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Z' mass limits and the naturalness of supersymmetry

P. Athron,[1,*] D. Harries,[2,†] and A. G. Williams[2,‡]

[1]*ARC Centre of Excellence for Particle Physics at the Terascale, School of Physics, Monash University, Melbourne VIC 3800, Australia*

[2]*ARC Centre of Excellence for Particle Physics at the Terascale, Department of Physics, The University of Adelaide, Adelaide, South Australia 5005, Australia*

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The discovery of a 125 GeV Higgs boson and rising lower bounds on the masses of superpartners have led to concerns that supersymmetric models are now fine-tuned. Large stop masses, required for a 125 GeV Higgs, feed into the electroweak symmetry breaking conditions through renormalization group equations forcing one to fine-tune these parameters to obtain the correct electroweak vacuum expectation value. Nonetheless, this fine-tuning depends crucially on our assumptions about the supersymmetry breaking scale. At the same time, $U(1)$ extensions provide the most compelling solution to the μ problem, which is also a naturalness issue, and allow the tree-level Higgs mass to be raised substantially above M_Z. These very well-motivated supersymmetric models predict a new Z' boson which could be discovered at the LHC, and the naturalness of the model requires that the Z' boson mass should not be too far above the TeV scale. Moreover, this fine-tuning appears at the tree level, making it less dependent on assumptions about the supersymmetry breaking mechanism. Here we study this fine-tuning for several $U(1)$ supersymmetric extensions of the Standard Model and compare it to the situation in the MSSM where the most direct tree-level fine-tuning can be probed through chargino mass limits. We show that future LHC Z' searches are extremely important for challenging the most natural scenarios in these models.

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I. INTRODUCTION

The discovery of an approximately 125 GeV Higgs [1,2] at the Large Hadron Collider (LHC) has interesting implications for physics beyond the Standard Model (SM) and supersymmetry (SUSY). On the one hand, it provides a light Higgs boson, as expected from supersymmetry, and can be fitted in the minimal supersymmetric standard model (MSSM). On the other hand, the Higgs mass is slightly heavier than the constrained version of the MSSM (cMSSM) can accommodate naturally [3,4].

In the MSSM the Higgs mass causes a naturalness problem because at tree level it has an upper bound of the mass of the Z boson, M_Z. The dominant higher-order corrections to the Higgs mass come from stops, and to obtain a 125 GeV Higgs they need to be rather heavy. Heavy stops will provide a large contribution to the low-energy value of $m^2_{H_u}$, the soft breaking mass for the up-type Higgs scalar, through the evolution of the renormalization group equations (RGEs) from the grand unification (GUT) scale to the electroweak (EW) scale. This affects the SUSY prediction of the EW vacuum expectation value (VEV), v, or M_Z. This naturalness problem motivates both further examination of nonminimal SUSY models that can raise the Higgs mass without the need for heavy stops and alternative possibilities for how the soft breaking parameters get generated, which might set them at lower energies, reducing the influence the stops have on $m^2_{H_u}$.

In addition to that naturalness issue, often referred to as the little hierarchy problem, the MSSM also suffers from the μ problem. This is also a naturalness problem since there should be a natural explanation of how the μ superpotential parameter can be set to the same scale as the soft breaking masses.

$U(1)$ extensions of the MSSM provide a very elegant solution to this μ problem [5–12] and also raise the Higgs mass with new F and D terms. Nonetheless, as was recently demonstrated in the context of the exceptional supersymmetric standard model (E_6SSM) [13–15], such models can still suffer from naturalness problems with the mass of the new Z' associated with the break down of the new $U(1)$ appearing in the EW symmetry breaking (EWSB) conditions at tree level [16]. Despite this the constrained version of the E_6SSM (cE_6SSM) [17,18] was still found to be significantly less tuned than the cMSSM. Tree-level fine-tuning from the Z' mass was also considered previously [19].

However, this comparison of fine-tuning depends crucially upon the assumptions of these gravity mediated SUSY breaking motivated constrained models and, in particular, the universality constraints being applied at the GUT scale. As mentioned above, given the findings at the LHC, it is worth considering other possibilities, which may allow the soft masses to be set at lower energies.

*peter.athron@monash.edu
†dylan.harries@adelaide.edu.au
‡anthony.williams@adelaide.edu.au

As the scale at which the parameters fulfill some breaking inspired constraints is lowered the stop masses contribute less to the fine-tuning.

At the same time in $U(1)$ extensions, lowering the UV boundary scale for the RGE evolution also allows even larger F-term contributions to the Higgs mass, so long as one only requires λ, the coupling between the Singlet Higgs, S and the up- and down-type Higgs bosons, H_u and H_d, to remain perturbative up to the UV scale and not all of the way up to the GUT scale.

However, the tuning from the Z' mass limit does not disappear as the UV boundary condition is lowered. This tuning appears in the EWSB conditions at tree level and is quite difficult to avoid without introducing a pure gauge singlet [20].

In this paper we investigate how big this tuning is if we bring this scale all the way down to 20 TeV, effectively minimizing the contribution from the stops. We find that the Z' limit is enough to already require moderate fine-tuning in the E_6SSM. We also show this is comparable to the situation in the MSSM defined at the same scale if charginos could be ruled out below 700 GeV. We then show how this tuning from the Z' mass looks for different $U(1)$ extensions, finding that the current severity depends upon the charges but that Z' limits are important in constraining the most natural scenarios of these models. Therefore, the Z' constraint is amongst the most important in terms of tuning and attacking natural supersymmetry experimentally and the next run of the LHC will be crucial in this respect.

Finally, we make a case study, for a few benchmarks, of the impact of raising the high-scale boundary condition, M_X, at which the SUSY breaking parameters must be fixed by some SUSY breaking mechanism. We show that which model has less fine-tuning depends on M_X. We also see rather complicated behavior in the tuning for the E_6SSM points due to the combination of different sources of tuning.

As mentioned earlier, the fine-tuning of the cE_6SSM was recently studied [16] and there it was revealed that the associated Z' boson leads to a new source of fine-tuning since its mass appears in the EWSB conditions.

However, in this study we will examine this source of fine-tuning in more detail by considering low-energy constructions where the usual fine-tuning problem from the Higgs is minimized. We will also consider alternate charges for the extra $U(1)$ symmetry to relax the focus on the E_6SSM and demonstrate that this is quite a generic result.

To quantify the fine-tuning, we will employ the traditional Barbieri-Giudice measure [21,22]. This has been used extensively within the literature e.g. Refs. [16,23–49].

A number of alternative measures have also been applied in the literature [50–65] with varying motivations. A very different approach is to work within a Bayesian analysis. There the concept of naturalness is automatically incorporated since in models where one must fine- tune parameters to fit measured values of the observables, the region with high likelihood will occupy a tiny prior volume [4,66–70] suppressing the posterior. Indeed in the MSSM and the next-to-MSSM (NMSSM) if one transforms GUT scale parameters to the VEVs, the inverse of the Jacobian for this transformation looks quite like the derivatives that appear in the traditional fine-tuning measure [66,67,70]. If one thinks more generally, then a model without fine-tuning is one where the parameterization is such that all the parameters are observables [69,70]. This provides a quite general definition of fine-tuning as $1/|J|$ where $|J|$ is the determinant of the Jacobian for the coordinate transformation between the parameters and the observables. Interestingly this means the tuning is the ratio of the infinitesimal observable space volume element to the infinitesimal parameter space element and coincides with the measure proposed in Ref. [63] when the interval of variation is taken to zero.

While this approach has many merits here we will employ the traditional measure of fine-tuning because it is both simple to apply and easy to compare with previous results due to it's widespread use. Fortunately the derivatives which appear in these tunings are also similar to the Bayesian motivated measure so there should not be too large a discrepancy between the two approaches.

The structure of this paper is as follows. In Sec. II we review the models we consider. In Sec. III we specify the EWSB conditions of the models, with particular focus on how the Z' mass influences the prediction of M_Z. Then in Sec. IV we introduce our fine-tuning measure and our approach to evaluating it to obtain the individual sensitivities. The results are then given in Sec. V.

II. $U(1)$ EXTENSIONS AND THE E_6SSM

In this paper we consider $U(1)$ extensions of the MSSM where the gauge group at low energies is

$$SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)'. \qquad (1)$$

$U(1)'$ is the new gauge group beyond that of the SM and MSSM. The minimal superfield content of $U(1)$ extensions which solve the μ problem should be ordinary left-handed quark $\hat{Q}_i$ and lepton $\hat{L}_i$ $(i = 1, 2, 3)$ superfields along with right-handed superfields $\hat{u}^c_i$, $\hat{d}^c_i$, $\hat{e}^c_i$ $(i = 1, 2, 3)$ for the up-type (s)quarks, down-type (s)quarks and charged (s)leptons respectively and three Higgs superfields, up-type $\hat{H}_u$, down-type $\hat{H}_d$ and a singlet under the SM gauge group $\hat{S}$.

Here we will refer to $U(1)$ extensions of the MSSM, which solve the μ problem, as the USSM [8–12]. The couplings for the $U(1)'$ gauge group should allow the following renormalizable superpotential terms required in the USSM,

$$W_{\rm USSM} = y^U_{ij}\hat{u}^c_i\hat{H}_u\cdot\hat{Q}_j + y^D_{ij}\hat{d}^c_i\hat{Q}_j\cdot\hat{H}_d + y^E_{ij}\hat{e}^c_i\hat{L}_j\cdot\hat{H}_d + \lambda\hat{S}\hat{H}_d\cdot\hat{H}_u, \quad (2)$$

with $i, j \in \{1,2,3\}$. For the $SU(2)$ dot product we follow the convention $\hat{A}\cdot\hat{B} \equiv \epsilon_{\alpha\beta}\hat{A}^\alpha\hat{B}^\beta = \hat{A}^2\hat{B}^1 - \hat{A}^1\hat{B}^2$.

The $U(1)'$ charges should allow for cancellations of gauge anomalies. The most elegant way to do this is to use an extra $U(1)$ gauge symmetry that can be obtained from the break down of the E_6 gauge symmetry which is anomaly free and have all matter fields that fill the three generations of 27-plet representations of E_6 survive down to low energies. Such models are often referred to in the literature as E_6 inspired, and we will adopt this here.

The breaking of E_6 into $SO(10)$ gives rise to $E_6 \to SO(10)\times U(1)_\psi$, and the subsequent breaking of $SO(10)$ into $SU(5)$ gives $SO(10) \to SU(5)\times U(1)_\chi$ (this is reviewed in e.g. Ref. [71]). The extra $U(1)$ gauge symmetry at low energies should then be a linear combination of these in the E_6 inspired case,

$$U(1)' = U(1)_\chi\cos\theta + U(1)_\psi\sin\theta. \quad (3)$$

In Table I the charges for several popular E_6 inspired $U(1)$ extensions are shown.

$U(1)$ and E_6 inspired extensions of the MSSM have been studied very widely in the literature [21,72–94] (or, for reviews, see Refs. [71,95]). There has also been a lot of work recently including investigations of the neutralino sector [96–99]; the relic density of dark matter [100]; GUT scale family symmetries which can explain the hierarchy of masses in the fermion sector and their associated mixings [101]; neutrino physics [102]; explanations of the matter-antimatter asymmetry of the Universe though EW baryogenesis or leptogenesis [93,94,103]; decays of the Z' boson [104–107]; dipole moments [108]; anomaly mediated SUSY breaking with D-term contributions [109] and the (extended) Higgs sectors [110,111].

Here we will focus most on the special case where the gauge symmetry is $U(1)_N$, under which the right-handed neutrino $\hat{N}^c$ does not participate in gauge interactions. This is the case in the E$_6$SSM [13–15], and closely related variants [20,112–116]. Since the right-handed neutrino has no gauge symmetry protecting it's mass from becoming extremely heavy such models may explain the tiny observed masses of neutrinos via the see-saw mechanism and the baryon asymmetry in the Universe via leptogenesis [93,117,118]. Recently it has also been studied in the context of EW baryogenesis [119].

The gauge coupling running in the E$_6$SSM at the two-loop level leads to unification more precisely than in the MSSM [120] or, in slightly modified scenarios, two-step unification can take place [112,121]. If the exotic particles are light in these models this can open up nonstandard decays of the SM-like Higgs boson [20,122,123].

The correct relic density could be obtained entirely through an almost decoupled "inert" neutralino sector [124]. However, this is no longer phenomenologically viable due to limits from direct detection of dark matter [125–127] and due to a significant suppression of the decay of the lightest Higgs boson into SM states, due to a new channel into inert singlinos opening up.

There are still several remaining options. One may specialize to scenarios known as the EZSSM [115] where the inert singlinos that cause these problems are entirely decoupled and the relic abundance is fitted with a binolike candidate with a novel mechanism involving back-scattering into a heavier inert Higgsino. Another well motivated scenario admits two possible dark matter candidates [116], where one will be an inert singlino and the other will have a similar composition to MSSM neutralinos. The simplest phenomenologically viable solution in that case is to make the singlinos extremely light hot dark matter candidates, in which case the lightest ordinary neutralino accounts for almost all of the observed relic abundance.

The impact of gauge kinetic mixing in the case where both of the extra $U(1)$ symmetries appearing from the breakdown of E_6 are present at low energy was studied in Ref. [128]. The E$_6$SSM was also included in studies looking at how first- or second-generation sfermion masses can be used to constrain the GUT scale parameters [129] and the renormalization of VEVs [130,131]. The particle spectrum and collider signatures of the cE$_6$SSM have been studied in a series of papers, [17,18,106,132]. The threshold corrections to the $\overline{\rm DR}$ gauge and Yukawa couplings in the E$_6$SSM have also been calculated and their numerical impact in the constrained version examined [133].

TABLE I. The $U(1)_Y$, $U(1)_\psi$, $U(1)_\chi$ and $U(1)_N$ charges of the chiral superfields in the E_6 model. The specific case of $U(1)_N$, corresponding to the E$_6$SSM, is obtained for $\theta = \arctan\sqrt{15}$.

	$\hat{Q}$	$\hat{u}^c$	$\hat{d}^c$	$\hat{L}$	$\hat{e}^c$	$\hat{N}^c$	$\hat{S}$	$\hat{H}_2$	$\hat{H}_1$	$\hat{D}$	$\hat{\bar{D}}$	$\hat{H}'$	$\hat{\bar{H}}'$
$\sqrt{\frac{5}{3}}Q_i^Y$	$\frac{1}{6}$	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$	1	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{2}$
$2\sqrt{6}Q_i^\psi$	1	1	1	1	1	1	4	−2	−2	−2	−2	1	−1
$2\sqrt{10}Q_i^\chi$	−1	−1	3	3	−1	−5	0	2	−2	2	−2	3	−3
$\sqrt{40}Q_i^N$	1	1	2	2	1	0	5	−2	−3	−2	−3	2	−2

With three generations of matter 27-plet representations of E_6 surviving to low energies, the low-energy matter content in each generation, after integrating out the heavy right-handed neutrinos, includes,

$$(\hat{Q}_i, \hat{u}_i^c, \hat{d}_i^c, \hat{L}_i, \hat{e}_i^c) + (\hat{D}_i, \hat{\bar{D}}_i) + (\hat{S}_i) + (\hat{H}_i^u) + (\hat{H}_i^d), \tag{4}$$

where the $\hat{S}_i$, $\hat{H}_i^u$ and $\hat{H}_i^d$ have the quantum numbers of a SM singlet, and up-, down-type Higgs fields, respectively, and the $\hat{D}_i$ and $\hat{\bar{D}}_i$ are $SU(3)_C$ triplets that reside in the same $SU(5)$ multiplets as these Higgs-like states.

If one wishes to maintain gauge coupling unification this set of states should be augmented by two extra $SU(2)$ doublet states H' and $\bar{H}'$ belonging to other $27'$ and $\overline{27}'$ multiplets that must be incomplete at low energies.

The full superpotential for E_6 inspired models coming from $27 \otimes 27 \otimes 27$ decomposition of the fundamental E_6 representation will then be

$$W_{E_6} = W_0 + W_1 + W_2, \tag{5}$$

where

$$\begin{aligned} W_0 = {}& \lambda_{ijk} \hat{S}_i \hat{H}_j^d \cdot \hat{H}_k^u + \kappa_{ijk} \hat{S}_i \hat{D}_j \hat{\bar{D}}_k + h_{ijk}^N \hat{N}_i^c \hat{H}_j^u \cdot \hat{L}_k \\ & + y_{ijk}^U \hat{u}_i^c \hat{H}_j^u \cdot \hat{Q}_k + y_{ijk}^D \hat{d}_i^c \hat{Q}_k \cdot \hat{H}_j^d + y_{ijk}^E \hat{e}_i^c \hat{L}_k \cdot \hat{H}_j^d, \end{aligned} \tag{6}$$

$$W_1 = g_{ijk}^Q \hat{D}_i \hat{Q}_j \cdot \hat{Q}_k + g_{ijk}^q \hat{\bar{D}}_i \hat{d}_j^c \hat{u}_k^c, \tag{7}$$

$$W_2 = g_{ijk}^N \hat{N}_i^c \hat{D}_j \hat{d}_k^c + g_{ijk}^E \hat{e}_i^c \hat{D}_j \hat{u}_k^c + g_{ijk}^D \hat{Q}_i \cdot \hat{L}_j \hat{\bar{D}}_k. \tag{8}$$

Nonetheless, while this model is very elegant so far, the superpotential of Eq. (5) contains dangerous terms which can induce proton decay and lead to large flavor changing neutral currents (FCNCs). There are a number of approaches to suppress these terms, involving the use of different discrete symmetries. Here for the purposes of renormalization group running we will simply include the following unsuppressed superpotential terms, which follows the approach taken in work on the cE$_6$SSM [17,18],

$$\begin{aligned} W \approx {}& y_\tau \hat{L}_3 \cdot \hat{H}_d \hat{e}_3^c + y_b \hat{Q}_3 \cdot \hat{H}_d \hat{d}_3^c + y_t \hat{H}_u \cdot \hat{Q}_3 \hat{u}_3^c \\ & + \lambda_i \hat{S} \hat{H}_i^d \cdot \hat{H}_i^u + \kappa_i \hat{S} \hat{D}_i \hat{\bar{D}}_i + \mu' \hat{H}' \cdot \hat{\bar{H}}', \end{aligned} \tag{9}$$

where we denote by $\hat{H}_d^3 \equiv \hat{H}_d$, $\hat{H}_u^3 \equiv \hat{H}_u$ and $\hat{S}_3 \equiv \hat{S}$ the third-generation Higgs and SM singlet fields that are assumed to acquire nonzero VEVs. In addition to the terms coming from the $27 \otimes 27 \otimes 27$ interactions given in Eq. (5), this superpotential also contains a bilinear term $\mu' \hat{H}' \cdot \hat{\bar{H}}'$, arising from $27' \otimes \overline{27}'$, which is invariant with respect to the low-energy SM gauge group and the additional $U(1)'$ symmetry and also anomaly free. This term is responsible for setting the masses of the components of the superfields $\hat{H}'$, $\hat{\bar{H}}'$, included to ensure gauge coupling unification, but it is not involved in the process of EWSB. Consequently, the impact on the fine-tuning of the value of μ' is much smaller than that coming from other sectors, and so can be safely neglected in our study. In all of the scans we present below the value of μ' is fixed to $\mu' = 5$ TeV.

III. ELECTROWEAK SYMMETRY BREAKING

The Higgs scalar potential for the E_6 models considered can be written as [13]

$$V = V_F + V_D + V_{\text{soft}} + \Delta V, \tag{10}$$

where

$$V_F = \lambda^2 |S|^2 (|H_d|^2 + |H_u|^2) + \lambda^2 |H_d \cdot H_u|^2, \tag{11}$$

$$\begin{aligned} V_D = {}& \frac{\bar{g}^2}{8} (|H_u|^2 - |H_d|^2)^2 + \frac{g_2^2}{2} |H_d^\dagger H_u|^2 \\ & + \frac{g_1'^2}{2} (Q_1 |H_d|^2 + Q_2 |H_u|^2 + Q_S |S|^2)^2, \end{aligned} \tag{12}$$

$$\begin{aligned} V_{\text{soft}} = {}& m_S^2 |S|^2 + m_{H_d}^2 |H_d|^2 + m_{H_u}^2 |H_u|^2 \\ & + [\lambda A_\lambda S H_d \cdot H_u + \text{H.c.}]. \end{aligned} \tag{13}$$

In these expressions g_2, $g' = \sqrt{3/5} g_1$, and g_1' are the $SU(2)$, (non-GUT normalized) $U(1)_Y$ and $U(1)'$ gauge couplings, respectively, and $\bar{g}^2 = g_2^2 + g'^2$. The charges Q_1, Q_2 and Q_S are effective $U(1)'$ charges for H_d, H_u and S, respectively, and $\lambda \equiv \lambda_3$. In the case of the $U(1)_\chi$ model, V_F may also contain an elementary μ term, as occurs in the MSSM. The term ΔV contains the Coleman-Weinberg contributions to the effective potential. For the purposes of this study, we include in ΔV only the one-loop contributions from the top quark and stop squarks,

$$\begin{aligned} \Delta V = {}& \frac{3}{32\pi^2} \left[m_{\tilde{t}_1}^4 \left(\ln \frac{m_{\tilde{t}_1}^2}{Q^2} - \frac{3}{2} \right) + m_{\tilde{t}_2}^4 \left(\ln \frac{m_{\tilde{t}_2}^2}{Q^2} - \frac{3}{2} \right) \right. \\ & \left. - 2 m_t^4 \left(\ln \frac{m_t^2}{Q^2} - \frac{3}{2} \right) \right]. \end{aligned} \tag{14}$$

Explicit expressions for the running $\overline{\text{DR}}$ top mass m_t and stop masses $m_{\tilde{t}_{1,2}}$ are given below.

Demanding that the Higgs fields H_1, H_2 and the singlet S acquire real VEVs of the form

$$\langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \langle S \rangle = \frac{s}{\sqrt{2}}, \tag{15}$$

at the physical minimum leads to the minimization conditions

$$f_1 = m_{H_d}^2 v_1 + \frac{\lambda^2}{2}(v_2^2 + s^2)v_1 - \frac{\lambda A_\lambda}{\sqrt{2}} s v_2 - \frac{\bar{g}^2}{8}(v_2^2 - v_1^2)v_1 + D_{H_d} v_1 + \frac{\partial \Delta V}{\partial v_1} = 0, \tag{16a}$$

$$f_2 = m_{H_u}^2 v_2 + \frac{\lambda^2}{2}(v_1^2 + s^2)v_2 - \frac{\lambda A_\lambda}{\sqrt{2}} s v_1 + \frac{\bar{g}^2}{8}(v_2^2 - v_1^2)v_2 + D_{H_u} v_2 + \frac{\partial \Delta V}{\partial v_2} = 0, \tag{16b}$$

$$f_3 = m_S^2 s + \frac{\lambda^2}{2}(v_2^2 + v_1^2)s - \frac{\lambda A_\lambda}{\sqrt{2}} v_2 v_1 + D_S s + \frac{\partial \Delta V}{\partial s} = 0. \tag{16c}$$

The quantities D_{H_d}, D_{H_u} and D_S appearing above are $U(1)'$ D-term contributions that are absent in the MSSM and NMSSM and are given by

$$D_\phi \equiv \frac{g_1'^2}{2}(Q_1 v_1^2 + Q_2 v_2^2 + Q_S s^2)Q_\phi. \tag{17}$$

We also include these $U(1)'$ D-term contributions in the diagonalized stop masses,

$$m_{\tilde{t}_{1,2}}^2 = \frac{1}{2}\left\{ m_{Q_3}^2 + m_{u_3}^2 + \frac{\bar{g}^2}{8}(v_1^2 - v_2^2) + D_Q + D_u + 2m_t^2 \mp \sqrt{\left[m_{Q_3}^2 - m_{u_3}^2 + \frac{1}{8}(g_2^2 - g_1^2)(v_1^2 - v_2^2) + D_Q - D_u\right]^2 + 4m_t^2 X_t^2}\right\}, \tag{18}$$

where $m_t^2 = y_t^2 v_2^2/2$, $X_t = A_t - \frac{\lambda s v_1}{\sqrt{2} v_2}$, $m_{Q_3}^2$, $m_{u_3}^2$ are soft breaking scalar masses and A_t is a soft trilinear coupling. By definition we take $m_{\tilde{t}_1}$ to correspond to the lighter of the two states.

As was noted in Ref. [16], the first two of the conditions in Eq. (16) may be rewritten in the form

$$\frac{M_Z^2}{2} = -\frac{\lambda^2 s^2}{2} + \frac{\tilde{m}_{H_d}^2 - \tilde{m}_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} + \frac{D_{H_d} - D_{H_u}\tan^2\beta}{\tan^2\beta - 1}, \tag{19}$$

$$\sin 2\beta = \frac{\sqrt{2}\lambda A_\lambda s}{\tilde{m}_{H_d}^2 + \tilde{m}_{H_u}^2 + \lambda^2 s^2 + D_{H_d} + D_{H_u}}, \tag{20}$$

with $M_Z^2 = \bar{g}^2 v^2/4$, $v^2 = v_1^2 + v_2^2$ and $\tan\beta = v_2/v_1$ and where we have for convenience absorbed the effects of the loop corrections into the soft masses,

$$\tilde{m}_{H_d}^2 = m_{H_d}^2 + \frac{1}{v_1}\frac{\partial \Delta V}{\partial v_1},$$
$$\tilde{m}_{H_u}^2 = m_{H_u}^2 + \frac{1}{v_2}\frac{\partial \Delta V}{\partial v_2}.$$

Written in the form of Eq. (19), we see the potential new source of fine-tuning alluded to above, in the form of the third term on the right-hand side. For large values of the VEV s, the D-term contributions can be quite a bit larger than M_Z^2. In particular, recent experimental limits [134] require that the Z' mass be large, with for example bounds of $M_{Z'} \gtrsim 2.51$ TeV in $U(1)_\psi$ models and $M_{Z'} \gtrsim 2.62$ TeV in $U(1)_\chi$ models. To satisfy these limits typically requires large values of the singlet VEV s. For example, $s \gtrsim 6$ TeV is required in the E$_6$SSM with $U(1)' = U(1)_N$, so that $|D_{H_1}|, |D_{H_2}| \gg M_Z^2$ for E_6 models with $Q_S \neq 0$. As a result the remaining terms on the right-hand side of Eq. (19) must be tuned to cancel this very large contribution to M_Z. Moreover, because this is a large tree-level fine-tuning, it may negate the improvement in naturalness that is associated with having a reduced need for heavy superpartners. In $U(1)$ extended models for which $Q_S \neq 0$, the importance of the Z' mass to the fine-tuning in these models can be made even clearer by writing Eq. (19) in the form given in Ref. [16],

$$c(\theta, \tan\beta)\frac{M_Z^2}{2} = -\frac{\lambda^2 s^2}{2} + \frac{\tilde{m}_{H_d}^2 - \tilde{m}_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} + d(\theta, \tan\beta)\frac{M_{Z'}^2}{2}, \tag{21}$$

where

$$c(\theta, \tan\beta) = 1 - \frac{4}{(\tan^2\beta - 1)}\frac{g_1'^2}{\bar{g}^2}(Q_1 - Q_2\tan^2\beta)(Q_1\cos^2\beta + Q_2\sin^2\beta), \tag{22}$$

$$d(\theta, \tan\beta) = \frac{Q_1 - Q_2\tan^2\beta}{Q_S(\tan^2\beta - 1)}. \tag{23}$$

Written like so, it is evident that the fine-tuning contribution coming from the new D terms depends both on the $U(1)'$ charges and the Z' mass, and that the tuning can be expected to increase with $M_{Z'}$. As shall be shown below, the exact size of this tuning then depends strongly on the choice of $U(1)'$ charges, via the coefficient d.

