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Parton density function uncertainties on the W boson mass measurement from the lepton transverse momentum distribution

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We study the charged-current Drell–Yan process, and we evaluate the proton parton densities uncertainties on the lepton transverse momentum distribution and their impact on the determination of the W boson mass. We consider the global parton density function sets CT10, MSTW2008CPdeut, NNPDF2.3, NNPDF3.0, and MMHT2014 and apply the PDF4LHC recipe to combine the individual results, obtaining an uncertainty on m_W that ranges between ± 18 and ± 24 MeV, depending on the final state, collider energy, and kind. We discuss the dependence of the uncertainty on the acceptance cuts and the role of the individual parton densities in the final result. We remark that some parton density function sets predict an uncertainty on m_W of $\mathcal{O}(10\ \mathrm{MeV})$; this encouraging result is spoiled, in the combined analysis of the different sets, by an important spread of the central values predicted by each group.

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I. INTRODUCTION

The very accurate measurement of the W boson mass performed at the Tevatron experiments CDF ($m_W = 80.387 \pm 0.019$ GeV) [1] and D0 ($m_W = 80.375 \pm 0.023$ GeV) [2], with a world average now equal to $m_W = 80.385 \pm 0.015$ GeV [3], offers the possibility of a high-precision test of the gauge sector of the Standard Model (SM). There are prospects of a further reduction of the total experimental uncertainty, with a final error of $\mathcal{O}(10)$ MeV for the combination of LHC, Tevatron, and LEP results [4,5].

The current best prediction in the SM is $m_W = 80.357 \pm 0.009 \pm 0.003$ GeV [6] and has been computed including the full two-loop corrections [7], augmented by higher-order QCD corrections [8] and by resumming reducible contributions. The uncertainty on this evaluation is mostly due to parametric uncertainties of the inputs of the calculation, the top mass value, the hadronic contribution to the running of the electromagnetic coupling, and theoretical uncertainties.

The simultaneous indirect determination of the top quark mass and the W mass, together with the direct determination of the Higgs boson mass, provides an important consistency check for the Standard Model [9]; in turn the comparison of an accurate experimental M_W measurement with the predictions of different models might provide an indirect signal of physics beyond the SM.

The W boson mass is extracted by means of a template fit technique applied to different observables of the charged-current (CC) Drell–Yan (DY) process, namely the lepton and neutrino transverse momenta (p_T^l, p_T^ν) and the lepton pair transverse mass m_T (see, for instance, Refs. [1,2]). The differential distributions are computed with Monte Carlo simulation codes for different values of m_W and are subsequently compared with the corresponding data: the value that maximizes the agreement is chosen as the preferred value for m_W.

The present CDF and D0 results are affected by a systematic error obtained as the combination of several elements, both of experimental and theoretical origin (see Tables IX and X in Ref. [1] for CDF and Table VI in Ref. [2] for D0).

Among the experimental items, the most problematic ones are the determination of the lepton energy scale [$\mathcal{O}(7\text{–}17)$ MeV] and of the recoil scale and resolution [$\mathcal{O}(5\text{–}8)$ MeV], where the recoil $\vec{u}_T$ is defined as the sum of the momenta of all the measured charged tracks, with the exception of the ones associated to the lepton(s). The mismeasurement of the recoil affects the determination of the W transverse momentum and the application of the selection cuts on this variable; in turn it affects the determination of the leptons transverse momenta, with an impact on the final m_W value. The largest contribution of theoretical systematic error is due to the parametrization of the proton parton density functions (PDFs), which will be the main subject of the present paper, while another theoretical item present in these tables is the size of the missing QED effects not included in the available simulation tools, estimated to be of $\mathcal{O}(4)$ MeV.

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The recoil modeling is an important element of the analysis of charged-current DY events, for it enters in the determination of the neutrino and of the charged-lepton transverse momenta. The model is validated on neutral-current (NC) DY data of the lepton-pair transverse momentum distribution, thanks to the fact that, in this latter case, the full information about the kinematics in the transverse plane can be reconstructed. The propagation of this calibration of the recoil model to the m_W measurement is estimated to be of $\mathcal{O}(3)$ MeV. The description of the recoil was optimized at the Tevatron experiments in the region of small lepton-pair transverse momenta, so that the DY events used for the m_W determination are eventually selected imposing a cut $\vec{u}_T < 15$ GeV. In the following we will discuss the impact of this selection criterion, which will be implemented in our analysis as a cut on the transverse momentum of the lepton-neutrino pair.

The theoretical contribution to the final systematic error enters in the analysis of the data via the templates. In fact the latter are computed with codes based on perturbative calculations, which are truncated at a finite order in the expansion parameter. The proton PDFs, which are used to describe the partonic content of the proton, are affected by the error of the data from which they are extracted; this error propagates to the templates, inducing an additional systematic uncertainty of the fitting tool. The interplay between the W and Z transverse momentum distributions and the proton PDFs has been discussed in Refs. [10,11], without a discussion on the consequence for the m_W determination.

The aim of the present paper is to provide a quantitative assessment of the error induced by our imperfect knowledge of the proton PDFs, in the preparation of the templates used to fit m_W, in the case of the charged-lepton transverse momentum (p_T^l) distribution, as it is measured in the hadron collider processes $p \overset{(-)}{p} \to W^\pm \to l\nu + X$.

In Ref. [12,13] an analysis based on the lepton-pair transverse mass shows that PDF uncertainties do not challenge a measurement of M_W at 10 MeV accuracy. As discussed above, the measurement of the lepton p_T has different, and to a certain extent complementary, systematic uncertainties, compared to the lepton-pair transverse mass. On the other hand, getting theoretical predictions for the p_T^l distribution can be quite challenging due to the high sensitivity of this exclusive observable to the details of the description of the radiation emitted. In this respect, also the PDFs parametrization has a direct impact on the shape of the distribution and, in turn, on m_W.

A. Lepton transverse momentum distribution and m_W

The sensitivity of the lepton transverse momentum distribution to the precise value of the W boson mass is

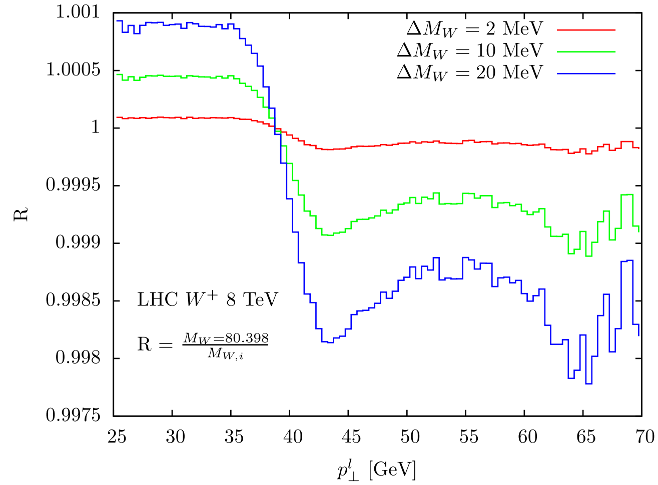

FIG. 1 (color online). Ratio of lepton transverse momentum distributions which have been generated with different W boson masses.

due to its Jacobian peak which has its maximum for $p_\perp^l \sim m_W/2$. A change of m_W in the simulation codes by 2, 10, and 20 MeV, with respect to a fixed reference value, yields a distortion of the distribution in the per mill range, as illustrated in Fig. 1. Any effect (perturbative QCD corrections, PDF uncertainties, etc.) that induces a change of the shape of this distribution of similar size represents a source of systematic theoretical uncertainty on the m_W determination; in particular a measurement at the 10 MeV level requires a control of the shape of the templates at the 1 per mill level or better.

