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Green's function method for handling radiative effects on false vacuum decay

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We introduce a Green's function method for handling radiative effects on false vacuum decay. In addition to the usual thin-wall approximation, we achieve further simplification by treating the bubble wall in the planar limit. As an application, we take the $\lambda\Phi^4$ theory, extended with N additional heavier scalars, wherein we calculate analytically both the functional determinant of the quadratic fluctuations about the classical soliton configuration and the first correction to the soliton configuration itself.

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I. INTRODUCTION

The association of the still recently discovered 125 GeV scalar particle [1,2] with the Higgs boson of the Standard Model (SM) places the stability of the electroweak vacuum under question [3–6]. This instability, arising at an energy scale around 10^{11} GeV [7,8], results from the renormalization-group (RG) running of the Higgs self-coupling, whose value is driven negative by contributions dominated by top-quark loops. State-of-the-art calculations suggest that the electroweak vacuum is metastable, having a lifetime longer than the present age of the Universe and lying at the edge of the stable region [7–10], where seemingly small corrections may have a material impact upon predictions. The uncertainty in such predictions remains, at present, dominated by that of the top-quark pole mass [11,12]. Even so, it has been suggested [13–17] that, regardless of any improved precision in the experimental determination of the latter, the presence of Planck-scale operators may weaken the claim of metastability. Nevertheless, having, as yet, no experimental evidence of additional stabilizing physics between the electroweak and Planck scales, it is provident to consider approaches to the calculation of tunneling rates that can consistently account for radiative corrections.

The degree of vacuum metastability provides a strong criterion for the phenomenological viability of extensions to the SM. For example, supersymmetric scenarios can be ruled out if the electroweak symmetry-breaking vacuum decays into a color-breaking one in a timescale shorter than the age of the Universe [18–25]. In addition, transitions between vacua can also occur at finite temperature [26,27]. In the context of early-Universe cosmology, this is of interest because the corresponding first-order phase transitions may leave behind relic gravitational waves [28–30]. Moreover, such phase transitions may turn out to be pivotal for generating the cosmic matter-antimatter asymmetry [31,32]. As a consequence of these applications and the wide range of phenomenological models, there are now routine methods for computing transition rates at both vanishing and finite temperature [33,34].

Vacuum transitions in scalar theories can be described in the following way [35–38]. In the event that there are two nondegenerate vacua, an initially homogeneous system lying in the false vacuum will spontaneously nucleate bubbles of true vacuum, leading to the production of domain walls or "kinks." The latter are the topological solitons that interpolate between regions of true and false vacuum. The study of these "solitary wave" solutions to nonlinear equations of motion (see e.g. Ref. [39]) has a long history [40–45], and archetypal examples of such field configurations arise in the sine-Gordon model [46,47] and the $\lambda\Phi^4$ theory with tachyonic mass $m^2 < 0$. The semiclassical [38] and quantum [48] descriptions of false vacuum decay in the latter theory were presented in the seminal works by Coleman and Callan (see also Ref. [49]). Early expansion on these works included induced vacuum decay [50] and the incorporation of gravity [51].

In order to decide whether a vacuum configuration is unstable, i.e. whether there exists a lowest-lying true vacuum, it is often necessary to account for the impact of radiative corrections. This is of particular relevance when the appearance or disappearance of minima is entirely a radiative effect [52], such as occurs for the Coleman-Weinberg (CW) mechanism of spontaneous symmetry breaking [53] or in symmetry restoration at finite temperature [54–56]. In phenomenological studies, this is commonly done by calculating the tunneling rate from the effective potential [57,58] of a *homogeneous* field configuration [34,59,60], which is subsequently promoted to a space-time-dependent configuration. This practice is problematic for two reasons. First, the temporal and spatial inhomogeneity of the solitonic background is not fully taken into account [61]. Second, in the presence of tachyonic instabilities, e.g. when there are nonconvex

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regions in the tree-level potential, the perturbatively calculated effective potential receives a seemingly pathological imaginary part. The latter has been shown [62] to have a physical interpretation as a decay rate for an initially homogeneous field configuration (see also Ref. [63]). However, this subtlety can be circumvented by using constructions such as the coarse-grained effective action [64–71] or, as is often employed in lattice simulations, the constraint effective potential [72] (cf. Ref. [73]). In addition, within the context of the SM, the standard RG improvement of the effective potential has recently been questioned [74]. For the above reasons, it is reasonable to conclude that the use of the effective potential to calculate transition rates is neither satisfactory nor justifiable, and it is desirable to consider alternative methods for determining quantum corrections, which can be applied to a wide range of models that feature vacuum decay.

The first quantum corrections [48] to the tunneling rate are those arising from the functional determinant over the quadratic fluctuations about the classical soliton configuration. In the case of one-dimensional operators, these determinants may be calculated using the Gel'fand-Yaglom theorem [75], which may be generalized to higher dimensions in the case of radially symmetric operators [76–78]. General numerical techniques may then be obtained [79,80] for calculating tunneling rates beyond the so-called thin-wall approximation, in which the width of the bubble wall is much smaller than its radius. These approaches have also been applied to radially separable Yang-Mills backgrounds [81,82] and scenarios in curved spacetime [83]. Alternatively, as we will employ, the functional determinant may be calculated by means of the so-called heat kernel method (see e.g. Refs. [84–87]), based upon the Schwinger proper-time representation and zeta function regularization [88]. Previously, this approach has been used to derive approximate analytic results for the one-loop fluctuation determinant beyond the thin-wall approximation [89], as well as one-loop corrections to sphaleron rates [90–92]. The latter have also been calculated by direct integration of the Green's function [93–97].

Recently, it has been shown that properties of topological solitons may be studied nonperturbatively using Monte Carlo and lattice simulations by considering correlation functions directly [98–100]. Other authors have proposed methods for calculating quantum corrections based upon functional renormalization techniques [101].

In this article, we derive an *analytic* result for the Green's function of the $\lambda\Phi^4$ theory in the background of the classical kink solution. Within the thin- and planar-wall approximations, we illustrate that this Green's function may be used to determine *analytically* the leading quantum corrections to both the semiclassical bounce action and the kink solution itself, thereby allowing us to compute the tunneling rate at the two-loop level, while isolating its diagrammatic interpretation. The latter calculation is performed within the context of a toy model extended with an additional N heavier scalars, where the parametric dependence on N allows the identification of a concrete example in which the calculated two-loop corrections dominate over the neglected higher-loop corrections. We illustrate that the problem of calculating these radiative corrections may be reduced to one of solving one-dimensional ordinary differential equations and integrals. Thus, we anticipate that this methodical development may have numerical applications in the study of the decay rates of radiatively generated metastable vacua, such as occur in the massless CW model [53] or the Higgs potential of the SM. Similar methods based upon Green's function techniques have been used previously to determine self-consistent bounce solutions *numerically* in the Hartree approximation of the pure $\lambda\Phi^4$ theory in both two and four dimensions [102–105].

The remainder of this article is organized as follows: In Sec. II, we review the calculation, *à la* Coleman and Callan [38,48], of the classical "bounce" configuration, describing the semiclassical tunneling rate between two quasi-degenerate vacua and its first quantum corrections. In Sec. III, we outline a Green's function method for the evaluation of the functional determinant over the quantum fluctuations about the classical bounce, making comparison with existing calculations. Subsequently, in Sec. IV, we illustrate that this Green's function method may be used to calculate analytically and self-consistently the first quantum corrections to the bounce itself. In Sec. V, we conclude our discussions and highlight potential applications and future directions. Finally, a number of mathematical appendices are included, outlining the technical details of the calculations summarized in Secs. III and IV.

II. SEMICLASSICAL BOUNCE

We consider a real scalar field $\Phi \equiv \Phi(x)$, with four-dimensional Euclidean Lagrangian $\mathcal{L} = (\partial_\mu \Phi)^2/2 + U$ and classical potential

$$U = \frac{1}{2!} m_\Phi^2 \Phi^2 + \frac{g}{3!}\Phi^3 + \frac{\lambda}{4!}\Phi^4 + U_0. \tag{1}$$

The mass squared is $m_\Phi^2 = -\mu^2 < 0$, g is of mass dimension 1, λ is dimensionless, U_0 is a constant, and $\partial_\mu \equiv \partial/\partial x_\mu$ denotes the derivative with respect to the Euclidean spacetime coordinate $x_\mu \equiv (\mathbf{x}, x_4)$. Throughout, we omit spacetime and field arguments for notational convenience when no ambiguity results.

The classical potential in Eq. (1) has nondegenerate minima at

$$\varphi = v_\pm = \pm v\left(1 + \frac{\bar{v}^2}{v^2}\right)^{\frac{1}{2}} - \bar{v}, \tag{2}$$

as depicted in Fig. 1 (left panel), where we have defined $v = \sqrt{6\mu^2/\lambda}$ and $\bar{v} = (3g)/(2\lambda)$. The separation of the minima $\Delta v = v_+ - v_- = 2d$ and the difference in their energy densities $\Delta U = U_{v_+} - U_{v-} = 2\varepsilon$ may be written in terms of the parameters

$$d = v\left(1 + \frac{\bar{v}^2}{v^2}\right)^{\frac{1}{2}} \approx v, \tag{3a}$$

$$\frac{\varepsilon}{d} = \frac{gv^2}{6}\left(1 + 3\frac{\bar{v}^2}{v^2}\right) \approx \frac{gv^2}{6}, \tag{3b}$$

where the approximations are valid in the limit $v \gg \bar{v}$, i.e. $g^2/\mu^2 \ll 8\lambda/3$. For $g \to 0$, $\varepsilon \to 0$, and the minima at $\varphi = \pm v$ become degenerate, as we would expect.

Finally, the constant U_0 is chosen so that the potential vanishes in the false vacuum at $\varphi = +v$, requiring $U_0 = (\mu v/2)^2 - gv^3/6$ and thus giving the barrier height to be $h = U_0 + 2\varepsilon \approx (\mu v/2)^2 + \varepsilon$ and $U(-v) = -gv^3/3$.

The semiclassical probability for tunneling between the false ($\varphi = +v$) and true ($\varphi = -v$) vacua and its first quantum corrections were described in the seminal works by Coleman and Callan [38,48], which we now review. The classical equation of motion

$$-\partial^2\varphi + U'(\varphi) = 0, \tag{4}$$

where $'$ denotes the derivative with respect to the field φ, is analogous to that of a particle moving in a potential $-U$. The boundary conditions of the "bounce" are $\varphi|_{x_4\to\pm\infty} = +v$ and $\dot{\varphi}|_{x_4=0} = 0$, where $\cdot$ denotes the derivative with respect to x_4. These correspond to a particle initially at $+v$ rolling through the valley in $-U$, reaching a turning point close to $-v$, before rolling back to $+v$, see Fig. 1 (right panel). Finally, in order to ensure that the action of the bounce is finite, we require $\varphi|_{|\mathbf{x}|\to\infty} = +v$.

Translating to four-dimensional hyperspherical coordinates, Eq. (4) takes the form

$$-\frac{\mathrm{d}^2}{\mathrm{d}r^2}\varphi - \frac{3}{r}\frac{\mathrm{d}}{\mathrm{d}r}\varphi + U'(\varphi) = 0, \tag{5}$$

where $r^2 = \mathbf{x}^2 + x_4^2$. The boundary conditions become $\varphi|_{r\to\infty} = +v$ and $\mathrm{d}\varphi/\mathrm{d}r|_{r=0} = 0$, where the latter ensures that the solution is well defined at the origin. Thus, the bounce corresponds to a four-dimensional bubble of some radius R, which separates the false vacuum ($\varphi = +v$) outside from the true vacuum inside ($\varphi = -v$). Analytically continuing to Minkowski spacetime ($x_4 = ix_0$), the $O(4)$ symmetry of the bounce becomes an $SO(1,3)$ symmetry, with the bubble expanding along the hyperbolic trajectory $R^2 = \mathbf{x}^2 - c^2t^2$.

