Degenerate spectrum in the neutrino mass anarchy with Wishart matrices and implications for $0\nu\beta\beta$ and δ_{CP}

Kwang Sik Jeong,[1,2,*] Naoya Kitajima,[3,†] and Fuminobu Takahashi[3,4,‡]

[1]*Department of Physics, Pusan National University, Busan 609-735, Korea*
[2]*Center for Theoretical Physics of the Universe, IBS, Daejeon 305-811, Korea*
[3]*Department of Physics, Tohoku University, Sendai 980-8578, Japan*
[4]*Kavli IPMU, TODIAS, University of Tokyo, Kashiwa 277-8583, Japan*

(Received 22 January 2015; revised manuscript received 19 May 2015; published 16 June 2015)

We show that a degenerate neutrino mass spectrum can be realized in the neutrino mass anarchy hypothesis, if the neutrino Yukawa and right-handed neutrino mass matrices are given by the Wishart matrix, i.e., products of $N \times 3$ rectangular random matrices, whose eigenvalue distribution tends to degenerate for large N. The mixing angle and charge-parity (CP) phase distributions are determined by either the Haar measure of U(3) or that of SO(3). We study how large N is allowed to be without tension with the observed neutrino mass squared differences and find that the predicted value of m_{ee} can be within the reach of future $0\nu\beta\beta$ experiments especially for N on the high side of the allowed range.

DOI: 10.1103/PhysRevD.91.113010 PACS numbers: 14.60.Pq

I. INTRODUCTION

The standard model (SM) of particle physics has been overwhelmingly successful for decades, and the long-sought Higgs boson, the last missing piece of the SM, was finally discovered at the LHC [1,2]. Despite the great success of the SM, there are many puzzles left unanswered; one of them is the origin of the flavor structure.

While neutrinos are massless in the SM, atmospheric and solar neutrino oscillation experiments revealed that neutrinos have tiny but nonzero masses (see, e.g., Refs. [3,4] for the latest results). In particular, a mild mass hierarchy and large mixing angles for the neutrino sector are in sharp contrast with quarks and charged leptons. If we are to understand the neutrino flavor structure based on symmetry principles, it seems to require rather contrived flavor models.[1] The observed large mixing angles rather suggest a structureless mass matrix for neutrinos, implying that all the neutrino species have the same quantum number.

The squared mass differences and mixing angles are measured by various neutrino oscillation experiments [6–11], the recent best-fit values for normal (inverted) hierarchy are [3]

$$\begin{aligned} \Delta m_{21}^2 &= 7.60 \times 10^{-5}\ \mathrm{eV}^2, \\ |\Delta m_{31}^2| &= 2.48(2.38) \times 10^{-3}\ \mathrm{eV}^2 \\ \sin^2\theta_{12} &= 0.323, \qquad \sin^2\theta_{23} = 0.567(0.573), \\ \sin^2\theta_{13} &= 0.0234(0.0240), \end{aligned} \tag{1}$$

and the favored value of the Dirac charge-parity (CP) phase is around $3\pi/2$. Besides the neutrino oscillation experiments, further information can be obtained from the cosmic microwave background (CMB) observations and the neutrinoless double beta decay ($0\nu\beta\beta$) experiments. In particular, the CMB observations by Planck, WMAP, and other ground-based experiments set the upper limit on the sum of the neutrino masses as $\sum m_i < 0.66$ eV (95% C.L.) [12].

One of the attractive explanations for the observed large neutrino mixing is the neutrino mass anarchy [13–16], which gained momentum especially after the discovery of a nonzero value of θ_{13} by the Daya-Bay experiment [6]. The basic idea of the neutrino mass anarchy is simple. Suppose that all the Yukawa couplings and/or right-handed neutrino masses are determined by a UV theory, which has a sufficiently large landscape of vacua. If each coupling is allowed to take values of order unity in the landscape, the Yukawa couplings and/or right-handed neutrino masses may be modeled by some functions of random matrices. Note that, as emphasized in Ref. [14], the neutrino mass anarchy tells us nothing about the weighting functions, and therefore, one has to choose an appropriate one to evaluate the probability distribution of the neutrino masses. The simplest and the most studied form is the linear measure

$$h, M \sim X, \tag{2}$$

*ksjeong@pusan.ac.kr
†kitajima@tuhep.phys.tohoku.ac.jp
‡fumi@tuhep.phys.tohoku.ac.jp

[1]While it is possible to understand the hierarchical mass pattern of quarks and charged leptons based on symmetry principles, a variety of flavor symmetries and charge assignments are allowed. For an alternative approach without flavor symmetry, see, e.g., Ref. [5].

where the neutrino Yukawa matrix h as well as the right-handed neutrino mass matrix M are proportional to 3×3 random matrices represented by X. Phenomenological and cosmological aspects of the neutrino mass anarchy have been studied; e.g., two of the present authors (K. S. J. and F. T.) studied the implications of neutrino mass anarchy for leptogenesis in Ref. [17], and it was also recently revisited in Ref. [18]. See also Refs. [19–21] for phenomenological study of the neutrino mass anarchy with a various number of right-handed neutrinos.

