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Origin of hierarchical structures of quark and lepton mass matrices

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It is the goal of the so-called "Yukawaon" model to give a unified description of masses, mixing, and *CP* violation parameters of quarks and leptons without using any hierarchical (family number-dependent) parameters besides the charged lepton masses. However, in the conventional model so far, we have compelled ourselves to introduce a phase matrix $P = \mathrm{daig}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ with the phase parameters (ϕ_1, ϕ_2, ϕ_3) as family number-dependent parameters other than the observed charged lepton mass values m_{ei}. In this paper, we present a revised model in which we are successful in describing the *CP* violating phase parameters ϕ_i in terms of m_{ei}. In the present model, by using the parameter values slightly changed from the previous version, we predict a value $\delta^{\ell}_{CP} \simeq -76°$ for the *CP* violating phase δ^{ℓ}_{CP} in the standard expression of the lepton mixing matrix U_{PMNS}, and $\delta^{q}_{CP} \simeq 72°$ for that of the quark mixing matrix.

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I. INTRODUCTION

A. What is the Yukawaon model?

One of the big subjects in particle physics is the investigation of the origin of flavors. There is an attractive idea that the flavor physics is understood from the point of view of a family symmetry [1]. However, the symmetry has to be explicitly broken by the Yukawa coupling constants Y_f ($f = \nu, e, u, d$) if we suppose the family symmetry to be a continuous symmetry. Therefore, the symmetry has been usually considered as a discrete symmetry. If we adhere to the basic idea that the flavor symmetry should be a continuous symmetry which is unbroken at the start, we are forced to consider that the Yukawa coupling constants are effective coupling constants Y_f^{eff} which are given by vacuum expectation values (VEVs) of scalars ("Yukawaons") Y_f with 3×3 components [2]:

$$(Y_f^{\mathrm{eff}})_i{}^j = \frac{y_f}{\Lambda}\langle Y_f\rangle_i{}^j \quad (f = u, d, \nu, e), \tag{1.1}$$

where Λ is an energy scale of the effective theory. In the Yukawaon model, all the flavons [3] are expressed by 3×3 components of U(3). We consider no substructures of it such as 2×2 and so on. Here, the Yukawaons are defined in the following would-be Yukawa interactions that are invariant under U(3) family symmetry:

$$\begin{aligned} H_Y = &\frac{y_\nu}{\Lambda}(\bar{\ell}_L)^i(\hat{Y}_\nu)_i{}^j(\nu_R)_j H_u \\ &+ \frac{y_e}{\Lambda}(\bar{\ell}_L)^i(\hat{Y}_e)_i{}^j(e_R)_j H_d + y_R(\bar{\nu}_R)^i(Y_R)_{ij}(\nu_R^c)^j \\ &+ \frac{y_u}{\Lambda}(\bar{q}_L)^i(\hat{Y}_u)_i{}^j(u_R)_j H_u + \frac{y_d}{\Lambda}(\bar{q}_L)^i(\hat{Y}_d)_i{}^j(d_R)_j H_d, \end{aligned} \tag{1.2}$$

where $\ell_L = (\nu_L, e_L)$ and $q_L = (u_L, d_L)$ are $\mathrm{SU}(2)_L$ doublets. H_u and H_d are two Higgs doublets. The third term in Eq. (1.2) leads to the so-called neutrino seesaw mass matrix [4] $M_\nu = \langle\hat{Y}_\nu^T\rangle\langle Y_R\rangle^{-1}\langle\hat{Y}_\nu\rangle$, where $\langle\hat{Y}_\nu\rangle$ and $\langle Y_R\rangle$ correspond to the Dirac and Majorana mass matrices of neutrinos, respectively. Hereafter, for convenience, we use notation $\hat{A}$, A, and $\bar{A}$ for fields with $\mathbf{8}+\mathbf{1}$, $\mathbf{6}$, and $\mathbf{6}^*$ of U(3), respectively.

VEV relations among Yukawaons have been obtained by supersymmetric vacuum conditions from the superpotentials that are invariant under *R*-charge conservation and family symmetries such as $\mathrm{U}(3) \times \mathrm{U}(3)'$, which are broken at $\mu = \Lambda$ and $\mu = \Lambda'$. (We assume $\Lambda \ll \Lambda'$.) We have noticed that the observed hierarchical structures of masses and mixings in the quarks and leptons can be generated by one common origin in the Yukawaon model because relations among Yukawaon VEVs (i.e., effective Yukawa coupling constants) are directly given by forms of multiplication (not by forms of the sum). Therefore, in order to check this idea, it is important to check whether the observed hierarchical structures of masses and mixings in the quarks and leptons can be expressed only in terms of the observed charged lepton masses without using any family number-dependent parameters, in other words, only by using the charged lepton masses and family number-independent parameters. Here, the family number-independent parameters mean, for example, coefficients of the following matrices:

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad X_3 = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{1.3}$$

In this paper, we propose a model in which the hierarchy of the observed structures of masses and mixings in the quarks and leptons comes from the common origin, i.e., that of the charged lepton masses (m_e, m_μ, m_τ), although

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we do not ask for the origin of the hierarchical structure of the observed charged lepton masses. The details of the other fundamental assumptions used in the Yukawaon model will be summarized in the Appendix A.

B. What is new?

In the present model, the VEV relations among flavons are essentially the same as those in the previous paper [5]. The VEV structures of the Yukawaons are universally given by a bilinear form [5]

$$\langle \hat{Y}_f \rangle_i{}^j = k_f \langle \Phi_f \rangle_{i\alpha} \langle \bar{\Phi}_f \rangle^{\alpha j} + \xi_f \mathbf{1}_i{}^j \quad (f = u, d, \nu, e), \tag{1.4}$$

where i and α are indices of U(3) and U(3)$'$, respectively. The VEV structures of Φ_f and $\bar{\Phi}_f$ are also given a bilinear form of a fundamental VEV $\langle \Phi_0 \rangle$ (and $\langle \bar{\Phi}_0 \rangle$) of the fundamental Yukawaon Φ_0 (and $\bar{\Phi}_0$), which is $(\mathbf{3}, \mathbf{3})$ [and $(\mathbf{3}^*, \mathbf{3}^*)$] of U(3) $\times$ U(3)$'$,

$$\begin{aligned} &\langle \bar{P}_f \rangle^{ik} \langle \Phi_f \rangle_{kl} \langle \bar{P}_f \rangle^{lj} = k'_f \langle \bar{\Phi}_0 \rangle^{i\alpha} \langle S_f \rangle_{\alpha\beta} \langle \bar{\Phi}_0 \rangle^{\beta j}, \\ &\langle P_f \rangle_{ik} \langle \bar{\Phi}_f \rangle^{kl} \langle P_f \rangle_{lj} = k'_f \langle \Phi_0 \rangle_{i\alpha} \langle \bar{S}_f \rangle^{\alpha\beta} \langle \Phi_0 \rangle_{\beta j}, \\ &(f = u, d, \nu, e), \end{aligned} \tag{1.5}$$

where

$$\begin{aligned} &\langle \bar{\Phi}_0 \rangle^{i\alpha} = \langle \Phi_0 \rangle_{i\alpha} = v_0 \mathrm{diag}(x_1, x_2, x_3)_{i\alpha} \\ &(x_1^2 + x_2^2 + x_3^2 = 1). \end{aligned} \tag{1.6}$$

