What we can learn from the running of the spectral index if no tensors are detected in the cosmic microwave background anisotropy

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In this paper we operate under the assumption that no tensors from inflation will be measured in the future by the dedicated experiments and argue that, while for single-field slow-roll models of inflation the running of the spectral index will be hard to detect, in multifield models the running can be large due to its strong correlation with non-Gaussianity. A detection of both the running and non-Gaussianity might therefore strengthen the conclusion that more than one scalar degree of freedom was active during inflation.

I. MOTIVATIONS

Inflation [1] provides a mechanism to explain the initial conditions for the large-scale structure (LSS) and for the cosmic microwave background (CMB) anisotropy. The seeds for the density scalar perturbations are created from the quantum fluctuations "redshifted" out of the Hubble radius during the early period of superluminal expansion of the Universe. At the same time, the generation of tensor (gravity-wave) fluctuations is a generic prediction of an accelerated de Sitter expansion of the Universe [2].

To characterize the scalar and tensor perturbations, several observables are introduced: the spectral index (or tilt) n_s of the scalar perturbations, currently measured to be $n_s = 0.9655 \pm 0.0062$ at 68% C.L.; the running with the wave number k of the spectral index $\alpha_s = \mathrm{d}n_s/\mathrm{d}\ln k$, currently consistent with zero, $\alpha_s = -0.00571 \pm 0.0071$ at 68% C.L. (assuming no tensors); and the level of local non-Gaussianities parametrized by $f_{\mathrm{NL}} = 0.8 \pm 5.0$ at 68% C.L. and $g_{\mathrm{NL}} = (-9.0 \pm 7.7) \times 10^4$ at 68% C.L. All these figures are quoted from the last Planck releases [3,4]. Finally, the tensor-to-scalar ratio is bounded to be $r < 0.12$ at 95% C.L. [5].

Many next-generation satellite missions—see, for instance, Refs. [6–8]—are dedicated to measuring the polarization of the CMB anisotropies. On large angular scales the B-mode polarization of the CMB carries the imprint of primordial gravitational waves, and its precise measurement would provide a powerful probe of the epoch of inflation. The goal of these missions is to achieve a measurement of r down to $\mathcal{O}(10^{-3})$.

While a detection of gravity waves from inflation will be a major milestone for the inflationary paradigm, on the other hand one needs to remember that going from $r = 10^{-1}$ down to $r = 10^{-3}$ means testing the energy scale of inflation E_{inf} only by a factor $(100)^{1/4} \simeq 3.1$ better than current bounds (r scales like E_{inf}^4).

One should therefore envisage the situation in which nature has not been so kind to us as to set the scale of the energy of inflation within a factor of 3 away from the current limits. In this paper, we will therefore operate under the working assumption that no tensors will be measured in the future by the dedicated experiments.

While we hope this hypothesis will prove to be wrong in the future, we believe it is reasonable to ask how else we could learn something more about the inflationary perturbations.

In this paper we turn our attention to the scale dependence of the tilt of the scalar perturbations. Our hypothesis on the tensor modes leads to a hardly detectable running within single-field slow-roll models. On the other hand, a negative result on tensors might indicate a low level of the Hubble rate, leaving open the possibility that the scalar perturbations are due to a light scalar field other than the inflaton field. In such a case, our findings indicate that a large running may be achieved thanks to an interesting correlation between the running and the level of non-Gaussianity. Such strong correlation exists only in multifield models of inflation, while it is missing in single-field slow-roll models. If no tensors will be observed, a measurement of a sizable level of non-Gaussianity will lead to the conclusion that more than one scalar degree of freedom was active during inflation. However, one has to keep in mind that nonlinear relativistic corrections in general relativity induce a signal in the observations that might be misinterpreted as primordial non-Gaussianity with a local shape [9,10]. The correlation between α_s and f_{NL} might help to disentangle the multifield contribution to f_{NL} from the general relativity effects.

Our considerations are particularly relevant for the goals of future missions designed to measure the three-dimensional structure of the Universe, such as the ESA mission EUCLID [11], the Square Kilometre Array [12], and the NASA SPHEREx mission [13], a proposed all-sky spectroscopic survey satellite. For instance, the combination of the galaxy power spectrum and bispectrum leads to the forecasted 68% C.L. errors $\sigma(n_s - 1) = 2.2 \times 10^{-3}$,

$\sigma(\alpha_s) = 6.5 \times 10^{-4}$, and $\sigma(f_{\rm NL}) = 0.2$ (for the local case) in the case of SPHEREx. This is an improvement with respect to EUCLID forecasts of a factor ~ 2 in α_s and a factor ~ 20 for the local $f_{\rm NL}$ (or ~ 6 with the galaxy power spectrum only). For the other parameters SPHEREx will deliver constraints comparable to those expected from the EUCLID spectroscopic survey.