In the neutrino mass anarchy hypothesis, the mixing angle and CP phase distributions are determined by the invariant Haar measure of the underlying symmetry group such as U(3) or SO(3) [14], independently of the adopted weighting function, and so, there are rather robust predictions. Interestingly, the observed large mixing angles can be nicely explained in the neutrino mass anarchy [16].[2] On the other hand, the neutrino mass spectrum depends sensitively on the weighting functions. In the case of the linear measure, normal mass hierarchy is highly favored over the inverted or quasidegenerate one. In addition, the observed mild hierarchy of the mass squared differences can be nicely explained by the neutrino mass anarchy together with the seesaw mechanism [13,14]. The estimated m_{ee} turned out to be too small to be detected by future $0\nu\beta\beta$ experiments [17], but this result can be modified for more general measure functions [24].[3]

In this article we study the next simplest possibility: the neutrino Yukawa couplings and the right-handed neutrino masses are given by the random matrix squared, or more precisely, the Wishart matrices

$$h, M \sim X^\dagger X \quad \text{or} \quad X^T X, \tag{3}$$

where X represents $N \times 3$ complex or real random matrices. In general, N does not have to be equal to 3. For $N > 3$, the neutrino Yukawa and right-handed neutrino mass matrices are given by products of rectangular matrices. We shall see that the observed neutrino mass squared differences can be explained for $N \lesssim 35$. Interestingly, the eigenvalue distribution of the Wishart matrix tends to be degenerate for large N.[4] Therefore, the quasidegenerate neutrino mass spectrum can be realized in the neutrino mass anarchy with the Wishart matrix if $N \gg 3$, which should be contrasted to the case of the linear measure (2). We will discuss its implications for the $0\nu\beta\beta$ experiments. We will also show that the mixing angle and CP phase distributions of our scenario are determined by either the Haar measure of U(3) or that of SO(3).

The rest of this article is organized as follows. In Sec. II we first explain our setup and see how the neutrino mass spectrum changes as the size of the rectangular matrices N increases. Then we study the implication for the Dirac CP phase and the $0\nu\beta\beta$ experiments. The last section is devoted to discussion and conclusions.

II. NEUTRINO MASS ANARCHY

In this section, we consider the neutrino mass anarchy based on the Wishart matrices as a simple extension of the linear measure. We focus on the case of the Majorana neutrino mass with the seesaw mechanism [25–28].[5]

A. Preliminaries

The seesaw Lagrangian is given by

$$\mathcal{L} = f_{ij}\bar{e}_{Ri}\ell_j\tilde{H} + h_{ij}\bar{N}_i\ell_j H + \frac{1}{2}M_{ij}\bar{N}_i\bar{N}_j + \text{H.c.}, \tag{4}$$

where ℓ, $H(\tilde{H})$, e_R, and N are, respectively, the left-handed lepton doublet, the Higgs doublet [its SU(2) conjugate], the right-handed charged leptons, and the right-handed neutrinos; f_{ij}, h_{ij} are Yukawa matrices for charged leptons and neutrinos, respectively; and M_{ij} represents the Majorana mass matrix for right-handed neutrinos. The subscripts represent the generation, $i, j = 1, 2, 3$.

Let us first diagonalize the charged lepton Yukawa matrix as

$$f = U_{fR}^\dagger D_e U_{fL} \tag{5}$$

with

$$D_e \equiv \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \tag{6}$$

where U_{fR} and U_{fL} are unitary matrices, and $y_{e,\mu,\tau}(>0)$ denote the charged lepton Yukawa couplings.[6] In the basis where the charged lepton Yukawa matrix is diagonal, the Lagrangian becomes

[2] See, however, Refs. [22,23] and references therein.

[3] Our analysis is different from Ref. [24] in which the adopted measure is not applicable to the case of the seesaw mechanism with neutrino mass anarchy.

[4] A similar behavior can be seen in the singular value distributions of the $n \times 3$ neutrino Yukawa matrix if one introduces $n(>3)$ right-handed neutrinos [20]. However, the eigenvalues of the $n \times n$ Majorana mass matrix obeying the linear measure are more repulsive than in the case of the Wishart matrix. Thus, while the resultant neutrino masses are also degenerate to some extent for large n, the degeneracy is weaker than in the case of the Wishart matrix.

[5] Our setup can be straightforwardly applied to the case of the Dirac neutrino mass, and most of our results (except for the $0\nu\beta\beta$) will remain qualitatively valid. In particular, the quasidegenerate spectrum can be realized.

[6] Throughout this article we do not try to interpret the charged lepton mass hierarchy in our scheme because there could be additional selection (anthropic) effects.

$$\mathcal{L} = (y_\alpha \delta_{\alpha\beta}) \bar{e}_{R\alpha} \ell_\beta \tilde{H} + h_{i\alpha} \bar{N}_i \ell_\alpha H + \frac{1}{2} M_{ij} \bar{N}_i \bar{N}_j + \text{H.c.}, \tag{7}$$

where α, β run over the lepton flavor indices (e, μ, τ), and we have defined

$$h_{i\alpha} \equiv h_{ij} (U_{fL}^\dagger)_{j\alpha}, \tag{8}$$

$$\ell_\alpha \equiv (U_{fL})_{\alpha i} \ell_i, \tag{9}$$

$$e_{R\alpha} \equiv (U_{fR})_{\alpha i} e_{Ri}. \tag{10}$$

After the Higgs field acquires the vacuum expectation value (VEV), one obtains the effective Lagrangian for active neutrinos by integrating out the heavy right-handed neutrinos,

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2} (m_\nu)_{\alpha\beta} \nu_\alpha \nu_\beta + \text{H.c.}, \tag{11}$$

where ν_α are the light left-handed neutrinos, and the neutrino mass matrix is given by

$$(m_\nu)_{\alpha\beta} = v^2 (h^T)_{\alpha i} (M^{-1})_{ij} h_{j\beta} \tag{12}$$

with $v \simeq 174$ GeV being the VEV of the Higgs field. The neutrino mass matrix m_ν is generically a complex-valued symmetric matrix, and it can be diagonalized by a unitary matrix U_{MNS} as

$$m_\nu = U_{\text{MNS}}^* \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_{\text{MNS}}^\dagger. \tag{13}$$

Here m_1, m_2, and m_3 are real and positive values with $m_1 < m_2 < m_3$. This numbering is for the normal hierarchy, whereas in the inverted hierarchy case, one should relabel them as $m_3 \to m_2$, $m_2 \to m_1$, and $m_1 \to m_3$ in order to compare our results with the observations (1). In fact, however, mostly either normal or quasidegenerate (normal-ordering) mass hierarchy is realized in our scheme, and so, the inverted hierarchy case is practically negligible.