The VEV structures of P_f and $\bar{P}_f$ in Eq. (1.5) will be discussed in the next section. In the VEV relations (1.5), the flavons S_f belong to $(\mathbf{1}, \mathbf{6})$ of U(3) $\times$ U(3)$'$. The VEV structures of them are given by the form[1]

$$\langle S_f \rangle_{\alpha\beta} = v_{Sf} (\mathbf{1}_{\alpha\beta} + a_f (X_3)_{\alpha\beta}). \tag{1.7}$$

We consider that the form (1.7) is due to a symmetry breaking U(3)$' \to$ S$_3$ at $\mu = \Lambda'$. Neutrino mass matrix M_ν is given by a seesaw form [4]

$$(M_\nu)^{ij} = \langle Y_\nu^T \rangle^i{}_k (\langle Y_R \rangle^{-1})^{kl} \langle Y_\nu \rangle_l{}^j, \tag{1.8}$$

where the VEV relation of Y_R is given by[2]

$$\langle Y_R \rangle_{ij} = k_R (\langle \hat{Y}_e \rangle_i{}^k \langle \Phi_u \rangle_{kj} + \langle \Phi_u \rangle_{ik} \langle \hat{Y}_e^T \rangle^k{}_j). \tag{1.9}$$

Since we discuss mass ratios and mixing only, we are interested only in the relative ratios among flavon VEVs. Therefore, hereafter, we will drop family-common coefficients k_f, k'_f, k_R, v_0, v_{Sf}, and so on for simplicity.

[1]The form (1.6) was suggested by a "democratic universal seesaw" mass matrix model [6], in which quark mass matrices are given by a form $\langle \Phi_e \rangle (\mathbf{1} + a_f X_3) \langle \Phi_e \rangle$.

[2]This relation was first found from the phenomenological investigation in Ref. [7].

The VEV relations (1.4)–(1.9) are almost the same as those in the previous paper [5]. Nevertheless, the present Yukawaon model has the following new characteristic features:

(i) We put a new selection rule for VEV structures $\langle P_f \rangle$. As we will discuss in Sec. II A, we take the following forms:

$$\begin{aligned} &\langle P_f \rangle = v_P P \equiv v_P \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}) \quad \text{for } P_u \text{ and } P_e, \\ &\langle P_f \rangle = v_E E \equiv v_E \mathrm{diag}(1, 1, 1) \quad \text{for } P_d \text{ and } P_\nu. \end{aligned} \tag{1.10}$$

(ii) As a result of (i), the fitting of CP violating parameters is considerably changed from the previous analysis [5], especially in the neutrino sector. Note that the phase matrix P_u $(= P)$ affects not only the Cabibbo-Kobayashi-Maskawa (CKM) [8] quark mixing matrix V_{CKM} but also the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [9] lepton mixing matrix U_{PMNS} through the VEV relation (1.8). As a result, as we will see in Sec. III, the CP violating phase parameters δ_{CP}^q and δ_{CP}^ℓ in the standard expression of V_{CKM} and U_{PMNS} become not independent, and we obtain $\delta_{CP}^\ell \simeq -\delta_{CP}^q \simeq -70°$ unlike the previous papers.

(iii) So far [5,10,11], we have inevitably needed to introduce a VEV matrix P defined by Eq. (1.10) in order to fit CKM mixing matrix V_{CKM} reasonably. On the other hand, it is our aim of the present paper to describe all the masses and mixing of the quarks and leptons in terms of family number-independent parameters and the charged lepton masses. Therefore, the introduction of the phase parameters ϕ_i is against our aim and unwelcome as they are. Thus, in the present paper, we try to denote the family number-dependent parameters ϕ_i in terms of the observed charged lepton masses m_{ei}. The details will be discussed in Sec. IV.

In Sec. II, we construct a mass matrix model base on a Yukawaon model in which the phase matrix P appears in the up-quark sector. In Sec. III, parameter fittings are discussed. In Sec. IV, we will propose a new relation between P and m_{ei}. Finally, Sec. V is devoted to concluding remarks.

II. MODEL

The formulation of model building is almost the same as that in the previous paper [5]. Therefore, let us discuss only items that are different from the previous model.

A. VEV structures of P_f

Prior to discussing VEV forms $\langle P_f \rangle$ and $\langle \bar{P}_f \rangle$ given in Eqs. (1.5) and (1.10), let us consider the following superpotential consisting of P and E:

$$W_P = \frac{\lambda_1}{\Lambda}\mathrm{Tr}[P\bar{P}E\bar{E}] + \frac{\lambda_2}{\Lambda}\mathrm{Tr}[P\bar{P}]\mathrm{Tr}[E\bar{E}]. \tag{2.1}$$

Here, in order to distinguish P from E, we assign R charges of P and E as

$$R(P) = R(\bar{P}) = \frac{1}{2}(1-\Delta), \quad R(E) = R(\bar{E}) = \frac{1}{2}(1+\Delta), \tag{2.2}$$

so that $R(P)+R(\bar{P})+R(E)+R(\bar{E}) = 2$. The supersymmetric vacuum conditions lead to

$$\langle P\rangle\langle\bar{P}\rangle = \mathbf{1}, \qquad \langle E\rangle\langle\bar{E}\rangle = \mathbf{1}. \tag{2.3}$$

We define specific solutions of (2.3) as

$$\langle P\rangle = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}), \qquad \langle E\rangle = \mathrm{diag}(1,1,1). \tag{2.4}$$

We consider that each $\langle P_f\rangle$ given in Eq. (1.5) is given by either $\langle P\rangle$ or $\langle E\rangle$ in Eq. (2.3) under the D-term condition as discussed in (2.5) below.

In general, VEV matrix $\langle\bar{A}\rangle$ is related to VEV matrix $\langle A\rangle$ under the D term condition as

$$\langle\bar{A}\rangle = \langle A\rangle^*, \quad \text{or} \quad \langle\bar{A}\rangle = \langle A\rangle. \tag{2.5}$$

For the VEV matrix $\langle S_f\rangle$, $\langle\bar{S}_f\rangle$, $\langle P\rangle$, and $\langle\bar{P}\rangle$ in Eq. (1.5), let us take

$$\langle S_f\rangle = \langle\bar{S}_f\rangle = \mathbf{1} + a_f e^{i\alpha_f} X_3, \tag{2.6}$$

while

$$\langle\bar{P}\rangle = \langle P\rangle^* = \mathrm{diag}(e^{-i\phi_1}, e^{-i\phi_2}, e^{-i\phi_3}), \tag{2.7}$$

in addition to the choice (1.6). Here parameters x_i, ϕ_i, a_f, and α_f are real. Then, from the relation (1.5), we obtain

$$\langle P_f\rangle^*\langle\Phi_f\rangle\langle P_f\rangle^* = \langle P_f\rangle\langle\bar{\Phi}_f\rangle\langle P_f\rangle = \langle\Phi_0\rangle\langle S_f\rangle\langle\Phi_0\rangle. \tag{2.8}$$

If we take $\langle\bar{\Phi}_f\rangle = \langle\Phi_f\rangle^*$, Eq. (2.8) leads to

$$\langle P_f\rangle^*\langle\Phi_f\rangle\langle P_f\rangle^* = \langle P_f\rangle\langle\bar{\Phi}_f\rangle\langle P_f\rangle = (\langle P_f\rangle^*\langle\Phi_f\rangle\langle P_f\rangle^*)^*, \tag{2.9}$$

so that $\langle\Phi_0\rangle\langle S_f\rangle\langle\Phi_0\rangle$ has to be real. In other words, the parameter α_f in Eq. (2.6) has to be zero. On the other hand, if we take $\langle\bar{\Phi}_f\rangle = \langle\Phi_f\rangle$, Eq. (2.8) leads to

$$\langle P_f\rangle^*\langle\Phi_f\rangle\langle P_f\rangle^* = \langle P_f\rangle\langle\bar{\Phi}_f\rangle\langle P_f\rangle = \langle P_f\rangle^*\langle\Phi_f\rangle\langle P_f\rangle^*, \tag{2.10}$$

so that $\langle\bar{P}_f\rangle = \langle P_f\rangle$; i.e., the parameters ϕ_i in Eq. (2.7) have to be zero.

