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Warped AdS_3, dS_3, and flows from $\mathcal{N} = (0,2)$ SCFTs

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We present the general form of all timelike supersymmetric solutions to three-dimensional $U(1)^3$ gauged supergravity, a known consistent truncation of string theory. We uncover a rich vacuum structure, including an infinite class of new timelike-warped AdS_3 (Gödel) and timelike-warped dS_3 critical points. We outline the construction of supersymmetric flows, driven by irrelevant scalar operators in the SCFT, which interpolate between critical points. For flows from AdS_3 to Gödel, the natural candidate for the central charge decreases along the flow. Flows to timelike-warped dS_3 exhibit topology change.

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I. INTRODUCTION

It is a remarkable fact that the geometrical Bekenstein-Hawking (BH) entropy of black holes with AdS_3 near-horizons can be derived from the central charge of a two-dimensional (2D) CFT [1,2]. This result, a key precusor to AdS/CFT [3], rests on the Brown-Henneaux analysis of the asymptotic symmetries of AdS_3 [4] and the Cardy formula [5], which permits one to determine the asymptotic density of states in a CFT in the semiclassical limit.

It is well known that Kerr black holes, candidates for astrophysical black holes, e.g. Cygnus X-1 [6], exhibit *warped* AdS_3 near-horizons [7]. In recent years, the matching of BH entropy through the Cardy formula led to a bold conjecture that there is a (warped) CFT dual to Kerr black holes [8] (see [9] for a review). A greater understanding of the putative dual QFT, if it is even a CFT [10–14], requires a theory with a UV completion, such as string theory.

In this work we take a step in this direction by identifying warped AdS_3 vacua of $\mathcal{N} = 2$ $U(1)^3$ gauged supergravity [15], a consistent truncation of string theory [16,17], and offering evidence that they can be connected to understood AdS_3 vacua by supersymmetric flows. This places holography on a firmer footing, since at one end of the flow, the supersymmetric AdS_3 vacua are dual to two-dimensional $\mathcal{N} = (0,2)$ SCFTs [18–21], whose central charge and R symmetry can be determined exactly using c-extremization [20,21] and agree with holographic calculations (see [22] for subleading terms).

Following a review in the next section, we make the following novel contributions. First, we present the general form—dictated by the Bogomol'nyi-Prasad-Sommerfield (BPS) conditions—of all supersymmetric timelike solutions to three-dimensional $U(1)^3$ gauged supergravity, including an infinite class of new half-BPS critical points, going under the moniker timelike-warped AdS_3 (Gödel) and timelike-warped dS_3 in the literature. Indeed, the latter is a known solution to topologically massive gravity with a positive cosmological constant [23], and here we provide potentially the first example in both a supersymmetric and string theory context. Being timelike-warped, the geometries exhibit characteristic closed-timelike-curves (CTCs), signaling a breakdown in unitarity in the dual theory. Along with the Gödel universe [24], which is not ruled out by supersymmetry [25,26], a version of Hawking's chronology protection conjecture [27] is expected for timelike-warped dS_3. See [28–31] for related works in the AdS/CFT context.

We construct numerical supersymmetric flows from AdS_3 to timelike-warped critical points and identify the flows as deformations of the two-dimensional SCFT by irrelevant scalar operators. We show that the inverse of the real superpotential monotonically decreases along flows to timelike-warped AdS_3 vacua and calculate an expression for the candidate central charge in terms of twist parameters. For flows to timelike-warped dS_3, the curvature of the Riemann surface changes sign and the topology changes. Since the two-dimensional SCFTs correspond to twisted compactifications of $\mathcal{N} = 4$ super-Yang-Mills, our three-dimensional flows can be uplifted to five dimensions, where they may be interpreted as deformations of $\mathcal{N} = 4$ super-Yang-Mills. This short letter highlights the existence of novel warped critical points; further examples of supersymmetric flows, the five-dimensional uplift and the generalization to null spacetimes can be found in [32].

II. THREE-DIMENSIONAL $U(1)^3$ GAUGED SUPERGRAVITY

We consider three-dimensional $\mathcal{N} = 2$ gauged supergravity [15], which uplifts on a constant curvature Riemann surface of genus $\mathfrak{g}$, $\Sigma_{\mathfrak{g}}$, to well-known five-dimensional $U(1)^3$ gauged supergravity, where it can be further embedded consistently in higher dimensions [16,17]. Examples of consistent truncations of string theory with warped AdS_3 vacua have appeared previously in [33] (see also [34]).

