Fermion masses and mixing in general warped extra dimensional models

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We analyze fermion masses and mixing in a general warped extra dimensional model, where all the Standard Model (SM) fields, including the Higgs, are allowed to propagate in the bulk. In this context, a slightly broken flavor symmetry imposed universally on all fermion fields, without distinction, can generate the full flavor structure of the SM, including quarks, charged leptons and neutrinos. For quarks and charged leptons, the exponential sensitivity of their wave functions to small flavor breaking effects yield hierarchical masses and mixing as it is usual in warped models with fermions in the bulk. In the neutrino sector, the exponential wave-function factors can be flavor blind and thus insensitive to the small flavor symmetry breaking effects, directly linking their masses and mixing angles to the flavor symmetric structure of the five-dimensional neutrino Yukawa couplings. The Higgs must be localized in the bulk and the model is more successful in generalized warped scenarios where the metric background solution is different than five-dimensional anti–de Sitter (AdS$_5$). We study these features in two simple frameworks, flavor complimentarity and flavor democracy, which provide specific predictions and correlations between quarks and leptons, testable as more precise data in the neutrino sector becomes available.

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I. INTRODUCTION

The discovery of a SM-like Higgs boson at the LHC with a mass of 125 GeV was a huge step forward in confirming the validity of the Standard Model (SM) and probing the electroweak symmetry breaking mechanism. But despite its experimental success, the SM still fails to provide an explanation, among other things, for the origin and observed pattern of fermion masses and mixings.

In the quark sector, the masses are extremely hierarchical, with the top much heavier than the rest of the quarks and with a strong ordering. In the up sector, the masses are separated by 3 orders of magnitude, while in the down sector the mass ratios are separated by 2 orders of magnitude. The quarks also exhibit mixing patterns given by three small mixing angles and a large (CP) phase. In the lepton sector, the charged lepton masses obey a similar hierarchical pattern as the down-type quarks. On the other hand, though not known exactly, neutrino masses are known to be very small and their square mass differences imply a closer mass pattern, $\Delta m^2_{31(32)} : \Delta m^2_{21} \sim 10^2 : 1$. Neutrinos appear to mix maximally and this has been long seen as pointing towards a different flavor origin between quarks and leptons, and also to the necessity of introducing

new physics. After the successful measurement of the neutrino mixing angle θ_{13} by the Daya Bay [1,2], T2K [3,4], MINOS [5,6], RENO [7] and Double Chooz [8,9] collaborations, the determination of the neutrino mass hierarchy has become a priority for theoretical studies and for future neutrino experiments. A great deal of theoretical work in this area has been trying to provide answers, based on such diverse frameworks as seesaw mechanisms [10–16], Abelian [17–19] and non-Abelian [20–37] symmetries imposed on the leptonic sector (both charged and neutral), and many texture structures for leptonic mass matrices, including modifications of accepted paradigms, such as tribimaximal [38–43], bimaximal [44–49] and democratic [50–52] neutrino mixing matrices. While various attempts to unify the description of quarks and leptons already exist (mostly based on quark-lepton complementarity [53–66]), an attractive possibility would be that quarks and leptons obey the same symmetry at a higher scale, which is then slightly broken at lower scales, yielding different patterns for masses and mixing for the quarks/leptons than for the neutrinos. This is the scenario we plan to investigate here, in the context of warped extra dimensions, where small symmetry breaking terms have very different effects on quarks and leptons due to the geometry of the model.

Introducing a warped extra dimension provides an elegant way to address both the hierarchy problem (to stabilize the Higgs mass against large radiative corrections) and the fermion flavor hierarchy problem. These models

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were first proposed to deal with the hierarchy problem of the SM [67,68], where introducing an extra dimension produced a five-dimensional anti–de Sitter (AdS$_5$) geometry bounded by two hard walls (branes) along the extra dimension, referred to as the Planck (or UV) brane and the TeV (or IR) brane. Allowing for exponential modulation from the gravity scale down to the weak scale along this compact extra dimension [67,68] yields the weak-Planck mass hierarchy.

The original Randall-Sundrum (RS) model localized all SM fields on the IR brane, leading to severe flavor violation bounds on the new physics scale. It was later shown that if the fermions were allowed to propagate in the bulk of the extra dimension [69–74], the same model could address the flavor hierarchy problem of the SM as well. The model thus emerged as a geometric theory of flavor. By localizing the Higgs on the IR brane with anarchic order-one couplings to the bulk fermions, the profiles of the fermion zero-modes can be adjusted to reproduce the observed Yukawa couplings in the low energy theory. Since the first and second generation fermions are localized towards the UV brane, they inherit substantial flavor protection from the RS–Glashow-Iliopoulos-Maiani mechanism [75]. However, by allowing the SM fields to propagate in the bulk, from an effective four-dimensional point of view, a tower of Kaluza-Klein (KK) fermions exists for all the SM fields, yielding enhanced contributions to electroweak and flavor observables in the SM [73,75–82]. This effect imposes a stringent bound on the scale of new physics of some ∼10 TeV [83,84] and hence renders these models completely out of the reach of current experiments. Some solutions were proposed to relieve these restrictions. One way was to extend the gauge symmetry of the model by introducing an $SU(2)_R$ gauge sector with custodial protection [73,76,85]. Here the basic idea is to align the down-type Yukawa couplings using the additional symmetry, so that the primary sources of intergenerational mixing are the up-type Yukawa couplings. Since the dominant constraints on flavor-changing neutral currents come from the down-type sector, the constraint on KK masses is substantially relaxed. A different approach to address the issue is through a slight modification of the warping factor along the extra dimension, allowing it to deviate slightly from the AdS$_5$ metric [86–91]. This deviation is such that the warping is more drastic near the TeV brane, while the background becomes more AdS$_5$-like near the Planck brane. These types of metric solutions can help suppress dangerous contributions to the electroweak and flavor observables by reducing the constraints on new physics down to about ∼1 TeV.

Warped extra dimensional models were shown to provide new contributions to the Higgs production rate through gluon fusion which could conflict with the collider data [74,92–100].

Interestingly, in the modified five-dimensional (5D) metric scenarios the region of parameter space in which the dangerous contributions to flavor and electroweak precision observables are small is the same as the region where the new contributions to the Higgs production cross section are also small (and thus safe) [94]. Moreover, this is achievable only when the Higgs field in these models is allowed to leak considerably into the bulk.

In the context of warped extra dimensional models with $\mathcal{O}(1)$ 5D Yukawa couplings and with no *a priori* structure (i.e., flavor anarchy), one can easily generate the hierarchical structure of the quark and charged lepton sectors [101] while, due to large mixing angles, the neutrino sector must be treated differently. In particular, in [102], it was shown that if the Higgs field leaks sufficiently into the bulk it is possible that the (exponentially small) neutrino wave functions become independent on the flavor structure of the 5D neutrino mass parameters (c_ν^i), and thus the 4D neutrino flavor structure depends directly on the flavor structure of the 5D neutrino Yukawa couplings.

In a previous work [103], we proposed a unified picture of fermion masses and mixings in the context of a warped extra dimensional model with AdS$_5$ background metric, and with all the SM fields in the bulk, including the Higgs. In that picture, the same flavor symmetric structure is imposed on all the fermions of the SM, including neutrinos. Small flavor breaking effects are exponentially enhanced in the quark and charged lepton sectors, thus producing hierarchical masses and mixings. With a sufficiently delocalized Higgs field, the neutrino wave functions are flavor blind and the flavor structure is governed by the 5D neutrino Yukawa flavor structure. Previous analyses on masses in warped space are available in the literature [104–121].

In this work, we revisit this idea in the context of the modified AdS$_5$ metric solutions. We show that the SM masses and mixing can be generated successfully, and the mass generation in the neutrino sector appears to be less fine tuned than in the case where a pure AdS$_5$ background metric is assumed.

Our work is organized as follows. We summarize the features of the modified AdS$_5$ model in Sec. II, with particular emphasis on fermion mass generation, Sec. III. We explore an explicit implementation of the scenario, flavor complementarity, in Sec. IV and another, of a democratic flavor symmetry, in Sec. V. We summarize our results and conclude in Sec. VI. We leave the details of some calculations for the Appendix.

II. WARPED SPACE MODELS WITH FIELDS IN THE BULK

We consider a 5D warped space with the extra dimension compactified and allow all SM fields to propagate in the following generalized warped space-time metric:

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2, \qquad (2.1)$$

$\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ being the flat metric. The fifth dimension, y, is bounded by two branes localized at $y = 0$ and $y = y_1$ and $A(y)$ is a model-dependent function. As mentioned in the Introduction, some generalized warped models can be safe from precision electroweak tests and flavor bounds for very low KK masses. Motivated by this, we consider such a model within a modified AdS_5 scenario with the following warp exponent [90,122,123]:

$$A(y) = ky + \frac{1}{\nu^2} \ln\left(1 - \frac{y}{y_s}\right), \qquad (2.2)$$

where $k \sim M_{Pl}$ is the AdS_5 curvature, expected to be of the order of the Planck mass scale, y_s is the position of the metric singularity, always chosen to be outside of the physical region, $y_s > y_1$, and $\nu > 0$ is a model parameter taken to be real. The ν parameter, alongside $\Delta = y_s - y_1$, the distance between the location of the metric singularity and the IR brane, measures the departure of the metric from the pure AdS_5 background. The smaller the values of Δ and ν, the more modified the metric; intuitively, the singularity has a larger effect on the physics at the IR brane the closer it gets to it. One can calculate the curvature along the fifth dimension and obtain

$$R(y) = 8A''(y) - 20(A'(y))^2. \qquad (2.3)$$

The curvature radius, $L(y) = \sqrt{-20/R}$, in units of k along the fifth dimension is then given by

$$kL(y) = \frac{k\Delta}{\sqrt{1 - 2\nu^2/5 + 2\nu^2 k\Delta + (\nu^2 k\Delta)^2}}. \qquad (2.4)$$

One can see that for values of $\nu > \sqrt{5/2}$, this function has a minimum before the singularity and therefore the curvature can change sign within the physical region. Following [91], we impose that this minimum is located outside of the physical region and hence the curvature radius is a monotonically decreasing function between the UV and the IR branes.