In conclusion, a case in which $\alpha_f \neq 0$ and $\phi_i \neq 0$ are simultaneously satisfied is ruled out. Only the following two cases are allowed: either a case that $\alpha_f = 0$ and $\phi_i \neq 0$ ($P_f = P$) or a case that $\alpha_f \neq 0$ and $\phi_i = 0$ ($P_f = E$), although this is nothing but a result due to our postulations (2.6) and (2.7).

It should be noted that whether $\alpha_f \neq 0$ or $\alpha_f = 0$ (i.e., whether P_f is E or P) is determined from the phenomenological point of view as we discuss below: The parameters α_f affect not only CP violation but also mass ratios. In the up-quark sector, as we will discuss in Sec. III we can fit up-quark mass ratios m_u/m_c and m_c/m_t by taking two parameters a_u and ξ_u (keeping $\alpha_u = 0$). Therefore, we regard the up-quark sector as the case of $\alpha_f = 0$, so that we regard $\langle P_f\rangle$ as $\langle P_f\rangle = \langle P\rangle$. Also, since we have taken $a_e = 0$ as we will discuss in the Appendix A, we have to regard $\langle P_e\rangle$ as $\langle P_e\rangle = \langle P\rangle$. Note that $\langle P\rangle$ and $\langle\hat{Y}_e\rangle$ are diagonal, so that they are commutable to each other. Therefore, $\langle P_e\rangle$ does not play any essential physical role in the parameter fitting of the masses and mixing of quarks. Hereafter, we denote $\langle P_e\rangle$ as $\langle E\rangle$ from the practical point of view, except for a case of counting the R charge. On the other hand, in the down-quark sector, we cannot fit down-quark mass ratios m_d/m_s and m_s/m_b without the help of $\alpha_d \neq 0$. Therefore, we regard the down-quark sector as a case of $\langle P_f\rangle = \langle E\rangle$. Thus, we have the selection rule, $\langle P_f\rangle = \langle P\rangle$ or $\langle P_f\rangle = \langle E\rangle$, as a phenomenological one. For the neutrino sector, we have no phenomenological information. For simplicity, we take a fewer parameter scheme ($\alpha_\nu \neq 0$ rather than $\phi_i^\nu \neq 0$). Hereafter, we will use the notation P_f as

$$P_f = P \quad \text{for } f = u, e, P_f = E \quad \text{for } f = d, \nu. \tag{2.11}$$

Sometimes, for convenience, we use notations P_u, P_e, and so on, although we identify P_u and P_e as one flavon P, and also P_d and P_ν as one flavon E.

The phase matrix $\langle P\rangle$ does not affect mass ratios. The phase parameters in $\langle P_u\rangle = \langle P\rangle$ affect the CP violating phase δ_{CP}^q in the CKM mixing matrix V_{CKM} and the CP violating phase δ_{CP}^ℓ in the PMNS mixing matrix U_{PMNS}, because the phase in P_u can affect Y_R through Φ_u as shown in Eq. (1.9).

B. R-charge assignments

To distinguish each Yukawaon from the others, we assume that $\hat{Y}_f$ have different R charges from each other by considering R-charge conservation [a global U(1) symmetry in $N = 1$ supersymmetry]. Of course, the R-charge conservation is broken at an energy scale Λ, at which the U(3) family symmetry is broken. For R parity assignments, we inherit those in the standard supersymmetry model; i.e., R parities of Yukawaons $\hat{Y}_f$ (and all flavons) are the same as those of Higgs particles [i.e., $P_R(\text{fermion}) = -1$ and $P_R(\text{scalar}) = +1$], while quarks and leptons are assigned to $P_R(\text{fermion}) = +1$ and $P_R(\text{scalar}) = -1$.

In the Yukawaon model, the R-charge assignment is essential from the phenomenological point of view: If we give unsuitable R charges, we will meet unwelcome combinations among flavons and numerous terms with an unwelcome higher dimension in the superpotential. Therefore, even if it is a minor change of the R-charge assignment, it will give a considerable change of the phenomenological results. In the present Yukawaon model, the number of flavons is larger than that of VEV relations. Therefore, in general, we cannot uniquely determine R charges of flavons. Since we demand to assign R charges as simple as possible, we put the following rules for simplicity:

(i) We assign the same R charge to flavons A and $\bar{A}$,

$$R(A) = R(\bar{A}), \tag{2.12}$$

independently whether $\langle\bar{A}\rangle = \langle A\rangle^*$ or $\langle\bar{A}\rangle = \langle A\rangle$. Then, we obtain R-charge relations

$$R(\hat{Y}_f) = 2R(\Phi_f) \equiv 2r_f \quad (f = u, d, \nu, e), \tag{2.13}$$

and

$$R(\Phi_f) = R(\bar{\Phi}_f) = R(S_f) + 2R(\Phi_0) - 2R(P_f) \quad (f = u, d, \nu, e), \tag{2.14}$$

from Eqs. (1.4) and (1.5), and

$$R(P_u) = R(P_e) = R(P) \equiv \frac{1}{2}(1+\Delta),$$
$$R(P_d) = R(P_\nu) = R(E) \equiv \frac{1}{2}(1-\Delta), \tag{2.15}$$

from Eqs. (2.2) and (2.11). Therefore, from Eq. (2.14), we obtain the following relations:

$$R(\Phi_e) - R(S_e) = R(\Phi_u) - R(S_u) = 2R(\Phi_0) - (1+\Delta),$$
$$R(\Phi_\nu) - R(S_\nu) = R(\Phi_d) - R(S_d) = 2R(\Phi_0) - (1-\Delta). \tag{2.16}$$

(ii) We can regard that R charges of $\hat{Y}_f$ are determined only by those of the $SU(2)_L$ singlet fermions f^c. Therefore, we simply assign

$$R(\ell H_u) = R(\ell H_d) = R(qH_u) = R(qH_d) \equiv r_H + 2. \tag{2.17}$$

Since those have different quantum numbers of $U(1)_Y$, we can distinguish those from each other in spite of the relation (2.17). Then, we obtain a simple R-charge relation

$$R(\hat{Y}_f) + R(f^c) = -r_H. \tag{2.18}$$

For Y_R, we obtain

$$R(Y_R) = 2 - 2R(\nu^c) = 2r_H + 2 + 2R(\hat{Y}_\nu), \tag{2.19}$$

from Eq. (2.18). On the other hand, from Eq. (1.9), $R(Y_R)$ must be the satisfied relation

$$R(Y_R) = R(\Phi_u) + 2R(\Phi_e). \tag{2.20}$$

From Eqs. (2.19) and (2.20), we have the following constraint:

$$2R(\Phi_e) - 4R(\Phi_\nu) + R(\Phi_u) = 2r_H + 2. \tag{2.21}$$

Even under these constraints, we cannot still completely fix the R charges of whole flavons. In the present model, R-charge assignments are not so essential, so that it is enough to assign R charges to distinguish flavons with the same U(3) from each other. That is, we are satisfied with any R-charge numbers that satisfy the relations (2.13)–(2.21). Nevertheless, it is desirable to have explicit R-charge assignments as simple as possible. Therefore, let us go on our search for explicit R-charge assignments.