There are two mechanisms that yield a distribution of the lepton transverse momentum in the DY processes: the decay of the gauge boson and its recoil against QCD (and in smaller amount QED) radiation. The initial-state radiation collinear divergences make any fixed-order prediction for this quantity unreliable, because of the important contributions in the region of small gauge boson transverse momenta, which have to be resummed to all orders. It is thus necessary to use a code that implements the resummation to all orders of multiple gluon emissions, either analytically or in a numerical approach via a parton shower (PS), to obtain a physically sensible prediction. In this study we use the POWHEG Monte Carlo event generator [14], matched with the PYTHIA [15] QCD PS. The accuracy of this code on the inclusive DY cross section is next-to-leading-order (NLO) QCD, while from the point of view of the enhancement due to the logarithms of the transverse momentum of the lepton pair, the lepton transverse momentum distribution has leading logarithmic (LL) accuracy. We consistently choose NLO-QCD PDF distributions. The POWHEG event generator is currently used by the ATLAS and CMS (see e.g., respectively, Refs. [16] and [17]) collaborations to study the DY processes and provides a good description of the data.

We thus consider it as a valid starting point to study the propagation of the PDF uncertainty in the preparation of the templates eventually used to fit the data.

A more accurate description of the charged-lepton transverse momentum distribution can be obtained by the use of codes that include higher-order QCD corrections, like `ResBos` [18,19] or `DYRes` [20], or electroweak and QED multiple photon effects, like `POWHEG` [21–23]. As it is well known, final-state QED radiation plays a crucial role in the precise determination of m_W [24]. However, since in this study our main focus is on the assessment of the impact on the m_W determination of the PDF uncertainty, we choose a fast code that yields a basic realistic description of the shape of this distribution. The difference with the predictions that one could obtain adopting one of the other above listed codes belongs to the class of mixed PDF $\times$ higher-order effects, and it can be estimated as a perturbative correction to the results of the present study; the use of a code that includes final-state QED effects modifies the basic shape of the templates, with a shift of the central m_W value of $\mathcal{O}(200)$ MeV [25]; our *ansatz*, in the absence of an explicit check, is that this modification of the shape yields also a rescaling of all the PDF uncertainties at most of $\mathcal{O}(10\%)$ of the results discussed in this paper; this estimate follows from the comparison between the QED shift and the size of the region of the $p_\perp^l$ distribution sensitive to an m_W variation, which is of at least of one, but more realistically of a few W decay widths. We would thus obtain a change in our results, for a given PDF set, by $\mathcal{O}(2\ \mathrm{MeV})$, which is comparable to the statistical accuracy that we can claim in the template fit and to the error, of experimental origin, on the PDF uncertainty itself [26].

The plan of the paper is the following. In Sec. II we recall the definition of some basic theoretical tools which will be used in the study. In Sec. III we present our numerical results, discussing the PDF uncertainty on the lepton transverse momentum distribution and on the m_W determination; we also consider the dependence of the m_W PDF uncertainty on the acceptance cuts and comment on possible future developments. In Sec. IV we draw our conclusions.

II. THEORETICAL TOOLS

In this section, we briefly outline the strategy adopted to estimate the PDF uncertainty in the determination of m_W at hadron colliders; we refer the interested reader to Ref. [12] for more details.

A. Template fit

In this paper we discuss the uncertainty on m_W of different PDF sets. To make a quantitative evaluation, we follow some basic steps:

(1) We generate the lepton transverse momentum $\mathcal{O}$ with different PDF replicas, keeping the W mass fixed at a given value m_{W0}, and we treat each distribution as a set of *pseudodata*.

(2) We compute the *templates*, i.e. the distributions that are used to fit the pseudodata, with one specific choice for the PDF set (`NNPDF2.3`, replica 0), and we let m_W assume all the values in the interval [80.312, 80.470] in steps of 2 MeV.

(3) We compare a given pseudodata distribution, obtained with a given PDF replica labelled by i, with all the templates labelled by j; in each comparison we compute an indicator

$$\chi^2_{i,j} = \frac{1}{N_{\mathrm{bins}}} \sum_{k=1}^{N_{\mathrm{bins}}} \frac{(\mathcal{O}_k^{j,\mathrm{template}} - \mathcal{O}_k^{i,\mathrm{data}})^2}{(\sigma_k^{i,\mathrm{data}})^2 + (\sigma_k^{j,\mathrm{template}})^2 - 2\mathrm{Cov}(\mathrm{data},\mathrm{template})}, \tag{1}$$

where $\mathcal{O}_k$ and σ_k are, respectively, the value of the distribution and its associated error in the bin k and Cov is the covariance between the two distributions.

(4) The template $\bar{j}$ that yields the minimum value of $\chi^2_{i,j}$ is the one that best describes the pseudodata, and its associated $m_{W,\bar{j}}$ value is thus the preferred value associated to the replica i; the difference $\Delta m_{W,i} = m_{W,\bar{j}} - m_{W0}$, is the shift induced by the PDF replica i chosen for that set of pseudodata; in other words it is the difference between the results that we would obtain when fitting the real data if we prepared the templates with the replica i instead of the replica 0 of `NNPDF2.3`.

B. PDF uncertainties

The proton PDF sets considered in this study are `MSTW2008CPdeut` [27], `CT10` [28], `NNPDF2.3` [29], `NNPDF3.0` [30], and `MMHT2014` [31]. They are called global sets because they include all the available relevant hard scattering data. Each collaboration provides a prescription to estimate the PDF uncertainties[1]: in particular we recall the formula for the symmetric error in the Hessian approach (`CT10, MSTW2008CPdeut, MMHT2014`) for a generic observable X

[1] We refer to the original publications for more details.

$$\Delta X = \frac{1}{2}\sqrt{\sum_{i=1}^{N_{\text{eigenvectors}}} [X_i^+ - X_i^-]^2}, \tag{2}$$

where the sum runs over the $N_{\text{eigenvectors}}$ eigenvectors in parameter space, with the associated pairs of replicas ($+$ and $-$). Instead with NNPDF the average and the standard deviation over the ensemble $\{q\}$ of N_{rep} PDF replicas provide the estimate of the best value and of the error on the observable $\mathcal{F}$:

$$\langle \mathcal{F}[\{q\}]\rangle = \frac{1}{N_{\text{rep}}}\sum_{k=1}^{N_{\text{rep}}} \mathcal{F}[\{q^{(k)}\}], \tag{3}$$

$$\sigma_{\mathcal{F}} = \left(\frac{1}{N_{\text{rep}}-1}\sum_{k=1}^{N_{\text{rep}}} (\mathcal{F}[\{q^{(k)}\}] - \langle \mathcal{F}[\{q\}]\rangle)^2\right)^{1/2}. \tag{4}$$

The results obtained with these PDF sets can be combined according to the current PDF4LHC recommendation [32], to find a conservative estimate of the PDF uncertainty.

In this paper we apply this procedure to two observables, namely the lepton transverse momentum distribution and the W mass determined with the template fit procedure.

C. Correlation functions

A useful quantity to evaluate the role of the different parton densities in the hadronic cross section is the correlation function ρ between the parton-parton luminosities and the charged-lepton distribution at a given value of the transverse momentum. The parton-parton luminosity is defined as $\mathcal{P}_{ij}(x,\tau) = f_i(x,\mu_F^2) f_j(\frac{\tau}{x},\mu_F^2)$ where $f_i(x,\mu_F^2)$ is the density describing a parton i at a scale μ_F and $\tau = \frac{M^2}{S}$ with M the final-state invariant mass and S the hadronic Mandelstam invariant. The correlation ρ is defined as

$$\rho(x,\tau) = \frac{\langle \mathcal{P}_{ij}(x,\tau)\frac{d\sigma}{dp_\perp^l}\rangle - \langle \mathcal{P}_{ij}(x,\tau)\rangle\langle\frac{d\sigma}{dp_\perp^l}\rangle}{\sigma^{\text{PDF}}_{\mathcal{P}_{ij}}\sigma^{\text{PDF}}_{d\sigma/dp_\perp^l}}, \tag{5}$$

where the angle brackets indicate the average with respect to the different PDF replicas.