The action for the theory may be written as

$$\mathcal{L}_3 = R *_3 \mathbf{1} - \frac{1}{2}\sum_{i=1}^{3}[\mathrm{d}W_i \wedge *_3\mathrm{d}W_i + e^{2W_i}G^i \wedge *_3 G^i] + 8\left(T^2 - \sum_{i=1}^{3}(\partial_{W_i}T)^2\right) *_3 \mathbf{1} - a_1 B^2 \wedge \mathrm{d}B^3 - a_2 B^3 \wedge \mathrm{d}B^1 - a_3 B^1 \wedge \mathrm{d}B^2, \quad (1)$$

with the field content comprising three scalars, W_i, and three gauge fields, $G^i = \mathrm{d}B^i$, which may be rewritten in the canonical form of three-dimensional gauged supergravity [35]. T denotes the superpotential

$$T = \sum_{i=1}^{3}\left(\frac{1}{2}e^{-W_i} - \frac{a_i}{4}e^{W_i+K}\right), \quad (2)$$

$K = -\sum_i W_i$ is the Kähler potential of the scalar manifold, and a_i, $i = 1, 2, 3$ denote constants that are constrained by the curvature κ of $\Sigma_{\mathfrak{g}}$:

$$a_1 + a_2 + a_3 = -\kappa. \quad (3)$$

We note there is the freedom to change the sign of T and the potential does not change [36].

Through AdS/CFT [3], AdS_3 vacua of the above supergravity correspond to two-dimensional SCFTs arising through twisted compactifications of four-dimensional $\mathcal{N} = 4$ super Yang-Mills with gauge group U(N) on $\Sigma_{\mathfrak{g}}$ [37,38]. To preserve supersymmetry, one "twists" the theory by turning on gauge fields coupled to the SO(6) R symmetry of the four-dimensional theory. For twists involving the $SO(2)^3$ Cartan subgroup of the R symmetry, the twist parameters, a_i, must satisfy (3), a necessary condition for $\mathcal{N} = 2$ supergravity. Supersymmetry is enhanced to $\mathcal{N} = (2,2)$ and $\mathcal{N} = (4,4)$, when one or two of the a_i vanish, respectively.

Supersymmetric AdS_3 vacua of the action (1) correspond to the critical points, $\partial_{W_i}T = 0$ [34],

$$e^{W_i} = -\frac{\prod_{j\neq i}a_j}{\kappa + 2a_i}. \quad (4)$$

These vacua were featured in a series of works [18–21,39]. From extrema of T, one can see there is no good AdS_3 vacuum dual to $\mathcal{N} = (4,4)$ SCFTs and that $\mathcal{N} = (2,2)$ vacua only exist when $\mathfrak{g} > 1$.

III. ALL TIMELIKE SOLUTIONS

Given a supergravity theory, it is feasible to invoke Killing spinor techniques to find all supersymmetric solutions, e. g. [25,40,41] in five dimensions. Here we present the timelike solutions to the theory (1). Full details of the classification exercise appear elsewhere [32].

The class of supersymmetric geometries is characterized by a real timelike Killing vector P_0, $\mathcal{L}_{P_0}W_i = \mathcal{L}_{P_0}G^i = 0$ and an additional complex vector, $P_z = P_1 + iP_2$. Suitably normalized, we have $P_a \cdot P_b = \eta_{ab}$, $\eta_{ab} = (-1, 1, 1)$ and $a = 0, 1, 2$. The existence of a Killing spinor is equivalent to the differential conditions [32]:

$$\mathrm{d}P_0 = 4T *_3 P_0, \quad (5)$$

$$e^{-\frac{1}{2}K}\mathrm{d}[e^{\frac{1}{2}K}P_z] = \sum_{i=1}^{3}(e^{-W_i} *_3 + iB^i) \wedge P_z. \quad (6)$$

The field strengths G^i are completely determined and break supersymmetry by one-half when nonzero,

$$G^i = e^{-W_i}(-4\partial_{W_i}T *_3 P_0 + P_0 \wedge \mathrm{d}W_i), \quad (7)$$

meaning $G^i = 0$ at AdS_3 vacua. This appears to contradict the existence of well-known holographic flows from AdS_5 to AdS_3 [18], but in our conventions these fall into the null class of spacetimes, $P_a \cdot P_b = 0$.