The extra $U(1)'$ gauge symmetry may mix with the $U(1)_Y$ gauge symmetry associated with hypercharge through gauge kinetic mixing,

$$\mathcal{L}^{\text{kin}}_{\text{mix}} = -\frac{\sin\chi}{2} F^Y_{\mu\nu} F^N_{\mu\nu}, \tag{24}$$

where $F^Y_{\mu\nu}$ and $F^N_{\mu\nu}$ are field strengths associated with the $U(1)_Y$ and $U(1)_N$, respectively. The gauge kinetic mixing can have a significant impact on the phenomenology [135–137] and may reduce the Z' mass limit.

However, if the extra $U(1)$ gauge symmetry appears from the breakdown of E_6, then $\sin\chi$ should be zero at the GUT scale. Nonetheless, even if this term is zero at the outset, it will still be radiatively generated if the trace of the $U(1)$ charges, $\sum_i Q^Y_i Q'_i$, is nonzero. In the cases studied here, the trace of the charges over states in the complete 27-plets vanishes, but to be consistent with single-step gauge coupling unification, we also included $\hat{H}'$ and $\hat{\overline{H}}'$ which lead to a nonzero value for $\sum_i Q^Y_i Q'_i$. The value induced by this at the EW scale though is rather small as can be seen[1] in Fig. 3 of Ref. [135], and this was also checked with two-loop RGEs in the E$_6$SSM [13,18]. For this reason and due to the huge expansion in the number of terms in the two-loop RGEs when one allows for gauge kinetic mixing, we will neglect this in our analysis here and throughout this paper.

In general, though, it is possible for gauge kinetic mixing to be much larger, which can be the case if one considers an additional $5 + \bar{5}$ pair of $SU(5)$ multiplets [135] or which has been looked at in the $U(1)_{B-L}$ [136,137]. In such a case, this will impact the results in two ways, firstly by altering the Z' limit from experiment and secondly by altering the charges which appear in the EWSB condition, which can be seen from examining Eqs. (21)–(23).

IV. THE FINE-TUNING MEASURE

As stated above, to quantify the resulting fine-tuning, we apply the traditional Barbieri-Giudice measure [21,22]. A specific model is characterized by a set of n model parameters $\{p_i\}$ and is defined at some input scale M_X. For a given parameter p in this set, one computes an associated sensitivity,

$$\Delta_p = \left|\frac{\partial \ln M_Z^2}{\partial \ln p}\right| = \left|\frac{p}{M_Z^2}\frac{\partial M_Z^2}{\partial p}\right|. \tag{25}$$

The coefficient Δ_p measures the fractional variation in M_Z^2 resulting from a given variation in the parameter p. The overall fine-tuning is then taken to be $\Delta = \max_i\{\Delta_{p_i}\}$.

The sensitivity Δ_p may be calculated directly from the expression for M_Z^2 in terms of the p_i for a particular model, which leads to a so-called master formula for calculating the fine-tuning. A master formula for the E$_6$SSM, obtained from the tree-level scalar potential, was presented in Ref. [16]. In order to derive the expression presented there, the fact that $s \gg v$ was made use of to neglect certain $O(v^2)$ terms in the EWSB conditions, greatly simplifying the final result. For the purposes of exploring a wider class of E_6 inspired models, we have derived the master formula without neglecting any $O(v^2)$ terms. The more complete tree-level master formula is somewhat complicated. This is because, unlike in the MSSM, even at tree level it is not possible to solve explicitly for the VEVs v_1, v_2 in terms of the Lagrangian parameters. It may be written in the form

$$\Delta_p = |C|^{-1} \times \frac{|p|}{M_Z}\left|\sum_q \tilde{\Delta}_q \frac{\partial q}{\partial p}\right|, \tag{26}$$

where the sum is over all low-energy running parameters appearing in the tree-level EWSB conditions, i.e., $q \in \{\lambda, A_\lambda, m^2_{H_d}, m^2_{H_u}, m^2_S, g_1, g_2, g'_1\}$. Expressions for the quantity C and the $\tilde{\Delta}_q$ appearing above are given in Appendix A. It should be noted that the effects of $U(1)$ mixing are neglected in deriving Eq. (26).

However, it is well known in the MSSM that radiative corrections can significantly change (indeed, reduce) the fine-tuning [138]. It is, therefore, important when studying the fine-tuning to include loop corrections to the effective potential in the fine-tuning measure. To do so it is most convenient to work in terms of the EWSB conditions Eq. (16), rather than Eq. (19). The general procedure that we use is as follows (this method has also previously been applied in the NMSSM; see, for example, Ref. [139]). For a model in which m fields develop real VEVs (e.g. $m = 2$ in the MSSM, $m = 3$ in the NMSSM and in the E_6 models considered), we require that the m minimization conditions,

$$f_1 = f_2 = \cdots = f_m = 0, \tag{27}$$

continue to hold under an arbitrary variation in a model parameter $p \to p + \delta p$, so that the variations δf_i satisfy

$$\delta f_1 = \delta f_2 = \cdots = \delta f_m = 0. \tag{28}$$

Each f_i is a function of the VEVs v_j and l running parameters q_k evaluated at the scale of EWSB, $f_i = f_i(v_j, q_k)$. Thus for each f_i we find that

[1]The specific incomplete multiplets we consider here correspond to the third of the four possible embeddings referred to in Ref. [135].

$$\sum_{j=1}^{m}\frac{\partial f_i}{\partial v_j}\frac{\partial v_j}{\partial p}+\sum_{k=1}^{l}\frac{\partial f_i}{\partial q_k}\frac{\partial q_k}{\partial p}=0. \tag{29}$$

The quantities $\frac{\partial f_i}{\partial v_j}$ are the elements of the CP-even Higgs squared mass matrix M_h^2 of the model before rotating into the mass eigenstate basis. When evaluated for all n model parameters, the above system of equations can be concisely expressed as

$$M_h^2\begin{pmatrix}\frac{\partial v_1}{\partial p_1} & \cdots & \frac{\partial v_1}{\partial p_n}\\ \vdots & \ddots & \vdots\\ \frac{\partial v_m}{\partial p_1} & \cdots & \frac{\partial v_m}{\partial p_n}\end{pmatrix} = -\begin{pmatrix}\frac{\partial f_1}{\partial q_1} & \cdots & \frac{\partial f_1}{\partial q_l}\\ \vdots & \ddots & \vdots\\ \frac{\partial f_m}{\partial q_1} & \cdots & \frac{\partial f_m}{\partial q_l}\end{pmatrix}\begin{pmatrix}\frac{\partial q_1}{\partial p_1} & \cdots & \frac{\partial q_1}{\partial p_n}\\ \vdots & \ddots & \vdots\\ \frac{\partial q_l}{\partial p_1} & \cdots & \frac{\partial q_l}{\partial p_n}\end{pmatrix}. \tag{30}$$

The quantities forming the first matrix on the right-hand side, along with M_h^2, are easily calculated by differentiating the conditions in Eq. (16) with respect to the VEVs and the running parameters. The remaining derivatives $\partial q_k/\partial p$ must be determined using the RGEs. Once these have been obtained, it is straightforward to solve for the $\partial v_i/\partial p$. The sensitivities Δ_p are then simply linear combinations of the $\partial v_i/\partial p$ and $\partial q_k/\partial p$. The effects of radiative corrections may be easily included by including the Coleman-Weinberg potential contributions ΔV in the EWSB conditions. Here we use the one-loop corrections given in Eq. (14).

Evaluating the derivatives $\partial q_k/\partial p$ must, in general, be done by numerically integrating the two-loop RGEs. This is time consuming and presents an obstacle to doing large scans of the parameter space. For studying models defined at low energies, as we do here, we can take advantage of the fact that the running is over much smaller scales than when evolving up to the GUT scale. This makes it possible to use approximate analytic solutions to the RGEs that exhibit good accuracy over the range of scales considered. Given the two-loop RG equation for a parameter q,

$$\frac{\mathrm{d}q}{\mathrm{d}t}\equiv\beta_q=\frac{1}{16\pi^2}\beta_q^{(1)}+\frac{1}{(16\pi^2)^2}\beta_q^{(2)}, \qquad t\equiv\ln\frac{Q}{M_X}, \tag{31}$$

a Taylor series expansion of the solution may be used to obtain the parameter at the scale Q,

$$q(Q)=q(M_X)+\int_0^t\beta_q(t')\mathrm{d}t' \approx q(M_X)+\frac{t}{16\pi^2}\left(\beta_q^{(1)}+\frac{\beta_q^{(2)}}{16\pi^2}\right)+\frac{t^2}{32\pi^2}\frac{\mathrm{d}\beta_q^{(1)}}{\mathrm{d}t}+O(t^2). \tag{32}$$

Expanded to this order, we obtain the leading log (LL) and next-to-LL (NLL) contributions at two-loop order. The $O(t^2)$ terms not displayed above are formally of three-loop order and are neglected. The derivative of the one-loop β function is given by

$$\frac{\mathrm{d}\beta_q^{(1)}}{\mathrm{d}t}=\frac{1}{16\pi^2}\sum_{q_k}\beta_{q_k}^{(1)}\frac{\partial\beta_q^{(1)}}{\partial q_k}, \tag{33}$$

where the sum is over all running parameters appearing in $\beta_q^{(1)}$. The β functions appearing on the right-hand side of Eqs. (32) and (33) are evaluated at the scale M_X, giving a simple analytic expression for the parameters at the scale of EWSB in terms of the model parameters at M_X. Explicit results for the relevant series expansions in the MSSM and E_6 models are presented in Appendix B.

V. RESULTS

Using the approach outlined above, we are able to scan the low-energy parameter space of the MSSM and $\mathrm{E_6}$SSM and calculate the fine-tuning in each. To do so, we implemented the above expressions for computing the fine-tuning in a modified version of the $\mathrm{E_6}$SSM spectrum generator that was used in Ref. [16]. This code implemented two-loop RGEs for all parameters except the soft scalar masses. In order to properly include the fine-tuning impact of the $SU(3)$ gaugino soft mass M_3, we have extended the original code to make use of the two-loop RGEs generated by SARAH [140–143] and FlexibleSUSY [144], which also makes use of SOFTSUSY [145,146]. The CP-even Higgs masses are calculated including the leading one-loop effective potential contributions given in Ref. [18] and for the light Higgs we use the leading two-loop[2] contributions from Ref. [13] which are a generalization of the corrections in the MSSM and NMSSM calculated using effective field theory techniques [149,150]. To scan over the MSSM parameter space, the equivalent MSSM fine-tuning expressions were implemented into a modified version of SOFTSUSY 3.3.10 [145]. For consistency with the results produced in the E_6 models, and for computational speed, for our main scans only the dominant one- and

[2]Two-loop corrections calculated for a nonminimal SUSY model may now also be obtained from SARAH [147,148]. However, this was not available when the numerical work for this paper was carried out, and such corrections go beyond the required precision for studying fine-tuning here.

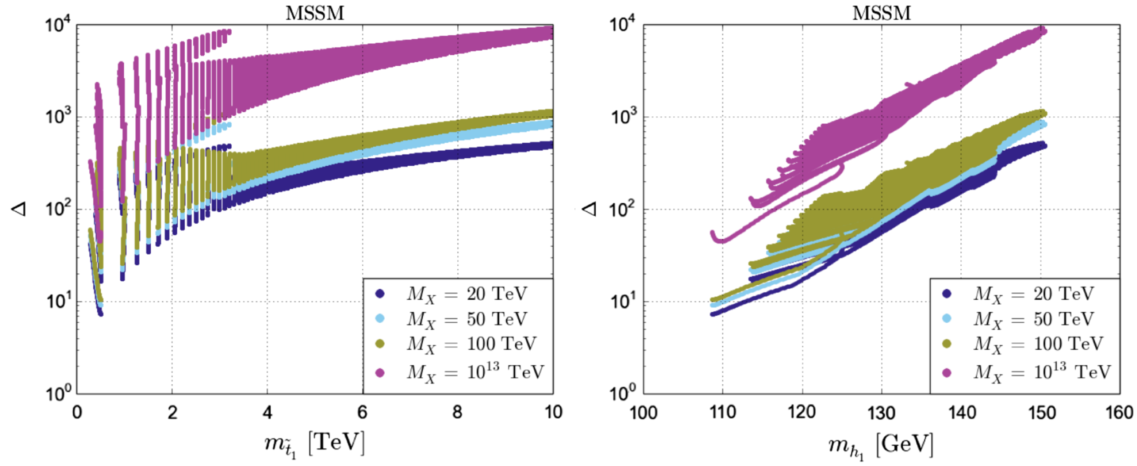

FIG. 1 (color online). Left panel: Scatter plot of fine-tuning in the MSSM as a function of the lightest stop mass, $m_{\tilde{t}_1}$, for the cutoff scales (from bottom to top) $M_X = 20$ TeV, $M_X = 50$ TeV, $M_X = 100$ TeV and $M_X = 10^{16}$ GeV. Right panel: Scatter plot of fine-tuning in the MSSM as a function of the lightest Higgs mass, m_{h_1}, for the cutoff scales (from bottom to top) $M_X = 20$ TeV, $M_X = 50$ TeV, $M_X = 100$ TeV and $M_X = 10^{16}$ GeV.

two-loop corrections to the CP-even Higgs masses were included. Finally, in all of the results below the fine-tuning was evaluated at the scale $Q = M_{\mathrm{SUSY}} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, where $m_{\tilde{t}_{1,2}}$ are the running $\overline{\mathrm{DR}}$ stop masses evaluated at $Q = M_{\mathrm{SUSY}}$.

As discussed in the Introduction, many recent papers interested in natural supersymmetry have focused on light stops, with much theoretical effort to find models where it is easier to get a 125 GeV Higgs boson and light stops simultaneously and much experimental effort to search for light stops. This is entirely appropriate since there are many good reasons to expect the soft masses to be set at high energies. However, that is not the only possibility and the fine-tuning problem depends strongly on the RG evolution from the GUT scale, as the soft Higgs masses that appear in the EWSB conditions pick up contributions from the soft squark masses.

To illustrate this, in the left panel of Fig. 1 we show the variation in fine-tuning for $M_X = 20$ TeV, 50 TeV, 100 TeV and 10^{16} GeV when we scan over the stop masses and mixing, with 500 GeV $\leq m_{Q_3}$, $m_{u_3} \leq 10$ TeV and -3810 GeV $\leq A_t \leq -20$ GeV. The remaining parameters we fix such that at M_{SUSY} they have the values $\mu = -97.5$, $B = -84.8$, $M_1 = 92.1$, $M_2 = 95.9$, $M_3 = 352$, $A_b = -117.9$, $A_\tau = -7.8$, $m_{L_i} = 400$, $m_{e_i} = 204$, $m_{Q_{1,2}} = 438$, $m_{u_{1,2}} = 436$ and $m_{d_i} = 438$ GeV ($i = 1, 2, 3$). Here we denote by M_1, M_2 and M_3 the soft gaugino masses for $U(1)$, $SU(2)$ and $SU(3)$, while A_b and A_τ are soft trilinear couplings and the m_ϕ are soft scalar masses for the indicated fields. The soft bilinear coupling B is defined such that at tree level the mass of the CP-odd MSSM Higgs boson reads $m_A^2 = 2B\mu/\sin 2\beta$. All off-diagonal couplings and scalar masses are set to zero, as are the first- and second-generation Yukawa couplings and soft trilinears. Although we should stress that making this choice will lead to a spectrum which is in conflict with the LHC limits, doing so ensures that fine-tuning due to the other parameters is small, so that we avoid washing out the fine-tuning impact of the stops when the tuning is small[3] as can be the case when the stop masses are less than 1 TeV. Note that the Higgs mass is also allowed to vary in this scan, as shown in the right panel of Fig. 1. This illustrates the tuning problem which people have been worrying about since the discovery of the 125 GeV Higgs boson as we see that raising the stop masses is also pushing up the Higgs mass, meaning that heavier Higgs masses require more fine-tuning. However, for a low value of the UV scale this tuning is not so severe unless the stops are very heavy, and a 125 GeV Higgs can be obtained without much tuning in this unrealistic case where we have minimized other sources of tuning. On the other hand, the tuning becomes more severe as we increase the cutoff such that for $M_X = 10^{16}$ GeV a lightest stop mass of 1–3 TeV can result in a fine-tuning of ≈100–1000 and the minimum tuning we find[4] for a 125 GeV Higgs is ≈200, as shown in Fig. 1.

Since the stop mass does not have such a large impact on the fine-tuning when the cutoff scale is very low we can use this to see more clearly the impact of the Z' mass on fine-tuning. To do so we select a fixed low cutoff of $M_X = 20$ TeV and compare the fine tuning between the MSSM and E_6SSM for two different values of the Z' mass. We

[3]For models in which the spectrum is heavier, when the stop masses are small the fine tuning reaches a lower bound imposed by other heavier parameters.

[4]Note that in the calculation of the Higgs mass there is a significant theoretical error, even with leading two-loop corrections, which should be considered when thinking about what the results imply for the minimum fine tuning in the model consistent with the recent discovery of a 125 GeV Higgs.

TABLE II. The parameters scanned over and the ranges of values used in the MSSM and the E_6SSM models.

MSSM	E_6SSM
$2 \leq \tan\beta \leq 50$	$2 \leq \tan\beta \leq 50$
-1 TeV $\leq \mu \leq 1$ TeV	$-3 \leq \lambda \leq 3$
-1 TeV $\leq B \leq 1$ TeV	-10 TeV $\leq A_\lambda \leq 10$ TeV
200 GeV $\leq m_{Q_3} \leq 2000$ GeV	200 GeV $\leq m_{Q_3} \leq 2000$ GeV
200 GeV $\leq m_{u_3} \leq 2000$ GeV	200 GeV $\leq m_{u_3} \leq 2000$ GeV
-10 TeV $\leq A_t \leq 10$ TeV	-10 TeV $\leq A_t \leq 10$ TeV
$M_2 = 100$, 1050, 2000 GeV	$M_2 = 100$, 1050, 2000 GeV

choose to look at $M_{Z'} = 2.5$ TeV, which is just above the current limits, and $M_{Z'} = 4.5$ TeV, which should be in reach in run II at the LHC [151] and then compare the fine tuning calculated in each case to the tuning in the MSSM. For this, we have performed a six-dimensional parameter space scan in both the MSSM and E_6SSM, varying those parameters most relevant for the fine tuning and the Higgs mass. Therefore, the set of parameters which we vary includes μ, B and $\tan\beta$ for the MSSM, and λ, A_λ and $\tan\beta$, for the E_6SSM, which appear at tree level in the EWSB conditions of the models. While the RGE contribution from large stop masses to the fine tuning is small for such a low cutoff scale, the stop contributions to the effective potential can play a significant role in reducing the fine tuning. For this reason it is still important to properly treat the tuning associated with stop contributions to the one-loop effective potential, and so we also scan over the soft masses $m_{Q_3}^2$, $m_{u_3}^2$ and the stop mixing A_t. The relevant parameters and ranges that were scanned over are summarized in Table II. In addition to this we also repeat each scan for three different values of M_2 to allow more variation in the chargino masses.

In this case, we now consider realistic scenarios, where the parameters that are not scanned over are set to values which keep the associated states comfortably above their experimental limits. So in both the MSSM and E_6SSM, all other soft scalar masses are set to 5 TeV. We require a valid spectrum with no tachyonic states to exclude points which would have an unrealistic minimum, for example due to the appearance of charge or color breaking (CCB) minima. We work in the third family approximation, taking the first- and second-generation Yukawa couplings to be zero, and we also assume that their associated soft trilinears vanish. Similarly, we take $A_b = A_\tau = 0$ GeV. The $U(1)$ gaugino soft mass M_1 was fixed to $M_1 = 300$ GeV, and we fix $M_3 = 2000$ GeV. Additionally, in the E_6SSM the $U(1)_N$ gaugino soft mass M_1' is held fixed with $M_1' = M_1 = 300$ GeV, and $\mu' = 5$ TeV.

In Fig. 2, results from the scan are plotted showing the tuning for each case against the lightest Higgs mass. As expected, the dependence on the Higgs mass is now quite weak, while the minimum tuning in the model for the

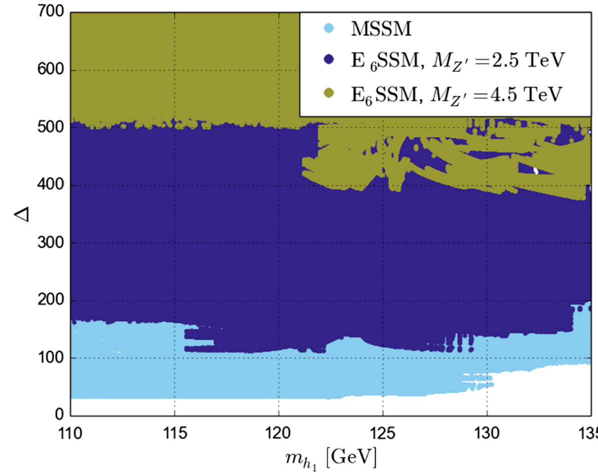

FIG. 2 (color online). Scatter plot of fine-tuning vs lightest Higgs mass for the MSSM (light blue, bottom band), E_6SSM with $M_{Z'} = 2.5$ TeV (dark blue, middle band) and E_6SSM with $M_{Z'} = 4.5$ TeV (dark yellow, top band). Note that there are points for which the fine-tuning in the MSSM and E_6SSM with $M_{Z'} = 2.5$ TeV is larger than is visible on this plot and those below; however, these points are obscured by the overlaid data for the E_6SSM with $M_{Z'} = 4.5$ TeV, and it is the lower bound on the achievable tuning that is of interest here.

E_6SSM is increased by the mass of the Z' boson. So in the case of a very low cutoff the tuning required to get a 125 GeV Higgs is not so large. However, the tuning from the Z' mass appears already at tree level and is, therefore, not suppressed when the cutoff scale is low. In our scan we find that, for the points satisfying the current limit on the mass of the Z' boson and having an approximately 125 GeV Higgs, the minimum fine-tuning that can be achieved is $\Delta_{\rm min} \approx 121$. If run II of the LHC further pushes up the limit on the Z' mass to be above 4.5 TeV then the fine-tuning in the model will be greater than at least $\Delta_{\rm min} \approx 394$ for a Higgs mass between 124.5 and 125.5 GeV.

This demonstrates two important points about these $U(1)$ extensions—first, that limits on the Z' mass play an incredibly important role in constraining natural scenarios in such models and, second, that the tuning from the Z' limits in these models depends less on assumptions about SUSY breaking than the tuning required by the 125 GeV Higgs measurement which concerns people in the MSSM.

There is another limit which plays a similar role. Chargino limits directly constrain the μ parameter (or effective μ parameter in these $U(1)$ extensions). The LEP bound [152] on chargino masses, excluding $m_{\tilde{\chi}_1^\pm} \lesssim 104$ GeV, implies that $|\mu|$ should only be greater than $\sim$100 GeV, which is not substantially larger than M_Z. Consequently, the bound from LEP is not high enough to have an impact on the fine-tuning obtained in the models and parameter space regions that we have studied, as we have checked explicitly. Significantly larger lower bounds on the μ parameter, and therefore on the fine-tuning, may

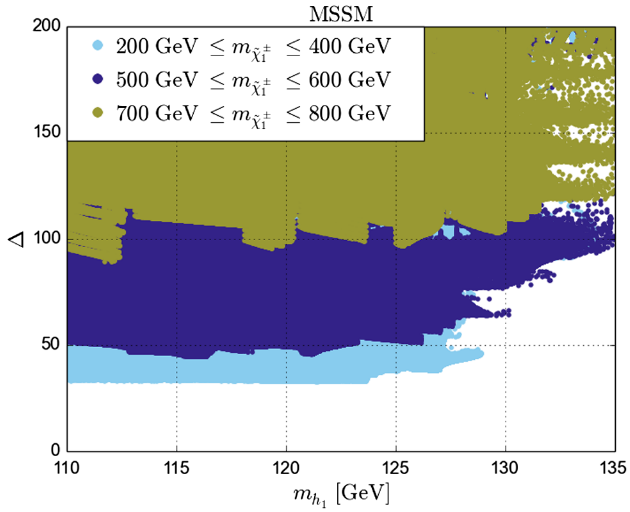

FIG. 3 (color online). Scatter plot of fine-tuning vs lightest Higgs mass in the MSSM with 200 GeV $\leq m_{\tilde{\chi}_1^\pm} \leq$ 400 GeV shown in light blue (bottom band), 500 GeV $\leq m_{\tilde{\chi}_1^\pm} \leq$ 600 GeV in dark blue (middle band), and 700 GeV $\leq m_{\tilde{\chi}_1^\pm} \leq$ 800 GeV in dark yellow (top band).

arise from chargino limits coming from LHC searches. However, the chargino limits from the LHC depend on whether there are light sleptons or sneutrinos and the mass difference between the lightest chargino and lightest neutralino. Current limits placed by CMS and ATLAS extend up to $m_{\tilde{\chi}_1^\pm} \approx 700$–740 GeV if there are light sleptons [153,154] with much weaker bounds if there are no light sleptons or sneutrinos.[5]

Nonetheless, for the MSSM the impact of potential chargino mass limits is shown in Fig. 3. There we see that if the full parameter space with $m_{\tilde{\chi}_1^\pm} < 700$ GeV was excluded, the impact would be to make the tuning in the MSSM with a 20 TeV cutoff similar to that of the E$_6$SSM with the same cutoff and a Z' mass just larger than current limits. In the E$_6$SSM, while raising the chargino limit can have the same impact in principle, due to current limits on the Z' mass already imposing a significant degree of tuning, chargino masses do not make much of a noticeable change.

The exact level of tuning from the Z' depends on the charges of the extra $U(1)$ gauge symmetry it is associated with. In Fig. 4 we look at the fine-tuning for other $U(1)$ extensions for the same Z' masses as we did for the E$_6$SSM. To simplify the analysis we fix $\tan\beta = 10$, but scan over the remaining parameters as in Table II and fix the rest to the same values we did in the scan carried out for Fig. 2. In order to more clearly identify the lower bound on the obtainable tuning in each model, the parameter values for points in these main grid scans with a low fine-tuning were then used as the starting points for smaller scans about those values. In these smaller scans the parameters were more finely varied to populate the low fine-tuning regions.