As EUCLID and (possibly) SPHEREx will strongly narrow down the allowed parameter space in the $(\alpha_s, f_{\rm NL})$ plane, it is therefore extremely interesting and timely to understand what are the theoretical expectations in terms of α_s and its connections to non-Gaussianity.

II. SINGLE-FIELD SLOW-ROLL INFLATION

Let us first consider the case in which the cosmological perturbations are generated by the same scalar field ϕ driving inflation. The power spectrum of scalar perturbations is given in such a case by

$$\mathcal{P}_s(k) = \frac{1}{2M_{\rm p}^2 \epsilon_*}\left(\frac{H_*}{2\pi}\right)^2, \qquad (1)$$

where the subindex $_*$ indicates that quantities have to be computed at Hubble crossing, H_* is the Hubble rate during inflation, $M_{\rm p} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass, and $\epsilon_* = -\dot{H}_*/H_*^2$ is one of the slow-roll parameters (dots indicate derivatives with respect to time).

Let us now assume that no tensors will be observed in the future and see what are the consequences we can draw. Measuring no tensors implies that $r = 16\epsilon_* \ll 10^{-3}$, or $\epsilon_* \ll 10^{-4}$. Irrespectively of the form of the inflaton potential, this means that $\phi \ll M_{\rm p}$. To see this, take the cosmological range of scales to span four decades, corresponding to $\ln k \sim 10$. This corresponds to ~ 10 e-folds of inflation. In the first approximation in slow-roll inflation, ϵ_* has negligible variation over one e-fold, and in typical models it has only small variation over the ~ 10 e-folds. Taking that to be the case, one finds (this is nothing else that the reverse of the so-called Lyth bound [14]) $\phi \ll 0.4 M_{\rm p}$, where we have used (primes indicate derivative with respect to the inflaton field) $N'(\phi) = \pm(1/\sqrt{2\epsilon}M_{\rm p})$, and N is the number of e-folds till the end of inflation. A more detailed computation leads to sub-Planckian excursions of the inflation field for $r \lesssim 2 \times 10^{-5}$ [15].

The first consequence of observing no tensors is therefore that the model of inflation will be characterized by a potential of the form $V(\phi) = V_0 + \cdots$ with the constant first term dominating.

The second consequence is that one can simplify the formulas for the spectral index and its running [2]

$$n_s - 1 = 2\eta_*, \qquad \alpha_s = 2\xi_*^2, \qquad (2)$$

where $\eta_* = M_{\rm p}^2 V''/V$ and $\xi_*^2 = M_{\rm p}^4 (V'V'''/V^2)$. Therefore, observing no tensors implies that the running of the spectral

index will be sizable only if the third derivative of the potential is nonvanishing. Also, since $(n_s - 1) = 2\eta_*$ is measured to be negative by Planck, we conclude that the second derivative of the potential must be negative.

A prototype of inflationary models summarizing all these properties (domination of the vacuum energy, a nonvanishing V''' and $V'' < 0$) is represented by the following form of the potential ($c > 0$):

$$V(\phi) = V_0(1 - c\phi^p), \qquad -\infty < p < 0 \quad \text{or} \quad p > 1. \tag{3}$$

This parametrization has the virtue of reproducing for $p \to -\infty$ the exponential models $V = V_0(1 - e^{-q\phi})$, among which are the popular Starobinsky $\mathcal{R}^2$ model [16] and the Higgs inflation model [17]. Related to the expression (3), as far as the predictions are concerned, are the logarithmic supersymmetry-inspired models $V = V_0(1 + c(\ln \phi/\mu))$ obtained for $p \to 0$ and the brane inflation models obtained for $p = -4$. Potentials of the form $V(\phi) = V_0(1 \pm c\phi^2)$ give a scale-independent spectral index. In all these cases the predictions are [2] (for $p \neq 2$)

$$n_s - 1 = -\left(\frac{p-1}{p-2}\right)\frac{2}{N}, \qquad \alpha_s = -\left(\frac{p-1}{p-2}\right)\frac{2}{N^2}. \tag{4}$$