First, for simplicity, we put

$$R(\Phi_0) = \frac{1}{2}. \tag{2.22}$$

Then. Eq. (2.16) becomes the simpler relations

$$R(S_f) = R(\Phi_f) + \Delta \quad (f = e, u),$$
$$R(S_f) = R(\Phi_f) - \Delta \quad (f = \nu, d). \tag{2.23}$$

Now, let us discuss possible R-charge assignments for Yukawaons $\hat{Y}_f$ under the conditions discussed above. If we have $R(\hat{Y}_f) = 0$, then we can attach the field $\hat{Y}_f$ on any term in superpotential. Therefore, we require $R(\hat{Y}_f) \neq 0$ for any $f = e, \nu, d, u$. Also, we have to require $R(\hat{Y}_f\hat{Y}_{f'}) \neq 0$ for any combination of f and f'. As a result, we have to consider that whole R values of $\hat{Y}_f$ are positive. Furthermore, we speculate that the values of R will be described by simple integers. Of course, the R charges have to satisfy the relation (2.21). Therefore, we assign simpler R charges to the Yukawaons $\hat{Y}_f$ on trial as follows:

$$(R(\hat{Y}_e), R(\hat{Y}_u), R(\hat{Y}_\nu), R(\hat{Y}_d)) = (1, 2, 3, 4), \tag{2.24}$$

that is,

TABLE I. Assignments of $SU(2)_L \times SU(3)_c \times U(3) \times U(3)'$. For R charges, see Sec. II C. We assign the same R charges for flavons A and $\bar{A}$, e.g., $R(A) = R(\bar{A})$. For a special choice, r_e, r_ν, r_u, and r_d are taken as $r_e = 1/2$, $r_\nu = 3/2$, $r_u = 2/2$, and $r_d = 4/2$.

	$\ell = (\nu, e)$	$f^c = \nu^c, e^c$	$q = (u, d)$	$f^c = u^c, d^c$	H_u	H_d
$SU(2)_L$	**2**	**1**	**2**	**1**	**2**	**2**
$SU(3)_c$	**1**	**1**	**3**	$\mathbf{3}^*$	**1**	**1**
$U(3)$	**3**	$\mathbf{3}^*$	**3**	$\mathbf{3}^*$	**1**	**1**
$U(3)'$	**1**	**1**	**1**	**1**	**1**	**1**
R	2	$-(2r_f + r_H)$	2	$-(2r_f + r_H)$	r_H	r_H

$\hat{Y}_f$	Y_R	$\bar{\Phi}_f$	Φ_f	$\bar{\Phi}_0$	$\bar{\Phi}_0$	$S_{e,u}$	$\bar{S}_{e,u}$	$S_{\nu,d}$	$\bar{S}_{\nu,d}$
1	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
1	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$\mathbf{8} + \mathbf{1}$	**6**	**6**	$\mathbf{6}^*$	**3**	$\mathbf{3}^*$	**1**	**1**	**1**	**1**
1	**1**	**1**	**1**	**3**	$\mathbf{3}^*$	**6**	$\mathbf{6}^*$	**6**	$\mathbf{6}^*$
$2r_f$	r_R	r_f		$1/2$		$r_{e,u} - \Delta$		$r_{\nu,d} + \Delta$	

P	$\bar{P}$	E	$\bar{E}$	$\hat{\Theta}_f$	$\bar{\Theta}_R$	$\Theta_{\Phi f}$	$\bar{\Theta}_{\Phi f}$
1	**1**	**1**	**1**	**1**	**1**	**1**	**1**
1	**1**	**1**	**1**	**1**	**1**	**1**	**1**
6	$\mathbf{6}^*$	**6**	$\mathbf{6}^*$	$\mathbf{8} + \mathbf{1}$	$\mathbf{6}^*$	**6**	$\mathbf{6}^*$
1	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$\frac{1}{2}(1 + \Delta)$		$\frac{1}{2}(1 - \Delta)$		$2 - 2r_f$	$2 - r_R$	$1 - R(S_f)$	

$$(R(\Phi_e), R(\Phi_u), R(\Phi_\nu), R(\Phi_d)) = \left(\frac{1}{2}, 1, \frac{3}{2}, 2\right). \tag{2.25}$$

This assignment satisfies the condition (2.20) for $R(Y_R)$ with $r_H = -3$.

In Table I, as a summary of Sec. II, we present the assignments of $SU(2)_L \times SU(3)_c \times U(3) \times U(3)'$ and the R charges of the fields in the present model.

III. PARAMETER FITTING

A. How many parameters?

We have already given the form of the effective Yukawa coupling constants $\langle Y_f \rangle$ $(f = e, \nu, d, u)$ in Eqs. (1.4)–(1.10). For the convenience of phenomenological parameter fitting, let us summarize again such VEV relations. In this subsection, for convenience, the notations "$\langle$" and "$\rangle$" are dropped. The notations that distinguish transformation property, A, $\bar{A}$, and $\hat{A}$ are also dropped. When $\langle \bar{A} \rangle = \langle A \rangle^*$, we denote $\langle \bar{A} \rangle$ as A^*. Under such simplified expressions, the VEV relations (1.4)–(1.10) are given as follows;

$$\begin{aligned} Y_e &= \Phi_e \Phi_e^*, \\ \Phi_e &= P^* \Phi_0 \Phi_0 P^*, \\ \Phi_e^* &= P \Phi_0 \Phi_0 P, \\ \Phi_0 &= \mathrm{diag}(x_1, x_2, x_3), \end{aligned} \tag{3.1}$$

$$\begin{aligned} Y_\nu &= \Phi_\nu \Phi_\nu + \xi_\nu e^{i\beta_\nu} \mathbf{1}, \\ \Phi_\nu &= E \Phi_0 (\mathbf{1} + a_\nu e^{i\alpha_\nu} X_3) \Phi_0 E, \\ \beta_\nu &= \mathrm{Arg}(\mathrm{Tr}[\Phi_\nu \Phi_\nu]), \end{aligned} \tag{3.2}$$

$$\begin{aligned} Y_u &= \Phi_u \Phi_u^* + \xi_u e^{i\beta_u} \mathbf{1}, \\ \Phi_u &= P^* \Phi_0 (\mathbf{1} + a_u X_3) \Phi_0 P^*, \\ \Phi_u^* &= P \Phi_0 (\mathbf{1} + a_u X_3) \Phi_0 P, \\ \beta_u &= \mathrm{Arg}(\mathrm{Tr}[\Phi_u \Phi_u^*]), \end{aligned} \tag{3.3}$$

$$\begin{aligned} Y_d &= \Phi_d \Phi_d + \xi_d e^{i\beta_d} \mathbf{1}, \\ \Phi_d &= E \Phi_0 (\mathbf{1} + a_d e^{i\alpha_d} X_3) \Phi_0 E, \\ \beta_d &= \mathrm{Arg}(\mathrm{Tr}[\Phi_d \Phi_d]), \end{aligned} \tag{3.4}$$

$$\begin{aligned} M_\nu &= Y_\nu Y_R^{-1} Y_\nu, \\ Y_R &= Y_e \Phi_u + \Phi_u Y_e. \end{aligned} \tag{3.5}$$