III. NUMERICAL RESULTS

A. Input parameters and setup

We simulate the processes $p\overset{(-)}{p} \to W^+ \to \mu^+\nu_\mu + X$ and $p\overset{(-)}{p} \to W^- \to \mu^-\bar{\nu}_\mu + X$ in proton-antiproton collisions with $\sqrt{S} = 1.96$ TeV and in proton-proton collisions with $\sqrt{S} = 8, 13, 33, 100$ TeV energies. In the absence of QED effects, not considered here, our results will be identical to those obtained with electrons instead of muons. We consider the PDF sets MSTW2008CPdeut [27], CT10 [28], NNPDF2.3 [29], NNPDF3.0 [30], and MMHT2014 [31] and use the corresponding values of $\alpha_s(m_Z)$. We use the following values for the input parameters in the Monte Carlo codes:

$$\begin{array}{lll}
G_\mu = 1.16637\times 10^{-5}\ \text{GeV}^{-2} & m_W = 80.398\ \text{GeV} & m_Z = 91.1876\ \text{GeV} \\
\sin^2\theta_W = 1 - m_W^2/m_Z^2 & \Gamma_W = 2.141\ \text{GeV} & \Gamma_Z = 2.4952\ \text{GeV} \\
V_{cd} = 0.222 & V_{cs} = 0.975 & V_{cb} = 0 \\
V_{ud} = 0.975 & V_{us} = 0.222 & V_{ub} = 0 \\
V_{td} = 0 & V_{ts} = 0 & V_{tb} = 1.
\end{array}$$

The charm quark in the partonic cross section is treated as a massless particle, while the bottom quark does not contribute because of the vanishing top density in the proton. As for the kinematic cuts, we used those summarized in Table I, similar to those used in the corresponding experimental analysis: the main difference between the Tevatron and LHC is the wider acceptance for the rapidity of the leptons in the latter case. The p_T^l distribution has been studied in the interval 29 GeV $\le p_\perp^l \le$ 49 GeV, with a bin size of 0.5 GeV. All the following analyses are performed with bare leptons both in the pseudodata and in the templates.

The Monte Carlo simulation requires a specific, technical comment. The effects under study are deformations of the shape of the lepton transverse momentum distribution at the per mill level, either due to a variation of the m_W value or to a different PDF replica choice. This distribution receives contributions from a large fraction of the available final-state phase space, making very difficult an accurate dedicated sampling. As a consequence, Monte Carlo

TABLE I. Selection criteria for DY $W \to l\nu$ events for the Tevatron and the LHC.

Tevatron	LHC
$p_\perp^\mu \ge 25$ GeV	$p_\perp^\mu \ge 25$ GeV
$E_T \ge 25$ GeV	$E_T \ge 25$ GeV
$\vert\eta_\mu\vert < 1.0$	$\vert\eta_\mu\vert < 2.5$
$p_\perp^W < 15$ GeV	$p_\perp^W < 15$ GeV

statistical fluctuations at the per mill level are present also with hundreds of millions of simulated unweighted events. The solution to this problem is found using a reweighting technique, based on the remark that both the dependence on the PDFs and the dependence on m_W factorize from the rest of the fixed-order partonic cross section. Only one simulation, i.e. only one sequence of events, is used to generate all the templates and all the pseudodata: the weight w_0 associated to each event is corrected by an appropriate reweighting factor to account for different replica or, separately, m_W value choices,

$$w_0 \to w_j = w_0 \frac{(\hat{s} - m_{W0}^2)^2 + \Gamma_W^2 m_{W0}^2}{(\hat{s} - m_{W,j}^2)^2 + \Gamma_W^2 m_{W,j}^2} \quad \text{template } j$$
$$w_0 \to w_i = w_0 \frac{f_i(x_1) g_i(x_2)}{f_0^{\mathrm{NNPDF}}(x_1) g_0^{\mathrm{NNPDF}}(x_2)} \quad \text{replica } i, \tag{6}$$

where f, g are two generic parton densities. In the POWHEG formulation, this rescaling spoils the exact NLO accuracy of the final result, by terms generated by the POWHEG Sudakov form factor. The size of the latter could not be distinguished from Monte Carlo statistical fluctuations, when we compared two distributions, one obtained with an exact simulation and the other introducing the new PDF replica via the above rescaling. The main results of this study should thus not be affected. An update of the POWHEG generator is in progress [33] to restore the NLO accuracy after reweighting.

Since the events used are exactly the same, the statistical fluctuations of the different distributions (templates and pseudodata) are highly correlated (correlation is about 0.987, almost constant over the bins, with 80 millions of events) and cancel to a large extent when we compute the difference of two cross sections in a bin. A statistical error of ± 2 MeV can be obtained with the simulation of 1620 millions of events. We checked that, with increasing statistics, the result of each individual fit is stable, because of the uniform reduction of the statistical error in the different $p_\perp^l$ bins. We stress that, rather than an absolute prediction of the m_W value, we are interested in the quantitative assessments i) of the relative difference between various PDF sets and ii) of the PDF uncertainty within one set, which is defined as the spread with respect to a central value. In both cases we have to provide a solid estimate of differences bin by bin, and the reweighting procedure allows us to efficiently remove the Monte Carlo fluctuation effects, leaving only the physically relevant shifts.

B. PDF uncertainty of the distribution

We study the percentage PDF uncertainty on the lepton transverse momentum distribution and also on the associated normalized distribution defined as

$$\frac{d\bar{\sigma}}{dp_\perp^l} = \frac{1}{\left(\int_{p_\perp^{\min}}^{p_\perp^{\max}} dp_\perp^l \frac{d\sigma}{dp_\perp^l}\right)} \cdot \frac{d\sigma}{dp_\perp^l}. \tag{7}$$

As it is well known from the Tevatron experiments [1,2], the uncertainty is in general reduced in the normalized observable. In fact, each PDF replica in a set contributes in a different way to the shape and to the overall normalization of the physical distributions; by considering the normalized distributions of Eq. (7), we are sensitive mostly to the shape change; the latter is the most relevant item in the determination of m_W, because we associate the precise position and shape of the Jacobian peak to the value of the gauge boson mass. In Fig. 2 we show the PDF uncertainty of the lepton transverse momentum distribution and of the associated normalized distribution, computed at the Tevatron, at the LHC 8 and 13 TeV with different PDF sets, both in the cases of W^+ and W^- production, in the presence of the additional cut $p_\perp^W < 15$ GeV with respect to the choices indicated in Table I. The percentage uncertainty of the normalized distributions is at the few per mill level at the Jacobian peak and could mimic the effect of a m_W shift by $\mathcal{O}(10)$ MeV. In Fig. 3 we use the PDF set NNPDF3.0 and study the change of the PDF uncertainty with the collider energy, in the presence of the additional cut $p_\perp^W < 15$ GeV. The uncertainty of the distribution increases, as function of the collider energy, from 1.5% to 2.5%, while in the normalized case, the uncertainty is almost independent of the energy.