Given these conditions, it is a straightforward exercise to introduce coordinates $P_0 \equiv \partial_\tau$, $P_z = e^{D-\frac{1}{2}K}(\mathrm{d}x_1 + i\mathrm{d}x_2)$, so that the spacetimes take the form

$$\mathrm{d}s_3^2 = -(\mathrm{d}\tau + \rho)^2 + e^{2D-K}(\mathrm{d}x_1^2 + \mathrm{d}x_2^2),$$
$$G^i = e^{-W_i}[-4\partial_{W_i}T e^{2D-K}\mathrm{d}x_1 \wedge \mathrm{d}x_2 + (\mathrm{d}\tau + \rho) \wedge \mathrm{d}W_i], \quad (8)$$

where ρ is a one-form connection on the Riemann surface parametrized by (x_1, x_2), with $\mathrm{d}\rho = 4Te^{2D-K}\mathrm{d}x_1 \wedge \mathrm{d}x_2$. Here $D(x_1, x_2)$ is modulo a convenient factor of the Kähler potential, a warp factor parametrizing the vector P_z, and in turn the Riemann surface. Inserting the expressions for G^i into the flux equations of motion (EOMs), one can derive the following equation for the scalars:

$$\nabla^2 e^{W_i} = 2e^{2D}\left[\frac{4T}{e^K} - \sum_{j\neq i}a_j(e^{W_j} + e^{W_i}) + \prod_{j\neq i}a_j\right]. \quad (9)$$

From (6), one derives a differential condition for the warp factor,

$$\nabla^2 D = 4\sum_{i=1}^{3}e^{-W_i}(\partial_{W_i}T + T)e^{2D-K}. \quad (10)$$

We note for a given constant value of W_i, this equation reduces to the Liouville equation on the Riemann surface, $\nabla^2 D = -\mathcal{K}e^{2D}$, with Gaussian curvature $\mathcal{K}$.

Using these four equations, it is possible to show that the scalar and Einstein EOMs are satisfied. It can be independently checked that the EOMs are consistent with the integrability conditions [32], as expected.

IV. NEW CRITICAL POINTS

In this section, we get oriented by recovering the AdS_3 vacua (4). For simplicity, we introduce a radial direction $r = \sqrt{x_1^2 + x_2^2}$ and a U(1) isometry parametrized by φ. A general solution to (10) exists where

$$e^D = \frac{2\sqrt{|\mathcal{K}|}}{|\mathcal{K}| + \mathcal{K} r^2}, \tag{11}$$

resulting in a spacetime metric of the form

$$\mathrm{d}s_3^2 = -\ell^2 \left(\mathrm{d}\tau - \frac{\mathrm{sgn}(\mathcal{K}) r^2}{[1 + \mathrm{sgn}(\mathcal{K}) r^2]} \mathrm{d}\varphi \right)^2 + \frac{e^{-K}}{|\mathcal{K}|} \left[\frac{4(\mathrm{d}r^2 + r^2 \mathrm{d}\varphi^2)}{(1 + \mathrm{sgn}(\mathcal{K}) r^2)^2} \right]. \tag{12}$$

For AdS_3 vacua, $\mathcal{K} = -4T e^{-K} \sum_i e^{-W_i}|_{\partial_{W_i} T=0}$, which upon redefinition, $r = \tanh \rho$ and a shift $\varphi \to \varphi - \tau$, leads to the usual form of global AdS_3 (radius $\ell = \frac{2}{T}|_{\partial_{W_i} T=0}$),

$$\mathrm{d}s_3^2 = \ell^2 [-\cosh^2 \rho \mathrm{d}\tau^2 + \mathrm{d}\rho^2 + \sinh^2 \rho \mathrm{d}\varphi^2]. \tag{13}$$

We now present a key observation of this paper, namely that (9) has a second critical point, i.e. solutions with $\partial_a W_i = 0$, supported by fluxes. In addition to (4), the rhs of (9) vanishes when

$$e^{W_i} = \sum_{j \neq i} a_j + \frac{\kappa}{2} + \frac{\prod_{j \neq i} a_j}{\kappa}. \tag{14}$$

This exhausts the possibility for additional critical points beyond the supersymmetric AdS_3 vacuum. For $W_i \in \mathbb{R}$, a requirement for real solutions, necessarily $\kappa < 0$, so without loss of generality we set $\kappa = -1$. Furthermore, the range in parameter space where good vacua is constrained, as depicted in Fig. 1. From Fig. 1, suppressing a_1 through the supersymmetry condition $a_1 = 1 - a_2 - a_3$, we recognize that within the range of parameters where good AdS_3 vacua exist (cream), there are regions where additional new critical points exist (green). Points in parameter space where supersymmetry is enhanced to $\mathcal{N} = (2,2)$ ($\mathcal{K} = 0$) e. g. $(a_2, a_3) = (\frac{1}{2}, 0), (0, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2})$ on the dashed red locus are excluded, meaning new critical points only exist for $\mathcal{N} = (0,2)$ supersymmetry.