The running can be written as

$$\begin{aligned}\alpha_s &= -\frac{1}{2}\left(\frac{p-2}{p-1}\right)(n_s-1)^2 = -\frac{72}{25}\left(\frac{p-2}{p-1}\right)f_{\rm NL}^2 \\ &\simeq -6\left(\frac{p-2}{p-1}\right)\times 10^{-4},\end{aligned} \tag{5}$$

where in the second passage we have made use of the known relation $f_{\rm NL} = -5(n_s - 1)/12$ valid for single-field models [18], and the numerical estimate has been done using the Planck central value for $(n_s - 1)$ (as we will do in the following). The factor $(p-2)/(p-1)$ is of order unity, and the most favorable case is achieved for the logarithmic models, for which $p \simeq 0$ and $\alpha_s \simeq 1.2 \times 10^{-3}$.

Since the SPHEREx measurements of the power spectrum and the bispectrum may provide 1σ-level detection of the running spectral index at the level of 6.5×10^{-4} [13], a nondetection of the running will rule out models with $p < -12$ at the 1σ level. On the other hand, a 2σ detection of the running would require a running larger than 1.3×10^{-3}, which is almost impossible to get in single-field slow-roll models of inflation if no tensors are observed, unless $(n_s - 1)$ is redder than the Planck central value by more than two Planck standard deviations. Of course, a large running can be obtained by abandoning the slow-roll regime [19], but in this case no detection of $f_{\rm NL}$ is expected.

III. MULTIFIELD INFLATION

In the single-field inflation scenario, the total curvature perturbation is generated during inflation and remains constant on super-Hubble scales. However, in many inflationary models, there are usually a plethora of scalar fields which may play a significant role during inflation. Indeed, if these fields do not dominate the energy density during inflation but are light enough to be quantum-mechanically excited during the de Sitter stage, they provide a source of isocurvature perturbation.

If the curvature perturbation associated with the field driving inflation is suppressed, the total curvature perturbation may have originated from such isocurvature perturbation. From Eq. (1), one deduces that this is naturally achieved if the Hubble rate is small: no significant amount of tensor modes are expected to be detected in the case in which the seeds of the LSS and CMB anisotropies are due to a light field other than the inflaton (this is in fact not a no-go theorem [20], but it is a very well-educated guess). More precisely, r below the expected sensitivity of 10^{-3} implies H_* smaller than about 7.7×10^{12} GeV.

We will concentrate on the most studied example of such a mechanism, the so-called curvaton mechanism [21–24], even though our findings hold as well for other mechanisms [25–28]. In the curvaton mechanism, during inflation the curvaton energy density is negligible, and isocurvature perturbations with a nearly flat spectrum are produced in the curvaton field σ.

After the end of inflation, the curvaton field oscillates during some radiation-dominated era, causing its energy density to grow and thereby converting the initial isocurvature into curvature perturbation. The comoving curvature perturbation ζ can be computed through the δN formalism [29] $\zeta(\vec{x}, t) = N(\vec{x}, t) - N(t)$, where $N(\vec{x}, t)$ is the amount of expansion along the worldline of a comoving observer from a spatially flat slice at some initial time to a generic slice at time t. Since the number of e-folds from the end of inflation to the beginning of the oscillations is unperturbed because the radiation energy density $\rho_{\rm rad} = (\rho_{\rm tot} - \rho_\sigma)$ dominates during this time, for the curvaton case, one can redefine N as the number of e-folds from the beginning of the sinusoidal oscillations (that is, when the quadratic part of the curvaton potential with mass m starts dominating) to the curvaton decay [30]:

$$N[\rho(t_{\rm d}), \rho(t_{\rm osc}), \sigma_*] = \frac{1}{4} \ln \frac{\rho_{\rm rad}(t_{\rm osc})}{\rho_{\rm tot}(t_{\rm d}) - \rho_\sigma(t_{\rm d})}, \quad (6)$$

where

$$\rho_\sigma(t_{\rm d}) = \frac{1}{2} m^2 g^2(\sigma_*) \left(\frac{\rho_{\rm tot}(t_{\rm d}) - \rho_\sigma(t_{\rm d})}{\rho_{\rm rad}(t_{\rm osc})} \right)^{3/4}. \quad (7)$$

We have highlighted the role of the function $g = g(\sigma_*)$. It represents the initial amplitude of curvaton oscillations,

which is some function of the field value at the Hubble exit. The generic function $g(\sigma_*)$ parametrizes the nonlinear evolution on large scales if the curvaton potential deviates from a purely quadratic potential with mass m away from its minimum [31,32].