The neutrino oscillation experiments provide us with only the squared mass differences, $\Delta m_{ij}^2 = m_i^2 - m_j^2$. To compare our results with observations, we use the dimensionless parameter R defined by the ratio of the squared mass difference between the heaviest and the second heaviest neutrinos to that between the second heaviest and the lightest ones,

$$R = \frac{\Delta m_{21}^2}{\Delta m_{32}^2} \ (\text{normal}) \quad \text{or} \quad \frac{\Delta m_{13}^2}{\Delta m_{21}^2} \ (\text{inverted}). \tag{14}$$

The observed value of R is given by $R \sim 1/30$ for normal-ordering hierarchy and $R \sim 30$ for inverted hierarchy.

The neutrino mixing matrix U_{MNS} can be expressed in terms of the mixing angles, θ_{ij}, with $(i, j) = (1, 2), (2, 3)$, and (3,1), and the Dirac and Majorana CP phases, δ, α_{21}, and α_{31} after absorbing the unphysical phases by redefinition of the fields, and it is conventionally written as

$$U_{\text{MNS}} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix} \times \text{diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}), \tag{15}$$

where we abbreviate $\sin\theta_{ij}$ and $\cos\theta_{ij}$ as s_{ij} and c_{ij}, respectively, and the mixing angles and the CP phases satisfy $\theta_{ij} \in [0, \pi/2)$ and $\delta, \alpha_{21}, \alpha_{31}, \in [0, 2\pi)$.

B. Neutrino mass anarchy based on the Wishart matrices

In the neutrino mass anarchy hypothesis with the linear measure, both $h_{i\alpha}$ and M_{ij} are taken to be proportional to 3×3 complex-valued (or real-valued) random matrices [cf. Eq. (2)]. The unitary matrix U_{fL} does not affect the probability distributions of the mixing angles and the CP phases, as they are fixed by the Haar measure of U(3) [SO(3)]. This is the simplest possibility, but it remains unknown how the randomness for these matrices is originated in the landscape. In fact, there are various other basis-independent choices for these matrices. Here we consider the next-to-simplest setup, in which the neutrino Yukawa matrix and the Majorana mass matrix consist of products of random matrices,[7]

$$h_{ij} = \frac{y_\nu}{N} (F^\dagger F)_{ij}, \qquad M_{ij} = \frac{M_0}{2N} (G^\dagger G + G^T G^*)_{ij}, \tag{16}$$

where F and G are $N \times 3$ complex (or real) random matrices of order unity, and y_ν and M_0 represent the typical neutrino Yukawa couplings and the right-handed neutrino masses. For $y_\nu = \mathcal{O}(1)$, $M_0 \sim 10^{15}$ GeV is suggested by the neutrino oscillation experiments and the seesaw

[7]If the neutrino Yukawa couplings and the right-handed neutrino masses are given by $h \sim F^T F$ and $M \propto G^T G$, where F and G are complex-valued $N \times 3$ random matrices of order unity, there is no degeneracy in the eigenvalues. We do not pursue this case in this article.

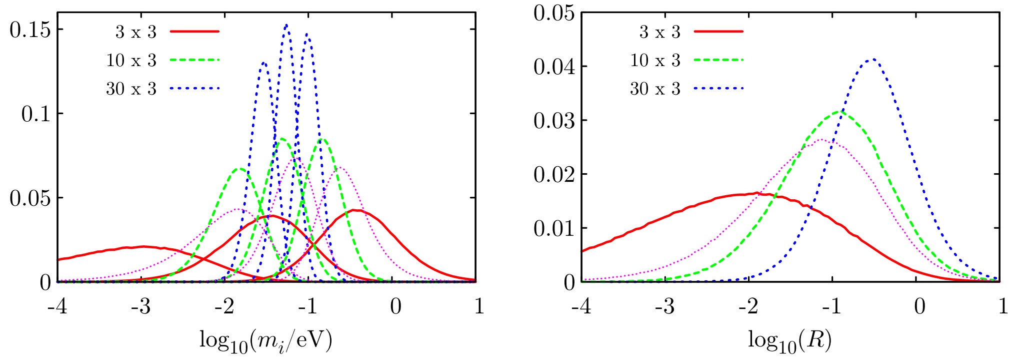

FIG. 1 (color online). Probability distributions of the neutrino masses (left) and R (right) for complex Wishart matrices are shown. The solid red, dashed green, and dotted blue lines correspond to the cases with $N = 3$, 10, and 30, respectively, while the magenta lines represent the anarchy with the linear measure. Here we have taken $y_\nu = 1$ and $M_0 = 10^{15}$ GeV.

mechanism. Note that the above form of the neutrino Yukawa couplings is given in the original basis, and one has to multiply it with the unitary matrix U_{fL} in the basis where the charged lepton Yukawa matrix is diagonal [see Eq. (8)]. This, however, does not affect the final mixing and CP phase distributions just as in the previous case.[8]

