QCD dynamics of tetraquark production

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We use the twist dimensions of the operators underlying the dynamical behavior of exclusive production processes as a tool for determining the structure of exotic heavy-quark states such as the $Z_c^+(4430)$ tetraquark. The resulting counting rules predict distinctive falloffs of the cross sections in center-of-mass energy, thus distinguishing whether the tetraquarks are segregated into di-meson molecules, diquark-antidiquark pairs, or more democratically arranged four-quark states. In addition, we propose straightforward methods of experimentally producing additional exotic multiquark states.

I. INTRODUCTION

Hadronic physics has reached an important milestone in the past year: the experimental confirmation by LHCb [1] of the *tetraquark* $(\bar{c}c\bar{d}u)$ state $Z_c^+(4430)$ with spin-parity $J^P = 1^+$. Its interpretation as a true resonance is confirmed by the observation that the phase shift δ of its complex production amplitude increases by $\frac{\pi}{2}$ radians as the energy crosses the resonant mass. The tetraquark thus joins the $\bar{q}q$ meson and qqq baryon as a third class of hadrons. The $Z_c^+(4430)$ represents just one of a growing collection of unexpected charmoniumlike states, beginning with the famous $X(3872)$ first seen by Belle in 2003 [2].

The discovery of the charged (Z_c^+) states requires a valence quark content of at least four quarks, $\bar{c}c\bar{d}u$, and the strength of the observed transitions amongst the X, Y (charmoniumlike states appearing in initial-state radiation processes $e^+e^- \to \gamma Y$), and Z_c strongly suggest a common tetraquark nature for all of these novel states [3]. The $J^{PC} = 1^{++}$ $X(3872)$, for example, is almost certainly a $\bar{c}c\bar{q}q$ state, in which $\bar{q}q$ is a linear combination of $\bar{u}u$ and $\bar{d}d$.

In this paper we will discuss two essential questions: (a) the color composition of the tetraquarks in QCD, and (b) the dynamics underlying their production. For example, we shall argue that tetraquarks such as the Z_c can be produced near threshold in both hadronic collisions and electroproduction. We will also discuss the possible existence of other novel multiquark hadronic states that are natural extensions of tetraquark states.

The first, and still most widely known, ansatz proposed for the structure of tetraquarks is one of di-meson molecules bound by pion exchange or color van der Waals forces (as reviewed in, e.g., Ref. [4]). The proximity of several of

these states to the corresponding two-meson thresholds is quite remarkable: For instance, $m_{X(3872)} - m_{D^{*0}} - m_{D^0} = -0.11 \pm 0.21$ MeV. On the other hand, a number of other tetraquark candidates lie just above such thresholds, suggesting some sort of potential barrier to allow bound states with *positive* binding energy, while others—prominently, the $Z_c^+(4430)$—have no obvious nearby threshold with the appropriate quantum numbers, in this case $J^P = 1^+$.

The prompt production cross section of the $X(3872)$ at e^+e^- colliders is substantial, indicating that the $X(3872)$ can be created with high relative momentum between its components; however, since the binding energy between its meson components is very small in the di-meson molecular picture, this empirical observation creates great difficulty for this picture [5–7], even when substantial final-state interactions between the mesons [8,9] are taken into account.

An alternative to the molecular picture, *hadro-charmonium* [10], assumes that a compact charmonium state is located at the center of a light-quark cloud; in this case, one must question why such states would be quasistable, and to what degree they would be obscured through mixing with conventional charmonium.

In this work, we shall argue that tetraquarks are primarily diquark-antidiquark $(\delta\text{-}\bar{\delta})$ bound states. Thus, the $Z_c^+(4430)$ can be considered as a $[\bar{c}\,\bar{d}]_{3C}[cu]_{\bar{3}C}$ bound state of a color-(anti)triplet charmed diquark δ and an anti-diquark $\bar{\delta}$. The same diquark clusters appear in the valence Fock state structure of baryons; e.g., the $\Lambda_c(cud)$ can be considered as a color-singlet $[cu]_{\bar{3}C}d_{3C} + [cd]_{\bar{3}C}u_{3C}$ composite. Inasmuch as the diquarks can be considered pointlike color sources, the confining potential that confines the color-triplet δ and $\bar{\delta}$ into color-singlet tetraquarks is thus identical to the confinement potential underlying $q\bar{q}$ mesons and $q\bar{\delta}$ baryons. Thus, three strong binding interactions are present in the diquark-antidiquark picture: the interactions creating the δ

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and $\bar{\delta}$, and the δ-$\bar{\delta}$ binding. This is also the natural interpretation obtained from anti-de Sitter space (AdS)/ QCD and superconformal algebra, an approach which accounts remarkably well for the observed Regge spectroscopy of mesons and baryons in terms of bound states of color-triplet and color-antitriplet constituents [11,12].