The bounce action is

$$B = \int \mathrm{d}^4x\left[\frac{1}{2}\left(\frac{\mathrm{d}\varphi}{\mathrm{d}r}\right)^2 + U(\varphi)\right], \tag{6}$$

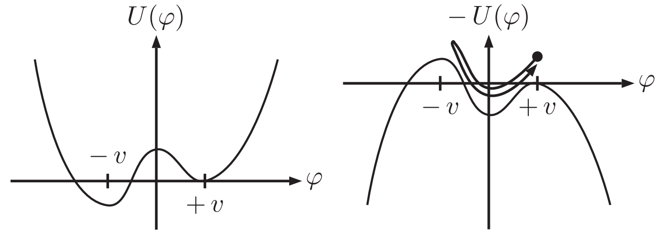

FIG. 1. The classical potential U (left panel) and the inverted potential $-U$ (right panel). The arrow (right panel) indicates the trajectory of the "bounce" in imaginary time τ.

which can be written as $B = B_{\mathrm{surface}} + B_{\mathrm{vacuum}}$, where

$$B_{\mathrm{surface}} = 2\pi^2R^3\int_{-v}^{+v}\mathrm{d}\varphi\frac{\mathrm{d}\varphi}{\mathrm{d}r}, \tag{7a}$$

$$B_{\mathrm{vacuum}} = 2\pi^2\int_0^R \mathrm{d}r r^3 U(-v) \tag{7b}$$

are the contributions from the surface tension of the bubble and the energy of the true vacuum, respectively. In writing Eq. (7a), we have used the fact that for $\partial_\mu\varphi \neq 0$, i.e. for $r \sim R$, we may show that the bounce φ satisfies the virial theorem

$$\left(\frac{\mathrm{d}\varphi}{\mathrm{d}r}\right)^2 - 2U(\varphi) = 0, \tag{8}$$

i.e. it is the configuration of zero total energy density. Notice that there is no contribution to the bounce action [Eq. (6)] from the exterior of the bubble, since the choice of the potential Eq. (1), viz. U_0, ensures that the false vacuum has zero energy density.

In the thin-wall approximation, we may safely neglect the damping term in Eq. (5) and the contribution from the cubic self-interaction $g\varphi^3$, as will be the case for the remainder of this article. We then obtain the well-known kink solution [40]

$$\varphi(r) = v\tanh[\gamma(r - R)], \tag{9}$$

with $\gamma = \mu/\sqrt{2}$. The radius R of the bubble is then obtained by extremizing the bounce action [Eq. (6)], that is by minimizing the energy difference between the surface tension of the bubble and the true vacuum. This gives

$$R = \frac{12\gamma}{gv}. \tag{10}$$

By considering the invariance of the bounce action in Eq. (6) under general coordinate transformations, i.e. $\varphi \to \varphi + x_\mu\partial_\mu\varphi$, we may show that

$$B = \frac{1}{2}\pi^2R^3\int_{-v}^{+v}\mathrm{d}\varphi\frac{\mathrm{d}\varphi}{\mathrm{d}r}. \tag{11}$$

This is to say that

$$B_{\text{vacuum}} = -\frac{3}{4}B_{\text{surface}}, \tag{12}$$

in which case we find

$$B = -\frac{1}{3}B_{\text{vacuum}} = -\frac{\pi^2}{6}R^4U(-v) = \frac{8\pi^2R^3\gamma^3}{\lambda}. \tag{13}$$

The decay rate of the false vacuum, i.e. the probability per unit time for the nucleation of a bubble of true vacuum, has the generic form [38,48]

$$\Gamma = AVe^{-B/\hbar}. \tag{14}$$

Here, V is the three-volume within which the bounce may occur, arising from integrating over the center of the bounce, and A contains the quantum corrections to the classical bounce action B that are the subject of the remainder of this article.

The tunneling probability in Eq. (14) may be obtained from the path integral

$$Z[0] = \int[\mathrm{d}\Phi]e^{-S[\Phi]/\hbar}, \tag{15}$$

via

$$\Gamma = 2|\mathrm{Im}Z[0]|/T, \tag{16}$$

where T is the Euclidean time of the bounce.

In order to evaluate the functional integral over Φ, we first expand around the classical bounce φ, whose equation of motion [Eq. (4)] is obtained from

$$\left.\frac{\delta S[\Phi]}{\delta\Phi}\right|_{\Phi=\varphi} = 0. \tag{17}$$

Writing $\Phi = \varphi + \hbar^{1/2}\hat{\Phi}$, where the factor of $\hbar^{1/2}$ is written explicitly for bookkeeping purposes, we find

$$S[\Phi] = S[\varphi] + \frac{\hbar}{2}\int \mathrm{d}^4x\hat{\Phi}(x)G^{-1}(\varphi;x)\hat{\Phi}(x) + \mathcal{O}(\hbar^{3/2}), \tag{18}$$

where $S[\varphi] \equiv B$ is the classical bounce action and

$$G^{-1}(\varphi;x) \equiv \left.\frac{\delta^2S[\Phi]}{\delta\Phi^2(x)}\right|_{\Phi=\varphi} = -\Delta^{(4)} + U''(\varphi;x), \tag{19}$$

in which $\Delta^{(4)}$ is the four-dimensional Laplacian.

Before proceeding to perform the functional integration over the quadratic fluctuations about the bounce, we must consider the spectrum of the operator $G^{-1}(\varphi;x)$, which is not positive definite. By differentiating the equation of motion [Eq. (4)] with respect to x_μ and comparing with the eigenvalue equation

$$(-\Delta^{(4)} + U''(\varphi))\phi_{\{n\}} = \lambda_{\{n\}}\phi_{\{n\}}, \tag{20}$$

it is straightforward to show that there exist 4 zero eigenmodes $\phi_\mu = \mathcal{N}\partial_\mu\varphi$, transforming as a vector of $SO(4)$ and resulting from the translational invariance of the bounce. The normalization $\mathcal{N}$ follows from Eq. (11), since

$$\int \mathrm{d}^4x\phi_\mu^*\phi_\nu = \frac{1}{4}\mathcal{N}^2\delta_{\mu\nu}\int \mathrm{d}^4x(\partial_\lambda\varphi)^2 = \mathcal{N}^2B\delta_{\mu\nu}. \tag{21}$$

Thus, $\phi_\mu = B^{-1/2}\partial_\mu\varphi$.

Differentiating Eq. (5) with respect to r and subsequently setting $r = R$ in those terms originating from the damping term, we can show that there also exists a discrete eigenmode $\phi_0 = B^{-1/2}\partial_r\varphi$. This eigenmode transforms as a scalar of $SO(4)$, corresponding to dilatations of the classical bounce solution, and has the negative eigenvalue

$$\lambda_0 = \frac{1}{B}\frac{\delta^2B}{\delta R^2} = -\frac{3}{R^2}. \tag{22}$$

It is this lowest mode that is responsible for the path integral in Eq. (15) obtaining the nonzero imaginary part in Eq. (16) [106].

Alternatively, we may solve the eigenvalue problem directly in hyperspherical coordinates (see Appendix B), by making the substitution $\phi_{\{n\}} = \phi_{nj}/r^3$.[1] Neglecting the damping term and setting $r = R$ in the centrifugal potential, we obtain the eigenspectrum

$$\lambda_{nj} = \gamma^2(4-n^2) + \frac{j(j+2)-3}{R^2}. \tag{23}$$

The radial parts of the eigenfunctions are the associated Legendre polynomials of the first kind and of order 2, i.e. $P_2^n(\varphi/v)$. Thus, demanding normalizability, the quantum number n is restricted to the set $\{1,2\}$.

From Eq. (23), we see that the negative mode corresponds to $\lambda_0 = \lambda_{20}$ $(n=2,\ j=0)$, and the zero modes correspond to λ_{21} $(n=2,\ j=1)$, having degeneracy $(j+1)^2 = 4$. The lowest two positive-definite eigenvalues are $\lambda_{10} = 2\gamma^2 - 3/R^2$ $(n=1, j=0)$ and $\lambda_{11} = 2\gamma^2$ $(n=1, j=1)$. Thus, for R large, the "continuum" of positive-definite modes begins at $\lambda_{10} \approx \lambda_{11} = 2\gamma^2$, cf. Ref. [86].

In order to perform the functional integral over the five negative-semidefinite discrete modes, we expand $\hat{\Phi} = \sum_{i=0}^4 a_i\phi_i + \phi_+$, where ϕ_+ comprises the continuum

[1] We note that this substitution differs from that used in Ref. [48].

of positive-definite eigenmodes. The functional measure then becomes

$$[\mathrm{d}\Phi] = [\mathrm{d}\phi_+] \prod_{i=0}^{4} (2\pi\hbar)^{-1/2} \mathrm{d}a_i. \tag{24}$$

The functional integral over the 4 zero eigenmodes ($i = 1, \ldots, 4$) is traded for an integral over the collective coordinates of the bounce [107] (see Appendix A) and yields a factor

$$VT\left(\frac{B}{2\pi\hbar}\right)^2. \tag{25}$$

The integral over the negative eigenmode ($i = 0$) may be performed using the method of steepest descent, giving an overall factor of $-i|\lambda_0|^{-1/2}/2$. Here, the overall sign is unphysical [48] and depends on the choice of analytic continuation, thereby justifying the modulus sign in Eq. (16).

Finally, the Gaussian integral over the continuum of positive eigenmodes ϕ_+ may be performed in the usual manner, and we obtain

$$iZ[0] = e^{-B/\hbar}\left|\frac{\lambda_0 \mathrm{det}^{(5)} G^{-1}(\varphi)}{\frac{1}{4}(VT)^2(\frac{B}{2\pi\hbar})^4(4\gamma^2)^5 \mathrm{det}^{(5)} G^{-1}(v)}\right|^{-\frac{1}{2}}, \tag{26}$$

in which $\mathrm{det}^{(5)}$, cf. Ref. [86], denotes the determinant calculated only over the continuum of positive-definite eigenmodes, i.e. omitting the zero and negative eigenmodes, whose contributions are included explicitly. In addition, we have normalized the determinant to that of the operator $G^{-1}(v)$, evaluated in the false vacuum. Substituting Eq. (26) into Eq. (16), we find the tunneling rate per unit volume

$$\Gamma/V = \left(\frac{B}{2\pi\hbar}\right)^2 (2\gamma)^5 |\lambda_0|^{-\frac{1}{2}} \exp\left[-\frac{1}{\hbar}(B + \hbar B^{(1)})\right], \tag{27}$$

where

$$B^{(1)} = \frac{1}{2}\mathrm{tr}^{(5)}(\ln G^{-1}(\varphi) - \ln G^{-1}(v)) \tag{28}$$

contains the one-loop corrections from the quadratic fluctuations around the classical bounce. Here, $\mathrm{tr}^{(5)}$ indicates that we are to trace over only the positive-definite eigenmodes.

III. GREEN'S FUNCTION METHOD

In this section, we outline the derivation of the Green's function of the operator in Eq. (19). The technical details are included for completeness in Appendix B. Subsequently, we use this Green's function to evaluate the functional determinant in Eq. (26) and obtain the correction from quadratic fluctuations. In addition, we calculate analytically the tadpole contribution to the effective equation of motion and point out that this may be used to calculate the first quantum corrections to the bounce.