The above form of h_{ij} and M_{ij} imply that they are given by the so-called Wishart matrix. Specifically, we will take F and G as a chiral Gaussian unitary (orthogonal) ensemble, i.e., the Gaussian measure, where each element follows a complex-valued (real-valued) Gaussian distribution with zero mean and a variance of unity. In this case, the basis independence is automatically assured [18]. The measure for the eigenvalues (λ_i) of the complex and real Wishart matrix composed of $N \times 3$ random matrices are, respectively, known as

$$\prod_{i>j}^{3} |\lambda_i - \lambda_j|^2 \prod_{i=1}^{3} \lambda_i^{N-3} d\lambda_i \quad \text{(complex)},$$
$$\prod_{i>j}^{3} |\lambda_i - \lambda_j| \prod_{i=1}^{3} \lambda_i^{(N-4)/2} d\lambda_i \quad \text{(real)}. \tag{17}$$

The first factor $|\lambda_i - \lambda_j|$ represents the repulsive nature, and this effect is (partially) canceled by the second factor λ_i^{N-3} or $\lambda_i^{(N-4)/2}$. For large N, the eigenvalues of h and M tend to be highly degenerate due to the second factor proportional to λ_i^{N-3} or $\lambda_i^{(N-4)/2}$.[9] As a result, the light neutrino masses are also expected to be degenerate, which is difficult to realize in the case of the linear measure. As we shall see, however, N cannot be arbitrarily large because the predicted value of R tends to be too large compared to the observed value, $R \sim 1/30$, for large N.

C. Mass spectrum, mixing angles, and CP phases

We have performed numerical calculations of the neutrino mass anarchy based on the Wishart matrices. Specifically, we have generated 10^6 $N \times 3$ complex (real) random matrices, F and G, to obtain the distributions of neutrino masses, mixing angles, and CP violating phases. The results are shown in Figs. 1 and 2 corresponding to the complex and real Wishart matrices, respectively. We have varied N as $N = 3$ (solid red curves), $N = 10$ (dashed green curves), $N = 30$ (dotted blue curves), and we have set $y_\nu = 1$ and $M_0 = 10^{15}$ GeV. Note that the distribution of R in the right panel is independent of the choice of y_ν and M_0. For comparison, we show the results of the neutrino anarchy with the linear measure as the small-dotted magenta lines in each figure. One can see that the neutrino mass distribution (Figs. 1 and 2) tends to be more degenerate as N increases. The probability distribution of R is suppressed at $R > 1$, implying that the inverted hierarchy ($R \sim 30$) is highly disfavored. Thus, the neutrino mass hierarchy is either normal or quasidegenerate (normal ordering) in the anarchy based on the Wishart matrices.

Figure 3 shows the mean value of R as a function of N with 1 and 2σ error bands. It shows that the normal hierarchy ($R \sim 1/30$) is preferred over the inverted hierarchy ($R \sim 30$) and N is bounded from above as $N \lesssim 35$ ($N \lesssim 70$ for real Wishart matrices) in order to be consistent with the observations. This implies that, even if one considers the Wishart matrices, there is an upper bound on the degeneracy of the neutrino masses. We will discuss its implications for the $0\nu\beta\beta$ experiments in the next subsection.

[8]In general, any Yukawa matrix can be written as a product of a Hermitian matrix and a unitary matrix by the polar decomposition theorem. Here we consider a case where the Hermitian matrix is of the Wishart-type random matrix.

[9]Instead of the Gaussian measure, one can adopt an arbitrary basis independent measure for the Wishart matrix. For instance, one may multiply $(\mathrm{tr}[G^\dagger G])^p$ with the measure. In this case, the eigenvalue distributions are modified, but the eigenvalues remain to be degenerate for large N as long as the measure contains positive powers of the eigenvalues.

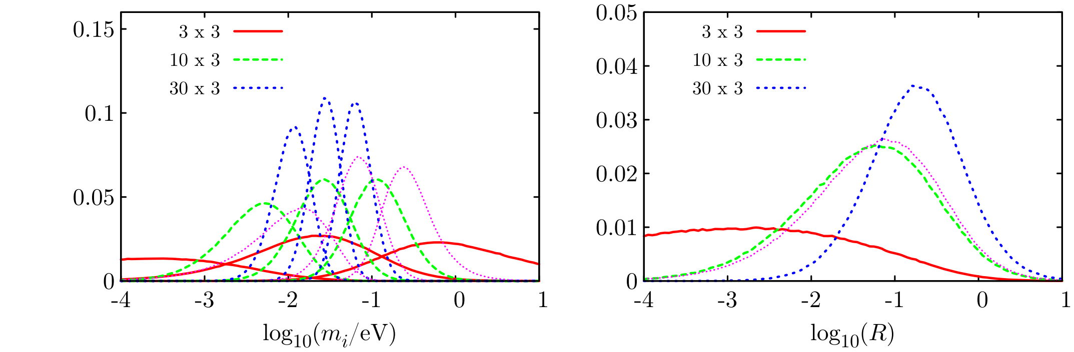

FIG. 2 (color online). Same as Fig. 1 but for real Wishart matrices.