C. Impact of the PDF uncertainty on the m_W determination

The template fit procedure, described in Sec. II A, has been applied to the distributions computed with all the replicas of the different PDF sets under study; the corresponding preferred m_W values have been combined, according to the rules described in Sec. II B, to derive the uncertainty on the m_W extraction due to the PDFs. The fit interval has been chosen to be $p_\perp^l \in [29, 49]$ GeV, in order to minimize the contribution to the PDF uncertainty from the tails of the distribution above and below the Jacobian peak. The template fit has been applied to our pseudodata generated with a fixed value of the W boson decay width Γ_W. We have checked that our results are weakly dependent on the choice of this parameter: we repeated the fit using for Γ_W a value modified by $\pm\sigma_\Gamma$, where $\sigma_\Gamma = 0.042$ GeV is the current experimental error, and we found that the prediction for the PDF uncertainty on m_W gets modified by 1–2 MeV, depending on the selection cuts. The results for the Tevatron and for the LHC 8 and 13 TeV are presented in Table V and are also summarized in Fig. 4. In the upper half of Table V (and in Fig. 4, left plot), no additional cut on $p_\perp^W$ has been imposed on the lepton pair, whereas in the lower half of the same Table (and in Fig. 4, right plot), a cut $p_\perp^W < 15$ GeV has been applied.

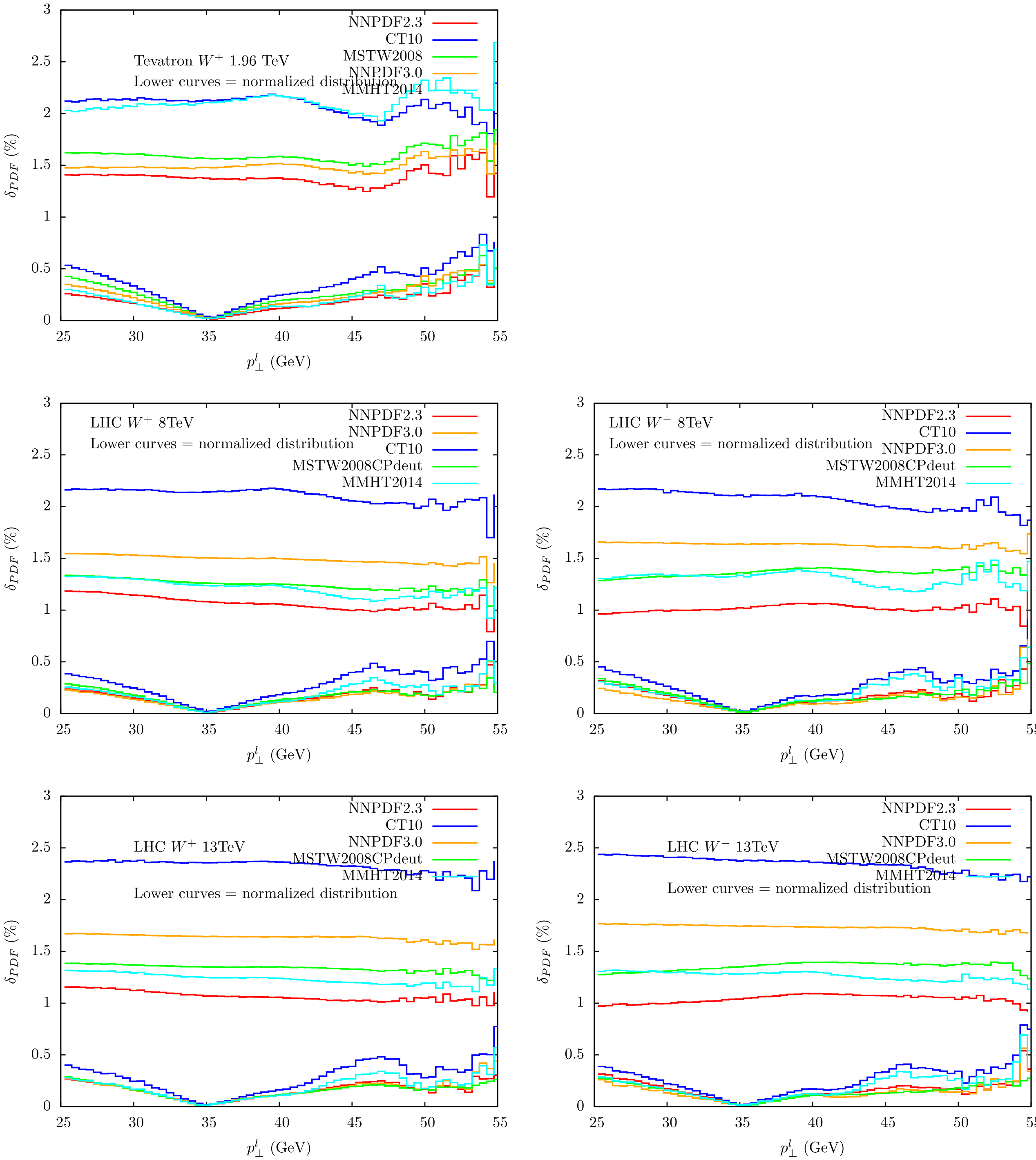

FIG. 2 (color online). Percentage size of the PDF uncertainty on the lepton transverse momentum distribution, computed with different PDF sets. In addition to the basic acceptance criteria of Table I, a cut $p_\perp^W < 15$ GeV on the lepton pair has been applied. The lower lines refer to the normalized distributions of Eq. (7), and the upper lines refer to the standard ones. Results for W^+ (left) and W^- (right) production at the LHC 8 TeV (middle plots) and 13 TeV (lower plots); results for the Tevatron in the upper plot.

The PDF uncertainty reflects the experimental error of the data from which the parton densities are extracted and also the different methodologies used in their determination. As it can be observed from Fig. 4, the estimate of the PDF uncertainty on m_W predicted by the different PDF collaborations differs by a factor up to 3 between the different groups. The uncertainty on m_W extracted from normalized distributions, with the basic selection criteria of

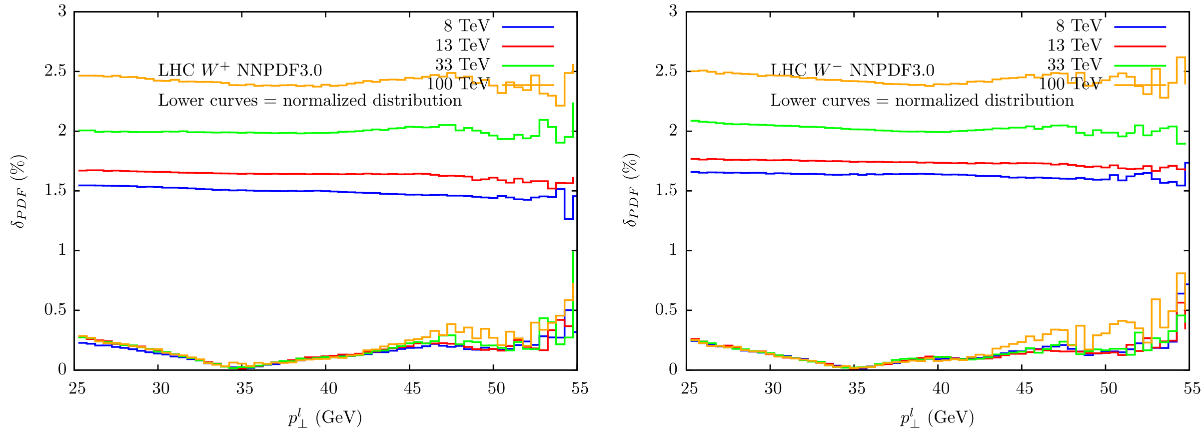

FIG. 3 (color online). Percentage size of the PDF uncertainty on the lepton transverse momentum distribution, computed with the NNPDF3.0 set at the LHC at different energies. Results for W^+ (left) and W^- (right) production, in the case of absolute (upper lines) or normalized (lower lines) distributions.

