Magnetic moments of heavy $J^P = \frac{1}{2}^+$ baryons in light cone QCD sum rules

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The magnetic moments of heavy sextet $J^P = \frac{1}{2}^+$ baryons are calculated in the framework of the light cone QCD sum rules method. Linearly independent relations among the magnetic moments of these baryons are obtained. The results for the magnetic moments of heavy baryons obtained in this work are compared with the predictions of the other approaches.

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I. INTRODUCTION

The quark model predicts the existence of heavy baryons composed of single, double and triple quarks. Essential improvement has been achieved in heavy baryon spectroscopy in recent years. All baryons with a single charm quark that are predicted by the quark model have been observed in experiments. Moreover, heavy baryons with a single bottom quark, such as Λ_b, Σ_b, Ξ_b and Ω_b, have also been discovered (for a review see [1]).

These progresses in experiments stimulated future investigation for the properties of these baryons at LHC, as well as further theoretical studies on this subject.

Remarkable information about the internal structure of baryons can be gained by studying their magnetic moments. Magnetic moments of the heavy baryons have long been under the focus of theoretical physicists, and they have been studied in framework of various approaches. So far, magnetic moments of charmed heavy baryons have been calculated in framework of the naive quark model [2,3], relativistic quark model [4], chiral perturbation model [5], hypercentral model [6], soliton model [7], skyrmion model [8], bag model [9], QCD sum rules method [10], nonrelativistic quark model [11], phenomenological relativistic model [12], quark model with confinement law potential [13], and chiral constituent quark model, respectively [14].

Magnetic moments of heavy hadrons have also been studied in numerous works within the QCD sum rules method. Magnetic moments of the Λ_c and Σ_c baryons have been calculated in the framework of the traditional QCD sum rules method in [10]; of the Λ_Q and Ξ_Q ($Q = c$ or b) baryons within the light version of the QCD sum rules method in [15,16], respectively. It should be noted here that magnetic moments of the spin-3/2 heavy baryons have already been analyzed within the same approach in [17].

The goal of the present paper is calculation of the magnetic moments of heavy Σ_Q, Ξ'_Q and Ω_Q baryons within the light cone QCD sum rules method.

The paper is organized as follows. In Sec. II, sum rules for the magnetic moments of the above-mentioned baryons are constructed. Numerical results of our calculations are presented in Sec. III. This section further contains comparison of our results with the predictions of other approaches, and concluding remarks.

II. LIGHT CONE SUM RULES FOR THE SEXTET HEAVY BARYONS

We start this section by giving a brief summary of the classification of heavy baryons in SU(3) symmetry. According to SU(3) symmetry, heavy baryons with single heavy quark belong to either symmetric sextet or antisymmetric antitriplet flavor representations. As has already been noted, magnetic moments of the Λ_Q and Ξ_Q baryons belonging to antisymmetric flavor representations of SU(3) are calculated within the light QCD sum rules method in [15] and [16], respectively. In the present paper we calculate magnetic moments of the heavy baryons belonging to sextet representation of the SU(3) group.

In order to calculate magnetic moments of the heavy sextet baryons, we start by considering the following correlation function:

$$\Pi(p, q) = i \int d^4x e^{ipx} \langle 0|T\{\eta_Q(x)\bar{\eta}_Q(0)\}|0\rangle_\gamma, \quad (1)$$

where η_Q is the interpolating current of the heavy spin-1/2 baryon, and γ is the external magnetic field. The main task in constructing the sum rules for magnetic moments of the heavy sextet baryons is the calculation of the correlation function in terms of the photon distribution amplitude (DA) by using the operator product expansion (OPE) over twist from one side, and in terms of the hadrons from the other side, and then equating both representations. Calculation of the correlation function from the hadronic side is

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accomplished by inserting a complete set of hadrons carrying the same quantum numbers of the interpolating current η_Q. Isolating the ground state's contribution, we obtain

$$\Pi = \frac{\langle 0|\eta_Q|B_Q(p_2)\rangle}{p_2^2 - m_{B_Q}^2} \langle B_Q(p_2)|B_Q(p_1)\rangle_\gamma \frac{\langle B_Q(p_1)|\bar\eta_Q|0\rangle}{p_1^2 - m_{B_Q}^2}$$
$$+ \cdots, \tag{2}$$

where dots correspond to the contributions of higher states and continuum, $p_1 = p_2 + q$. In further analysis we shall make the replacement $p_2 = p$. The matrix elements in Eq. (2) are determined as

$$\langle 0|\eta_Q|B_Q(p)\rangle = \lambda_Q u_{B_Q}(p), \tag{3}$$

$$\langle B_Q(p)|B_Q(p_1)\rangle_\gamma = \varepsilon^\mu \bar u_{B_Q}(p)\left[f_1\gamma_\mu - f_2\frac{i\sigma_{\mu\nu}}{2m_{B_Q}}q^\nu\right]u_{B_Q}(p_1), \tag{4}$$

where ε^μ is the photon polarization vector, f_1 and f_2 are the form factors. Using the equation of motion, Eq. (4) can be written as

$$\langle B_Q(p)|B_Q(p_1)\rangle_\gamma$$
$$= \bar u_{B_Q}(p)\left[(f_1 + f_2)\gamma_\mu + \frac{(p + p_1)_\mu}{2m_{B_Q}}\right]u_{B_Q}(p_1)\varepsilon^\mu. \tag{5}$$

Since the photon involved in these transitions is a real photon, we set $q^2 = 0$, and hence in calculation of the magnetic moments of the heavy sextet baryons the values of the form factors are needed only at $q^2 = 0$.