As can be seen in Fig. 4, the severity of the tunings varies quite a bit. This is because the charges appear as coefficients in front of the Z' mass in the EWSB condition. These charges change the value of the coefficient d in Eq. (21). The values of the coefficient d in each model, for $\tan\beta = 10$, is $\{-0.01, 0.40, 0.50, 0.81\}$ for $\{U(1)_I, U(1)_N, U(1)_\psi, U(1)_\eta\}$ and this determines which of the models are most tuned.

Interestingly, the coefficient d is very small (and negative) in the case of the $U(1)_I$. This allows a dramatic reduction in the fine-tuning from the $U(1)_I$ symmetry. This is a result of the H_u charge associated with $U(1)_I$ vanishing, which means that the D terms to the lightest Higgs which is predominantly H_u at large $\tan\beta$ are suppressed, making it difficult to raise the Higgs mass in the same way as happens in the other models and explaining why heavier Higgs values in this model can't be obtained. Therefore, the fine-tuning behavior in this model is closer to that of the MSSM, and in this case raising the Z' mass limit to 4.5 TeV will have little impact on naturalness. From naively estimating the tuning, using the d coefficient one can estimate that Z' limits need to be around 15 TeV before they will raise the tuning in this model.

Finally we want to emphasize that while in Fig. 2 the E$_6$SSM looks more fine-tuned than the MSSM this depends on the high scale boundary, M_X, where the parameters are assumed to be set by some SUSY breaking mechanism. Indeed in Ref. [16] a constrained version of the E$_6$SSM, with the high scale boundary at the GUT scale, is considered and there the cE$_6$SSM was found to be less tuned than the cMSSM. Since a 125 GeV Higgs can be achieved in the E$_6$SSM with lighter stops, then if the cutoff is large, the larger stop masses of the MSSM can make that model more fine-tuned due to large RGE effects.

To further illustrate this point, we looked at how the tuning varies with M_X for low tuning benchmarks in the MSSM and E$_6$SSM. These benchmarks are defined in Table III and the results are shown in Fig. 5. Since the behavior is quite complicated we now discuss these in detail as it provides some insight into the many differences in the tuning between the two models.

In the top panel one can see that the MSSM BM1 tuning (dotted curve) steadily climbs as the cutoff scale is increased, as one would expect when the tuning originates from large soft masses entering from the RGEs. The panel on the middle left confirms this, showing that the largest tuning contributions come from Δ_{A_t} and $\Delta_{m^2_{H_u}}$ with the former being the larger sensitivity until $M_X \approx 10^8$ GeV at which point $\Delta_{m^2_{H_u}}$ takes over, leading to the small kink in overall tuning which can be seen in the dotted curve in the

[5]Useful summary plots of these limits may be found on the public pages of ATLAS, https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CombinedSummaryPlots/SUSY/ATLAS_SUSY_EWSummary/ATLAS_SUSY_EWSummary.png and CMS http://cms.web.cern.ch/sites/cms.web.cern.ch/files/styles/large/public/field/image/Image_03_exclusion_Combined.png?itok=8FMBpu_1.

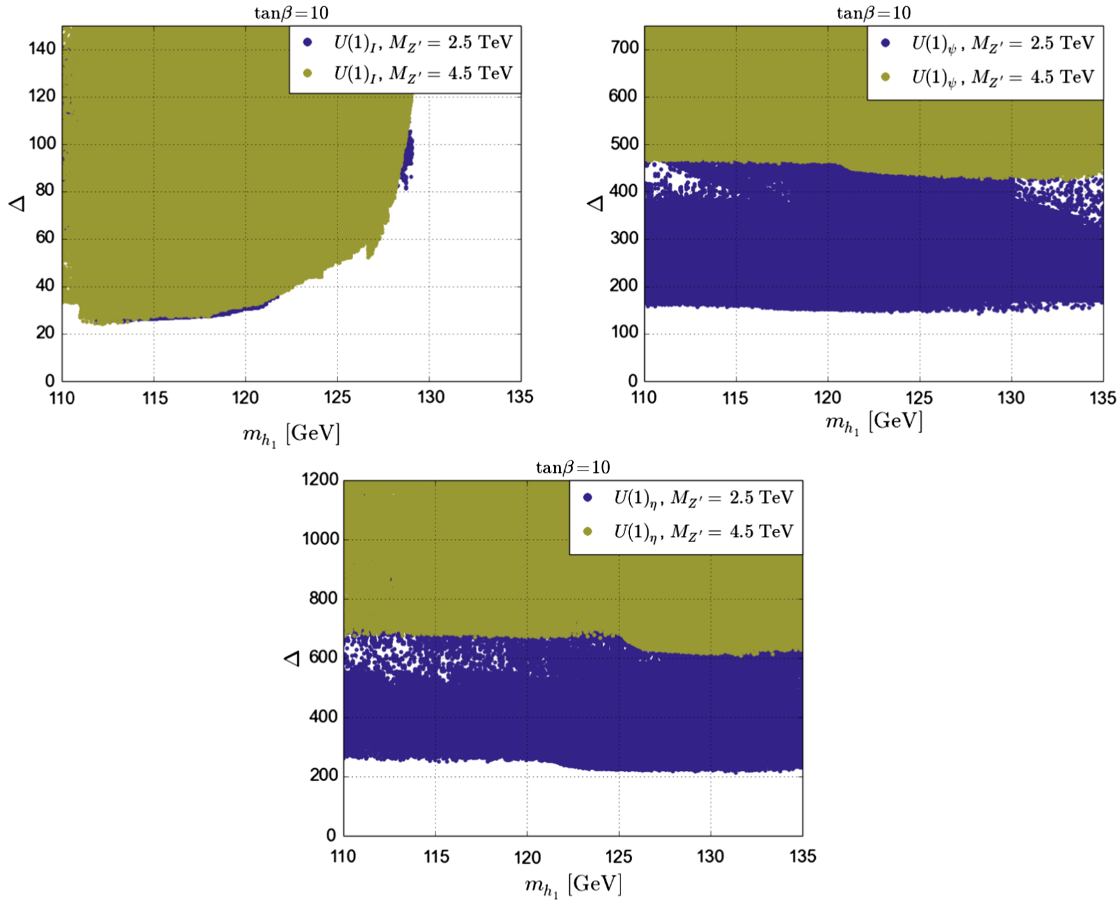

FIG. 4 (color online). Top left panel: Scatter plot of the fine-tuning vs lightest Higgs mass in the $U(1)_I$ model. Top right panel: Scatter plot of the fine-tuning vs lightest Higgs mass in the $U(1)_\psi$ model. Bottom panel: Scatter plot of the fine-tuning vs lightest Higgs mass in the $U(1)_\eta$ model. In each plot points with $M_{Z'} = 2.5$ TeV are shown in dark blue (bottom band), and points with $M_{Z'} = 4.5$ TeV are shown in dark yellow (top band).

top panel. In this case we have chosen a point with large mixing, which is known to reduce the MSSM tuning. We found this does not eliminate the tuning as there is still a strong sensitivity to A_t, but we did find that large mixing lead to less fine-tuning overall for the points we examined.

Comparing the MSSM tunings to the E_6SSM tunings one can see that which point is more fine-tuned depends on the scale at which the parameters are defined. This illustrates that any statement about which model is more tuned depends on the high scale boundary, M_X.

For E_6SSM BM1 the fine-tuning is shown by the solid curve in the top panel of Fig. 5 and the individual sensitivities are given in the middle right panel. The tuning actually reduces initially as the cutoff is increased from 20 TeV. This occurs because the largest sensitivity is initially Δ_λ (shown in solid light blue in the middle right panel). This contains some terms proportional to $M_Z'^2$, which provide the dominant contribution to this sensitivity at very low M_X. However, as M_X is increased contributions from the soft masses become more important and these actually start to cancel the large contribution to Δ_λ coming from $M_{Z'}$ until Δ_λ passes through zero. At the same time though these large soft masses also cause other sensitivities to grow, in particular Δ_{M_3}. The fine-tuning rises with M_X once $M_X \gtrsim 10^5$–10^6 GeV, but remains lower than that of the other points, until $M_X \approx 10^8$ GeV. Eventually the Δ_{M_3} sensitivity leads to this point being the most fine-tuned of the four shown in Fig. 5.

Although the gluino mass and $M_3(M_{\rm SUSY})$ have similar values to those in the MSSM BM1 point, in the E_6SSM $M_3(M_X)$ is larger due to the altered RGE running from exotic matter.[6] This is why this E_6SSM BM1 has a larger tuning at larger values of M_X, coming from Δ_{M_3}.

Interestingly other sensitivities are suppressed by this effect since at the same time larger M_3 at higher scales reduces the soft squark masses at M_X. Therefore, the stop mass contributions are ameliorated, compared to the

[6]This altered RGE running is a result of the exotic matter introduced to keep the extra $U(1)$ anomaly free.

TABLE III. Parameters for the MSSM and E_6SSM benchmark points. In the E_6SSM, we define $\mu_{\text{eff}} \equiv \lambda s/\sqrt{2}$ and $B_{\text{eff}} = A_\lambda$. The soft masses $m^2_{H_d}$, $m^2_{H_u}$ and m^2_S are those that satisfy the EWSB conditions including one-loop Coleman-Weinberg corrections involving the top and stops. For E_6SSM BM1 (BM2) we also set $\mu' = 5000.0$ (897.9) GeV, $B\mu' = 5000.0$ (-4.21×10^5) GeV^2, $A_{\kappa_{1,2,3}} = 0$ (-1389.2) GeV, $A_{\lambda_{1,2}} = 0$ (-52.9) GeV, $m^2_{D_{1,2,3}} = 2.5 \times 10^7$ (4.81×10^6) GeV^2, $m^2_{\bar{D}_{1,2,3}} = 2.5 \times 10^7$ (4.90×10^6) GeV^2, $m^2_{H^d_{1,2}} = 2.5 \times 10^7$ (4.46×10^6) GeV^2, $m^2_{H^u_{1,2}} = 2.5 \times 10^7$ (4.81×10^6) GeV^2, $m^2_{S_{1,2}} = 2.5 \times 10^7$ (5.28×10^6) GeV^2, $m^2_{H'} = 2.5 \times 10^7$ (4.94×10^6) GeV^2 and $m^2_{\overline{H'}} = 2.5 \times 10^7$ (4.87×10^6) GeV^2.

	MSSM BM1	MSSM BM2	E_6SSM BM1	E_6SSM BM2
$\tan\beta(M_Z)$	10	10	10	10
$s(M_{\text{SUSY}})$ [GeV]	...	...	6700	6700
$\kappa_{1,2,3}(M_{\text{SUSY}})$	...	...	0.6	0.52
$\lambda_{1,2}(M_{\text{SUSY}})$	...	...	0.2	0.13
$\mu_{\text{eff}}(M_{\text{SUSY}})$ [GeV]	689.7	1013.5	1093.3	1313.0
$B_{\text{eff}}(M_{\text{SUSY}})$ [GeV]	345.7	1032.5	3792.7	817.8
$A_\tau(M_{\text{SUSY}})$ [GeV]	0	−5057.9	0	−88.5
$A_b(M_{\text{SUSY}})$ [GeV]	0	−5707.2	0	−1720.7
$A_t(M_{\text{SUSY}})$ [GeV]	−3335.7	−2734.8	−1100	−1103.2
$m^2_{L_{1,2}}(M_{\text{SUSY}})$ [GeV^2]	2.5×10^7	6.35×10^6	2.5×10^7	4.94×10^6
$m^2_{L_3}(M_{\text{SUSY}})$ [GeV^2]	2.5×10^7	6.22×10^6	2.5×10^7	4.90×10^6
$m^2_{e_{1,2}}(M_{\text{SUSY}})$ [GeV^2]	2.5×10^7	6.27×10^6	2.5×10^7	5.21×10^6
$m^2_{e_3}(M_{\text{SUSY}})$ [GeV^2]	2.5×10^7	6.03×10^6	2.5×10^7	5.11×10^6
$m^2_{Q_{1,2}}(M_{\text{SUSY}})$ [GeV^2]	2.5×10^7	7.37×10^6	2.5×10^7	5.76×10^6
$m^2_{Q_3}(M_{\text{SUSY}})$ [GeV^2]	4.45×10^6	3.97×10^6	4.50×10^5	3.61×10^6
$m^2_{u_{1,2}}(M_{\text{SUSY}})$ [GeV^2]	2.5×10^7	7.30×10^6	2.5×10^7	5.54×10^6
$m^2_{u_3}(M_{\text{SUSY}})$ [GeV^2]	4.0×10^6	6.60×10^5	5.86×10^5	2.04×10^6
$m^2_{d_{1,2}}(M_{\text{SUSY}})$ [GeV^2]	2.5×10^7	7.30×10^6	2.5×10^7	5.88×10^6
$m^2_{d_3}(M_{\text{SUSY}})$ [GeV^2]	2.5×10^7	7.03×10^6	2.5×10^7	5.78×10^6
$m^2_{H_d}(M_{\text{SUSY}})$ [GeV^2]	1.82×10^6	8.96×10^6	4.06×10^7	1.04×10^7
$m^2_{H_u}(M_{\text{SUSY}})$ [GeV^2]	-3.60×10^5	-9.35×10^5	5.0×10^5	-2.66×10^5
$m^2_S(M_{\text{SUSY}})$ [GeV^2]	...	...	-3.10×10^6	-3.17×10^6
$M_1(M_{\text{SUSY}})$ [GeV]	300	260.8	300	173.4
$M_2(M_{\text{SUSY}})$ [GeV]	2000	479.2	1050	281.4
$M_3(M_{\text{SUSY}})$ [GeV]	2000	1312.3	2000	1200
$M'_1(M_{\text{SUSY}})$ [GeV]	...	...	300	175.2
$M_{Z'}$ [GeV]	...	...	2473.2	2512.7
m_{h_1} [GeV]	124.3	124.4	125.0	126.2
$m_{\tilde{t}_1}$ [GeV]	1942.1	861.6	993.8	1665.0
$m_{\tilde{t}_2}$ [GeV]	2220.1	2023.9	1174.8	2094.4
$m_{\tilde{g}}$ [GeV]	2259.8	1472.9	2290.0	1407.4
$\Delta(M_X = 20$ TeV)	157.3	242.8	165.3	402.1
$\Delta(M_X = 10^{16}$ GeV)	1089.0	949.0	1722.3	546.7

MSSM, both by allowing lighter stops at M_{SUSY} and by the modified RGE running. Nonetheless the stops still do lead to $\Delta_{m^2_{H_u}}$ increasing with the cutoff through the usual mechanism.[7]

[7]Wherein $m^2_{H_u}(M_{\text{SUSY}})$ receives a positive contribution to it's mass from $m^2_{H_u}(M_X)$ and a negative contribution from $m^2_{Q_3}(M_X)$ and $m^2_{u_3}(M_X)$, allowing heavy stop masses to cause fine-tuning. In this case $m^2_{H_u}(M_{\text{SUSY}})$ is held fixed so as the cutoff increases the values of these soft masses at M_X will be larger and there will be a bigger cancellation between them, increasing the sensitivity of M_Z to both $m^2_{H_u}$ and the soft scalar masses for the stops.

By contrast the tuning for E_6SSM BM2 is very different, as is shown by the dashed line in the top panel, with the individual sensitivities given in the bottom right panel. This point was chosen as it had a much lighter gluino mass that is just above the experimental limit of 1.4 TeV [155]. At 20 TeV this benchmark is not amongst the lowest tuned points, since at that scale the tree-level tuning from $M_{Z'}$ dominates. However, the reduction in M_3 means that Δ_{M_3} is substantially lower and only becomes the dominant tuning at a much larger scale of $M_X \gtrsim 10^{12}$–10^{13} GeV, giving a tuning at 10^{16} GeV of ≈546, which is far below that of the other three benchmark points.

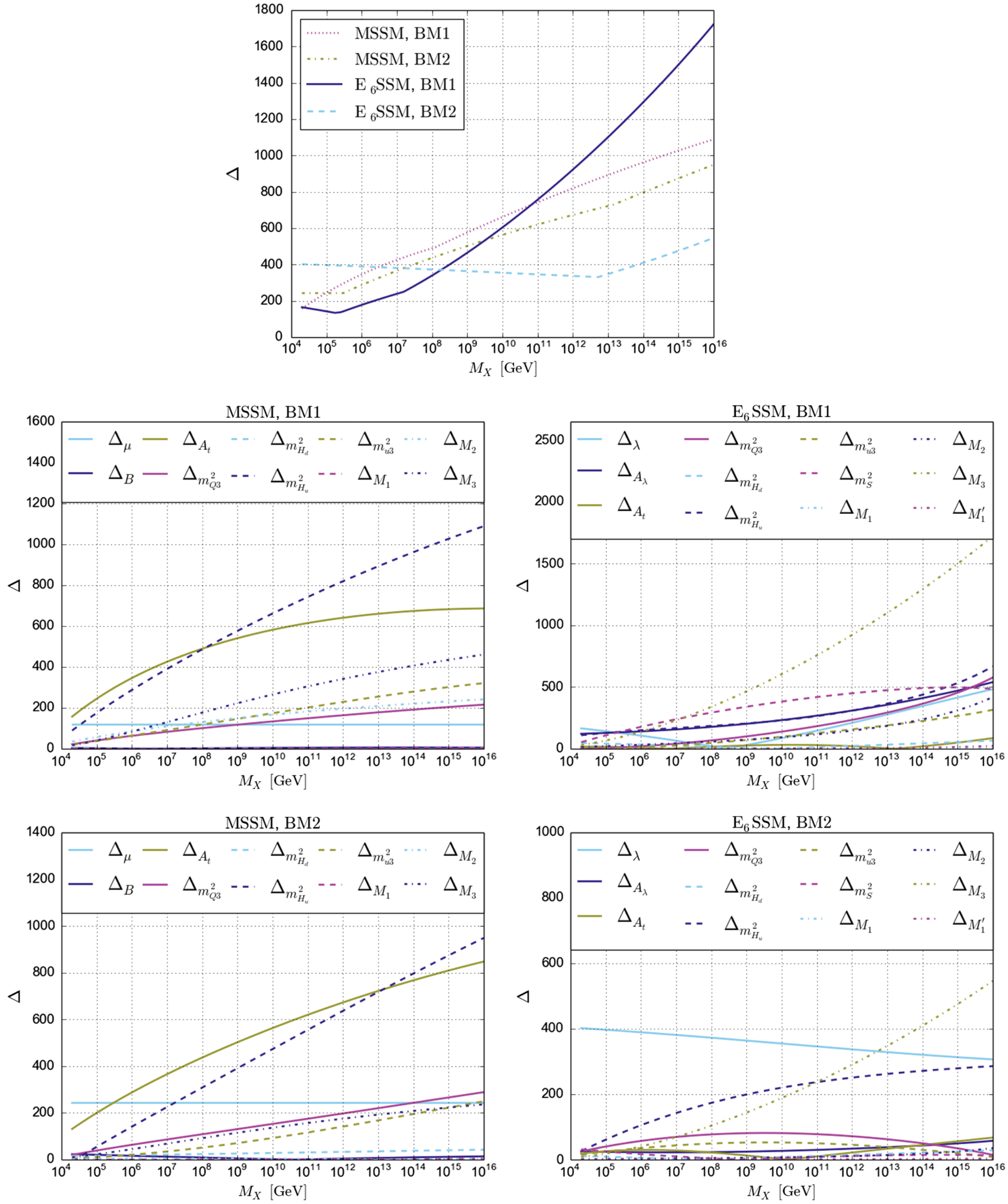

FIG. 5 (color online). Top panel: Scatter plot of the fine-tuning as a function of the cutoff scale M_X for the four benchmark points given in Table III. Middle left panel: Individual sensitivities for MSSM BM1 plotted against the high scale M_X which give the overall tuning shown by the dotted line in the top panel. Middle right panel: Individual sensitivities for E_6SSM BM1 plotted against the high scale M_X which give the overall tuning shown by the solid line in the top panel. Bottom left panel: Individual sensitivities for MSSM BM2 plotted against the high scale M_X which give the overall tuning shown by the dash-dotted line in the top panel. Bottom right panel: Individual sensitivities for E_6SSM BM2 plotted against the high scale M_X which give the overall tuning shown by the dashed line in the top panel.

In addition to this, the soft parameters in E_6SSM BM2 follow a pattern similar to that found in the constrained model. With the exception of the parameters $m^2_{Q_3}$, $m^2_{u_3}$, $m^2_{H_d}$, $m^2_{H_u}$, and M_3, the values of which are given in Table III, the soft masses at the SUSY scale correspond to the values that result in the cE_6SSM with $m_0 = 2.2$ TeV, $M_{1/2} = 1003$ GeV, $A_0 = 500$ GeV, $\kappa_{1,2,3}(M_X) = 0.1923$, $\lambda(M_X) = 0.2646$ and $\lambda_{1,2}(M_X) = 0.1$. This leads to a significant reduction in the contributions to the RG running of $m^2_{H_u}$ and $m^2_{Q_3}$ coming from terms of the form $g_1^2\Sigma_1$ and, to a lesser extent, $g_1'^2\Sigma_1'$. Here we define for the E_6SSM (see also Eqs. (B8)–(B9) for general $U(1)$ inspired models)

$$\Sigma_1 = \sum_{i=1}^{3}(m^2_{Q_i} - 2m^2_{u_i} + m^2_{d_i} + m^2_{e_i} - m^2_{L_i} + m^2_{H^u_i} - m^2_{H^d_i} + m^2_{\bar{D}_i} - m^2_{D_i}) - m^2_{H'} + m^2_{\overline{H'}},$$

$$\Sigma_1' = \sum_{i=1}^{3}(6m^2_{Q_i} + 3m^2_{u_i} + 6m^2_{d_i} + m^2_{e_i} + 4m^2_{L_i} - 4m^2_{H^u_i} - 6m^2_{H^d_i} + 5m^2_{S_i} - 9m^2_{\bar{D}_i} - 6m^2_{D_i}) + 4m^2_{H'} - 4m^2_{\overline{H'}}.$$

In the unconstrained case, this contribution acts to drive up the values of $m^2_{Q_3}$ and $m^2_{H_u}$, and thus the associated tuning sensitivities, at the cutoff scale M_X. In the case of E_6SSM BM2, on the other hand, the reduced splitting between the soft masses leads to a much smaller contribution from these terms. Together with the reduction in M_3 described above, this allows to maintain the observed low fine-tuning at very large values of M_X.

MSSM benchmark BM2 (dash-dotted in top panel, individual sensitivities in bottom left panel) is designed to be similar to E_6SSM BM2, for a reasonable comparison. However, from the individual sensitivities one can see that the behavior is quite similar to MSSM BM1, though in this case $\Delta_{m^2_{H_u}}$ becomes the largest tuning at a higher M_X and does not reach such large values, since more of the tuning is from the mixing in this case.

VI. CONCLUSIONS

Prior to stringent experimental constraints on the mass of the lightest Higgs boson and squarks in supersymmetric models, a simple picture of a natural SUSY model emerged from theoretical reasoning, with soft masses set to similar values at the GUT scale through local gravity interactions with the hidden sector. Through the use of renormalization group running, one can then see that at the EW scale the stops enter the EWSB condition for M_Z; therefore, it was expected that these masses should not be much bigger than 100 GeV. However, to disturb this elegant picture, first LEP placed constraints on the Higgs mass, requiring it to be above 114.4 GeV [156,157], which already introduced significant tuning for constrained models since heavy stops are required to raise the lightest Higgs mass above its tree-level bound. Then, recently, this problem got much worse since the LHC measured the Higgs mass to be around 125 GeV.

$U(1)$ extensions motivated by the μ problem, E_6 GUT theories and the connection to string theory contain both F- and D-term contributions to the light Higgs mass which can raise the tree-level mass, evading the need for large radiative corrections to increase it. However, such models come with their own fine-tuning problem, where the Z' mass appears in the EWSB condition for M_Z at tree level. While in a previous study of the constrained E_6SSM it was found that the tuning is less severe than the MSSM, it was still significant.

In light of such difficulties it is worth considering whether the simple picture which emerged is wrong in some way and if there are other possibilities that allow naturalness. Or to phrase this in a more challenging manner are there ways to constrain the naturalness of these models that do not rely upon assumptions about how SUSY is broken?

We have investigated this question here in the context of the MSSM and $U(1)$ extensions. Since the RG evolution links the soft masses together and causes these problems from stop and gluino masses the most conservative approach to placing naturalness limits is to choose a low cutoff. We find that in the MSSM the most direct way to constrain naturalness in the model without making assumptions about the SUSY breaking scale is through limits on the chargino masses. Current LHC limits on charginos are not model independent and thereby leave many gaps where one can have light charginos.

In contrast we find that in $U(1)$ extensions of the MSSM there is an additional way to constrain the naturalness of the models, which is through the Z' mass limit. We find when we impose a low cutoff of 20 TeV for setting the soft masses, the lowest tuning in the E_6SSM compatible with a Z' mass of 2.5 TeV was $\Delta \approx 121$, while if the LHC run II can place a limit of 4.5 TeV on M_Z' then the tuning would be approximately 394. By comparison the current situation in the MSSM only requires a tuning of around 38. This should be interpreted as saying that in the most conservative limits one can place on naturalness in these models, the tuning in the E_6SSM is worse. However, if there are no charginos below 700 GeV then the situation in the two models would be the same.

This should also be contrasted with what happens as we raise the high scale boundary, M_X. We showed that for our benchmark points, which one is more tuned depends very strongly on M_X. The E_6SSM tuning is sufficiently complicated by the interplay of these different sources of tension in the EWSB conditions that a small reduction in fine-tuning can even occur for a moderate increase in M_X. However, as M_X increases towards the scale where the gauge couplings unify, the familiar tunings do dominate, though with tunings from the gluino mass appearing to be more significant relative to those from soft scalar masses.

We also looked at the tuning in different $U(1)$ extensions for fixed $\tan\beta = 10$. We found that in every case except for the $U(1)_I$ the fine-tuning was much worse for the larger Z' mass, further emphasizing the importance of this in $U(1)$

extensions. The $U(1)_I$ model showed the least tuning due to the vanishing charge of the H_u state. This model is quite interesting in the sense that it provides a solution to the μ problem while avoiding the large tuning (with current limits) from the Z' mass. However, one should remember we are looking at conservative limits on naturalness here and there is no solution to the usual tuning coming from the large stops needed to get a 125 GeV Higgs in this model, which will be a problem as the UV scale is raised.