The diquark-antidiquark picture was first proposed for charmoniumlike tetraquarks in Ref. [13]. Since the diquarks are color triplets, this description leads to new insights into confined color dynamics. However, unless the model is constrained, it also predicts many more tetraquark states than are seen experimentally. Nevertheless, recent work [14] shows that if the spin-spin couplings within each diquark dominate, one can explain many empirical features of the observed tetraquark spectroscopy. The significance of novel color correlations in exotics such as tetraquarks is discussed in Ref. [15].

An important question is why the component quarks in a δ-$\bar{\delta}$ bound state do not immediately reorganize themselves (either dynamically, or simply using group-theory identities) into color-singlet $\bar{q}q$ pairs, thus recreating the molecular picture. To address this objection, we note a well-known fact of color dynamics: Two color-**3** quarks have an attractive color-**$\bar{3}$** channel that is fully half as strong from gluon exchange as the attraction of a $\bar{q}(\bar{3})q(3)$ pair into a color singlet. This result follows from simple SU(3) color group theory: The coupling of two representations R_1 and R_2 to a representation R is proportional to the combination of quadratic Casimirs given by $C_2(R) - C_2(R_1) - C_2(R_2)$. For two quarks, the only attractive channels are the $\bar{q}q$ singlet ($R_1 = \bar{3}$, $R_2 = 3$, $R = 1$) and the qq antitriplet ($R_1 = R_2 = 3$, $R = \bar{3}$), and the latter is half as strong as the former. In this sense, diquarks are special entities in QCD; if two quarks are created closer to each other than to any antiquarks, then it is natural to expect them to form a bound quasiparticle. In fact, the greater the energy available in a system (as often occurs in heavy-quark processes), the more opportunities arise for such channels to occur.

As an explicit example, we recently proposed [16] a new paradigm: The tetraquarks are not simple quasistatic δ-$\bar{\delta}$ bound states, but instead arise as modes of a rapidly separating δ-$\bar{\delta}$ pair, remaining confined, a color flux tube stretching between them with its length determined by the available energy. Many, but not all, of the narrow-width tetraquarks lie near hadronic thresholds because these are the energies at which the color string can easily break. A case in point is the $X(4632)$, the first exotic state found above the 4573 MeV charmed-baryon ($\Lambda_c^+ \bar{\Lambda}_c^-$) threshold; it decays dominantly to $\Lambda_c^+ \bar{\Lambda}_c^-$ (indeed, this is the only mode yet seen), and this decay is precisely what one would expect from flux-tube fragmentation to a single light $\bar{q}q$ pair. Below this threshold, the exotic state widths are not particularly large because they can hadronize only by forming mesons with wave functions stretching from the

quarks in $\bar{\delta}$ to the antiquarks in the δ. As suggested above, the δ-$\bar{\delta}$ pair can separate a significant distance (>1 fm) if it is produced with enough relative momentum; for instance, a B-meson decay can produce this circumstance.

The decay of the $Z_c^+(4430)$ suggests that it is a spatially extended state: Even though the $\psi(2S)$ and J/ψ have precisely the same $J^{PC} = 1^{--}$ quantum numbers, the $Z_c^+(4430)$ prefers by a large margin to decay to $\psi(2S)\pi$ instead of $J/\psi\pi$ [3]; the $\psi(2S)$ mode has much less phase space, but is spatially much larger than J/ψ, matching the expected size from the diquark decay mechanism.