Substituting Eqs. (3) and (5) in Eq. (2), and performing summations over spins of the heavy baryons, for the hadronic of the correlation function we get

$$\Pi = \lambda_{B_Q}^2 \varepsilon^\mu \frac{(\not p + m_{B_Q})}{p^2 - m_{B_Q}^2}\left[(f_1 + f_2)\gamma_\mu + \frac{(p + p_1)_\mu}{2m_{B_Q}}\right]$$
$$\times \frac{(\not p_1 + m_{B_Q})}{p_1^2 - m_{B_Q}^2}. \tag{6}$$

TABLE I. Quark contents of the heavy sextet baryons.

	$\Sigma_{b(c)}^{+(++)}$	$\Sigma_{b(c)}^{0(+)}$	$\Sigma_{b(c)}^{-(0)}$	$\Xi_{b(c)}^{\prime -(0)}$	$\Xi_{b(c)}^{\prime 0(+)}$	$\Omega_{b(c)}^{-(0)}$
q_1	u	u	d	d	u	s
q_2	u	d	d	s	s	s

We observe from Eq. (6) that the correlation function contains many structures; any of them can be chosen in calculating magnetic moments of the sextet baryons, and in this respect we choose the structure $\not p \varepsilon \not q$. This structure envelopes the magnetic form factor $f_1 + f_2$, and at $q^2 = 0$ it gives the magnetic moment μ_{B_Q} of the heavy baryons in units of $e\hbar/2m_{B_Q}$. As a result, the correlation function can be written in terms of the magnetic moment of heavy baryons as

$$\Pi = \lambda_{B_Q}^2 \frac{1}{m_{B_Q}^2 - p^2}\mu_{B_Q}\frac{1}{m_{B_Q}^2 - (p + q)^2}. \tag{7}$$

In order to calculate the correlation function in terms of quark and gluon degrees of freedom and photon distribution amplitudes, the expressions of the interpolating currents of the heavy baryons are needed. The general form of the interpolating currents of the heavy spin-1/2 positive parity baryons is given as (see for example [18])

$$\eta_{B_Q} = -\frac{1}{\sqrt{2}}\epsilon^{abc}\{(q_1^{aT}CQ^b)\gamma_5 q_2^c + t(q_1^{aT}C\gamma_5 Q^b)q_2^c$$
$$- (Q^{aT}Cq_2^b)\gamma_5 q_1^c - t(Q^{aT}C\gamma_5 q_2^b)q_1^c\}, \tag{8}$$

where a, b, c are the color indices, C is the charge conjugation operator, and t is an arbitrary parameter whose value at $t = -1$ gives the so-called Ioffe current. The quark contents of the sextet heavy sextet baryons are given in Table I.

Using the expression for the interpolating current and Wick's theorem, the theoretical part of the correlation function for the Σ_b^0 can be written as

$$\Pi^{\Sigma_b^0} = -3\{\gamma_5 S_d(x)S_b'(x)S_u(x)\gamma_5 + \gamma_5 S_u(x)S_b'(x)S_d(x)\gamma_5 + \gamma_5 S_u(x)\gamma_5\mathrm{Tr}[S_b(x)S_d'(x)] + \gamma_5 S_d(x)\gamma_5\mathrm{Tr}[S_u(x)S_b'(x)]$$
$$+ t(\gamma_5 S_d(x)\gamma_5 S_b'(x)S_u(x) + \gamma_5 S_u(x)\gamma_5 S_b'(x)S_d(x) + S_d(x)S_b'(x)\gamma_5 S_u(x)\gamma_5 + S_u(x)S_b'(x)\gamma_5 S_d(x)\gamma_5$$
$$+ \gamma_5 S_u(x)\mathrm{Tr}[S_b(x)\gamma_5 S_d'(x)] + S_u(x)\gamma_5\mathrm{Tr}[S_b(x)S_d'(x)\gamma_5] + \gamma_5 S_d(x)\mathrm{Tr}[S_u(x)\gamma_5 S_b'(x)] + S_d(x)\gamma_5\mathrm{Tr}[S_u(x)S_b'(x)\gamma_5])$$
$$+ t^2(S_d(x)\gamma_5 S_b'(x)\gamma_5 S_u(x) + S_u(x)\gamma_5 S_b'(x)\gamma_5 S_d(x) + S_d(x)\mathrm{Tr}[S_b(x)\gamma_5 S_u'(x)\gamma_5] + S_u(x)\mathrm{Tr}[S_d(x)\gamma_5 S_b'(x)\gamma_5])\}, \tag{9}$$

where $S' = CS^T C$, T symbolizes transposition operator, and S is the quark (light or heavy) propagator. The corresponding expressions of the correlation functions for the other members of the sextet baryons can be found from the Σ_{B_Q} by making the following replacements:

$$\Pi^{\Sigma_c^0} = \Pi^{\Sigma_c^+}(u \to d),$$

$$\Pi^{\Sigma_c^{++}} = \Pi^{\Sigma_c^+}(d \to u),$$

$$\Pi^{\Sigma_b^-} = \Pi^{\Sigma_b^0}(u \to d),$$

$$\Pi^{\Sigma_b^+} = \Pi^{\Sigma_b^0}(d \to u),$$

$$\Pi^{\Xi_c'^+} = \Pi^{\Sigma_c^+}(d \to s),$$

$$\Pi^{\Xi_c'^0} = \Pi^{\Sigma_c^+}(u \to d, d \to s),$$

$$\Pi^{\Xi_b'^0} = \Pi^{\Sigma_b^0}(d \to s),$$

$$\Pi^{\Xi_b'^-} = \Pi^{\Sigma_b^0}(u \to d, d \to s),$$

$$\Pi^{\Omega_c^0} = \Pi^{\Sigma_c^+}(u \to s, d \to s),$$

$$\Pi^{\Omega_b^-} = \Pi^{\Sigma_b^0}(u \to s, d \to s). \quad (10)$$

The correlation function described by Eq. (9) contains three different parts: (a) perturbative part, when a photon is radiated from a short distance (perturbative contribution); (b) a photon is radiated from a short distance from the quark propagators, and light quarks form a condensate (mixed contribution); (c) a photon is radiated from a long distance (nonperturbative contribution). The diagrams corresponding to these three different contributions are presented in Fig. 1.