We have the inhomogeneous Klein-Gordon equation

$$(-\Delta^{(4)} + U''(\varphi; x))G(\varphi; x, x') = \delta^{(4)}(x - x'), \tag{29}$$

where $\delta^{(4)}(x - x')$ is the four-dimensional Dirac delta function. Working in hyperspherical coordinates and writing $x_\mu^{(\prime)} = r^{(\prime)}\mathbf{e}_{r^{(\prime)}}$, where $\mathbf{e}_{r^{(\prime)}}$ are four-dimensional unit vectors, the Green's function may be expanded as

$$G(\varphi;x,x') = \frac{1}{2\pi^2}\sum_{j=0}^{\infty}(j+1)G_j(\varphi;r,r')U_j(\cos\theta), \tag{30}$$

where $\cos\theta = \mathbf{e}_r \cdot \mathbf{e}_{r'}$ and $U_j(z)$ are the Chebyshev polynomials of the second kind (see Appendix B). The radial functions $G_j(r, r')$ satisfy the inhomogeneous equation

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{3}{r}\frac{\mathrm{d}}{\mathrm{d}r} + \frac{j(j+2)}{r^2} + U''(r)\right]G_j(\varphi; r, r') = \frac{\delta(r - r')}{r'^3}. \tag{31}$$

For the thin wall, we safely neglect the damping term and approximate the centrifugal term by $j(j+2)/R^2$. For self-consistency of this approximation, we also replace the discontinuity on the rhs of Eq. (31) with $\delta(r - r')/R^3$. For generality of notation in what follows, it is then convenient to define

$$G(u, u', m) \equiv R^3 G_j(\varphi; r, r'), \tag{32}$$

being a function only of the normalized bounce

$$u^{(\prime)} \equiv \frac{\varphi(r^{(\prime)})}{v} = \tanh[\gamma(r^{(\prime)} - R)] \tag{33}$$

and the parameter

$$m = 2\left(1 + \frac{j(j+2)}{4\gamma^2 R^2}\right)^{\frac{1}{2}}. \tag{34}$$

The full Green's function may then be written

$$G(\varphi;x,x') \equiv G(u,u',\theta) = \frac{1}{2\pi^2 R^3}\sum_{j=0}^{\infty}(j+1)U_j(\cos\theta)G(u,u',m). \tag{35}$$

With the above approximations, the lhs of Eq. (31) is of Pöschl-Teller form [108], having general solutions that may be expressed in terms of the associated Legendre functions (see Appendix B). We are then able to find the full analytic solution

$$G(u,u',m) = \frac{1}{2\gamma m}\left[\vartheta(u-u')\left(\frac{1-u}{1+u}\right)^{\frac{m}{2}}\left(\frac{1+u'}{1-u'}\right)^{\frac{m}{2}} \times\left(1-3\frac{(1-u)(1+m+u)}{(1+m)(2+m)}\right) \times\left(1-3\frac{(1-u')(1-m+u')}{(1-m)(2-m)}\right)+(u\leftrightarrow u')\right], \quad (36)$$

where $\vartheta(z)$ is the generalized unit-step function.

Taking the coincidence limit $u=u'$, $\theta=0$, the local contribution to the Green's function $G(u)\equiv G(u,u,0)$ in Eq. (35) takes the form

$$G(u) = \frac{1}{2\pi^2 R^3}\sum_{j=0}^{\infty}(j+1)^2 G(u,m), \quad (37)$$

where

$$G(u,m)\equiv G(u,u,m) = \frac{1}{2\gamma m}\left[1+3(1-u^2)\sum_{n=1}^{2}\frac{(-1)^n(n-1-u^2)}{m^2-n^2}\right]. \quad (38)$$

In Eq. (38), the summation over $n=1,2$ corresponds to the contributions from the two towers of positive-definite eigenmodes of the operator $G^{-1}(\varphi;x)$, see Eq. (23).

For R large, we may approximate the summation over j by an integral over a continuous variable $k\sim\frac{j+1}{R}$ (see Appendix B). In which case, we obtain

$$G(u) = \frac{1}{2\pi^2}\int_0^{\infty}\mathrm{d}k k^2 G(u,m), \quad (39)$$

with

$$m = 2\left(1+\frac{k^2}{4\gamma^2}\right)^{\frac{1}{2}}. \quad (40)$$

The continuum limit described above is entirely equivalent to the so-called planar-wall approximation. Therein, for R large, we align a set of coordinates $(z_\perp,\mathbf{z}_\parallel)$ with the bubble wall, as shown in Fig. 2. We may then Fourier-transform with respect to the coordinates $\mathbf{z}_\parallel$ that lie within the three-dimensional wall, introducing a three-momentum $\mathbf{k}$, i.e.

$$G(\varphi;x,x') = \int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}e^{i\mathbf{k}\cdot(\mathbf{z}_\parallel-\mathbf{z}'_\parallel)}G(\varphi;z,z',\mathbf{k}), \quad (41)$$

where we have let $z=z_\perp$ for notational convenience. The three-momentum-dependent Green's function $G(\varphi;z,z',\mathbf{k})$ satisfies the inhomogeneous Klein-Gordon equation

$$(-\partial_z^2+k^2+U''(\varphi;z))G(\varphi;z,z',\mathbf{k}) = \delta(z-z'). \quad (42)$$

We may then show straightforwardly that

$$G(\varphi;z,z',\mathbf{k}) = G(u,u',m), \quad (43)$$

where $G(u,u',m)$ is as presented in Eq. (36), with m given by Eq. (40). This planar-wall approximation is employed for the remainder of this article.

A. Quantum-corrected bounce

Before making use of the Green's function calculated in the preceding section, we first derive the equation of motion for the quantum-corrected bounce. This calculation was first suggested by Goldstone and Jackiw [42] and, in the following sections, we will illustrate that, within the thin- and planar-wall approximations, it may be completed analytically.

The one-particle irreducible (1PI) effective action [57] is given by the Legendre transform

$$\Gamma[\phi] = -\hbar\ln Z[J]+\int\mathrm{d}^4xJ(x)\phi(x), \quad (44)$$

where

$$\phi(x) = \hbar\frac{\delta\ln Z[J]}{\delta J(x)} \quad (45)$$

is a functional of the source $J(x)=\frac{\delta\Gamma[\phi]}{\delta\phi(x)}$ and

$$Z[J] = \int[\mathrm{d}\Phi]\exp\left[-\frac{1}{\hbar}\left(S[\Phi]-\int\mathrm{d}^4xJ(x)\Phi(x)\right)\right]. \quad (46)$$

In order to obtain the quantum corrections to the bounce φ, we wish to evaluate the functional integral in Eq. (46) by expanding around the configuration $\varphi^{(1)}$, which is the solution to the quantum equation of motion

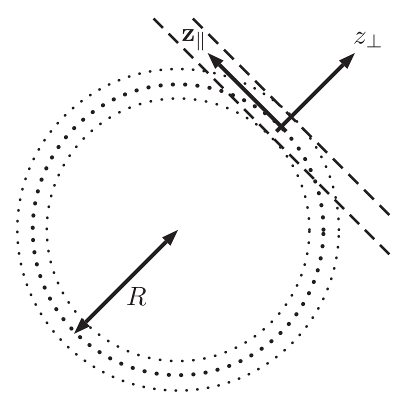

FIG. 2. Schematic representation of the planar-wall approximation, illustrating the alignment of the three-dimensional hypersurface and associated coordinate system.

$$\left.\frac{\delta\Gamma[\phi]}{\delta\phi}\right|_{\phi=\varphi^{(1)}} = 0. \tag{47}$$

Here, the superscript "(1)" indicates that $\varphi^{(1)}$ contains the first quantum corrections to φ. It follows from Eq. (47) that $\varphi^{(1)}$ cannot extremize the classical action in the absence of the source J, i.e.

$$\left.\frac{\delta S[\Phi]}{\delta\Phi(x)}\right|_{\Phi=\varphi^{(1)}} = J(x) \neq 0. \tag{48}$$

Writing $\Phi = \varphi^{(1)} + \hbar^{1/2}\hat{\Phi}^{(1)}$, where the factor of $\hbar^{1/2}$ is again written explicitly for bookkeeping, we proceed as in Sec. II, expanding

$$S[\Phi] = S[\varphi^{(1)}] + \hbar^{1/2}\int \mathrm{d}^4x J(x)\hat{\Phi}^{(1)}(x) + \frac{\hbar}{2}\int \mathrm{d}^4x\hat{\Phi}^{(1)}(x)G^{-1}(\varphi^{(1)};x)\hat{\Phi}^{(1)}(x) + \cdots, \tag{49}$$

where

$$G^{-1}(\varphi^{(1)};x) \equiv \left.\frac{\delta^2 S[\Phi]}{\delta\Phi^2(x)}\right|_{\Phi=\varphi^{(1)}} = -\Delta^{(4)} + U''(\varphi^{(1)};x). \tag{50}$$

We now write $\varphi^{(1)} = \varphi + \hbar\delta\varphi$ and expand about the quadratic fluctuations evaluated around the classical bounce φ. Thus, in performing the functional integral, we consider the same spectrum of negative and zero eigenmodes as in Sec. II. Finally, by expanding the effective action $\Gamma[\phi]$ in Eq. (44) around $\varphi^{(1)} = \phi - \hbar\delta\varphi$ (see Ref. [109]), we obtain

$$\Gamma[\varphi^{(1)}] = S[\varphi^{(1)}] + \frac{i\pi\hbar}{2} + \hbar^2 B^{(2)\prime}[\varphi] + \frac{\hbar}{2}\ln\left|\frac{\lambda_0 \det^{(5)} G^{-1}(\varphi)}{\frac{1}{4}(VT)^2(\frac{B}{2\pi\hbar})^4(4\gamma^2)^5\det^{(5)}G^{-1}(v)}\right| + \cdots. \tag{51}$$

where

$$B^{(2)\prime}[\varphi] = \frac{1}{2}\int \mathrm{d}^4x\delta\varphi(x) \times \frac{\delta}{\delta\varphi^{(1)}(x)}\ln\left.\frac{\det^{(5)}G^{-1}(\varphi^{(1)})}{\det^{(5)}G^{-1}(v)}\right|_{\varphi^{(1)}=\varphi}. \tag{52}$$

Functionally differentiating Eq. (51) with respect to $\varphi^{(1)}$, we obtain the equation of motion for the corrected bounce

$$-\partial^2\varphi^{(1)}(x) + U'_{\mathrm{eff}}(\varphi^{(1)};x) = 0, \tag{53}$$

where

$$U'_{\mathrm{eff}}(\varphi^{(1)};x) \equiv U'(\varphi^{(1)};x) + \hbar\Pi(\varphi;x)\varphi(x), \tag{54}$$

containing the tadpole contribution

$$\Pi(\varphi;x) = \frac{\lambda}{2}G(\varphi;x,x). \tag{55}$$

Comparing the functional derivative of Eq. (51) with Eqs. (47) and (48), we see that this evaluation of the effective action is self-consistent so long as the source

$$J(x) = -\hbar\Pi(\varphi;x)\varphi(x), \tag{56}$$

which is, as expected, nonvanishing.

We may show that the correction to the classical bounce $\delta\varphi$ satisfies the equation of motion

$$G^{-1}(\varphi;x)\delta\varphi(x) = -\Pi(\varphi;x)\varphi(x). \tag{57}$$

The corrected bounce action $S[\varphi^{(1)}]$ contains contributions at order $\hbar^2$. Specifically,

$$S[\varphi^{(1)}] = S[\varphi] + \frac{\hbar^2}{2}\int \mathrm{d}^4x\delta\varphi(x)G^{-1}(\varphi;x)\delta\varphi(x) + \mathcal{O}(\hbar^3), \tag{58}$$

where we have used Eqs. (17) and (19). Thus, using Eq. (57), we may write

$$S[\varphi^{(1)}] = B + \hbar^2 B^{(2)}, \tag{59}$$

where

$$B^{(2)} = -\frac{1}{2}\int \mathrm{d}^4x\varphi(x)\Pi(\varphi;x)\delta\varphi(x). \tag{60}$$

Hence, we obtain the tunneling rate per unit volume

$$\begin{aligned}\Gamma/V &= 2|\mathrm{Im}e^{-\Gamma[\varphi^{(1)}]/\hbar}|/(VT)\\ &= \left(\frac{B}{2\pi\hbar}\right)^2(2\gamma)^5|\lambda_0|^{-\frac{1}{2}}\\ &\quad\times\exp\left[-\frac{1}{\hbar}(B + \hbar B^{(1)} + \hbar^2 B^{(2)} + \hbar^2 B^{(2)\prime})\right],\end{aligned} \tag{61}$$

where B is the classical bounce action; $B^{(1)}$, given in Eq. (28), contains the corrections from quadratic fluctations about the classical bounce; and $B^{(2)}$, given in Eq. (60), contains the contribution arising from the quantum corrections to the bounce itself. We note that

$$B^{(2)\prime} = -2B^{(2)}, \tag{62}$$

such that the $\mathcal{O}(\hbar)$ corrections to the quadratic fluctuations flip the sign of the contribution to the bounce action from the $\mathcal{O}(\hbar)$ corrections to the bounce itself.