The diquark interpretation of tetraquarks naturally leads to the possibility of more complex hadronic states in QCD, such as *hexaquarks* [17], which can arise as $\bar{\delta}_{3C}\bar{\delta}_{3C}\bar{\delta}_{3C}$ color-singlets analogous to $q_{\bar{3}C}q_{\bar{3}C}q_{\bar{3}C}$ (anti)baryonic bound states. An example would be the charmed, charge $Q = 2$, baryon-number $B = 2$ state $[uu]_{\bar{3}C}[cu]_{\bar{3}C}[uu]_{\bar{3}C}$. In this case, two of the diquarks, e.g., $[uu]_{\bar{3}C}$ and $[cu]_{\bar{3}C}$, can arrange themselves into a color-triplet ($3C$) four-quark cluster—precisely by the same analysis of attractive color channels described above—which then, in turn, binds to the $[uu]_{\bar{3}C}$ diquark. Thus, one can consider such multiquark states as a sequence of two-body bound-state clusters of color-triplet and antitriplet states. Another example is the $B = 2$ *octoquark* resonance $|\bar{c}cuuduud\rangle$, which can explain the dramatic spin dependence seen in elastic $pp \rightarrow pp$ scattering at the charm production threshold $\sqrt{s} \simeq 5$ GeV [18]. Again, this state can be considered as sequential binding of four diquarks. The light-front wave function in this case satisfies a cluster decomposition analogous to that in the structure of the deuteron [19].

In this paper, we put the δ-$\bar{\delta}$ tetraquark picture to a new dynamical test, using the well-known constituent counting rules [20–30], to predict the fixed-θ_{cm} power-law scaling in Mandelstam s of exclusive processes at high energies. The counting rules determine the s power-law dependence of cross sections and form factors for processes at high s and fixed scattering angle θ_{cm}, where the power of s is determined by the total number of fundamental constituents—incoming plus outgoing—appearing in the hard scattering. The first application of the counting rules in this tetraquark picture, a study of the so-called "cusp effect" of threshold-induced shifts to resonance masses, appeared in a very recent paper [31] by one of the present authors.

The counting rules, derived from the twist of the interpolating fields controlling each hadron at short distances, provide two especially interesting probes of tetraquark states. First, as originally pointed out in Refs. [32,33], the high-s production data for exotic states should follow the s power dependence predicted based on their expected valence quark structure (i.e., four-quark tetraquarks). Second, and original to this work, the counting rules should be sensitive to the presence of diquarks. If particularly strongly bound diquarks are formed in the production process, and dissociate only after their

production (so that the diquarks effectively function for a time as single dynamical units) then the counting rules will treat the diquarks effectively as elementary fundamental color-triplet constituents at intermediate energies.

We will also discuss a simple method of producing numerous exotic states, via electroproduction near the charm (or other heavy-quark) threshold. While not directly dependent upon the diquark hypothesis, the production mechanism also addresses the formation and dynamics of QCD multiquark exotics.

This paper is organized as follows: In Sec. II, we briefly review the constituent counting rules, how they are derived, and their limitations. Section III presents our principal predictions for the production cross sections and the form factors of exotic charmonium and bottomonium states. Section IV proposes straightforward experimental methods of producing numerous exotic states, particularly via electroproduction processes. In Sec. V we summarize and indicate future directions.

II. CONSTITUENT COUNTING RULES

The constituent counting rules, which we briefly review in this section, were developed in the decade subsequent to the creation of perturbative QCD (pQCD) [20–30]. In essence, they represent the conformality and scale invariance of QCD at high energies, and therefore are applicable to a wide variety of field theories; for example, they have been derived nonperturbatively in AdS/QCD [34]. The summary presented here follows the more detailed introduction in Ref. [32], which also was the first work to apply counting rules to exotic multiquark hadrons.

The counting rules find their most incisive applications in fixed-$\theta_{\rm cm}$ exclusive scattering processes at high $\sqrt{s}$, for which none of the particles are accidentally close to being collinear. Constituent masses can then be neglected, and all of the Mandelstam variables s, t, and $u \simeq -(s + t)$ are large. To maintain the integrity of the exclusive states, each of the constituents must undergo a large momentum transfer to be deflected through the same finite angle $\theta_{\rm cm}$, i.e., fixed t/s; therefore, all large energy scales may be expressed in terms of s. In pQCD, hard gluon exchanges are responsible for the momentum transfers, while if leptons also appear in the process (e.g., in electroproduction), then hard electroweak gauge boson exchanges must also be taken into account. In the AdS/QCD picture, the "operator dictionary" relates the counting rules to the short-distance twist dimension of interpolating fields.

