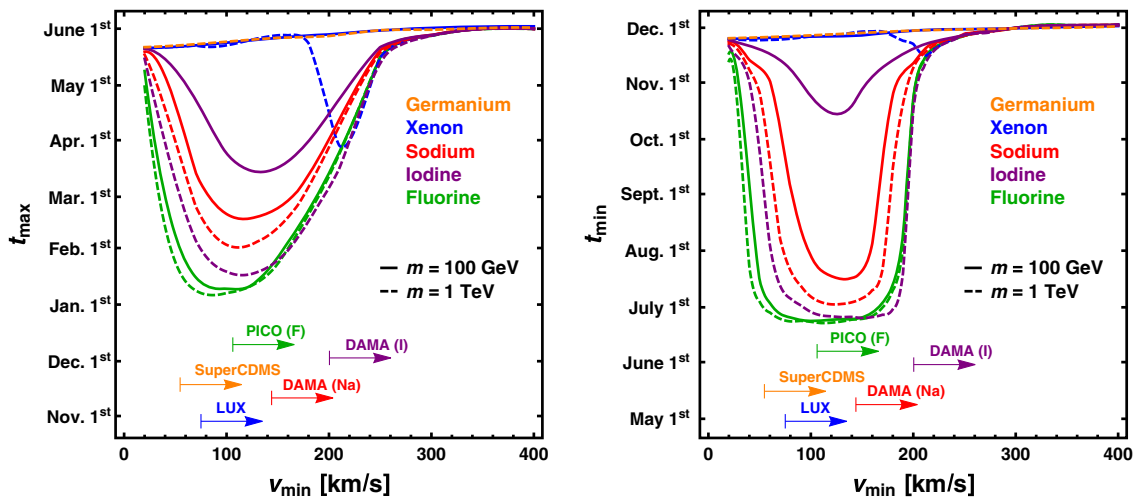


FIG. 3 (color online). Time of maximum t_{\max} (left) and minimum t_{\min} (right) of the scattering rate as functions of v_{\min} for a 100 GeV (solid lines) and 1 TeV (dashed lines) magnetic dipole DM particle scattering elastically off different targets, assuming the SHM. For germanium, the two lines would overlap (only dashed is shown). Also shown are the low-energy thresholds for several current direct detection experiments, for $m = 100$ GeV (for larger m , the thresholds shift to lower v_{\min} values).

At sufficiently small values of v_{\min} , the rate is always dominated by \tilde{r} (i.e. $\tilde{f} \approx 1$ and $f \approx 0$), as shown in Fig. 2. This is due to the $1/v_{\min}^2$ factor appearing in Eq. (6). Therefore, in the small v_{\min} limit, one can disregard the contribution of r and correctly assume t_{\max} and t_{\min} coincide with the τ_{\max} and τ_{\min} of $\tilde{\eta}$ shown in Fig. 1. This explains why t_{\max} in Fig. 3 occurs in May at small v_{\min} values regardless of the target.

Assuming at least one target isotope has a nonzero nuclear magnetic moment, the dipole-dipole part of the interaction becomes dominant, and thus $r > \tilde{r}$, at large values of v_{\min} . This is due to the fact that the spin-independent charge form factor decreases faster than the magnetic form factor. Figure 2 confirms that for the elements and DM masses considered, there is a v_{\min} value above which r dominates, and below which \tilde{r} dominates. In Fig. 3, this corresponds to the time variation of the rate being determined by η or $\tilde{\eta}$, respectively. For germanium, this switch occurs at large v_{\min} values because of its small average magnetic moment. How and where this switch in v_{\min} occurs determine the main features of t_{\max} and t_{\min} in Fig. 3.

For each element, the features in Fig. 2 move to smaller v_{\min} values as the DM particle mass increases. This is in part because the v_{\min} value corresponding to a particular E_R decreases, but also because the $1/\mu_T^2$ and μ_T^2/m^2 factors in Eq. (6) decrease. Notice that, as m increases, the v_{\min} value above which r becomes the dominant term in the rate may fall below 300 km/s, leading to the appearance of a feature in Fig. 3. This happens with xenon when m goes from 100 GeV to 1 TeV.

We emphasize that the interplay between η and $\tilde{\eta}$ does not only affect observables associated with the modulation of the rate, such as t_{\max} and t_{\min} , but also the extent to which the standard approximation of the modulation given in Eq. (1) holds. Figure 4 shows that the difference between

t_{\min} and $\hat{t}_{\min} \equiv t_{\max} - 6$ months is target and DM particle mass dependent, and can be large, e.g. $t_{\min} - \hat{t}_{\min}$ for $m = 100$ GeV could be as large as ± 45 days. This implies that higher-order terms in the Fourier expansion of the rate beyond Eq. (1) cannot be neglected.