B. Tadpole contribution

We will now proceed to calculate explicitly the tadpole contribution appearing in Eq. (55).

Introducing an ultraviolet cutoff Λ, the k integral can be performed in Eq. (39), and we obtain

$$G(u) = \frac{\gamma^2}{8\pi^2}\left[\frac{\Lambda^2}{\gamma^2} + 2 - (1-3u^2)\ln\frac{\gamma^2}{\Lambda^2} - \sqrt{3}\pi u^2(1-u^2)\right]. \tag{63}$$

We choose to define the physical mass and coupling in the homogeneous nonsolitonic background.[2] The renormalization conditions are then as follows:

$$\left.\frac{\partial^2 U_{\rm eff}(\varphi)}{\partial\varphi^2}\right|_{\varphi=v} = -\mu^2 + \frac{\lambda}{2}v^2 = 2\mu^2, \tag{64a}$$

$$\left.\frac{\partial^4 U_{\rm eff}(\varphi)}{\partial\varphi^4}\right|_{\varphi=v} = \lambda, \tag{64b}$$

where $U_{\rm eff}$ is the CW effective potential [53]. The resulting mass and coupling counterterms are

$$\delta m^2 = -\frac{\lambda\gamma^2}{16\pi^2}\left(\frac{\Lambda^2}{\gamma^2} - \ln\frac{\gamma^2}{\Lambda^2} - 31\right), \tag{65a}$$

$$\delta\lambda = -\frac{3\lambda^2}{32\pi^2}\left(\ln\frac{\gamma^2}{\Lambda^2} + 5\right). \tag{65b}$$

We then arrive at the renormalized tadpole correction

$$\Pi^R(u) = \frac{\lambda}{2}G(u) + \delta m^2 + \frac{2\gamma^2}{\lambda}\delta\lambda u^2 = \frac{3\lambda\gamma^2}{16\pi^2}\left[6 + (1-u^2)\left(5 - \frac{\pi}{\sqrt{3}}u^2\right)\right]. \tag{66}$$

C. Functional determinant

We may calculate the traces appearing in the exponent of Eq. (27), which arise from the functional determinant of the operator $G^{-1}(\varphi)$ in Eq. (26), by using the heat kernel method (see e.g. Ref. [87]). Specifically, the trace may be written in the form

$$\mathrm{tr}^{(5)}\ln G^{-1}(\varphi;x) = -\int \mathrm{d}^4x\int_0^\infty\frac{\mathrm{d}\tau}{\tau}K(\varphi;x,x|\tau). \tag{67}$$

The heat kernel $K(\varphi;x,x'|\tau)$ is the solution to the heat-flow equation

$$\partial_\tau K(\varphi;x,x'|\tau) = G^{-1}(\varphi;x)K(\varphi;x,x'|\tau) \tag{68}$$

and satisfies the condition $K(\varphi;x,x'|0) = \delta^{(4)}(x-x')$.

It is convenient to work in terms of the Laplace transform of the heat kernel

$$\mathcal{K}(\varphi;x,x'|s) = \int_0^\infty \mathrm{d}\tau e^{s\tau}K(\varphi;x,x'|\tau), \tag{69}$$

which is the solution to

$$(-\partial^2 + s + U''(\varphi;x))\mathcal{K}(\varphi;x,x'|s) = \delta^{(4)}(x-x'). \tag{70}$$

In the planar-wall approximation, we take

$$\mathcal{K}(\varphi;x,x'|s) = \int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}e^{i\mathbf{k}\cdot(\mathbf{z}_\parallel - \mathbf{z}'_\parallel)}\mathcal{K}(\varphi;z,z',\mathbf{k}|s), \tag{71}$$

where $\mathcal{K}(\varphi;z,z',\mathbf{k}|s)$ satisfies

$$(-\partial_z^2 + k^2 + s + U''(\varphi;z))\mathcal{K}(\varphi;z,z',\mathbf{k}|s) = \delta(z-z'). \tag{72}$$

Comparing Eq. (72) with Eq. (42), we see that $\mathcal{K}(\varphi;z,z',\mathbf{k}|s)$ is nothing other than the Green's function $G(u,u',m)$ in Eq. (36) with the replacement $k^2 \to k^2 + s$ in m, see Eq. (40). Thus, we may write

$$B^{(1)} = -\frac{1}{2}\int_0^\Lambda \mathrm{d}k k^2\int_0^\infty\frac{\mathrm{d}\tau}{\tau}\int_0^\infty \mathrm{d}r r^3\mathcal{L}_s^{-1}[\tilde{G}(u,m)](\tau), \tag{73}$$

where we have defined

$$\tilde{G}(u,m) = G(u,m) - G(1,m), \tag{74}$$

and

$$\mathcal{L}_s^{-1}[\tilde{G}(u,m)](\tau) = \int_{\mathcal{C}}\frac{\mathrm{d}s}{2\pi i}e^{-s\tau}\tilde{G}(u,m) \tag{75}$$

is the inverse Laplace transform with respect to s, with $\mathcal{C}$ indicating the Bromwich contour.

We may perform the integrals in Eq. (73) analytically, proceeding in order from right to left and beginning with the inverse Laplace transform. We then obtain the unrenormalized correction to the bounce action

[2] It is natural to define the renormalized quantities in the false vacuum, since this is where the physical measurements of these quantities are performed. If it were the case that such measurements were taking place in the true vacuum, or indeed within the wall itself, then the decay rate would be of little concern.

$$B^{(1)} = -B\left(\frac{3\lambda}{16\pi^2}\right)\left(\frac{\pi}{3\sqrt{3}} + \frac{\Lambda^2}{\gamma^2} + \ln\frac{\gamma^2}{\Lambda^2}\right). \tag{76}$$

The technical details of the relevant integrations are included in Appendix B. Adding the counterterm

$$\delta B^{(1)} = \int \mathrm{d}^4 x\left(\frac{1}{2!}\delta m^2(\varphi^2 - v^2) + \frac{1}{4!}\delta\lambda(\varphi^4 - v^4)\right)$$
$$= B\left(\frac{3\lambda}{16\pi^2}\right)\left(\frac{\Lambda^2}{\gamma^2} + \ln\frac{\gamma^2}{\Lambda^2} - 21\right), \tag{77}$$

we obtain the final renormalized result

$$B^{(1)} = -B\left(\frac{3\lambda}{16\pi^2}\right)\left(\frac{\pi}{3\sqrt{3}} + 21\right). \tag{78}$$

In Appendix B, we reproduce this result by the method presented in Ref. [86].[3]

IV. RADIATIVE CORRECTIONS TO THE BOUNCE

We now discuss an example of the role played by loop corrections to the bounce itself. Within the perturbation expansion, one should expect that these lead to second-order corrections to the classical action of the soliton simply because the latter is evaluated for a stationary path. There are, however, important situations, in which all one-loop contributions must be resummed in order to capture the leading quantum corrections to the action. Examples include situations where the symmetry-breaking minima of the potential emerge radiatively through the CW mechanism [53]. In the absence of a soliton, this implies that the classical solution, i.e. the homogeneous expectation value of the field, has to be found consistently by minimizing the one-loop effective potential as a *function* of the field expectation value itself. Analogously, in order to find the decay rate of the false vacuum, the bounce must be computed consistently from the one-loop effective action, which is a *functional* of the bounce itself. The methods presented in this article reduce the problem of tunneling in radiatively generated potentials to one-dimensional ordinary differential equations and integrals. It is anticipated that it should be possible to derive numerical solutions in future work.

For the purpose of illustration, however, we remain herein on the ground of analytic and perturbative approximations. In order to enhance the corrections to the bounce compared to other quantum effects that appear at second order in perturbation theory, we extend the model in Eq. (1) with N copies of an additional scalar field χ by adding to the Lagrangian the terms

$$\mathcal{L}\chi = \sum_{i=1}^{N}\left\{\frac{1}{2!}(\partial_\mu\chi_i)^2 + \frac{1}{2!}m_\chi^2\chi_i^2 + \frac{\lambda}{4}\Phi^2\chi_i^2\right\}. \tag{79}$$

Here, we have chosen the coupling λ to be identical to the self-coupling of Φ for the sake of simplicity in the Green's function of the χ fields. Since $\langle\chi_i\rangle = 0$, the additional scalars do not impact upon the classical bounce in Sec. II or the discussion of the Green's function in Sec. III.

The Klein-Gordon equation for χ_i takes the form

$$\left[-\partial^2 + m_\chi^2 + \frac{\lambda}{2}\varphi^2\right]S(\varphi; x, x') = \delta^{(4)}(x - x'). \tag{80}$$

Comparing with that of Φ in Eq. (29), we see that the Green's function $S(u, u', m)$ may be obtained straightforwardly from $G(u, u', m)$ in Eq. (36) by making the replacement

$$m \to \sqrt{6}\left(1 + \frac{k^2 + m_\chi^2}{6\gamma^2}\right)^{\frac{1}{2}}. \tag{81}$$

The renormalized tadpole contribution from each χ_i field, integrated over the three-momentum $\mathbf{k}$, is given by

$$\Sigma^R(u) = \frac{\lambda\gamma^2}{8\pi^2}\frac{\gamma^2}{m_\chi^2}[72 + (1 - u^2)(40 - 3u^2)], \tag{82}$$

where we have assumed $m_\chi^2 \gg \gamma^2$ for simplicity. The full form of $S(u)$ and the relevant counterterms are provided in Appendix C.

The renormalized correction to the classical bounce $\delta\varphi$ is governed by the equation of motion

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \mu^2 - \frac{\lambda}{2}\varphi^2\right]\delta\varphi = (\Pi^R(u) + N\Sigma^R(u))\varphi, \tag{83}$$

cf. Eq. (57). We obtain the solution by making use of the Green's function $G(u, u', 2) \equiv G(u, u', m)|_{k=0}$, writing

$$\delta\varphi(u) = -\frac{v}{\gamma}\int_{-1}^{1}\mathrm{d}u'\,\frac{u' G(u, u', 2)}{1 - u'^2}(\Pi^R(u') + N\Sigma^R(u')), \tag{84}$$

where we have used Eq. (33) in order to substitute φ.