Since we are interested only in the mass ratios and mixings, we use dimensionless expressions $\Phi_0 = \mathrm{diag}(x_1, x_2, x_3)$ (with $x_1^2 + x_2^2 + x_3^2 = 1$), $P = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, 1)$, and $E = \mathbf{1} = \mathrm{diag}(1, 1, 1)$. Therefore, the parameters a_e, a_ν, a_u, a_d, ξ_ν, ξ_u, and ξ_d are redefined by Eqs. (3.1)–(3.5). Note that the parameters ξ_f given in Eq. (1.4) are, in general, not real. Therefore, in the present subsection, we denote the complex parameter ξ_f as $\xi_f e^{i\beta_f}$. The phase parameter β_f is given by $\beta_f = \mathrm{Arg}(\mathrm{Tr}[\Phi_f \bar{\Phi}_f])$ as shown in the previous paper [5], so that the phase parameters β_f are not free parameters.

TABLE II. Predicted values vs observed values.

	$\|V_{us}\|$	$\|V_{cb}\|$	$\|V_{ub}\|$	$\|V_{td}\|$	δ^q_{CP}	r^u_{12}	r^u_{23}	r^d_{12}	r^d_{23}
Pred	0.2261	0.0426	0.00360	0.00920	72.4°	0.0458	0.0600	0.0611	0.0312
Obs	0.22536	0.0414	0.00355	0.00886	69.4°	0.045	0.060	0.053	0.019
	± 0.00061	± 0.0012	± 0.00015	$^{+0.00033}_{-0.00032}$	$\pm 3.4°$	$^{-0.010}_{+0.013}$	± 0.005	$^{-0.003}_{+0.005}$	$^{-0.006}_{+0.006}$
	$\sin^2 2\theta_{12}$	$\sin^2 2\theta_{23}$	$\sin^2 2\theta_{13}$	$R_\nu [10^{-2}]$	δ^ℓ_{CP}	$m_{\nu 1}$ [eV]	$m_{\nu 2}$ [eV]	$m_{\nu 3}$ [eV]	$\langle m \rangle$ [eV]
Pred	0.857	0.993	0.0964	3.16	−76.0°	0.00046	0.00879	0.0502	0.00377
Obs	0.846	0.999	0.093	3.09	⋯	⋯	⋯	⋯	$< \mathrm{O}(10^{-1})$
	± 0.021	$^{-0.018}_{+0.001}$	± 0.008	± 0.15					

In the phase matrix P defined by Eq. (2.7), physical values are only differences among (ϕ_1, ϕ_2, ϕ_3), i.e., $\phi_i - \phi_j$, so that we can take one of ϕ_i $(i = 1, 2, 3)$ as zero in the parameter fitting for $V_{\rm CKM}$. In this paper, we put $\phi_3 = 0$, so that free parameters are (ϕ_1, ϕ_2). Note that $\langle P \rangle$ and $\langle \bar{P} \rangle$ do not affect $\langle Y_e \rangle$ practically, because $\langle \Phi_0 \rangle$ and $\langle Y_e \rangle$ are diagonal, so that $\langle P \rangle$ and $\langle \bar{P} \rangle$ are commutable with $\langle \Phi_0 \rangle$ and $\langle Y_e \rangle$. Therefore, in the present model, we have ten adjustable parameters, $(a_\nu, \alpha_\nu, \xi_\nu)$, (a_u, ξ_u), (a_d, α_d, ξ_d), and (ϕ_1, ϕ_2) for the 16 observable quantities, except for the parameters (x_1, x_2, x_3) that are determined from the charged lepton masses as shown in the Appendix A.

B. Quark mass ratios

The ways of the parameter fitting and input values are essentially the same as those in the previous paper [5]. Therefore, we give only the results of parameter values,

$$(a_u, \xi_u) = (-1.4715, -0.001521), \qquad (3.6)$$

from the observed values m_u/m_c and m_c/m_t at $\mu = M_Z$ [12], and

$$-(a_d, \alpha_d, \xi_d) = (-1.4733, 15.694°, +0.004015), \qquad (3.7)$$

from the observed values m_d/m_s and m_s/m_b at $\mu = M_Z$ [12] (and also [13]). Those values are slightly changed because of the minor change of the model. The details of input values and predicted values are summarized in Table II.

C. CKM mixing

Parameter fitting for the observed CKM matrix parameters is also substantially the same as in the previous paper [5]. In Fig. 1, with taking $\xi_u = -0.001521$, $a_u = -1.4715$, $a_d = -1.47312$, $\alpha_d = 15.7°$, and $\xi_d = 0.004091$, we draw allowed regions in the (ϕ_1, ϕ_2) parameter plane, which are obtained from the observed values [14] of the CKM mixing matrix elements and the observed value [15] of the CP violating phase parameter δ^q_{CP} in the standard expression of $V_{\rm CKM}$. These input values and fitting results are also given in Table II.

As shown in Fig. 1, all the experimental constraints on CKM parameters are satisfied by fine-tuning the parameters ϕ_1 and ϕ_2 as

$$(\phi_1, \phi_2) = (-41.815°, -15.128°), \qquad (3.8)$$

which leads to the predicted values for the CKM mixing matrix elements and the CP violating phase parameter δ^q_{CP} as shown in Table II.

D. PMNS mixing

Now let us present the result for the neutrino sector. Substantial differences between the present and previous papers appear in the parameter fitting of the PMNS lepton mixing.

We have already fixed the four parameters a_u, ξ_u, ϕ_1, and ϕ_2 as Eqs. (3.6)–(3.8). The remaining free parameters in the

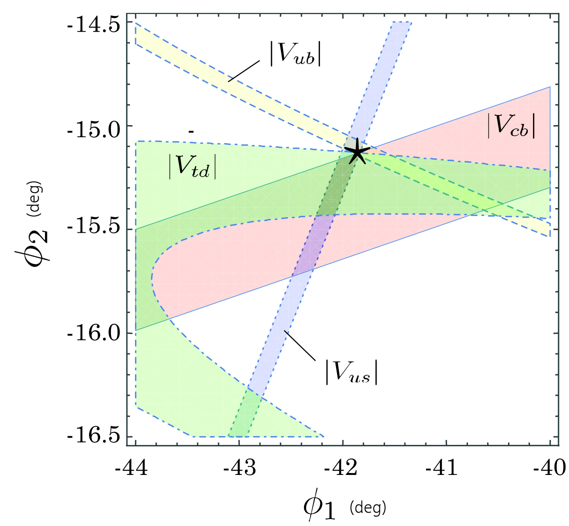

FIG. 1 (color online). Allowed region in the (ϕ_1, ϕ_2) parameter plane obtained by the observed values of the CKM mixing matrix elements $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, and $|V_{td}|$. We draw allowed regions obtained from the observed constraints of the CKM mixing matrix elements shown in Table II, with taking $(a_u, \xi_u) = (-1.4715, -0.001521)$ and $(a_d, \alpha_d, \xi_d) = (-1.4733, 15.694°, +0.004015)$. We find that the parameter set around $(\phi_1, \phi_2) = (-41.815°, -15.128°)$ indicated by a star ($\star$) is consistent with all the observed values.