In order to calculate the perturbative contribution of the correlation function, it is enough to make the replacement

$$S \to \int d^4 y S^{\text{free}}(x - y) \mathcal{A}(y) S^{\text{free}}(y), \quad (11)$$

for one of the quark propagators, where S_{free} is the free quark operator; and the other two are being the free propagators. The expressions of the free light and heavy quarks in coordinate representation are given as

$$S_q^{\text{free}} = \frac{i \rlap{/}{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2},$$

$$S_Q^{\text{free}} = \frac{m_{B_Q}^2}{4\pi^2} \frac{K_1(m_{B_Q} \sqrt{-x^2})}{\sqrt{-x^2}} + i \frac{m_{B_Q}^2}{4\pi^2} \frac{\rlap{/}{x} K_2(m_{B_Q} \sqrt{-x^2})}{(\sqrt{-x^2})^2}, \quad (12)$$

where K_i are the Bessel functions.

The contribution of part (c) can easily be calculated by replacing one of the light quark operator with

$$(S_q^{ab})_{\alpha\beta} \to -\frac{1}{4} \delta^{ab} \bar{q}^a \Gamma_i q^b (\Gamma_i)_{\alpha\beta}, \quad (13)$$

where Γ_j are the full set of Dirac matrices, and other quark operators that involve perturbative, as well as nonperturbative terms. The explicit forms of the "full" quark propagators can be found in [19] and [20].

Nonperturbative contribution is realized as the matrix element of the nonlocal operator $\bar{q}\Gamma_i q$ between the vacuum and one-photon states. These matrix elements are described in terms of photon distribution amplitudes, as are given below (see [21]):
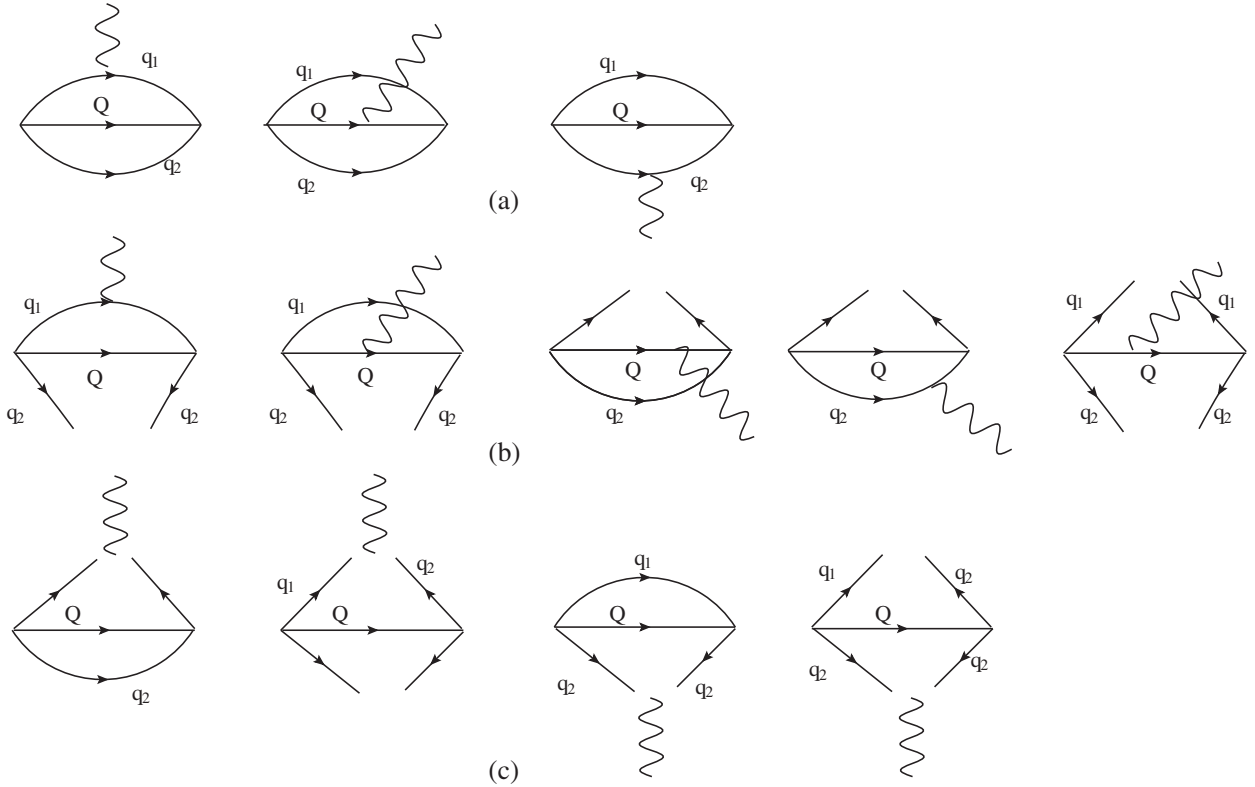

FIG. 1. Diagrams (a), (b), and (c) correspond to the perturbative, "mixed," and nonperturbative contributions, respectively.