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APPENDIX A: FINE-TUNING MASTER FORMULA

To write down the tree-level master formula, it is convenient to define the quantities

$$z_i = \epsilon_{ijk} \frac{\partial f_j}{\partial s} \frac{\partial f_k}{\partial \tan\beta} \tag{A1}$$

with f_1, f_2, f_3 as given in Eq. (16). The relevant partial derivatives are

$$\begin{aligned}
\frac{\partial f_1}{\partial \tan\beta} &= -\frac{2M_Z}{\bar{g}}\cos^2\beta\Bigg\{\frac{\lambda A_\lambda s}{\sqrt{2}}\cos\beta + \sin\beta\bigg[m_{H_d}^2 + \frac{s^2}{2}(\lambda^2 + g_1'^2 Q_1 Q_S) \\
&\quad + M_Z^2\bigg(\frac{5}{2} - \frac{4\lambda^2}{\bar{g}^2} - \frac{4g_1'^2}{\bar{g}^2}Q_1 Q_2 + \frac{6g_1'^2}{\bar{g}^2}Q_1^2\bigg)\bigg] + 3M_Z^2\sin^3\beta\bigg[\frac{2\lambda^2}{\bar{g}^2} - 1 + \frac{2g_1'^2}{\bar{g}^2}(Q_1 Q_2 - Q_1^2)\bigg]\Bigg\}, \\
\frac{\partial f_1}{\partial s} &= \frac{2M_Z}{\bar{g}}\bigg[s(\lambda^2 + g_1'^2 Q_1 Q_S)\cos\beta - \frac{\lambda A_\lambda}{\sqrt{2}}\sin\beta\bigg], \\
\frac{\partial f_2}{\partial \tan\beta} &= \frac{2M_Z}{\bar{g}}\cos^2\beta\Bigg\{\frac{\lambda A_\lambda s}{\sqrt{2}}\sin\beta + \cos\beta\bigg[m_{H_u}^2 + \frac{s^2}{2}(\lambda^2 + g_1'^2 Q_2 Q_S) \\
&\quad + M_Z^2\bigg(\frac{5}{2} - \frac{4\lambda^2}{\bar{g}^2} - \frac{4g_1'^2}{\bar{g}^2}Q_1 Q_2 + \frac{6g_1'^2}{\bar{g}^2}Q_2^2\bigg)\bigg] + 3M_Z^2\cos^3\beta\bigg[\frac{2\lambda^2}{\bar{g}^2} - 1 + \frac{2g_1'^2}{\bar{g}^2}(Q_1 Q_2 - Q_2^2)\bigg]\Bigg\}, \\
\frac{\partial f_2}{\partial s} &= \frac{2M_Z}{\bar{g}}\bigg[s(\lambda^2 + g_1'^2 Q_2 Q_S)\sin\beta - \frac{\lambda A_\lambda}{\sqrt{2}}\cos\beta\bigg], \\
\frac{\partial f_3}{\partial \tan\beta} &= \frac{2M_Z^2}{\bar{g}^2}\cos^2\beta[g_1'^2 Q_S s(Q_2 - Q_1)\sin 2\beta - \sqrt{2}\lambda A_\lambda \cos 2\beta], \\
\frac{\partial f_3}{\partial s} &= m_S^2 + \frac{2\lambda^2 M_Z^2}{\bar{g}^2} + \frac{g_1'^2}{2}Q_S\bigg[\frac{4M_Z^2}{\bar{g}^2}(Q_1\cos^2\beta + Q_2\sin^2\beta) + 3Q_S s^2\bigg].
\end{aligned}$$

For a running parameter q appearing in the tree-level EWSB conditions, the corresponding contribution to the individual sensitivity can then be written

$$\tilde{\Delta}_q = z_1 \frac{\partial f_1}{\partial q} + z_2 \frac{\partial f_2}{\partial q} + z_3 \frac{\partial f_3}{\partial q}. \tag{A2}$$

It is straightforward to compute the appropriate derivatives directly from the EWSB conditions, Eq. (16). Similarly, the quantity C appearing in Eq. (26) is given by

$$C = \frac{1}{2}\bigg(z_1 \frac{\partial f_1}{\partial M_Z} + z_2 \frac{\partial f_2}{\partial M_Z} + z_3 \frac{\partial f_3}{\partial M_Z}\bigg), \tag{A3}$$

with

$$\frac{\partial f_1}{\partial M_Z} = \frac{2}{\bar{g}}\cos\beta\left(m_{H_d}^2 + \frac{\lambda^2 s^2}{2} + \frac{g_1'^2}{2}Q_1 Q_S s^2 + \frac{6g_1'^2}{\bar{g}^2}Q_1^2 M_Z^2\right) - \sqrt{2}\frac{\lambda A_\lambda s}{\bar{g}}\sin\beta + \frac{3M_Z^2}{\bar{g}}\cos\beta\cos 2\beta + \frac{6}{\bar{g}^3}M_Z^2\sin\beta\sin 2\beta[\lambda^2 + g_1'^2(Q_1 Q_2 - Q_1^2)],$$

$$\frac{\partial f_2}{\partial M_Z} = \frac{2}{\bar{g}}\sin\beta\left(m_{H_u}^2 + \frac{\lambda^2 s^2}{2} + \frac{g_1'^2}{2}Q_2 Q_S s^2 + \frac{6g_1'^2}{\bar{g}^2}Q_2^2 M_Z^2\right) - \sqrt{2}\frac{\lambda A_\lambda s}{\bar{g}}\cos\beta - \frac{3M_Z^2}{\bar{g}}\sin\beta\cos 2\beta + \frac{6}{\bar{g}^3}M_Z^2\cos\beta\sin 2\beta[\lambda^2 + g_1'^2(Q_1 Q_2 - \tilde{Q}_2{}^2)],$$

$$\frac{\partial f_3}{\partial M_Z} = \frac{4M_Z}{\bar{g}^2}\left[\lambda^2 s - \frac{\lambda A_\lambda}{\sqrt{2}}\sin 2\beta + g_1'^2 Q_S s(Q_1\cos^2\beta + Q_2\sin^2\beta)\right].$$

APPENDIX B: RGE CONTRIBUTIONS

Provided that one does not run over too large a range of scales, the solutions to the RG equations for a model can be reasonably well approximated by a Taylor series, Eq. (32). For a parameter p, this reads

$$q(Q) = q(M_X) + \frac{t}{16\pi^2}\left(\beta_q^{(1)} + \frac{\beta_q^{(2)}}{16\pi^2}\right) + \frac{t^2}{(16\pi^2)^2}b_q^{(2)}(M_X),$$

where we have for convenience defined

$$b_q^{(2)}(M_X) = \frac{1}{2!}\sum_{q_k}\beta_{q_k}^{(1)}\frac{\partial\beta_q}{\partial q_k}\bigg|_{M_X}.$$

We have constructed the necessary series solutions in both the MSSM and the $U(1)$ extended models. Due to the smallness of the first- and second-generation Yukawa couplings, we neglect them in our calculations. The corresponding soft SUSY breaking trilinears are likewise omitted. Additionally, all soft mass matrices are assumed diagonal, and the gaugino masses are taken to be real.

In the MSSM, the relevant parameters for the fine-tuning calculation are μ, B, $m_{H_u}^2$, $m_{H_d}^2$ at tree level. For the renormalization group running of the relevant parameters SOFTSUSY uses the one- and two-loop RGEs from [158,159]. The corresponding $O(t^2)$ contributions are

$$b_\mu^{(2)} = \frac{\mu}{2}\left[45y_t^4 + 45y_b^4 + 9y_\tau^4 + 30y_t^2 y_b^2 + 6y_t^2 y_\tau^2 + 18y_b^2 y_\tau^2 - 32g_3^2(y_t^2 + y_b^2) - 12g_2^2(3y_t^2 + 3y_b^2 + y_\tau^2) - \frac{4}{5}g_1^2(11y_t^2 + 8y_b^2 + 6y_\tau^2) + 3g_2^4 - \frac{189}{25}g_1^4 + \frac{18}{5}g_1^2 g_2^2\right], \tag{B1a}$$

$$b_B^{(2)} = 72y_t^4 A_t + 72y_b^4 A_b + 16y_\tau^4 A_\tau + 12y_t^2 y_b^2(A_t + A_b) + 12y_\tau^2 y_b^2(A_b + A_\tau) - 32g_3^2 y_t^2(A_t - M_3) - 32g_3^2 y_b^2(A_b - M_3) - 18g_2^2 y_t^2(A_t - M_2) - 18g_2^2 y_b^2(A_b - M_2) - 6g_2^2 y_\tau^2(A_\tau - M_2) - \frac{26}{5}g_1^2 y_t^2(A_t - M_1) - \frac{14}{5}g_1^2 y_b^2(A_b - M_1) - \frac{18}{5}g_1^2 y_\tau^2(A_\tau - M_1) + 12g_2^4 M_2 + \frac{396}{25}g_1^4 M_1, \tag{B1b}$$

$$b_{m_{H_d}^2}^{(2)} = 72y_b^4(m_{H_d}^2 + m_{Q_3}^2 + m_{d_3}^2 + 2A_b^2) + 6y_t^2 y_b^2(m_{H_u}^2 + m_{H_d}^2 + 2m_{Q_3}^2 + m_{u_3}^2 + m_{d_3}^2 + (A_t + A_b)^2) + 12y_\tau^2 y_b^2(2m_{H_d}^2 + m_{Q_3}^2 + m_{d_3}^2 + m_{L_3}^2 + m_{e_3}^2 + (A_\tau + A_b)^2) + 16y_\tau^4(m_{H_d}^2 + m_{L_3}^2 + m_{e_3}^2 + 2A_\tau^2) - 32g_3^2 y_b^2(m_{H_d}^2 + m_{Q_3}^2 + m_{d_3}^2 + A_b^2 - 2M_3 A_b + 2M_3^2) - 18g_2^2 y_b^2(m_{H_d}^2 + m_{Q_3}^2 + m_{d_3}^2 + A_b^2 - 2M_2 A_b + 2M_2^2) - 6g_2^2 y_\tau^2(m_{H_d}^2 + m_{L_3}^2 + m_{e_3}^2 + A_\tau^2 - 2M_2 A_\tau + 2M_2^2) - \frac{14}{5}g_1^2 y_b^2(m_{H_d}^2 + m_{Q_3}^2 + m_{d_3}^2 + A_b^2 - 2M_1 A_b + 2M_1^2) - \frac{18}{5}g_1^2 y_\tau^2(m_{H_d}^2 + m_{L_3}^2 + m_{e_3}^2 + A_\tau^2 - 2M_1 A_\tau + 2M_1^2) - 18g_2^4 M_2^2 - \frac{198}{25}g_1^4(\mathcal{S} + 3M_1^2), \tag{B1c}$$

$$\begin{aligned}b^{(2)}_{m^2_{H_u}} &= 72y_t^4(m^2_{H_u}+m^2_{Q_3}+m^2_{u_3}+2A_t^2)+6y_t^2y_b^2(m^2_{H_u}+m^2_{H_d}+2m^2_{Q_3}+m^2_{u_3}+m^2_{d_3}+(A_t+A_b)^2)\\ &\quad -32g_3^2y_t^2(m^2_{H_u}+m^2_{Q_3}+m^2_{u_3}+A_t^2-2A_tM_3+2M_3^2)-18g_2^2y_t^2(m^2_{H_u}+m^2_{Q_3}+m^2_{u_3}+A_t^2-2A_tM_2+2M_2^2)\\ &\quad -\frac{26}{5}g_1^2y_t^2(m^2_{H_u}+m^2_{Q_3}+m^2_{u_3}+A_t^2-2A_tM_1+2M_1^2)+\frac{198}{25}g_1^4(\mathcal{S}-3M_1^2)-18g_2^4M_2^2. \qquad \text{(B1d)}\end{aligned}$$

In these expressions the quantity $\mathcal{S}$ is defined by

$$\mathcal{S}=m^2_{H_u}-m^2_{H_d}+\sum_{i=1}^{3}(m^2_{Q_i}-m^2_{L_i}-2m^2_{u_i}+m^2_{d_i}+m^2_{e_i}). \qquad \text{(B2)}$$

If, in addition, the one-loop contributions to the effective potential from top and stop loops are included, it is also necessary to construct the expansions for $m^2_{Q_3}$, $m^2_{u_3}$ and A_t. The coefficients read

$$\begin{aligned}b^{(2)}_{m^2_{Q_3}} &= 24y_t^4(m^2_{H_u}+m^2_{Q_3}+m^2_{u_3}+2A_t^2)+24y_b^4(m^2_{H_d}+m^2_{Q_3}+m^2_{d_3}+2A_b^2)\\ &\quad +4y_t^2y_b^2(m^2_{H_u}+m^2_{H_d}+2m^2_{Q_3}+m^2_{u_3}+m^2_{d_3}+(A_t+A_b)^2)+2y_b^2y_\tau^2(2m^2_{H_d}+m^2_{Q_3}+m^2_{L_3}+m^2_{d_3}+m^2_{e_3}+(A_b+A_\tau)^2)\\ &\quad -\frac{32}{3}g_3^2y_t^2(m^2_{H_u}+m^2_{Q_3}+m^2_{u_3}+A_t^2-2M_3A_t+2M_3^2)-\frac{32}{3}g_3^2y_b^2(m^2_{H_d}+m^2_{Q_3}+m^2_{d_3}+A_b^2-2M_3A_b+2M_3^2)\\ &\quad -6g_2^2y_t^2(m^2_{H_u}+m^2_{Q_3}+m^2_{u_3}+A_t^2-2M_2A_t+2M_2^2)-6g_2^2y_b^2(m^2_{H_d}+m^2_{Q_3}+m^2_{d_3}+A_b^2-2M_2A_b+2M_2^2)\\ &\quad -\frac{26}{15}g_1^2y_t^2(m^2_{H_u}+m^2_{Q_3}+m^2_{u_3}+A_t^2-2M_1A_t+2M_1^2)-\frac{14}{15}g_1^2y_b^2(m^2_{H_d}+m^2_{Q_3}+m^2_{d_3}+A_b^2-2M_1A_b+2M_1^2)\\ &\quad +96g_3^4M_3^2-18g_2^4M_2^2+\frac{66}{25}g_1^4(\mathcal{S}-M_1^2), \qquad \text{(B3a)}\end{aligned}$$

$$\begin{aligned}b^{(2)}_{m^2_{u_3}} &= 48y_t^4(m^2_{H_u}+m^2_{Q_3}+m^2_{u_3}+2A_t^2)+4y_t^2y_b^2(m^2_{H_u}+m^2_{H_d}+2m^2_{Q_3}+m^2_{u_3}+m^2_{d_3}+(A_t+A_b)^2)\\ &\quad -\frac{64}{3}g_3^2y_t^2(m^2_{H_u}+m^2_{Q_3}+m^2_{u_3}+A_t^2-2M_3A_t+2M_3^2)-12g_2^2y_t^2(m^2_{H_u}+m^2_{Q_3}+m^2_{u_3}+A_t^2-2M_2A_t+2M_2^2)\\ &\quad -\frac{52}{15}g_1^2y_t^2(m^2_{H_u}+m^2_{Q_3}+m^2_{u_3}+A_t^2-2M_1A_t+2M_1^2)+96g_3^4M_3^2-\frac{264}{25}g_1^4(\mathcal{S}+4M_1^2), \qquad \text{(B3b)}\end{aligned}$$

$$\begin{aligned}b^{(2)}_{A_t} &= 144y_t^4A_t+24y_b^4A_b+14y_t^2y_b^2(A_t+A_b)+2y_b^2y_\tau^2(A_b+A_\tau)-64g_3^2y_t^2(A_t-M_3)-36g_2^2y_t^2(A_t-M_2)-\frac{52}{5}g_1^2y_t^2(A_t-M_1)\\ &\quad -\frac{32}{3}g_3^2y_b^2(A_b-M_3)-6g_2^2y_b^2(A_b-M_2)-\frac{14}{15}g_1^2y_b^2(A_b-M_1)-64g_3^4M_3+12g_2^4M_2+\frac{572}{25}g_1^4M_1. \qquad \text{(B3c)}\end{aligned}$$

We can similarly obtain the two-loop β functions and coefficients $b_p^{(2)}$ for a general set of $U(1)'$ charges. Two-loop RGEs for the gauge and Yukawa couplings, gaugino masses and soft trilinears, along with the one-loop RGEs for the soft scalar masses, were originally obtained in Ref. [18] for the particular case of the E_6SSM. FlexibleSUSY uses full one- and two-loop RGEs from SARAH, which for the models considered here are based on Ref. [159] and the recent extension[8] in Ref. [160] to include models with multiple $U(1)$ gauge groups, in the most general case where the trace of the matrix formed from the charges Q_i^Y of the $U(1)_Y$ gauge symmetry and Q_i of the extra $U(1)'$ symmetry does not vanish, i.e. $\sum_i Q_i^Y Q_i \neq 0$.

When this trace is nonzero, it will also induce gauge kinetic mixing to be generated during RGE evolution and this is the case in the models we consider here.[9] However, when these models are evolved down from the

[8]In the version of SARAH which we used the extra terms from this extension were included for all terms except the trilinear and bilinear soft masses. We have been in contact with the SARAH author about this and understand they will be included in future versions.

[9]In E_6 inspired models with only complete 27-plet matter multiplets this trace would vanish. However, since we assume some incomplete multiplets so that our models are consistent with gauge coupling unification this trace doesn't vanish.

GUT scale, the radiatively generated gauge kinetic mixing gives an off-diagonal gauge coupling, g_{11}, of just ≈ 0.02 at the EW scale [13] and so it does not play a large role. At the same time if gauge kinetic mixing is included the RGE expressions become very large and unmanageable, so we neglected the gauge kinetic mixing by setting the SARAH flag NoU1Mixing to true.

At tree level in the EWSB conditions the parameters that must be considered are λ, A_λ, $m_{H_u}^2$, $m_{H_d}^2$, m_S^2 and g_1, g_2 and g_1'. Neglecting kinetic mixing, the two-loop β functions for the relevant gauge couplings read

$$\beta_{g_1}^{(1)} = \frac{48}{5} g_1^3, \tag{B4a}$$

$$\beta_{g_1}^{(2)} = \frac{2}{25} g_1^3 (30 g_1'^2 \Pi_Q^Y + 117 g_1^2 + 135 g_2^2 + 300 g_3^2 - 10\Sigma_\kappa - 15\Sigma_\lambda - 65 y_t^2 - 35 y_b^2 - 45 y_\tau^2), \tag{B4b}$$

$$\beta_{g_2}^{(1)} = 4 g_2^3, \tag{B4c}$$

$$\beta_{g_2}^{(2)} = \frac{2}{5} g_2^3 (5 g_1'^2 \Pi_Q^L + 9 g_1^2 + 115 g_2^2 + 60 g_3^2 - 5\Sigma_\lambda - 15 y_t^2 - 15 y_b^2 - 5 y_\tau^2), \tag{B4d}$$

$$\beta_{g_1'}^{(1)} = g_1'^3 \Sigma_Q, \tag{B4e}$$

$$\begin{aligned}\beta_{g_1'}^{(2)} = {} & \frac{2}{5} g_1'^3 (-15\Sigma_\kappa (Q_{\bar{D}}^2 + Q_D^2 + Q_S^2) - 30 y_b^2 (Q_d^2 + Q_1^2 + Q_Q^2) \\ & + 120 g_3^2 \Pi_Q^C - 10 y_\tau^2 (Q_e^2 + Q_L^2 + Q_1^2) + 15 g_2^2 \Pi_Q^L \\ & + 10 g_1'^2 \Pi_Q + 6 g_1^2 \Pi_Q^Y - 10\Sigma_\lambda (Q_S^2 + Q_1^2 + Q_2^2) \\ & - 30 y_t^2 (Q_u^2 + Q_2^2 + Q_Q^2)).\end{aligned} \tag{B4f}$$

In order to keep these expressions compact, we have used the notation

$$\Sigma_Q = \sum_i Q_i^2 = \frac{321}{40}\cos^2\theta + \frac{217}{24}\sin^2\theta + \frac{27}{8\sqrt{15}}\sin 2\theta$$

to denote the trace over the $U(1)'$ charges, along with[10]

$$\Sigma_Q^Y = \sum_i \sqrt{\frac{5}{3}} Q_i^Y Q_i = -\frac{3}{\sqrt{10}}\cos\theta - \frac{1}{\sqrt{6}}\sin\theta,$$

$$\begin{aligned}\Pi_Q = {} & \sum_i Q_i^4 \\ = {} & \frac{2049}{1600}\cos^4\theta + \frac{483}{80\sqrt{15}}\cos^3\theta\sin\theta + \frac{681}{160}\cos^2\theta\sin^2\theta \\ & + \frac{9}{16\sqrt{15}}\cos\theta\sin^3\theta + \frac{1297}{576}\sin^4\theta,\end{aligned}$$

$$\begin{aligned}\Pi_Q^Y = {} & \sum_i \left(\sqrt{\frac{5}{3}} Q_i^Y\right)^2 Q_i^2 = \frac{59}{40}\cos^2\theta + \frac{31}{24}\sin^2\theta \\ & + \frac{3}{8\sqrt{15}}\sin 2\theta,\end{aligned}$$

$$\begin{aligned}\Pi_Q^L = {} & 3Q_1^2 + 3Q_2^2 + Q_{H'}^2 + Q_{\overline{H'}}^2 + 3Q_L^2 + 9Q_Q^2 \\ = {} & \frac{39}{20}\cos^2\theta + \frac{19}{12}\sin^2\theta + \frac{3}{4\sqrt{15}}\sin 2\theta,\end{aligned}$$

$$\Pi_Q^C = Q_d^2 + Q_D^2 + Q_{\bar{D}}^2 + 2Q_Q^2 + Q_u^2 = \frac{1}{2}.$$

Note that in these expressions the $U(1)_Y$ and $U(1)'$ charges are assumed to be GUT-normalized. The expressions in terms of the E_6 mixing angle θ follow from the charge assignments given in Table I and hold provided that $U(1)$ mixing is neglected. Similarly, we write

$$\Sigma_\lambda = \lambda_1^2 + \lambda_2^2 + \lambda_3^2, \qquad \Sigma_\kappa = \kappa_1^2 + \kappa_2^2 + \kappa_3^2,$$
$$\Pi_\lambda = \lambda_1^4 + \lambda_2^4 + \lambda_3^4, \qquad \Pi_\kappa = \kappa_1^4 + \kappa_2^4 + \kappa_3^4.$$

The corresponding $O(t^2)$ coefficients for the gauge couplings are simply

$$b_{g_1}^{(2)} = \frac{3456}{25} g_1^5, \tag{B5a}$$

$$b_{g_2}^{(2)} = 24 g_2^5, \tag{B5b}$$

$$b_{g_1'}^{(2)} = \frac{3}{2} g_1'^5 \Sigma_Q^2. \tag{B5c}$$

The one- and two-loop contributions to the β function for λ and the $O(t^2)$ coefficient in the series expansion are

$$\beta_\lambda^{(1)} = \lambda \left[2\lambda^2 + 2\Sigma_\lambda + 3\Sigma_\kappa + 3y_t^2 + 3y_b^2 + y_\tau^2 - 3g_2^2 - \frac{3}{5} g_1^2 - 2(Q_1^2 + Q_2^2 + Q_S^2) g_1'^2 \right], \tag{B6a}$$

[10] The first of these is the trace which is assumed to vanish in Ref. [159]. Although we use the NoU1Mixing flag to neglect gauge kinetic mixing, SARAH does this by removing the RGE for the off diagonal gauge couplings and effectively setting them to zero at all scales by removing all terms involving them from the RGEs. Therefore, some terms with this trace remain and the RGEs shown here do not reduce to those which one would obtain from Ref. [159] or Ref. [18] unless $\Sigma_Q^Y = 0$. Note, however, these contributions do not appear in the corresponding trilinears due to the version of SARAH used.

$$\beta_\lambda^{(2)} = \lambda\Big\{-2\lambda^2(\lambda^2+2\Sigma_\lambda+3\Sigma_\kappa)-4\Pi_\lambda-6\Pi_\kappa-3\lambda^2(3y_t^2+3y_b^2+y_\tau^2)-3(3y_t^4+3y_b^4+2y_t^2y_b^2+y_\tau^4)+6g_2^2\Sigma_\lambda$$
$$+\frac{2}{5}g_1^2(2y_t^2-y_b^2+3y_\tau^2+2\Sigma_\kappa+3\Sigma_\lambda)+g_1'^2[4Q_S^2\lambda^2-6(Q_2^2-Q_Q^2-Q_u^2)y_t^2-6(Q_1^2-Q_Q^2-Q_d^2)y_b^2$$
$$-2(Q_1^2-Q_L^2-Q_e^2)y_\tau^2-6(Q_S^2-Q_D^2-Q_{\bar{D}}^2)\Sigma_\kappa-4(Q_S^2-Q_1^2-Q_2^2)\Sigma_\lambda]$$
$$+16g_3^2(y_t^2+y_b^2+\Sigma_\kappa)+\frac{33}{2}g_2^4+\frac{297}{50}g_1^4+2g_1'^4[2Q_1^4+2Q_2^4+2Q_S^4+(Q_1^2+Q_2^2+Q_S^2)\Sigma_Q]$$
$$+\frac{9}{5}g_1^2g_2^2+6g_1'^2g_2^2(Q_1^2+Q_2^2)+\frac{6}{5}g_1^2g_1'^2[Q_1^2+Q_2^2+(Q_2-Q_1)\Sigma_Q^Y]\Big\}, \tag{B6b}$$

$$b_\lambda^{(2)} = \lambda\Big\{2\lambda^2(3\lambda^2+4\Sigma_\lambda+6\Sigma_\kappa)+4\Pi_\lambda+6\Pi_\kappa+6\left(\Sigma_\lambda+\frac{3}{2}\Sigma_\kappa\right)^2+7\lambda^2(3y_t^2+3y_b^2+y_\tau^2)+(3y_t^2+3y_b^2+y_\tau^2)(2\Sigma_\lambda+3\Sigma_\kappa)$$
$$+3\left(\frac{15}{2}y_t^4+\frac{15}{2}y_b^4+\frac{3}{2}y_\tau^4+5y_b^2y_t^2+3y_b^2y_\tau^2+y_t^2y_\tau^2\right)-\frac{1}{5}g_1^2(12\lambda^2+16y_b^2+22y_t^2+12y_\tau^2+12\Sigma_\lambda+13\Sigma_\kappa)$$
$$-3g_2^2(4\lambda^2+6y_t^2+6y_b^2+2y_\tau^2+4\Sigma_\lambda+3\Sigma_\kappa)-16g_3^2(y_t^2+y_b^2+\Sigma_\kappa)-2g_1'^2[4(Q_1^2+Q_2^2+Q_S^2)(\lambda^2+\Sigma_\lambda)$$
$$+3\Sigma_\kappa(Q_1^2+Q_2^2+Q_S^2+Q_D^2+Q_{\bar{D}}^2)+3y_b^2(2Q_1^2+Q_2^2+Q_d^2+Q_Q^2+Q_S^2)+3y_t^2(Q_1^2+2Q_2^2+Q_Q^2+Q_S^2+Q_u^2)$$
$$+y_\tau^2(2Q_1^2+Q_2^2+Q_e^2+Q_L^2+Q_S^2)]-\frac{15}{2}g_2^4-\frac{279}{50}g_1^4+2g_1'^4(Q_1^2+Q_2^2+Q_S^2)(Q_1^2+Q_2^2+Q_S^2-\Sigma_Q)$$
$$+\frac{9}{5}g_1^2g_2^2+6g_1'^2g_2^2(Q_1^2+Q_2^2+Q_S^2)+\frac{6}{5}g_1^2g_1'^2(Q_1^2+Q_2^2+Q_S^2)\Big\}. \tag{B6c}$$