The more familiar RS metric is recovered by taking the limits $\nu \to \infty$ and $y_s \to \infty$, yielding $A(y) = ky$, with the curvature radius being constant, $kL = 1$. In Fig. 1, we compare the two metrics and plot the warp exponent function $A(y)$ for the AdS_5 and the modified AdS_5 cases. We can see that the amount of warping near the IR brane at around $ky = 35$ is larger for the modified AdS_5. Thus, as the figure indicates, the same amount of warping from the UV brane to the IR brane in the modified scenario requires a slightly smaller length of the fifth dimension and hence an IR brane slightly closer to the UV brane. The curvature radius [Eq. (2.4)] at the UV brane is approximately equal for the pure and modified AdS_5 spaces with, $ky(y) \simeq 1$. On the contrary, at the IR brane, as $kL(y)$ is a monotonically decreasing function, $ky(y)$ assumes its minimal value for the modified AdS_5 space and hence $kL_1 \equiv kL(y_1)$ is a
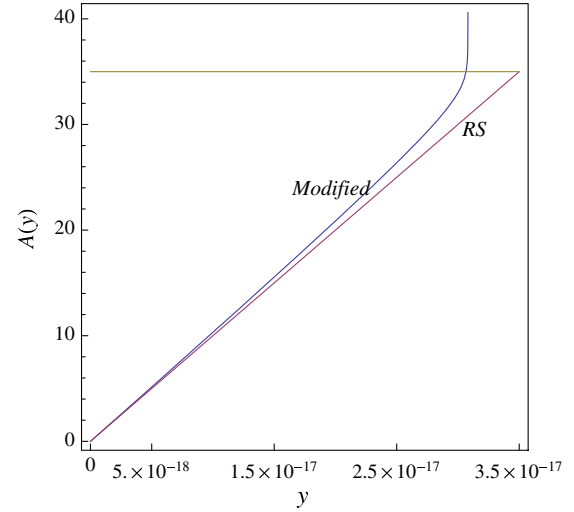

FIG. 1 (color online). The modified AdS_5 warp factor $A(y)$ versus the standard RS warp exponent, y. The horizontal line corresponds to $ky = 35$. For the same amount of warping, the modified scenario requires a shorter length scale along the fifth dimension.

good measure of the amount of deviation from the pure AdS_5 space with constant curvature radius.

The 5D fermion Lagrangian density with Dirac neutrinos is

$$\begin{aligned} \mathcal{L}_q = \mathcal{L}_{\text{kinetic}} &+ M_{q_i}\bar{Q}_iQ_i + M_{u_i}\bar{U}_iU_i + M_{d_i}\bar{D}_iD_i \\ &+ (Y_{ij}^{u5D}H\bar{Q}_iU_j + \text{H.c.}) + (Y_{ij}^{d5D}H\bar{Q}_iD_j + \text{H.c.}) \\ &+ (Q_i \to L_i, U_i \to N_i, D_i \to E_i), \end{aligned} \qquad (2.5)$$

where, i, j are flavor indices and the 5D Yukawa parameters, Y_{ij}^{5D}, are dimensionful quantities of $\mathcal{O}(1) \times \sqrt{k}$. Q_i (L_i) are 5D quark (lepton) fields for $SU(2)$ doublets while U_i (N_i) and D_i (E_i) are $SU(2)$ singlet quark (lepton) fields. The bulk mass, M_{ψ_i}, originating from the momentum along the fifth dimension, can be taken in general to be y dependent. To be able to compare, we choose it such that it coincides with its usual definition in RS models, and express it in units of the fifth dimension curvature, k, as $M_{\psi_i} = c_\psi^i k$, where c_ψ^i [1] are localization parameters, dimensionless quantities of $\mathcal{O}(1)$, and ψ_i runs over all SM quarks and leptons.[2] Dimensional reduction then yields the normalized profile for the fermion and the Higgs fields along the bulk of the extra dimension, $q_L^{0,i}(y)$, $u_R^{0,i}(y)$ and $h(y)$, which are given by

$$q_L^{0,i}(y) = q_0^i \, e^{(2-c_q^i)A(y)}, \qquad (2.6)$$

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[1]We use throughout c_q for the doublets (c_q and c_l) and c_u for the singlets (c_u, c_d, c_ν and c_e).

[2]Alternative fermion and Higgs profiles can be found in [94,122], where different bulk mass conventions are adopted.

$$u_R^{0,i}(y) = u_0^i\, e^{(c_u^i+2)A(y)}, \qquad (2.7)$$

$$h(y) = h_0\, e^{aky}, \qquad (2.8)$$

with $q_0^i = f(c_q^i)$, $u_0^i = f(-c_u^i)$ and $h_0 = e^{-(a-1)ky_s}\mathfrak{h}_0$, and where $f(c)$ and $\mathfrak{h}_0$ are normalization factors which depend on c, ν and y_s, given explicitly in the Appendix, along with their limiting expressions for the usual RS (AdS$_5$) metric background. From these profiles, one can check that localization of the fields in the bulk of the extra dimension is determined by the values of the c_ψ^i for the fermion fields, such that a value of $c_\psi^i > 1/2$ indicates an UV localized field, while a value of $c_\psi^i < 1/2$ localizes the field near the IR brane.[3] The Higgs field localization along the fifth dimension is given by the parameter a, the dimension of the Higgs condensate operator. A completely IR brane localized field corresponds to the limit $a \to \infty$, while for a delocalized Higgs field a is small. However, in order to maintain the original Randall-Sundrum solution to the hierarchy problem without fine-tuning, the Higgs field localization should be such that $a \gtrsim 2$ (for an AdS$_5$ metric background). If the 5D Higgs potential is of the form $V_{\text{bulk}}(H) = M_{5d}^2 H^2$, with associated brane potentials at each boundary, the Higgs profile has two solutions, one growing towards the IR and another one decaying at the IR brane. This last one is proportional to $e^{(4-a)ky}$ in the AdS$_5$ background. In order to maintain the RS solution to the hierarchy problem, the decaying solution has to be subdominant, and this happens for $a > 2$. In that situation, the Higgs field profile [see Eq. (2.10)] is localized near the IR brane and mass terms depending on the Higgs profile vacuum expectation value are warped down to the energy scale of the IR brane as

$$m_{\text{Higgs}} \simeq M_{5d}\, e^{-A(y_1)}. \qquad (2.9)$$

This resolution of the hierarchy problem depends on the actual length of the extra dimension, as well as the choice of the 5D metric parameters, ν and y_s, which are usually set such that $A(y_1) \simeq 35$.

When $a < 2$, and in order to maintain a Higgs profile localization towards the IR brane, some fine-tuning between the parameters of the bulk scalar potential and the brane potentials is necessary in order to suppress the unwanted solution. In the modified AdS$_5$ scenario the lowest value of a that does not require fine-tuning depends on the various new metric parameters [82]. In this case, the Higgs profile is given by

$$h(y) = h_0 e^{aky}[1 + (M_0/k - a)[F(y) - F(0)]], \qquad (2.10)$$

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[3]This convention is for left-handed doublets. For right-handed singlet fields, our convention is such that $c_u^i > -1/2$ for an UV localized field and $c_u^i < -1/2$ for an IR localized field.

where M_0 is the brane Higgs mass term [coefficient of the $|H|^2\delta(y - y_1)$ term at the IR brane] and the function $F(y)$ is given by

$$F(y) = e^{-2(a-2)ky_s}ky_s[-2(a-2)ky_s]^{-1+4/\nu^2}$$
$$\times \Gamma\left[1 - \frac{4}{\nu^2}, -2(a-2)k(y_s - y)\right]. \qquad (2.11)$$

The decaying term at the IR brane is the second term in Eq. (2.10) and, forcing $M_0/k \simeq a$ (fine-tuning parameters), the solution can become subdominant. In order to avoid this fine-tuning of parameters, we note that $F(y)$ is a monotonically increasing function. We can thus introduce $\delta \equiv |F(y_1)|$ as a measurement of the amount of fine-tuning needed for the RS solution to survive the hierarchy problem. When $\delta \sim \mathcal{O}(1)$, no fine-tuning is needed to guarantee that the increasing solution for the Higgs profile dominates. When the parameter $\delta = F(y_1)$ becomes larger, this solution needs fine-tuning of parameters to suppress its value.

As expressed in the Introduction, scenarios of generalized warped metric backgrounds can yield lower bounds on KK masses coming from precision and flavor constraints. The precision electroweak oblique parameters, S and T are proportional to the dimension-6 operators

$$|H^\dagger D_\mu h|^2, \qquad H^\dagger W_{\mu\nu} H B^{\mu\nu}.$$

Due to the deviations of the metric from AdS$_5$ near the IR brane all the KK modes, including the gauge KK modes, become more localized near the IR brane. Intuitively, for a Higgs field that is sufficiently delocalized from the IR brane, couplings to these KK modes will be suppressed, which will result in lower bounds for the KK mode masses in these models (which can be as low as $m_{KK} \gtrsim 1$ TeV). As the Higgs field becomes more localized towards the IR brane, these couplings become large (even larger than in the pure AdS$_5$ case) and the bounds can become much worse. Therefore the success of this model is crucially dependent on the localization of the Higgs field along the fifth dimension [86,124].

Figure 2 shows the no-fine-tuning region (above the red solid $\delta = 1$ curve) in the (a, ν) plane, where a is the Higgs localization parameter and ν is the metric parameter of the modified metric solution. The region below is the one fine tuned, requiring an adjustment of Lagrangian parameters with a tuning precision growing exponentially. (The close dashed curve locates the points where $\delta = 10$, i.e., where the tuning is already 10%.)

In producing these graphs, for each case we first set the value of the IR brane position, ky_1, which in turn fixes the value of y_s, the position of the singularity. Then for each value of the parameter ν we solved for a in $\delta(a, \nu, y_1, y_s) = 1$.

