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Gravitational waves: A test for modified gravity

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In a modified gravity theory, the propagation equation of gravitational waves will be presented in a nonstandard way. Therefore this tensor mode perturbation of time-space, as a complement to the scalar mode perturbation, provides a unique character distinguishing modified gravity from general relativity. To avoid the model-dependent issue, in this paper, we propose a parametrized modification to the propagation of gravitational waves. We show the effects on the angular power spectrum of cosmic microwave background radiation due to the parametrized modification and its degeneracy to the tensor mode power spectrum index n_t and its running α_t. Last, we report the current status on the detection of modified gravity through the currently available cosmic observations. Our results show no significant deviation to general relativity.

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I. INTRODUCTION

Commonly realizing a late time accelerated expansion of our Universe needs a modification to general relativity (GR) at large scales dubbed as modified gravity (MG), or an addition of exotic energy component named dark energy (DE). However, these realizations are totally different in nature. Finding out a modification to GR at large scales means the discovery of new gravity theory beyond GR. Confirming the existence of DE implies the discovery of new particle(s) beyond the standard particle physics model. The issue is how to distinguish MG theories from DE models. It is believed that cosmic observations provide the finally experimental judgement in addition to the fundamental physics theory.

Due to the existence of a great diversity of MG theories and DE models, it is almost impossible to test every model. One possibility is finding out a general formalism which can grasp the main characteristics of MG theories and DE models, for example, a feasibly parametrized MG and DE model that are consistent with cosmic observations but may be independent of any concrete fundamental physics. Based on this spirit, a parametrized modification to GR in the scalar mode perturbation was studied in the literature; see [1–3] and references therein for examples. The modification was mainly focused on the Poisson equation and the slip of the Newtonian potentials of Φ and Ψ, but keeps the background evolution to a standard ΛCDM cosmology, say,

$$k^2\Psi = -\mu(k,a)4\pi Ga^2[\rho\Delta + 3(\rho+P)\sigma], \qquad (1)$$

$$k^2[\Phi - \gamma(k,a)\Psi] = \mu(k,a)12\pi Ga^2(\rho+P)\sigma, \qquad (2)$$

in Fourier k-space as an example borrowed from Ref. [2], where $\Delta = \delta + 3\mathcal{H}(1+w)\theta/k^2$ is the gauge-invariant overdensity and $\delta \equiv \delta\rho/\rho$ is the overdensity of energy component ρ; $\mathcal{H} \equiv \dot{a}/a$ is the conformal Hubble parameter, where the dot denotes the derivative with respect to the conformal time τ, and a is the scale factor; w is the equation of state of energy component ρ; θ is the divergence of the velocity perturbation, i.e., the peculiar velocity; and σ is the anisotropic stress. Ψ and Φ are the Newtonian potentials in the conformal Newtonian gauge

$$ds^2 = a(\tau)^2\{-(1+2\Psi)d\tau^2 + [(1-2\Phi)\delta_{ij} + 2h^T_{ij}]dx^i dx^j\}, \qquad (3)$$

where h^T_{ij} is a traceless ($h^{Ti}_i = 0$), divergence-free ($\nabla^i h^T_{ij}$), symmetric ($h^T_{ij} = h^T_{ji}$) tensor field. The two $\mu(k,a)$ and $\gamma(k,a)$ are scale- and time-dependent functions encoding any modification to gravity theory in scalar mode. Note that GR is recovered in the $\mu = \gamma = 1$ limit. When considering the locality, general covariance and the quasistatic approximation, the physically acceptable forms of $\mu(k,a)$ and $\gamma(k,a)$ should be the ratios of polynomials in even k, with a numerator of μ set by the denominator of γ [3]. Therefore, for the scalar part modification, one obtains [3] (see also Refs. [4–6])

$$\gamma(k,a) = \frac{p_1(a) + p_2(a)k^2}{1 + p_3(a)k^2}, \qquad \mu(k,a) = \frac{1 + p_3(a)k^2}{p_4(a) + p_5(a)k^2}, \qquad (4)$$

where $p_i(a), i = 1\ldots5$ are functions of a. The $\mu(k,a)$ and $\gamma(k,a)$ can be fixed for a specific MG model; see Refs. [7,8] for examples. However, the *Planck* 2015 DE and MG paper

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has shown that the scale dependence of μ and γ does not lead to a significantly small χ^2 with respect to the scale-independent case [9]. This may come from the insufficiency of the large scale structure information. Therefore in this paper, we only consider the scale-independent forms. After eliminating the scale dependence, the final form for μ and γ should be

$$\gamma(k,a) = p_\gamma(a), \qquad \mu(k,a) = \frac{1}{p_\mu(a)}; \tag{5}$$

for explicitness we propose $p_i(a) = \lambda_i a^{s_i}, (i = \gamma, \mu)$ as a working example. In phenomena, it can be a function of $w_{de}(a)$ and $\Omega_{de}(a)$ effectively, say, $p_i(a) = \lambda_{s_i} w_{de}(a)\Omega_{de}(a)^{s_i}$, etc.