We note at this point that $G(u, u', m)$ is singular as $k \to 0$ (or, equivalently, $m \to 2$). Nonetheless, the integral in Eq. (84) remains finite, since $G(u, u', m)$ is multiplied with an odd function, whereas the singularity resides in its even part. It is therefore useful to define

[3] Using the same renormalization conditions as in Eq. (64), Ref. [86] finds (in the notation employed here)

$$B^{(1)} = -B\left(\frac{3\lambda}{16\pi^2}\right)\left(\frac{\pi}{3\sqrt{3}} + \frac{50}{3}\right).$$

Repeating the analysis presented therein, as outlined in Appendix B, we find a result in agreement with Eq. (78) reported here, suggesting a numerical error in the factor of 50 above.

$$G^{\rm odd}(u,u') \equiv \frac{1}{2}(G(u,u',2) - G(u,-u',2)). \tag{85}$$

Within the domain $0 \le u, u' \le 1$, this function can be expressed as

$$G^{\rm odd}(u,u') = \vartheta(u-u')\frac{1}{32\gamma}\frac{1-u^2}{1-u'^2}\left[2u'(5-3u'^2) + 3(1-u'^2)^2\ln\frac{1+u'}{1-u'}\right] + (u \leftrightarrow u'). \tag{86}$$

Defining in addition

$$p_0(u) = \gamma\int_{-1}^{1}\mathrm{d}u'\,\frac{u'}{1-u'^2}G^{\rm odd}(u,u') = \frac{1-u^2}{8}\left[\frac{2u}{1-u^2} + \ln\frac{1+u}{1-u}\right], \tag{87a}$$

$$p_1(u) = \gamma\int_{-1}^{1}\mathrm{d}u'\,u'G^{\rm odd}(u,u') = \frac{1-u^2}{8}\ln\frac{1+u}{1-u}, \tag{87b}$$

$$p_2(u) = \gamma\int_{-1}^{1}\mathrm{d}u'\,u'^3G^{\rm odd}(u,u') = \frac{1-u^2}{8}\left[\ln\frac{1+u}{1-u} - \frac{4}{3}u\right], \tag{87c}$$

we find the result

$$\delta\varphi(u) = -\frac{3\lambda v}{16\pi^2}\left[6\left(\frac{8\gamma^2}{m_\chi^2}N + 1\right)p_0(u) + 5\left(\frac{16\gamma^2}{3m_\chi^2}N + 1\right)p_1(u) - \left(\frac{2\gamma^2}{m_\chi^2}N + \frac{\pi}{\sqrt{3}}\right)p_2(u)\right]. \tag{88}$$

In Fig. 3, we plot $\delta\varphi$ as a function of $\gamma(r-R)$ for a range of values of $N\gamma^2/m_\chi^2$. We see from Fig. 4, which plots the corrected bounce $\varphi + \delta\varphi$ ($\hbar = 1$) for the same range, that the impact of this correction is to lower the height and broaden the width of the bubble wall. We note that this behavior is in qualitative agreement with the results of the self-consistent numerical analysis in Ref. [102], presented there for the pure $\lambda\Phi^4$ theory in $1+1$ dimensions.

Substituting Eq. (88) into Eq. (60), we find the correction to the bounce action

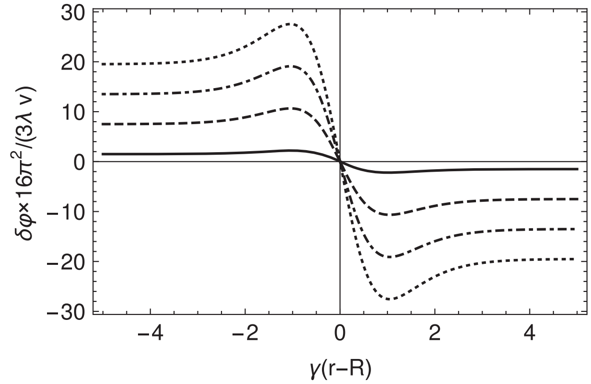

FIG. 3. The correction to the bounce $\delta\varphi$ as a function of $\gamma(r-R)$ for $N\gamma^2/m_\chi^2 = 0$ (solid), 0.5 (dashed), 1 (dash-dotted) and 1.5 (dotted).

$$B^{(2)} + B^{(2)\prime} = \frac{1}{2}\int \mathrm{d}^4x\,\varphi(u)(\Pi^R(u) + N\Sigma^R(u))\delta\varphi(u) = -\frac{B}{3}\left(\frac{3\lambda}{16\pi^2}\right)^2\left[\frac{291}{8} - \frac{37}{4}\frac{\pi}{\sqrt{3}} + \frac{5}{56}\frac{\pi^2}{3} + \left(\frac{667}{2} - \frac{2897}{42}\frac{\pi}{\sqrt{3}}\right)\frac{\gamma^2}{m_\chi^2}N + \frac{5829}{14}\frac{\gamma^4}{m_\chi^4}N^2\right]. \tag{89}$$

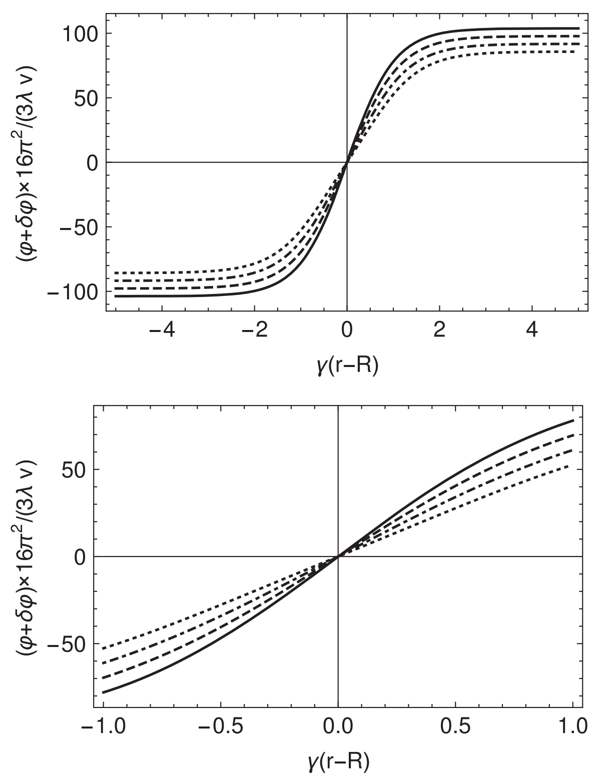

FIG. 4. The corrected bounce $\varphi + \delta\varphi$ as a function of $\gamma(r-R)$ for $N\gamma^2/m_\chi^2 = 0$ (solid), 0.5 (dashed), 1 (dash-dotted) and 1.5 (dotted). We see clearly that the impact of the tadpole correction is to broaden the bubble wall.

In order to obtain a finite result for Eq. (89), we have added to U_0 the correction

$$\delta U_0 = \frac{9}{4}\left(\frac{3\lambda}{16\pi^2}\right)^2 \gamma^2 v^2 \left(\frac{8\gamma^2}{m_\chi^2}N + 1\right)^2, \quad (90)$$

ensuring that the potential continues to vanish in the false vacuum.

The corrections appearing in Eq. (89) should be compared to the renormalized logarithm of the determinants of the Klein-Gordon operators of the χ fields in the background given by φ, which are given by

$$B_\chi^{(1)} = -B\left(\frac{3\lambda}{16\pi^2}\right)\frac{2542}{15}\left[\frac{\gamma^2}{m_\chi^2} + \mathcal{O}\left(\frac{\gamma^4}{m_\chi^4}\right)\right]N. \quad (91)$$

In comparison, the leading term in Eq. (89) is suppressed by a factor $\sim\lambda\mu^2/m_\chi^2/(16\pi^2)$. The one-loop corrections $B^{(1)}$ and $B_\chi^{(1)}$ are both negative, thereby increasing the tunneling rate. It is interesting to note that, although the contribution $B^{(2)}$ to the tunneling action from the corrections to the bounce itself is positive, the net contribution of $B^{(2)} + B^{(2)\prime}$ is still negative, again increasing the tunneling rate.

In Fig. 5, we present a diagrammatic representation of the corrections to the bounce action. It is also useful in order to see that there appear no contributions of $\mathcal{O}(\lambda^2 N^2)$ relative to the bounce action B in addition to those from $B^{(2)}$. In order to avoid proliferation, we only show the leading contributions in $1/N$ for a given type of diagram. At one-loop order, there is the vacuum bubble in terms of the propagator S of the χ fields, Fig. 5(a), which gives the contribution $\mathcal{O}(\lambda N)$ relative to B from $B_\chi^{(1)}$ in Eq. (91). On substituting $\delta\varphi$ in the form of Eq. (84) into the action [Eq. (58)], we see that the diagram corresponding to the $\mathcal{O}(\lambda^2 N^2)$ term in $B^{(2)}/B$ is given by Fig. 5(b), where, when counting the powers of λ, one should note that each explicit insertion of φ contributes a factor of $1/\sqrt{\lambda}$. Finally, at two-loop order, there are the diagrams Figs. 5(c) and 5(d), which we do not compute, but yield contributions of $\mathcal{O}(\lambda^2 N)$ relative to B. These contributions are therefore suppressed by a relative factor of $1/N$ relative to the $\mathcal{O}(\lambda^2 N^2)$ in $B^{(2)}/B$, as is familiar from the standard approximation scheme known as the $1/N$ expansion [110]. We should remark that these arguments do not hold, of course, for the contribution to $B^{(2)}$ from the Φ tadpole, which we include here for completeness. The latter is formally the same order as other two-loop diagrams, involving only Φ, that are not captured in the 1PI approximation employed here. This observation is true also of the Hartree approximation for the pure $\lambda\Phi^4$ theory analyzed numerically in Refs. [102–105]. Nevertheless, these additional two-loop diagrams remain subdominant compared to the $\mathcal{O}(\lambda^2 N)$ and $\mathcal{O}(\lambda^2 N^2)$ contributions from the χ tadpole in Eq. (89).

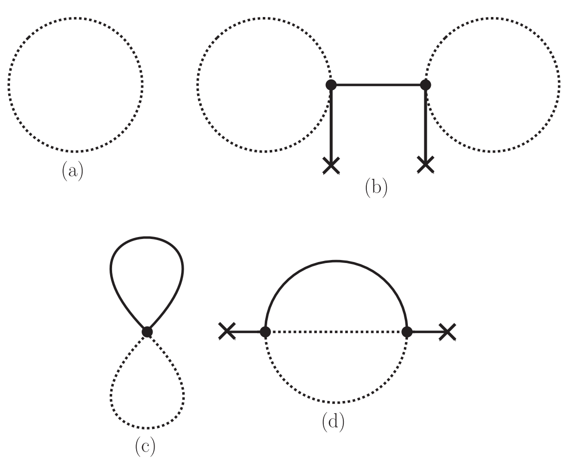

FIG. 5. Diagrammatic representation of various contributions to the effective action: (a) is the one-loop term $B_\chi^{(1)}$, (b) is the $\mathcal{O}(\lambda^2 N^2)$ contribution to $B^{(2)}$, and (c) and (d) are $\mathcal{O}(\lambda^2 N)$ terms. Solid lines represent the propagator $G(\varphi; x, x')$, dotted lines $S(\varphi; x, x')$. Crosses denote insertions of the bounce φ.

Finally, we note that approximating $\delta\varphi$ as a small perturbation to φ, using Eq. (84), requires for consistency that $6N\lambda\gamma^2/(m_\chi^2\pi^2) \ll 1$, such that within the range of validity of present approximations, we cannot obtain $|B^{(2)} + B^{(2)\prime}| > |B^{(1)}|$. Nevertheless, for large N, $B^{(2)} + B^{(2)\prime}$ can be the dominant two-loop contribution to the effective action.

V. CONCLUSIONS

Within the context of $\lambda\Phi^4$ theory, we have described a Green's function method for handling radiative effects on false vacuum decay. By this means and employing the thin- and planar-wall approximations, we have been able to calculate analytically and in a straightforward manner both the functional determinant of the quadratic fluctuations about the classical soliton configuration and the first correction to the configuration itself.

This Green's function method is well suited to numerical evaluation and, as a consequence, should be applicable to potentials of more general form. As such, we anticipate that it may be of particular use when the nondegeneracy of minima is purely radiatively generated. Examples of the latter include the spontaneous symmetry breaking of the massless CW model [53] or the instability of the electroweak vacuum. Other applications might include the calculation of corrections to inflationary potentials in the time-dependent inflaton background, for instance in inflection-point or A-term inflation [111–114], which exploit the flat directions and saddle points of the MSSM potential. Furthermore, the use of Green's functions naturally admits the introduction of finite-temperature effects or extension to nontrivial background spacetimes.

Green's functions have proved to be central objects within perturbative calculations throughout quantum field theory, and it is therefore unsurprising that we find these suitable to treat solitons in $\lambda\Phi^4$ theory as well. We take this as an encouragement that further theoretically and phenomenologically interesting systematic results on false vacuum decay may be within reach.