As one crosses the dashed red locus in Fig. 1, the topology of the Riemann surface parmetrized by (x_1, x_2) changes from H^2 externally to S^2 internally. We note that $\frac{\ell^2}{4} e^K |\mathcal{K}| \geq 0$ for critical points, so that the timelike fibration in the metric (12) is stretched, and thus warped. This inequality is saturated only for the supersymmetric AdS_3 vacua, where no stretching occurs. Moreover, the Ricci scalar of the overall three-dimensional spacetime, $R = 2(4T^2 + \mathcal{K} e^K)$, changes sign as one crosses this locus, an observation that justifies the billing "de Sitter" in the internal region, but not de Sitter in the conventional sense, since the geometry is supersymmetric. Uplifting the warped critical points to ten or eleven dimensions [16,17], one can show that CTCs appear for large values of r [32]. Finally, again suppressing a_1, we remark that there is an external locus, illustrated in Fig. 2, where critical points coalesce and only the supersymmetric AdS_3 vacuum exists,

$$a_2 = \frac{-1 + 2a_3 - a_3^2 \pm \sqrt{a_3 - 2a_3^2 + a_3^4}}{2(a_3 - 1)}. \tag{15}$$

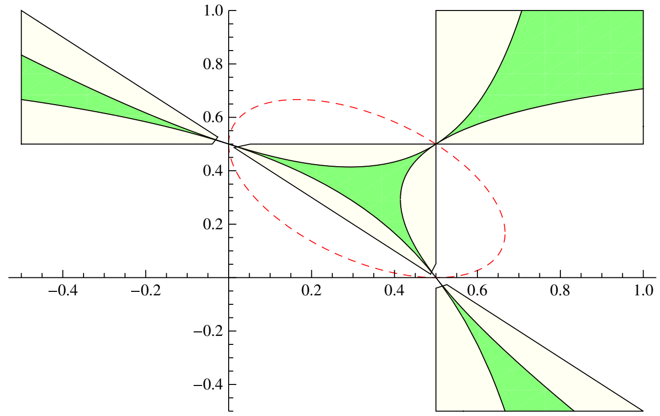

FIG. 1 (color online). The range of parameters in the (a_2, a_3) plane where the scalars W_i remain real for AdS_3 vacua (cream) and warped AdS_3 vacua (green). The dotted red line separates external ($\mathcal{K} < 0$) from internal regions ($\mathcal{K} > 0$).

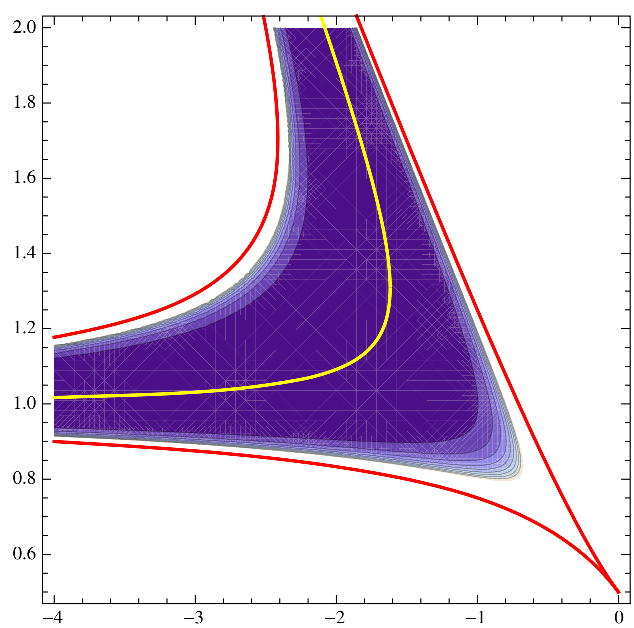

FIG. 2 (color online). A contour plot in the (a_2, a_3) plane of stretching, $4\Omega^2 - m^2$, in a sample $\mathcal{K} < 0$ region. Yellow curve corresponds to the AdS_3 locus.