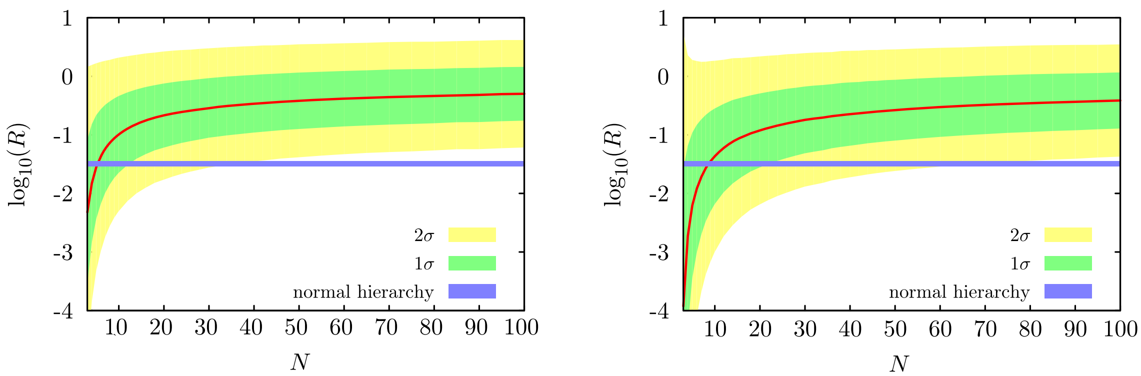

FIG. 3 (color online). The mean value of R with 1σ (green region) and 2σ (yellow region) error as a function of N corresponding to complex (left) and real (right) random matrices. The blue-shaded region represents the experimental value with 2σ uncertainty for the normal hierarchy.

We can also see from Fig. 4 that the mixing angle and CP phase distributions are determined by the Haar measure of U(3). If the random matrices F as well as the charged lepton Yukawa matrix are taken to be real, the resultant distribution is given by the Haar measure of SO(3). (The right-handed neutrino mass matrix is real by construction.) In this case the Majorana CP phases vanish, and the Dirac CP phase δ takes a value of either 0 or π. We note that the

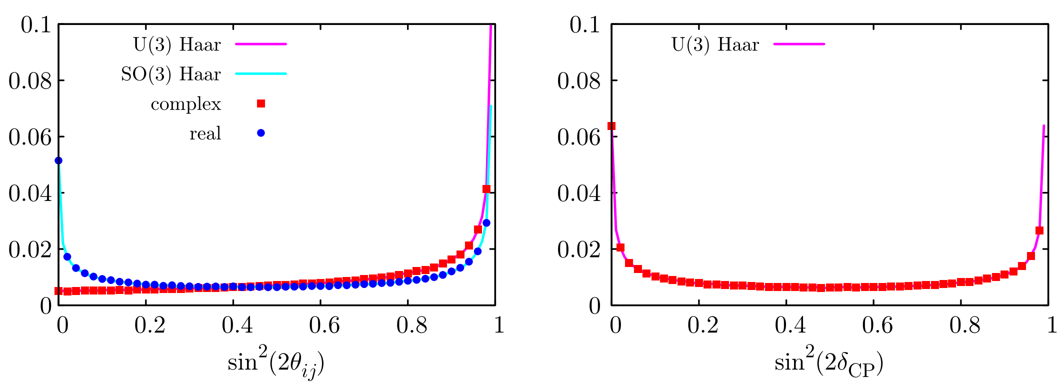

FIG. 4 (color online). Probability distributions of mixing angles (left) and CP violating phases (right). θ_{ij} represents θ_{12}, θ_{23}, and θ_{13} and δ_{CP} represents δ, α_{21}, and α_{31}. The red squares and blue circles correspond to complex and real Wishart matrices, respectively, and the magenta and cyan lines correspond to the U(3) and SO(3) Haar measures, respectively. We have taken $N = 30$, but the distributions are the same for a different value of N.

currently favored value of δ is about $3\pi/2$ according to Ref. [4], which corresponds to $\sin^2 2\delta = 0$. Interestingly, the U(3) Haar measure results in the probability distribution of δ peaked at $\sin^2 2\delta = 0$.

D. Neutrinoless double beta decay

The Majorana nature of the neutrinos can be probed by the $0\nu\beta\beta$ experiments, which is sensitive to m_{ee} defined by

$$m_{ee} \equiv \left| \sum_{i=1}^{3} (U_{\rm MNS})^2_{ei} m_i \right| = |m_1(c_{12}c_{13})^2 + m_2(s_{12}c_{13})^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i(\alpha_{31}-2\delta)}|. \quad (18)$$

The current upper bound on m_{ee} by the GERDA experiment using ^{76}Ge reads [29]

$$m_{ee} \lesssim (0.2\text{–}0.4)\ \text{eV} \quad (90\%\ \text{C.L.}). \quad (19)$$

A similar bound was obtained by EXO-200 using ^{136}Xe [30], and a slightly better bound has been recently obtained by the KamLAND-Zen experiment as [31]

$$m_{ee} \lesssim (0.14\text{–}0.28)\ \text{eV} \quad (90\%\ \text{C.L.}). \quad (20)$$

The next-generation experiment is expected to reach the level of $m_{ee} \simeq 0.01$ eV [32].