It is sufficient for our purposes to consider the trilinear coupling $a_\lambda \equiv \lambda A_\lambda$, rather than A_λ, for which the relevant expressions read

$$\beta_{a_\lambda}^{(1)} = a_\lambda\left[2\lambda^2+2\Sigma_\lambda+3\Sigma_\kappa+3y_t^2+3y_b^2+y_\tau^2-3g_2^2-\frac{3}{5}g_1^2-2(Q_1^2+Q_2^2+Q_S^2)g_1'^2\right]$$
$$+\lambda\left[4\lambda a_\lambda+4\Sigma_{a_\lambda}+6\Sigma_{a_\kappa}+6y_ta_t+6y_ba_b+2y_\tau a_\tau+6g_2^2M_2+\frac{6}{5}g_1^2M_1+4g_1'^2M_1'(Q_1^2+Q_2^2+Q_S^2)\right], \tag{B7a}$$

$$\beta_{a_\lambda}^{(2)} = a_\lambda\Big\{-2\lambda^2(\lambda^2+2\Sigma_\lambda+3\Sigma_\kappa)-4\Pi_\lambda-6\Pi_\kappa-3\lambda^2(3y_t^2+3y_b^2+y_\tau^2)-3(3y_t^4+3y_b^4+2y_t^2y_b^2+y_\tau^4)$$
$$+6g_2^2\Sigma_\lambda+\frac{2}{5}g_1^2(2y_t^2-y_b^2+3y_\tau^2+2\Sigma_\kappa+3\Sigma_\lambda)+g_1'^2[4Q_S^2\lambda^2-6(Q_2^2-Q_Q^2-Q_u^2)y_t^2-6(Q_1^2-Q_Q^2-Q_d^2)y_b^2$$
$$-2(Q_1^2-Q_L^2-Q_e^2)y_\tau^2-6(Q_S^2-Q_D^2-Q_{\bar{D}}^2)\Sigma_\kappa-4(Q_S^2-Q_1^2-Q_2^2)\Sigma_\lambda]+16g_3^2(y_t^2+y_b^2+\Sigma_\kappa)+\frac{33}{2}g_2^4+\frac{297}{50}g_1^4$$
$$+2g_1'^4[2Q_1^4+2Q_2^4+2Q_S^4+(Q_1^2+Q_2^2+Q_S^2)\Sigma_Q]+\frac{9}{5}g_1^2g_2^2+6g_1'^2g_2^2(Q_1^2+Q_2^2)+\frac{6}{5}g_1^2g_1'^2(Q_1^2+Q_2^2)\Big\}$$
$$+\lambda\Big\{-4\lambda a_\lambda(\lambda^2+2\Sigma_\lambda+3\Sigma_\kappa)-4\lambda^2(\lambda a_\lambda+2\Sigma_{a_\lambda}+3\Sigma_{a_\kappa})-16\Pi_{a_\lambda}-24\Pi_{a_\kappa}-6\lambda a_\lambda(3y_t^2+3y_b^2+y_\tau^2)$$
$$-6\lambda^2(3y_ta_t+3y_ba_b+y_\tau a_\tau)-12[3y_t^3a_t+3y_b^3a_b+y_ty_b(y_ta_b+y_ba_t)+y_\tau^3a_\tau]$$
$$+32g_3^2[y_ta_t+y_ba_b+\Sigma_{a_\kappa}-(y_t^2+y_b^2+\Sigma_\kappa)M_3]+12g_2^2(\Sigma_{a_\lambda}-\Sigma_\lambda M_2)$$
$$+\frac{2}{5}g_1^2[4y_ta_t-2y_ba_b+6y_\tau a_\tau+4\Sigma_{a_\kappa}+6\Sigma_{a_\lambda}-2(2y_t^2-y_b^2+3y_\tau^2+2\Sigma_\kappa+3\Sigma_\lambda)M_1]$$
$$+4g_1'^2[3y_t(Q_Q^2-Q_2^2+Q_u^2)(a_t-y_tM_1')+3y_b(Q_Q^2-Q_1^2+Q_d^2)(a_b-y_bM_1')+y_\tau(Q_L^2-Q_1^2+Q_e^2)(a_\tau-y_\tau M_1')$$
$$+2Q_S^2\lambda(a_\lambda-\lambda M_1')+2(Q_1^2+Q_2^2-Q_S^2)(\Sigma_{a_\lambda}-\Sigma_\lambda M_1')+3(Q_D^2+Q_{\bar{D}}^2-Q_S^2)(\Sigma_{a_\kappa}-\Sigma_\kappa M_1')]$$
$$-66g_2^4M_2-\frac{594}{25}g_1^4M_1-8g_1'^4M_1'[2(Q_1^4+Q_2^4+Q_S^4)+(Q_1^2+Q_2^2+Q_S^2)\Sigma_Q]$$
$$-\frac{18}{5}g_2^2g_1^2(M_2+M_1)-12g_2^2g_1'^2(Q_1^2+Q_2^2)(M_2+M_1')-\frac{12}{5}g_1^2g_1'^2(Q_1^2+Q_2^2)(M_1+M_1')\Big\}, \tag{B7b}$$

$$
\begin{aligned}
b_{a_\lambda}^{(2)} = \lambda\bigg[&4\lambda a_\lambda + 4\Sigma_{a_\lambda} + 6\Sigma_{a_\kappa} + 6y_t a_t + 6y_b a_b + 2y_\tau a_\tau + 6g_2^2 M_2 + \frac{6}{5}g_1^2 M_1 + 4g_1'^2 M_1'(Q_1^2 + Q_2^2 + Q_S^2)\bigg] \\
&\times\bigg[2\lambda^2 + 2\Sigma_\lambda + 3\Sigma_\kappa + 3y_t^2 + 3y_b^2 + y_\tau^2 - 3g_2^2 - \frac{3}{5}g_1^2 - 2(Q_1^2 + Q_2^2 + Q_S^2)g_1'^2\bigg] \\
&+ a_\lambda\bigg\{2\lambda^2(3\lambda^2 + 4\Sigma_\lambda + 6\Sigma_\kappa) + 4\Pi_\lambda + 6\Pi_\kappa + 6\left(\Sigma_\lambda + \frac{3}{2}\Sigma_\kappa\right)^2 + 7\lambda^2(3y_t^2 + 3y_b^2 + y_\tau^2) + (3y_t^2 + 3y_b^2 + y_\tau^2)(2\Sigma_\lambda + 3\Sigma_\kappa) \\
&+ 3\left(\frac{15}{2}y_t^4 + \frac{15}{2}y_b^4 + \frac{3}{2}y_\tau^4 + 5y_b^2 y_t^2 + 3y_b^2 y_\tau^2 + y_t^2 y_\tau^2\right) - \frac{1}{5}g_1^2(12\lambda^2 + 16y_b^2 + 22y_t^2 + 12y_\tau^2 + 12\Sigma_\lambda + 13\Sigma_\kappa) \\
&- 3g_2^2(4\lambda^2 + 6y_t^2 + 6y_b^2 + 2y_\tau^2 + 4\Sigma_\lambda + 3\Sigma_\kappa) - 16g_3^2(y_t^2 + y_b^2 + \Sigma_\kappa) - 2g_1'^2[4(Q_1^2 + Q_2^2 + Q_S^2)(\lambda^2 + \Sigma_\lambda) \\
&+ 3\Sigma_\kappa(Q_1^2 + Q_2^2 + Q_S^2 + Q_D^2 + Q_{\bar{D}}^2) + 3y_b^2(2Q_1^2 + Q_2^2 + Q_d^2 + Q_Q^2 + Q_S^2) + 3y_t^2(Q_1^2 + 2Q_2^2 + Q_Q^2 + Q_S^2 + Q_u^2) \\
&+ y_\tau^2(2Q_1^2 + Q_2^2 + Q_e^2 + Q_L^2 + Q_S^2)] - \frac{15}{2}g_2^4 - \frac{279}{50}g_1^4 + 2g_1'^4(Q_1^2 + Q_2^2 + Q_S^2)(Q_1^2 + Q_2^2 + Q_S^2 - \Sigma_Q) + \frac{9}{5}g_1^2 g_2^2 \\
&+ 6g_1'^2 g_2^2(Q_1^2 + Q_2^2 + Q_S^2) + \frac{6}{5}g_1^2 g_1'^2(Q_1^2 + Q_2^2 + Q_S^2)\bigg\} + \lambda\bigg[16\lambda^3 a_\lambda + 16\Pi_{a_\lambda} + 24\Pi_{a_\kappa} + 8\lambda\sum_{i=1}^{3}\lambda_i(\lambda_i a_\lambda + a_{\lambda_i}\lambda) \\
&+ 12\lambda\sum_{i=1}^{3}\kappa_i(\kappa_i a_\lambda + a_{\kappa_i}\lambda) + 8\sum_{i=1}^{3}\sum_{j=1}^{3}\lambda_i\lambda_j(a_{\lambda_i}\lambda_j + \lambda_i a_{\lambda_j}) + 18\sum_{i=1}^{3}\sum_{j=1}^{3}\kappa_i\kappa_j(\kappa_i a_{\kappa_j} + a_{\kappa_i}\kappa_j) \\
&+ 24\sum_{i=1}^{3}\sum_{j=1}^{3}\lambda_i\kappa_j(\lambda_i a_{\kappa_j} + a_{\lambda_i}\kappa_j) + 72y_t^3 a_t + 72y_b^3 a_b + 16y_\tau^3 a_\tau + 12y_t y_b(a_t y_b + y_t a_b) + 12y_b y_\tau(a_b y_\tau + y_b a_\tau) \\
&+ 30\lambda y_t(a_\lambda y_t + \lambda a_t) + 30\lambda y_b(a_\lambda y_b + \lambda a_b) + 10\lambda y_\tau(a_\lambda y_\tau + \lambda a_\tau) - 32g_3^2 y_t(a_t - y_t M_3) - 32g_3^2 y_b(a_b - y_b M_3) \\
&- 32g_3^2(\Sigma_{a_\kappa} - M_3\Sigma_\kappa) - 12g_2^2\lambda(a_\lambda - \lambda M_2) - 18g_2^2 y_t(a_t - y_t M_2) - 18g_2^2 y_b(a_b - y_b M_2) \\
&- 6g_2^2 y_\tau(a_\tau - y_\tau M_2) - 12g_2^2(\Sigma_{a_\lambda} - M_2\Sigma_\lambda) - \frac{12}{5}g_1^2\lambda(a_\lambda - \lambda M_1) - \frac{26}{5}g_1^2 y_t(a_t - y_t M_1) - \frac{14}{5}g_1^2 y_b(a_b - y_b M_1) \\
&- \frac{18}{5}g_1^2 y_\tau(a_\tau - y_\tau M_1) - \frac{12}{5}g_1^2(\Sigma_{a_\lambda} - M_1\Sigma_\lambda) - \frac{8}{5}g_1^2(\Sigma_{a_\kappa} - M_1\Sigma_\kappa) - 8g_1'^2\lambda(Q_1^2 + Q_2^2 + Q_S^2)(a_\lambda - \lambda M_1') \\
&- 12g_1'^2 y_t(Q_2^2 + Q_Q^2 + Q_u^2)(a_t - y_t M_1') - 12g_1'^2 y_b(Q_1^2 + Q_Q^2 + Q_d^2)(a_b - y_b M_1') - 4g_1'^2 y_\tau(Q_1^2 + Q_L^2 + Q_e^2)(a_\tau - y_\tau M_1') \\
&- 8g_1'^2(Q_1^2 + Q_2^2 + Q_S^2)(\Sigma_{a_\lambda} - M_1'\Sigma_\lambda) - 12g_1'^2(Q_S^2 + Q_D^2 + Q_{\bar{D}}^2)(\Sigma_{a_\kappa} - M_1'\Sigma_\kappa) \\
&+ 48g_2^4 M_2 + \frac{576}{25}g_1^4 M_1 + 8g_1'^4 M_1'\Sigma_Q(Q_1^2 + Q_2^2 + Q_S^2)\bigg],
\end{aligned}
\tag{B7c}
$$

where

$$
\begin{aligned}
\Sigma_{a_\lambda} &= \lambda_1 a_{\lambda_1} + \lambda_2 a_{\lambda_2} + \lambda_3 a_{\lambda_3}, \qquad & \Sigma_{a_\kappa} &= \kappa_1 a_{\kappa_1} + \kappa_2 a_{\kappa_2} + \kappa_3 a_{\kappa_3}, \\
\Pi_{a_\lambda} &= \lambda_1^3 a_{\lambda_1} + \lambda_2^3 a_{\lambda_2} + \lambda_3^3 a_{\lambda_3}, \qquad & \Pi_{a_\kappa} &= \kappa_1^3 a_{\kappa_1} + \kappa_2^3 a_{\kappa_2} + \kappa_3^3 a_{\kappa_3}.
\end{aligned}
$$

Note that $a_{\lambda_i} \equiv \lambda_i A_{\lambda_i}$, $a_{\kappa_i} \equiv \kappa_i A_{\kappa_i}$, $a_t \equiv y_t A_t$, $a_b \equiv y_b A_b$ and $a_\tau \equiv y_\tau A_\tau$. Defining

$$
\Sigma_1 = \sum_{i=1}^{3}(m_{Q_i}^2 - 2m_{u_i}^2 + m_{d_i}^2 + m_{e_i}^2 - m_{L_i}^2 + m_{H_i^u}^2 - m_{H_i^d}^2 + m_{\bar{D}_i}^2 - m_{D_i}^2) - m_{H'}^2 + m_{\bar{H}'}^2,
\tag{B8}
$$

$$
\begin{aligned}
\Sigma_1' = \sum_{i=1}^{3}(&6Q_Q m_{Q_i}^2 + 3Q_u m_{u_i}^2 + 3Q_d m_{d_i}^2 + Q_e m_{e_i}^2 + 2Q_L m_{L_i}^2 + 2Q_2 m_{H_i^u}^2 + 2Q_1 m_{H_i^d}^2 \\
&+ Q_S m_{S_i}^2 + 3Q_{\bar{D}} m_{\bar{D}_i}^2 + 3Q_D m_{D_i}^2) + 2Q_{H'} m_{H'}^2 + 2Q_{\bar{H}'} m_{\bar{H}'}^2,
\end{aligned}
\tag{B9}
$$

the one- and two-loop β functions and the $O(t^2)$ coefficients for $m_{H_d}^2$ are

$$\beta^{(1)}_{m^2_{H_d}} = 2\lambda^2(m^2_{H_d} + m^2_{H_u} + m^2_S) + 2a^2_\lambda + 6y^2_b(m^2_{H_d} + m^2_{Q_3} + m^2_{d_3}) + 6a^2_b + 2y^2_\tau(m^2_{H_d} + m^2_{L_3} + m^2_{e_3}) + 2a^2_\tau - 6g^2_2M^2_2 - \frac{6}{5}g^2_1M^2_1 - 8Q^2_1g'^2_1M'^2_1 - \frac{3}{5}g^2_1\Sigma_1 + 2Q_1g'^2_1\Sigma'_1, \quad \text{(B10a)}$$

$$\begin{aligned}
\beta^{(2)}_{m^2_{H_d}} = & -36y^4_b(m^2_{H_d} + m^2_{Q_3} + m^2_{d_3}) - 12y^4_\tau(m^2_{H_d} + m^2_{L_3} + m^2_{e_3}) - 72y^2_ba^2_b - 24y^2_\tau a^2_\tau \\
& - 6y^2_ty^2_b(m^2_{H_d} + m^2_{H_u} + 2m^2_{Q_3} + m^2_{u_3} + m^2_{d_3}) - 6(y_ta_b + y_ba_t)^2 \\
& - 4\lambda^4(m^2_{H_d} + m^2_{H_u} + m^2_S) - 8\lambda^2a^2_\lambda - \lambda^2\sum_{i=1}^3[4\lambda^2_i(m^2_{H_d} + m^2_{H_u} + 2m^2_S + m^2_{H^d_i} \\
& + m^2_{H^u_i}) + 6\kappa^2_i(m^2_{H_d} + m^2_{H_u} + 2m^2_S + m^2_{D_i} + m^2_{\bar{D}_i})] - \sum_{i=1}^3[4(\lambda_ia_\lambda + \lambda a_{\lambda_i})^2 \\
& + 6(\kappa_ia_\lambda + \lambda a_{\kappa_i})^2] - 6\lambda^2y^2_t(m^2_{H_d} + 2m^2_{H_u} + m^2_S + m^2_{Q_3} + m^2_{u_3}) - 6(\lambda a_t + y_ta_\lambda)^2 \\
& + 32g^2_3y^2_b(m^2_{H_d} + m^2_{Q_3} + m^2_{d_3} + 2M^2_3) + 32g^2_3(a^2_b - 2y_ba_bM_3) \\
& + \frac{6}{5}g^2_1y^2_t(3m^2_{H_u} + m^2_{Q_3} - 4m^2_{u_3}) - \frac{2}{5}g^2_1y^2_b(11m^2_{H_d} - m^2_{Q_3} - 4m^2_{d_3} + 4M^2_1) \\
& - \frac{4}{5}g^2_1(a^2_b - 2y_ba_bM_1) + \frac{6}{5}g^2_1y^2_\tau(m^2_{H_d} + m^2_{L_3} + 4m^2_{e_3} + 4M^2_1) + \frac{12}{5}g^2_1(a^2_\tau - 2y_\tau a_\tau M_1) \\
& + \frac{6}{5}g^2_1\sum_{i=1}^3[\lambda^2_i(m^2_{H^d_i} - m^2_{H^u_i}) + \kappa^2_i(m^2_{\bar{D}_i} - m^2_{D_i})] + 12g'^2_1y^2_b(Q^2_Q + Q^2_d - Q^2_1) \\
& \times(m^2_{H_d} + m^2_{Q_3} + m^2_{d_3} + 2M'^2_1) + 12g'^2_1(Q^2_Q + Q^2_d - Q^2_1)(a^2_b - 2y_ba_bM'_1) \\
& - 24Q_1g'^2_1y^2_b(Q_1m^2_{H_d} + Q_Qm^2_{Q_3} + Q_dm^2_{d_3}) + 4g'^2_1y^2_\tau(Q^2_L + Q^2_e - Q^2_1) \\
& \times(m^2_{H_d} + m^2_{L_3} + m^2_{e_3} + 2M'^2_1) + 4g'^2_1(Q^2_L + Q^2_e - Q^2_1)(a^2_\tau - 2y_\tau a_\tau M'_1) \\
& - 8Q_1g'^2_1y^2_\tau(Q_1m^2_{H_d} + Q_Lm^2_{L_3} + Q_em^2_{e_3}) - 24Q_1g'^2_1y^2_t(Q_2m^2_{H_u} + Q_Qm^2_{Q_3} + Q_um^2_{u_3}) \\
& + 4g'^2_1\lambda^2(Q^2_2 + Q^2_S - Q^2_1)(m^2_{H_d} + m^2_{H_u} + m^2_S + 2M'^2_1) + 4g'^2_1(Q^2_2 + Q^2_S - Q^2_1) \\
& \times(a^2_\lambda - 2\lambda a_\lambda M'_1) - 4Q_1g'^2_1\sum_{i=1}^3[2\lambda^2_i(Q_1m^2_{H^d_i} + Q_2m^2_{H^u_i} + Q_Sm^2_S) \\
& + 3\kappa^2_i(m^2_S + m^2_{D_i} + m^2_{\bar{D}_i})] - \frac{16}{5}g^2_3g^2_1\sum_{i=1}^3(m^2_{Q_i} - 2m^2_{u_i} + m^2_{d_i} + m^2_{\bar{D}_i} - m^2_{D_i}) \\
& + 32Q_1g^2_3g'^2_1\sum_{i=1}^3(2Q_Qm^2_{Q_i} + Q_um^2_{u_i} + Q_dm^2_{d_i} + Q_{\bar{D}}m^2_{\bar{D}_i} + Q_Dm^2_{D_i}) \\
& + 3g^4_2\left[29M^2_2 + m^2_{H'} + m^2_{\bar{H}'} + \sum_{i=1}^3(3m^2_{Q_i} + m^2_{L_i} + m^2_{H^d_i} + m^2_{H^u_i})\right] \\
& + \frac{9}{5}g^2_2g^2_1\left[2(M^2_1 + M_1M_2 + M^2_2) + m^2_{H'} - m^2_{\bar{H}'} - \sum_{i=1}^3(m^2_{Q_i} - m^2_{L_i} + m^2_{H^u_i} - m^2_{H^d_i})\right] \\
& + 12Q_1g^2_2g'^2_1\left[2Q_1(M'^2_1 + M'_1M_2 + M^2_2) + Q_{H'}m^2_{H'} + Q_{\bar{H}'}m^2_{\bar{H}'}\right. \\
& \left. + \sum_{i=1}^3(3Q_Qm^2_{Q_i} + Q_Lm^2_{L_i} + Q_1m^2_{H^d_i} + Q_2m^2_{H^u_i})\right] + \frac{1}{25}g^4_1\left[891M^2_1 + 18m^2_{H'}\right.
\end{aligned}$$

$$
\begin{aligned}
&+ 2\sum_{i=1}^{3}(m_{d_i}^2 + 5m_{D_i}^2 + m_{\bar{D}_i}^2 - 9m_{e_i}^2 + 9m_{H_i^d}^2 + 9m_{L_i}^2 + m_{Q_i}^2 + 28m_{u_i}^2)\Bigg] \\
&- \frac{4}{5}g_1^2 g_1'^2\Bigg[6Q_1(3Q_d + 3Q_{\bar{D}} - 3Q_D + 3Q_e - 4Q_1 + 3Q_2 + Q_{\bar{H}'} - Q_{H'} - 3Q_L \\
&+ 3Q_Q - 6Q_u)(M_1^2 + M_1 M_1' + M_1'^2) + 3Q_{\bar{H}'}^2 m_{\bar{H}'}^2 - 3Q_{H'}^2 m_{H'}^2 \\
&+ 3\sum_{i=1}^{3}(Q_d^2 m_{d_i}^2 + Q_{\bar{D}}^2 m_{\bar{D}_i}^2 - Q_D^2 m_{D_i}^2 + Q_e^2 m_{e_i}^2 - Q_1^2 m_{H_i^d}^2 + Q_2^2 m_{H_i^u}^2 - Q_L^2 m_{L_i}^2 \\
&+ Q_Q^2 m_{Q_i}^2 - 2Q_u^2 m_{u_i}^2) + 3Q_1 Q_{\bar{H}'} m_{\bar{H}'}^2 - 9Q_1 Q_{H'} m_{H'}^2 + Q_1\sum_{i=1}^{3}(4Q_d m_{d_i}^2 + 4Q_{\bar{D}} m_{\bar{D}_i}^2 \\
&- 8Q_D m_{D_i}^2 - 9Q_1 m_{H_i^d}^2 + 3Q_2 m_{H_i^u}^2 - 9Q_L m_{L_i}^2 + 5Q_Q m_{Q_i}^2 - 20Q_u m_{u_i}^2)\Bigg] \\
&+ 8Q_1 g_1'^4\Bigg[3Q_1 M_1'^2(9Q_d^2 + 9Q_{\bar{D}}^2 + 9Q_D^2 + 3Q_e^2 + 8Q_1^2 + 6Q_2^2 + 2Q_{\bar{H}'}^2 + 2Q_{H'}^2 + 6Q_L^2 \\
&+ 18Q_Q^2 + 3Q_S^2 + 9Q_u^2) + 2Q_{\bar{H}'}^3 m_{\bar{H}'}^2 + 2Q_{H'}^3 m_{H'}^2 + \sum_{i=1}^{3}(3Q_d^3 m_{d_i}^2 + 3Q_{\bar{D}}^3 m_{\bar{D}_i}^2 + 3Q_D^3 m_{D_i}^2 \\
&+ Q_e^3 m_{e_i}^2 + 2Q_1^3 m_{H_i^d}^2 + 2Q_2^3 m_{H_i^u}^2 + 2Q_L^3 m_{L_i}^2 + 6Q_Q^3 m_{Q_i}^2 + Q_S^3 m_{S_i}^2 + 3Q_u^3 m_{u_i}^2) \\
&+ 2Q_1 Q_{\bar{H}'}^2 m_{\bar{H}'}^2 + 2Q_1 Q_{H'}^2 m_{H'}^2 + Q_1\sum_{i=1}^{3}(3Q_d^2 m_{d_i}^2 + 3Q_{\bar{D}}^2 m_{\bar{D}_i}^2 + 3Q_D^2 m_{D_i}^2 + Q_e^2 m_{e_i}^2 \\
&+ 2Q_1^2 m_{H_i^d}^2 + 2Q_2^2 m_{H_i^u}^2 + 2Q_L^2 m_{L_i}^2 + 6Q_Q^2 m_{Q_i}^2 + Q_S^2 m_{S_i}^2 + 3Q_u^2 m_{u_i}^2)\Bigg], \qquad \text{(B10b)}
\end{aligned}
$$