To first order, the effective SM Yukawa couplings are obtained from the overlap integral
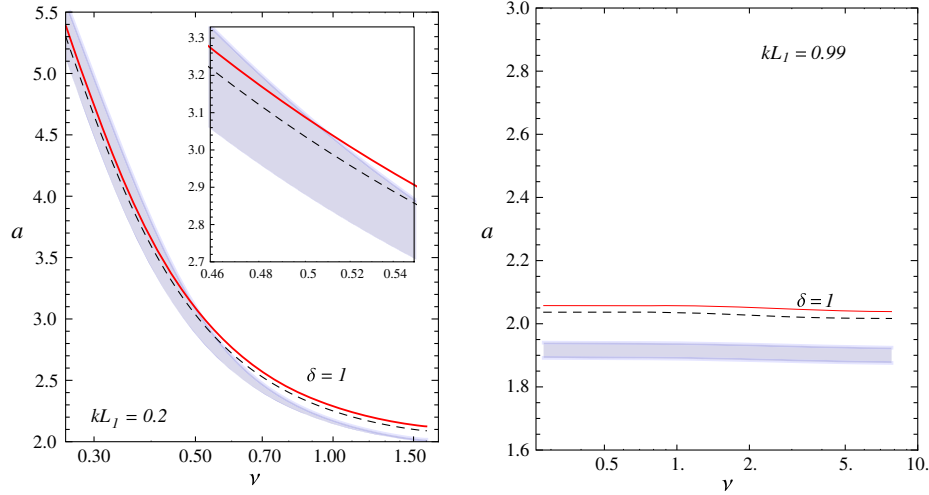
FIG. 2 (color online). The $\delta = 1$ plots and the connection with neutrino masses in the $\nu - a$ plane for (left panel) $kL_1 = 0.2$ (large deviation) and (right panel) $kL_1 = 0.99$ (more RS-like). The red (solid) curve locates the no-fine-tuning threshold in which $\delta = 1$. Above this curve, $\delta < 1$, and below the curve, $\delta > 1$ (corresponding to the unwanted fine-tuned region). The shaded area corresponds to a heavy neutrino mass (m_3 in *normal* ordering), $m_\nu \sim 5 \times 10^{-11}$ GeV for different values of the c parameters, but such that the mass expression has no exponential sensitivity to the c parameters. Note that in the RS-like case (right panel) it is not possible to obtain neutrino masses without fine-tuning, or quitting the neutrino plateau parameter region (as would be the case in usual RS scenarios). With modified-AdS$_5$ metric (left panel) it is possible to find nontuned points with neutrino masses in the plateau (corresponding to no exponential c-dependence).

$$y_{ij}^u = \frac{Y_{ij}^u}{\sqrt{k}} \int_0^{y_1} dy \, e^{-4A(y)} h(y) q_L^{0,i}(y) u_R^{0,j}(y), \qquad (2.12)$$

where the index u denotes the four types of Yukawa couplings of the SM, i.e., $u = u, d, e, N$ and we have defined the *dimensionless* 5D Yukawa couplings as $Y_{ij}^u = Y_{ij}^{5D}/\sqrt{k} \sim \mathcal{O}(1)$. Given the profiles (2.6), (2.7) and (2.8) and the metric (2.2), the integral above can be performed analytically and written as

$$y_{ij}^u = \tilde{Y}_{ij}^u h_0 f(c_q^i) f(-c_u^j), \qquad (2.13)$$

where the factor $\tilde{Y}_{ij}^u$, defined in the Appendix, has very mild c^i dependence. The function $f(c)$ is such that depending on the value of c, the Yukawa couplings can be exponentially sensitive to c, or only mildly dependent. Finally h_0 is the bulk Higgs normalization factor and does not depend on the fermion mass parameters c^i. Note that, although throughout the paper we have suppressed the explicit dependence of the fields on the metric parameters, ν, y_s, y_1 and k, and in particular all of the factors of Eq. (2.13), are metric dependent. As shown in the Appendix, one can always retrieve the RS limit for these terms by taking the limit $\nu, y_s \to \infty$.

III. FERMION MASSES IN WARPED SPACE

We now return to a more detailed discussion of fermion masses in this model. These masses are given, to first order, by the eigenvalues of the 3×3 Yukawa matrices of the form

$$v \, y^u = v \begin{pmatrix} y_{11}^u & y_{12}^u & y_{13}^u \\ y_{21}^u & y_{22}^u & y_{23}^u \\ y_{31}^u & y_{32}^u & y_{33}^u \end{pmatrix}. \qquad (3.1)$$

In general, in order to get the correct SM masses, no exponential c-dependence is needed for the top quark, which corresponds to $c_q^t \lesssim 1/2$ and $c_u^t > -1/2$ (this region of parameter space corresponds to the top plateau shown in Fig. 3). For the rest of the SM particles $c_q^i \gtrsim 1/2$ and $c_u^i < -1/2$, which implies that the Yukawa couplings will depend exponentially on the values of the c^i. In the case of neutrinos however, in order to accommodate their tiny masses, one needs localization parameters $c_\nu < -1$. It was shown [102] that, for a delocalized Higgs with a parameter small compared to the localization parameter c^i, the 4D effective neutrino masses depend exponentially on a but lose their dependence on the c^i's. This region of parameter space corresponds to the neutrino plateau shown in Fig. 3. In other words, the limit of the function in Eq. (2.13) for large c_ν's is given by

$$y_{ij}^u \sim \tilde{Y}_{ij}^u h_0. \qquad (3.2)$$

To make this more transparent, in the formula for fermion masses given by [cf. Eq. (2.13)]

$$(M_f)_{ij} = v \tilde{Y}_{ij}^u h_0 f(c_q^i) f(-c_u^j), \qquad (3.3)$$

we factor the exponential behaviors and write them in the following two distinct limits:
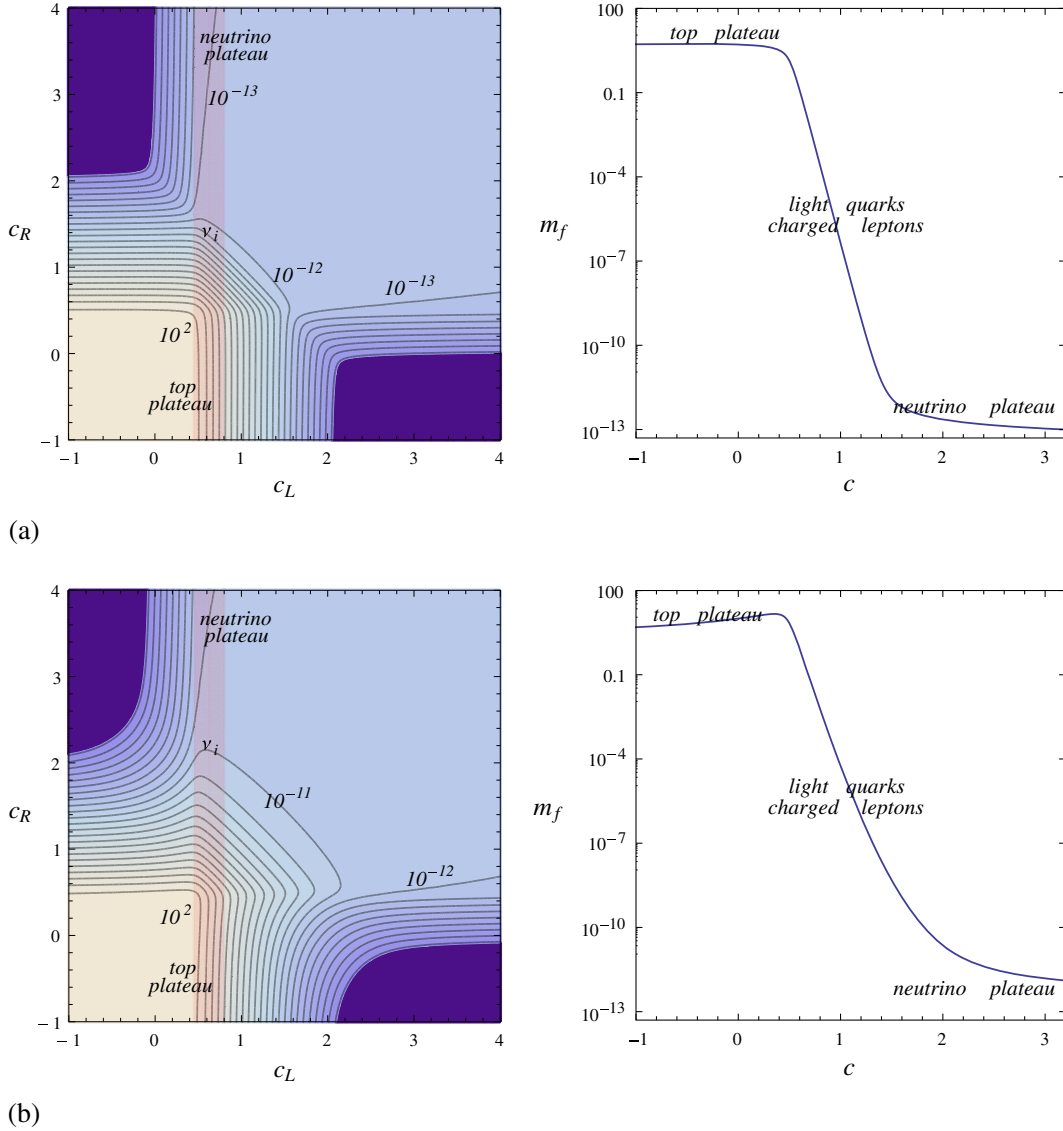
FIG. 3 (color online).   Effective 4D Yukawa couplings for fermions as functions of the fermion bulk mass parameter c for (a) the RS and (b) general warped space-time metric. The plots on the right-hand side are produced by taking the c parameters for the doublet and the singlet to be equal. In the left-handed plots, each contour depicts 1 order of magnitude difference with respect to its adjacent contour. A typical location for neutrino masses of order $m_\nu \sim 10^{-11}$ GeV is also shown. The shaded vertical band shows the region where all fermions of the SM should be located. (a) RS with all SM elds including the Higgs in the bulk, $a = 2.04$ and $\delta = 1$ and (b) Modied AdS5 with $\nu = 0.32$, $kL(y_1) = 0.2$, $A(y_1) = 35$, $a = 4.46$ and $\delta = 1$.

$$(M_f)_{ij} \sim vY^f_{ij}\epsilon^{(c^i_q-\frac{1}{2})}\epsilon^{-(c^j_u+\frac{1}{2})} \quad \text{for } c_q > 1/2, \quad c_u < -1/2,$$

$$(M_\nu)_{ij} \sim vY^N_{ij}e^{-ky_1(a-1)} \quad \text{for } c_q - c_u > a, \quad (3.4)$$

where $\epsilon = e^{-A(y)}$ (see the Appendix for details).[4]

The limits described in Eq. (3.4) imply that, as the quark and charged lepton mass matrix elements are exponentially dependent on the c^i parameters, any structure in the 5D Yukawa matrix elements Y^f_{ij} will

be largely washed out and will always produce generically hierarchical fermion masses as well as small mixing angles. For the neutrinos, on the other hand, since there is no exponential sensitivity on the flavorful c^i parameters, any structure inherent in the 5D Yukawa matrix elements will survive in the 4D effective theory. This is the region of neutrino parameter space we are interested in, shown in Fig. 3 as the neutrino plateau. In RS models, the actual height of the neutrino plateau is determined exclusively by the value of the a parameter. For the value of the warp factor required to solve the hierarchy problem, the highest possible neutrino masses in the plateau are too small by 1 to 2 orders of magnitude