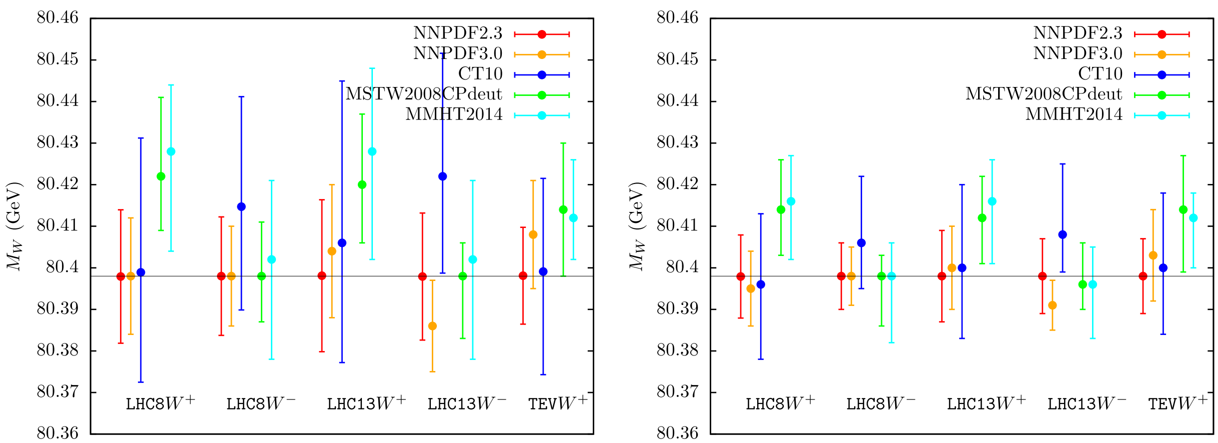

FIG. 4 (color online). Summary of the PDF uncertainty on m_W computed with different PDF sets, colliders, and final states. The basic acceptance criteria have been used in the left plot, while in the right plot an additional cut $p_\perp^W < 15$ GeV has been applied.

Table I, ranges from 12 to 23 MeV at the Tevatron, from 12 to 29 MeV at the LHC 8 TeV, and from 11 to 34 MeV at the LHC 13 TeV. Imposing on the lepton pair a cut $p_\perp^W < 15$ GeV modifies these results; the ranges of the PDF uncertainties become from 11 to 17 MeV at the Tevatron, from 7 to 17 MeV at the LHC 8 TeV, and from 6 to 18 MeV at the LHC 13 TeV.

In addition, the PDF sets under study differ in the parametrization that they adopt to describe the proton

TABLE II. Half-width $\delta_{\rm PDF}$ of the envelope of the PDF uncertainty intervals by CT10, MSTW2008CPdeut, and NNPDF2.3. Corresponding spread $\Delta_{\rm sets}$ of the central predictions.

	No $p_\perp^W$ cut		$p_\perp^W < 15$ GeV	
	$\delta_{\rm PDF}$ (MeV)	$\Delta_{\rm sets}$ (MeV)	$\delta_{\rm PDF}$ (MeV)	$\Delta_{\rm sets}$ (MeV)
Tevatron 1.96 TeV	27	16	21	15
LHC 8 TeV W^+	33	26	24	18
W^-	29	16	18	8
LHC 13 TeV W^+	34	22	20	14
W^-	34	24	18	12

TABLE III. Same as in Table II, now considering only the two recent PDF sets NNPDF3.0 and MMHT2014.

	No $p_\perp^W$ cut		$p_\perp^W < 15$ GeV	
	$\delta_{\rm PDF}$ (MeV)	$\Delta_{\rm sets}$ (MeV)	$\delta_{\rm PDF}$ (MeV)	$\Delta_{\rm sets}$ (MeV)
Tevatron 1.96 TeV	16	4	13	9
LHC 8 TeV W^+	32	33	21	21
W^-	22	6	12	0
LHC 13 TeV W^+	30	24	18	16
W^-	23	16	11	5

TABLE IV. Estimate of the central values and of the PDF uncertainty on m_W, extracted from the lepton transverse momentum distributions simulated with the NNPDF3.0 set at different proton-proton collider energies.

Normalized distribution, additional cut $p_\perp^W < 15$ GeV				
	8 TeV	13 TeV	33 TeV	100 TeV
W^+	80.395 ± 0.009	80.400 ± 0.010	80.402 ± 0.010	80.404 ± 0.013
W^-	80.398 ± 0.007	80.391 ± 0.006	80.385 ± 0.007	80.398 ± 0.011

structure; the latter affects the best description of the PDFs and in turn the best prediction of the central m_W value. The spread $\Delta_{\rm sets}$ of the central values, defined as the difference between the largest and the smallest central values, is a second component of the final PDF uncertainty on m_W.

A conservative estimate of the uncertainty on m_W, that combines the two elements of uncertainty described above, can be obtained by computing the envelope of the predictions under study, according to the PDF4LHC recipe [32] and by measuring the half-width $\delta_{\rm PDF}$ of the resulting band. We include, in the evaluation of the envelope, the results of the sets CT10, MSTW2008CPdeut and NNPDF2.3, because they are based on the same sets of data, making their comparison homogeneous. These results are presented in Table 2. We observe that the spread $\Delta_{\rm sets}$ represents a large contribution, up to 35% of the overall uncertainty . In Table III we compute the envelope of the results obtained with two more modern PDF sets, namely NNPDF3.0 and MMHT2014, which include recent public data from the LHC. We observe that the width of the envelope ranges between 16 and 32 MeV, depending on the collider energy and kind and on the final state; more interesting, the spread of the two central values is below 5 MeV in the W^- case at the LHC, while it is above 15 MeV in the W^+ case and at the Tevatron.

From Table V we can appreciate the impact of the inclusion of the new LHC data, which have been used in the determination of the NNPDF3.0 set. Beside a few MeV offset for the central values, it is possible to observe a small (few MeV) reduction of the PDF uncertainty, which is roughly 20% smaller than the one computed with NNPDF2.3. For MMHT2014 the uncertainties are similar or slightly larger than the ones obtained with MSTW2008CPdeut.

The dependence of the PDF uncertainty with the collider energy is illustrated in Table IV, using the NNPDF3.0 PDF set.

D. PDF uncertainty dependence on the acceptance cuts

The results presented in Sec. III C have been obtained imposing on the leptons the basic cuts of Table I. The dependence of the m_W PDF uncertainty on additional cuts on the lepton-pair transverse momentum $p_\perp^W$ or on the charged-lepton pseudorapidity acceptance interval is presented in Table VI. This study suggests possible optimizations of the event selection, to minimize the PDF uncertainty impact. It also offers a link to the dependence of the PDF uncertainty on the different flavors in the proton and on the most problematic range in partonic-x.

We observe that the region at large $p_\perp^W$ yields an important contribution to the PDF uncertainty, which can be reduced by a suitable cut on this variable. A tight cut like $p_\perp^W < 10$ GeV could bring the uncertainty below the 10 MeV level. The experimental problem to accurately select the events that pass the cut can be a limiting factor for the improvement in this direction.

The impact of the cut on the lepton-pair transverse momentum can be explained by studying the change of the relative contribution of the medium- vs the large-x PDF region, where x is the fraction of momentum of the parent hadron carried by the incoming parton. In Fig. 5 (left plot), we show the normalized $d\sigma/dx$ distributions, where x is the fraction of longitudinal momentum carried by the partons of one given hadron in the scattering;[2] they are computed with different $p_\perp^W$ cuts and express the relative contribution of a given partonic x to the cross section. In Fig. 5 (right plot), we show the ratio of the previous distributions, computed with different $p_\perp^W$ cuts, with respect to the inclusive (no $p_\perp^W$ cut) normalized distribution. These ratios express the relative change of the weight of the various x intervals, in the presence of a cut. We thus recognize that the $p_\perp^W < 15$ GeV cut enhances the $x < 0.004$ region and suppresses the contribution at $x > 0.004$. Since the PDF uncertainty of all the densities rapidly increases for $x > 0.1$ (cf. Fig. 7, left plot), the effect of the $p_\perp^W$ cut is a reduction of the global PDF uncertainty affecting the m_W determination. A second effect of the cut is a change of the basic shape of the distribution, which becomes steeper and closer to the leading-order one, above the Jacobian peak, as it is shown in Fig. 6: this modification increases the sensitivity of the fitting procedure, which becomes more stable, because large shifts are more penalized with respect to the case of a broader distribution. In the right panel of Fig. 6, we show the normalized lepton pseudorapidity distribution, computed for different values of the $p_\perp^W$ cut. We observe that with tighter cuts the distribution develops

[2]The choice of the hadron is not relevant, because the contribution of the partonic subprocesses is symmetric for exchange of hadrons 1 and 2.