$$\langle\gamma(q)|\bar{q}(x)\sigma_{\mu\nu}q(0)|0\rangle = -ie_q\bar{q}q(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int_0^1 due^{i\bar{u}qx}\left(\chi\varphi_\gamma(u) + \frac{x^2}{16}\mathbb{A}(u)\right)$$

$$-\frac{i}{2(qx)}e_q\langle\bar{q}q\rangle\left[x_\nu\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right) - x_\mu\left(\varepsilon_\nu - q_\nu\frac{\varepsilon x}{qx}\right)\right]\int_0^1 due^{i\bar{u}qx}h_\gamma(u),$$

$$\langle\gamma(q)|\bar{q}(x)\gamma_\mu q(0)|0\rangle = e_q f_{3\gamma}\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right)\int_0^1 due^{i\bar{u}qx}\psi^v(u),$$

$$\langle\gamma(q)|\bar{q}(x)\gamma_\mu\gamma_5 q(0)|0\rangle = -\frac{1}{4}e_q f_{3\gamma}\epsilon_{\mu\nu\alpha\beta}\varepsilon^\nu q^\alpha x^\beta\int_0^1 due^{i\bar{u}qx}\psi^a(u),$$

$$\langle\gamma(q)|\bar{q}(x)g_s G_{\mu\nu}(vx)q(0)|0\rangle = -ie_q\langle\bar{q}q\rangle(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{S}(\alpha_i),$$

$$\langle\gamma(q)|\bar{q}(x)g_s\tilde{G}_{\mu\nu}i\gamma_5(vx)q(0)|0\rangle = -ie_q\langle\bar{q}q\rangle(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\tilde{\mathcal{S}}(\alpha_i),$$

$$\langle\gamma(q)|\bar{q}(x)g_s\tilde{G}_{\mu\nu}(vx)\gamma_\alpha\gamma_5 q(0)|0\rangle = e_q f_{3\gamma}q_\alpha(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{A}(\alpha_i),$$

$$\langle\gamma(q)|\bar{q}(x)g_s G_{\mu\nu}(vx)i\gamma_\alpha q(0)|0\rangle = e_q f_{3\gamma}q_\alpha(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{V}(\alpha_i),$$

$$\langle\gamma(q)|\bar{q}(x)\sigma_{\alpha\beta}g_s G_{\mu\nu}(vx)q(0)|0\rangle = e_q\langle\bar{q}q\rangle\Bigg\{\left[\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right)\left(g_{\alpha\nu} - \frac{1}{qx}(q_\alpha x_\nu + q_\nu x_\alpha)\right)q_\beta\right.$$

$$-\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right)\left(g_{\beta\nu} - \frac{1}{qx}(q_\beta x_\nu + q_\nu x_\beta)\right)q_\alpha$$

$$-\left(\varepsilon_\nu - q_\nu\frac{\varepsilon x}{qx}\right)\left(g_{\alpha\mu} - \frac{1}{qx}(q_\alpha x_\mu + q_\mu x_\alpha)\right)q_\beta$$

$$+\left(\varepsilon_\nu - q_\nu\frac{\varepsilon x}{q.x}\right)\left(g_{\beta\mu} - \frac{1}{qx}(q_\beta x_\mu + q_\mu x_\beta)\right)q_\alpha\Bigg]\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_1(\alpha_i)$$

$$+\left[\left(\varepsilon_\alpha - q_\alpha\frac{\varepsilon x}{qx}\right)\left(g_{\mu\beta} - \frac{1}{qx}(q_\mu x_\beta + q_\beta x_\mu)\right)q_\nu\right.$$

$$-\left(\varepsilon_\alpha - q_\alpha\frac{\varepsilon x}{qx}\right)\left(g_{\nu\beta} - \frac{1}{qx}(q_\nu x_\beta + q_\beta x_\nu)\right)q_\mu$$

$$-\left(\varepsilon_\beta - q_\beta\frac{\varepsilon x}{qx}\right)\left(g_{\mu\alpha} - \frac{1}{qx}(q_\mu x_\alpha + q_\alpha x_\mu)\right)q_\nu$$

$$+\left(\varepsilon_\beta - q_\beta\frac{\varepsilon x}{qx}\right)\left(g_{\nu\alpha} - \frac{1}{qx}(q_\nu x_\alpha + q_\alpha x_\nu)\right)q_\mu\Bigg]\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_2(\alpha_i)$$

$$+\frac{1}{qx}(q_\mu x_\nu - q_\nu x_\mu)(\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_3(\alpha_i)$$

$$+\frac{1}{qx}(q_\alpha x_\beta - q_\beta x_\alpha)(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_4(\alpha_i)\Bigg\}. \tag{14}$$

In the definitions given above, χ is the magnetic susceptibility of the quarks, $\varphi_\gamma(u)$ is the leading twist-2, $\psi^v(u)$, $\psi^a(u)$, $\mathcal{A}$ and $\mathcal{V}$ are the twist-3, and $h_\gamma(u)$, $\mathbb{A}$, $\mathcal{T}_i$ ($i = 1, 2, 3, 4$) are the twist-4 photon DAs, respectively, whose explicit expressions are given in the Appendix. The measure $\mathcal{D}\alpha_i$ is defined as

$$\int\mathcal{D}\alpha_i = \int_0^1 d\alpha_{\bar{q}}\int_0^1 d\alpha_q\int_0^1 d\alpha_g\delta(1 - \alpha_{\bar{q}} - \alpha_q - \alpha_g). \tag{15}$$

In constructing the sum rules for the magnetic moments of heavy sextet baryons it is necessary to equate the coefficients of the structure $\not{p}\not{\varepsilon}\not{q}$ from the phenomenological and theoretical representations of the correlation function. The following steps in obtaining the final form of the sum rules for the magnetic moment are: Fourier transformation, Borel transformation over the variables p^2 and $(p + q)^2$ variables, and continuum subtraction in order to suppress the contribution of the higher states

and continuum. After these procedures we get the sum rules for the magnetic moment of the sextet heavy baryons, which can schematically be written in the following form:

$$\lambda_{B_{Q_i}}^2 \mu_{B_{Q_i}} e^{m_{B_{Q_i}}^2/M^2} = \Pi^{B,\text{theor}}, \qquad (16)$$

where the function $\Pi^{B,\text{theor}}$ contain perturbative and non-perturbative contributions.