At (14) the Gaussian curvature may be written as follows:

$$\mathcal{K} = 2(a_1a_2 + a_2a_3 + a_3a_1) - a_1^2 - a_2^2 - a_3^2 = \frac{2a_1a_2a_3}{\ell}. \tag{16}$$

For $\mathcal{K} < 0$, the critical points are easy to identify and correspond to supersymmetric Gödel spacetimes [24], a healthy collection of which can be found in three dimensions [42–45]. To see this, we can recast the solution in the form [46]

$$ds_3^2 = -\left(d\tau + \frac{4\Omega}{m^2}\sinh^2\left(\frac{m\rho}{2}\right)d\varphi\right)^2 + d\rho^2 + \frac{\sinh^2(m\rho)}{m^2}d\varphi^2, \tag{17}$$

where in our notation, one has $\rho = \frac{2}{m}\tanh^{-1}(r)$, $\Omega = \frac{\ell}{4}|\mathcal{K}|e^K|_{\text{crit}}$, $m^2 = |\mathcal{K}|e^K|_{\text{crit}}$. Written in the above form (17), the homogeneity and causal structure of the Gödel solution holds in the range $0 \le m^2 < 4\Omega^2$ [47], with the original Gödel solution at $m^2 = 2\Omega^2$ and AdS_3 at $m^2 = 4\Omega^2$. We plot $4\Omega^2 - m^2$ in Fig. 2, noting the yellow (zero valued) AdS_3 locus and steadily increasing contours outwards towards the boundaries.

For $\mathcal{N} = (0,2)$ SCFTs, the central charge is proportional to T^{-1} [15], making it the natural candidate for a holographic c-function [48,49]. Indeed, $T^{-1}_{AdS_3} > T^{-1}_{\text{Gödel}}$, so for flows from AdS_3 to Gödel, this observation suggests an analogue of Zamolodchikov's c-theorem [50]. Recent work [14] on the asymptotic symmetries of warped AdS_3, including Gödel, demonstrates that one can find two copies of the Virasoro symmetry, resulting in the expected central charge $c = \frac{3\ell}{2G}$ for a two-dimensional CFT. Unfortunately, the analysis in [14] and earlier [51] only holds for warped AdS_3 vacua where the cosmological constant does not change. In the language of three-dimensional gauged supergravity, this means we are confined to warped AdS_3 vacua that coexist with unwarped AdS_3 partners at constant value of the scalars in the potential.

Here, our setting is more general since the vacua exist at different values of the scalar fields, and these results do not apply. It is an open problem to repeat the analysis of Ref. [14] to see how the central charge depends on the scalar potential. Given the limitations of the literature, it is fitting to speculate that the inverse of the superpotential, as in the AdS_3 case, is the relevant quantity that encodes the central charge of the dual QFT. On this assumption [52], we can determine c at Gödel fixed points in terms of twist parameters of $\mathcal{N} = 4$ super-Yang-Mills:

$$c = 3|\mathfrak{g} - 1|N^2\prod_{i=1}^{3}\frac{1}{a_i}\left(2a_i^2 - \sum_{k=1}^{3}a_k^2\right). \tag{18}$$

It will be interesting to repeat the analysis of Ref. [14] to determine the central charge for gauged supergravities.

When $\mathcal{K} > 0$, little is known about these solutions, other than that they exist as solutions to topologically massive gravity [23] and suffer from CTCs. Since they are topologically $\mathbb{R} \times H^2$, it is possible that they can be analytically continued along the lines of [53] to give spacelike-warped AdS_3 with topology $S^1 \times AdS_2$, on the proviso we change the sign of T. We now show that this is not possible. To see this, we send $ds^2(S^2) = -ds^2(AdS_2)$ through redefining $r \to i\rho$, $\varphi \to i\tilde{\tau}$ and $\tau \to i\tilde{\varphi}$. This would leave us with a signature problem; however, this is overcome by $W_i \to \tilde{W}_i + i\pi$, an analytic continuation that allows us to (with $a_i \to -a_i$) flip the sign of T. Note, this leaves (9), (10) and (16) unchanged. From the uplifted five-dimensional perspective, this analytic continuation sends $\kappa \to -\kappa$, thus changing the genus $\mathfrak{g}$ of the Riemann surface used in the five-dimensional to three-dimensional reduction. Unfortunately, the price one pays for this operation is that G^i becomes complex, so the solution is not real. One can potentially overcome this by sending $T \to iT$, but then one sacrifices the consistent truncation, essentially by complexifying the theory.