The resulting power spectrum of the scalar perturbations reads [30]

$$\mathcal{P}_s(k) = \frac{4 r_{\rm d}^2}{9} \left(\frac{H_*}{2\pi} \right)^2 \left(\frac{g'}{g} \right)^2, \quad (8)$$

where the prime now denotes differentiation with respect to σ_* and

$$r_{\rm d} = \frac{3\rho_\sigma(t_{\rm d})}{3\rho_\sigma(t_{\rm d}) + 4\rho_{\rm rad}(t_{\rm d})} \quad (9)$$

parametrizes the curvaton energy density at the time of curvaton decay with respect to the radiation component.

We neglect for the time being the scale dependence of H_* and write therefore $d/d\ln k = H_*^{-1} d/dt_* = (\dot{\sigma}_*/H_*)\partial/\partial\sigma_*$. We then find

$$(n_s - 1) = 2\frac{\dot{\sigma}_*}{H_*} \frac{g'}{g} \left(\frac{g'' g}{g'^2} - 1 \right) + 2\frac{\dot{\sigma}_*}{H_*} \frac{r_{\rm d}'}{r_{\rm d}} \quad (10)$$

and

$$\begin{aligned} \alpha_s = &\, 2 \left(\frac{\dot{\sigma}_*}{H_*} \frac{g'}{g} \right)^2 \left(\frac{g''' g^2}{g'^3} - \frac{g''^2 g^2}{g'^4} - \frac{g'' g}{g'^2} + 1 \right) \\ &+ 2 \left(\frac{\dot{\sigma}_*}{H_*} \right)^2 \left(\frac{r_{\rm d}''}{r_{\rm d}} - \frac{r_{\rm d}'^2}{r_{\rm d}^2} \right) \\ &- \frac{2}{3} \frac{V''}{H_*^2} \frac{\dot{\sigma}_*}{H_*} \left[\frac{g'}{g} \left(\frac{g'' g}{g'^2} - 1 \right) + \frac{r_{\rm d}'}{r_{\rm d}} \right], \quad (11) \end{aligned}$$

with $r_{\rm d}' = (2g'/3g) r_{\rm d}(1 - r_{\rm d})(3 + r_{\rm d})$ and $V = V(\sigma)$ the curvaton potential. The expressions for the quadratic and cubic non-Gaussianities are [33,34]

$$f_{\rm NL} = \frac{5}{4r_{\rm d}} \left(1 + \frac{g'' g}{g'^2} \right) - \frac{5}{3} - \frac{5r_{\rm d}}{6},$$

$$\begin{aligned} g_{\rm NL} = &\, \frac{25}{54} \left[\frac{9}{4r_{\rm d}^2} \left(\frac{g''' g^2}{g'^3} + 3\frac{g'' g}{g'^2} \right) - \frac{9}{r_{\rm d}} \left(1 + \frac{g'' g}{g'^2} \right) \right. \\ &\left. + \frac{1}{2} \left(1 - 9\frac{g'' g}{g'^2} \right) + 10 r_{\rm d} + 3 r_{\rm d}^2 \right]. \quad (12) \end{aligned}$$

Notice that $r_{\rm d}$ depends on σ_* through $\rho_\sigma(t_{\rm d})$ in Eq. (7). This dependence is essential to obtain the correct $f_{\rm NL}$ and $g_{\rm NL}$ given above [30].

We can recast the running of the spectral index in a rather appealing and compact form:

$$\alpha_s = \frac{3}{4}(n_s - 1)^2 \left(\frac{g_{NL}}{f_{NL}^2} - \frac{2}{3} \right) - \frac{V''}{3H_*^2}(n_s - 1). \quad (13)$$

The same expression can be derived more directly in terms of derivatives of the number of e-folds using the δN formalism [29]. Indeed, knowing that

$$\mathcal{P}_s = \left(\frac{H_*}{2\pi} \right)^2 N'^2, \qquad n_s - 1 = 2\frac{\dot{\sigma}_*}{H_*}\frac{N''}{N'},$$

$$f_{NL} = \frac{5}{6}\frac{N''}{N'^2}, \qquad g_{NL} = \frac{25}{54}\frac{N'''}{N'^3}, \quad (14)$$

one can write

$$\alpha_s = 2 \left(\frac{\dot{\sigma}_*}{H_*} \right)^2 \left(-\frac{(N'')^2}{N'^2} + \frac{N'''}{N'} \right) - \frac{V''}{3H_*^2}(n_s - 1). \quad (15)$$