The counting rules in their simplest form simply enumerate s factors in propagators and spinor normalizations. For simplicity, let us begin with processes in which all n external constituents, $n = n_{\rm in} + n_{\rm out}$, are fermions. In order for every constituent to share an $O(1)$ fraction of the total s, the leading-order Feynman diagrams for the scattering must have at least $\frac{n}{2} - 1$ hard gauge boson propagators ($\sim 1/s$), and the associated vertices much be connected by at least

$\frac{n}{2} - 2$ internal constituent propagators ($\sim 1/\sqrt{s}$). Since each external constituent fermion field carries a spinor normalization scaling as $s^{\frac{1}{4}}$, the fermion scaling factors cancel except for an overall factor s, leaving a total invariant amplitude $\mathcal{M}$ scaling as

$$\mathcal{M} \propto 1/s^{\frac{n}{2}-2}. \qquad (1)$$

The cross section for a scattering process in which the constituents form two initial-state and two final-state particles may then be written

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} |\mathcal{M}|^2 \equiv \frac{1}{s^{n-2}} f\left(\frac{t}{s}\right). \qquad (2)$$

As fixed by the mass dimension M^{-4} of the left-hand side of this equation, the function f has mass dimension M^{2n-8}. However, f itself is constructed so as not to scale with s; its dimensionful factors instead arise from the amplitude overlaps between the fundamental constituent fields and their external composite states, such as those defining decay constants.

Modifying this result to allow for external gauge bosons is straightforward: Each external boson line introduced replaces two external fermion lines [$\sim (\sqrt{s})^2$] and one hard gauge boson propagator ($\sim 1/s$), which cancel and therefore give precisely the same scaling expressions for $\mathcal{M}$ and $d\sigma/dt$ as in Eqs. (1)–(2).

In the case of this work, we consider the scenario in which diquarks are treated as fundamental constituents for purposes of counting, since they are tightly bound to each other compared to their binding with the other quarks; in scattering processes, they may be redirected as a single unit. Moreover, as discussed above, they form overall color triplets in their channel of greatest attraction and therefore can interact with the other quarks via single-gluon exchange. If one replaces the two dynamical quarks with a single diquark in the counting argument, one loses the hard gluon connecting them ($\sim 1/s$), while two hard (fermionic) quark propagators are replaced by a single (bosonic) diquark propagator [$(1/\sqrt{s})^2 \rightarrow 1/s$], and the four external spinor normalizations [$(s^{\frac{1}{4}})^4$] are replaced with two external diquark normalizations [$(\sqrt{s})^2$]. The latter two actions do not change the net counting of s factors, while the removal of the extra gluon leads to a scaling equivalent to that of reducing the number of constituents from n to $n - 2$, exactly as one would have from treating the diquark as a fundamental constituent. One must note, however, that this scaling holds only if the diquark propagates intact through the scattering process, or if a δ-$\bar{\delta}$ pair is created in a pointlike configuration.

The original thrust of Ref. [32] actually points to a contrary but complementary direction: If an exotic multiquark state is produced without a diquark component, then the scattering cross section receives a contribution from all

of the component valence quarks. The primary example discussed in [32] is $\pi^- + p \to K^0 + \Lambda(1405)$, where the small $\Lambda(1405)$ mass relative to nonstrange analogues such as $N(1535)$ has led to the proposal that the former is a pentaquark state. In this case, one can test this hypothesis by measuring whether $d\sigma/dt$ at large s scales according to Eq. (2) as $s^{-(2+3+2+3-2)} = s^{-8}$ or $s^{-(2+3+2+5-2)} = s^{-10}$. The same authors subsequently applied the large-s scaling counting rules to examine the properties of generalized parton distributions and distribution amplitudes that appear in processes such as these [33]. Whether diquarks act as one or two fundamental constituents, thus, becomes an experimentally testable prospect.

By applying Eq. (1), one can determine the large-s behavior of hadronic form factors from the corresponding amplitude $\mathcal{M}$. To give one explicit example, the Z_c^+ electromagnetic form factor $F_{Z_c}(s)$ should scale as [31]

$$F_{Z_c}(s) \to \frac{1}{s^{\frac{1}{2}(1+1+4+4)-2}} = \frac{1}{s^3}, \qquad (3)$$

but scale as $\sim 1/s^1$ if the δ and $\bar{\delta}$ are very tightly bound.