We show the predicted range of m_{ee} in the m_{ee}-m_1 plane in Fig. 5 (complex Wishart) and Fig. 6 (real Wishart), where we have taken $N = 10$ and 30. We have generated 10^7 Wishart matrices and extracted the subset satisfying the observed R (within 2σ), and M_0 is adjusted to realize the best fit value of Δm_{21}^2. The mixing angles are also adjusted to the best fit values. The thick red (blue) lines are contours of equal probability in which 68% (95%) of the data points are contained. For comparison, we similarly show the

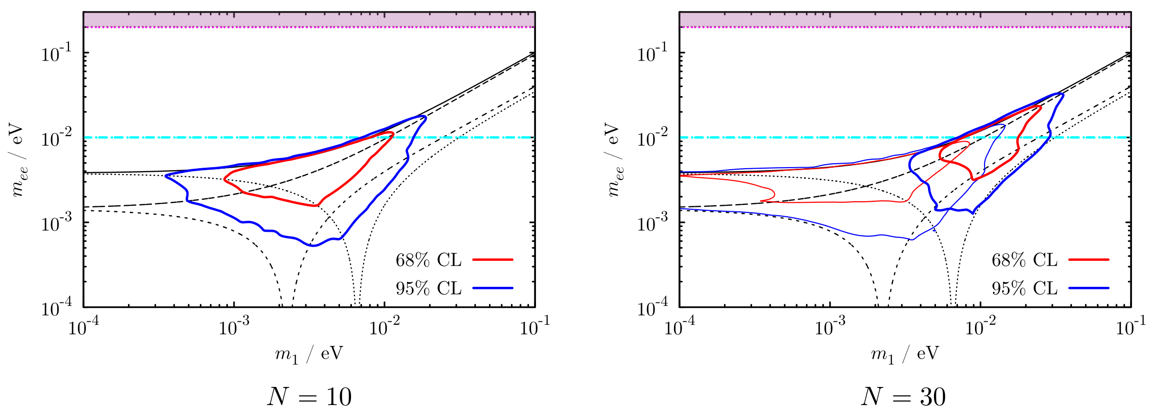

FIG. 5 (color online). Contours of probability distribution on the m_{ee}-m_1 plane for $N = 10$ (left) and $N = 30$ (right), where the mixing angles are set to be the best-fit values. The red and blue contours correspond to 68% and 95% C.L., respectively, and for comparison, the case of the linear measure is shown by the thin red and blue contours in the right panel. The black curves with various line types correspond to the normal hierarchy for best fit values of the neutrino mass differences and mixing angles with vanishing CP phases; $(e^{i\alpha_{21}}, e^{i(\alpha_{31}-2\delta)}) = (+1,+1), (+1,-1), (-1,+1)$, and $(-1,-1)$ from top to bottom at $m_1 \gtrsim 10^{-2}$ eV. The horizontal dashed (cyan) line represents the sensitivity of a future experiment, while the shaded (magenta) region is excluded by the current experiments.

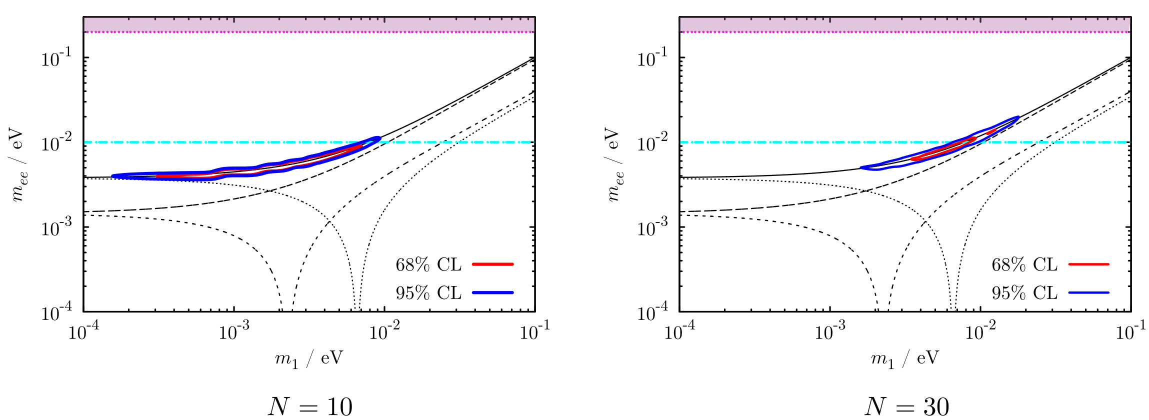

FIG. 6 (color online). Same as Fig. 5 but for real Wishart matrices. Here we have chosen the case of $(e^{i\alpha_{21}}, e^{i(\alpha_{31}-2\delta)}) = (+1,+1)$.

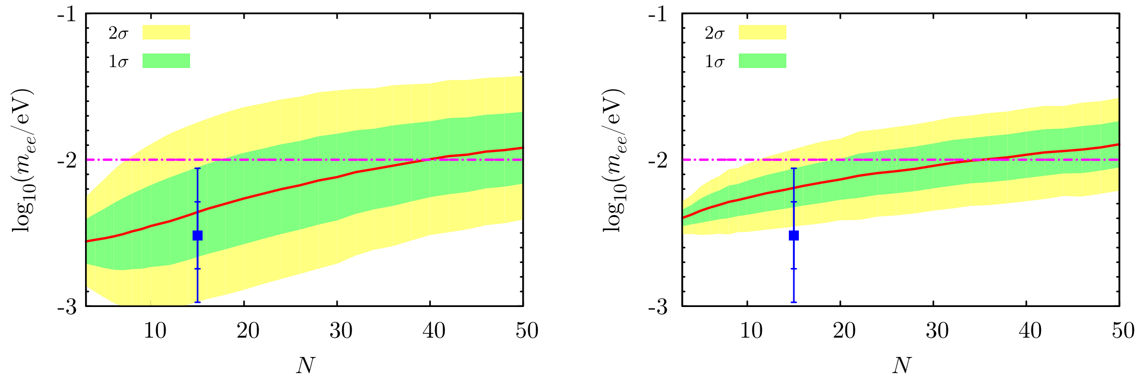

FIG. 7 (color online). The mean value of m_{ee} (red solid line) with 1σ (green) and 2σ (yellow) uncertainties as a function of N, for complex (left) and real (right) random matrices. The horizontal dashed (magenta) line represents the sensitivity of a future experiment. The blue point with an error bar represents the one for the linear measure with 1σ and 2σ uncertainties. (The position in the horizontal axis is arbitrary.)