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[4]For modified AdS$_5$, $\epsilon_{\text{mod}} = e^{-ky_1}(1 - \frac{y_1}{y_s})^{\frac{1}{\nu^2}} \sim 10^{-15}$ while for RS $\epsilon_{\text{RS}} = e^{-ky_1} \sim 10^{-15}$.
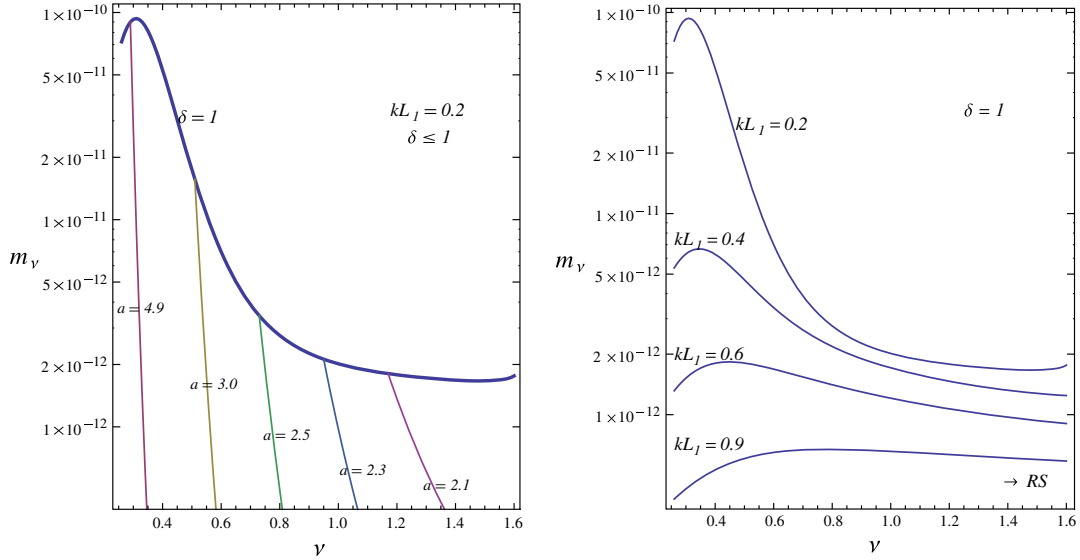
FIG. 4 (color online).   (Left panel) Neutrino masses as functions of the metric parameter ν for different values of the Higgs localization parameter a and fixed $kL_1 = 0.2$. The fermion mass parameters (c's) are fixed to a region where there is no exponential dependence on them (the neutrino plateau). The curves end whenever the fine-tuning threshold ($\delta = 1$) is reached (thick overlapping curve). Note that for small values of ν the neutrino masses become larger while still remaining in the nontuned regime and in the neutrino plateau. (Right panel) Same plot for different values of kL_1 and fixed $\delta = 1$. The graph corresponding to $kL_1 = 0.9$ remains constant at larger values of ν and (approximately) coincides with the RS limit.

(without fine-tuning) to be phenomenologically viable (see the upper panels in Fig. 3). Some tuning, or some enhancement of the 5D Yukawa couplings, and/or trespassing the edge of the plateau would then be required, making the scenario less attractive for our purposes. On the other hand, in modified AdS$_5$ scenarios, although the level of the plateau is still highly sensitive to the value of a, the masses could actually be increased by some 2 orders of magnitude and thus allow for phenomenologically acceptable neutrino masses in the desired region of the model (see Fig. 2 and the lower panels in Fig. 3). This feature occurs because the modified AdS$_5$ metric [see Eq. (2.2)] exhibits a richer parameter space. In the RS metric, in order to produce the correct neutrino masses in the plateau, we need $a \sim 1.85$, a value which amounts to about 1/10000 fine-tuning of parameters in the 5D Higgs potential. While for modified AdS$_5$ scenarios, one can produce a neutrino plateau within the experimental bounds for some range of a parameter values without fine-tuning. It is interesting that the parameter space for which the neutrino plateau is most favorable coincides with the region where small KK masses satisfy bounds coming from flavor and electroweak precision measurements [125,126] *and* constraints from Higgs phenomenology [93].

In order to further illustrate this issue, in Fig. 4 we show the resulting neutrino masses in the plateau region as a function of the parameter ν, for different values of a (left panel) and kL_1 (right panel). The area below the curves is the no-fine-tuning region, and one can see

that the largest neutrino masses (in the plateau) occur for $\nu \sim 0.3$ and $kL_1 = 0.2$, whereas in the RS limit, i.e., $kL_1 \sim 1$ and ν large, the masses are some 2 orders of magnitude lower (slightly too low). This makes the extended metric scenarios a framework for the flavor mechanism investigated here, and adds to the advantages of these scenarios (i.e., a much lower KK scale consistent with electroweak and flavor bounds, and with Higgs production).

Qualitatively the general features of the flavor structure of these modified AdS$_5$ models are very similar to these features in the pure AdS$_5$ models for the bulk mass c_ψ^i parameters. Therefore, in order to illustrate our flavor setup, it is useful to consider the simpler case of the RS metric. In this case the fermion mass formulas in Eq. (3.4) can be simplified dramatically. As usual, we consider the mass matrix for the neutrino sector separately from the case of quarks and charged lepton mass matrices. Consider the case with $c_{q,u}^i > \frac{1}{2}$. In this case we have,

$$(M_f)_{ij} \simeq v \epsilon^{(c_q^i - \frac{1}{2})} \epsilon^{-(c_u^j + \frac{1}{2})} \sqrt{2(a-1)|1 - 2c_q^i||1 + 2c_u^j|} \tilde{Y}_{ij}^f$$

$$(M_\nu)_{ij} \simeq v \epsilon^{a-1} \frac{\sqrt{2(a-1)|1 - 2c_l^i||1 + 2c_\nu^j|}}{\sqrt{\epsilon^{(1-2c_l)} - 1}\sqrt{\epsilon^{(1+2c_\nu)} - 1}} \tilde{Y}_{ij}^\nu, \qquad (3.5)$$

where the 5D Yukawa couplings are given by[5]

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[5] For exact formulas see the Appendix.

$$\tilde{Y}^u_{ij} \simeq \frac{1}{a - c^i_q + c^j_u} Y^u_{ij}. \tag{3.6}$$

From Eqs. (2.6), (2.7) and (2.12) one can see that there are two sources of flavor structure: the 5D dimensionless Yukawa couplings Y^u_{ij} and the bulk mass coefficients c^i_ψ. We are interested in scenarios in which all Yukawa matrices $(Y_F = Y^u_{ij}, Y^d_{ij}, Y^\nu_{ij}$ and $Y^e_{ij})$ and fermion bulk mass matrices from the 5D Lagrangian $(\mathbf{c}_f = \mathbf{c}_q, \mathbf{c}_u, \mathbf{c}_d, \mathbf{c}_l, \mathbf{c}_\nu$ and $\mathbf{c}_e)$ share the same symmetry structure, which is then slightly broken through some high energy mechanism according to

$$Y_F = Y^0_F + \delta Y_F, \tag{3.7}$$

$$\mathbf{c}_f = \mathbf{c}^0_f + \delta \mathbf{c}_f, \tag{3.8}$$

where the matrices Y^0_F and $\mathbf{c}^0_f$ are flavor symmetric while the perturbation matrices δY_F and $\delta \mathbf{c}_f$ are random. Inserting these perturbations in Eq. (3.5), the fermion masses receive corrections to leading order as follows:

$$
\begin{aligned}
m_t &= m^0_t + \delta m_t c^3_q, \qquad c^3_u < 1/2, \\
(m_f)_{ij} &= (m_f)^0_{ij} f(\delta c^i_q) f(\delta c^j_u)) \sim (m_f)^0_{ij} \epsilon^{(\delta c^i_q + \delta c^j_u)} \\
&\qquad a > c^i_l + c^j_u, \\
(m_\nu)_{ij} &= (m_\nu)^0_{ij} + \delta(m_\nu)_{ij} \qquad a < c^i_l + c^j_\nu.
\end{aligned}
\tag{3.9}
$$

Therefore the same exponential sensitivity on the bulk mass c^i_ψ parameters, $\epsilon \sim 10^{-15}$, responsible for producing the SM hierarchy in the standard RS, is now translated into exponential sensitivity of the symmetry breaking terms. As a consequence, small symmetry breaking terms $(|\delta c^i| \sim 0.1)$ can produce mass corrections of order $10^{-15(\delta c^i + \delta c^j)}$ (i.e., a hierarchy of order $\sim 10^6$) to the quark and charged lepton mass matrices. This is in complete agreement with the observed hierarchy in these sectors. As mentioned before, the neutrinos and the top quark fields live in the two plateaus (see Fig. 3) with mild c^i_ψ sensitivity.

For the mixing angles, the eigenvector matrix that diagonalizes the neutrino sector should be very close to the eigenvector matrix of the 5D Yukawa matrix. However, in the quark and charged lepton sectors the mixing matrices are generically close to the unit matrix with off-diagonal entries hierarchically small as[6]

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[6]Similar expressions are obtained for V^d_L and V^q_L (see for example [112]).

$$V^u_L \simeq \begin{pmatrix} 1 & \frac{f^1_q(\tilde{M}_u)_{21}}{f^2_q(\tilde{M}_u)_{11}} & \frac{f^1_q(\tilde{Y}_u)_{13}}{f^3_q(\tilde{Y}_u)_{33}} \\ -\frac{f^1_q(\tilde{M}^*_u)_{21}}{f^2_q(\tilde{M}^*_u)_{11}} & 1 & \frac{f^2_q(\tilde{Y}_u)_{23}}{f^3_q(\tilde{Y}_u)_{33}} \\ \frac{f^1_q(\tilde{M}^*_u)_{31}}{f^3_q(\tilde{M}^*_u)_{11}} & -\frac{f^2_q(\tilde{Y}^*_u)_{23}}{f^3_q(\tilde{Y}^*_u)_{33}} & 1 \end{pmatrix}, \tag{3.10}$$

where f^i_q is shorthand for the profile functions $f(c^i_q)$, $(\tilde{M}_u)_{ij}$ is the (ij) minor of the matrix in parenthesis, and $(\tilde{Y}_u)_{ij}$ is the ij element of the Yukawa matrix. We define the Cabibbo-Kobayashi-Maskawa (CKM) and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrices as the following:

$$V_{\text{CKM}} \equiv V^u_L V^{d\dagger}_L \quad \text{and} \quad V_{\text{PMNS}} \equiv V^\nu_L V^{e\dagger}_L. \tag{3.11}$$

As mentioned after Eq. (3.4), for the quarks and charged leptons, the off-diagonal mixing angles are also exponentially sensitive to the c^i_ψ parameters and hence to the small symmetry breaking terms. The V^f_L matrix elements in Eq. (3.10) can then be written as

$$(V^{u,d,e}_L)_{ij} \sim (V^{u,d,e}_L)^0_{ij} \epsilon^{(\delta c^i_q - \delta c^j_q)} \quad \text{(for } u, d \text{ and } e), \tag{3.12}$$

where $(V^{u,d,e}_L)^0_{ij}$ are the matrix elements before the flavor symmetry breaking and $\{i, j\}$ can be $\{1, 2\}$, $\{1, 3\}$ and $\{2, 3\}$. We see again that, due to exponential warping, all original symmetries present in the high energy theory are washed out in the quark and charged lepton sectors. Contrary to this, in the Dirac neutrino sector, the terms in the V^ν_L are much less sensitive to the symmetry breaking terms, since their own dependence on the c^i_ψ is mild in the region of parameters of interest. If we define the eigenvalues of the 5D neutrino Yukawa matrix as

$$Y^{\text{diag}}_\nu = V_{Y_L} Y_\nu V_{Y_R}, \tag{3.13}$$

then the matrix diagonalizing the 4D effective neutrino mass matrix is

$$(V^\nu_L)_{ij} \sim (V_{Y_L})_{ij}. \tag{3.14}$$

So far the framework is general, and we did not specify the type of symmetry imposed. In the next two sections we will consider two concrete implementations, one within flavor complementarity and then a second within flavor democracy. The background metric considered will always be the modified AdS$_5$ solution, in the most favorable region of parameter space.