To illustrate how important the target element dependence of the rate modulation can be, consider the signal due to a 100 GeV DM particle being detected with both xenon and fluorine near the present LUX and PICO thresholds. Were the modulation due solely to η or $\tilde{\eta}$, the two experiments should observe nearly the same value of t_{\max} ; see Fig. 1. Instead, due to the target-dependent interplay of η and $\tilde{\eta}$, the t_{\max} observed with the two target elements could differ by more than four months, and the modulation in xenon would be better described by Eq. (1) than the modulation in fluorine.

As we already mentioned, in order to observe the target-dependent effects described so far, it is essential that the experimental threshold in v_{\min} , which depends on the threshold in E_R , the DM particle mass and the scattering kinematics, be below 300 km/s. Figure 3 shows that $m = 100$ GeV is already large enough with present thresholds to observe this effect. For lower m , the effect will only be present with the light targets, for elastic scattering.

Should DM scatter inelastically off nuclei, the scattering kinematics would be different from that of elastic scattering. Inelastic scattering [51,52] can happen if there are at least two almost degenerate DM particles with masses m and $m + \delta$ ($\delta \ll m$). If the particle with mass m scatters into the $m + \delta$ particle, $v_{\min} = |(m_T E_R / \mu_T) + \delta| / \sqrt{2m_T E_R}$. In particular, if $\delta < 0$ (exothermic scattering [52]), the v_{\min} value corresponding to a given E_R and m can be much smaller than in the case of elastic scattering.

All the effects we have described here rely on having a DM-nucleus differential cross section with a particular v dependence. The issue remains of how such a cross section could be identified experimentally. We believe that this would require observing an annual modulation in at least two experiments with different target materials. If the velocity and target dependence in the differential cross section factorize, the observables associated with the modulation as functions of v_{\min} would be independent of the target element for any DM distribution. However, experiments do not measure their signal in v_{\min} , but in energy, and the values of m and δ entering the E_R - v_{\min} relation are not known *a priori*. This problem could be overcome by comparing observables of the modulation, like t_{\max} and t_{\min} , of at least two experiments employing different target materials, and trying to find values of m and δ that reconcile the differences between observed modulations as functions of v_{\min} . Should there exist no E_R - v_{\min} relation that would make the modulations as functions of v_{\min} compatible across experiments, one may infer the differential cross section contains a nonfactorizable velocity and target dependence.

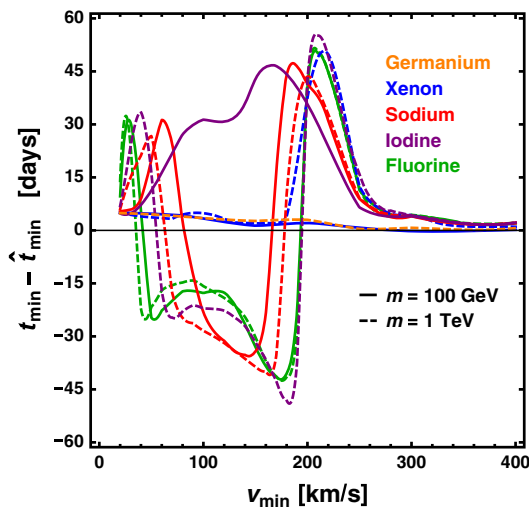


FIG. 4 (color online). $t_{\min} - \hat{t}_{\min}$, with \hat{t}_{\min} the time six months apart from t_{\max} . See Fig. 3 for details.

IV. CONCLUSIONS

It is usually assumed that the modulation of the expected differential rate in direct DM detection experiments, expressed as a function of v_{\min} (the minimum DM speed necessary to impart a certain recoil energy to a target nucleus), does not depend on the target. We have shown instead that experiments employing different target materials could observe an entirely different annual modulation of their differential rate as a function of v_{\min} . This would be a signature of DM interactions with more than one velocity-dependent term in the scattering cross section, in particular terms proportional to $1/v^2$ and v^n with $n \geq 0$. In order to identify experimentally this type of cross section, we believe at least two experiments employing different target materials should observe an annual modulation. Should no E_R - v_{\min} relation be found that reconciles the modulated signals as functions of v_{\min} , one may infer the differential

cross section contains a nonfactorizable velocity and target dependence regardless of the DM distribution.

As an example, we have shown explicitly the target dependence of the times of maximum t_{\max} and minimum t_{\min} of the rate for a 100 GeV and a 1 TeV magnetic dipole DM scattering elastically, assuming the SHM. We found that the values of t_{\max} observed with e.g. xenon and fluorine close to the present LUX and PICO thresholds could disagree by as much as four months (see Fig. 3), and the modulation in xenon could be better described by the sinusoidal time dependence usually assumed than that in fluorine.

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