Using Eq. (14), we get

$$\alpha_s = \frac{108}{25} \left(\frac{\dot{\sigma}_*}{H_*} \right)^2 N'^2 f_{NL}^2 \left(\frac{g_{NL}}{f_{NL}^2} - \frac{2}{3} \right) - \frac{V''}{3H_*^2}(n_s - 1) \quad (16)$$

and

$$N' f_{NL} = \frac{5}{12}\frac{H_*}{\dot{\sigma}_*}(n_s - 1), \quad (17)$$

which allow us to reobtain the expression (13). We have decided to include the first and slightly more involved derivation of (13) to stress the importance of the non-linearities in the curvaton potential.

Equation (13) is the main result of this paper and indicates that the running of the spectral index can be sizable if $g_{NL} \gtrsim f_{NL}^2$. In general, this is true in those models for which $r_d \simeq 1$ and $g''g/g'^2 \simeq 1$ (or $r_d \lesssim 1$ and $g''g/g'^2 \simeq -1$) when f_{NL} becomes small. This can happen if the curvaton potential has nonlinearities in it [35,36]. As a rule of thumb, being $3(n_s - 1)^2/4 \simeq 9 \times 10^{-4}$, we deduce that SPHEREx could detect a running of the spectral index at 2σ if $g_{NL}/f_{NL}^2 = \mathcal{O}(\pm 2)$. Since SPHEREx can detect at the 1σ level a local f_{NL} larger than 0.2, the situation seems optimistic.

Consider, for instance, the case in which the curvaton potential has a self-interacting piece $\lambda\sigma^4$, where λ is a dimensionless coupling, besides the quadratic term. In the limit in which the nonlinear term dominates over the quadratic piece during inflation $g(\sigma_*) \sim \sigma_*^{3/4}$ [35,36], one obtains

$$n_s - 1 = \frac{4}{3}\lambda(r_d(2 + r_d) - 1)\frac{\sigma_*^2}{H_*^2},$$

$$f_{NL} = \frac{5}{6r_d}(1 - r_d^2 - 2r_d). \quad (18)$$

For instance, for $r_d = 1/3$ we get a red tilt, $f_{NL} = 5/9$, $g_{NL} \simeq -32 f_{NL}^2$, and $\alpha_s \simeq -13 \times 10^{-3}$. This running can certainly be measured by EUCLID and SPHEREx.

To describe further the correlation between a sizable running α_s and the level of non-Gaussianity, we present some illustrative examples obtained numerically in Fig. 1. For completeness, we have restored the scale dependence of H_* through an inflaton potential of the form
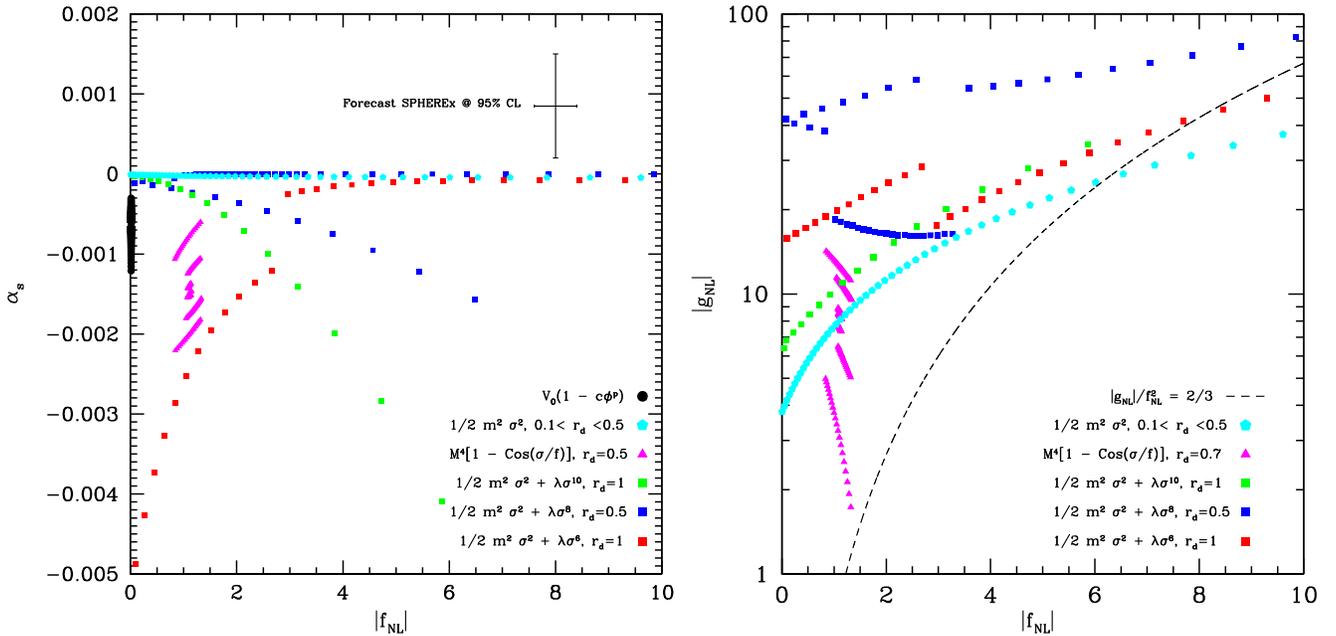