IV. FLAVOR COMPLEMENTARITY

In the first symmetry implementation, in this section we study the effects of enforcing a strong correlation between 5D quark and 5D lepton parameters. This scenario is

motivated by the observation that charged lepton masses and down-quark masses obey similar patterns. In warped space models, masses are obtained from exponentially small overlap integrals, and having similar mass patterns can indicate that both the 5D Yukawa structure and the 5D bulk fermion masses (c parameters) are very similar for the down quarks and for the charged leptons. We therefore assume that $Y_d = Y_e$ and $\mathbf{c}_q = \mathbf{c}_l$ to a high order of precision, so that the differences in observed masses between down quarks and charged leptons come from deviations in the right-handed bulk parameters $\mathbf{c}_d$ and $\mathbf{c}_e$.

At the same time, inspired by neutrino phenomenology, we implement a popular 2-3 ($\mu - \tau$) symmetry [22,24,127–148] within the general flavor structure of the model. We require that all the 5D Yukawa matrices must be symmetric under two to three permutations and consider, for simplicity, the case where the 5D fermion bulk mass matrices $\mathbf{c}_f^0$ are diagonal and degenerate in that basis [maybe a remnant of some global $U(3)$ flavor symmetry broken in the Yukawa sector], i.e.,

$$\mathbf{c}_f^0 = \begin{pmatrix} c_f^0 & 0 & 0 \\ 0 & c_f^0 & 0 \\ 0 & 0 & c_f^0 \end{pmatrix}, \qquad (4.1)$$

where c_f^0 are real parameters, and f denotes both doublets and singlets $f = q, l, u, d, e, \nu$. In 4D scenarios, discrete permutation symmetries of the neutrino mass matrix are known to lead to interesting (and phenomenology viable) neutrino masses and mixing patterns (such as the tribimaximal mixing matrix [38–43], or the bimaximal mixing matrix [44–49]).

We will impose the zeroth order Yukawa matrices to be

$$Y_{u,\nu}^0 = y_u \begin{pmatrix} \sqrt{6}d & 0 & 0 \\ d & d+1 & d-1 \\ d & d-1 & d+1 \end{pmatrix} \quad \text{and}$$

$$Y_{d,e}^0 = y_d \begin{pmatrix} a & 1 & 1 \\ c & g & e \\ c & e & g \end{pmatrix}, \qquad (4.2)$$

where the parameters y_u, y_d, a, c, d, g and e are complex.[7] As explained above, we impose the down-type Yukawa couplings to match the charged lepton ones, i.e., $Y_d = Y_e \equiv Y_{d,e}$. We further require that the up-type Yukawa couplings be the same as the neutrino Yukawa

couplings, i.e., $Y_u = Y_\nu \equiv Y_{u,\nu}$, in order to reduce free parameters and enhance the degree of correlation between quarks and leptons.

The structure of the down-type Yukawa $Y_{d,e}^0$ is set only by the 2-3 symmetry, but the structure of neutrino-type Yukawa couplings $Y_{u,\nu}^0$ in Eq. (4.2) requires more explanation. The number of parameters has been reduced to two complex parameters, y_u and d, such that the rotation matrix that diagonalizes Y_ν^0 is the bimaximal mixing matrix[8] and the eigenvalues of $Y_{u,\nu}^0$ are simply given by $3.08d$, $1.59d$ and 2 (in units of y_u). When $|d| \sim 1/6$, we should naively generate a hierarchy between the eigenvalues of the matrix matching the solar and atmospheric neutrino mass hierarchies, for the case of a normal ordering in the neutrino masses. Of course, in warped scenarios, the 5D Yukawa matrices are in general not directly proportional to the effective 4D fermion mass matrices. In the neutrino sector however, since we are near the neutrino mass "plateau" of the c parameters, the structure of the effective 4D mass matrix should be very similar to the 5D Yukawa matrix. We therefore expect that the effective 4D neutrino structure should be diagonalized by a matrix close to the bimaximal mixing matrix, with a normal ordering in the masses matching the observed mass differences. 5D wave function effects, as well as random, but small, symmetry breaking terms, would perturb this structure, but the main features survive.

Finally, the two texture zeros of Y_u^0 in Eq. (4.2) cause the vanishing of the (21) minor of that matrix, which in turn contributes to the quark-lepton correlations which we want to address here; the vanishing, or the smallness of those entries, is therefore a critical requirement in the setup, as it links the value of the Cabibbo angle to the PMNS element V_{13} (i.e., to θ_{13}). To see how this occurs, let us write the approximate expression for the Cabibbo angle V_{us} in warped extra dimensions,

$$V_{us} \simeq \left| \frac{f(c_q^1)}{f(c_q^2)} \right| \left| \frac{(\tilde{M}_d)_{21}}{(\tilde{M}_d)_{11}} - \frac{(\tilde{M}_u)_{21}}{(\tilde{M}_u)_{11}} \right|, \qquad (4.3)$$

where $(\tilde{M}_u)_{ij}$ and $(\tilde{M}_d)_{ij}$ are the (ij) minors of the Y_u and Y_d matrices. We can see that the texture zeros in the 5D Yukawa matrix Y_u ensure that $(\tilde{M}_u)_{12} = 0$, so that the Cabibbo angle is controlled, to first order, by the down sector Yukawa couplings and by the quark doublet bulk c parameters, i.e.,

$$V_{us} \simeq \frac{f(c_q^1)}{f(c_q^2)} \frac{(\tilde{M}_d)_{21}}{(\tilde{M}_d)_{11}}. \qquad (4.4)$$

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[7]We also assume that the symmetry is broken by some small perturbations, i.e., $Y_i = Y_i^0 + \delta Y_i$ and $\mathbf{c}_f = \mathbf{c}_f^0 + \delta\mathbf{c}_f$, although we require that the constraints $Y_d = Y_e$ and $\mathbf{c}_q = \mathbf{c}_l$ survive the flavor symmetry breaking in order to maintain the quark lepton complementarity.

[8]The tribimaximal scheme predicts the CP phase in the PMNS matrix, δ, to be zero, contrary to phenomenological conjectures which favor $\delta = -\frac{\pi}{2}$.

Since we imposed $Y_d^0 = Y_e^0$ and $\mathbf{c}_q = \mathbf{c}_l$, the left mixing matrices W_L^d and W_L^e for the down sector and charged lepton sector are very similar, and in particular the 12 entries have the exact same first order expansion $(W_L^d)_{12} \simeq (W_L^e)_{12} \simeq V_{us}$. This is the origin of the quark-lepton complementarity relations we wish to study here.

In the lepton sector, the effective 4D neutrino mass matrix should be diagonalized by a unitary matrix, almost bimaximal, with its third eigenvector (associated to m_3^ν) close to $(0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$. The charged lepton Yukawa coupling, on the other hand, will be diagonalized by W_L^d, since both the Yukawa couplings and the doublet c parameters are the same in the down quark and charged lepton sectors. The unitary matrices generating the charged flavor part of the PMNS matrix $V_{\mathrm{PMNS}} = (W_L^e)^\dagger W_L^\nu$ should be close to

$$W_L^e \simeq \begin{pmatrix} 1 & V_{us} & V_{ub} \\ -V_{us}^* & 1 & \frac{V_{ub}}{V_{us}} - \frac{f(c_q^2)}{f(c_q^3)} \\ -\frac{f(c_q^2)}{f(c_q^3)} V_{us}^* & -\frac{V_{ub}^*}{V_{us}^*} + \frac{f(c_q^2)}{f(c_q^3)} & 1 \end{pmatrix} \quad \text{and}$$

$$W_L^\nu \simeq \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} + \mathcal{O}(\epsilon), \quad (4.5)$$

where we have $W_L^d \simeq W_L^e$, and where the $\mathcal{O}(\epsilon)$ entries represent small corrections coming from wave-function effects and flavor symmetry breaking terms. The PMNS matrix elements V_{e3}, $V_{\mu3}$ and V_{e2} are then expected to be

$$V_{e3} \simeq \left(1 - \frac{f(c_q^2)}{f(c_q^3)}\right) \frac{V_{us}}{\sqrt{2}} + \mathcal{O}(\epsilon), \quad (4.6)$$

$$V_{\mu3} \simeq -\frac{1}{\sqrt{2}} \left[1 + \frac{V_{ub}}{V_{us}} - \frac{f(c_q^2)}{f(c_q^3)}\right] + \mathcal{O}(\epsilon), \quad (4.7)$$

$$V_{e2} \simeq \frac{1}{\sqrt{2}} - \frac{1}{2}\left(1 + \frac{f(c_q^2)}{f(c_q^3)}\right) V_{us} + \mathcal{O}(\epsilon). \quad (4.8)$$

Combining these equations we obtain the relations linking the PMNS and CKM mixing matrix elements:

$$V_{e3} \simeq -V_{\mu3} V_{us} - \frac{V_{ub}}{\sqrt{2}} + \mathcal{O}(\epsilon), \quad (4.9)$$

$$V_{e2} \simeq \frac{1}{\sqrt{2}}\left(1 - \frac{V_{ub}}{\sqrt{2}}\right) - V_{us}\left(1 + \frac{V_{\mu3}}{\sqrt{2}}\right) + \mathcal{O}(\epsilon). \quad (4.10)$$