ACKNOWLEDGMENTS

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APPENDIX A: ZERO-MODE FUNCTIONAL MEASURE

In order to perform the functional integration over the zero modes, we insert four copies of unity in Faddeev-Popov form [107]:

$$1 = \int \mathrm{d}y_\mu |\partial_\mu^{(y)} f(y_\mu)| \delta(f(y_\mu)). \tag{A1}$$

Here, μ is not summed over, and

$$f(y_\mu) = \int \mathrm{d}^4 x \Phi(x-y) \partial_\mu^{(x)} \varphi(x-y) = B^{1/2} a_\mu, \tag{A2}$$

where we recall that

$$\Phi = \varphi + \sum_{i=0}^{4} a_i \phi_i + \phi_+. \tag{A3}$$

It follows that

$$\partial_\mu^{(y)} f(y_\mu) = -\int \mathrm{d}^4 x (\partial_\mu^{(x)} \varphi(x-y))^2 = -B, \tag{A4}$$

ignoring terms that are formally $\mathcal{O}(\hbar^{1/2})$. Thus,

$$1 = B \int \mathrm{d}y_\mu \delta(B^{1/2} a_\mu) = B^{1/2} \int \mathrm{d}y_\mu \delta(a_\mu). \tag{A5}$$

We then have

$$\int \prod_{\mu=1}^{4} (2\pi\hbar)^{-1/2} \mathrm{d}a_\mu = \left(\frac{B}{2\pi\hbar}\right)^2 \int \mathrm{d}^4 y \prod_{\mu=1}^{4} \int \mathrm{d}a_\mu \delta(a_\mu) = VT \left(\frac{B}{2\pi\hbar}\right)^2. \tag{A6}$$

APPENDIX B: GREEN'S FUNCTION

In this appendix, we include the technical details of the calculations outlined in Secs. III and IV. All functional identities used in what follows may be found in Ref. [115].

1. Expansion in hyperspherical harmonics

In d dimensions, the Green's function satisfies the inhomogeneous Klein-Gordon equation

$$(-\Delta^{(d)} + U''(\varphi)) G^{(d)}(\varphi; x, x') = \delta^{(d)}(x - x'), \tag{B1}$$

where $\delta^{(d)}(x-x')$ is the Dirac delta function and $\Delta^{(d)}$ is the Laplacian. Given the $O(d)$ invariance of the bounce φ, it is convenient to work in hyperspherical coordinates, in which case the Laplacian takes the form

$$\Delta^{(d)} = r^{1-d} \partial_r r^{d-1} \partial_r + \Delta_{S^{d-1}}, \tag{B2}$$

where $\Delta_{S^{d-1}}$ is the Laplace-Beltrami operator on the $d-1$ sphere.

We proceed by performing a partial-wave decomposition of the Green's function:

$$G^{(d)}(\varphi; x, x') = \sum_{j\{\ell\}} G_j(\varphi; r, r') Y^*_{j\{\ell\}}(\mathbf{e}_{r'}) Y_{j\{\ell\}}(\mathbf{e}_r), \tag{B3}$$

where $x = r\mathbf{e}_r$, $x' = r'\mathbf{e}_{r'}$, and $Y_{j\{\ell\}}(\mathbf{e}_r)$ are the hyperspherical harmonics (see e.g. Ref. [116]), satisfying the eigenvalue equation

$$\Delta_{S^{d-1}} Y_{j\{\ell\}} = -j(j+d-2) Y_{j\{\ell\}}, \tag{B4}$$

with $\{\ell\} = \ell_1, \ell_2, \ldots, \ell_{d-2}$. The hyperradial function $G_j(\varphi; r, r')$ satisfies

$$\left[-r^{1-d} \frac{\mathrm{d}}{\mathrm{d}r} r^{d-1} \frac{\mathrm{d}}{\mathrm{d}r} + \frac{j(j+d-2)}{r^2} + U''(\varphi)\right] G_j(\varphi; r, r') = r'^{1-d} \delta(r - r'). \tag{B5}$$

Since, for each j, the $\{\ell\}$ modes are degenerate, we may use the sum rule [116]

$$\sum_{\{\ell\}} Y^*_{j\{\ell\}}(\mathbf{e}_{r'}) Y_{j\{\ell\}}(\mathbf{e}_r) = \frac{2}{(4\pi)^{\kappa+\frac{1}{2}}} \frac{j+\kappa}{j+2\kappa} \left(j+\kappa+\frac{1}{2}\right)_{\kappa+\frac{1}{2}} P_j^{(\kappa-\frac{1}{2},\kappa-\frac{1}{2})}(\cos\theta), \tag{B6}$$

where $\kappa = d/2 - 1$, $\cos\theta = \mathbf{e}_r \cdot \mathbf{e}_{r'}$,

$$(z)_n = \frac{\Gamma(z+n)}{\Gamma(z)} \tag{B7}$$

is the Pochhammer symbol, and the $P_j^{(\alpha,\beta)}(z)$ are the Jacobi polynomials.

For $d = 1$, $\kappa = -1/2$, $\cos\theta \in \{-1, +1\}$, and we have

$$P_j^{(-1,-1)}(+1) = 0, \tag{B8}$$

$$P_j^{(-1,-1)}(-1) = \frac{\sin \pi j}{\pi j} = \begin{cases} 1, & j = 0 \\ 0, & j \neq 0 \end{cases}. \tag{B9}$$

Hence, $G^{(1)}(\varphi; x, x') = G_0(\varphi; r, r')$, as we would expect.

For $d = 2$, $\kappa = 0$, and we have

$$P_j^{(-\frac{1}{2},-\frac{1}{2})}(z) = \frac{T_j(z)}{\sqrt{\pi}(j+\frac{1}{2})_{\frac{1}{2}}}, \tag{B10}$$

where $T_j(z)$ is the Chebyshev polynomial of the first kind. We then obtain

$$G^{(2)}(\varphi; x, x') = \frac{1}{\pi}\sum_{j=0}^{\infty} \cos j\theta \, G_j(\varphi; r, r'), \tag{B11}$$

where we have used the trigonometric form $T_j(\cos\theta) = \cos j\theta$.

For $d = 3$, $\kappa = 1/2$, and

$$P_j^{(0,0)}(z) = P_j(z), \tag{B12}$$

where $P_j(z)$ are the Legendre polynomials. Thus, we obtain the familiar three-dimensional expansion

$$G^{(3)}(\varphi; x, x') = \frac{1}{4\pi}\sum_{j=0}^{\infty}(2j+1)P_j(\cos\theta)G_j(\varphi; r, r'). \tag{B13}$$

Finally, for $d = 4$, $\kappa = 1$, and

$$P_j^{(\frac{1}{2},\frac{1}{2})}(z) = \frac{2}{\sqrt{\pi}}\frac{j+2}{(j+\frac{3}{2})_{\frac{3}{2}}}U_j(z), \tag{B14}$$

where $U_j(z)$ are the Chebyshev polynomials of the second kind. Hence, we find

$$G^{(4)}(\varphi; x, x') = \frac{1}{2\pi^2}\sum_{j=0}^{\infty}(j+1)U_j(\cos\theta)G_j(\varphi; r, r') \tag{B15}$$

as appearing in Eq. (30).

2. Continuum approximation

In the coincident limit $x = x'$, $\cos\theta = 1$, and we have

$$T_j(1) = 1, \qquad P_j(1) = 1, \qquad U_j(1) = j + 1. \tag{B16}$$

Alternatively, in d dimensions, we may use

$$P_j^{(\alpha,\beta)}(1) = \frac{(\alpha+1)_j}{\Gamma(j+1)} \tag{B17}$$

in Eq. (B6), giving

$$G^{(d)}(\varphi; x, x') = \frac{2(4\pi)^{-\frac{d-1}{2}}}{\Gamma(\frac{d-1}{2})} \times \sum_{j=0}^{\infty}(j + d/2 - 1)\frac{\Gamma(j+d-2)}{\Gamma(j+1)}G_j(\varphi; r, r'). \tag{B18}$$

Completing the square in the centrifugal potential in Eq. (B5), we make the following approximation for large R:

$$\frac{j(j+d-2)}{R^2} = \frac{(j+\kappa)^2}{R^2} - \frac{\lambda^2}{4R^2} \approx \frac{(j+\kappa)^2}{R^2}, \tag{B19}$$

where $\kappa = d/2 - 1$, as before. We may then promote $(j+\kappa)/R$ to a continuous variable k, obtaining

$$G^{(2)}(\varphi; x, x) = \frac{1}{\pi}\int_0^{\infty} \mathrm{d}k G(u, m), \tag{B20a}$$

$$G^{(3)}(\varphi; x, x) = \frac{1}{2\pi}\int_0^{\infty} \mathrm{d}k k G(u, m), \tag{B20b}$$

$$G^{(4)}(\varphi; x, x) = \frac{1}{2\pi^2}\int_0^{\infty} \mathrm{d}k k^2 G(u, m), \tag{B20c}$$

$$\vdots$$

$$G^{(d)}(\varphi; x, x) = \frac{2(4\pi)^{-\frac{d-1}{2}}}{\Gamma(\frac{d-1}{2})}\int_0^{\infty} \mathrm{d}k k^{d-2} G(u, m), \tag{B20d}$$

where we have used the general notation employed in Sec. III, see Eq. (38), with m given by Eq. (40). We note that for $d > 4$, we have dropped terms $\mathcal{O}(k/R)$ and higher within the integrand.

3. Radial function

For large R, we neglect the damping term in the radial equation [Eq. (B5)] and set $r = R$ in the centrifugal potential and discontinuity, giving

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{j(j+d-2)}{R^2}+U''(\varphi)\right]G_j(\varphi;r,r')=\frac{\delta(r-r')}{R^{d-1}}. \tag{B21}$$

Since the solution depends only on the normalized bounce $u=\tanh[\gamma(r-R)]$, it is convenient to define

$$G(u,u',m)\equiv R^{d-1}G_j(\varphi;r,r'), \tag{B22}$$

cf. Sec. III. Equation (B21) may then be recast in the form

$$\left[\frac{\mathrm{d}}{\mathrm{d}u}(1-u^2)\frac{\mathrm{d}}{\mathrm{d}u}-\frac{m^2}{1-u^2}+n(n+1)\right]G(u,u',m) = -\gamma^{-1}\delta(u-u'), \tag{B23}$$

where

$$n=2,\qquad m=2\left(1+\frac{j(j+d-2)}{4\gamma^2R^2}\right)^{\frac{1}{2}}. \tag{B24}$$

Splitting around the discontinuity at $u=u'$, we decompose

$$G(u,u',m)=\vartheta(u-u')G^{>}(u,u',m) + \vartheta(u'-u)G^{<}(u,u',m), \tag{B25}$$

where $G^{\gtrless}(u,u',m)$ are the solutions to the homogeneous equation

$$\left[\frac{\mathrm{d}}{\mathrm{d}u}(1-u^2)\frac{\mathrm{d}}{\mathrm{d}u}-\frac{m^2}{1-u^2}+n(n+1)\right]G^{\gtrless}(u,u',m)=0. \tag{B26}$$

The latter is the associated Legendre differential equation, and we obtain the general solutions

$$G^{\gtrless}(u,u',m)=A^{\gtrless}P_2^m(u)+B^{\gtrless}Q_2^m(u), \tag{B27}$$

where $P_n^m(z)$ and $Q_n^m(z)$ are the associated Legendre functions of the first and second kind, respectively.