The tensor mode perturbation, as a complement to the scalar mode perturbation, should also be altered in a modified gravity theory. In GR, the propagation of gravitational waves in Fourier k-space is written as

$$\ddot{h}^T_{ij} + 2\mathcal{H}\dot{h}^T_{ij} + c_T^2(k^2 + 2K)h^T_{ij} = 8\pi G a^2 \Pi_{ij}, \tag{6}$$

where the transverse-traceless tensor Π_{ij} is the anisotropic part of the stress tensor, K the three-dimensional curvature ($K = 0$ is adopted in this paper), and c_T^2 is the square of the speed of gravitational waves. In the literature [10–14], a parametrized modification to the tensor mode perturbation was proposed recently. In general, the propagation of gravitational waves is modified due to the interaction between the new degree of freedom (introduced for providing late time accelerated expansion of our Universe) and the curvature or metric [10],

$$\ddot{h}^T_{ij} + 2\mathcal{H}\left[1 + \frac{\chi(k,a)}{2}\right]\dot{h}^T_{ij} + c_T^2(k^2 + 2K)h^T_{ij} + a^2 m_g^2 h^T_{ij} = 8\pi G a^2 \Gamma(k,a) S_{ij}, \tag{7}$$

where $\chi \equiv \mathcal{H}^{-1}(d\ln M_*^2/dt)$ describes the running rate of the effective *Planck mass* M_*, and m_g is the mass of a graviton in a massive gravity theory. The transverse-traceless tensor S_{ij} is the source term for the gravitational waves. The form of this source term S_{ij} depends on MG theories or the properties of matter fluid [10,15]. It is very important to note that when the source term S_{ij} comes from the anisotropic stress of matter fluid, the homogenous part of Eq. (7) will never be modified [10]. Therefore the anisotropic stress plays the role of a signature of nonstandard propagation of gravitational waves. It also implies that any significant $\xi(k,a) \neq 0$ or $m_g \neq 0$ signals the detection of MG which cannot be disguised by DE. The $\Gamma(k,a)$ term modification would be related to the $\xi(k,a)$ term for a specific MG model; for example in $f(R)$ gravity the propagation of gravitational waves is given by [16]

$$\ddot{h}^T_{ij} + 2\mathcal{H}\left(1 + \frac{1}{2}\frac{d\ln F}{d\ln a}\right)\dot{h}^T_{ij} + c_T^2(k^2 + 2K)h^T_{ij} = \frac{8\pi G a^2 \Pi_{ij}}{F}, \tag{8}$$

where $F \equiv df/dR$. Sometimes c_T^2 will also deviate from the speed of light in a MG model, for example, the scalar-tensor and Einstein-aether models as shown in Ref. [10]. In Ref. [17], the speed of the cosmological gravitational waves was constrained by using the *Planck* 2013 and BICEP2 data sets, where no significant deviation from the standard values $c_T^2 = 1$ was probed: $c_T^2 = 1.30 \pm 0.79$. Therefore in this paper, we will fix c_T^2 to its standard value. But we also will show the possible degeneracy between the c_T^2 and the $\chi(k,a)$ term in Sec. II. Motivated by the modification coming from $f(R)$ gravity, say, Eq. (8), one can propose a modified equation in the following form:

$$\ddot{h}^T_{ij} + 2\xi(k,a)\mathcal{H}\dot{h}^T_{ij} + c_T^2(k^2 + 2K)h^T_{ij} = 8\pi G\mu(k,a)\Pi_{ij}, \tag{9}$$

where $\xi(k,a)$ and $\mu(k,a)$ are two functions encoding a modified gravity theory. Since the $\xi(k,a)$ characterize the running rate of the effective *Planck mass*, which should be scale independent, we therefore assume

$$\xi(k,a) = p_t(a), \tag{10}$$

and we will take $p_t(a) = \lambda_t a^{s_t}$ as a working example. In phenomena, it can be a function of $w_{de}(a)$ and $\Omega_{de}(a)$ effectively, say, $p_t(a) = \lambda_t w_{de}(a)\Omega_{de}(a)^t$, etc.

Recently, the Background Imaging of Cosmic Extragalactic Polarization (BICEP2) experiment [18,19] has detected the B-modes of polarization in the cosmic microwave background (CMB), where the tensor-to-scalar ratio $r = 0.20^{+0.07}_{-0.05}$ with $r = 0$ disfavored at 7.0σ of the lensed-ΛCDM model was found. Recently, *Planck 2015* released the polarization result and did not find significant signal of the primordial gravitational waves. However the *Planck 2015* temperature angular power spectrum (TT), E-mode polarization power spectrum (EE), and temperature-polarization cross-power spectrum (TE) data are not released now. So in this paper, we still use the *Plank* 2013 data points. But our analysis on the CMB TT and B-mode polarization power spectrum (BB) power spectrum does not depend on the data points.

This paper is structured as follows. At first, in Sec. II, we show the effects on the CMB TT and BB power spectrum due to the parametrized modification to GR along with Eqs. (5), (9), and (10). To confirm these effects purely coming from the parametrized modification, we also test the possible degeneracy to the tensor spectrum index n_t and its running $\alpha_t = dn_t/d\ln k$. We report the current probe of MG by performing a global Markov chain Monte Carlo (MCMC) analysis in Sec. III. Section IV is the conclusion.