In order to check these relations we perform a random scan of the free parameters in the Yukawa couplings, fixing only the absolute value of $|d| = 1/7$ and allowing its phase and the rest of the complex parameters of Eq. (4.2), a, c, e, g, to be random, with absolute values of $\mathcal{O}(1)$. Still at the level of the Yukawa couplings, we also implement full random perturbations of order 5% as a small flavor symmetry breaking effect. The quark and charged lepton c parameters are fixed (without a scan) in order to obtain good first order values of CKM angles and masses, and the precise numerical values used are such that the flavor symmetry is also slightly broken. In the neutrino sector we take random values of c_ν^1, c_ν^2 and c_ν^3, with the only constraint that the values are in the "neutrino plateau" (see Fig. 3) and that their values remain relatively degenerate. In particular we consider three windows, where we randomly scan, i.e., $c_\nu^i \in [-2.5, -2.3]$, $c_\nu^i \in [-2.2, -2.0]$ and $c_\nu^i \in [-2.0, -1.8]$. During the scan over random parameters, we keep only points that produce numerically correct CKM angles $|V_{us}| \simeq 0.22$, $|V_{cb}| \simeq 0.041$ and $|V_{ub}| \simeq 0.0035$ [149], as well as correct neutrino mass differences. In Fig. 5, we test the correlation between $|V_{e3}|$ and $|V_{\mu3}|$ and find it in very good agreement with Eq. (4.9). The bands in the graph represent the 3σ uncertainty around the central values obtained from experimental global fits [150,151]. It is quite remarkable that the theoretical correlation curve as well as most of the points generated for the intermediate window lie within these bounds. Higher values of c_ν^i and lower values of c_ν^i produce points with too high or too small $|V_{\mu3}|$.[9] Note that the lower values of c_ν^i lie at the end of the plateau and the beginning of the exponential sensitivity, and so these points start to show greater deviations from the expected correlation. For higher c_ν^i values however, the correlation is expected to be quite robust, although the experimental values might not agree with the obtained results. In order to further check this scenario, we perform the same scan as before but including all the points generated, without checking for correct CKM values (although in general they are quite CKM-like). This means that sometimes, the value of the Cabibbo angle is larger or smaller than expected due to accidental alignments or suppressions coming from the Yukawa couplings (taken randomly). Nevertheless the expectation is that the correlation between V_{e3} and V_{us} from Eq. (4.6) should survive, and indeed we see in Fig. 6 that this is the case. The exact values of the parameters used in the scan are obtained by adding small deviations to the zero-order values. In the case of the c parameters we have thus $c_f^i = 0.6 \pm \mathcal{O}(0.1)$ for quarks and leptons, and $c_\nu^i = 2 \pm \mathcal{O}(0.1)$

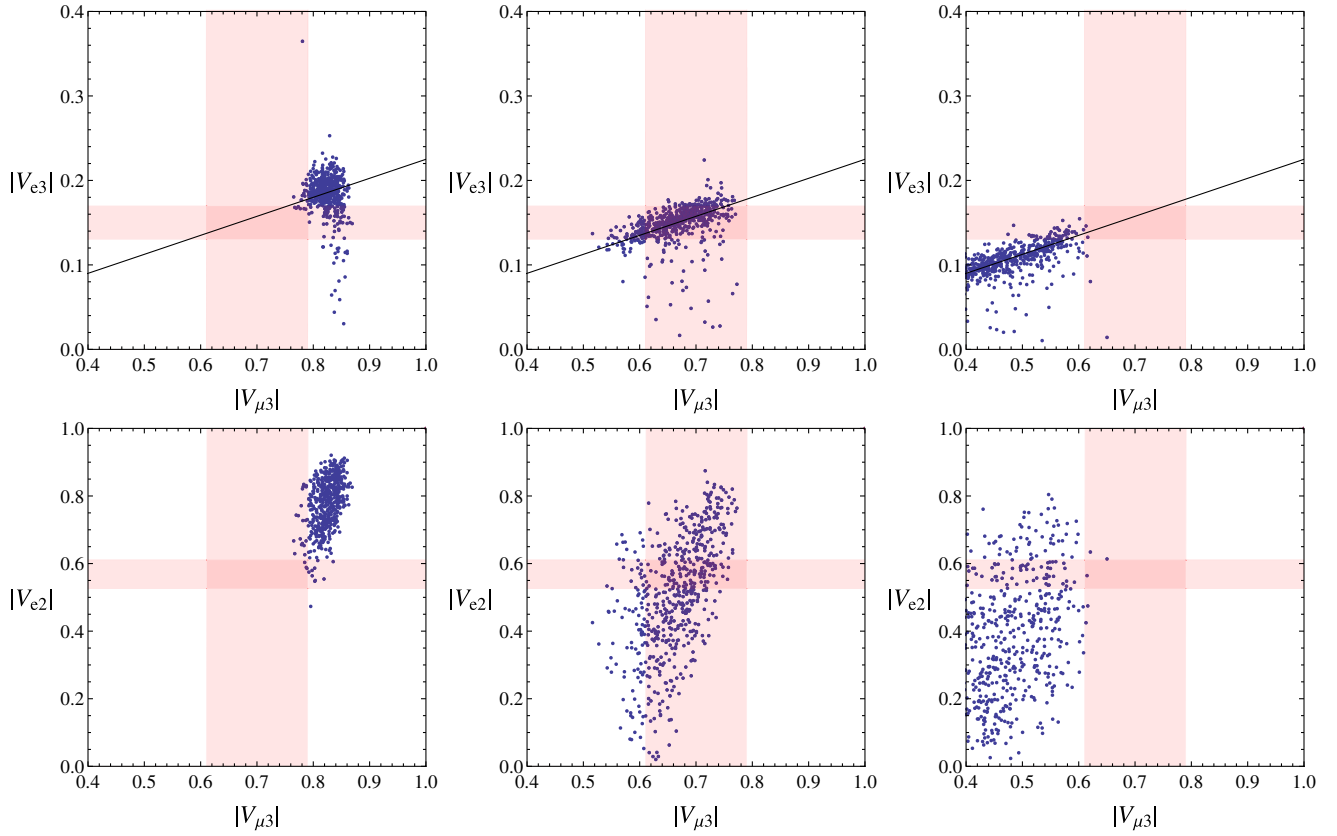[9]Breaking further the degeneracy in c_ν^i will increase the range of possible results.

FIG. 5 (color online). Scans of $|V_{e3}|$ versus $|V_{\mu3}|$ (upper panels) and $|V_{e2}|$ versus $|V_{\mu3}|$ (lower panels) using the 2-3 symmetric Yukawa couplings of Eq. (4.2), with $|d| = 1/7$ and random a, c, e, g, and their phases. Small random general perturbations of order 5% to these terms are also included. The right-hand neutrino c parameters are also taken randomly, within three different windows, i.e., $c_\nu^i \in [-2.5, -2.3]$ (left panels), $c_\nu^i \in [-2.2, -2.0]$ (center panels) and $c_\nu^i \in [-2.0, -1.8]$ (right panels). The points show agreement with the nontrivial correlation of Eq. (4.9) (represented by the diagonal line in the upper graphs).
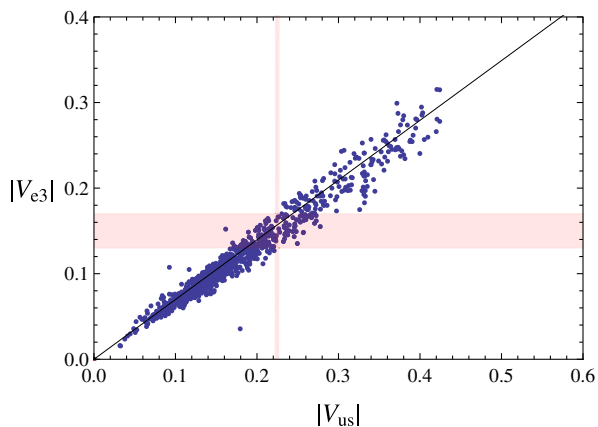

FIG. 6 (color online). Scan of the V_{us} versus V_{e3} with random perturbations around 2-3 symmetric Yukawa couplings as in the previous figures, with $c_\nu^i \in [-2.2, -2.0]$, and allowing for un-physical CKM entries (although still hierarchical), in order to test the quark-lepton correlation expressed in Eq. (4.6) and shown in the graph with the diagonal line.

for right-handed neutrinos. The Yukawa couplings are those given in Eq. (4.2), with the addition of small random perturbations.

V. FLAVOR DEMOCRACY

In the second symmetry implementation, in this section we assume a democratic structure [152–156] for all flavor parameters, meaning that in our case the 5D Yukawa couplings, Y_F^0 are invariant under $S_3 \times S_3$ symmetry, while the 5D fermion bulk mass matrices, $\mathbf{c}_f^0$ are invariant under S_3 permutations. Explicitly, the democratic 5D Yukawa couplings and 5D fermion bulk mass matrices are given by

$$Y_F^{\text{Dem}} \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{c}_f^{\text{Dem}} = \begin{pmatrix} a_f & b_f & b_f \\ b_f & a_f & b_f \\ b_f & b_f & a_f \end{pmatrix}.$$

$$(5.1)$$

Overall, there are four Yukawa matrices, Y_u^{Dem}, Y_d^{Dem}, Y_e^{Dem} and Y_ν^{Dem}, corresponding to the up-quark sector, the down-quark sector, the charged leptons and the neutrinos. There are six fermion bulk c matrices, namely $\mathbf{c}_q^{\text{Dem}}$, $\mathbf{c}_u^{\text{Dem}}$, $\mathbf{c}_d^{\text{Dem}}$, $\mathbf{c}_l^{\text{Dem}}$, $\mathbf{c}_e^{\text{Dem}}$ and $\mathbf{c}_\nu^{\text{Dem}}$.