$$
\begin{aligned}
b_{m_{H_d}^2}^{(2)} = {}& 72y_b^4(m_{H_d}^2 + m_{Q_3}^2 + m_{d_3}^2) + 144y_b^2 a_b^2 + 16y_\tau^4(m_{H_d}^2 + m_{L_3}^2 + m_{e_3}^2) + 32y_\tau^2 a_\tau^2 \\
&+ 8\lambda^4(m_{H_d}^2 + m_{H_u}^2 + m_S^2) + 16\lambda^2 a_\lambda^2 + 6y_t^2 y_b^2(m_{H_d}^2 + m_{H_u}^2 + 2m_{Q_3}^2 + m_{u_3}^2 + m_{d_3}^2) \\
&+ 6(y_t a_b + y_b a_t)^2 + 12y_b^2 y_\tau^2(2m_{H_d}^2 + m_{Q_3}^2 + m_{d_3}^2 + m_{L_3}^2 + m_{e_3}^2) + 12(y_b a_\tau + y_\tau a_b)^2 \\
&+ 6\lambda^2 y_t^2(m_{H_d}^2 + 2m_{H_u}^2 + m_S^2 + m_{Q_3}^2 + m_{u_3}^2) + 6(\lambda a_t + y_t a_\lambda)^2 \\
&+ 12\lambda^2 y_b^2(2m_{H_d}^2 + m_{H_u}^2 + m_S^2 + m_{Q_3}^2 + m_{d_3}^2) + 12(\lambda a_b + y_b a_\lambda)^2 \\
&+ 4\lambda^2 y_\tau^2(2m_{H_d}^2 + m_{H_u}^2 + m_S^2 + m_{L_3}^2 + m_{e_3}^2) + 4(\lambda a_\tau + y_\tau a_\lambda)^2 \\
&+ 4\sum_{i=1}^{3}[\lambda^2\lambda_i^2(m_{H_i^d}^2 + m_{H_i^u}^2 + m_{H_d}^2 + m_{H_u}^2 + 2m_S^2) + (\lambda a_{\lambda_i} + \lambda_i a_\lambda)^2] \\
&+ 6\sum_{i=1}^{3}[\lambda^2\kappa_i^2(m_{H_d}^2 + m_{H_u}^2 + 2m_S^2 + m_{D_i}^2 + m_{\bar{D}_i}^2) + (\lambda a_{\kappa_i} + \kappa_i a_\lambda)^2] \\
&- 32g_3^2 y_b^2(m_{H_d}^2 + m_{Q_3}^2 + m_{d_3}^2 + 2M_3^2) - 32g_3^2(a_b^2 - 2y_b a_b M_3) \\
&- 18g_2^2 y_b^2(m_{H_d}^2 + m_{Q_3}^2 + m_{d_3}^2 + 2M_2^2) - 18g_2^2(a_b^2 - 2y_b a_b M_2) \\
&- 6g_2^2 y_\tau^2(m_{H_d}^2 + m_{L_3}^2 + m_{e_3}^2 + 2M_2^2) - 6g_2^2(a_\tau^2 - 2y_\tau a_\tau M_2) \\
&- 6g_2^2\lambda^2(m_{H_d}^2 + m_{H_u}^2 + m_S^2 + 2M_2^2) - 6g_2^2(a_\lambda^2 - 2\lambda a_\lambda M_2) \\
&- \frac{14}{5}g_1^2 y_b^2(m_{H_d}^2 + m_{Q_3}^2 + m_{d_3}^2 + 2M_1^2) - \frac{14}{5}g_1^2(a_b^2 - 2y_b a_b M_1) \\
&- \frac{18}{5}g_1^2 y_\tau^2(m_{H_d}^2 + m_{L_3}^2 + m_{e_3}^2 + 2M_1^2) - \frac{18}{5}g_1^2(a_\tau^2 - 2y_\tau a_\tau M_1)
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{6}{5}g_1^2\lambda^2(m_{H_d}^2+m_{H_u}^2+m_S^2+2M_1^2)-\frac{6}{5}g_1^2(a_\lambda^2-2\lambda a_\lambda M_1)\\
&-12g_1'^2y_b^2(Q_1^2+Q_Q^2+Q_d^2)(m_{H_d}^2+m_{Q_3}^2+m_{d_3}^2+2M_1'^2)-12g_1'^2(Q_1^2+Q_Q^2+Q_d^2)\\
&\times(a_b^2-2y_ba_bM_1')+6g_1'^2y_b^2(Q_1+Q_Q+Q_d)(2Q_1m_{H_d}^2+2Q_1m_{Q_3}^2+2Q_1m_{d_3}^2+\Sigma_1')\\
&+12Q_1g_1'^2a_b^2(Q_1+Q_Q+Q_d)-4g_1'^2y_\tau^2(Q_1^2+Q_L^2+Q_e^2)(m_{H_d}^2+m_{L_3}^2+m_{e_3}^2+2M_1'^2)\\
&-4g_1'^2(Q_1^2+Q_L^2+Q_e^2)(a_\tau^2-2y_\tau a_\tau M_1')+2g_1'^2y_\tau^2(Q_1+Q_L+Q_e)\\
&\times(2Q_1m_{H_d}^2+2Q_1m_{L_3}^2+2Q_1m_{e_3}^2+\Sigma_1')+4Q_1g_1'^2a_\tau^2(Q_1+Q_L+Q_e)\\
&+12Q_1g_1'^2y_t^2(Q_2+Q_Q+Q_u)(m_{H_u}^2+m_{Q_3}^2+m_{u_3}^2)+12Q_1g_1'^2a_t^2(Q_2+Q_Q+Q_u)\\
&-4g_1'^2\lambda^2(Q_1^2+Q_2^2+Q_S^2)(m_{H_d}^2+m_{H_u}^2+m_S^2+2M_1'^2)-4g_1'^2(Q_1^2+Q_2^2+Q_S^2)\\
&\times(a_\lambda^2-2\lambda a_\lambda M_1')+2g_1'^2\lambda^2(Q_1+Q_2+Q_S)\Sigma_1'+4Q_1g_1'^2(Q_1+Q_2+Q_S)\\
&\times\sum_{i=1}^3[\lambda_i^2(m_{H_i^d}^2+m_{H_i^u}^2+m_S^2)+a_{\lambda_i}^2]+6Q_1g_1'^2(Q_S+Q_D+Q_{\bar D})\\
&\times\sum_{i=1}^3[\kappa_i^2(m_S^2+m_{D_i}^2+m_{\bar D_i}^2)+a_{\kappa_i}^2]-96Q_1g_3^2g_1'^2M_3^2(2Q_Q+Q_u+Q_d+Q_D+Q_{\bar D})\\
&-72g_2^4M_2^2-12Q_1g_2^2g_1'^2M_2^2(9Q_Q+3Q_L+3Q_1+3Q_2+Q_{\bar H'}+Q_{H'})\\
&-\frac{288}{25}g_1^4(\Sigma_1+3M_1^2)-\frac{3}{5}g_1^2g_1'^2[4Q_1M_1^2(2Q_d+2Q_{\bar D}+2Q_D+6Q_e+3Q_1+3Q_2\\
&+Q_{\bar H'}+Q_{H'}+3Q_L+Q_Q+8Q_u)-4M_1'^2(3Q_d^2+3Q_{\bar D}^2-3Q_D^2+3Q_e^2-3Q_1^2+3Q_2^2\\
&+Q_{\bar H'}^2-Q_{H'}^2-3Q_L^2+3Q_Q^2-6Q_u^2)+(\Sigma_1'-2Q_1\Sigma_1)\Sigma_Q^Y]-4Q_1g_1'^4[2M_1'^2(9Q_d^3\\
&+9Q_{\bar D}^3+9Q_D^3+3Q_e^3+6Q_1^3+6Q_2^3+2Q_{\bar H'}^3+2Q_{H'}^3+6Q_L^3+18Q_Q^3+3Q_S^3+9Q_u^3)\\
&+(6Q_1M_1'^2-\Sigma_1')\Sigma_Q].
\end{aligned}
\tag{B10c}
$$

Similarly, those for $m_{H_u}^2$ read

$$
\begin{aligned}
\beta_{m_{H_u}^2}^{(1)}&=2\lambda^2(m_{H_d}^2+m_{H_u}^2+m_S^2)+2a_\lambda^2+6y_t^2(m_{H_u}^2+m_{Q_3}^2+m_{u_3}^2)+6a_t^2\\
&\quad-6g_2^2M_2^2-\frac{6}{5}g_1^2M_1^2-8Q_2^2g_1'^2M_1'^2+\frac{3}{5}g_1^2\Sigma_1+2Q_2g_1'^2\Sigma_1',
\end{aligned}
\tag{B11a}
$$

$$
\begin{aligned}
\beta_{m_{H_u}^2}^{(2)}&=-36y_t^4(m_{H_u}^2+m_{Q_3}^2+m_{u_3}^2)-6y_t^2y_b^2(m_{H_u}^2+m_{H_d}^2+2m_{Q_3}^2+m_{u_3}^2+m_{d_3}^2)\\
&\quad-72y_t^2a_t^2-6(y_ta_b+y_ba_t)^2-4\lambda^4(m_{H_d}^2+m_{H_u}^2+m_S^2)-8\lambda^2a_\lambda^2\\
&\quad-\lambda^2\sum_{i=1}^3[4\lambda_i^2(m_{H_d}^2+m_{H_u}^2+2m_S^2+m_{H_i^d}^2+m_{H_i^u}^2)\\
&\quad+6\kappa_i^2(m_{H_d}^2+m_{H_u}^2+2m_S^2+m_{D_i}^2+m_{\bar D_i}^2)]-\sum_{i=1}^3[4(\lambda_ia_\lambda+\lambda a_{\lambda_i})^2\\
&\quad+6(\kappa_ia_\lambda+\lambda a_{\kappa_i})^2]-6\lambda^2y_b^2(2m_{H_d}^2+m_{H_u}^2+m_S^2+m_{Q_3}^2+m_{d_3}^2)\\
&\quad-2\lambda^2y_\tau^2(2m_{H_d}^2+m_{H_u}^2+m_S^2+m_{L_3}^2+m_{e_3}^2)-6(\lambda a_b+y_ba_\lambda)^2-2(\lambda a_\tau+y_\tau a_\lambda)^2\\
&\quad+32g_3^2y_t^2(m_{H_u}^2+m_{Q_3}^2+m_{u_3}^2+2M_3^2)+32g_3^2(a_t^2-2y_ta_tM_3)\\
&\quad+\frac{2}{5}g_1^2y_t^2(-5m_{H_u}^2+m_{Q_3}^2+16m_{u_3}^2+8M_1^2)+\frac{8}{5}g_1^2(a_t^2-2y_ta_tM_1)
\end{aligned}
$$

$$+\frac{6}{5}g_1^2y_b^2(3m_{H_d}^2-m_{Q_3}^2-2m_{d_3}^2)+\frac{6}{5}g_1^2y_\tau^2(m_{H_d}^2+m_{L_3}^2-2m_{e_3}^2)$$
$$+\frac{6}{5}g_1^2\sum_{i=1}^3[\lambda_i^2(m_{H_i^d}^2-m_{H_i^u}^2)+\kappa_i^2(m_{D_i}^2-m_{\bar{D}_i}^2)]$$
$$+12g_1'^2y_t^2(Q_Q^2+Q_u^2-Q_2^2)(m_{H_u}^2+m_{Q_3}^2+m_{u_3}^2+2M_1'^2)$$
$$+12g_1'^2(Q_Q^2+Q_u^2-Q_2^2)(a_t^2-2y_ta_tM_1')-24Q_2g_1'^2y_t^2(Q_2m_{H_u}^2+Q_Qm_{Q_3}^2+Q_um_{u_3}^2)$$
$$-24Q_2g_1'^2y_b^2(Q_1m_{H_d}^2+Q_Qm_{Q_3}^2+Q_dm_{d_3}^2)-8Q_2g_1'^2y_\tau^2(Q_1m_{H_d}^2+Q_Lm_{L_3}^2+Q_em_{e_3}^2)$$
$$+4g_1'^2\lambda^2(Q_1^2-Q_2^2+Q_S^2)(m_{H_d}^2+m_{H_u}^2+m_S^2+2M_1'^2)$$
$$+4g_1'^2(Q_1^2-Q_2^2+Q_S^2)(a_\lambda^2-2\lambda a_\lambda M_1')$$
$$-4Q_2g_1'^2\sum_{i=1}^3[2\lambda_i^2(Q_1m_{H_i^d}^2+Q_2m_{H_i^u}^2+Q_Sm_S^2)+3\kappa_i^2(Q_Sm_S^2+Q_Dm_{D_i}^2+Q_{\bar{D}}m_{\bar{D}_i}^2)]$$
$$+\frac{16}{5}g_3^2g_1^2\sum_{i=1}^3(m_{Q_i}^2-2m_{u_i}^2+m_{d_i}^2+m_{\bar{D}_i}^2-m_{D_i}^2)$$
$$+32Q_2g_3^2g_1'^2\sum_{i=1}^3(2Q_Qm_{Q_i}^2+Q_um_{u_i}^2+Q_dm_{d_i}^2+Q_{\bar{D}}m_{\bar{D}_i}^2+Q_Dm_{D_i}^2)$$
$$+3g_2^4\left[29M_2^2+m_{H'}^2+m_{\bar{H}'}^2+\sum_{i=1}^3(3m_{Q_i}^2+m_{L_i}^2+m_{H_i^d}^2+m_{H_i^u}^2)\right]$$
$$+\frac{9}{5}g_2^2g_1^2\left[2(M_1^2+M_1M_2+M_2^2)+m_{\bar{H}'}^2-m_{H'}^2+\sum_{i=1}^3(m_{Q_i}^2-m_{L_i}^2+m_{H_i^u}^2-m_{H_i^d}^2)\right]$$
$$+12Q_2g_2^2g_1'^2\left[2Q_2(M_1'^2+M_1'M_2+M_2^2)+Q_{H'}m_{H'}^2+Q_{\bar{H}'}m_{\bar{H}'}^2\right.$$
$$\left.+\sum_{i=1}^3(3Q_Qm_{Q_i}^2+Q_Lm_{L_i}^2+Q_1m_{H_i^d}^2+Q_2m_{H_i^u}^2)\right]+\frac{1}{25}g_1^4\left[891M_1^2+18m_{\bar{H}'}^2\right.$$
$$\left.+\sum_{i=1}^3(10m_{d_i}^2+2m_{D_i}^2+10m_{\bar{D}_i}^2+54m_{e_i}^2+18m_{H_i^u}^2+4m_{Q_i}^2-8m_{u_i}^2)\right]$$
$$+\frac{4}{5}g_1^2g_1'^2\left[6Q_2(3Q_d+Q_{\bar{D}}-3Q_D+3Q_e-3Q_1+4Q_2+Q_{\bar{H}'}-Q_{H'}-3Q_L\right.$$
$$+3Q_Q-6Q_u)(M_1^2+M_1M_1'+M_1'^2)+3Q_{\bar{H}'}^2m_{\bar{H}'}^2-3Q_{H'}^2m_{H'}^2$$
$$+3\sum_{i=1}^3(Q_d^2m_{d_i}^2+Q_{\bar{D}}^2m_{\bar{D}_i}^2-Q_D^2m_{D_i}^2+Q_e^2m_{e_i}^2-Q_1^2m_{H_i^d}^2+Q_2^2m_{H_i^u}^2-Q_L^2m_{L_i}^2$$
$$+Q_Q^2m_{Q_i}^2-2Q_u^2m_{u_i}^2)+9Q_2Q_{\bar{H}'}m_{\bar{H}'}^2-3Q_2Q_{H'}m_{H'}^2+Q_2\sum_{i=1}^3(8Q_dm_{d_i}^2+8Q_{\bar{D}}m_{\bar{D}_i}^2$$
$$\left.-4Q_Dm_{D_i}^2+12Q_em_{e_i}^2-3Q_1m_{H_i^d}^2+9Q_2m_{H_i^u}^2-3Q_Lm_{L_i}^2+7Q_Qm_{Q_i}^2-4Q_um_{u_i}^2)\right]$$
$$+8Q_2g_1'^4\left[3Q_2M_1'^2(9Q_d^2+9Q_{\bar{D}}^2+9Q_D^2+3Q_e^2+6Q_1^2+8Q_2^2+2Q_{\bar{H}'}^2+2Q_{H'}^2+6Q_L^2\right.$$
$$+18Q_Q^2+3Q_S^2+9Q_u^2)+2Q_{\bar{H}'}^3m_{\bar{H}'}^2+2Q_{H'}^3m_{H'}^2+\sum_{i=1}^3(3Q_d^3m_{d_i}^2+3Q_{\bar{D}}^3m_{\bar{D}_i}^2$$
$$+3Q_D^3m_{D_i}^2+Q_e^3m_{e_i}^2+2Q_1^3m_{H_i^d}^2+2Q_2^3m_{H_i^u}^2+2Q_L^3m_{L_i}^2+6Q_Q^3m_{Q_i}^2+Q_S^3m_{S_i}^2+3Q_u^3m_{u_i}^2)$$

$$+ 2Q_2 Q_{\bar{H}'}^2 m_{\bar{H}'}^2 + 2Q_2 Q_{H'}^2 m_{H'}^2 + Q_2 \sum_{i=1}^{3} (3Q_d^2 m_{d_i}^2 + 3Q_{\bar{D}}^2 m_{\bar{D}_i}^2 + 3Q_D^2 m_{D_i}^2 + Q_e^2 m_{e_i}^2 + 2Q_1^2 m_{H_i^d}^2 + 2Q_2^2 m_{H_i^u}^2 + 2Q_L^2 m_{L_i}^2 + 6Q_Q^2 m_{Q_i}^2 + Q_S^2 m_{S_i}^2 + 3Q_u^2 m_{u_i}^2)\Bigg], \quad \text{(B11b)}$$

$$\begin{aligned}
b_{m_{H_u}^2}^{(2)} ={}& 72 y_t^4 (m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2) + 144 y_t^2 a_t^2 + 8\lambda^4 (m_{H_d}^2 + m_{H_u}^2 + m_S^2) + 16\lambda^2 a_\lambda^2 \\
&+ 6 y_t^2 y_b^2 (m_{H_d}^2 + m_{H_u}^2 + 2m_{Q_3}^2 + m_{u_3}^2 + m_{d_3}^2) + 6(y_t a_b + y_b a_t)^2 \\
&+ 12\lambda^2 y_t^2 (m_{H_d}^2 + 2m_{H_u}^2 + m_S^2 + m_{Q_3}^2 + m_{u_3}^2) + 12(\lambda a_t + y_t a_\lambda)^2 \\
&+ 6\lambda^2 y_b^2 (2m_{H_d}^2 + m_{H_u}^2 + m_S^2 + m_{Q_3}^2 + m_{d_3}^2) + 6(\lambda a_b + y_b a_\lambda)^2 \\
&+ 2\lambda^2 y_\tau^2 (2m_{H_d}^2 + m_{H_u}^2 + m_S^2 + m_{L_3}^2 + m_{e_3}^2) + 2(\lambda a_\tau + y_\tau a_\lambda)^2 \\
&+ 4\sum_{i=1}^{3} [\lambda^2 \lambda_i^2 (m_{H_i^d}^2 + m_{H_i^u}^2 + m_{H_d}^2 + m_{H_u}^2 + 2m_S^2) + (\lambda a_{\lambda_i} + \lambda_i a_\lambda)^2] \\
&+ 6\sum_{i=1}^{3} [\lambda^2 \kappa_i^2 (m_{H_d}^2 + m_{H_u}^2 + 2m_S^2 + m_{D_i}^2 + m_{\bar{D}_i}^2) + (\lambda a_{\kappa_i} + \kappa_i a_\lambda)^2] \\
&- 32 g_3^2 y_t^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2 + 2M_3^2) - 32 g_3^2 (a_t^2 - 2y_t a_t M_3) \\
&- 18 g_2^2 y_t^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2 + 2M_2^2) - 18 g_2^2 (a_t^2 - 2y_t a_t M_2) \\
&- 6 g_2^2 \lambda^2 (m_{H_d}^2 + m_{H_u}^2 + m_S^2 + 2M_2^2) - 6 g_2^2 (a_\lambda^2 - 2\lambda a_\lambda M_2) \\
&- \frac{26}{5} g_1^2 y_t^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2 + 2M_1^2) - \frac{26}{5} g_1^2 (a_t^2 - 2y_t a_t M_1) \\
&- \frac{6}{5} g_1^2 \lambda^2 (m_{H_d}^2 + m_{H_u}^2 + m_S^2 + 2M_1^2) - \frac{6}{5} g_1^2 (a_\lambda^2 - 2\lambda a_\lambda M_1) \\
&- 12 g_1'^2 y_t^2 (Q_2^2 + Q_Q^2 + Q_u^2)(m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2 + 2M_1'^2) - 12 g_1'^2 (Q_2^2 + Q_Q^2 + Q_u^2) \\
&\times (a_t^2 - 2y_t a_t M_1') + 6 g_1'^2 y_t^2 (Q_2 + Q_Q + Q_u)(2Q_2 m_{H_u}^2 + 2Q_2 m_{Q_3}^2 + 2Q_2 m_{u_3}^2 + \Sigma_1') \\
&+ 12 Q_2 g_1'^2 a_t^2 (Q_2 + Q_Q + Q_u) + 12 Q_2 g_1'^2 y_b^2 (Q_1 + Q_Q + Q_d)(m_{H_d}^2 + m_{Q_3}^2 + m_{d_3}^2) \\
&+ 12 Q_2 g_1'^2 a_b^2 (Q_1 + Q_Q + Q_d) + 4 Q_2 g_1'^2 y_\tau^2 (Q_1 + Q_L + Q_e)(m_{H_d}^2 + m_{L_3}^2 + m_{e_3}^2) \\
&+ 4 Q_2 g_1'^2 a_\tau^2 (Q_1 + Q_L + Q_e) - 4 g_1'^2 \lambda^2 (Q_1^2 + Q_2^2 + Q_S^2)(m_{H_d}^2 + m_{H_u}^2 + m_S^2 + 2M_1'^2) \\
&- 4 g_1'^2 (Q_1^2 + Q_2^2 + Q_S^2)(a_\lambda^2 - 2\lambda a_\lambda M_1') + 2 g_1'^2 \lambda^2 (Q_1 + Q_2 + Q_S)\Sigma_1' \\
&+ 4 Q_2 g_1'^2 (Q_1 + Q_2 + Q_S) \sum_{i=1}^{3} [\lambda_i^2 (m_{H_i^d}^2 + m_{H_i^u}^2 + m_S^2) + a_{\lambda_i}^2] \\
&+ 6 Q_2 g_1'^2 (Q_S + Q_D + Q_{\bar{D}}) \sum_{i=1}^{3} [\kappa_i^2 (m_S^2 + m_{D_i}^2 + m_{\bar{D}_i}^2) + a_{\kappa_i}^2] \\
&- 96 Q_2 g_3^2 g_1'^2 M_3^2 (2Q_Q + Q_u + Q_d + Q_D + Q_{\bar{D}}) - 72 g_2^4 M_2^2 \\
&- 12 Q_2 g_2^2 g_1'^2 M_2^2 (9Q_Q + 3Q_L + 3Q_1 + 3Q_2 + Q_{\bar{H}'} + Q_{H'}) + \frac{288}{25} g_1^4 (\Sigma_1 - 3M_1^2) \\
&- \frac{3}{5} g_1^2 g_1'^2 [4Q_2 M_1^2 (2Q_d + 2Q_{\bar{D}} + 2Q_D + 6Q_e + 3Q_1 + 3Q_2 + Q_{\bar{H}'} + Q_{H'} + 3Q_L \\
&+ Q_Q + 8Q_u) + 4M_1'^2 (3Q_d^2 + 3Q_{\bar{D}}^2 - 3Q_D^2 + 3Q_e^2 - 3Q_1^2 + 3Q_2^2 + Q_{\bar{H}'}^2 - Q_{H'}^2 \\
&- 3Q_L^2 + 3Q_Q^2 - 6Q_u^2) - (2Q_2 \Sigma_1 + \Sigma_1')\Sigma_Q^Y] - 4 Q_2 g_1'^4 [2M_1'^2 (9Q_d^3 + 9Q_{\bar{D}}^3 + 9Q_D^3 \\
&+ 3Q_e^3 + 6Q_1^3 + 6Q_2^3 + 2Q_{\bar{H}'}^3 + 2Q_{H'}^3 + 6Q_L^3 + 18Q_Q^3 + 3Q_S^3 + 9Q_u^3) + (6Q_2 M_1'^2 - \Sigma_1')\Sigma_Q],
\end{aligned} \quad \text{(B11c)}$$

while those for m_S^2 are

$$\beta_{m_S^2}^{(1)} = \sum_{i=1}^{3} [4\lambda_i^2(m_{H_i^d}^2 + m_{H_i^u}^2 + m_S^2) + 4a_{\lambda_i}^2 + 6\kappa_i^2(m_S^2 + m_{D_i}^2 + m_{\bar{D}_i}^2) + 6a_{\kappa_i}^2] - 8Q_S^2 g_1'^2 M_1'^2 + 2Q_S g_1'^2 \Sigma_1', \tag{B12a}$$