II. EFFECTS ON THE CMB TT AND BB POWER SPECTRUM

To study the effects on the CMB TT and BB power spectrum arising from the parametrized modification to GR, we modified the MGCAMB code [2] to include the tensor perturbation equation as shown in Eqs. (9)–(10). In the MGCAMB code, the CMB TT source term from the scalar mode perturbation in terms of the synchronous gauge variables is given by [20]

$$S_T^{(S)}(k,\tau) = g\left(\Delta_{T0} + 2\dot{\alpha} + \frac{\dot{v}_b}{k} + \frac{\Pi}{4} + \frac{3\ddot{\Pi}}{4k^2}\right) + e^{-\kappa}(\dot{\eta} + \ddot{\alpha}) + \dot{g}\left(\alpha + \frac{v_b}{k} + \frac{3\dot{\Pi}}{2k^2}\right) + \frac{3\ddot{g}\Pi}{4k^2}, \tag{11}$$

where κ is the optical depth, g is the visibility function, and $\Pi = \Delta_2^T + \Delta_2^P + \Delta_0^P$ and $\Delta_\ell^T(\Delta_\ell^P)$ are the ℓth moments of $\Delta^T(\Delta^P)$ in terms of Legendre polynomials [20]. The α term is changed to [2]

$$\alpha = \left\{\eta + \frac{\mu 8\pi G a^2}{2k^2}[\gamma\rho\Delta + 3(\gamma - 1)(\rho + P)\sigma]\right\}/\mathcal{H}, \tag{12}$$

in terms of μ and γ for the parametrized MG. The integrated Sachs-Wolfe (ISW) effect term $e^{-\kappa}(\dot{\eta} + \ddot{\alpha})$ in (11) is modified to [2]

$$\dot{\eta} + \ddot{\alpha} = \frac{\kappa}{2k^2}\left\{-\left[(\gamma + 1)(\dot{\rho}\Delta + \rho\dot{\Delta}) + \gamma\frac{3}{2}(\rho + P)\dot{\sigma} + \gamma\frac{3}{2}(\dot{\rho} + \dot{P})\sigma\right] + \dot{\gamma}\mu\left[(\rho\Delta) + \frac{3}{2}(\rho + P)\sigma\right]\right\}, \tag{13}$$

which is the time derivative of the summation of the Newtonian potentials

$$\Psi + \Phi = \dot{\alpha} + \eta, \tag{14}$$

that is modified by the μ and γ terms through the variation of α. The CMB TT source term from the tensor mode perturbation in terms of the synchronous gauge variables is given by [20]

$$S_T^{(T)} = -\dot{h}e^{-\kappa} + g\tilde{\Psi}, \tag{15}$$

where $\tilde{\Psi}$ denotes the temperate and polarization perturbations generated by gravitational waves. Here $\dot{h}$ is related to α by

$$\dot{h} = 2k^2\alpha - 6\dot{\eta}. \tag{16}$$

Now we move to study the effects on the CMB TT and BB power spectrum as shown in Fig. 1, by fixing the relevant cosmological parameters to their mean values obtained by the *Planck* group [21] and $r = 0.2$ by the BICEP2 group [19], but varying the parameters contained in γ, μ, ξ, and c_T^2 freely. When $\lambda_i = 1$, $s_i = 0$ $(i = \mu, \gamma, t)$ and $c_T^2 = 1$ are respected, and the standard ΛCDM cosmology is recovered. It is called the corresponding standard value.

In the top two panels of Fig. 1, we show the effects on the CMB TT (left panel) and BB (right panel) power spectrum resulting from the variation of the μ term, by fixing $\gamma \equiv 1$, $\xi \equiv 1$, and $c_T^2 \equiv 1$. For the CMB TT power spectrum, since $\gamma \equiv 1$ is fixed, the ISW term is untouched (retained to the standard ΛCDM model). The change of the amplitude of the CMB TT power spectrum is mainly caused by the integration of μ and $\dot{\mu}$ terms through the SW effect (the $g\dot{\alpha}$ and $\dot{g}\alpha$ terms) along the line of sight, i.e., at the range of $20 < \ell < 200$. Actually it is the result of the competition between $S_T^{(S)}$ and $S_T^{(T)}$ source terms. For the CMB BB power spectrum, the tensor perturbation h is sourced by the μ term; therefore the amplitude of the CMB BB power spectrum is enlarged with the increase of μ through the lensing effects at the multipole $l > 100$ region. This modification keeps the CMB BB power spectrum almost untouched at low multipole $\ell < 10$, where the BB power spectrum is mainly dominated by the primordial gravitational waves.

In the middle two panels of Fig. 1, the effects on the CMB TT (left panel) and BB (right panel) power spectrum due to variation of the γ term are shown. The term contributes not only to the SW effect but also to the early and late ISW effect. Therefore it makes an observable change at the low multipole $\ell < 20$. This late ISW effect arises from the evolution of γ which cannot be produced by the μ term. Similar to the μ term, the contribution to the CMB BB power spectrum mainly comes from the lensing effects, but the γ term makes its amplitude change along the contrary direction.