The expression for $\Pi^{B,\text{theor}}$ is quite lengthy, and for this reason we do not present it here.

In order to determine the magnetic moments of heavy sextet baryons, the value of the overlap amplitude is needed, which can be found from the analysis of the two-point correlation function. The residues of the heavy sextet baryons are calculated in [22], and their expressions can be found in this paper.

III. RESULTS AND DISCUSSION

Having obtained the sum rules for the magnetic moments of the heavy sextet spin-1/2 baryons, we are now ready to perform the numerical analysis. The main input parameters of the light cone QCD sum rules for the magnetic moments are the photon distribution functions DAs [21]. Their explicit expressions are presented in the Appendix. The values of the other input parameters are: $\langle \bar{u}u \rangle_{\mu=1\text{GeV}} = \langle \bar{d}d \rangle_{\mu=1\text{GeV}} = -(0.243)^3 \text{GeV}^3$, $\langle \bar{s}s \rangle_{\mu=1\text{GeV}} = 0.8 \langle \bar{u}u \rangle_{\mu=1\text{GeV}}$ [23], $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$ [22], $\Lambda = (0.5 \pm 0.1) \text{ GeV}$ [24] $f_{3\gamma} = -0.0039 \text{ GeV}^2$ [21], $m_{s(\mu=2 \text{ GeV})} = (111 \pm 6) \text{ MeV}$ [25].

A few words about the magnetic susceptibility involved in the numerical analysis are in order. Its value is estimated to have the value $\chi(1 \text{ GeV}) = -4.4 \text{ GeV}^{-2}$ in [26]. In [21], using the vector dominance model ansatz and QCD sum rules its value is predicted to be $\chi(1 \text{ GeV}) = -(3.15 \pm 0.15) \text{ GeV}^{-2}$. Furthermore, from an analysis of the radiative decays of heavy mesons its value is found to be $\chi(1 \text{ GeV}) = -(2.85 \pm 0.50) \text{ GeV}^{-2}$ [27]. In numerical analysis we have used all these predicted values of the magnetic susceptibility.

It should also be remembered that the sum rules for the magnetic moments involve the following auxiliary parameters: the arbitrary parameter t appearing in the expressions of the interpolating currents, Borel mass parameter M^2, and the continuum threshold s_0. The magnetic moment must be independent of these parameters. In order to find "regions" of these parameters where magnetic moment exhibits good stability with respect to their variations we proceed as follows. First, we try to find the upper and lower bounds of Borel mass parameter at fixed values of s_0 and t. The upper bound of M^2 can be determined by requiring that the contribution due to the continuum threshold should be less than half of the contribution coming from the perturbative part. The lower bound of M^2 can be found by demanding

that the higher powers of $1/M^2$ contribute less than the leading twist contributions. Our numerical analysis shows that these two conditions are satisfied simultaneously if M^2 ranges in the following regions:

$$
\begin{aligned}
2.0 \text{ GeV}^2 &\leq M^2 \leq 3.0 \text{ GeV}^2 & \Sigma_c \\
2.2 \text{ GeV}^2 &\leq M^2 \leq 3.4 \text{ GeV}^2 & \Xi_c' \\
2.5 \text{ GeV}^2 &\leq M^2 \leq 4.0 \text{ GeV}^2 & \Omega_c \\
5.0 \text{ GeV}^2 &\leq M^2 \leq 6.0 \text{ GeV}^2 & \Sigma_b \\
5.0 \text{ GeV}^2 &\leq M^2 \leq 6.4 \text{ GeV}^2 & \Xi_b' \\
5.2 \text{ GeV}^2 &\leq M^2 \leq 7.0 \text{ GeV}^2 & \Omega_b.
\end{aligned} \qquad (17)
$$

In regard to the continuum threshold s_0 appearing in the sum rules, it is known that the values of this arbitrary parameter are related to the energy of first excited state. The difference $\sqrt{s_0} - m_{\text{ground}}$, where m_{ground} is the ground state mass of the baryon, is the energy needed to excite the particle to its first energy state. This quantity usually varies between 0.3 and 0.8 GeV. Analysis of the mass sum rules shows that, in order to reproduce the experimental mass of the sextet baryons the continuum threshold should vary in the following domain:

$$
\sqrt{s_0} = \begin{cases}
(3.1 \pm 0.1) \text{ GeV} & \Sigma_c \\
(3.2 \pm 0.1) \text{ GeV} & \Xi_c' \\
(3.3 \pm 0.1) \text{ GeV} & \Omega_c \\
(6.6 \pm 0.2) \text{ GeV} & \Sigma_b \\
(6.7 \pm 0.2) \text{ GeV} & \Xi_b' \\
(6.8 \pm 0.2) \text{ GeV} & \Omega_b.
\end{cases} \qquad (18)
$$