Both matrices can be simultaneously diagonalized by the same unitary transformation resulting in two zero eigenvalues for the Yukawa matrices, and two degenerate eigenvalues for the bulk mass matrices, $\mathbf{c}_f^{\text{Dem}}$. The 5D Yukawa and bulk mass matrices thus become, in their diagonal basis,

$$Y_F^0 = y_F^0 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{c}_f^0 = \begin{pmatrix} c_f^{1^0} & 0 & 0 \\ 0 & c_f^{1^0} & 0 \\ 0 & 0 & c_f^{3^0} \end{pmatrix},$$

$$(5.2)$$

where y_F^0 are complex Yukawa couplings and the index F runs over u, d, e and ν. The elements c_f^{i0} are real and the index f_i runs over doublets q_i, l_i as well as singlets u_i, d_i, ν_i and e_i, with i the flavor index. Note that in this flavor symmetric limit, all fermions except the t quark, b quark, τ lepton and ν_τ neutrino are massless. The 5D flavor structure of Eq. (5.2) yields the zeroth order CKM and PMNS matrices for this scenario:

$$V_i^0 = \begin{pmatrix} \cos\theta_i^0 & \sin\theta_i^0 & 0 \\ -\sin\theta_i^0 & \cos\theta_i^0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (5.3)$$

where $i = $ CKM, PMNS. The angle θ_i^0, depends on the detailed structure of the symmetry breaking terms, δY_F and $\delta \mathbf{c}_f$ in Eqs. (3.8) and (3.7), and is not fixed by the underlying $S_3 \times S_3$ symmetry. Addition of generic small perturbations, as in Eqs. (3.7) and (3.8), breaks the flavor symmetry and lifts the degeneracies to produce SM-like masses and mixing angles. In the neutrino sector, the two level degeneracy is lifted by a small amount $(\delta Y_F)_{ij}$. This suggests a normal hierarchy ordering with one heavier eigenstate, and with two lighter ones having similar masses. Using Eq. (3.4) and taking the generic size of the perturbations as $(\delta Y_F)_{ij} \simeq \delta Y^\nu$ for simplicity yields the following relations for the neutrino masses:

$$m_1 \sim \delta Y^\nu \, v \, e^{-ky_1(a-1)},$$
$$m_2 \sim \delta Y^\nu \, v \, e^{-ky_1(a-1)},$$
$$m_3 \sim (1 + \delta Y^\nu) v \, e^{-ky_1(a-1)}. \quad (5.4)$$

Neutrino mass data requires $v \, e^{-ky_1(a-1)} \simeq 0.3$ eV and explaining the hierarchy problem requires $ky_1 \simeq 35$, which means that the value of the Higgs localization parameter

should be about $a \simeq 1.8$. As explained in a previous section, this value of a requires some fine-tuning of parameters in the RS 5D Higgs potential. With modified AdS_5 metrics it is possible to remain in a nontuned region, and in particular we find that the best region is obtained for $\nu \sim 0.2$ and $kL_1 \sim 0.3$, where the parameter a can have values as large as 4.5.[10] In order to obtain the observed neutrino mass hierarchy ratio r, defined as $r = (|m_2|^2 - |m_1|^2)/(|m_3|^2 - |m_1|^2) \simeq 0.03$, the size of the Yukawa coupling perturbations δY must be fixed to $\delta Y^\nu \lesssim \sqrt{r} \simeq 0.17$. There are no restrictions on the values of bulk mass parameter c_ψ^i's (as long as they are within the bounds $a < c_\nu + c_l$).

Consider the elements V_{e2}, V_{e3} and $V_{\mu3}$ of the PMNS matrix. As mentioned before, due to the plateau in the neutrino sector, for a small a parameter, the 5D Yukawa matrix structure is more or less preserved and therefore these elements should be close to the ones shown in Eq. (5.3) plus some perturbations. This matrix predicts very small values for both $|V_{e3}|$ and $|V_{\mu3}|$ and so the perturbations should lift them (especially $|V_{\mu3}|$). The value of V_{e2} on the other hand is highly sensitive to the structure of the neutrino Yukawa flavor violating matrix δY_{ij}^ν and can be large or small. It turns out that when the c parameters are such that $a < c_l^3 + c_\nu^j$ and $c_l^3 < 1/2$ (a region not generic in usual warped extra dimension scenarios) the lifting of the zero-order PMNS can be successful and we have

$$V_{e2} \sim \sin\theta_\nu^0, \quad (5.5)$$

$$V_{e3} \propto \delta Y_{13}^\nu f(c_l^3), \quad (5.6)$$

$$V_{\mu3} \propto \delta Y_{23}^\nu f(c_l^3). \quad (5.7)$$

Note in particular that we expect that $V_{e3}/V_{\mu3} \propto \delta Y_{13}^\nu/\delta Y_{23}^\nu$. In Fig. 7 we present a scan of the model parameters to verify the validity of Eq. (5.5). In this scan we randomly perturbed the values of the 5D parameters Y_ν, Y_e, c_l and c_ν, but kept the values of the three relevant parameters c_l^3, δY_{13}^ν and δY_{23}^ν fixed. We see from the figure that the values obtained for the matrix elements $V_{\mu3}$ and V_{e3} can be made to lie within the experimental bounds by fixing only these three parameters with all other terms randomly perturbed. In particular, the formulas show the sensitivity of these two PMNS mixing angles to the flavor structure of the neutrino Yukawa matrix δY^ν, but not to the charged lepton Yukawa matrix δY^l or to the bulk masses δc_ψ^i, except for δc_l^3. Knowing that experimentally $V_{\mu3}^{\text{exp}} \simeq 0.65$ and $V_{e3}^{\text{exp}} \simeq 0.15$, for the numerical evaluations we took the bulk mass parameter of the third family lepton

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[10]This is due to the fact that in the modified AdS_5 case, Eq. (5.4) will be slightly modified and higher values of the a parameter become acceptable.
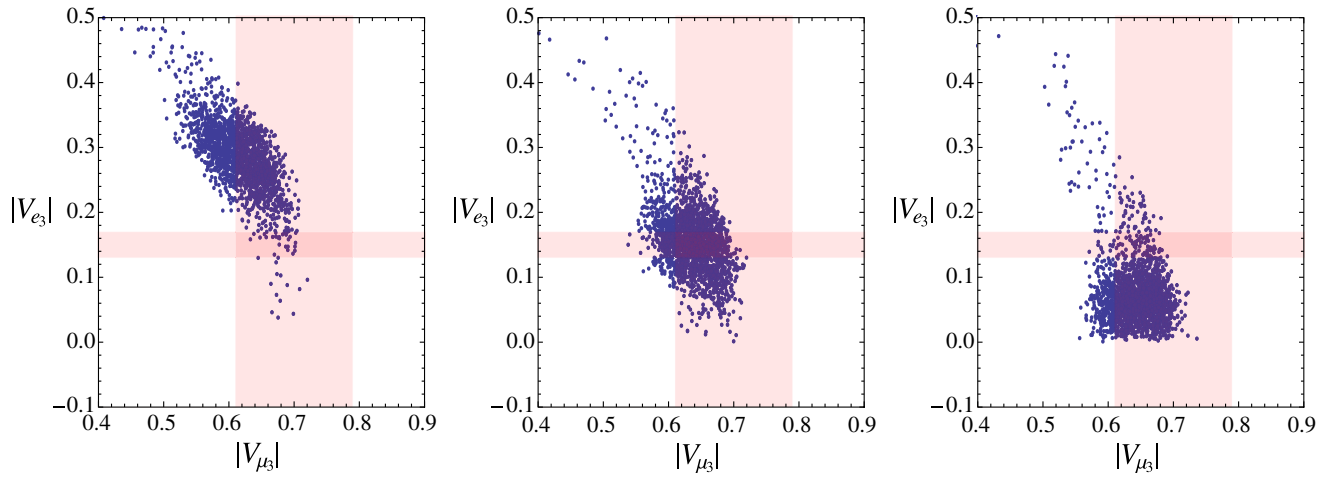
FIG. 7 (color online).   Scatter plots of V_{e3} versus $V_{\mu 3}$ with random values of δY_ν, δY_e, c_l and c_ν. We have fixed $c_l^3 = 0.41$ and $(\delta Y^\nu)_{23} = 0.13$ and have then taken $(\delta Y^\nu)_{13} = 0.002$ (left panel) $(\delta Y^\nu)_{13} = 0.008$ (center panel) and $(\delta Y^\nu)_{13} = 0.032$ (right panel). The concentration of points in a precise region shows that the mixing angles V_{e3} and $V_{\mu 3}$ are sensitive to only these three parameters. The obtained values of $V_{\mu 3}$ lie on the lower side of the experimental window, whereas V_{e3} depends on the ratio of Yukawa couplings $(\delta Y^\nu)_{13}/(\delta Y^\nu)_{23}$, so that its smallness is due to a slight hierarchy in these.

doublet $c_l^3 < 1/2$ to obtain larger mixing angles for small $\delta Y \simeq 0.1$, the same as in the quark sector, where $c_q^3 \lesssim 1/2$ is needed to obtain a large top quark mass, and thus it could be a hint of a possible additional family symmetry among the $SU(2)$ doublets of the third family. The particular values used in the scan are $c_l^3 = 0.41$, $\delta Y_{13}^\nu = 0.008$ and $\delta Y_{23}^\nu = 0.13$, which produce PMNS angles consistent with the experiment but with $|V_{\mu 3}|$ on the lower experimental side. When the experimental uncertainty decreases and if the central value of $|V_{\mu 3}^{\text{exp}}|$ ends up on the higher end of the current allowed region, this scenario will be under great pressure. We also show how indeed the ratio $V_{e3}/V_{\mu 3}$ depends on the ratio $\delta Y_{13}^\nu/\delta Y_{23}^\nu$ by increasing and decreasing the value of δY_{13}^ν, resulting in larger or smaller values of $|V_{e3}|$.

In the charged lepton sector and the up- and down-quark sectors, massless states are also lifted by the flavor symmetry breaking, and these masses emerge directly proportional to the generic size of the perturbations in the Yukawa matrix, δY, as described in [103].

TABLE I.   A parameter point for democratic flavor symmetry in the localization parameter space (out of many possible points) in the zeroth order 5D fermion c parameter space, consistent with all the experimental and model constraints. For this point, we have set all the zeroth order Yukawa coefficients to be universal, $y_u^0 = y_d^0 = y_\nu^0 = y_e^0 = 4.4$, and the Higgs localization parameter to $a = 2.1$. The modified AdS$_5$ metric parameters are $\nu = 1.1$, $y_1 = 2.8 \times 10^{-17}$ and $A(y_1) = 35$.

f	q	l	u	d	e	ν
$c_f^{1^0}$	0.55	0.55	0.55	0.65	0.65	5.00
$c_f^{3^0}$	0.45	0.45	0.45	0.65	0.65	3.00

Finally, in Table I we present a set of S_3 symmetric zeroth order bulk c parameters (many other points with the same symmetry in the parameter space are possible), which, with a small perturbation of order $\delta c \simeq \delta Y \simeq 10\%$, can lead to the SM in the modified AdS$_5$ scenario.[11] For this specific point we have taken the value of the warp exponent at the IR brane, $A(y_1)$, to be exactly 35. With this assignment, there is only one more free parameter in the model left to completely fix the metric. This parameter can be either ν or the position of the singularity, y_s. Therefore one has the freedom to choose the amount of departure from the pure AdS$_5$. For the point presented in Table I, we took $\nu \simeq 0.32$ and the value of the localization of the Higgs field operator a is 4.46 as in the previous example. With this starting point, the SM is then easily reproduced by breaking the symmetry through adding perturbations δc^i's and δY_{ij}'s to lift the degeneracies of the symmetric scenario. We require the $\delta Y_{ij} \lesssim 0.1$ and $\delta c_\psi^i \lesssim 0.1$ parameters to produce the SM masses, angles and phases, by systematically using the approximate formulas in this section up to any order of precision consistent with the SM. In general the sensitivity to the precise values of the 5D Yukawa couplings, δY_{ij}, is minimal and randomly chosen matrices with $\delta Y_{ij} \lesssim 0.1$ produce all the required SM features.