FIG. 1 (color online).   Left: The running α_s as a function of f_{NL} for a range of curvaton models. Right: Nonlinearity parameter g_{NL} as a function of f_{NL} for the same models as in the left plot.

$V(\phi) \sim \exp(\mu\phi/M_p^2)$, which provides a constant contribution $(-\mu^2/M_p^2)$ to $(n_s - 1)$. The various points have been obtained by finding the function $g(\sigma_*)$, scanning over the various parameters, and using the general expression (11). We have systematically checked that all the observables, such as the amplitude of the power spectrum, are in agreement with the current observations and that the contribution to the running from the V'' term in Eq. (13) and from the scale dependence of H_* is smaller than the one coming from the ratio g_{NL}/f_{NL}^2. The 2σ SPHEREx forecasted errors for α_s and f_{NL} are also shown. On the right panel, we present the corresponding values of g_{NL} as a function of f_{NL}. The dashed line illustrates that the running is mainly due to the correlation with the non-Gaussianity. Of course, one should keep in mind that there are regions of the parameter space where the correlation is not present.

Finally, let us consider the case of the modulated reheating scenario. The curvature perturbation is generated during reheating if the inflaton decay rate $\Gamma = \Gamma(\sigma)$ is a function of a light field σ. The corresponding power spectrum and non-Gaussianities are [34]

$$\mathcal{P}_s(k) = \left(\frac{H_*}{18\pi}\right)^2 \left(\frac{\Gamma'}{\Gamma}\right)^2, \qquad f_{NL} = 5\left(\frac{\Gamma''\Gamma}{\Gamma'^2} - 1\right),$$

$$g_{NL} = \frac{100}{3}\left(1 - \frac{3}{2}\frac{\Gamma''\Gamma}{\Gamma'^2} + \frac{1}{2}\frac{\Gamma'''\Gamma^2}{\Gamma'^3}\right). \tag{19}$$

One can easily show that the analytical expression for the running is exactly the same as in Eq. (13). Now, at the time of reheating, the field σ will have a value $\sigma_{reh} = \sigma_{reh}(\sigma_*)$, and the plausible dependence of the decay rate is $\Gamma(\sigma_{reh}) \propto \sigma_{reh}^q$ ($q > 0$). If the potential for the light field is quadratic, then $\sigma_{reh} = \sigma_*$ and we find $g_{NL}/f_{NL}^2 = 4/3$: the first term in the running turns out to be of the order of 6×10^{-4}, too small to be detected. Nevertheless, if there is a nonlinear relation between σ_{reh} and σ_* due to the fact that the potential of the field σ deviates from a purely quadratic potential,

then one can easily increase the ratio g_{NL}/f_{NL}^2 and make the running detectable.

IV. CONCLUSIONS

In this paper we have stressed that a sizable running of the spectral index may be connected to the primordial non-Gaussianity in the case in which the curvature perturbation is due to the presence of more than one scalar field during inflation. A future detection of a significant running of the spectral index and primordial non-Gaussianity may tell us a lot about the true mechanism, giving rise to the primordial perturbations. Of course, one could not disregard the possibility that the running of the spectral index might come from single-field models with a partial breaking of slow roll and the measured non-Gaussianity from the general relativity effects. Dedicated searches aiming at measuring with great accuracy the running and non-Gaussianity are therefore of vital importance.

Note added.—Recently, Ref. [37] appeared, also making the point that the running of the scalar spectral index is correlated with the statistical properties of non-Gaussianities. We thank the authors of Ref. [37] for correspondence.

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