Having determined the sum rules, input parameters, and working regions of all auxiliary parameters, we perform numerical analysis to calculate the magnetic moment μ_{B_Q} of the heavy sextet spin-1/2 baryons, whose results we can summarize as follows. As examples, in Figs. 2 and 3 we present the dependence of μ_{Σ_Q} on M^2 at several fixed values of t and at the fixed value of s_0 listed above for the baryons Σ_c^{++} and Σ_b^0, respectively. We observe from these figures that the magnetic moments of Σ_Q show good stability with respect to the variation of the Borel mass parameter in the aforementioned domains. The next step in finding the values of the magnetic moments of the baryons under consideration is to determine the working region of the arbitrary parameter t, where μ_{B_Q} is practically independent of its variation. For this purpose, in Figs. 4 and 5 the dependence of μ_{Σ_Q} on $\cos\theta$, where $\tan\theta = t$ for the Σ_c^{++} and Σ_b^0 baryons, at several fixed values of the Borel mass parameter M^2 and at the predetermined value of the continuum threshold s_0, are presented, respectively. It follows from these figures that the magnetic moments of
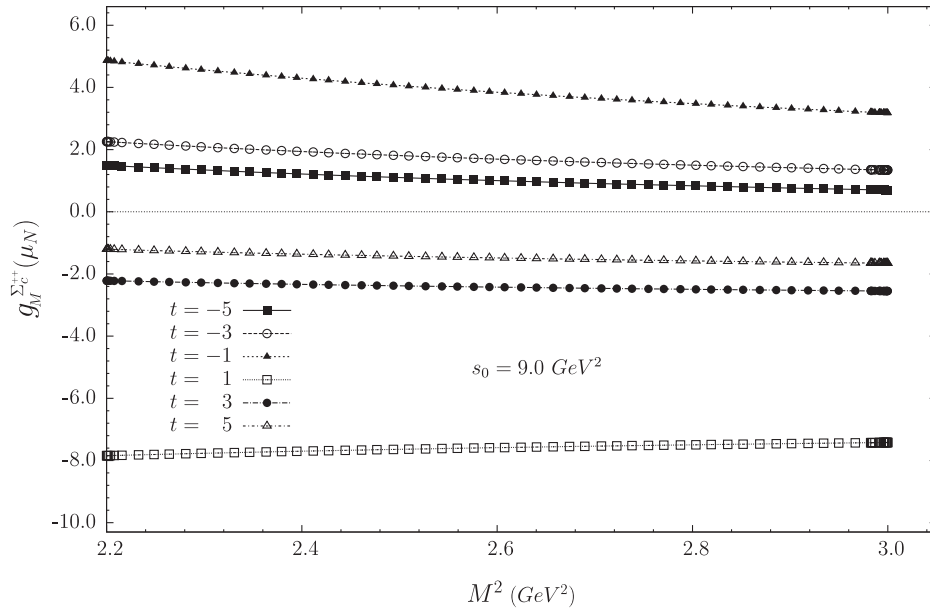
FIG. 2. The dependence of the magnetic moment of the Σ_c^{++} baryon on M^2, at several fixed values of t, and at $s_0 = 9.0$ GeV^2.

the heavy sextet spin-1/2 baryons seem to be practically independent of the arbitrary parameter t in the domain $-0.9 \leq \cos\theta \leq -0.6$. We also perform a similar analysis for the Σ_c^{++} and Σ_b^0 baryons at $s_0 = 11.0$ GeV^2 and $s_0 = 43.0$ GeV^2, respectively. It is observed that the results change about 5% in both cases, which can be concluded that the results are practically insensitive to the variation in the values of the continuum threshold s_0. Our final results on the magnetic moments of the heavy sextet spin-1/2 baryons are presented in Table II. For comparison, in the same Table we present the predictions of other approaches