TABLE V. Estimate of the central values and of the PDF uncertainty on m_W, extracted from the lepton transverse momentum distributions simulated with different PDF sets and acceptance cuts. The templates have been generated with `NNPDF2.3` replica 0. The pseudodata for the different PDF sets have been simulated by setting $m_W = 80.398$ GeV.

absolute distributions					
Collider/channel	CT10	MSTW2008CPdeut	NNPDF2.3	NNPDF3.0	MMHT2014
Tevatron, W^+	80.406 + 0.043 − 0.046	80.428 + 0.025 − 0.017	80.400 ± 0.030	80.427 ± 0.018	80.430 + 0.022 − 0.022
LHC 8 TeV, W^+	80.394 + 0.040 − 0.029	80.422 + 0.025 − 0.016	80.398 ± 0.020	80.406 ± 0.019	80.428 + 0.027 − 0.022
W^-	80.444 + 0.055 − 0.062	80.390 + 0.038 − 0.036	80.398 ± 0.030	80.441 ± 0.027	80.404 + 0.041 − 0.048
LHC 13 TeV, W^+	80.396 + 0.045 − 0.034	80.416 + 0.020 − 0.020	80.398 ± 0.022	80.414 ± 0.022	80.422 + 0.030 − 0.024
W^-	80.416 + 0.088 − 0.065	80.374 + 0.044 − 0.033	80.398 ± 0.031	80.426 ± 0.037	80.384 + 0.037 − 0.049
normalized distributions					
Collider/channel	CT10	MSTW2008CPdeut	NNPDF2.3	NNPDF3.0	MMHT2014
Tevatron, W^+	80.400 + 0.022 − 0.025	80.414 + 0.016 − 0.016	80.398 ± 0.012	80.408 ± 0.013	80.412 + 0.014 − 0.010
LHC 8 TeV, W^+	80.398 + 0.032 − 0.026	80.424 + 0.014 − 0.019	80.398 ± 0.016	80.395 ± 0.014	80.428 + 0.016 − 0.024
W^-	80.416 + 0.026 − 0.025	80.398 + 0.011 − 0.014	80.398 ± 0.014	80.396 ± 0.012	80.402 + 0.019 − 0.024
LHC 13 TeV, W^+	80.406 + 0.039 − 0.029	80.420 + 0.017 − 0.014	80.398 ± 0.018	80.404 ± 0.016	80.428 + 0.020 − 0.026
W^-	80.422 + 0.030 − 0.023	80.398 + 0.008 − 0.015	80.398 ± 0.015	80.386 ± 0.011	80.402 + 0.019 − 0.024
absolute distributions, additional cut $p_\perp^W < 15$ GeV					
Collider/channel	CT10	MSTW2008CPdeut	NNPDF2.3	NNPDF3.0	MMHT2014
Tevatron, W^+	80.412 + 0.024 − 0.024	80.424 + 0.018 − 0.017	80.398 ± 0.013	80.420 ± 0.014	80.426 + 0.009 − 0.021
LHC 8 TeV, W^+	80.392 + 0.026 − 0.021	80.414 + 0.020 − 0.011	80.398 ± 0.014	80.403 ± 0.014	80.418 + 0.019 − 0.017
W^-	80.422 + 0.039 − 0.034	80.394 + 0.019 − 0.023	80.398 ± 0.017	80.423 ± 0.017	80.400 + 0.023 − 0.028
LHC 13 TeV, W^+	80.392 + 0.028 − 0.022	80.410 + 0.012 − 0.016	80.398 ± 0.014	80.408 ± 0.014	80.414 + 0.016 − 0.019
W^-	80.408 + 0.042 − 0.037	80.386 + 0.019 − 0.021	80.398 ± 0.016	80.410 ± 0.018	80.388 + 0.021 − 0.025
normalized distributions, additional cut $p_\perp^W < 15$ GeV					
Collider/channel	CT10	MSTW2008CPdeut	NNPDF2.3	NNPDF3.0	MMHT2014
Tevatron, W^+	80.400 + 0.018 − 0.016	80.414 + 0.013 − 0.015	80.398 ± 0.009	80.403 ± 0.011	80.412 + 0.006 − 0.012
LHC 8 TeV, W^+	80.396 + 0.017 − 0.018	80.414 + 0.012 − 0.011	80.398 ± 0.010	80.395 ± 0.009	80.416 + 0.011 − 0.014
W^-	80.406 + 0.016 − 0.011	80.398 + 0.005 − 0.012	80.398 ± 0.010	80.398 ± 0.007	80.398 + 0.008 − 0.016
LHC 13 TeV, W^+	80.400 + 0.020 − 0.017	80.412 + 0.010 − 0.011	80.398 ± 0.011	80.400 ± 0.010	80.416 + 0.010 − 0.015
W^-	80.408 + 0.017 − 0.009	80.396 + 0.010 − 0.006	80.398 ± 0.009	80.391 ± 0.006	80.396 + 0.009 − 0.013

TABLE VI. LHC 8 TeV, W^+ production. Impact of different acceptance cuts. The two cuts $p_\perp^l > 25$ GeV and $\not{E}_T \geq 25$ GeV are always applied. In the first four rows, we vary the cut on $p_\perp^W$, for the fixed $|\eta_l|$ interval. In the second four rows, we vary the pseudorapidity acceptance, with $p_\perp^W < 15$ GeV.

Normalized distributions			
Cut on $p_\perp^W$	Cut on $\vert\eta_l\vert$	CT10	NNPDF3.0
Inclusive	$\vert\eta_l\vert < 2.5$	80.400 + 0.032 − 0.027	80.398 ± 0.014
$p_\perp^W < 20$ GeV	$\vert\eta_l\vert < 2.5$	80.396 + 0.027 − 0.020	80.394 ± 0.012
$p_\perp^W < 15$ GeV	$\vert\eta_l\vert < 2.5$	80.396 + 0.017 − 0.018	80.395 ± 0.009
$p_\perp^W < 10$ GeV	$\vert\eta_l\vert < 2.5$	80.392 + 0.015 − 0.012	80.394 ± 0.007
$p_\perp^W < 15$ GeV	$\vert\eta_l\vert < 1.0$	80.400 + 0.032 − 0.021	80.406 ± 0.017
$p_\perp^W < 15$ GeV	$\vert\eta_l\vert < 2.5$	80.396 + 0.017 − 0.018	80.395 ± 0.009
$p_\perp^W < 15$ GeV	$\vert\eta_l\vert < 4.9$	80.400 + 0.009 − 0.004	80.401 ± 0.003
$p_\perp^W < 15$ GeV	$1.0 < \vert\eta_l\vert < 2.5$	80.392 + 0.025 − 0.018	80.388 ± 0.012

two peaks at forward and backward rapidities. These regions are dominated by the contribution of at least one valence quark, the PDF uncertainty of which is smaller than the one of the corresponding sea component.