Matching around the delta function in the inhomogeneous equation, we require

$$(A^{>}-A^{<})P_2^m(u')+(B^{>}-B^{<})Q_2^m(u')=0, \tag{B28a}$$

$$(A^{>}-A^{<})\frac{\mathrm{d}}{\mathrm{d}u'}P_2^m(u') + (B^{>}-B^{<})\frac{\mathrm{d}}{\mathrm{d}u'}Q_2^m(u')=-\frac{1}{\gamma(1-u'^2)}. \tag{B28b}$$

Thus, we find

$$A^{>}-A^{<}=\frac{1}{\gamma(1-u'^2)}\frac{Q_2^m(u')}{W[P_2^m(u'),Q_2^m(u')]}, \tag{B29a}$$

$$B^{>}-B^{<}=\frac{1}{\gamma(1-u'^2)}\frac{P_2^m(u')}{W[Q_2^m(u'),P_2^m(u')]}, \tag{B29b}$$

where $W[P_n^m(z),Q_n^m(z)]$ is the Wronskian, having the explicit form

$$W[P_n^m(z),Q_n^m(z)]=\frac{(n-m+1)_{2m}}{1-u'^2}, \tag{B30}$$

with the Pochhammer symbol defined in Eq. (B7). We also require the boundary condition that $G(u,u',m)$ go to zero as $u\to\pm1$, giving

$$\frac{A_{>}}{B_{>}}=-\frac{\pi}{2}\cot m\pi,\qquad B_{<}=0. \tag{B31}$$

We may now solve for the remaining nonzero coefficients and obtain

$$G^{>}(u,u',m)=\frac{\pi}{2\gamma}\frac{1}{\sin m\pi}P_2^{-m}(u)P_2^m(u'), \tag{B32}$$

with $G^{<}(u,u',m)=G^{>}(u',u,m)$. Here, we have used the identity

$$\frac{\pi(n-m+1)_{2m}}{2\sin m\pi}P_n^{-m}(z)=\frac{\pi}{2}\cot m\pi\,P_n^m(z)-Q_n^m(z). \tag{B33}$$

Finally, we employ the representation

$$P_n^m(z)=\left(\frac{z+1}{z-1}\right)^{\frac{m}{2}}(n-m+1)_mP_n^{(-m,m)}(z) \tag{B34}$$

of the associated Legendre function of the first kind in terms of the Jacobi polynomials. For $n=2$, the polynomial expansion of the latter terminates, and we have

$$P_2^{(\pm m,\mp m)}(z)=\frac{1}{2}[(1\pm m)(2\pm m) - 3(2\pm m)(1-u)+3(1-u)^2]. \tag{B35}$$

After some algebraic simplification, we then arrive at the final analytic solution, as presented in Eq. (36) of Sec. III.

4. Functional determinant

The normalized heat kernel $\tilde{\mathcal{K}}(\varphi;z,z',\mathbf{k}|\tau)$, see Sec. III, is given in terms of the inverse Laplace transform

$$\tilde{K}(\varphi;z,z',\mathbf{k}|\tau)=\mathcal{L}_s^{-1}[\tilde{G}(u,m)](\tau), \tag{B36}$$

where

$$\tilde{G}(u,m) = \frac{3}{2\gamma m}(1-u^2)\sum_{n=1}^{2}(-1)^n\frac{(n-1-u^2)}{m^2-n^2}, \quad \text{(B37)}$$

with

$$m = 2\left(1+\frac{k^2+s}{4\gamma^2}\right)^{\frac{1}{2}}. \quad \text{(B38)}$$

The inverse Laplace transform may be performed by using the shift, scaling and division properties

$$\mathcal{L}_s^{-1}[F(s+b)](\tau) = e^{-b\tau}f(\tau), \quad \text{(B39a)}$$

$$\mathcal{L}_s^{-1}[F(as)](\tau) = \frac{1}{a}f(\tau/a), \quad \text{(B39b)}$$

$$\mathcal{L}_s^{-1}[s^{-1}F(s)](\tau) = \int_0^\tau \mathrm{d}\tau' f(\tau'), \quad \text{(B39c)}$$

where $f(\tau) = \mathcal{L}_s^{-1}[F(s)](\tau)$, as well as the elementary transformation

$$\mathcal{L}_s^{-1}[s^{-z}](\tau) = \frac{\tau^{z-1}}{\Gamma(z)}, \qquad \mathrm{Re}z > 0. \quad \text{(B40)}$$

We find

$$\mathcal{L}_s^{-1}[m^{-1}(m^2-n^2)^{-1}](\tau) = \frac{\gamma^2}{n}e^{[\gamma^2(n^2-4)-k^2]\tau}\mathrm{erf}(n\gamma\sqrt{\tau}), \quad \text{(B41)}$$

where

$$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z \mathrm{d}t e^{-t^2} \quad \text{(B42)}$$

is the error function. Hence, we have

$$\tilde{K}(\varphi; z, z', \mathbf{k}|\tau) = -\frac{3}{2}\gamma(1-u^2)e^{-k^2\tau} \times \sum_{n=1}^{2}(-1)^n\left(\frac{1+u^2}{n}-1\right)e^{\gamma^2(n^2-4)\tau}\mathrm{erf}(n\gamma\sqrt{\tau}). \quad \text{(B43)}$$

Generalizing to d dimensions, using the continuum limit in Eq. (B20d), the correction to the bounce action arising from the functional determinant is therefore

$$B^{(1)} = -\frac{3}{2}\frac{\Omega_d(4\pi)^{-\frac{d-1}{2}}}{\Gamma(\frac{d-1}{2})}\int_0^\infty \mathrm{d}k k^{d-2}\int_0^\infty\frac{\mathrm{d}\tau}{\tau}e^{-k^2\tau} \times \int_0^\infty \mathrm{d}r r^{d-1}\gamma(1-u^2) \times \sum_{n=1}^{2}(-1)^n\left(\frac{1+u^2}{n}-1\right)e^{\gamma^2(n^2-4)\tau}\mathrm{erf}(n\gamma\sqrt{\tau}), \quad \text{(B44)}$$

where $\Omega_d = 2\pi^{d/2}/\Gamma(d/2)$ is the solid angle subtended by the $(d-1)$-dimensional hypersphere. The integral over r is dominated by $r \sim R$, such that (for $n = 1, 2$)

$$(-1)^n\int_0^\infty \mathrm{d}r r^{d-1}\gamma(1-u^2)\left(\frac{1+u^2}{n}-1\right) \approx -\frac{2}{3}R^{d-1}. \quad \text{(B45)}$$

We are then left with

$$B^{(1)} = \frac{\Omega_d(4\pi)^{-\frac{d-1}{2}}R^{d-1}}{\Gamma(\frac{d-1}{2})}\int_0^\infty \mathrm{d}k k^{d-2}\int_0^\infty \frac{\mathrm{d}\tau}{\tau}e^{-k^2\tau} \times \sum_{n=1}^{2}e^{\gamma^2(n^2-4)\tau}\mathrm{erf}(n\gamma\sqrt{\tau}), \quad \text{(B46)}$$

cf. the form presented in Ref. [86].

We may now proceed in one of two ways: (i) performing the τ integration first, we must regularize the k integral, for instance by introducing an ultraviolet cutoff Λ; or (ii) performing the k integral first, we must instead regularize the τ integral. The latter is the approach presented in Ref. [86], which we reproduce in what follows for comparison.

(i) Performing the τ integral first gives

$$B^{(1)} = -\frac{2\Omega_d(4\pi)^{-\frac{d-1}{2}}R^{d-1}}{\Gamma(\frac{d-1}{2})}\int_0^\Lambda \mathrm{d}k k^{d-2} \times \sum_{n=1}^{2}\mathrm{arcsinh}\frac{n\gamma}{\sqrt{k^2-\gamma^2(n^2-4)}}. \quad \text{(B47)}$$

Subsequently, performing the k integral for $d = 4$, we obtain the result in Eq. (78).

(ii) Instead, performing the k integral first, we obtain

$$B^{(1)} = \frac{1}{2}\Omega_d R^{d-1}(4\pi)^{-\frac{d-1}{2}}\int_0^\infty \mathrm{d}\tau\tau^{-\frac{d+1}{2}} \times \sum_{n=1}^{2}e^{\gamma^2(n^2-4)\tau}\mathrm{erf}(n\gamma\sqrt{\tau}), \quad \text{(B48)}$$

which is regularized by introducing a large mass M as follows:

$$B^{(1)} = \frac{1}{2}\Omega_d R^{d-1}(4\pi)^{-\frac{d-1}{2}} \lim_{\epsilon\to 0}\frac{\mathrm{d}}{\mathrm{d}\epsilon}\frac{M^{2\epsilon}}{\Gamma(\epsilon)}\int_0^\infty \mathrm{d}\tau \times \tau^{-\frac{d+1}{2}+\epsilon}\sum_{n=1}^{2} e^{\gamma^2(n^2-4)\tau}\mathrm{erf}(n\gamma\sqrt{\tau}). \tag{B49}$$

We may proceed by using the series representation of the error function

$$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}e^{-z^2}\sum_{\ell=0}^{\infty}\frac{2^\ell}{(2\ell+1)!!}z^{2\ell+1}, \tag{B50}$$

where !! denotes the double factorial. The τ integral may now be performed, and we obtain

$$B^{(1)} = (\gamma R)^{d-1}\Omega_d \pi^{-\frac{d}{2}}\lim_{\epsilon\to 0}\frac{\mathrm{d}}{\mathrm{d}\epsilon}\frac{1}{\Gamma(\epsilon)}\left(\frac{M^2}{4\gamma^2}\right)^\epsilon \times \sum_{n=1}^{2}\sum_{\ell=0}^{\infty}\frac{2^\ell (n/2)^{2\ell+1}}{(2\ell+1)!!}\Gamma(\epsilon+\ell+1-d/2). \tag{B51}$$

Considering the derivative with respect to ϵ, we have

$$\frac{\mathrm{d}}{\mathrm{d}\epsilon}\frac{\Gamma(\epsilon+\ell+1-d/2)}{\Gamma(\epsilon)}\left(\frac{M^2}{4\gamma^2}\right)^\epsilon = \frac{\Gamma(\epsilon+\ell+1-d/2)}{\Gamma(\epsilon)}\left(\frac{M^2}{4\gamma^2}\right)^\epsilon \times\left[\ln\frac{M^2}{4\gamma^2} - \psi(\epsilon) + \psi(\epsilon+\ell+1-d/2)\right], \tag{B52}$$

where $\psi(z)$ is the digamma function. In order to take the limit $\epsilon\to 0$ safely, we must take note of the poles occurring in $\Gamma(z)$ and $\psi(z)$ for nonpositive integers. Such poles occur in even dimensions for $\ell = 0, 1, \ldots, d-3$.

After treating the limit $\epsilon\to 0$, we find for d odd (including $d = 1$)

$$B^{(1)} = -(\gamma R)^{d-1}\Omega_d\pi^{-\frac{d}{2}} \times\sum_{n=1}^{2}\sum_{\ell=0}^{\infty}\frac{2^\ell(n/2)^{2\ell+1}}{(2\ell+1)!!}\Gamma(\ell+1-d/2). \tag{B53}$$

On the other hand, for d even, we find

$$B^{(1)} = -(\gamma R)^{d-1}\Omega_d\pi^{-\frac{d}{2}} \times\sum_{n=1}^{2}\left[\sum_{\ell=d-2}^{\infty}\frac{2^\ell(n/2)^{2\ell+1}}{(2\ell+1)!!}\Gamma(\ell+1-d/2) + \sum_{\ell=0}^{d-3}\frac{2^\ell(n/2)^{2\ell+1}}{(2\ell+1)!!}\frac{(-1)^{d/2-\ell-1}}{(d/2-\ell-1)!} \times\left(\ln\frac{M^2}{4\gamma^2}+H_{d/2-\ell-1}\right)\right], \tag{B54}$$

where

$$H_n = \sum_{k=1}^{n}\frac{1}{k} \tag{B55}$$

are the harmonic numbers, which we have supplemented with $H_0 \equiv 0$ for notational simplicity.