on the magnetic moments of the relevant baryons, such as nonrelativistic quark model [11], bag model [9], phenomenological relativistic quark model [12], quark model with confinement law potential [13], relativistic quark model [4], hypercentral model [6], chiral constituent model [14], and QCD sum rules method [10]. The total uncertainty coming from the quark condensates, m_0^2, continuum threshold s_0, parameters entering into the photon DAs, variation of the Borel parameter M^2 is maximally around 35%. Moreover, in order to test the convergence of the operator product expansion terms, we perform a numerical analysis and find
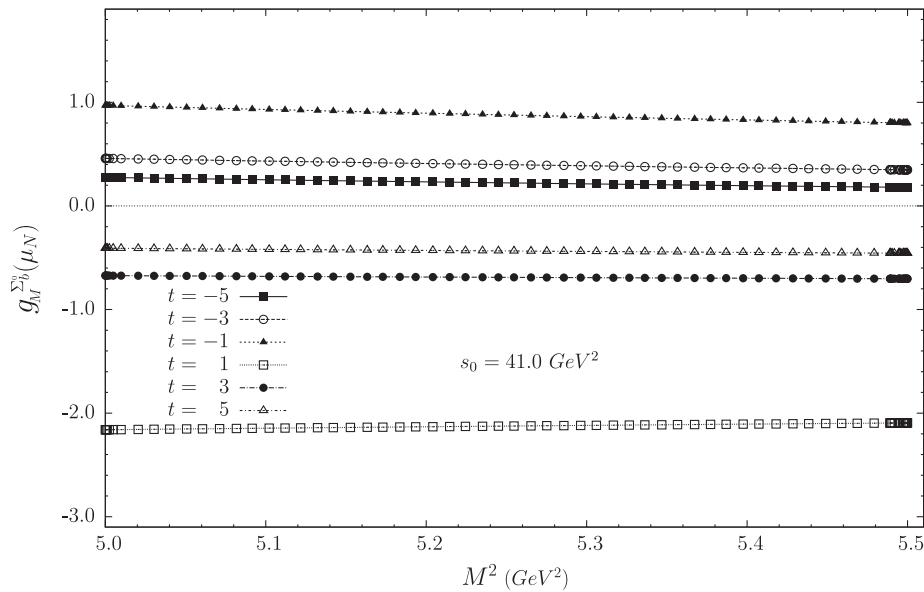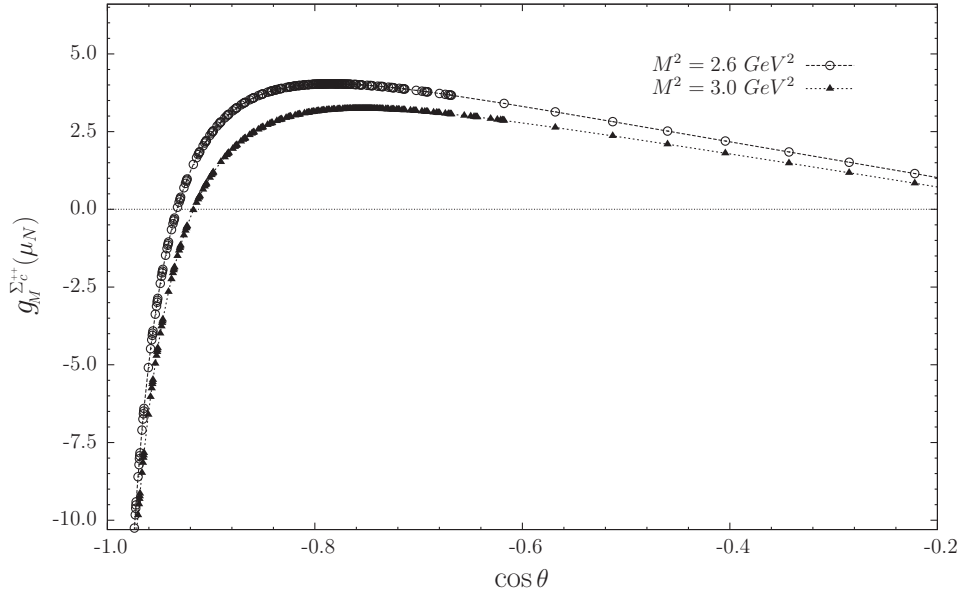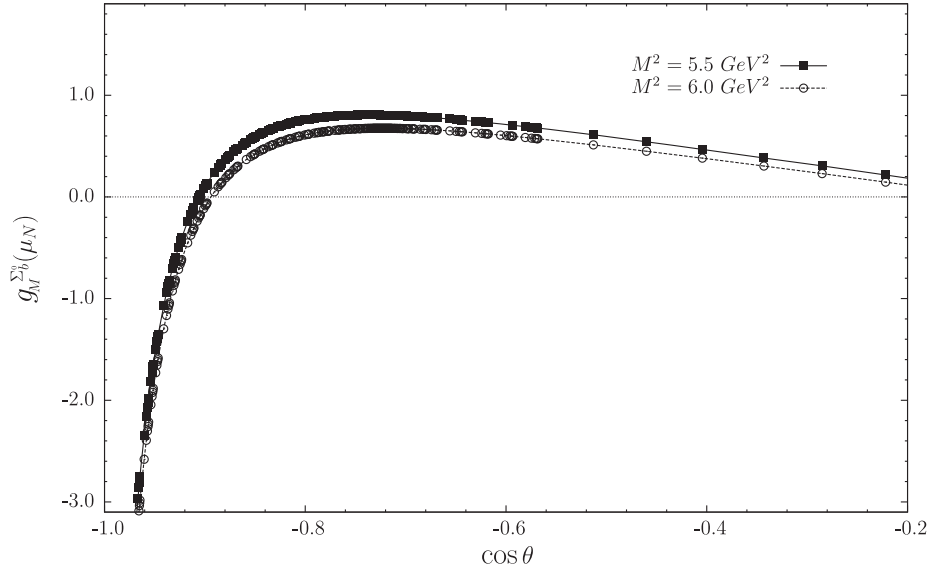

FIG. 3. The same as Fig. 1, but for the Σ_b^0 baryon, at $s_0 = 41.0$ GeV^2.

FIG. 4.   The dependence of the magnetic moment of the Σ_c^{++} baryon on $\cos\theta$, at several fixed values of M^2, and at $s_0 = 9.0$ GeV^2.

FIG. 5.   The same as Fig. 3, but for the Σ_b^0 baryon, at $s_0 = 41.0$ GeV^2.

out that the perturbative term (after subtraction of the continuum contribution) constitutes approximately 75% and 73% of the total result for the Σ_c^{++} (at $s_0 = 11$ GeV^2, $M^2 = 3$ GeV^2) and Σ_b^0 (at $s_0 = 43$ GeV^2, $M^2 = 6$ GeV^2), respectively. In other words we observe very good convergence of the operator product expansion terms.

We observe from this Table that almost all approaches give, more or less, similar predictions, except the results of [9] and [4] (especially for the charmed baryons), which are smaller.

Using our results one can easily deduce the following relations among the magnetic moments of heavy baryons:

$$\mu_{\Sigma_c^{++}} + \mu_{\Sigma_c^0} \simeq 2\mu_{\Sigma_c^+},$$

$$\mu_{\Sigma_c^{++}} + \mu_{\Omega_c^0} \simeq 2\mu_{\Xi_c'^+},$$

$$\mu_{\Sigma_c^{++}} + 2\mu_{\Xi_c'^0} \simeq \mu_{\Sigma_c^0} + 2\mu_{\Xi_c'^+},$$

$$\mu_{\Sigma_b^+} + \mu_{\Sigma_b^-} \simeq 2\mu_{\Sigma_b^0},$$

$$\mu_{\Sigma_b^+} + \mu_{\Omega_b^-} \simeq 2\mu_{\Xi_b'^0},$$

$$\mu_{\Sigma_b^+} + 2\mu_{\Xi_b'^-} \simeq \mu_{\Sigma_b^-} + 2\mu_{\Xi_b'^0}.$$

The direct measurement of the magnetic moments of heavy baryons is unlikely in the near future. Therefore, any indirect estimations of the magnetic moments of the heavy

TABLE II.   Magnetic moments of the heavy sextet, $J^P = \frac{1}{2}^+$ baryons in units of the nuclear magneton μ_N.