In the bottom two panels of Fig. 1, we show the effects on the CMB TT (left panel) and BB (right panel) power spectrum arising from the variation of the $\xi(k,a)$ term in the case of a fixed $c_T^2 = 1$. The $\xi(k,a)$ term only modifies the propagation of the primordial gravitational waves through the friction term $\dot{h}^T$, but keeps the CMB TT power

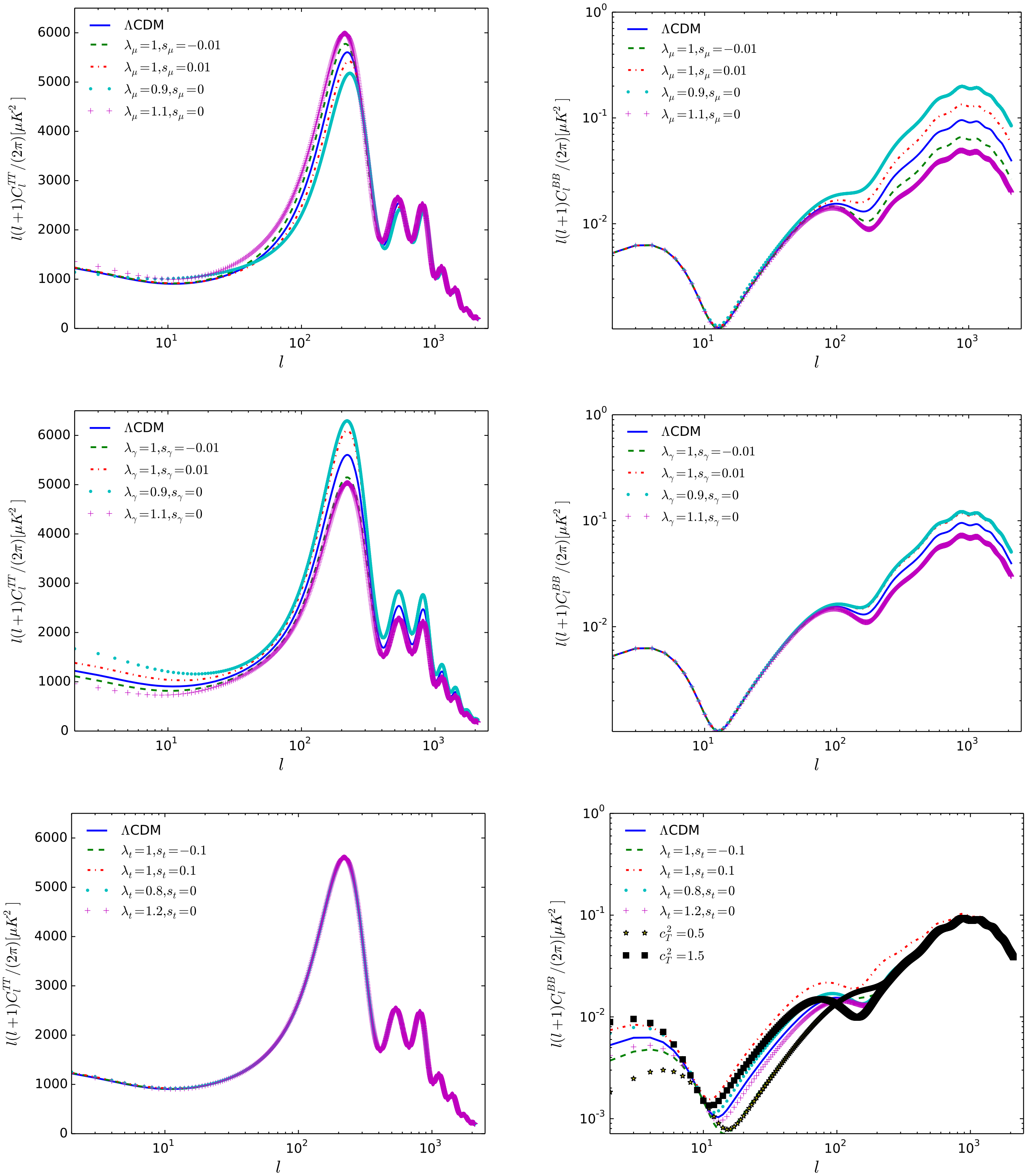

FIG. 1 (color online). The effects on the CMB TT (left panel) and BB (right panel) power spectrum arising from the variation of μ (top two panels), γ (middle two panels) and ξ (bottom two panels) terms, where the relevant cosmological parameters are fixed to their mean values obtained by the *Planck* group [21] and $r = 0.2$ by the BICEP2 group [19]. For every two panels, the other relevant MG parametrization terms are fixed to their standard values.

spectrum untouched. Increasing the friction depresses the amplitude and make it move to the right direction. This effect happens at the low multipole $\ell < 100$ and cannot be mimicked by the μ and ξ terms which usually modify the relations between Newtonian potentials. Therefore, the CMB BB power spectrum at low multipole provides a unique character, distinguishing MG from the DE model in principle. In the right panel of Fig. 1, we also show the