A number of technical complications of real QCD modify the simple s scaling naively obtained from perturbative Feynman diagrams. Included in this list are $\alpha_s(s)$ running and renormalization-group effects in the parton distribution amplitudes [35,36], Sudakov logarithms [37–39], "pinch" singularities from virtual gluons going on mass shell [40], and singularities of the "end point" type when one or more constituents carry only a $\ll 1$ fraction of the total s [41]. Even so, the current consensus view holds that the leading power-law scaling in s for exclusive processes remains the same as in the naive analysis.

III. SCALING OF TETRAQUARK CROSS SECTIONS

The scaling results and discussion of the previous section can be directly applied to make a number of simple and experimentally testable predictions for processes involving exotic states. The most straightforward applications use Eqs. (1)–(3) to predict cross sections at high s. Assuming that the Z_c^+ has four independent fundamental constituents that share $O(1)$ fractions of the total energy, one expects

$$\frac{d\sigma}{dt}(e^+e^- \to Z_c^+(\bar{c}c\bar{d}u) + \bar{Z}_c^-(\bar{c}c\bar{u}d)) \propto \frac{1}{s^8}, \qquad (4)$$

at finite scattering angle $\theta_{\text{c.m.}}$. The same scaling occurs if four ordinary mesons are produced in a direct e^+e^- annihilation process. On the other hand, if the Z_c states are formed from especially tightly bound δ and $\bar{\delta}$ quasi-particles, the scaling exponent drops to $1/s^4$, which is the same result as for the production of two ordinary mesons. In that case, it is then only because the exotic content of the Z_c is well established—the Z_c clearly contains hidden

charm, but is nevertheless charged—that one can unambiguously assert the Z_c is not an ordinary meson.

We note at this point an interesting distinction if one selects the production of neutral pairs $X\bar{X}$ or $Y\bar{Y}$, for which the scaling arguments are the same as for Z_c pairs. If the X or Y are comprised of tightly bound diquarks, then one cannot be certain from the scaling behavior alone whether or not the neutrals truly carry exotic quark content. Furthermore, by Z_c we do not mean just the $Z_c^+(4430)$, although its possession of a dominant decay mode $\psi(2S)\pi^+$ should make its reconstruction simpler.

Arguing against such a simple test is the fact that the scaling predicted by the counting rules only holds when s is "large enough." At very high values of s, one expects the scaling to work well, but then one is faced with both a paucity of data and a proliferation of final states from which to extract the exclusive two-particle events. At lower values of s, one is faced with the problem that having more constituents requires a larger total s before one is confident of all constituents carrying energies that lie in the perturbative regime; just from counting alone, one would expect the onset of scaling behavior for $e^+e^- \to Z_c^+\bar{Z}_c^-$ to occur at an s value about $(10/6)^2 \simeq 2.8$ times higher than for $e^+e^- \to \text{meson} + \text{meson}$. One can ameliorate this effect (not to mention greatly increase the rate) by considering semi-exotic processes such as $e^+e^- \to Z_c^+(\bar{c}c\bar{d}u) + \pi^-(\bar{u}d)$, but the question of the precise onset point for the asymptotic scaling regime remains.

A very simple modification, which extends the reach of the scaling to lower s is to form cross section ratios in order to eliminate systematic corrections. For example,

$$\frac{\sigma(e^+e^- \to Z_c^+(\bar{c}c\bar{d}u) + \bar{Z}_{\bar{c}}^-(\bar{c}c\bar{u}d))}{\sigma(e^+e^- \to \mu^+\mu^-)} = |F_{Z_c}(s)|^2$$

$$\propto \frac{1}{s^{n-4}}, \qquad (5)$$

where the exponent of $1/s$ is 6 if Z_c is a bound state of a two quarks and two antiquarks, and 2 if it is a bound state of two particularly tightly bound diquarks (n referring only to the number of constituents in the numerator process). Indeed, the first equality in Eq. (5) is effectively a definition of the form factor $F_{Z_c}(s)$, so it holds all the way down to the threshold, $s = 4m_{Z_c}^2$.