neutrino sector are only $(a_\nu, \alpha_\nu, \xi_\nu)$. We determine the parameter values of $(a_\nu, \alpha_\nu, \xi_\nu)$ as follows:

$$(a_\nu, \alpha_\nu, \xi_\nu) = (-2.59, -27.3°, -0.0115), \tag{3.9}$$

which are obtained so as to reproduce the observed values [14] of the following PMNS mixing angles and R_ν:

$$\sin^2 2\theta_{12} = 0.846 \pm 0.021, \qquad \sin^2 2\theta_{23} > 0.981,$$
$$\sin^2 2\theta_{13} = 0.093 \pm 0.008, \tag{3.10}$$

$$R_\nu \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{32}} = \frac{m^2_{\nu 2} - m^2_{\nu 1}}{m^2_{\nu 3} - m^2_{\nu 2}} = \frac{(7.53 \pm 0.18) \times 10^{-5}\ \mathrm{eV}^2}{(2.44 \pm 0.06) \times 10^{-3}\ \mathrm{eV}^2}$$
$$= (3.09 \pm 0.15) \times 10^{-2}. \tag{3.11}$$

In Fig. 2, we show the contour plots of the observed PMNS mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and R_ν in the (a_ν, α_ν) parameter space for the case of $\xi_\nu = -0.0115$ with taking $(\phi_1, \phi_2) = (-41.815°, -15.128°)$ and $(a_u, \xi_u) = (-1.4715, -0.001521)$. It is found from Fig. 2 that all the PMNS mixing parameters are well consistent with the observed values in Eqs. (3.10) and (3.11) by fine-tuning the parameters a_ν, α_ν, and ξ_ν as

$$(a_\nu, \alpha_\nu, \xi_\nu) = (-2.59, -27.3°, -0.0115), \tag{3.12}$$

which leads to the predicted values for the PMNS mixing angles, R_ν, and the Dirac CP violating phase parameter δ^ℓ_{CP} in the standard expression of U_{PMNS} as follows:

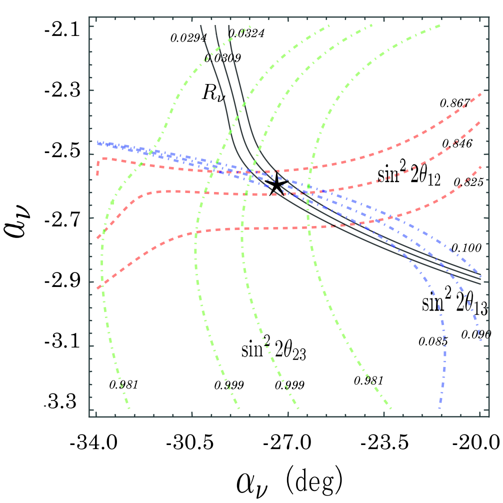

FIG. 2 (color online). Contour curves of the center, upper, and lower values of the observed PMNS mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and R_ν in the (a_ν, α_ν) parameter space. We draw the curves for the case of $\xi_\nu = -0.0115$ and $(\phi_1, \phi_2) = (-41.815°, -15.128°)$ with taking $(a_u, \xi_u) = (-1.4715, -0.001521)$. We find that the parameter set around $(a_\nu, \alpha_\nu) = (-2.59, -27.3°)$ indicated by a star ($\star$) is consistent with all the observed values.

$$\sin^2 2\theta_{12} = 0.857,$$
$$\sin^2 2\theta_{23} = 0.993,$$
$$\sin^2 2\theta_{13} = 0.0964,$$
$$R_\nu = 0.0316,$$
$$\delta^\ell_{CP} = -76.0°. \tag{3.13}$$

It should be noted that our model predicts $\delta^\ell_{CP} = -76.0°$ for the Dirac CP violating phase in the lepton sector. This is very interesting because the value shows a size similar to $\delta^q_{CP} = +72.4°$ in the CKM mixing matrix.

We can predict neutrino masses, for the parameters given by (3.6), (3.8), and (3.9), as follows:

$$m_{\nu 1} \simeq 0.00046\ \mathrm{eV}, \qquad m_{\nu 2} \simeq 0.00879\ \mathrm{eV},$$
$$m_{\nu 3} \simeq 0.0502\ \mathrm{eV}, \tag{3.14}$$

by using the input value [14] $\Delta m^2_{32} \simeq 0.00244\ \mathrm{eV}^2$. We also predict the effective Majorana neutrino mass [16] $\langle m \rangle$ in the neutrinoless double beta decay as

$$\langle m \rangle = |m_{\nu 1}(U_{e1})^2 + m_{\nu 2}(U_{e2})^2 + m_{\nu 3}(U_{e3})^2|$$
$$\simeq 3.8 \times 10^{-3}\,\mathrm{eV}. \tag{3.15}$$

In Table II, we summarize our predictions of the CKM and the PMNS mixing parameters, quark mass ratios, and neutrino masses together with the observed values.

IV. VEV RELATION BETWEEN P AND Φ_0

We have tried to describe all Yukawaon VEV matrices $\langle \hat{Y}_f \rangle$ by using only the observed charged lepton masses m_{ei} as input values. We have also tried to understand the CP violating phase only by using phase parameters α_f, which are phases of family number-independent parameters a_f. Nevertheless, all such attempts have failed because we always needed a phase matrix P in order to fit reasonable CKM mixings and the quark mass ratios. In this paper, we accept the existence of P, and we try to understand the values of the phase parameters ϕ_i in P from the charged lepton mass values m_{ei}.

In the present model, we have flavon VEVs with diagonal form, P, $\bar{P}$, E, $\bar{E}$, Φ_0, $\bar{\Phi}_0$, Φ_e, $\bar{\Phi}_e$, and $\hat{Y}_e$. (Here, we omit $\langle$ and $\rangle$.) In considering combinations of U(3) $\mathbf{8} + \mathbf{1}$ scalars out of those flavons, we have to consider a combination without the parameter Δ for E and P because the R charges of Φ_0 and Φ_e do not contain the parameter Δ. Only a combination with P whose R charge does not include the parameter Δ is

$$(P\bar{E} + E\bar{P})_i{}^j = \delta_i{}^j (e^{i\phi_i} + e^{-i\phi_i}) = \delta_i^j 2\cos\phi_i, \tag{4.1}$$

with R charge of $R = \frac{1}{2}(1 + \Delta) + \frac{1}{2}(1 - \Delta) = +1$. On the other hand, since we have R charges

$$R(\Phi_e) = \frac{1}{2}, \qquad R(\Phi_0) = \frac{1}{2}, \tag{4.2}$$

for Φ_e and Φ_0 as discussed in Sec. II C, we have only two combinations that have an R charge of $R = +1$, $(\Phi_e)_{ik}(\bar{\Phi}_e)^{kj}$ and $(\Phi_0)_{i\alpha}(\bar{\Phi}_0)^{\alpha j}$. [Note that $(\Phi_0\bar{\Phi}_e + \Phi_e\bar{\Phi}_0)$ cannot be a candidate because it has $R = +1$ but is not a U(3)′ singlet.] Therefore, we can take superpotential

$$W = \lambda_1 \mathrm{Tr}[(P\bar{E} + E\bar{P})\hat{\Theta}_P] + \lambda_2 \mathrm{Tr}[(\Phi_e\bar{\Phi}_e + b\Phi_0\bar{\Phi}_0)\hat{\Theta}_P], \tag{4.3}$$

so that we obtain

$$k(P\bar{E} + E\bar{P}) = \Phi_e\bar{\Phi}_e + b\Phi_0\bar{\Phi}_0, \tag{4.4}$$

i.e.,

$$2k\cos\phi_i = x_i^4 + bx_i^2, \tag{4.5}$$

where we have used the dimensionless expressions of P, E, Φ_0, and Φ_e, Eq. (2.4), Eq. (1.6) with $v_0 = 1$, and so on.