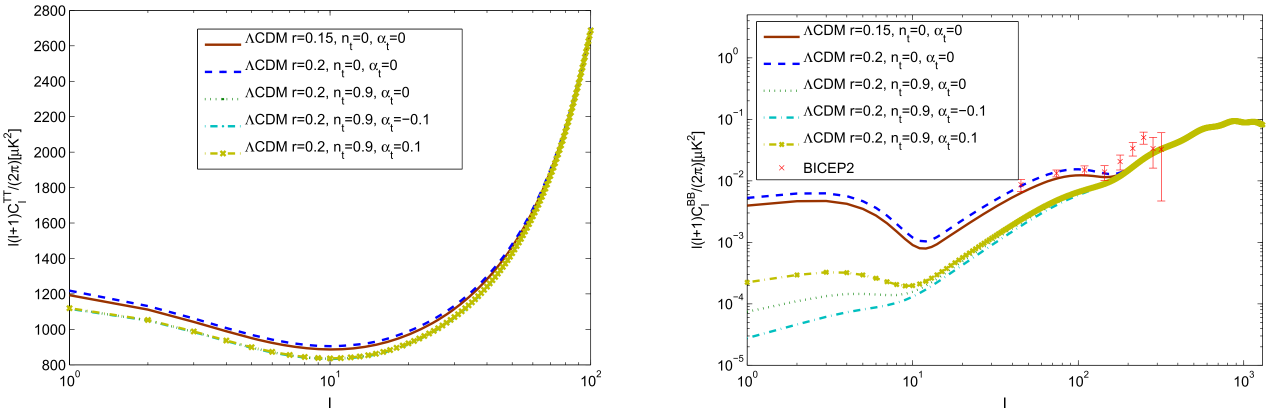

FIG. 2 (color online). The effects on the CMB TT and BB power spectrum with the variation of r, n_t, and α_t, where the relevant cosmological parameters are fixed to their mean values obtained by the *Planck* group [21].

effects on the CMB BB power spectrum with a varying c_T^2. When c_T^2 varies for a MG model with fixed $\xi \equiv 1$, it can mimic the effect on the CMB BB power spectrum as that of the ξ term. That is the degeneracy between $\xi(a)$ and c_T^2 terms as shown in Fig. 1. This confirms the results obtained in Ref. [17] and the reliability of our code. The degeneracy happens when $2\xi(k,a)\mathcal{H}\dot{h}_{ij}^T \sim c_T^2 k^2 h_{ij}^T$ is respected. And this degeneracy makes it more difficult to detect MG from the CMB BB power spectrum.

However, in the above investigation, the primordial power spectrum is specified by a fixed tensor-to-scalar ratio $r = 0.2$. But commonly the primordial power spectrum is characterized by the tensor-to-scalar ratio r, the tensor mode power spectrum index n_t, and its running $\alpha_t = dn_t/d\ln k$. So one should check whether the shift of the CMB BB power spectrum at the multipole $\ell < 100$ is caused by MG or modification of the propagation equation of gravitational waves. In doing so, we show the effects on the CMB TT and BB power spectrum arising from different values of r, n_t, and α_t in GR for the standard ΛCDM cosmology in Fig. 2. Here we only focus on the CMB BB power spectrum. Large r values increase the ratio of A_t/A_s and make the whole BB power spectrum move along the vertical direction at the multipole $\ell < 100$, but the spectrum retains its shape. The index n_t changes the amplitude and shape of the power spectrum simultaneously at the range $10 < \ell < 100$. The running α_t changes the shape at $\ell < 10$ and has little effect on the power spectrum in the range $10 < \ell < 100$. Therefore careful choices of r, n_t, and α_t can mimic the effects arising from the MG ξ term. Thus to distinguish MG from DE models, one still needs to understand the inflation very well. So in this work, we assume the inflation model is parametrized by r only.

TABLE I. The data points of $f\sigma_8(z)$ measured from RSD with the survey references.

♯	z	$f\sigma_8(z)$	Survey and refs.
1	0.067	0.42 ± 0.06	6dFGRS (2012) [30]
2	0.17	0.51 ± 0.06	2dFGRS (2004) [31]
3	0.22	0.42 ± 0.07	WiggleZ (2011) [32]
4	0.25	0.39 ± 0.05	SDSS LRG (2011) [33]
5	0.37	0.43 ± 0.04	SDSS LRG (2011) [33]
6	0.41	0.45 ± 0.04	WiggleZ (2011) [32]
7	0.57	0.43 ± 0.03	BOSS CMASS (2012) [34]
8	0.60	0.43 ± 0.04	WiggleZ (2011) [32]
9	0.78	0.38 ± 0.04	WiggleZ (2011) [32]
10	0.80	0.47 ± 0.08	VIPERS (2013) [35]

TABLE II. The mean values with 1σ errors and the best fit values of the model parameters and the derived cosmological parameters, where the *Planck* 2013, WMAP9, BAO, BICEP2, JLA, HST, and RSD data sets were used. "–" denotes the one which is not well constrained.