$$\begin{aligned}
\beta_{m_S^2}^{(2)} = {} & \sum_{i=1}^{3}[-16\lambda_i^4(m_{H_i^u}^2 + m_{H_i^d}^2 + m_S^2) - 24\kappa_i^4(m_{D_i}^2 + m_{\bar{D}_i}^2 + m_S^2) \\
& - 32\lambda_i^2 a_{\lambda_i}^2 - 48\kappa_i^2 a_{\kappa_i}^2] - 12\lambda^2 y_t^2(2m_{H_u}^2 + m_{H_d}^2 + m_S^2 + m_{Q_3}^2 + m_{u_3}^2) \\
& - 12\lambda^2 y_b^2(m_{H_u}^2 + 2m_{H_d}^2 + m_S^2 + m_{Q_3}^2 + m_{d_3}^2) \\
& - 4\lambda^2 y_\tau^2(m_{H_u}^2 + 2m_{H_d}^2 + m_S^2 + m_{L_3}^2 + m_{e_3}^2) - 12(\lambda a_t + y_t a_\lambda)^2 - 12(\lambda a_b + y_b a_\lambda)^2 \\
& - 4(\lambda a_\tau + y_\tau a_\lambda)^2 + 32g_3^2\sum_{i=1}^{3}[\kappa_i^2(m_S^2 + m_{D_i}^2 + m_{\bar{D}_i}^2 + 2M_3^2) + a_{\kappa_i}^2 - 2\kappa_i a_{\kappa_i} M_3] \\
& + 12g_2^2\sum_{i=1}^{3}[\lambda_i^2(m_{H_i^d}^2 + m_{H_i^u}^2 + m_S^2 + 2M_2^2) + a_{\lambda_i}^2 - 2\lambda_i a_{\lambda_i} M_2] \\
& + \frac{4}{5}g_1^2\sum_{i=1}^{3}[3\lambda_i^2(m_{H_i^d}^2 + m_{H_i^u}^2 + m_S^2 + 2M_1^2) + 2\kappa_i^2(m_S^2 + m_{D_i}^2 + m_{\bar{D}_i}^2 + 2M_1^2) \\
& + 3(a_{\lambda_i}^2 - 2\lambda_i a_{\lambda_i} M_1) + 2(a_{\kappa_i}^2 - 2\kappa_i a_{\kappa_i} M_1)] \\
& + 4g_1'^2\sum_{i=1}^{3}[2\lambda_i^2(Q_1^2 + Q_2^2 - Q_S^2)(m_{H_i^d}^2 + m_{H_i^u}^2 + m_S^2 + 2M_1'^2) \\
& - 2Q_S\lambda_i^2(Q_1 m_{H_i^d}^2 + Q_2 m_{H_i^u}^2 + Q_S m_S^2) + 3\kappa_i^2(Q_D^2 + Q_{\bar{D}}^2 - Q_S^2) \\
& \times (m_S^2 + m_{D_i}^2 + m_{\bar{D}_i}^2 + 2M_1'^2) - 3Q_S\kappa_i^2(Q_S m_S^2 + Q_D m_{D_i}^2 + Q_{\bar{D}} m_{\bar{D}_i}^2) \\
& + 2(a_{\lambda_i}^2 - 2\lambda_i a_{\lambda_i} M_1')(Q_1^2 + Q_2^2 - Q_S^2) + 3(a_{\kappa_i}^2 - 2\kappa_i a_{\kappa_i} M_1')(Q_D^2 + Q_{\bar{D}}^2 - Q_S^2)] \\
& - 24Q_S g_1'^2 y_t^2(Q_2 m_{H_u}^2 + Q_Q m_{Q_3}^2 + Q_u m_{u_3}^2) - 24Q_S g_1'^2 y_b^2(Q_1 m_{H_d}^2 + Q_Q m_{Q_3}^2 + Q_d m_{d_3}^2) \\
& - 8Q_S g_1'^2 y_\tau^2(Q_1 m_{H_d}^2 + Q_L m_{L_3}^2 + Q_e m_{e_3}^2) + 32Q_S g_3^2 g_1'^2\sum_{i=1}^{3}(2Q_Q m_{Q_i}^2 + Q_u m_{u_i}^2 \\
& + Q_d m_{d_i}^2 + Q_D m_{D_i}^2 + Q_{\bar{D}} m_{\bar{D}_i}^2) + 12Q_S g_2^2 g_1'^2\Bigg[Q_{\bar{H}'} m_{\bar{H}'}^2 + Q_{H'} m_{H'}^2 \\
& + \sum_{i=1}^{3}(3Q_Q m_{Q_i}^2 + Q_L m_{L_i}^2 + Q_1 m_{H_i^d}^2 + Q_2 m_{H_i^u}^2)\Bigg] + \frac{4}{5}Q_S g_1^2 g_1'^2\Bigg[3Q_{\bar{H}'} m_{\bar{H}'}^2 + 3Q_{H'} m_{H'}^2 \\
& + \sum_{i=1}^{3}(2Q_d m_{d_i}^2 + 2Q_{\bar{D}} m_{\bar{D}_i}^2 + 2Q_D m_{D_i}^2 + 6Q_e m_{e_i}^2 + 3Q_1 m_{H_i^d}^2 + 3Q_2 m_{H_i^u}^2 + 3Q_L m_{L_i}^2 \\
& + Q_Q m_{Q_i}^2 + 8Q_u m_{u_i}^2)\Bigg] + 8Q_S g_1'^4\Bigg[3Q_S M_1'^2(9Q_d^2 + 9Q_{\bar{D}}^2 + 9Q_D^2 + 3Q_e^2 + 6Q_1^2 + 6Q_2^2 \\
& + 2Q_{\bar{H}'}^2 + 2Q_{H'}^2 + 6Q_L^2 + 18Q_Q^2 + 5Q_S^2 + 9Q_u^2) + 2Q_{\bar{H}'}^3 m_{\bar{H}'}^2 + 2Q_{H'}^3 m_{H'}^2 \\
& + \sum_{i=1}^{3}(3Q_d^3 m_{d_i}^2 + 3Q_{\bar{D}}^3 m_{\bar{D}_i}^2 + 3Q_D^3 m_{D_i}^2 + Q_e^3 m_{e_i}^2 + 2Q_1^3 m_{H_i^d}^2 + 2Q_2^3 m_{H_i^u}^2 + 2Q_L^3 m_{L_i}^2 \\
& + 6Q_Q^3 m_{Q_i}^2 + Q_S^3 m_{S_i}^2 + 3Q_u^3 m_{u_i}^2) + 2Q_S Q_{\bar{H}'}^2 m_{\bar{H}'}^2 + 2Q_S Q_{H'}^2 m_{H'}^2
\end{aligned}$$

$$+ Q_S \sum_{i=1}^{3} (3Q_d^2 m_{d_i}^2 + 3Q_{\bar{D}}^2 m_{\bar{D}_i}^2 + 3Q_D^2 m_{D_i}^2 + Q_e^2 m_{e_i}^2 + 2Q_1^2 m_{H_i^d}^2 + 2Q_2^2 m_{H_i^u}^2 + 2Q_L^2 m_{L_i}^2 + 6Q_Q^2 m_{Q_i}^2 + Q_S^2 m_{S_i}^2 + 3Q_u^2 m_{u_i}^2)\Bigg], \quad \text{(B12b)}$$

$$\begin{aligned}
b_{m_S^2}^{(2)} = {} & 8\sum_{i=1}^{3}[2\lambda_i^4(m_{H_i^d}^2 + m_{H_i^u}^2 + m_S^2) + 4\lambda_i^2 a_{\lambda_i}^2 + 3\kappa_i^4(m_S^2 + m_{D_i}^2 + m_{\bar{D}_i}^2) + 6\kappa_i^2 a_{\kappa_i}^2] \\
& + 8\sum_{i=1}^{3}\sum_{j=1}^{3}[\lambda_i^2\lambda_j^2(m_{H_i^d}^2 + m_{H_i^u}^2 + m_{H_j^d}^2 + m_{H_j^u}^2 + 2m_S^2) + (\lambda_i a_{\lambda_j} + \lambda_j a_{\lambda_i})^2] \\
& + 24\sum_{i=1}^{3}\sum_{j=1}^{3}[\lambda_i^2\kappa_j^2(m_{H_i^d}^2 + m_{H_i^u}^2 + 2m_S^2 + m_{D_i}^2 + m_{\bar{D}_i}^2) + (\lambda_i a_{\kappa_j} + \kappa_j a_{\lambda_i})^2] \\
& + 18\sum_{i=1}^{3}\sum_{j=1}^{3}[\kappa_i^2\kappa_j^2(2m_S^2 + m_{D_i}^2 + m_{\bar{D}_i}^2 + m_{D_j}^2 + m_{\bar{D}_j}^2) + (\kappa_i a_{\kappa_j} + \kappa_j a_{\kappa_i})^2] \\
& + 12\lambda^2 y_t^2(2m_{H_u}^2 + m_{H_d}^2 + m_S^2 + m_{Q_3}^2 + m_{u_3}^2) + 12\lambda^2 y_b^2(m_{H_u}^2 + 2m_{H_d}^2 + m_S^2 \\
& + m_{Q_3}^2 + m_{d_3}^2) + 4\lambda^2 y_\tau^2(2m_{H_d}^2 + m_{H_u}^2 + m_S^2 + m_{L_3}^2 + m_{e_3}^2) + 12(\lambda a_t + y_t a_\lambda)^2 \\
& + 12(\lambda a_b + y_b a_\lambda)^2 + 4(\lambda a_\tau + y_\tau a_\lambda)^2 - 32g_3^2\sum_{i=1}^{3}[\kappa_i^2(m_S^2 + m_{D_i}^2 + m_{\bar{D}_i}^2 + 2M_3^2) \\
& + a_{\kappa_i}^2 - 2\kappa_i a_{\kappa_i} M_3] - 12g_2^2\sum_{i=1}^{3}[\lambda_i^2(m_{H_i^d}^2 + m_{H_i^u}^2 + m_S^2 + 2M_2^2) + a_{\lambda_i}^2 - 2\lambda_i a_{\lambda_i} M_2] \\
& - \frac{4}{5}g_1^2\sum_{i=1}^{3}[3\lambda_i^2(m_{H_i^d}^2 + m_{H_i^u}^2 + m_S^2 + 2M_1^2) + 3a_{\lambda_i}^2 - 6\lambda_i a_{\lambda_i} M_1 \\
& + 2\kappa_i^2(m_S^2 + m_{D_i}^2 + m_{\bar{D}_i}^2 + 2M_1^2) + 2a_{\kappa_i}^2 - 4\kappa_i a_{\kappa_i} M_1] \\
& + 2g_1'^2\sum_{i=1}^{3}[-4\lambda_i^2(Q_1^2 + Q_2^2 + Q_S^2)(m_{H_i^d}^2 + m_{H_i^u}^2 + m_S^2 + 2M_1'^2) \\
& + 2\lambda_i^2(Q_1 + Q_2 + Q_S)(Q_S m_{H_i^d}^2 + Q_S m_{H_i^u}^2 + Q_S m_S^2 + \Sigma_1') - 4(Q_1^2 + Q_2^2 + Q_S^2) \\
& \times (a_{\lambda_i}^2 - 2\lambda_i a_{\lambda_i} M_1') + 2Q_S a_{\lambda_i}^2(Q_1 + Q_2 + Q_S) - 6\kappa_i^2(Q_S^2 + Q_D^2 + Q_{\bar{D}}^2) \\
& \times (m_S^2 + m_{D_i}^2 + m_{\bar{D}_i}^2 + 2M_1'^2) + 3\kappa_i^2(Q_S + Q_D + Q_{\bar{D}})(Q_S m_S^2 + Q_S m_{D_i}^2 + Q_S m_{\bar{D}_i}^2 + \Sigma_1') \\
& - 6(Q_S^2 + Q_D^2 + Q_{\bar{D}}^2)(a_{\kappa_i}^2 - 2\kappa_i a_{\kappa_i} M_1') + 3Q_S a_{\kappa_i}^2(Q_S + Q_D + Q_{\bar{D}})] \\
& + 12Q_S g_1'^2 y_t^2(Q_2 + Q_Q + Q_u)(m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2) + 12Q_S g_1'^2 a_t^2(Q_2 + Q_Q + Q_u) \\
& + 12Q_S g_1'^2 y_b^2(Q_1 + Q_Q + Q_d)(m_{H_d}^2 + m_{Q_3}^2 + m_{d_3}^2) + 12Q_S g_1'^2 a_b^2(Q_1 + Q_Q + Q_d) \\
& + 4Q_S g_1'^2 y_\tau^2(Q_1 + Q_L + Q_e)(m_{H_d}^2 + m_{L_3}^2 + m_{e_3}^2) + 4Q_S g_1'^2 a_\tau^2(Q_1 + Q_L + Q_e) \\
& - 96Q_S g_3^2 g_1'^2 M_3^2(2Q_Q + Q_u + Q_d + Q_D + Q_{\bar{D}}) - 12Q_S g_2^2 g_1'^2 M_2^2(9Q_Q + 3Q_L + 3Q_1 \\
& + 3Q_2 + Q_{\bar{H}'} + Q_{H'}) - \frac{6}{5}Q_S g_1^2 g_1'^2[2M_1^2(2Q_d + 2Q_{\bar{D}} + 2Q_D + 6Q_e + 3Q_1 + 3Q_2 + Q_{\bar{H}'} \\
& + Q_{H'} + 3Q_L + Q_Q + 8Q_u) - \Sigma_1\Sigma_Q^Y] - 4Q_S g_1'^4[2M_1'^2(9Q_d^3 + 9Q_{\bar{D}}^3 + 9Q_D^3 + 3Q_e^3 + 6Q_1^3 \\
& + 6Q_2^3 + 2Q_{\bar{H}'}^3 + 2Q_{H'}^3 + 6Q_L^3 + 18Q_Q^3 + 3Q_S^3 + 9Q_u^3) + (6Q_S M_1'^2 - \Sigma_1')\Sigma_Q]. \quad \text{(B12c)}
\end{aligned}$$

If the one-loop contributions to the effective potential from top and stop loops are also included, it is necessary to consider the expansions for y_t, a_t, $m_{Q_3}^2$ and $m_{u_3}^2$. The required expressions for y_t read

$$\beta_{y_t}^{(1)} = y_t\left[\lambda^2 + 6y_t^2 + y_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 - 2g_1'^2(Q_2^2 + Q_Q^2 + Q_u^2)\right], \tag{B13a}$$

$$\begin{aligned}\beta_{y_t}^{(2)} = y_t\Bigg\{&-22y_t^4 - 5y_b^4 - 5y_t^2y_b^2 - y_b^2y_\tau^2 - \lambda^2(\lambda^2 + 3y_t^2 + 4y_b^2 + y_\tau^2 + 2\Sigma_\lambda + 3\Sigma_\kappa)\\
&+ 2g_1'^2[\lambda^2(Q_1^2 - Q_2^2 + Q_S^2) + 2y_t^2(2Q_Q^2 + Q_u^2) + y_b^2(Q_1^2 - Q_Q^2 + Q_d^2)]\\
&+ 16g_3^2y_t^2 + 6g_2^2y_t^2 + g_1^2\left(\frac{6}{5}y_t^2 + \frac{2}{5}y_b^2\right) + \frac{128}{9}g_3^4 + \frac{33}{2}g_2^4 + \frac{3913}{450}g_1^4\\
&+ 2g_1'^4[2(Q_2^4 + Q_Q^4 + Q_u^4) + (Q_2^2 + Q_Q^2 + Q_u^2)\Sigma_Q] + 8g_3^2g_2^2 + \frac{136}{45}g_3^2g_1^2\\
&+ \frac{32}{3}g_3^2g_1'^2(Q_Q^2 + Q_u^2) + g_2^2g_1^2 + 6g_2^2g_1'^2(Q_2^2 + Q_Q^2)\\
&+ \frac{2}{5}g_1^2g_1'^2\left[3Q_2^2 + \frac{1}{3}Q_Q^2 + \frac{16}{3}Q_u^2 + (3Q_2 + Q_Q - 4Q_u)\Sigma_Q^Y\right]\Bigg\},\end{aligned} \tag{B13b}$$

$$\begin{aligned}b_{y_t}^{(2)} = y_t\Bigg\{&54y_t^4 + \frac{13}{2}y_b^4 + 13y_t^2y_b^2 + y_b^2y_\tau^2 + \lambda^2\left(\frac{5}{2}\lambda^2 + 15y_t^2 + 5y_b^2 + y_\tau^2 + 2\Sigma_\lambda + 3\Sigma_\kappa\right)\\
&- \frac{16}{3}g_3^2(\lambda^2 + 2y_b^2 + 12y_t^2) - 6g_2^2(\lambda^2 + y_b^2 + 6y_t^2) - g_1^2\left(\frac{22}{15}\lambda^2 + \frac{4}{3}y_b^2 + \frac{52}{5}y_t^2\right)\\
&- 2g_1'^2[\lambda^2(Q_1^2 + 2Q_2^2 + Q_S^2 + Q_Q^2 + Q_u^2) + y_b^2(Q_1^2 + Q_2^2 + 2Q_Q^2 + Q_u^2 + Q_d^2)\\
&+12y_t^2(Q_2^2 + Q_Q^2 + Q_u^2)] + \frac{128}{9}g_3^4 - \frac{15}{2}g_2^4 - \frac{143}{18}g_1^4\\
&+ 2g_1'^4(Q_2^2 + Q_Q^2 + Q_u^2)(Q_2^2 + Q_Q^2 + Q_u^2 - \Sigma_Q) + 16g_3^2g_2^2 + \frac{208}{45}g_3^2g_1^2\\
&+ \frac{32}{3}g_3^2g_1'^2(Q_2^2 + Q_Q^2 + Q_u^2) + \frac{13}{5}g_2^2g_1^2 + 6g_2^2g_1'^2(Q_2^2 + Q_Q^2 + Q_u^2)\\
&+ \frac{26}{15}g_1^2g_1'^2(Q_2^2 + Q_Q^2 + Q_u^2)\Bigg\},\end{aligned} \tag{B13c}$$

and those for a_t read

$$\begin{aligned}\beta_{a_t}^{(1)} = a_t&\left[\lambda^2 + 6y_t^2 + y_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 - 2g_1'^2(Q_2^2 + Q_Q^2 + Q_u^2)\right]\\
&+ y_t\Bigg[2\lambda a_\lambda + 12y_ta_t + 2y_ba_b + \frac{32}{3}g_3^2M_3 + 6g_2^2M_2 + \frac{26}{15}g_1^2M_1\\
&+ 4g_1'^2M_1'(Q_2^2 + Q_Q^2 + Q_u^2)\Bigg],\end{aligned} \tag{B14a}$$

$$\begin{aligned}\beta_{a_t}^{(2)} = a_t\Bigg\{&-22y_t^4 - 5y_b^4 - 5y_t^2y_b^2 - y_b^2y_\tau^2 - \lambda^2(\lambda^2 + 3y_t^2 + 4y_b^2 + y_\tau^2 + 2\Sigma_\lambda + 3\Sigma_\kappa)\\
&+ 2g_1'^2[\lambda^2(Q_1^2 - Q_2^2 + Q_S^2) + 2y_t^2(2Q_Q^2 + Q_u^2) + y_b^2(Q_1^2 - Q_Q^2 + Q_d^2)]\\
&+ 16g_3^2y_t^2 + 6g_2^2y_t^2 + g_1^2\left(\frac{6}{5}y_t^2 + \frac{2}{5}y_b^2\right) + \frac{128}{9}g_3^4 + \frac{33}{2}g_2^4 + \frac{3913}{450}g_1^4\\
&+ 2g_1'^4[2(Q_2^4 + Q_Q^4 + Q_u^4) + (Q_2^2 + Q_Q^2 + Q_u^2)\Sigma_Q] + 8g_3^2g_2^2 + \frac{136}{45}g_3^2g_1^2\\
&+ \frac{32}{3}g_3^2g_1'^2(Q_Q^2 + Q_u^2) + g_2^2g_1^2 + 6g_2^2g_1'^2(Q_2^2 + Q_Q^2)\end{aligned}$$

$$
\begin{aligned}
&+\frac{2}{5}g_1^2g_1'^2\left[3Q_2^2+\frac{1}{3}Q_Q^2+\frac{16}{3}Q_u^2\right]\Bigg\}+y_t\Bigg\{-88y_t^3a_t-20y_b^3a_b-10y_ty_b(y_ba_t+y_ta_b)\\
&-2y_by_\tau(y_ba_\tau+y_\tau a_b)-2\lambda a_\lambda(2\lambda^2+3y_t^2+4y_b^2+y_\tau^2+2\Sigma_\lambda+3\Sigma_\kappa)\\
&-2\lambda^2(3y_ta_t+4y_ba_b+y_\tau a_\tau+2\Sigma_{a_\lambda}+3\Sigma_{a_\kappa})+32g_3^2y_t(a_t-y_tM_3)\\
&+12g_2^2y_t(a_t-y_tM_2)+\frac{2}{5}g_1^2[6y_ta_t+2y_ba_b-(6y_t^2+2y_b^2)M_1]\\
&+4g_1'^2[\lambda(Q_1^2-Q_2^2+Q_S^2)(a_\lambda-\lambda M_1')+2y_t(2Q_Q^2+Q_u^2)(a_t-y_tM_1')\\
&+y_b(Q_1^2-Q_Q^2+Q_d^2)(a_b-y_bM_1')]-\frac{512}{9}g_3^4M_3-66g_2^4M_2-\frac{7826}{225}g_1^4M_1\\
&-8g_1'^4M_1'[2(Q_2^4+Q_Q^4+Q_u^4)+(Q_2^2+Q_Q^2+Q_u^2)\Sigma_Q]-16g_3^2g_2^2(M_3+M_2)\\
&-\frac{272}{45}g_3^2g_1^2(M_3+M_1)-\frac{64}{3}g_3^2g_1'^2(Q_Q^2+Q_u^2)(M_3+M_1')-2g_2^2g_1^2(M_2+M_1)\\
&-12g_2^2g_1'^2(Q_2^2+Q_Q^2)(M_2+M_1')-\frac{4}{15}g_1^2g_1'^2(9Q_2^2+Q_Q^2+16Q_u^2)(M_1+M_1')\Bigg\},
\end{aligned}
\tag{B14b}
$$

$$
\begin{aligned}
b_{a_t}^{(2)}={}&y_t\left[2\lambda a_\lambda+12y_ta_t+2y_ba_b+\frac{32}{3}g_3^2M_3+6g_2^2M_2+\frac{26}{15}g_1^2M_1\right.\\
&\left.+4g_1'^2M_1'(Q_2^2+Q_Q^2+Q_u^2)\right]\times\left[\lambda^2+6y_t^2+y_b^2-\frac{16}{3}g_3^2-3g_2^2-\frac{13}{15}g_1^2\right.\\
&\left.-2g_1'^2(Q_2^2+Q_Q^2+Q_u^2)\right]+a_t\Bigg\{54y_t^4+\frac{13}{2}y_b^4+13y_t^2y_b^2+y_b^2y_\tau^2\\
&+\lambda^2\left(\frac{5}{2}\lambda^2+15y_t^2+5y_b^2+y_\tau^2+2\Sigma_\lambda+3\Sigma_\kappa\right)-\frac{16}{3}g_3^2(\lambda^2+2y_b^2+12y_t^2)\\
&-6g_2^2(\lambda^2+y_b^2+6y_t^2)-g_1^2\left(\frac{22}{15}\lambda^2+\frac{4}{3}y_b^2+\frac{52}{5}y_t^2\right)\\
&-2g_1'^2[\lambda^2(Q_1^2+2Q_2^2+Q_S^2+Q_Q^2+Q_u^2)+y_b^2(Q_1^2+Q_2^2+2Q_Q^2+Q_u^2+Q_d^2)\\
&+12y_t^2(Q_2^2+Q_Q^2+Q_u^2)]+\frac{128}{9}g_3^4-\frac{15}{2}g_2^4-\frac{143}{18}g_1^4\\
&+2g_1'^4(Q_2^2+Q_Q^2+Q_u^2)(Q_2^2+Q_Q^2+Q_u^2-\Sigma_Q)+16g_3^2g_2^2+\frac{208}{45}g_3^2g_1^2\\
&+\frac{32}{3}g_3^2g_1'^2(Q_2^2+Q_Q^2+Q_u^2)+\frac{13}{5}g_2^2g_1^2+6g_2^2g_1'^2(Q_2^2+Q_Q^2+Q_u^2)\\
&+\frac{26}{15}g_1^2g_1'^2(Q_2^2+Q_Q^2+Q_u^2)\Bigg\}+y_t\Bigg[144y_t^3a_t+24y_b^3a_b+14y_ty_b(y_ta_b+a_ty_b)\\
&+2y_by_\tau(y_ba_\tau+a_by_\tau)+18\lambda y_t(\lambda a_t+a_\lambda y_t)+8\lambda y_b(\lambda a_b+a_\lambda y_b)+2\lambda y_\tau(\lambda a_\tau+a_\lambda y_\tau)\\
&+8\lambda^3a_\lambda+4\lambda\sum_{i=1}^3\lambda_i(\lambda a_{\lambda_i}+a_\lambda\lambda_i)+6\lambda\sum_{i=1}^3\kappa_i(\lambda a_{\kappa_i}+a_\lambda\kappa_i)-64g_3^2y_t(a_t-y_tM_3)\\
&-\frac{32}{3}g_3^2y_b(a_b-y_bM_3)-36g_2^2y_t(a_t-y_tM_2)-6g_2^2y_b(a_b-y_bM_2)-6g_2^2\lambda(a_\lambda-\lambda M_2)\\
&-\frac{52}{5}g_1^2y_t(a_t-y_tM_1)-\frac{14}{15}g_1^2y_b(a_b-y_bM_1)-\frac{18}{15}g_1^2\lambda(a_\lambda-\lambda M_1)\\
&-24g_1'^2y_t(Q_2^2+Q_Q^2+Q_u^2)(a_t-y_tM_1')-4g_1'^2y_b(Q_1^2+Q_Q^2+Q_d^2)(a_b-y_bM_1')\\
&-4g_1'^2\lambda(Q_1^2+Q_2^2+Q_S^2)(a_\lambda-\lambda M_1')+48g_2^4M_2+\frac{832}{25}g_1^4M_1+8g_1'^4M_1'\Sigma_Q(Q_2^2+Q_Q^2+Q_u^2)\Bigg].
\end{aligned}
\tag{B14c}
$$