Eliminating the coefficient k in Eq. (4.5), we obtain two equations

$$\frac{\cos\phi_1}{\cos\phi_3} = \frac{x_1^4 + bx_1^2}{x_3^4 + bx_3^2}, \tag{4.6}$$

$$\frac{\cos\phi_2}{\cos\phi_3} = \frac{x_2^4 + bx_2^2}{x_3^4 + bx_3^2}. \tag{4.7}$$

In Sec. III, we have obtained the numerical results $\phi_1 = -41.815°$ and $\phi_2 = -15.128°$ by putting ϕ_3 as $\phi_3 = 0$. To avoid confusion, we use notation $\tilde{\phi}_i$ for these numerical results of ϕ_i. Since we can choose any value of ϕ_0 in $\phi_i \to \phi_i + \phi_0$, we define ϕ_i in Eq. (4.5) as

$$\phi_1 = \phi_0 + \tilde{\phi}_1, \qquad \phi_2 = \phi_0 + \tilde{\phi}_2, \qquad \phi_3 = \phi_0. \tag{4.8}$$

Equations (4.6) and (4.7) have two unknown parameters ϕ_0 and b under the input values $\tilde{\phi}_1$ and $\tilde{\phi}_2$. So, we obtain

$$\phi_0 = -45.903°, \qquad b = -1.11586, \tag{4.9}$$

which means

$$\phi_1 = -87.718°, \qquad \phi_2 = -61.031°, \qquad \phi_3 = -45.903°. \tag{4.10}$$

Regrettably, since we need two input parameters ϕ_0 and b in order to predict the values $\tilde{\phi}_1$ and $\tilde{\phi}_2$, the present model has no predictability for the phase parameters (ϕ_1, ϕ_2, ϕ_3). (If we use the fitting value $\tilde{\phi}_1 = -41.815°$ as an input value in addition to the input value $b = -1.11586$, we can predict the value $\tilde{\phi}_2$ together with the value of ϕ_0.) However, note that the parameters ϕ_i are family number-dependent parameters, while the parameters ϕ_0 and b are family number-independent parameters. Therefore, the aim of the Yukawaon model that we understand the mass spectra and mixings of all the quarks and leptons only in terms of the charged lepton mass spectrum and without using any other family number-dependent parameters has been achieved in this scenario.

V. CONCLUDING REMARKS

In the past Yukawaon models [5,11], we were forced to introduce the phase matrix P defined by Eq. (2.7) in order to fit the observed CKM mixing parameters. The most remarkable point in the present model is that we have succeeded in describing the family number-dependent parameters (ϕ_1, ϕ_2, ϕ_3) in the phase matrix P by using the family number-independent parameters ϕ_0 and b. Therefore, we can say that the main aims of the Yukawaon model have been achieved in the present work. Now, we may conclude that the observed hierarchical structures of quark and lepton masses and mixings are brought by a sole origin, namely the observed hierarchy of the charged lepton masses. [However, it is not the purpose of the present paper to reveal the origin of the charged lepton mass spectrum (m_e, m_μ, m_τ). We leave this investigation to our future task.]

The successful results in the present work suggests the following items: (i) The flavor basis in which the charged lepton mass matrix M_e is diagonal has a more fundamental basis in the flavor physics. (ii) The parameters (x_1, x_2, x_3) defined by Eq. (1.6) are fundamental parameters in quark and lepton physics. Note that the parameter values (m_e, m_μ, m_τ) are extremely hierarchical, while the parameter values (x_1, x_2, x_3) are mildly hierarchical. Understanding of the values of (x_1, x_2, x_3) will be left to our next task in future. Then, the relation $(m_e + m_\mu + m_\tau)/(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = 2/3$ [17] may play an essential role in investigating the origin of the parameter values (x_1, x_2, x_3). For reference, we have given a trial model on the charged lepton mass relation within the framework of the present Yukawaon model in the Appendix B, although this is only a trial one.

Related to the scenario given in Sec. IV, we have given a minor change of the R-charge assignments in the present paper. Even if it is a minor change, the change gives considerable effects on the VEV relations among the flavons. We have given reparameter fitting for the observed masses and mixings of quarks and neutrinos. In conclusion, as seen in Secs. III and IV, we have obtained reasonable results. Our predicted values are listed in Table II.

In the present model, there are four phase parameters α_ν, α_d, and $(\tilde{\phi}_1, \tilde{\phi}_2)$. The parameters α_ν and α_d play a role in giving mass ratios in the neutrino and down-quark sectors, respectively. The parameters $(\tilde{\phi}_1, \tilde{\phi}_2)$ contribute commonly to the CP violating phase parameters in the CKM and the PMNS mixing matrices, so that those play an essential role in the predicted values of both δ^q_{CP} and δ^ℓ_{CP}. It is interesting that, in spite of different values between α_d and α_ν, the CP violating parameters δ^q_{CP} and δ^ℓ_{CP} are predicted to take a similar magnitude, $\delta^q_{CP} \sim -\delta^\ell_{CP} \sim 70°$.

Of course, the present Yukawaon model has to be still improved with respect to the R-charge assignments, number of flavons, number of adjustable parameters, CP violating phase parameters, and so on. The unification of the CP violating phase parameters in the present paper into a simpler scenario and the investigation of the origin of the parameters (x_1, x_2, x_3) are our future tasks.

APPENDIX A: BASIC ASSUMPTIONS IN THE YUKAWAON MODEL

The framework of the Yukawaon model has been changed little by little. The framework of the present model is substantially the same as in the previous paper [5]. Although the previous model has been considerably changed from earlier versions of the Yukawaon model, the present paper did not mention any basic assumption of the model. Therefore, in this appendix, let us summarize basic assumptions in the recent Yukawaon model.

We consider that the charged lepton mass matrix is the most fundamental one compared with other mass matrices and that the charged lepton mass values play an essential role in understanding the flavor physics. Our postulations are as follows:

(i) There is a fundamental flavon Φ_0, and the reference basis in the flavor physics is defined by the diagonal basis of $\langle\Phi_0\rangle$ and $\langle\bar{\Phi}_0\rangle$:

$$\langle\Phi_0\rangle = \langle\bar{\Phi}_0\rangle \equiv v_0 \mathrm{diag}(x_1, x_2, x_3), \tag{A1}$$

where x_i are real parameters with $x_1^2 + x_2^2 + x_3^3 = 1$.

(ii) In the reference basis, the $\mathrm{U}(3)'$ family symmetry is broken into S_3 at $\mu = \Lambda'$ $(\Lambda' \gg \Lambda)$; i.e., VEVs of flavons S_f and $\bar{S}_f$ take the form (1.6).