Parameters	Priors	Mean with errors	Best fit
$\Omega_b h^2$	[0.005, 0.1]	$0.02201^{+0.00038}_{-0.00038}$	0.02200
$\Omega_c h^2$	[0.001, 0.99]	$0.1177^{+0.0017}_{-0.0017}$	0.1161
$100\theta_{MC}$	[0.5, 10]	$1.0436^{+0.0013}_{-0.0013}$	1.0431
τ	[0.01, 0.81]	$0.089^{+0.013}_{-0.014}$	0.091
$\ln(10^{10}A_s)$	[2.7, 4]	$3.113^{+0.089}_{-0.088}$	3.135
n_s	[0.9, 1.1]	$0.958^{+0.026}_{-0.042}$	0.970
r	[0, 1]	$0.045^{+0.008}_{-0.038}$	0.058
λ_μ	[0, 2]	$1.030^{+0.023}_{-0.023}$	1.021
s_μ	[−1, 1]	$0.0054^{+0.0037}_{-0.0037}$	0.0042
λ_γ	[0, 2]	$1.11^{+0.16}_{-0.15}$	1.07
s_γ	[−1, 1]	$0.007^{+0.027}_{-0.019}$	0.002
λ_t	[0, 2]	–	0.95
s_t	[−1, 1]	$0.62^{+0.31}_{-0.73}$	0.19
H_0	73.8 ± 2.4	$68.78^{+0.77}_{-0.76}$	69.15
Ω_Λ	…	$0.7032^{+0.0095}_{-0.0094}$	0.7098
Ω_m	…	$0.2968^{+0.0094}_{-0.0095}$	0.2902
σ_8	…	$0.798^{+0.021}_{-0.021}$	0.803
$z_{\rm re}$	…	$10.95^{+1.14}_{-1.13}$	11.12
Age/Gyr	…	$13.745^{+0.046}_{-0.046}$	13.753

III. DATA SET AND RESULTS

In this section, we probe the signal of MG parametrized by μ, γ, and ξ terms by using the currently available cosmic observations which are summarized in the following, based on the assumption that the inflation model is well understood and parametrized by r only, and c_T^2 is fixed to its standard value 1:

(i) The newly released BICEP2 CMB B-mode data [18,19]. They will be denoted by BICEP2. Although the BICEP2 data have been confirmed as dust polarization recently, we will still use these data points; then they can be taken as a test with significant primordial gravitational waves signals in the future.

(ii) The full information of CMB which includes the recently released *Planck* data sets, which include the high-l TT likelihood (*CAMSpec*) up to a maximum multipole number of $l_{\max} = 2500$ from $l = 50$, the low-l TT likelihood (*lowl*) up to $l = 49$ and the low-l TE, EE, BB likelihood up to $l = 32$ from WMAP9. The data sets are available online [21]. This data set combination will be denoted by P + W.

(iii) For the BAO data points as "standard ruler," we use the measured ratio of D_V/r_s, where r_s is the comoving sound horizon scale at the recombination