prediction of the linear measure case as thin red (blue) lines in the right panel of Fig. 5. The black lines with various line types represent m_{ee} for best-fit values of the neutrino mass differences and mixing angles with vanishing CP-violating phases: $(e^{i\alpha_{21}}, e^{i(\alpha_{31}-2\delta)}) = (+1,+1)$, $(+1,-1)$, $(-1,+1)$, and $(-1,-1)$ from top to bottom at $m_1 \gtrsim 10^{-2}$ eV. The horizontal dashed (cyan) line represents the sensitivity of the future experiment, while the shaded (magenta) region is excluded by the current experiments. We also show the statistical mean value of $\log_{10}(m_{ee}/\mathrm{eV})$ with 1 and 2σ uncertainties as a function of N in Fig. 7. Since a quasidegenerate mass spectrum is more likely for large values of N, relatively large $m_{ee}(\gtrsim 0.01$ eV$)$ is realized with a greater probability compared to the case of the linear measure, and a larger fraction of the parameter space will be accessible by the near future experiments. Note, however, that, since N is bounded from above in order to be consistent with observations, there is an upper bound on the neutrino mass degeneracy. As a result, m_{ee} cannot be arbitrarily large even in the case with the Wishart matrices (i.e., $m_{ee} \lesssim$ a few tens meV).

III. DISCUSSION AND CONCLUSIONS

In this article we have studied in detail the neutrino mass anarchy hypothesis with the Wishart matrices, where the neutrino Yukawa matrices and right-handed neutrino masses are given by products of $N \times 3$ random rectangular matrices. The mixing angle and CP phase distributions are determined by the Haar measure of U(3) or SO(3), depending on whether the Wishart matrices are complex or real. Interestingly, for $N \gg 3$, the eigenvalues of the Wishart matrix tend to be confined in a narrow range. As a result, compared to the case of the neutrino mass anarchy with the linear measure, the neutrino mass spectrum becomes more compressed; in particular, a quasidegenerate (normal-ordering) neutrino mass spectrum can easily be realized without resorting to introducing additional constraints (such as successful leptogenesis [17,18]) or an *ad hoc* choice of the weighting function. We have studied how large N is allowed to be in order to give a reasonable fit to the observed neutrino mass squared differences and found that N is allowed to be as large as 35 for complex Wishart matrices and 70 for real Wishart matrices. We have also studied implications of our scenario for the $0\nu\beta\beta$ experiment and have shown that the predicted m_{ee} can be within the reach of the future experiments with a larger probability than the case of the linear measure, especially if N is on the high side of the allowed range.

Let us discuss if we can understand the structure of the couplings based on symmetry principles. First let us regard the random matrices F and G as moduli fields whose VEVs can take various values determined by a UV theory. To be specific we assume that all the couplings are real and impose $\mathrm{O(N)} \times \mathrm{O(3)}$ flavor symmetry, under which the ordinary leptons and right-handed neutrinos transform as $\mathbf{1} \times \mathbf{3}$ while F and G transform as $\mathbf{N} \times \mathbf{3}$. The lepton doublets and the right-handed neutrinos are assumed to transform as 3 under O(3). Then, the following combinations

$$F^T F, \quad G^T G \tag{21}$$

are 3×3 matrices, which transform as bifundamental under $O(3)$. Once each component of F and G develops a nonzero VEV, the above matrices give rise to the neutrino Yukawa couplings and the Majorana masses. If the UV theory is sufficiently complicated, the VEVs of F and G may be modeled by random matrices. Thus, the above combination $F^T F$ and $G^T G$ plays the same role of the simple random matrix in the case of the linear measure. One can see that the above setup is more complicated than in the case of the linear measure.

In principle one can add a unit matrix to the Yukawa and the right-handed neutrino matrices, satisfying the flavor symmetries. If the contribution of the unit matrix is negligible compared to that of F and G, our results in the text approximately remain unchanged in this case. On the other hand, if the unit matrix contribution becomes significant, the mass eigenvalues become more degenerate, whereas the mixing angle distribution is still determined by the SO(3) Haar measure.[10]

We would like to emphasize here that the above argument explains only the structure of the interactions, not the reason why the measure is proportional to the random matrix squared. The essence of the neutrino mass anarchy hypothesis is the (statistical) equivalence between different neutrino flavors, and it tells us nothing about the weighting measure functions. The simplest and most studied function is the linear measure, but there is no compelling reason to choose this measure other than simplicity. In general, the weighting measure could be some complicated function of the random matrices. In this sense, our choice of the measure is the next simplest possibility.

So far we have focused on the neutrino mixing, mass, and CP phase distributions in the neutrino mass anarchy with the Wishart matrices. It will be interesting to study cosmological aspects of our scenario, especially in context with leptogenesis, as an extension of the analysis of Ref. [17]. In particular, in contrast to the case of the linear measure, the right-handed neutrinos tend to be degenerate in mass, leading to resonant leptogenesis [33]. The typical mass difference scales as $(M_2 - M_1)/(M_2 + M_1) \sim 1/\sqrt{N}$, and so, we expect an enhancement of the lepton asymmetry by a factor of 5 or so for $N = 30$. If the value of N is different between the neutrino Yukawa and right-handed neutrino mass matrices, this factor may be even more enhanced. We, however, expect that it is hard to realize the enhancement by many orders of magnitude in our scenario because the eigenvalues still repel each other even in the limit of large N. This difficulty may be eased by allowing a contribution proportional to the unit matrix. We leave the detailed analysis of leptogenesis in this case for future work.