Before outlining the construction of numerical solutions in the next section, we end with a final remark that we have only discussed classical supergravity vacua and the a_i should be quantized. To see this, we recall that the embedding in string theory is through a $U(1)^3$ fibration of S^5. For each $U(1)$ isometry ∂_{φ_i}, the corresponding gauge field, A^i, must be a connection on a bona fide $U(1)$ fibration. This is equivalent to the condition that the periods of the first Chern class be integer valued, or

$$\frac{1}{2\pi}\int_{\Sigma_{\mathfrak{g}}} dA^i = 2a_i(\mathfrak{g} - 1) \in \mathbb{Z}. \tag{19}$$

For $\mathfrak{g} > 1$ ($\kappa = -1$), where new critical points exist, this constraint poses few obstacles since we can ensure that the regions in Fig. 1 are populated by increasing the genus.

V. SUPERSYMMETRIC FLOWS

In this section we focus solely on parameters in the internal region of Fig. 1, where the timelike-warped de Sitter vacua exist, and construct a sample numerical solution to show flows from $\mathcal{N} = (0,2)$ fixed points exist. In this region topology changes from H^2 to S^2, and T changes sign making its c-function interpretation problematic. We note that linearizing (9) about its AdS_3 values, there is an instability to flows in the direction of the timelike-warped dS_3 point. In contrast, flows to Gödel are perturbatively stable.

Given a sample point in this region $(a_1, a_2, a_3) = (\frac{3}{10}, \frac{3}{10}, \frac{4}{10})$, we can use a shooting method, i.e. varying the initial conditions in the vicinity of $r = 0$, so that the interpolating solution arrives at the second critical point at

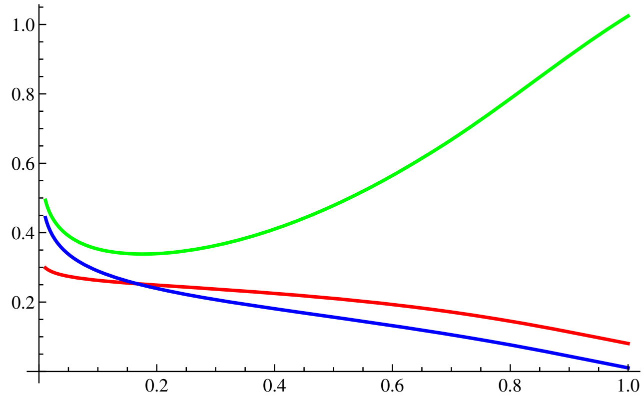

FIG. 3 (color online). Supersymmetric flow for $(a_1, a_2, a_3) = (\frac{3}{10}, \frac{3}{10}, \frac{2}{5})$, interpolating between AdS_3 values $W_1 = W_2 = \frac{3}{10}$ (red), $W_3 = \frac{9}{20}$ (blue) at $r = 0$ and warped AdS_3 values, $W_1 = W_2 = \frac{2}{25}$, $W_3 = \frac{1}{100}$, at $r = 1$. The green curve corresponds to D.

$r = 1$. We have checked that the output of MATHEMATICA in Fig. 3 leads to an error of order 1×10^{-7} when reinserted in the EOMs over the same range. Stiffness is encountered beyond $r = 1$, but this is due to T blowing up as $e^{W_i} \to 0$.

We have linearized the scalar EOMs about the AdS_3 vacuum to extract the masses, $m^2_{\phi_\pm} \ell^2 = \frac{1}{2}(4 \pm 3\sqrt{3})$, for scalars $\phi_\pm = \pm \frac{1}{\sqrt{3}} W_1 + \frac{1}{2}(1 \mp \frac{1}{\sqrt{3}}) W_3$, and they are consistent with one relevant and one irrelevant operator. Tracing the fluctuation to the boundary of AdS_3 at $r = 1$, it can be shown that the flows correspond to an irrelevant deformation of the SCFT [32]. We anticipate a rich class of supersymmetric flows both between critical points. It remains to be seen how these solutions are related to black holes, so that the CFT interpretation can be elucidated. We hope to report on these in future work.

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