In fact, one can perform a verification of the constituent content of the Z_c^+ with a significantly larger rate by measuring a ratio that gives the transition form factor

$$\frac{\sigma(e^+e^- \to Z_c^+(\bar{c}c\bar{d}u) + \pi^-(\bar{u}d))}{\sigma(e^+e^- \to \mu^+\mu^-)} = |F_{Z_c,\pi}(s)|^2$$

$$\propto \frac{1}{s^{n-4}}, \qquad (6)$$

where the exponent of $1/s$ is 4 if Z_c^+ is a bound state of two quarks and two antiquarks, and 2 if it is a bound state of two tightly bound diquarks.

Another type of e^+e^- exclusive annihilation cross section ratio is particularly interesting. Consider the archetype process ratio:

$$\frac{\sigma(e^+e^- \to Z_c^+(\bar{c}c\bar{d}u) + \pi^-(\bar{u}d))}{\sigma(e^+e^- \to \Lambda_c(cud)\bar{\Lambda}_c(\bar{c}\,\bar{u}\,\bar{d}))}, \tag{7}$$

both of which have the same number of constituents, as well as the same heavy-quark ($\bar{c}c$) constituents. Therefore, both the corrections due to high-s scaling and corrections due to the total heavy-quark mass cancel in this ratio. While independent of s at leading order, whether the ratio is numerically large or small should be sensitive to the fundamental QCD composition of the Z_c^+ state; to wit, if Z_c^+ is primarily a di-meson "molecular" hadro-charmonium ($[\bar{c}c] + [\bar{d}u]$) or $\bar{D}$-D ($[\bar{c}u][\bar{d}c]$) state, it should be bound by weaker color-singlet van der Waals forces and, thus, be numerically smaller than if the four quarks in the Z_c^+ remain coupled through color-nonsinglet hard gluon exchanges. Indeed, one can envisage truly peculiar scenarios: If Z_c^+ contains tightly bound diquarks but Λ_c does not, then the ratio of Eq. (7) could actually grow with s (in this case, as s^2).

Finally, we have to this point considered only e^+e^- collider processes. Similar final states produced through $\bar{p}p$ annihilation have identical ratios as powers of s. The absolute high-energy cross sections fall with an additional power of s^4 (three quarks in each hadron, as compared with a lepton-antilepton pair), but the rates can also be greatly enhanced due to $\bar{p}p$ already containing the light quarks required by the final state. To be explicit,

$$\frac{d\sigma}{dt}(\bar{p}(\bar{u}\,\bar{u}\,\bar{d})p(uud) \to Z_c^+(\bar{c}c\bar{d}u) + \pi^-(\bar{u}d)) \propto \frac{1}{s^{10}} \tag{8}$$

should be numerically substantial near threshold and fall off very quickly for large s, while the ratio

$$\frac{\sigma(\bar{p}p \to Z_c^+\pi^-)}{\sigma(\bar{p}p \to \Lambda_c\bar{\Lambda}_c)} \tag{9}$$

has again the same quark content in the numerator and denominator and, therefore, again has canceling scaling and heavy-quark content factors; in particular, its dependence on s should be much gentler.

Of course, many of the charmed processes analogous to those described here and below have direct analogues in the $\bar{b}b$ threshold region; for a discussion from a different theoretical perspective, see Refs. [42–44].

IV. ELECTROPRODUCTION OF EXOTIC STATES NEAR THRESHOLD

With the existence of the first genuine QCD exotics having apparently been experimentally confirmed, one is led to ask what other exotics await discovery and what processes can most effectively be used to produce them. Here we argue that electroproduction near the charm threshold provides a natural laboratory for creating such states. This kinematical region represents an obvious energy regime for the formation of exotic states, because the slowly-movi ng $\bar{c}$ and c quarks produced readily coalesce with comoving valence quarks of the target. The diquark hypothesis is not essential to the analysis presented in this section.