The one- and two-loop β functions and the resulting $O(t^2)$ coefficient for $m_{Q_3}^2$ are

$$\beta_{m_{Q_3}^2}^{(1)} = 2y_t^2(m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2) + 2a_t^2 + 2y_b^2(m_{H_d}^2 + m_{Q_3}^2 + m_{d_3}^2) + 2a_b^2 - \frac{32}{3}g_3^2M_3^2 - 6g_2^2M_2^2 - \frac{2}{15}g_1^2M_1^2 - 8Q_Q^2g_1'^2M_1'^2 + \frac{1}{5}g_1^2\Sigma_1 + 2Q_Qg_1'^2\Sigma_1', \tag{B15a}$$

$$\begin{aligned}\beta_{m_{Q_3}^2}^{(2)} = &-20y_t^4(m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2) - 20y_b^4(m_{H_d}^2 + m_{Q_3}^2 + m_{d_3}^2)\\
&- 2y_\tau^2y_b^2(2m_{H_d}^2 + m_{Q_3}^2 + m_{L_3}^2 + m_{d_3}^2 + m_{e_3}^2) - 40y_t^2a_t^2 - 40y_b^2a_b^2\\
&- 2(y_ba_\tau + y_\tau a_b)^2 - 2\lambda^2y_t^2(2m_{H_u}^2 + m_{H_d}^2 + m_S^2 + m_{Q_3}^2 + m_{u_3}^2)\\
&- 2\lambda^2y_b^2(m_{H_u}^2 + 2m_{H_d}^2 + m_S^2 + m_{Q_3}^2 + m_{d_3}^2) - 2(\lambda a_t + y_ta_\lambda)^2 - 2(\lambda a_b + y_ba_\lambda)^2\\
&+ \frac{2}{5}g_1^2y_t^2(m_{H_u}^2 + 3m_{Q_3}^2 + 8m_{u_3}^2 + 8M_1^2) + \frac{8}{5}g_1^2(a_t^2 - 2y_ta_tM_1)\\
&+ \frac{2}{5}g_1^2y_b^2(5m_{H_d}^2 + m_{Q_3}^2 + 4M_1^2) + \frac{4}{5}g_1^2(a_b^2 - 2y_ba_bM_1) + \frac{2}{5}g_1^2y_\tau^2(m_{H_d}^2 + m_{L_3}^2 - 2m_{e_3}^2)\\
&+ \frac{2}{5}g_1^2\sum_{i=1}^{3}[\kappa_i^2(m_{D_i}^2 - m_{\bar{D}_i}^2) + \lambda_i^2(m_{H_i^d}^2 - m_{H_i^u}^2)]\\
&+ 4g_1'^2y_t^2(Q_2^2 - Q_Q^2 + Q_u^2)(m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2 + 2M_1'^2)\\
&+ 4g_1'^2(Q_2^2 - Q_Q^2 + Q_u^2)(a_t^2 - 2y_ta_tM_1') - 24Q_Qg_1'^2y_t^2(Q_2m_{H_u}^2 + Q_Qm_{Q_3}^2 + Q_um_{u_3}^2)\\
&+ 4g_1'^2y_b^2(Q_1^2 - Q_Q^2 + Q_d^2)(m_{H_d}^2 + m_{Q_3}^2 + m_{d_3}^2 + 2M_1'^2)\\
&+ 4g_1'^2(Q_1^2 - Q_Q^2 + Q_d^2)(a_b^2 - 2y_ba_bM_1') - 24Q_Qg_1'^2y_b^2(Q_1m_{H_d}^2 + Q_Qm_{Q_3}^2 + Q_dm_{d_3}^2)\\
&- 8Q_Qg_1'^2y_\tau^2(Q_1m_{H_d}^2 + Q_Lm_{L_3}^2 + Q_em_{e_3}^2)\\
&- 4Q_Qg_1'^2\sum_{i=1}^{3}[3\kappa_i^2(Q_Sm_S^2 + Q_Dm_{D_i}^2 + Q_{\bar{D}}m_{\bar{D}_i}^2) + 2\lambda_i^2(Q_1m_{H_i^d}^2 + Q_2m_{H_i^u}^2 + Q_Sm_S^2)]\\
&+ \frac{16}{3}g_3^4\left[10M_3^2 + \sum_{i=1}^{3}(2m_{Q_i}^2 + m_{u_i}^2 + m_{d_i}^2 + m_{D_i}^2 + m_{\bar{D}_i}^2)\right] + 32g_3^2g_2^2(M_2^2 + M_2M_3 + M_3^2)\\
&+ \frac{16}{45}g_3^2g_1^2\left[2(M_1^2 + M_1M_3 + M_3^2) + 3\sum_{i=1}^{3}(m_{Q_i}^2 + m_{d_i}^2 - 2m_{u_i}^2 - m_{D_i}^2 + m_{\bar{D}_i}^2)\right]\\
&+ \frac{32}{3}Q_Qg_3^2g_1'^2\left[4Q_Q(M_1'^2 + M_1'M_3 + M_3^2) + 3\sum_{i=1}^{3}(2Q_Qm_{Q_i}^2 + Q_um_{u_i}^2 + Q_dm_{d_i}^2\right.\\
&\left. + Q_Dm_{D_i}^2 + Q_{\bar{D}}m_{\bar{D}_i}^2)\right] + 3g_2^4\left[29M_2^2 + m_{\bar{H}'}^2 + m_{H'}^2 + \sum_{i=1}^{3}(3m_{Q_i}^2 + m_{L_i}^2 + m_{H_i^d}^2 + m_{H_i^u}^2)\right]\\
&+ \frac{1}{5}g_2^2g_1^2\left[2(M_1^2 + M_1M_2 + M_2^2) + 3m_{\bar{H}'}^2 - 3m_{H'}^2 + 3\sum_{i=1}^{3}(m_{Q_i}^2 - m_{L_i}^2 + m_{H_i^u}^2 - m_{H_i^d}^2)\right]\\
&+ 12Q_Qg_2^2g_1'^2\left[2Q_Q(M_1'^2 + M_1'M_2 + M_2^2) + Q_{\bar{H}'}m_{\bar{H}'}^2 + Q_{H'}m_{H'}^2\right.\\
&\left. + \sum_{i=1}^{3}(3Q_Qm_{Q_i}^2 + Q_Lm_{L_i}^2 + Q_1m_{H_i^d}^2 + Q_2m_{H_i^u}^2)\right] + \frac{1}{75}g_1^4\left[289M_1^2 + 12m_{\bar{H}'}^2 - 6m_{H'}^2\right.\\
&\left. + \sum_{i=1}^{3}(6m_{d_i}^2 - 2m_{D_i}^2 + 6m_{\bar{D}_i}^2 + 42m_{e_i}^2 - 6m_{H_i^d}^2 + 12m_{H_i^u}^2 - 6m_{L_i}^2 + 2m_{Q_i}^2 - 24m_{u_i}^2)\right]\end{aligned}$$

$$
\begin{aligned}
&+\frac{4}{15}g_1^2g_1'^2\Big[2Q_Q(9Q_d+9Q_{\bar{D}}-9Q_D+9Q_e-9Q_1+9Q_2+3Q_{\bar{H}'}-3Q_{H'}-9Q_L\\
&+10Q_Q-18Q_u)(M_1^2+M_1M_1'+M_1'^2)+3Q_{\bar{H}'}^2m_{\bar{H}'}^2-3Q_{H'}^2m_{H'}^2\\
&+3\sum_{i=1}^{3}(Q_d^2m_{d_i}^2+Q_{\bar{D}}^2m_{\bar{D}_i}^2-Q_D^2m_{D_i}^2+Q_e^2m_{e_i}^2-Q_1^2m_{H_i^d}^2+Q_2^2m_{H_i^u}^2\\
&-Q_L^2m_{L_i}^2-Q_Q^2m_{Q_i}^2-2Q_u^2m_{u_i}^2)+15Q_QQ_{\bar{H}'}m_{\bar{H}'}^2+3Q_QQ_{H'}m_{H'}^2\\
&+Q_Q\sum_{i=1}^{3}(12Q_dm_{d_i}^2+12Q_{\bar{D}}m_{\bar{D}_i}^2+24Q_em_{e_i}^2+3Q_1m_{H_i^d}^2+15Q_2m_{H_i^u}^2+3Q_Lm_{L_i}^2\\
&+15Q_Qm_{Q_i}^2+12m_{u_i}^2)\Big]+8Q_Qg_1'^4\Big[3Q_QM_1'^2(9Q_d^2+9Q_{\bar{D}}^2+9Q_D^2+3Q_e^2+6Q_1^2\\
&+6Q_2^2+2Q_{\bar{H}'}^2+2Q_{H'}^2+6Q_L^2+20Q_Q^2+3Q_S^2+9Q_u^2)+2Q_{\bar{H}'}^3m_{\bar{H}'}^2+2Q_{H'}^3m_{H'}^2\\
&+\sum_{i=1}^{3}(3Q_d^3m_{d_i}^2+3Q_{\bar{D}}^3m_{\bar{D}_i}^2+3Q_D^3m_{D_i}^2+Q_e^3m_{e_i}^2+2Q_1^3m_{H_i^d}^2+2Q_2^3m_{H_i^u}^2+2Q_L^3m_{L_i}^2\\
&+6Q_Q^3m_{Q_i}^2+Q_S^3m_{S_i}^2+3Q_u^3m_{u_i}^2)+2Q_QQ_{\bar{H}'}^2m_{\bar{H}'}^2+2Q_QQ_{H'}^2m_{H'}^2\\
&+Q_Q\sum_{i=1}^{3}(3Q_d^2m_{d_i}^2+3Q_{\bar{D}}^2m_{\bar{D}_i}^2+3Q_D^2m_{D_i}^2+Q_e^2m_{e_i}^2+2Q_1^2m_{H_i^d}^2+2Q_2^2m_{H_i^u}^2\\
&+2Q_L^2m_{L_i}^2+6Q_Q^2m_{Q_i}^2+Q_S^2m_{S_i}^2+3Q_u^2m_{u_i}^2)\Big],
\end{aligned}
\tag{B15b}
$$

$$
\begin{aligned}
b_{m_{Q_3}^2}^{(2)}&=24y_t^4(m_{H_u^2}+m_{Q_3}^2+m_{u_3}^2)+24y_b^4(m_{H_d}^2+m_{Q_3}^2+m_{d_3}^2)+48y_t^2a_t^2+48y_b^2a_b^2\\
&+4y_t^2y_b^2(m_{H_u}^2+m_{H_d}^2+2m_{Q_3}^2+m_{u_3}^2+m_{d_3}^2)+4(y_ta_b+y_ba_t)^2\\
&+2y_b^2y_\tau^2(2m_{H_d}^2+m_{Q_3}^2+m_{d_3}^2+m_{L_3}^2+m_{e_3}^2)+2(y_\tau a_b+y_ba_\tau)^2\\
&+2\lambda^2y_t^2(m_{H_d}^2+2m_{H_u}^2+m_S^2+m_{Q_3}^2+m_{u_3}^2)+2(\lambda a_t+y_ta_\lambda)^2\\
&+2\lambda^2y_b^2(2m_{H_d}^2+m_{H_u}^2+m_S^2+m_{Q_3}^2+m_{d_3}^2)+2(\lambda a_b+y_ba_\lambda)^2\\
&-\frac{32}{3}g_3^2y_t^2(m_{H_u}^2+m_{Q_3}^2+m_{u_3}^2+2M_3^2)-\frac{32}{3}g_3^2(a_t^2-2y_ta_tM_3)\\
&-\frac{32}{3}g_3^2y_b^2(m_{H_d}^2+m_{Q_3}^2+m_{d_3}^2+2M_3^2)-\frac{32}{3}g_3^2(a_b^2-2y_ba_bM_3)\\
&-6g_2^2y_t^2(m_{H_u}^2+m_{Q_3}^2+m_{u_3}^2+2M_2^2)-6g_2^2(a_t^2-2y_ta_tM_2)\\
&-6g_2^2y_b^2(m_{H_d}^2+m_{Q_3}^2+m_{d_3}^2+2M_2^2)-6g_2^2(a_b^2-2y_ba_bM_2)\\
&-\frac{26}{15}g_1^2y_t^2(m_{H_u}^2+m_{Q_3}^2+m_{u_3}^2+2M_1^2)-\frac{26}{15}g_1^2(a_t^2-2y_ta_tM_1)\\
&-\frac{14}{15}g_1^2y_b^2(m_{H_d}^2+m_{Q_3}^2+m_{d_3}^2+2M_1^2)-\frac{14}{15}g_1^2(a_b^2-2y_ba_bM_1)\\
&-4g_1'^2y_t^2(Q_2^2+Q_Q^2+Q_u^2)(m_{H_u}^2+m_{Q_3}^2+m_{u_3}^2+2M_1'^2)-4g_1'^2(Q_2^2+Q_Q^2+Q_u^2)\\
&\times(a_t^2-2y_ta_tM_1')+2g_1'^2y_t^2(Q_2+Q_Q+Q_u)(6Q_Qm_{H_u}^2+6Q_Qm_{Q_3}^2+6Q_Qm_{u_3}^2+\Sigma_1')\\
&+12Q_Qg_1'^2a_t^2(Q_2+Q_Q+Q_u)-4g_1'^2y_b^2(Q_1^2+Q_Q^2+Q_d^2)(m_{H_d}^2+m_{Q_3}^2+m_{d_3}^2+2M_1'^2)\\
&-4g_1'^2(Q_1^2+Q_Q^2+Q_d^2)(a_b^2-2y_ba_bM_1')+2g_1'^2y_b^2(Q_1+Q_Q+Q_d)\\
&\times(6Q_Qm_{H_d}^2+6Q_Qm_{Q_3}^2+6Q_Qm_{d_3}^2+\Sigma_1')+12Q_Qg_1'^2a_b^2(Q_1+Q_Q+Q_d)
\end{aligned}
$$

$$
\begin{aligned}
&+ 4Q_Q g_1'^2 y_\tau^2 (Q_1 + Q_L + Q_e)(m_{H_d}^2 + m_{L_3}^2 + m_{e_3}^2) + 4Q_Q g_1'^2 a_\tau^2 (Q_1 + Q_L + Q_e) \\
&+ 2Q_Q g_1'^2 \sum_{i=1}^{3} [2\lambda_i^2 (Q_1 + Q_2 + Q_S)(m_{H_i^d}^2 + m_{H_i^u}^2 + m_S^2) + 2a_{\lambda_i}^2 (Q_1 + Q_2 + Q_S) \\
&+ 3\kappa_i^2 (Q_S + Q_D + Q_{\bar{D}})(m_S^2 + m_{D_i}^2 + m_{\bar{D}_i}^2) + 3a_{\kappa_i}^2 (Q_S + Q_D + Q_{\bar{D}})] \\
&- 96 Q_Q g_3^2 g_1'^2 M_3^2 (2Q_Q + Q_u + Q_d + Q_D + Q_{\bar{D}}) - 72 g_2^4 M_2^2 \\
&- 12 Q_Q g_2^2 g_1'^2 M_2^2 (9Q_Q + 3Q_L + 3Q_1 + 3Q_2 + Q_{\tilde{H}'} + Q_{H'}) + \frac{96}{25} g_1^4 (\Sigma_1 - M_1^2) \\
&- \frac{1}{5} g_1^2 g_1'^2 [12 Q_Q M_1^2 (2Q_d + 2Q_{\bar{D}} + 2Q_D + 6Q_e + 3Q_1 + 3Q_2 + Q_{\tilde{H}'} + Q_{H'} + 3Q_L \\
&+ Q_Q + 8Q_u) + 4M_1'^2 (3Q_d^2 + 3Q_{\bar{D}}^2 - 3Q_D^2 + 3Q_e^2 - 3Q_1^2 + 3Q_2^2 + Q_{\tilde{H}'}^2 - Q_{H'}^2 - 3Q_L^2 \\
&+ 3Q_Q^2 - 6Q_u^2) - (6Q_Q \Sigma_1 + \Sigma_1')\Sigma_Q^Y] - 4Q_Q g_1'^4 [2M_1'^2 (9Q_d^3 + 9Q_{\bar{D}}^3 + 9Q_D^3 + 3Q_e^3 + 6Q_1^3 \\
&+ 6Q_2^3 + 2Q_{\tilde{H}'}^3 + 2Q_{H'}^3 + 6Q_L^3 + 18Q_Q^3 + 3Q_S^3 + 9Q_u^3) + (6Q_Q M_1'^2 - \Sigma_1')\Sigma_Q]. \qquad \text{(B15c)}
\end{aligned}
$$

Finally, the relevant expressions for the soft mass $m_{u_3}^2$ read

$$
\begin{aligned}
\beta_{m_{u_3}^2}^{(1)} &= 4y_t^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2) + 4a_t^2 - \frac{32}{3} g_3^2 M_3^2 - \frac{32}{15} g_1^2 M_1^2 - 8Q_u^2 g_1'^2 M_1'^2 \\
&\quad - \frac{4}{5} g_1^2 \Sigma_1 + 2Q_u g_1'^2 \Sigma_1', \qquad \text{(B16a)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_{u_3}^2}^{(2)} &= -32 y_t^4 (m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2) - 4y_t^2 y_b^2 (m_{H_u}^2 + m_{H_d}^2 + 2m_{Q_3}^2 + m_{u_3}^2 + m_{d_3}^2) \\
&\quad - 64 y_t^2 a_t^2 - 4(y_t a_b + y_b a_t)^2 - 4\lambda^2 y_t^2 (2m_{H_u}^2 + m_{H_d}^2 + m_S^2 + m_{Q_3}^2 + m_{u_3}^2) \\
&\quad - 4(\lambda a_t + y_t a_\lambda)^2 + 12 g_2^2 y_t^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2 + 2M_2^2) + 12 g_2^2 (a_t^2 - 2y_t a_t M_2) \\
&\quad - \frac{4}{5} g_1^2 y_t^2 (-5m_{H_u}^2 - m_{Q_3}^2 + 9m_{u_3}^2 + 2M_1^2) - \frac{4}{5} g_1^2 (a_t^2 - 2y_t a_t M_1) \\
&\quad - \frac{8}{5} g_1^2 y_b^2 (3m_{H_d}^2 - m_{Q_3}^2 - 2m_{d_3}^2) - \frac{8}{5} g_1^2 y_\tau^2 (m_{H_d}^2 + m_{L_3}^2 - 2m_{e_3}^2) \\
&\quad - \frac{8}{5} g_1^2 \sum_{i=1}^{3} [\kappa_i^2 (m_{D_i}^2 - m_{\bar{D}_i}^2) + \lambda_i^2 (m_{H_i^d}^2 - m_{H_i^u}^2)] \\
&\quad + 8 g_1'^2 y_t^2 (Q_2^2 + Q_Q^2 - Q_u^2)(m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2 + 2M_1'^2) \\
&\quad + 8 g_1'^2 (Q_2^2 + Q_Q^2 - Q_u^2)(a_t^2 - 2y_t a_t M_1') - 24 Q_u g_1'^2 y_t^2 (Q_2 m_{H_u}^2 + Q_Q m_{Q_3}^2 + Q_u m_{u_3}^2) \\
&\quad - 24 Q_u g_1'^2 y_b^2 (Q_1 m_{H_d}^2 + Q_Q m_{Q_3}^2 + Q_d m_{d_3}^2) - 8 Q_u g_1'^2 y_\tau^2 (Q_1 m_{H_d}^2 + Q_L m_{L_3}^2 + Q_e m_{e_3}^2) \\
&\quad - 12 Q_u g_1'^2 \sum_{i=1}^{3} \kappa_i^2 (Q_S m_S^2 + Q_D m_{D_i}^2 + Q_{\bar{D}} m_{\bar{D}_i}^2) \\
&\quad - 8 Q_u g_1'^2 \sum_{i=1}^{3} \lambda_i^2 (Q_1 m_{H_d}^2 + Q_2 m_{H_u}^2 + Q_S m_S^2) \\
&\quad + \frac{16}{3} g_3^4 \left[10M_3^2 + \sum_{i=1}^{3} (m_{d_i}^2 + m_{D_i}^2 + m_{\bar{D}_i}^2 + 2m_{Q_i}^2 + m_{u_i}^2) \right] \\
&\quad + \frac{64}{45} g_3^2 g_1^2 \left[8(M_1^2 + M_1 M_3 + M_3^2) - 3\sum_{i=1}^{3} (m_{d_i}^2 - m_{D_i}^2 + m_{\bar{D}_i}^2 + m_{Q_i}^2 - 2m_{u_i}^2) \right] \\
&\quad + \frac{8}{3} Q_u g_3^2 g_1'^2 \Big[16 Q_u (M_1'^2 + M_1' M_3 + M_3^2)
\end{aligned}
$$

$$
\begin{aligned}
&+12\sum_{i=1}^{3}(Q_d m_{d_i}^2 + Q_D m_{D_i}^2 + Q_{\bar{D}} m_{\bar{D}_i}^2 + 2Q_Q m_{Q_i}^2 + Q_u m_{u_i}^2)\Big]\\
&+\frac{12}{5}g_2^2 g_1^2\Big[m_{H'}^2 - m_{\bar{H}'}^2 + \sum_{i=1}^{3}(m_{H_i^d}^2 - m_{H_i^u}^2 + m_{L_i}^2 - m_{Q_i}^2)\Big]\\
&+12Q_u g_2^2 g_1'^2\Big[Q_{H'} m_{H'}^2 + Q_{\bar{H}'} m_{\bar{H}'}^2 + \sum_{i=1}^{3}(Q_1 m_{H_i^d}^2 + Q_2 m_{H_i^u}^2 + Q_L m_{L_i}^2 + 3Q_Q m_{Q_i}^2)\Big]\\
&+\frac{4}{75}g_1^4\Big[1261M_1^2 + 21m_{H'}^2 + 3m_{\bar{H}'}^2\\
&+\sum_{i=1}^{3}(4m_{d_i}^2 + 12m_{D_i}^2 + 4m_{\bar{D}_i}^2 - 12m_{e_i}^2 + 21m_{H_i^d}^2 + 3m_{H_i^u}^2 + 21m_{L_i}^2 + 3m_{Q_i}^2 + 64m_{u_i}^2)\Big]\\
&+\frac{4}{15}g_1^2 g_1'^2\Big[-8Q_u(9Q_d + 9Q_{\bar{D}} - 9Q_D + 9Q_e - 9Q_1 + 9Q_2 + 3Q_{\bar{H}'} - 3Q_{H'} - 9Q_L\\
&+9Q_Q - 22Q_u)(M_1^2 + M_1M_1' + M_1'^2) + 12Q_{H'}^2 m_{H'}^2 - 12Q_{\bar{H}'}^2 m_{\bar{H}'}^2\\
&+12\sum_{i=1}^{3}(-Q_d^2 m_{d_i}^2 - Q_{\bar{D}}^2 m_{\bar{D}_i}^2 + Q_D^2 m_{D_i}^2 - Q_e^2 m_{e_i}^2 + Q_1^2 m_{H_i^d}^2 - Q_2^2 m_{H_i^u}^2 + Q_L^2 m_{L_i}^2\\
&-Q_Q^2 m_{Q_i}^2 + Q_u^2 m_{u_i}^2) - 15Q_{\bar{H}'} Q_u m_{\bar{H}'}^2 + 33Q_{H'} Q_u m_{H'}^2 + Q_u\sum_{i=1}^{3}(-18Q_d m_{d_i}^2\\
&-18Q_{\bar{D}} m_{\bar{D}_i}^2 + 30Q_D m_{D_i}^2 - 6Q_e m_{e_i}^2 + 33Q_1 m_{H_i^d}^2 - 15Q_2 m_{H_i^u}^2 + 33Q_L m_{L_i}^2\\
&-21Q_Q m_{Q_i}^2 + 84Q_u m_{u_i}^2)\Big] + 8Q_u g_1'^4\Big[3Q_u M_1'^2(9Q_d^2 + 9Q_{\bar{D}}^2 + 9Q_D^2 + 3Q_e^2 + 6Q_1^2\\
&+6Q_2^2 + 2Q_{\bar{H}'}^2 + 2Q_{H'}^2 + 6Q_L^2 + 18Q_Q^2 + 3Q_S^2 + 11Q_u^2) + 2Q_{H'}^3 m_{H'}^2 + 2Q_{\bar{H}'}^3 m_{\bar{H}'}^2\\
&+\sum_{i=1}^{3}(3Q_d^3 m_{d_i}^2 + 3Q_{\bar{D}}^3 m_{\bar{D}_i}^2 + 3Q_D^3 m_{D_i}^2 + Q_e^3 m_{e_i}^2 + 2Q_1^3 m_{H_i^d}^2 + 2Q_2^3 m_{H_i^u}^2 + 2Q_L^3 m_{L_i}^2\\
&+6Q_Q^3 m_{Q_i}^2 + Q_S^3 m_{S_i}^2 + 3Q_u^3 m_{u_i}^2) + 2Q_u Q_{H'}^2 m_{H'}^2 + 2Q_u Q_{\bar{H}'}^2 m_{\bar{H}'}^2\\
&+Q_u\sum_{i=1}^{3}(3Q_d^2 m_{d_i}^2 + 3Q_{\bar{D}}^2 m_{\bar{D}_i}^2 + 3Q_D^2 m_{D_i}^2 + Q_e^2 m_{e_i}^2 + 2Q_1^2 m_{H_i^d}^2 + 2Q_2^2 m_{H_i^u}^2 + 2Q_L^2 m_{L_i}^2\\
&+6Q_Q^2 m_{Q_i}^2 + Q_S^2 m_{S_i}^2 + 3Q_u^2 m_{u_i}^2)\Big],
\end{aligned}
\tag{B16b}
$$

$$
\begin{aligned}
b_{m_{u_3}^2}^{(2)} &= 48y_t^4(m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2) + 4y_t^2y_b^2(m_{H_u}^2 + m_{H_d}^2 + 2m_{Q_3}^2 + m_{u_3}^2 + m_{d_3}^2) + 96y_t^2a_t^2\\
&+4(y_ta_b + y_ba_t)^2 + 4\lambda^2y_t^2(2m_{H_u}^2 + m_{H_d}^2 + m_S^2 + m_{Q_3}^2 + m_{u_3}^2) + 4(\lambda a_t + y_ta_\lambda)^2\\
&-\frac{64}{3}g_3^2y_t^2(m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2 + 2M_3^2) - \frac{64}{3}g_3^2(a_t^2 - 2y_ta_tM_3)\\
&-12g_2^2y_t^2(m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2 + 2M_2^2) - 12g_2^2(a_t^2 - 2y_ta_tM_2)\\
&-\frac{52}{15}g_1^2y_t^2(m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2 + 2M_1^2) - \frac{52}{15}g_1^2(a_t^2 - 2y_ta_tM_1)\\
&-8g_1'^2y_t^2(Q_2^2 + Q_Q^2 + Q_u^2)(m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2 + 2M_1'^2)\\
&-8g_1'^2(Q_2^2 + Q_Q^2 + Q_u^2)(a_t^2 - 2y_ta_tM_1')
\end{aligned}
$$

$$
\begin{aligned}
&+ 4g_1'^2 y_t^2 (Q_2 + Q_Q + Q_u)(3Q_u m_{H_u}^2 + 3Q_u m_{Q_3}^2 + 3Q_u m_{u_3}^2 + \Sigma_1') \\
&+ 12 Q_u g_1'^2 a_t^2 (Q_2 + Q_Q + Q_u) + 12 Q_u g_1'^2 y_b^2 (Q_1 + Q_Q + Q_d)(m_{H_d}^2 + m_{Q_3}^2 + m_{d_3}^2) \\
&+ 12 Q_u g_1'^2 a_b^2 (Q_1 + Q_Q + Q_d) \\
&+ 4 Q_u g_1'^2 y_\tau^2 (Q_1 + Q_L + Q_e)(m_{H_d}^2 + m_{L_3}^2 + m_{e_3}^2) + 4 Q_u g_1'^2 a_\tau^2 (Q_1 + Q_L + Q_e) \\
&+ 2 Q_u g_1'^2 \sum_{i=1}^{3} [3\kappa_i^2 (Q_S + Q_D + Q_{\bar{D}})(m_S^2 + m_{D_i}^2 + m_{\bar{D}_i}^2) + 3a_{\kappa_i}^2 (Q_S + Q_D + Q_{\bar{D}}) \\
&+ 2\lambda_i^2 (Q_1 + Q_2 + Q_S)(m_{H_i^d}^2 + m_{H_i^u}^2 + m_S^2) + 2a_{\lambda_i}^2 (Q_1 + Q_2 + Q_S)] \\
&- 96 Q_u g_3^2 g_1'^2 M_3^2 (2Q_Q + Q_u + Q_d + Q_D + Q_{\bar{D}}) \\
&- 12 Q_u g_2^2 g_1'^2 M_2^2 (9Q_Q + 3Q_L + 3Q_1 + 3Q_2 + Q_{H'} + Q_{\bar{H}'}) - \frac{384}{25} g_1'^4 (\Sigma_1 + 4M_1^2) \\
&+ \frac{2}{5} g_1^2 g_1'^2 [8M_1'^2 (3Q_d^2 + 3Q_{\bar{D}}^2 - 3Q_D^2 + 3Q_e^2 - 3Q_1^2 + 3Q_2^2 + Q_{\bar{H}'}^2 - Q_{H'}^2 - 3Q_L^2 \\
&+ 3Q_Q^2 - 6Q_u^2) + 6Q_u M_1^2 (2Q_d + 2Q_{\bar{D}} + 2Q_D + 6Q_e + 3Q_1 + 3Q_2 + Q_{\bar{H}'} + Q_{H'} \\
&+ 3Q_L + Q_Q + 8Q_u) + (3Q_u \Sigma_1 - 2\Sigma_1')\Sigma_Q^Y] - 4Q_u g_1'^4 [2M_1'^2 (9Q_d^3 + 9Q_D^3 + 9Q_{\bar{D}}^3 \\
&+ 3Q_e^3 + 6Q_1^3 + 6Q_2^3 + 2Q_{\bar{H}'}^3 + 2Q_{H'}^3 + 6Q_L^3 + 18Q_Q^3 + 3Q_S^3 + 9Q_u^3) \\
&+ (6Q_u M_1'^2 - \Sigma_1')\Sigma_Q].
\end{aligned} \tag{B16c}
$$

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