As pointed out in Refs. [13,14], one can impose a flavor symmetry without modifying the predictions for the light neutrino masses: for instance, we can introduce a flavor symmetry on the right-handed neutrinos. Then, while the right-handed neutrinos are hierarchical due to the nontrivial flavor charges, the light neutrinos remain degenerate.

We can consider a possibility that the neutrino Yukawa and the right-handed neutrino mass matrices are given by a more complicated function(s) of random matrices, such as the Wishart matrices squared, and so on. Alternatively, one may consider sparse random matrices. It may be interesting to study these possibilities and their implications for the neutrino masses and CP phases.

[10] This argument suggests another extension of the neutrino mass anarchy with the linear measure: one may add a unit matrix (with a numerical coefficient) to the neutrino Yukawa and the right-handed neutrino mass matrices, leading to degenerate mass spectra while the mixing angle and CP phase distribution are still given by the U(3) or SO(3) Haar measure.

ACKNOWLEDGMENTS

This work was supported by JSPS Grant-in-Aid for Young Scientists (B) (No. 24740135 [F. T.]), Scientific Research (A) (No. 26247042 [F. T.]), Scientific Research (B) (No. 26287039 [F. T.]), the Grant-in-Aid for Scientific Research on Innovative Areas (No. 23104008 [N. K., F. T.]), and Inoue Foundation for Science [F. T.]. This work was also supported by World Premier International Center Initiative (WPI Program), MEXT, Japan [FT]. K. S. J. was supported by IBS under Project Code IBS-R018-D1.

[1] G. Aad *et al.* (ATLAS Collaboration), Phys. Lett. B **716**, 1 (2012).
[2] S. Chatrchyan *et al.* (CMS Collaboration), Phys. Lett. B **716**, 30 (2012).
[3] D. V. Forero, M. Tortola, and J. W. F. Valle, Phys. Rev. D **90**, 093006 (2014).
[4] M. C. Gonzalez-Garcia, M. Maltoni, and T. Schwetz, J. High Energy Phys. 11 (2014) 052.
[5] L. J. Hall, M. P. Salem, and T. Watari, Phys. Rev. D **76**, 093001 (2007).
[6] F. P. An *et al.* (DAYA-BAY Collaboration), Phys. Rev. Lett. **108**, 171803 (2012).
[7] K. Abe *et al.* (T2K Collaboration), Phys. Rev. Lett. **107**, 041801 (2011).
[8] P. Adamson *et al.* (MINOS Collaboration), Phys. Rev. Lett. **107**, 181802 (2011).
[9] Y. Abe *et al.* (DOUBLE-CHOOZ Collaboration), Phys. Rev. Lett. **108**, 131801 (2012).
[10] Y. Abe *et al.* (Double Chooz Collaboration), Phys. Rev. D **86**, 052008 (2012).
[11] J. K. Ahn *et al.* (RENO Collaboration), Phys. Rev. Lett. **108**, 191802 (2012).
[12] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **571**, A16 (2014).
[13] L. J. Hall, H. Murayama, and N. Weiner, Phys. Rev. Lett. **84**, 2572 (2000).
[14] N. Haba and H. Murayama, Phys. Rev. D **63**, 053010 (2001).

[15] A. de Gouvêa and H. Murayama, Phys. Lett. B **573**, 94 (2003).
[16] A. de Gouvea and H. Murayama, arXiv:1204.1249.
[17] K. S. Jeong and F. Takahashi, J. High Energy Phys. 07 (2012) 170.
[18] X. Lu and H. Murayama, J. High Energy Phys. 08 (2014) 101.
[19] B. Feldstein and W. Klemm, Phys. Rev. D **85**, 053007 (2012).
[20] J. Heeck, Phys. Rev. D **86**, 093023 (2012).
[21] Y. Bai and G. Torroba, J. High Energy Phys. 12 (2012) 026].
[22] G. Altarelli, F. Feruglio, I. Masina, and L. Merlo, J. High Energy Phys. 11 (2012) 139.
[23] J. Bergstrom, D. Meloni, and L. Merlo, Phys. Rev. D **89**, 093021 (2014).
[24] J. Jenkins, Phys. Rev. D **79**, 113003 (2009).
[25] P. Minkowski, Phys. Lett. **67B**, 421 (1977).
[26] T. Yanagida, in *Proceedings of the Workshop on the Unified Theories and Baryon Number in the Universe, Tsukuba, Japan, 1979*, edited by O. Sawada and A. Sugamoto (KEK Report No. 79–18, Tsukuba, 1979), p. 95.
[27] P. Ramond, arXiv:hep-ph/9809459.
[28] S. L. Glashow, NATO Sci. Ser. B **59**, 687 (1980).
[29] M. Agostini *et al.* (GERDA Collaboration), Phys. Rev. Lett. **111**, 122503 (2013).
[30] J. B. Albert *et al.* (EXO-200 Collaboration), Nature (London) **510**, 229 (2014).
[31] The KamLAND-Zen Collaboration, arXiv:1409.0077.
[32] S. Dell'Oro, S. Marcocci, and F. Vissani, Phys. Rev. D **90**, 033005 (2014).
[33] A. Pilaftsis, Nucl. Phys. **B504**, 61 (1997); Phys. Rev. D **56**, 5431 (1997); A. Pilaftsis and T. E. J. Underwood, Nucl. Phys. **B692**, 303 (2004).