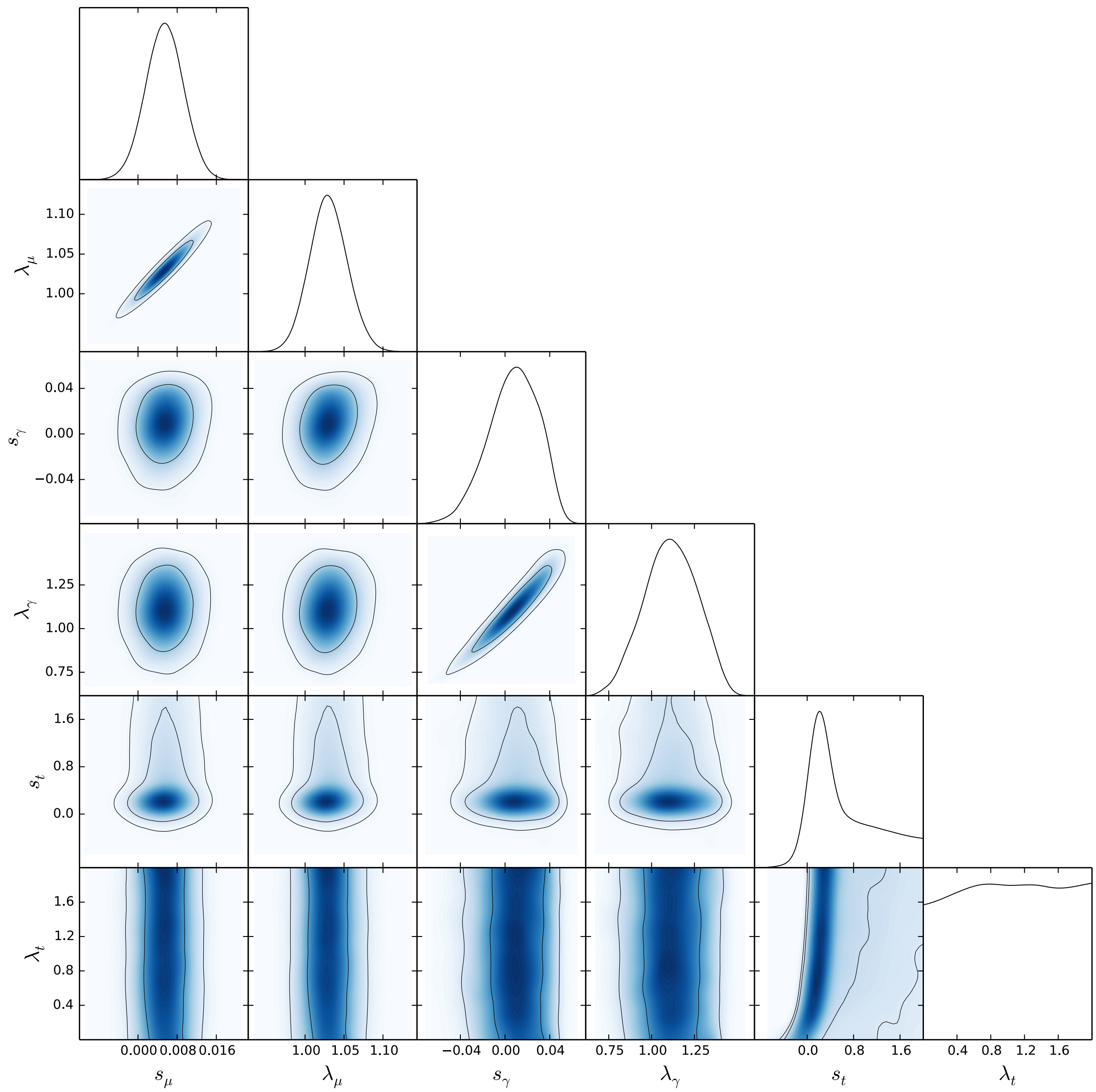

FIG. 3 (color online). The 1D marginalized distribution and 2D contours for interested model parameters with 68% C.L., 95% C.L. by using the *Planck* 2013, WMAP9, BAO, BICEP2, JLA, HST, and RSD data sets.

epoch, and D_V is the "volume distance," which is defined as

$$D_V(z) = [(1+z)^2 D_A^2(z) cz/H(z)]^{1/3}, \quad (17)$$

where D_A is the angular diameter distance. The BAO data include $D_V(0.106) = 456 \pm 27$ [Mpc] from the 6dF Galaxy Redshift Survey [22]; $D_V(0.35)/r_s = 8.88 \pm 0.17$ from SDSS DR7 data [23]; and $D_V(0.57)/r_s = 13.62 \pm 0.22$ from BOSS DR9 data [24]. This data set combination will be denoted by BAO.

(iv) The ten $f\sigma_8(z)$ data points from the redshift space distortion (RSD) are used; they are summarized in Table I.

The scale dependence of the growth rate $f = d\ln\Delta_m/d\ln a$ in a gravity theory beyond GR at linear scale was reported in Refs. [25,26], where $\Delta_m = \delta_m + 3\mathcal{H}(1+w_m)\theta_m/k^2$. Thus the product $f(z,k)$ and $\sigma_8(z)$, i.e., $f\sigma_8(z,k)$, depends on the scale k obviously, since $\sigma_8(z)$ is the function only of redshift z. To remove this explicit scale dependence of $f\sigma_8(z)$, we should define it in theory as

$$f\sigma_8(z) = \frac{d\sigma_8(z)}{d\ln a}, \quad (18)$$

which is scale independent for any gravity theory and cosmological model. The conventional definition is recovered for GR. Here we would like to warn the reader that the observed values of $f\sigma_8(z)$ are obtained based on the standard ΛCDM model, and are still unavailable for MG. With the observations on Fig. 11 in Ref. [27], say, in the regime $k < 0.1h/\mathrm{Mpc}$ at $z = 0$ for $|f_{R0}| = 10^{-4}$, the linear theory prediction for the growth rate almost matches the N-body simulation results for the $f(R)$ model, but deviates from the GR ones by about 20%. Therefore, we naively assume that the underlying complication [including the scale dependence of the growth rate $f(z,k)$] can enlarge the error bars listed in Table I to 20%, when the model parameter space is constrained. Therefore, we can take it as a preliminary result from the RSD constraint.

(v) The consistence of Ω_m between Ia supernovae and *Planck* 2013 was shown by the SDSS-II/SNLS3 joint light-curve analysis; for details, please see [28].

(vi) The present Hubble parameter $H_0 = 73.8 \pm 2.4$ [km s^{-1} Mpc^{-1}] from HST [29] is used.

We perform a global fitting to the model parameter space

$$P = \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, \ln(10^{10}A_s), n_s, r, \lambda_\mu, s_\mu, \lambda_\gamma, s_\gamma, \lambda_t, s_t\}, \quad (19)$$

on the Computing Cluster for Cosmos by using the publicly available package CosmoMC [36]. The priors for the model parameters are shown in the second column of Table II. The running was stopped when the Gelman and Rubin $R-1$ parameter $R-1 \sim 0.02$ was satisfied; that guarantees the accurate confidence limits. The obtained results are shown in Table II for the data combinations: *Planck* 2013, WMAP9, BAO, BICEP2, JLA, HST, and RSD. The obtained contour plots for the model parameters r, s_t and λ_t are shown in Fig. 3.