	[4]	[6]	[9]	[10]	[11]	[12]	[13]	[14]	Our paper
Σ_c^0	−1.04	−1.015	−1.043	−1.6 ± 0.2	−1.37	−1.38	−1.39	−1.6	−1.5 ± 0.35
Σ_c^+	0.36	0.5	0.318	0.6 ± 0.1	0.49	0.49	0.525	0.3	0.5 ± 0.15
Σ_c^{++}	1.76	2.279	1.679	2.1 ± 0.3	2.35	2.36	2.44	2.22	2.4 ± 0.5
$\Xi_c'^0$	−0.95	−0.966	−0.914	⋯	−1.18	−1.12	−1.12	−1.32	−1.2 ± 0.3
$\Xi_c'^+$	0.47	0.711	0.591	⋯	0.89	0.75	0.796	0.76	0.8 ± 0.2
Ω_c^0	−0.85	−0.96	−0.774	⋯	−0.94	−0.86	−0.85	−0.9	−0.9 ± 0.2
Σ_b^-	−1.01	−1.047	−0.778	⋯	−1.22	⋯	−1.256	⋯	−1.3 ± 0.3
Σ_b^0	0.53	0.592	0.422	⋯	0.64	⋯	0.659	⋯	0.6 ± 0.2
Σ_b^+	2.07	2.229	1.622	⋯	2.5	⋯	2.575	⋯	2.4 ± 0.5
$\Xi_b'^-$	−0.91	−0.902	−0.66	⋯	−1.02	⋯	−0.985	⋯	−1.2 ± 0.3
$\Xi_b'^0$	0.66	0.766	0.556	⋯	0.9	⋯	0.930	⋯	0.7 ± 0.2
Ω_b^-	−0.82	−0.96	−0.545	⋯	−0.79	⋯	−0.714	⋯	−0.8 ± 0.2

baryons could be very useful. For example, it could help extracting information about the mass spectrum of the heavy baryons. As is well known, experimentally measured mass difference is attributed to the hyperfine splitting. Moreover, the magnetic moments of quarks are proportional to the chromomagnetic moments, which determine the hyperfine splitting in the baryon spectrum. Following this reasoning, the magnetic moments of the Λ_c and Λ_b baryons are estimated in [28]. Hence, this approach could be an essential tool in estimating the magnetic moments of the heavy baryons.

In conclusion, the magnetic moments of the heavy sextet $J^P = \frac{1}{2}^+$ baryons are calculated in the framework of the light cone QCD sum rules method. Empirically, linearly independent relations among the magnetic moments of the sextet baryons are obtained. Comparison of our results with the predictions of other approaches is presented.

APPENDIX: PHOTON DISTRIBUTION AMPLITUDES

Explicit forms of the photon DAs [21]:

$$\varphi_\gamma(u) = 6u\bar{u}[1 + \varphi_2(\mu)C_2^{\frac{3}{2}}(u - \bar{u})],$$

$$\psi^v(u) = 3[3(2u-1)^2 - 1] + \frac{3}{64}(15w_\gamma^V - 5w_\gamma^A)[3 - 30(2u-1)^2 + 35(2u-1)^4],$$

$$\psi^a(u) = [1 - (2u-1)^2][5(2u-1)^2 - 1]\frac{5}{2}\left(1 + \frac{9}{16}w_\gamma^V - \frac{3}{16}w_\gamma^A\right),$$

$$\mathcal{A}(\alpha_i) = 360\alpha_q\alpha_{\bar{q}}\alpha_g^2\left[1 + w_\gamma^A\frac{1}{2}(7\alpha_g - 3)\right],$$

$$\mathcal{V}(\alpha_i) = 540w_\gamma^V(\alpha_q - \alpha_{\bar{q}})\alpha_q\alpha_{\bar{q}}\alpha_g^2,$$

$$h_\gamma(u) = -10(1 + 2\kappa^+)C_2^{\frac{1}{2}}(u - \bar{u}),$$

$$\mathbb{A}(u) = 40u^2\bar{u}^2(3\kappa - \kappa^+ + 1) + 8(\zeta_2^+ - 3\zeta_2)[u\bar{u}(2 + 13u\bar{u}) + 2u^3(10 - 15u + 6u^2)\ln(u) + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln(\bar{u})],$$

$$\mathcal{T}_1(\alpha_i) = -120(3\zeta_2 + \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g,$$

$$\mathcal{T}_2(\alpha_i) = 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)[(\kappa - \kappa^+) + (\zeta_1 - \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)],$$

$$\mathcal{T}_3(\alpha_i) = -120(3\zeta_2 - \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g,$$

$$\mathcal{T}_4(\alpha_i) = 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)[(\kappa + \kappa^+) + (\zeta_1 + \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)],$$

$$\mathcal{S}(\alpha_i) = 30\alpha_g^2\{(\kappa + \kappa^+)(1 - \alpha_g) + (\zeta_1 + \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) + \zeta_2[3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)]\},$$

$$\tilde{\mathcal{S}}(\alpha_i) = -30\alpha_g^2\{(\kappa - \kappa^+)(1 - \alpha_g) + (\zeta_1 - \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) + \zeta_2[3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)]\}.$$

The parameters entering the above DA's are borrowed from [21] whose values are $\varphi_2(1 \text{ GeV}) = 0$, $w_\gamma^V = 3.8 \pm 1.8$, $w_\gamma^A = -2.1 \pm 1.0$, $\kappa = 0.2$, $\kappa^+ = 0$, $\zeta_1 = 0.4$, $\zeta_2 = 0.3$, $\zeta_1^+ = 0$, and $\zeta_2^+ = 0$.

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