We observe that, for a fixed cut on $p_\perp^W$, the PDF uncertainty decreases from 17 (26) to 3 (6) MeV with `NNPDF3.0` (`CT10`), as one enlarges the charged-lepton rapidity cut, from 1.0 to 4.9. This reduction is consistent with the smaller PDF uncertainty of the lepton transverse momentum distribution with the cut $|\eta_l| < 4.9$ shown in Fig. 8 (left plot). In this case the problematic point is the possibility of an accurate

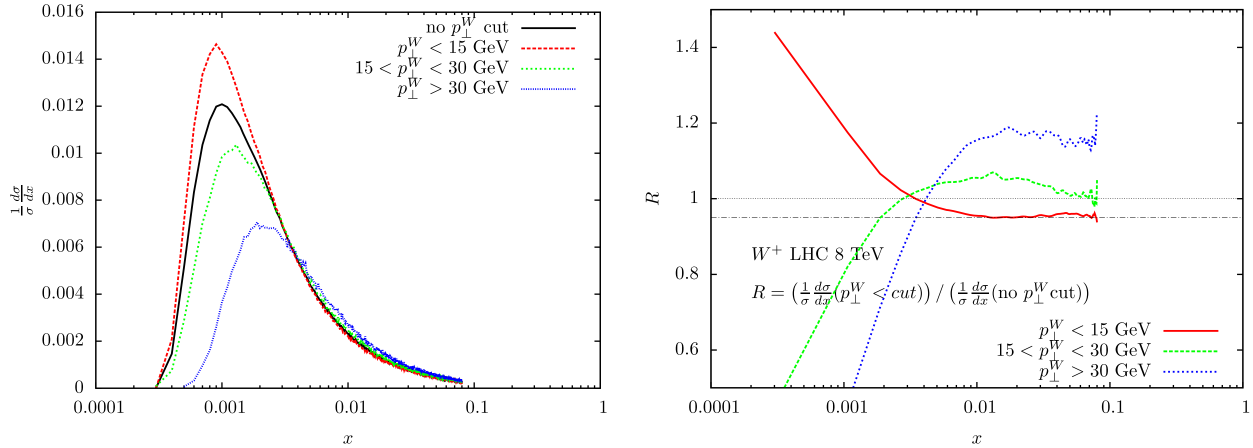

FIG. 5 (color online). Shape of the normalized differential distribution $d\sigma/dx$ for different $p_\perp^W$ cuts (left plot). Ratio of the previous shapes with different $p_\perp^W$ cuts with respect to the inclusive (no $p_\perp^W$ cut) distribution (right plot).

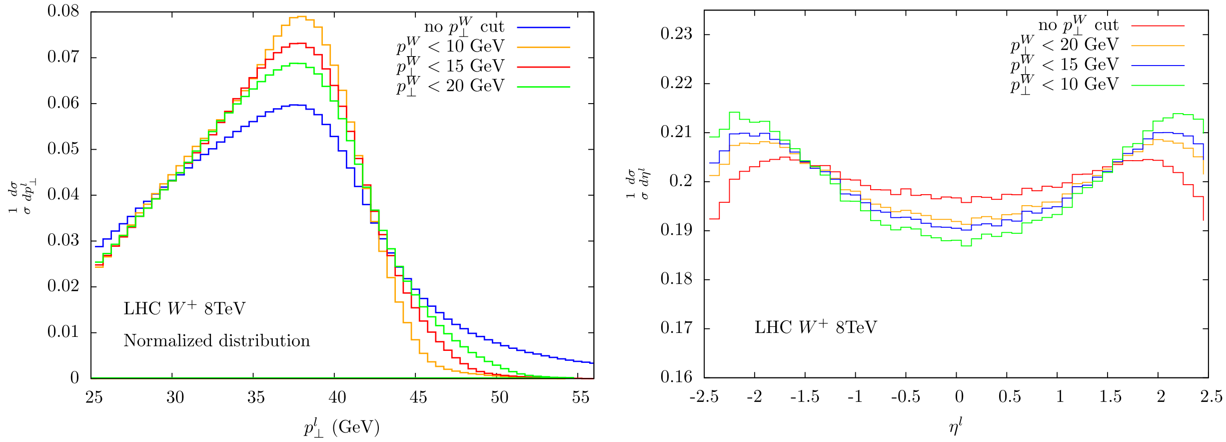

FIG. 6 (color online). Shape of the lepton transverse momentum (left panel) and of the lepton pseudorapidity (right panel) distributions, in the presence of different additional cuts on the lepton-pair transverse momentum $p_\perp^W$.

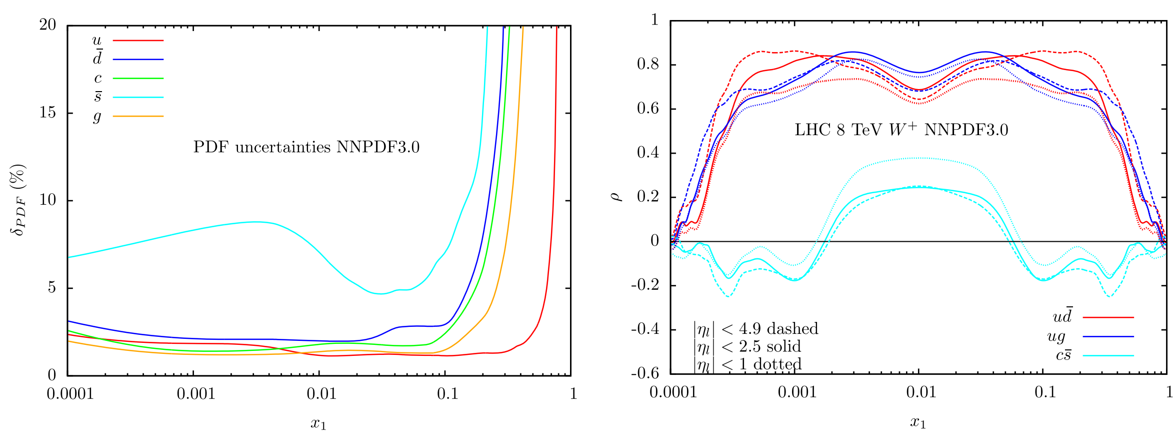

FIG. 7 (color online). Percentage PDF uncertainty of the charged-lepton $p_\perp^l$ distribution (left plot) and shape of the differential distribution $d\sigma/dx$ (right plot), computed with different acceptance cuts on $|\eta_l|$ and with $p_\perp^W < 15$ GeV.

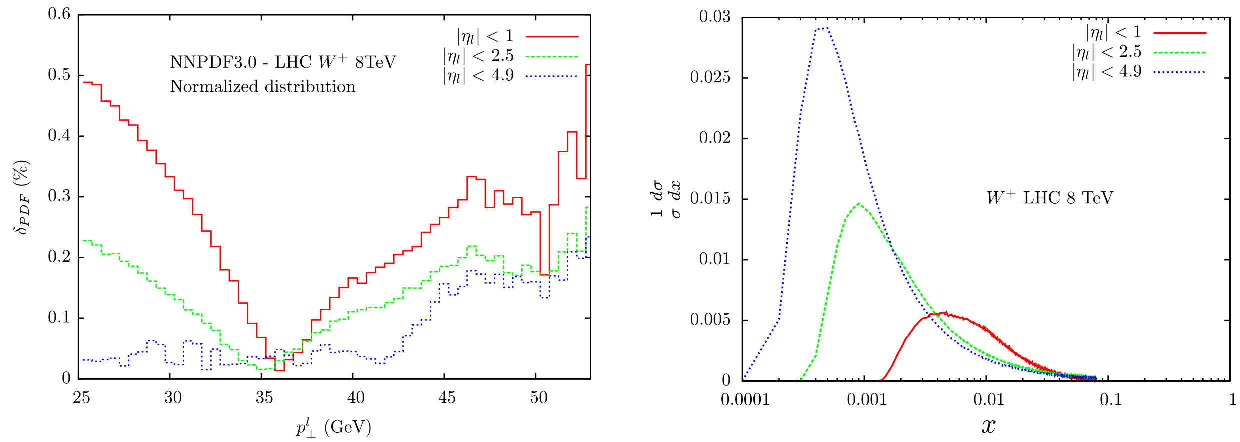

FIG. 8 (color online). Percentage uncertainty of the individual parton densities $f(x, m_W^2)$ of NNPDF3.0 (left plot). Correlation of different parton-parton luminosities with the charged-lepton $p_\perp^l$ distribution at $p_\perp^l = 40.5$ GeV, computed with different acceptance cuts on $|\eta_l|$ and with $p_\perp^W < 15$ GeV.

measurement of the lepton properties in the large rapidity regions of the detector.