For example, consider the process $ep \to e'X$ in the target-proton rest frame, which is most naturally considered a γ^*p collision. The virtual photon produces a $\bar{c}c$ pair a significant fraction of the time. A simple estimate for the ratio of $\bar{c}c$ to $\bar{u}u$ pair-production events is the ratio of s-channel squared masses at threshold, which is $\sim \frac{(m_p+m_\pi)^2}{(m_{\Lambda_c}+m_D)^2} \simeq 7\%$ at fixed γ^* momentum transfer above the charm threshold; note that using the slightly lower hidden-charm threshold $(m_p + m_{J/\Psi})^2$ in the denominator produces almost the same result. For energies slightly above the charm-production threshold, $s = (q + p)^2 \simeq (4.2 \text{ GeV})^2$ where q and p are, respectively, the photon and proton momenta, then the charm quarks coalesce with the uud valence quarks moving at the same rapidity to produce open-charm states such as $\gamma^*p \to \bar{D}^0(\bar{c}u)\Lambda_c^+(cud)$. For $s > (4.0 \text{ GeV})^2$, the hidden-charm process $\gamma^*p \to J/\Psi + p$ remains possible. Analysis such as that described in Refs. [45,46] for $\gamma^*\gamma \to \rho\rho$ may then be performed.

But electroproduction also provides a direct way to produce exotic hadronic states such as the $Z_c^+(\bar{c}c\bar{d}u)$ tetraquark and the $\mathcal{O}(\bar{c}cuuduud)$ octoquark. For example, if a low-mass *pentaquark* $\mathcal{P}(\bar{c}cuud)$ exists below the $\bar{D}\Lambda_c$ threshold, then the process $ep \to e'\mathcal{P}(\bar{c}cuud)$ would occur, with $\mathcal{P}$ manifesting as a peak in the missing-mass (M_X) distribution of $ep \to e'X$. If $4.2 \text{ GeV} > m_\mathcal{P} > 4.0 \text{ GeV}$, then $\mathcal{P}$ would likely appear as a resonance decaying to $J/\psi + p$. And if $m_\mathcal{P} < 4.0 \text{ GeV}$, then $\mathcal{P}$ would exist as a bound state and appear as a sharp peak in the M_X distribution of $ep \to e'X$.

Alternately, one can perform an indirect search for a $\mathcal{P}$ thus produced; if $\mathcal{P}$ has a sufficiently long lifetime, it can collide with a second nucleon in a fixed target downstream from the initial collision point and materialize as a hidden- or open-charm state,

$$e + p \to e' + \mathcal{P},$$
$$\Downarrow$$
$$\mathcal{P} + N \to NJ/\psi, \bar{D}\Lambda_c. \tag{10}$$

Even more exotic states could be produced this way, if they indeed exist. A very interesting experimental signal dating back three decades [47] is the surprisingly large spin-spin correlation in pp elastic scattering, sometimes called the "Krisch effect." The polarized cross sections for scattering of protons with spins normal to the scattering plane have a remarkable asymmetry: At $s = (5 \text{ GeV})^2$,

$$\frac{\frac{d\sigma}{dt}(p_\uparrow p_\uparrow \to pp)}{\frac{d\sigma}{dt}(p_\uparrow p_\downarrow \to pp)} \simeq 4. \tag{11}$$

Such an asymmetry is strongly at odds with the expectations of pQCD, since at such high energies one expects spin differences to be washed out. Note, however, that such an effect can occur if the high-energy process interferes with a resonance lying right at $s = (5 \text{ GeV})^2$. Since the baryon-number $B = 2$ hidden-charm threshold is $\simeq 2m_p + m_{J/\Psi} = 5.0$ GeV, the production of an *octoquark* state $\mathcal{O}^{++} = |\bar{c}cuuduud\rangle$ has been proposed as a resolution [48]. Should such a state exist, one can use the electroproduction methods to search for its isospin partner $\mathcal{O}^+$ in the missing-mass spectrum of collisions on a deuteron d target,

$$e + d \to e' + \mathcal{O}^+(\bar{c}cuudduud), \tag{12}$$

which would appear in the missing-mass spectrum of $ed \to e'X$. In this case, the open- and hidden-charm thresholds lie at $M_X \simeq m_{\Lambda_c} + m_n + m_{D^0} = 5.1$ GeV and $m_p + m_n + m_{J/\Psi} = 5.0$ GeV, respectively, and comments analogous to the ones above for the $\mathcal{P}$, regarding whether the $\mathcal{O}^+$ would appear as a resonance or a bound state, apply here as well. Furthermore, if $\mathcal{O}^+$ is long-lived, it could decay to $J/\psi + p + n$ or could be dissociated as $\mathcal{O}^+ + A \to J/\psi + p + n + A'$ in subsequent collisions in a nuclear target.