For $d = 4$, we then obtain

$$B^{(1)} = -2R^3\gamma^3\sum_{n=1}^{2}\left[\sum_{\ell=2}^{\infty}\frac{2^\ell(n/2)^{2\ell+1}}{(2\ell+1)!!}(\ell-2)! + \sum_{\ell=0}^{1}\frac{2^\ell(n/2)^{2\ell+1}}{(2\ell+1)!!}\frac{(-1)^{1-\ell}}{(1-\ell)!}\left(\ln\frac{M^2}{4\gamma^2}+H_{1-\ell}\right)\right]. \tag{B56}$$

Lastly, performing the summations, we arrive at the result

$$B^{(1)} = -B\left(\frac{3\lambda}{16\pi^2}\right)\left(\frac{\pi}{3\sqrt{3}} - 2 + \ln\frac{4\gamma^2}{M^2}\right). \tag{B57}$$

Defining the counterterms in the proper-time representation, see Ref. [86], and fixing the renormalization conditions as in Eq. (64), we find the counterterms

$$\delta m^2 = \frac{\lambda\gamma^2}{16\pi^2}\left(\ln\frac{4\gamma^2}{M^2}+29\right), \tag{B58a}$$

$$\delta\lambda = -\frac{3\lambda^2}{32\pi^2}\left(\ln\frac{4\gamma^2}{M^2}+3\right), \tag{B58b}$$

giving

$$\delta B^{(1)} = B\left(\frac{3\lambda}{16\pi^2}\right)\left(\ln\frac{4\gamma^2}{M^2}-23\right). \tag{B59}$$

Adding these to Eq. (B57), we obtain agreement with Eq. (78).

APPENDIX C: RENORMALIZATION OF THE N-FIELD MODEL

In this final appendix, we highlight the main technical details of the derivation of the Green's function and corrections to the bounce from the χ fields.

Proceeding as for the isolated φ case, see Sec. III, the renormalization is fixed using the CW effective potential [53], evaluated in a homogeneous false vacuum. The renormalization conditions are then

$$\left.\frac{\partial^2 U_{\mathrm{eff}}}{\partial\varphi^2}\right|_{\varphi=v,\chi_i=0} = 4\gamma^2, \tag{C1a}$$

$$\left.\frac{\partial^2 U_{\mathrm{eff}}}{\partial\chi_i^2}\right|_{\varphi=v,\chi_i=0} = 6\gamma^2 + m_\chi^2, \tag{C1b}$$

$$\left.\frac{\partial^4 U_{\rm eff}}{\partial\varphi^4}\right|_{\varphi=v,\chi_i=0} = \lambda, \tag{C1c}$$

$$\left.\frac{\partial^4 U_{\rm eff}}{\partial\varphi^2\partial\chi_i^2}\right|_{\varphi=v,\chi_i=0} = \lambda, \tag{C1d}$$

$$\left.\frac{\partial^4 U_{\rm eff}}{\partial\chi_i^2\partial\chi_j^2}\right|_{\varphi=v,\chi_i=0} = 0, \tag{C1e}$$

where the effective potential is

$$U_{\rm eff} = U(\varphi,\chi) + \delta U(\varphi,\chi) + \frac{N-1}{4\pi^2}\int_0^\Lambda {\rm d}k k^2\left(\sqrt{k^2+M_\chi^2} - k\right) + \frac{1}{4\pi^2}\int_0^\Lambda {\rm d}k k^2\left(\sqrt{k^2+M_+^2} + \sqrt{k^2+M_-^2} - 2k\right), \tag{C2}$$

with

$$M_\chi^2 = m_\chi^2 + \frac{\lambda}{2}\varphi^2, \tag{C3a}$$

$$M_\varphi^2 = -2\gamma^2 + \frac{\lambda}{2}\varphi^2 + \frac{\lambda}{2}\chi_i^2, \tag{C3b}$$

$$M_\pm^2 = \frac{M_\varphi^2 + M_\chi^2}{2} \pm \left[\left(\frac{M_\varphi^2 - M_\chi^2}{2}\right)^2 + \lambda^2\varphi^2\chi_i^2\right]^{1/2}, \tag{C3c}$$

and

$$U(\varphi,\chi) = -\frac{1}{2!}\mu^2\varphi^2 + \frac{1}{2!}m_\chi^2\chi_i^2 + \frac{1}{4!}\lambda\varphi^4 + \frac{1}{4}\lambda\varphi^2\chi_i^2, \tag{C4a}$$

$$\delta U(\varphi,\chi) = +\frac{1}{2!}\delta m_\varphi^2\varphi^2 + \frac{1}{2!}\delta m_\chi^2\chi_i^2 + \frac{1}{4!}\delta\lambda_\varphi\varphi^4 + \frac{1}{4}\delta\lambda_{\varphi\chi}\varphi^2\chi_i^2 + \frac{1}{4}\delta\lambda_\chi\chi_i^2\chi_j^2. \tag{C4b}$$

In Eqs. (C3) and (C4), the summations over $i, j = 1, \ldots, N$ have been left implicit for notational convenience.

Solving the resulting system, we obtain the set of counterterms

$$\delta m_\chi^2 = -\frac{\lambda\gamma^2}{16\pi^2}\left(\frac{\Lambda^2}{\gamma^2} - \ln\frac{\gamma^2}{\Lambda^2} - 13 + \frac{216\gamma^2}{m_\chi^2+6\gamma^2} - \frac{360\gamma^2}{m_\chi^2+2\gamma^2}\ln\frac{6\gamma^2+m_\chi^2}{4\gamma^2}\right), \tag{C5a}$$

$$\delta m_\varphi^2 = -\frac{\lambda\gamma^2}{16\pi^2}\left[(N+1)\left(\frac{\Lambda^2}{\gamma^2} - 30\right) - \left(\ln\frac{\gamma^2}{\Lambda^2} + 1\right) + N\frac{m_\chi^2}{2\gamma^2}\left(\ln\frac{6\gamma^2+m_\chi^2}{4\Lambda^2} + 1\right) + 27N\left(\frac{m_\chi^2+2\gamma^2}{m_\chi^2+6\gamma^2}\right)^2\right], \tag{C5b}$$

$$\delta\lambda_\varphi = -\frac{3\lambda^2}{32\pi^2}\left[\ln\frac{\gamma^2}{\Lambda^2} + 5(N+1) + N\ln\frac{6\gamma^2+m_\chi^2}{4\Lambda^2} - 3N\left(\frac{m_\chi^2+2\gamma^2}{m_\chi^2+6\gamma^2}\right)^2\right], \tag{C5c}$$

$$\delta\lambda_{\varphi\chi} = -\frac{\lambda^2}{32\pi^2}\left(\ln\frac{\gamma^2}{\Lambda^2} + 4\ln\frac{6\gamma^2+m_\chi^2}{4\Lambda^2} + \frac{136\gamma^2}{m_\chi^2+2\gamma^2}\ln\frac{6\gamma^2+m_\chi^2}{4\gamma^2} + 9\frac{m_\chi^2-2\gamma^2}{m_\chi^2+6\gamma^2}\right), \tag{C5d}$$

$$\delta\lambda_\chi = -\frac{\lambda^2}{32\pi^2}\left(\frac{1}{2}\ln\frac{\gamma^2}{\Lambda^2} + \frac{(m_\chi^2-10\gamma^2)^2+432\gamma^4}{(m_\chi^2+2\gamma^2)^2} + 24\gamma^2\frac{(m_\chi^2-2\gamma^2)^2-112\gamma^4}{(m_\chi^2+2\gamma^2)^3}\ln\frac{6\gamma^2+m_\chi^2}{4\gamma^2}\right). \tag{C5e}$$

Proceeding as for φ, we find the unrenormalized tadpole contribution of the χ fields

$$\Sigma(u) = \frac{\gamma^2\lambda}{16\pi^2}\left[\frac{\Lambda^2}{\gamma^2} + \frac{6\gamma^2+m_\chi^2}{2\gamma^2}\left(\ln\frac{6\gamma^2+m_\chi^2}{4\Lambda^2}+1\right) - 3(1-u^2)\ln\frac{6\gamma^2+m_\chi^2}{4\Lambda^2} - 6(1-u^2)\sum_{n=1}^{2}(-1)^n(n-1-u^2) \times\left(\frac{6\gamma^2+m_\chi^2}{n^2\gamma^2}-1\right)^{\frac{1}{2}}{\rm arccot}\left(\frac{6\gamma^2+m_\chi^2}{n^2\gamma^2}-1\right)^{\frac{1}{2}}\right]. \tag{C6}$$

After adding the counterterms, we obtain

$$\Sigma^R(u) = \frac{3\gamma^2\lambda}{16\pi^2}\left[11 - 5u^2 - 3(3-u^2)\left(\frac{m_\chi^2+2\gamma^2}{m_\chi^2+6\gamma^2}\right)^2 - 2(1-u^2)\sum_{n=1}^{2}(-1)^n(n-1-u^2) \times\left(\frac{6\gamma^2+m_\chi^2}{n^2\gamma^2}-1\right)^{\frac{1}{2}}{\rm arccot}\left(\frac{6\gamma^2+m_\chi^2}{n^2\gamma^2}-1\right)^{\frac{1}{2}}\right]. \tag{C7}$$

We note that the expression in Eq. (C7) agrees with the renormalized tadpole contribution from Φ in Eq. (66) for $m_\chi^2 = -\mu^2$, as we would expect. Assuming $m_\chi^2 \gg \gamma^2$, we may expand Eq. (C7) to leading order in γ^2/m_χ^2, giving Eq. (82).

The one-loop correction to the bounce action from the determinant over the quadratic fluctuations in the χ fields is given by

$$B_\chi^{(1)} = -\frac{N}{2}\int_0^\Lambda \mathrm{d}k k^2 \int_0^\infty \frac{\mathrm{d}\tau}{\tau}\int_0^\infty \mathrm{d}r r^3 \times \mathcal{L}_s^{-1}[\tilde{S}(u,m)](\tau), \tag{C8}$$

where $\tilde{S}(u,m)$ is obtained from Eqs. (38) and (74) with

$$m = \sqrt{6}\left(1 + \frac{k^2 + s + m_\chi^2}{6\gamma^2}\right)^{\frac{1}{2}}. \tag{C9}$$

Continuing as in Sec. III, we find

$$B_\chi^{(1)} = -N\frac{R^3\gamma^3}{2}\left[3\frac{\Lambda^2}{\gamma^2} + 3\frac{m_\chi^2 + 4\gamma^2}{2\gamma^2}\ln\frac{6\gamma^2 + m_\chi^2}{4\Lambda^2} - \frac{m_\chi^2 + 2\gamma^2}{2\gamma^2} + \frac{2}{3}\sum_{n=1}^{2} n^3\left(\frac{6\gamma^2 + m_\chi^2}{n^2\gamma^2} - 1\right)^{\frac{3}{2}} \times \mathrm{arccot}\left(\frac{6\gamma^2 + m_\chi^2}{n^2\gamma^2} - 1\right)^{\frac{1}{2}}\right]. \tag{C10}$$

Adding the counterterm

$$\delta B_\chi^{(1)} = \frac{3}{2}NR^3\gamma^3\left[\frac{\Lambda^2}{\gamma^2} - 20 + \frac{m_\chi^2}{2\gamma^2} + 21\left(\frac{m_\chi^2 + 2\gamma^2}{m_\chi^2 + 6\gamma^2}\right)^2 + \frac{m_\chi^2 + 4\gamma^2}{2\gamma^2}\ln\frac{6\gamma^2 + m_\chi^2}{4\Lambda^2}\right], \tag{C11}$$

obtained in analogy with Eq. (77), we find

$$B_\chi^{(1)} = -N\frac{R^3\gamma^3}{2}\left[63 - 4\frac{m_\chi^2 + 2\gamma^2}{2\gamma^2} - 63\left(\frac{m_\chi^2 + 2\gamma^2}{m_\chi^2 + 6\gamma^2}\right)^2 + \frac{2}{3}\sum_{n=1}^{2} n^3\left(\frac{6\gamma^2 + m_\chi^2}{n^2\gamma^2} - 1\right)^{\frac{3}{2}}\mathrm{arccot}\left(\frac{6\gamma^2 + m_\chi^2}{n^2\gamma^2} - 1\right)^{\frac{1}{2}}\right]. \tag{C12}$$

The result in Eq. (C12) reduces to that found in Eq. (78) for $m_\chi^2 = -2\gamma^2$ and $N = 1$, as we would expect. Instead, taking $m_\chi^2 \gg \gamma^2$, we obtain the expression in Eq. (91).

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