The impact of the cut on the charged-lepton pseudorapidity can be explained first of all by recalling that a lepton transverse momentum distribution fully integrated over the lepton-pair rapidity (without acceptance cuts) would depend on the PDFs only via a single numerical factor, which drops out when we study the normalized distributions. This ideal limit can be reached, in a realistic setup, by enlarging the charged-lepton pseudorapidity acceptance. More in detail, with different maximal values of η_l, we observe a corresponding change of the shape of the $d\sigma/dx$ distribution, shown in Fig. 7 (right plot): the bulk of the distribution is peaked around $5\times10^{-3}, 1\times10^{-3}$,and 5×10^{-4}, respectively, for $|\eta_l| < 1, 2.5, 4.9$.

First of all we observe in Table VI that the two PDF uncertainties on m_W extracted imposing the cuts $|\eta_l| < 1.0$ or $1.0 < |\eta_l| < 2.5$ are separately larger than the one obtained with $|\eta_l| < 2.5$. Indeed, the sensitivity to partonic x obtained by varying the cut on η_l makes evident the presence of the momentum sum rules, which have to be fulfilled by all the replicas; the more inclusive setup is thus more stable with respect to a PDF replica variation than the more exclusive cases. This uncertainty reduction is even more pronounced with $|\eta_l| < 4.9$.

Second, in the region $5\times10^{-3} \le x \le 1\times10^{-2}$, the strange density has its maximal uncertainty, which is more than three times larger than the one of all the other parton densities, as shown in Fig. 8 (left plot); in this same region, the parton-parton luminosity $c\bar{s}$ has a weak positive correlation with respect to the PDFs, defined in Eq. (5), with the charged-lepton transverse momentum distribution at $p_\perp^l = 40.5$ GeV, so that its contribution to the cross section and to the total PDF uncertainty sums together with the ones of the other channels. In the interval $3\times10^{-4} \le x \le 7\times10^{-4}$, the strange density still has a PDF uncertainty 2.5 times larger than the others, but in this region the $c\bar{s}$ luminosity has a negative correlation with the distribution. In this case there are nontrivial compensations between the contributions of the various partonic subprocesses, yielding a more stable result with respect to the PDF replica choice. This behavior of the parton densities, in the case of this quite inclusive observable, with the acceptance cut $|\eta_l| < 4.9$, reflects the enforcement in the global fit of the sum rules that have to be satisfied by the PDFs.

E. Comparison with previous studies

The PDF uncertainty affecting the m_W determination from the study of the lepton transverse momentum distribution has been estimated in Refs. [1] and in [2] to be, respectively, 12 and 11 MeV. This evaluation was based on the simulation code ResBos and on the use of the PDF sets cteq6.6 [34] and MSTW2008 [27]. We repeated the estimate of the uncertainty in the Tevatron setup also with cteq6.6, using POWHEG+PYTHIA and templates computed with NNPDF2.3 replica 0; we obtain $m_W = 80.396 + 0.015 - 0.016$, i.e. a slightly larger PDF uncertainty compared to the previous estimate.

At variance with the transverse mass case [12], the study presented in this paper, with events treated at generator level, is moderately sensitive to detector effects and should thus represent a realistic estimate of the overall size of the PDF uncertainty and of the relative behavior of the different PDF sets.

F. Role of the lepton-pair transverse momentum distribution in NC-DY

The description of the lepton transverse momentum distribution depends on the treatment of initial-state QCD radiation, to obtain a correct lepton-pair transverse momentum distribution and in turn the correct contribution to the lepton transverse momentum. At low lepton-pair transverse momenta, there are nonvanishing nonperturbative effects, which can be accounted for by means of *ad hoc* models, upon which the final result of m_W depends. The uncertainties of the PDFs and of the modeling of an intrinsic component $k_\perp$ of the transverse momentum of the partons inside the proton are entangled in the lepton-pair transverse momentum distribution, because of the different contribution of the various flavors to the transverse momentum spectrum; in other words, it is not possible, in principle, to derive a universal, flavor independent, model of the intrinsic $k_\perp$. This statement has been investigated in the past (see e.g. Ref. [35]) where the Tevatron data were described by universal flavor-independent nonperturbative functions.

A reduction of this dependence can be obtained by considering new observables, defined as ratios of the CC-DY observables with respect to their analogous ones in the case of NC-DY [36]. The similarities in the initial-state QCD radiation patterns determine a correlation between the CC-DY and the NC-DY quantities, which in turn yields a reduction of the error that affects the ratio. One should, however, keep in mind that it is not possible to expect a full correlation between the CC-DY and the NC-DY observables, because of the different flavor structure of the subprocesses in the two cases and because of the different phase spaces available.

Given the entanglement between PDF and intrinsic $k_\perp$ uncertainties, the estimate of the PDF uncertainty alone presented in this paper, for the CC-DY case, could be a slight overestimate of its contribution to the total nonperturbative uncertainty. On the other hand, the estimate of the PDF uncertainty for the ratio of CC-DY with respect to NC-DY observables should offer a more reliable result, thanks to the weaker model dependence. A detailed study of these ratios and of the associated theoretical uncertainties will be presented elsewhere.

IV. CONCLUSIONS

We presented a quantitative assessment of the PDF uncertainty affecting the extraction of the W boson mass from the study of the charged-lepton transverse momentum distribution in the charged-current Drell–Yan process, at different hadron colliders, for different collider energies. The study, conducted at the generator level, is based on the Monte Carlo code `POWHEG` interfaced with the `PYTHIA` QCD parton shower and uses the `NNPDF2.3` PDF set (replica 0) to prepare the templates used in the fitting procedure. The results are summarized in Fig. 4 and in Table II. The study provides information about the relative distance between the `NNPDF2.3` and the other sets considered (`CT10, MSTW2008CPdeut`); this distance is expressed by the difference between the best predictions of the various sets and ranges between 8 and 15 MeV, depending on the collider, on the energy and on the final state considered; these results rely on the application of a cut on the lepton-pair transverse momentum, $p_\perp^W < 15$ GeV. The study provides an estimate of the PDF uncertainty according to the prescriptions of each PDF group: the individual values range between 6 and 18 MeV, again depending on the considered setup and always in the presence of the cut on the lepton-pair transverse momentum. The combination of the two previous uncertainties, according to the PDF4LHC recipe, leads to a global PDF uncertainty that ranges between 18 and 24 MeV. The analysis of more modern sets, like `NNPDF3.0` and `MMHT2014`, does not change this overall picture but makes evident some differences in the description of W^+ with respect to W^- production.

We remark that the differences between the PDF sets considered here are large compared to an accuracy goal of 10 MeV in the m_W measurement. On the other hand, the fact that the individual sets predict uncertainties in the 10 MeV ballpark leaves hope that an improvement of the global PDF analysis will remove this bottleneck toward a precise m_W measurement.

The variation of the acceptance cut on the lepton pseudorapidity offers the possibility to scrutinize the dependence of the uncertainty on the flavor content of the proton and on the values of partonic-x. The preliminary results are not trivial, because of the correlations among the densities enforced by the PDF sum rules. Increasing the value of the cut on $|\eta_l|$ reduces the PDF uncertainty on m_W.

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