These methods for finding exotic states in electroproduction can be extended to the production of *nuclear-bound quarkonium* [49,50] states $[\bar{c}c]A$, in which quarkonium is bound to nuclei by QCD van der Waals interactions—the nuclear analogue to hadro-charmonium. Such states could be produced in $eA \to e'X$ collisions.

What of the original tetraquarks? If the outgoing baryon in the electroproduction process $ep \to e'X$ is a p, then the extra inelastically-produced state is neutral. Since the $X(3872)$ is the best-characterized exotic state, perhaps a natural place to start the electroproduction program is by observing the process $ep \to e'p'X(3872)$ near its $s = (4.8 \text{ GeV})^2$ threshold. As for the charged tetraquarks such as $Z_c^+(4430)$, a charge-exchange electroproduction process $ep \to e'nZ_c^+(4430)$ is required, which at its core can be considered a $\gamma^*\pi^+$ collision. Neutral $\mathcal{P}$ and $\mathcal{O}$ states

could be created analogously, at correspondingly higher thresholds.

To date, all of the observed exotic candidates contain either hidden charm or bottom. Is incorporating heavy quarks a necessary feature of observable tetraquark states? The original δ-$\bar{\delta}$ mechanism presented in Ref. [16] depends upon having sufficient energy release in the production process that the δ and $\bar{\delta}$ separate far enough so as to be considered distinct particles; perhaps for lighter systems the diquarks try to form but dissolve immediately into meson pairs. In any case, all of the searches described above apply to the strange sector as well, such as a $uud\bar{s}s$ pentaquark. One possible result is that hidden-charm exotics emerge naturally from these electroproduction experiments but hidden-strangeness ones do not.

As an intermediate case, one can also study open-charm, open-strangeness states using $ep \to e'\Lambda X$ as a $\gamma^*K^+(u\bar{s})$ collider. In this case, one would produce charged charmed-strange pentaquarks such as $\bar{c}c\bar{s}u$. Here, one would look for peaks in the M_X distribution after tagging the final-state electron e' and Λ baryon.

Another interesting case is e^+e^- annihilation to four heavy quarks. For example, just above the $\bar{c}c\bar{c}c$ threshold, one can produce $e^+e^- \to J/\psi\eta_c$. Just below threshold, the four heavy quarks can rearrange to form a exclusively charmed tetraquark as a bound state of $[cc]_{\bar{3}_C}$ and $[\bar{c}\bar{c}]_{3_C}$ diquarks.

V. DISCUSSION AND CONCLUSIONS

We have proposed a number of experimentally straightforward and feasible tests of the exotic nature of the recently discovered tetraquark candidates such as $X(3872)$ and $Z_c^+(4430)$. Scenarios in which the four quarks independently carry $\mathcal{O}(1)$ fractions of the hadron momentum, and scenarios in which the four quarks are segregated into tightly bound diquark and antidiquark pairs, have been explored utilizing constituent counting rules, which are normally limited to tests at high momentum transfer. By forming ratios of cross sections to different exclusive states, one can extend the usefulness of the counting rules to the threshold domain for producing heavy exotic states.

We have also discussed several promising methods to produce other exotic multiquark states in near-threshold electroproduction and electron-positron annihilation. One can also confirm the existence of known exotic $\bar{c}c$ states by creating them just above the threshold for production of the charm-quark pair, where the limited phase space makes the formation of exotics likely, through coalescing the soft heavy quarks with the light valence quarks moving at similar rapidities.

The exotic hadron production processes discussed in this paper lead to many new experimental opportunities at e^+e^- colliders such as BES and Belle, at electroproduction facilities such as the 12 GeV Upgrade of JLab and proposed

ep colliders, and at new hadronic beam facilities such as P̄ANDA at FAIR and AFTER@LHC.

QCD, now in its fifth decade, continues to present us with surprises. Even the full extent of its basic hadronic spectrum remains an open question. However, given the results of sufficiently ingenious experiments, an ever deepening understanding of the theory and its novel features will inevitably follow.

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