VI. CONCLUSION

In this work, we have provided a warped extra dimensional framework in which all of the fermions, including neutrinos, are treated on equal footing, and where the SM fermion flavor structure can still emerge out of a slightly

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[11] A similar point for the pure AdS$_5$ was presented in [103].

broken universal flavor symmetry. Essential for this scenario is that all the matter fields must be in the bulk and, in particular, the Higgs field should be as delocalized as possible from the IR boundary. We have explored a general warped scenario in which the metric is modified from the usual AdS_5 background. This setup has the advantage of allowing lower KK masses (~ 1 to 2 TeV), while still safe from precision electroweak tests and flavor bounds. For our study, the modified metric presents a further advantage, as the neutrino mass generation in our framework is less fine tuned. This is due to a numerical accident by which the neutrino masses generated in the AdS_5 background are too small (at most $\sim 10^{-4}$ eV) in the parameter region of interest (the neutrino plateau), while in the modified metric neutrino masses can be up to 2 orders of magnitude larger, in the same qualitative region of parameter space. A way out in AdS_5 would be to further delocalize the Higgs field, although requiring such a delocalized Higgs to generically solve the hierarchy problem, some degree of fine-tuning must be introduced in the Higgs potential. These tensions disappear when we use a modified AdS_5 geometry, where our flavor setup can be successfully implemented without further fine-tuning.

In the parameter region of interest (the neutrino plateau) the effective 4D neutrino masses do not have exponential dependence on the bulk mass parameters c^i, in contrast with quark and charged lepton masses. Once a universal flavor symmetry is slightly broken, the SM flavor structure emerges due to the inherent features of warped space models, i.e., the wave-function profiles of light quarks and charged leptons are exponentially sensitive to the symmetry violating terms, resulting in masses and mixing controlled by small flavor violating terms. In the neutrino sector, in the plateau region with a highly delocalized Higgs field, the wave functions are not exponentially sensitive to Lagrangian parameters and thus the original flavor symmetry is essentially preserved. Overall results are similar in both RS and modified AdS_5 types of scenarios, indicating that the precise nature of the flavor symmetry or the precise nature of the metric solution is not crucial for the main property of the scenarios.

For illustration, we choose two simple examples of different symmetries which can provide an implementation for this mechanism. In the first scenario, we study quark-lepton complementarity associated with μ-τ symmetry (or 2-3 symmetry), known to lead to phenomenologically viable neutrino masses and mixings. Assuming equal Yukawa matrix couplings in the charged lepton and down-quark sector, and equal Yukawa matrix couplings

in the neutrino and up-quark sector, as well as identical localization for the doublet quark and lepton representations, mass differences emerge entirely from singlet representation localization. The predictions of this implementation are definite connections between PMNS and CKM matrix elements, as given by Eqs. (4.9) and (4.10).

In the second example, flavor democracy, the 5D Yukawa couplings for the fermions are invariant under the $S_3 \times S_3$ symmetry, while the 5D fermion bulk mass matrices are invariant under S_3 family permutation invariance. Small perturbations of $\mathcal{O}(10\%)$ are enough to generate the full flavor structure of the SM in both quark and lepton sectors, and the ratio of PMNS matrix elements can be simply expressed in terms of these perturbations. Explicit expressions appear in Eq. (5.5). Localization of the third lepton family follows localization of the third quark family, hinting at an additional quark-lepton symmetry. As distinctive predictions and correlations appear in each implementation, the model studied here would yield a very promising novel laboratory for studying fermion flavor symmetries.

APPENDIX: EXPLICIT EXPRESSIONS FOR THE FIELD PROFILES

Here we derive the explicit expressions for the fermion and Higgs profiles, as well as for the Yukawa couplings for the modified AdS_5 scenario. The fermion profiles are given by

$$q_L^{0,i}(y) = q_0^i e^{(2 - c_q^i) A(y)}, \qquad u_R^{0,i} = u_0^i e^{(c_u^i + 2) A(y)}, \quad \text{(A1)}$$

while the Higgs profile is

$$h(y) = h_0 e^{aky}, \qquad \text{(A2)}$$

with

$$q_0^i = \sqrt{k} \epsilon^{\frac{1}{2} - c_q^i} f(c_q^i) \equiv \sqrt{k} \mathfrak{f}(c_q^i),$$
$$u_0^i = \sqrt{k} \epsilon^{\frac{1}{2} + c_u^i} f(-c_u^i) \equiv \sqrt{k} \mathfrak{f}(-c_u^i), \qquad \text{(A3)}$$

and

$$h_0 \equiv \sqrt{k} e^{-(a-1)ky_s} \mathfrak{h}_0$$

$$\mathfrak{h}_0 = \frac{1}{\sqrt{ky_s(2(a-1)ky_s)^{-\frac{2}{\nu^2}-1}(\Gamma(1 + \frac{2}{\nu^2}, 2(a-1)k(y_s - y_1)) - \Gamma(1 + \frac{2}{\nu^2}, 2(a-1)ky_s))}}, \qquad \text{(A4)}$$

where

$$\epsilon = e^{-A(y)} = e^{-ky_1}\left(1 - \frac{y_1}{y_s}\right)^{\frac{1}{\nu^2}}. \tag{A5}$$

The reduced profile functions $\mathfrak{f}(c)$ are defined as $\mathfrak{f}(c) \equiv \epsilon^{\frac{1}{2}-c}f(c)$, with

$$f(c) \equiv \frac{\epsilon^{c-\frac{1}{2}}}{\sqrt{ky_s e^{(1-2c)ky_s}((1-2c)ky_s)^{\frac{1-2c}{\nu^2}-1}(\Gamma(1-\frac{1-2c}{\nu^2},(1-2c)k(y_s-y_1))-\Gamma(1-\frac{1-2c}{\nu^2},(1-2c)ky_s))}}. \tag{A6}$$

Note that the Higgs and fermion profiles are defined differently, due to the specific Higgs potential we have considered [157]. We can now write down the most general form for the Yukawa couplings as

$$y_{ij}^u = \tilde{Y}_{ij}^u h_0 f(c_q^i)f(-c_u^j), \tag{A7}$$

where the $\tilde{Y}_{ij}$'s are related to the 5D Yukawa couplings via the equation

$$\tilde{Y}_{ij}^u \equiv Y_{ij}^u \sqrt{k}\epsilon^{1-c_q+c_u}y_s e^{ky_s(a-c_q^i+c_u^j)}[ky_s(a-c_q^i+c_u^j)]^{\frac{c_u^j-c_q^i}{\nu^2}-1}$$

$$\times \left[\Gamma\left(\frac{c_q^i-c_u^j}{\nu^2}+1,(a-c_q^i+c_u^j)k(y_s-y_1)\right)-\Gamma\left(\frac{c_q^i-c_u^j}{\nu^2}+1,(a-c_q^i+c_u^j)ky_s\right)\right]. \tag{A8}$$

Before switching to the expressions in the RS metric, we use the asymptotic expansion of the incomplete gamma function

$$\Gamma(a,z) \sim z^{a-1}e^{-z}\left(1+\frac{a-1}{z}+\frac{(a-1)(a-2)}{z^2}+\mathcal{O}(z^{-3})\right). \tag{A9}$$

Then we get the following asymptotic behavior, up to the first order in $\mathcal{O}(z^{a-1})$, with ϵ defined as in Eq. (A5):

$$f(c) \sim \sqrt{\frac{1-2c}{1-\epsilon^{1-2c}}}, \tag{A10}$$

$$h_0 \sim \sqrt{\frac{2(1-a)}{1-e^{2aky_1}\epsilon^2}}, \tag{A11}$$

$$\tilde{Y}_{ij}^u \sim \frac{e^{aky_1}\epsilon^{c_q^i-c_u^j}-1}{a-c_q^i+c_u^j}Y_{ij}^u. \tag{A12}$$

Keeping the $\mathcal{O}(z^{a-2})$ term not only gives a much better approximation for the top and neutrino plateaus, but it also gives a more transparent expression for these functions:

$$f(c) \sim \epsilon^{c-\frac{1}{2}}\sqrt{\frac{(1-2c)ky_s\nu^2}{ky_s\nu^2(\epsilon^{2c-1}-1)-\epsilon^{2c-1}+(\frac{y_s}{y_s-y_1})}}, \tag{A13}$$

$$h_0 \sim \epsilon^{-1}e^{-aky_1}\sqrt{\frac{2(1-a)^2ky_s\nu^2}{ky_s\nu^2(1-a)(\epsilon^{-2}e^{-2aky_1}-1)-\epsilon^{-2}e^{-2aky_1}+(\frac{y_s}{y_s-y_1})}}. \tag{A14}$$

From these formulas, valid for the general modified AdS$_5$ metric, one can see that, taking the limits $\nu \to \infty$ and $y_s \to \infty$ we obtain the expressions for the profiles in the RS metric:

$$f^{RS}(c) = \sqrt{\frac{1-2c}{1-\epsilon^{1-2c}}} \equiv \epsilon^{c-\frac{1}{2}}\mathfrak{f}^{RS}(c), \qquad h_0^{RS} = e^{(1-a)ky_1}\sqrt{\frac{2(1-a)}{\epsilon^{2(a-1)}-1}}, \tag{A15}$$

$$\tilde{Y}_{ij}^{RS,u} \equiv \frac{\epsilon^{-(a-c_q^i+c_u^j)}-1}{a-c_q^i+c_u^j}Y_{ij}^{RS,u}, \qquad y_{ij}^{RS,u} = \tilde{Y}_{ij}^{RS,u}h_0^{RS}f^{RS}(c_q^i)f^{RS}(-c_u^j), \tag{A16}$$

where $\epsilon \equiv e^{-ky_1}$.

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