One can clearly see that no significant deviation from GR was detected for the scaler perturbations in 2σ regions. However, for the scalar perturbations modeling the modification to the Poisson equation, slight deviations to the standard values in 1σ regions for model parameters $\lambda_\mu = 1.030^{+0.023}_{-0.023}$ and $s_\mu = 0.0054^{+0.0037}_{-0.0037}$ are shown. This tension was also reported in the *Planck* 2015 paper for dark energy and modified gravity [37]. And this tension can be reconciled by including the CMB lensing [37]. It is interesting to show the correlations for model parameter pairs $\lambda_\mu - s_\mu$ and $\lambda_\gamma - s_\gamma$. The lack of correlation between the μ and γ terms implies that they have different sources and cannot mimic each other.

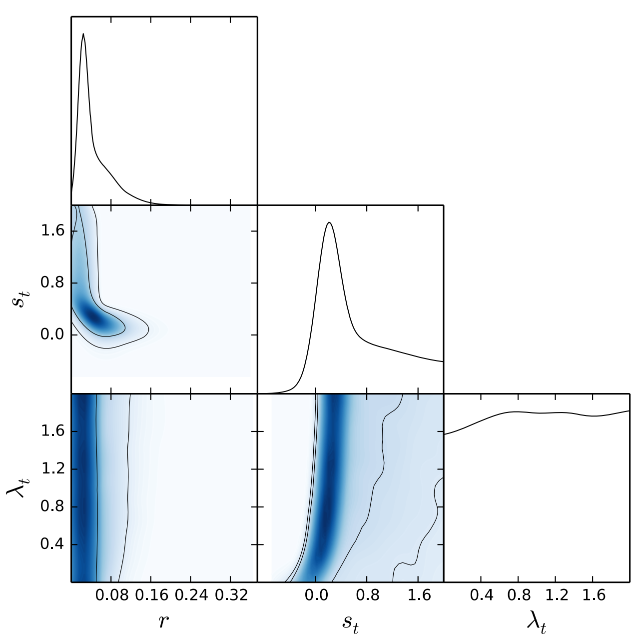

FIG. 4 (color online). The same as in Fig. 3, but for the model parameters r, λ_t, and s_t.

For the tensor perturbations, as shown in Fig. 4, one can see the anticorrelation between model parameters s_t and r. This can explain the small values of r. The model parameter λ_t cannot be well constrained due to the lack of data points below $l < 10$. A detection of the deviation to GR in this region is a tough task due to the domination of cosmic variance.

In the whole global fitting process, we have fixed c_T^2 to its standard value 1. If it is taken as another free model parameter, one cannot obtain a tight constraint to c_T^2 based on currently available data points due to the degeneracy to the ξ term which is not well constrained, as shown in Fig. 3 and Table II. Therefore, the introduction of free c_T^2 will not change the main results of our analysis.

IV. CONCLUSION

In this paper, we proposed a parametrized time-dependent modification to the propagation of gravitational waves, since the scale dependence does not lead to a significantly small χ^2 with respect to the scale-independent case [9]. Taking this specific form as a working example, we showed the effects on the CMB TT and BB power spectrum due to this kind of modification to GR by adopting different values of the model parameters. We also showed the possible degeneracy to the tensor mode power spectrum index n_t and its running α_t. Our analysis reveals that the modification to GR at tensor mode perturbations has effects on the CMB BB power spectrum at low multipole $l < 10$, i.e., the large scale, and keeps the shape of the CMB BB power spectrum. The tensor mode power spectrum index n_t and its running α_t have effects on the CMB BB power spectrum in the range $l \in (1, 100)$ and change the shape of the CMB BB power spectrum. It implies that precise data points below $l \sim 10$ can break this degeneracy between modification to GR and the power spectrum index and its running. However it is a tough task due to the domination of cosmic variance in this region.

We also used the currently available cosmic observational data sets, which include *Planck* 2013, WMAP9, BAO, BICEP2, JLA, HST, and RSD, to detect the possible deviation to GR. The results were gathered in Table II and Figs. 3 and 4. We did not find any significant deviation to GR in 2σ regions. But for the scalar perturbation part, we found the same tension as that reported in the *Planck* 2015 paper for dark energy and modified gravity [37]: the slight deviations to the standard values in 1σ regions for model parameters $\lambda_\mu = 1.030^{+0.023}_{-0.023}$ and $s_\mu = 0.0054^{+0.0037}_{-0.0037}$. The lack of correlation between the μ and γ terms implies that they have different sources and cannot mimic each other. For the tensor perturbation part, the model parameter λ_t is not well constrained due to the lack of data points. The anticorrelation between model parameters λ_t and r was also shown. This anticorrelation explains the small values of r.

Although in this paper, we have used the BICEP2 data points which are already confirmed as dust polarization, the analysis on the effects to the CMB TT and BB power spectrum is still robust. Also, the correlation and anticorrelation of the model parameters are irrelevant to the BICEP2 data points.

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