(iii) In the reference basis, the charged lepton mass matrix $\langle\hat{Y}_e\rangle$ is diagonal and real as well as $\langle\Phi_0\rangle$ and $\langle\bar{\Phi}_0\rangle$, and it should be described only in terms of the fundamental parameters x_i. Therefore, with demanding simplicity too, we require

$$a_e = 0, \qquad \xi_e = 0. \tag{A2}$$

This means $x_i \propto m_{ei}^{1/4}$ $[m_{ei} = (m_e, m_\mu, m_\tau)]$. In Sec. III, we use the following parameter values of x_i:

$$(x_1, x_2, x_3) = (0.115144, 0.438873, 0.891141). \tag{A3}$$

In (A3), we have used running mass values $m_e(\mu) = 0.000486847\,\mathrm{GeV}$, $m_\mu(\mu) = 0.102751\,\mathrm{GeV}$, and $m_\tau(\mu) = 1.7467$ GeV as the charged lepton mass values at $\mu = M_Z$, because our numerical predictions in the quark mass ratios are done at $\mu = M_Z$. Note that the mass values (m_e, m_μ, m_τ) have a large hierarchical structure, i.e., $m_e/m_\tau \sim 10^{-4}$, while the values (A3) have a mild hierarchical structure, i.e., $x_1/x_3 \sim 10^{-1}$.

In this paper, we do not ask the origin of the value (x_1, x_2, x_3). However, for reference, in the Appendix B, we will demonstrate an example of the charged lepton mass relation in the present Yukawaon model.

APPENDIX B: CHARGED LEPTON MASS RELATION IN THE YUKAWAON MODEL

The charged lepton mass relation [17]

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3} \tag{B1}$$

is one of the main motives of the Yukawaon model in the earlier stage [18]. The relation (B1) can be understood from the VEV of the U(3) $\mathbf{8}+\mathbf{1}$ scalar, $\langle\hat{\Phi}_e\rangle = \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m\tau})$, as

$$K = \frac{\mathrm{Tr}[\hat{\Phi}_e\hat{\Phi}_e]}{\mathrm{Tr}^2[\hat{\Phi}_e]}, \tag{B2}$$

where we have omitted VEV notation $\langle$ and $\rangle$ for simplicity. However, in the present scenario of the Yukawaon model, there is no $\mathbf{8}+\mathbf{1}$ scalar $\hat{\Phi}_e$, but we have only $\mathbf{6}$ and $\mathbf{6}^*$ scalars Φ_e and $\bar{\Phi}_e$. The purpose of the present paper is to understand mass ratios and mixings of quarks and leptons under the given parameters (m_e, m_μ, m_τ), and it is not to investigate the origin of the values (m_e, m_μ, m_τ).

However, in this appendix, let us try to understand the mass relation (B1) according to an idea suggested in Ref. [18]. First, let us introduce $\mathbf{8}+\mathbf{1}$ scalar $\hat{\Phi}_e$. By using the following superpotential:

$$W = \mu\mathrm{Tr}[\hat{\Phi}_e\hat{\Theta}_e] + \lambda_e\mathrm{Tr}[(\Phi_e\bar{E} + E\bar{\Phi}_e)\hat{\Theta}_e], \tag{B3}$$

we obtain a relation

$$\hat{\Phi}_e = \Phi_e\bar{E} + E\bar{\Phi}_e. \tag{B4}$$

Since $R(E) = \frac{1}{2}(1-\Delta)$ as seen in Eq. (2.16), $\hat{\Phi}_e$ has the R charge as

$$R(\hat{\Phi}_e) = 1 - \frac{1}{2}\Delta. \tag{B5}$$

Let us take $\Delta = +1$, so that we have

$$R(\hat{\Phi}_e) = R(\Phi_e) = \frac{1}{2}. \tag{B6}$$

This choice (B6) causes no problem because $\hat{\Phi}_e$ and Φ_e have different transformations under $\mathrm{U}(3)\times\mathrm{U}(3)'$. We will comment on the choice $R(E) = 0$ later.

Since $R(\hat{\Phi}_e) = 1/2$, we assume the following superpotential:

$$W = \frac{1}{\Lambda}(\lambda \mathrm{Tr}^2[\hat{\Phi}_e\hat{\Phi}_e] + \lambda' \mathrm{Tr}^2[\hat{\Phi}_e]\mathrm{Tr}[\hat{\Phi}_8\hat{\Phi}_8]), \tag{B7}$$

where $\hat{\Phi}_8$ is an octet part of the nonet $\hat{\Phi}_e$ defined by

$$\hat{\Phi}_8 \equiv \hat{\Phi}_e - \frac{1}{3}\mathrm{Tr}[\hat{\Phi}_e]\mathbf{1}. \tag{B8}$$

The first term in Eq. (B7) is the conventional nonet-nonet term. The second term is an (octet-octet) $\times$ (singlet-singlet) interaction term [18] although the second term is still SU(3) invariant. To derive the relation (B1), the assumption of the second term is essential. By noticing that the second term can be expressed as

$$\mathrm{Tr}[\hat{\Phi}_e\hat{\Phi}_e]\mathrm{Tr}^2[\hat{\Phi}_e] - \frac{1}{3}\mathrm{Tr}^4[\hat{\Phi}_e], \tag{B9}$$

we obtain

$$\frac{\partial W}{\partial \hat{\Phi}_e} = \frac{1}{\Lambda}\left\{2(2\lambda \mathrm{Tr}[\hat{\Phi}_e\hat{\Phi}_e] + \lambda' \mathrm{Tr}^2[\hat{\Phi}_e])\hat{\Phi}_e + 2\lambda'\left(\mathrm{Tr}[\hat{\Phi}_e\hat{\Phi}_e] - \frac{2}{3}\mathrm{Tr}^2[\hat{\Phi}_e]\right)\mathrm{Tr}[\hat{\Phi}_e]\mathbf{1}\right\}. \tag{B10}$$

The coefficients of $\hat{\Phi}_e$ and $\mathbf{1}$ must be zero in order to have a nontrivial solution of $\hat{\Phi}_e$ (nonzero and nonunit matrix form). Thus, we demand

$$2\lambda \mathrm{Tr}[\hat{\Phi}_e\hat{\Phi}_e] + \lambda' \mathrm{Tr}^2[\hat{\Phi}_e] = 0, \tag{B11}$$

and

$$\mathrm{Tr}[\hat{\Phi}_e\hat{\Phi}_e] - \frac{2}{3}\mathrm{Tr}^2[\hat{\Phi}_e] = 0. \tag{B12}$$

Equation (B11) requires a special relation between λ and λ'. Note that the relation (B12) is independent of the explicit value of λ'.

Let us comment on the choice of $\Delta = +1$. This choice means that $R(E) = 0$, so that a U(3) nonet $(E\bar{E})$ takes $R(E\bar{E}) = 0$. Therefore, the factor $E\bar{E}$ can be inserted into any terms with $R = 2$ in the superpotential. However, since $\langle E\bar{E}\rangle = \mathbf{1}$, this does not affect the obtained VEV relations practically. The choice $\Delta = +1$ also gives R charges of S_f as

$$(R(S_\nu), R(S_d), R(S_e), R(S_u)) = \left(\frac{1}{2}, 1, \frac{3}{2}, 2\right). \tag{B13}$$

It is interesting that the values $(1/2, 1, 3/2, 2)$ in (B13) are the same as the values for Φ_f as seen in Eq. (2.25), but the arrangements are different, i.e., a (e, u, ν, d) for $R(\Phi_f)$, while (ν, d, e, u) for $R(S_f)$.

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